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RESEARCH MEMORANDUM

ANALYTICAL STUDY OF THE EFFECT OF CENTER-OF-GRAVITY POSITION ON THE RESPONSE TO LONGITUDINAL CONTROL IN LANDING APPROACHES OF A SWEPT-WING AIRPLANE OF LOW ASPECT RATIO HAVING NO HORIZONTAL TAIL

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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ANALYTICAL STUDY OF THE EFFECT OF CENTER-OF-GRAVITY POSITION ON THE RESPONSE TO LONGITUDINAL CONTROL IN LANDING APPROACHES OF A SWEPT-WING AIRPLANE OF LOW ASPECT RATIO HAVING NO HORIZONTAL TAIL

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SUMMARY

An analytical study of the short-time response characteristics to longitudinal control movements of a swept-wing airplane of low aspect ratio having no horizontal tail has indicated that difficulty reported in landings aboard an aircraft carrier may have been the result of a relatively larger time lag in the response of the airplane in changing its flight-path angle than exists on more conventional type airplanes. It had been reasoned that reduction of the airplane's static longitudinal stability might improve the response characteristics. Accordingly, an analytical study of the effect of reducing the static stability by a practical center-of-gravity shift has been made.

The results of the limited analytical investigation using both three-degrees-of-freedom analog computations and two-degrees-of-freedom calculations indicated that for the airplane considered changes in longitudinal static stability by a practical movement of the center of gravity had relatively small influence on the short-time response characteristics when compared with the response characteristics of a conventional configuration. In a push-pull maneuver, one in which a rate of descent is induced by control movement and then checked, the effects of the stability changes tended to be compensated, and the response time to check the descent was relatively unaffected. In a single control movement, such as may be used in a flareout from a steady glide, the response time required to check the initial rate of descent was shortened somewhat by reduced static stability. An increase in the amount of available up elevator resulting from trim changes due to reduced static stability was as significant in changing the response characteristics as was the reduction in stability. The response time, the time from a final control movement until a rate of descent is stopped during short-time maneuvers, was adequately estimated by calculations based on an analytical solution of the equations for two degrees of freedom.

INTRODUCTION

An analytical study of the response to longitudinal control of three different airplane configurations in landing approaches is presented in reference 1. These results indicated that airplanes which have either separately or in combination large relative densities or wing loadings, large pitching moments of inertia, small lift-curve slopes, small elevator effectiveness, and limited up-elevator travel tend to have poorer response characteristics than airplanes considered as conventional in the past decade.

The results of reference 1 indicate that an airplane having no horizontal tail required more time to respond to elevator control and lost more altitude in a push-pull stick movement than did a conventional airplane. The results of reference 1, however, do not include the effects of center-of-gravity movement on the response characteristics. Movement of the center of gravity reduces the static stability of the airplane and also increases the amount of elevator available for control manipulation because of a change in control required for trim. It might be presumed that either or both of these factors could cause the airplane to pitch more rapidly and thus to require less time to respond to elevator movement. The effects of center-of-gravity position on the response characteristics of the airplane having no horizontal tail therefore were studied.

SYMBOLS

The longitudinal motions presented herein were calculated about the stability axes. A diagram of the axes showing the positive directions of the forces and moment is presented in figure 1.

S	wing area, sq ft
$\bar{c}$	mean aerodynamic chord, ft
W	weight of airplane, lb
m	mass of airplane, W/g, slugs
k_Y	radius of gyration about Y body axis, ft
ρ	air density, 0.002378 slug/cu ft
μ	airplane relative-density coefficient, $m/\rho S\bar{c}$

V velocity, ft/sec

g acceleration due to gravity, 32.2 ft/sec^2

L lift, lb

D drag, lb

M pitching moment, ft-lb

C_L lift coefficient, $\frac{L}{\frac{1}{2}\rho V^2 S}$

C_D drag coefficient, $\frac{D}{\frac{1}{2}\rho V^2 S}$

C_m pitching-moment coefficient, $\frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$

C_{L_o} hypothetical lift coefficient at $\alpha = 0^\circ$ based on an extrapolation from approach α, for lift-curve slope in the vicinity of approach α and with an elevator deflection which would be required to trim at approach α

C_{m_o} hypothetical pitching-moment coefficient at $\alpha = 0^\circ$ based on an extrapolation from approach α, for pitching-moment slope in the vicinity of approach α and with an elevator deflection which would be required to trim at approach α

T thrust, lb

Z height, $\int_0^t V \sin \gamma \, dt$, ft

α angle of attack, $\theta - \gamma$, deg

γ flight-path angle, deg

θ angle of pitch, deg

δ_e elevator deflection, deg

ΔZ increment of height from trimmed level-flight condition

$\Delta\alpha$ increment of angle of attack from trimmed level-flight condition

$\Delta\gamma$ increment of flight-path angle from trimmed level-flight condition

$\Delta\theta$ increment of angle of pitch from trimmed level-flight condition

ΔV increment of velocity from trimmed level-flight condition

$\Delta\delta_e$ increment of elevator deflection from trimmed level-flight condition

$\dot{\theta}$ or q pitching angular velocity, radians/sec

$\dot{\gamma}$ rate of change of flight-path angle with time

$\dot{V}$ rate of change of velocity V with time

t time after first control motion, sec

τ time after second control motion, sec

$C_{D(\alpha)}$ coefficient of drag as a nonlinear function of α

$C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e}$ per deg

$C_{D_{\delta_e}} = \frac{\partial C_D}{\partial \delta_e}$ per deg

$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$ per deg

$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}$ per deg

$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$ per deg

$C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}}$

Dots over symbols represent derivatives with respect to time, for example, $\ddot{\gamma} = \frac{\partial^2 \gamma}{\partial t^2}$.

AIRPLANE CONDITIONS

The airplane, having no horizontal tail, is an airplane similar in configuration to one that reportedly had poor response to longitudinal control and for which response calculations are presented in reference 1 (airplane B). The new results computed for this paper are for a center-of-gravity position of $0.20\bar{c}$ (a practical rearward limit for this airplane considering its entire speed range). These results are compared in this paper with those of reference 1 for which the center-of-gravity position was $0.14\bar{c}$. The results also are compared with those of a conventional airplane reportedly having good response characteristics in landing approaches, airplane A of reference 1. For convenience the designations of airplanes A and B as used in reference 1 will be maintained in this paper. The configurations of the airplanes are shown in figure 2. Pertinent aerodynamic, mass, and dimensional characteristics for the landing configurations of airplanes A and B for both center-of-gravity positions are given in table I.

PROCEDURE

The procedure used for calculations on the analog computer was the same as that described in reference 1. In brief the three longitudinal equations of motion were used in the analog calculations. The lift and pitching moment were introduced as linear functions of angle of attack. The drag coefficient $C_{D(\alpha)}$ was introduced as a nonlinear function of angle of attack because of its large nonlinear variations. Variations of lift, drag, and pitching moment with elevator deflection were introduced as linear functions of elevator deflection. Deflections of the elevator and, therefore, values of $C_{L_{\delta_e}}\Delta\delta_e$, $C_{D_{\delta_e}}\Delta\delta_e$, and $C_{m_{\delta_e}}\Delta\delta_e$ were introduced as step functions. The thrust and C_{m_q} were held constant.

The airplanes were initially trimmed for steady level flight at a landing approach speed of 185.8 ft/sec (110 knots). The initial trim values are given in table II. A disturbance from steady level flight was initiated by deflecting the elevator down and holding the down deflection for 2 seconds after which an attempt to stop the ensuing descent was made by deflecting the elevator full-up. As noted in reference 1 the amount of down elevator movement used for airplane A was such as to result in a loss of altitude (about 10 feet) that might be desired for a final correction during a landing approach aboard a carrier. The amount of down elevator used on airplane B for both center-of-gravity positions was such as to cause descent paths similar to that obtained on airplane A. The down-elevator deflections required for airplane B were based on the total elevator effectiveness parameter $\left(\frac{VC_{L_\alpha}}{2\mu\bar{c}} \frac{V^2 C_{m_{\delta_e}}}{2\mu {k_y}^2} \Delta\delta_e\right)$ presented in reference 1. As previously mentioned the elevator was then deflected from its down deflection to full-up. This procedure was employed to get the maximum response that would be theoretically possible for a given airplane configuration, although it was realized that such a control manipulation would not generally be used by a pilot. The elevator deflection was reduced from full-up to a deflection that would trim the airplane at the angle of maximum lift and in time to prevent any appreciable overshoot of the angle of attack of maximum lift. For airplane B with the center of gravity at $0.14\bar{c}$ full-up elevator trimmed the airplane at maximum lift.

The motion in response to these control manipulations was recorded and is presented in terms of variations from initial conditions of velocity, angle of pitch, angle of attack, flight-path angle, and height or altitude with respect to time. The lift due to elevator deflection for airplane B was relatively large when compared with that for airplane A, primarily because airplane B had no horizontal tail and used trailing-edge flaps for longitudinal control. In that the change in lift due to elevator deflection is undesirable, being in a direction opposite to that desired when the elevator is moved, the effects of eliminating the change in lift due to elevator deflection were investigated. Previous studies (ref. 1) have indicated that the total elevator effectiveness parameter is an important parameter regarding short-time responses to longitudinal control in landing approaches and an investigation was made of increasing this parameter in the pull-up part of the motion for airplane B. The total elevator effectiveness parameter was increased arbitrarily by increasing the up-elevator deflection so that the parameter for airplane B equalled that for airplane A. Changing the total elevator effectiveness parameter in this manner caused airplane B to have approximately the same initial rate of change of normal acceleration $V\dot{\gamma}$ at the time of the second control motion as did airplane A. The amounts of elevator deflection from the trimmed deflection for the various test conditions studied are given in table III.

The study presented in this paper is primarily concerned with the short-time response characteristics because in landing approaches and particularly in landing approaches aboard a carrier, where difficulties have been encountered, small altitude or height corrections are needed in very short times. In general, the results in this paper are for times up to when the altitude lost in the pushover is regained. Brief results for relatively larger time periods are also presented.

RESULTS AND DISCUSSION

Comparison of Airplanes A and B Showing Effects of Center-of-Gravity Position

A comparison of the response of airplane B for both the $0.14\bar{c}$ and $0.20\bar{c}$ center-of-gravity positions with the response of airplane A, as calculated on the analog computer is shown in figure 3. For this comparison an attempt was made to make the pushover flight paths for airplane B approximately the same as that for airplane A by the use of a different amount of down-elevator deflection, as has previously been mentioned. The same amount of down elevator, however, was used for both center-of-gravity positions of airplane B in that the amount of elevator used was based on the total elevator effectiveness parameter which, in itself, is not affected by changes in static stability. There appears to be only little influence of center-of-gravity position on the response characteristics of airplane B. In the pushover part of the motion, airplane B with the rearward center of gravity does pitch somewhat more rapidly than with the forward center-of-gravity position, resulting in somewhat larger changes in flight-path angle. This difference in flight-path angle causes a slightly more rapid loss in height. The time lag in response to up elevator, however, is about the same for both center-of-gravity positions. The airplane with the rearward center-of-gravity position does regain height more rapidly than with the forward center-of-gravity position. This is caused in part by more available up elevator as well as the reduced stability. The more available up elevator results from the fact that less elevator is needed to trim the airplane in the initial level flight for the rearward center-of-gravity position (see tables II and III).

Effect of Lift Due to Elevator Deflection

The effect of eliminating the lift due to elevator deflection for both center-of-gravity positions of airplane B is shown in figure 4. For these comparisons the amount of down elevator used in the pushover was the same for both center-of-gravity positions and was not adjusted in a manner to give consistent pushover flight paths. The effect of eliminating the lift due to elevator deflection is similar for both center-of-gravity positions in that somewhat more height is lost in the pushover because of a more rapid response to a down elevator of the airplane in flight-path angle when the lift due to elevator deflection is zero. The response time or lag following elevator deflection is not appreciably affected, however, by the elimination of lift due to elevator deflection. As discussed previously, the effects of moving the center of gravity rearward are to cause the flight-path angle to change more rapidly with the same elevator deflection and to lose somewhat more height in the pushover than for the forward center-of-gravity position. The height is regained more rapidly, however, again because of the somewhat more rapid response in flight-path angle associated in part to more available control deflection.

Effect of Increased Total Elevator Effectiveness

The results with an increase of maximum up-elevator deflection for both center-of-gravity positions for airplane B are shown in figure 5. The amount of down-elevator deflection used in the pushover was the same for both center-of-gravity positions and was not adjusted to give consistent pushover flight paths. The amount used was the same as used for the data of figures 3 and 4 (table III). The primary effect of the increased elevator deflection was to reduce the response time to up elevator movement and reduce the height lost in the maneuver. This is, of course, the same effect as reported in reference 1. As discussed previously, the effect of rearward movement of the center-of-gravity position was to cause a somewhat more rapid change of the flight-path angle in the pushover (using the same amount of down elevator) with somewhat more height being lost. The rates of change of flight-path angle in the pullup are similar primarily because the increment of up-elevator deflection used has been made the same for both center-of-gravity positions (57.81°). This amount of up-elevator deflection, as previously noted, was used to make the total elevator effectiveness parameter of airplane B the same as that of airplane A for the pull-up part of the maneuver. The response time, that is, the time when the descent is stopped, is again not appreciably affected by center-of-gravity position, actually being somewhat larger for the rearward than for the forward center-of-gravity position. The use of less down elevator in the pushover for the rearward center-of-gravity position would have made the flight paths more alike for the two center-of-gravity positions, although it probably would not have influenced the response time appreciably, primarily because proportional changes in elevator deflection probably would have no influence on response times.

The results for airplane B for both center-of-gravity positions, with the maximum up-elevator deflection increased and with the lift due to elevator deflection set equal to zero, are compared with the results for airplane A in figure 6. The results indicate that the response time is about the same for all cases - being of the order of 1 second after up-elevator movement, whereas for the original conditions shown in figure 3 the response time for airplane B for both center-of-gravity positions was almost 2 seconds - nearly 1 second longer than for airplane A. The improvements caused by increased elevator deflection and eliminating the lift due to elevator deflection result fundamentally from the fact that the angle of attack and thus the lift and flight-path angle were changed more rapidly. Following the movement of the elevator to full-up on airplane B, the maximum rate of change of the angle of attack was increased about 50 percent by increased elevator deflection; whereas, the center-of-gravity movement increased the maximum rate of change of angle of attack by only about 10 percent (fig. 6), which is, in part, the result of more available up-elevator deflection for the rearward center-of-gravity position.

Airplane A undoubtedly has more than minimum acceptable response characteristics and the comparisons shown are not intended to indicate any large deficiencies in airplane B; as a matter of fact, as noted in reference 1, the differences shown may not be greatly significant and only flight experience can establish any criteria of minimum acceptable response.

Effects of Response Characteristics Over Long Time Periods

The relatively long time characteristics, as calculated on the analog computer, for airplane B for both center-of-gravity positions are compared with those for airplane A in figures 7 and 8. In figure 8 the total elevator effectiveness parameter has been made the same for airplane B as for airplane A and the lift increment due to elevator deflection has been eliminated. All cases respond to the up-elevator deflection in that after the descent is stopped and the lost height is regained the airplanes continue to increase height over a considerable time. As was noted in reference 1, it is apparent that if an airplane is not trimmed at maximum lift, height can be gained generally by an exchange of kinetic for potential energy and that only impractically large drag-coefficient changes could prevent such an exchange of energy.

In figure 7, the effect of center-of-gravity position was the attainment of a much larger increase in flight-path angle for the rearward center-of-gravity position primarily because the airplane pitched up for a longer time. The result of the difference in flight-path angle was a larger change in height. The additional amount of up-elevator deflection available for the rearward center-of-gravity position influenced this motion to some extent as well as did the differences in stability.

In figure 8 with increased elevator deflection (the amount of up elevator used being the same for both center-of-gravity positions, see table III), the effect of center-of-gravity position is not as great. The differences shown are the apparent effects of the change in stability as well as the influence of the change on the transient or early part of the motion wherein the angle of pitch and flight-path angle assume somewhat different variations with center-of-gravity position.

Some Additional Considerations

A factor for the evaluation of the short-time characteristics of airplanes in response to longitudinal control appears to be the response or lag time which is required for the airplane to respond to control movement in the sense that an induced or existing rate of descent can be stopped. In short time periods, differences in response time appear to be the primary difference between airplanes A and B reported herein and in reference 1. The response time used herein is defined as the time from final control movement to when the rate of change of height is zero. The rate of change of height is the vertical velocity or rate of sink, $V \sin \gamma \approx V\gamma$ which can only be zero when the flight-path angle γ is zero.

A solution of the equations of two degrees of freedom for a single elevator control movement such as is used in a pushover is presented in the appendix of reference 1. A similar solution of the equations of two degrees of freedom for a second control movement as would be used in a pullup following a pushover are presented herein. For this solution the conditions which exist at the time of the second control motion as a result of a previous control motion are used to establish constants of the equation. This solution is as follows:

$$\gamma = \left[C_2\left(\Delta\delta_{e_1} - \Delta\delta_{e_2}\right) - C_1\,\Delta\delta_{e_2}b\right]\left[e^{-\frac{a\tau}{2}}\left(\frac{2}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\tau - \zeta\right)\right] +$$

$$\left(C_1\,\Delta\delta_{e_2}b + C_2\,\Delta\delta_{e_2}\right)\frac{a}{b^2} - \left(C_2\,\Delta\delta_{e_2}\right)\frac{\tau}{b} +$$

$$\left(C_3\,\Delta\delta_{e_2}\right)\left[\frac{1}{b} - e^{-\frac{a\tau}{2}}\left(\frac{2\sqrt{b}}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\tau - Z\right)\right] + \gamma_1 + \dot{\gamma}_1\frac{a}{b} +$$

$$\ddot{\gamma}_1\frac{1}{b} - \left(C_1\,\Delta\delta_{e_1}b + C_2\,\Delta\delta_{e_1}\right)\left[e^{-\frac{at}{2}}\left(\frac{2}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}t - \zeta\right)\right] -$$

$$\left(C_3\,\Delta\delta_{e_1}\right)\left[e^{-\frac{at}{2}}\left(\frac{2\sqrt{b}}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}t + Z\right)\right]$$

where

$$a = \frac{VC_{L_\alpha}}{2\mu\bar{c}} - \frac{VC_{m_q}\bar{c}}{4\mu k_y^2}$$

$$b = \frac{V^2 C_{L_\alpha} C_{m_q}}{8\mu^2 k_y^2} - \frac{V^2 C_{m_\alpha}}{2\mu k_y^2}$$

$$c_1 = \frac{VC_{L_{\delta_e}}}{2\mu\bar{c}}$$

$$c_2 = \frac{VC_{L_{\delta_e}}}{2\mu\bar{c}} \frac{V^2 C_{m_\alpha}}{2\mu k_y^2} - \frac{VC_{L_\alpha}}{2\mu\bar{c}} \frac{V^2 C_{m_{\delta_e}}}{2\mu k_y^2}$$

$$c_3 = \frac{VC_{L_{\delta_e}}}{2\mu\bar{c}} \frac{VC_{L_\alpha}}{2\mu\bar{c}}$$

$$\zeta = \tan^{-1} \frac{a^2 - 2b}{a\left|\sqrt{a^2 - 4b}\right|}$$

$$Z = \tan^{-1} \frac{a}{\left|\sqrt{a^2 - 4b}\right|}$$

and

$\Delta\delta_{e_1}$ first elevator movement

$\Delta\delta_{e_2}$ second elevator movement measured from first movement

t time from beginning of motion or first elevator movement

τ time from second elevator movement

γ_1 flight-path angle which exists at time t_1 when second control movement is made

$\dot{\gamma}_1$ and $\ddot{\gamma}_1$ flight-path-angle derivatives which exist at time t_1 when second control movement is made

The formulas for γ_1, $\dot{\gamma}_1$, and $\ddot{\gamma}_1$ are

$$\gamma_1 = \left(C_2\,\Delta\delta_{e_1} + C_1\,\Delta\delta_{e_1}b\right)\left[\frac{a}{b^2} - e^{-\frac{at_1}{2}}\left(\frac{2}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1 - \zeta\right)\right] -$$

$$\left(C_2\,\Delta\delta_{e_1}\right)\frac{t_1}{b} + \left(C_3\,\Delta\delta_{e_1}\right)\left[\frac{1}{b} - e^{-\frac{at_1}{2}}\left(\frac{2\sqrt{b}}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1 - Z\right)\right]$$

$$\dot{\gamma}_1 = \left(C_2\,\Delta\delta_{e_1} + C_1\,\Delta\delta_{e_1}b\right)\left[e^{-\frac{at_1}{2}}\left(\frac{2\sqrt{b}}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1 - Z\right)\right] -$$

$$\left(C_2\,\Delta\delta_{e_1}\right)\frac{1}{b} + \left(C_3\,\Delta\delta_{e_1}\right)\left[e^{-\frac{at_1}{2}}\left(\frac{2}{\left|\sqrt{a^2 - 4b}\right|}\right)\sin\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1\right)\right]$$

and

$$\ddot{\gamma}_1 = -\left(C_2\,\Delta\delta_{e_1} + C_1\,\Delta\delta_{e_1}b\right)\left[e^{-\frac{at_1}{2}}\left(\frac{2}{\left|\sqrt{a^2 - 4b}\right|}\right)\sin\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1\right)\right] +$$

$$\left(C_3\,\Delta\delta_{e_1}\right)\left[e^{-\frac{at_1}{2}}\left(\frac{2\sqrt{b}}{\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2}\,t_1 + Z\right)\right]$$

These expressions for γ_1, $\dot{\gamma}_1$, and $\ddot{\gamma}_1$ differ somewhat from those presented in appendix A of reference 1 by the additional C_3 terms making these expressions more complete even though the C_3 terms are normally not significantly large.

A solution for τ in equation (1) with the equation set to zero will give the lag or response time required from the time of the second control motion until the rate of descent is stopped. Equation 1, however, is not amenable to a general or analytic solution because of its transcendental form and only graphical or numerical solutions for specific cases are practical. It must be pointed out that in two-degrees-of-freedom solutions, γ becomes a divergent quantity with time because of the assumption that the velocity is constant; therefore any solutions of equation (1) must be limited to very short time periods after any final control movement.

Solutions for γ in equation (1) for airplanes A and B for both center-of-gravity positions for the conditions presented in figure 3 are presented in figure 9. The differences in the lag times between the two airplanes and the small differences in lag time for the two center-of-gravity positions of airplane B are indicated as they were on figure 3. The various components of γ made up from the constant (independent of time), oscillatory (function of $e^{at} \cos \omega t$), and time proportional (direct function of time) parts of equation (1) are also shown on figure 9. In figure 9 (and on the other remaining figures) these various components are designated as γ_a, γ_b, and γ_c, respectively, where from equation (1),

$$\gamma_a = \left(C_1 \Delta\delta_{e_2} b + C_2 \Delta\delta_{e_2}\right)\frac{a}{b^2} + \left(C_3 \Delta\delta_{e_2}\right)\frac{1}{b} + \gamma_1 + \dot{\gamma}_1 \frac{a}{b} + \ddot{\gamma}_1 \frac{1}{b}$$

$$\gamma_b = \left[C_2\left(\Delta\delta_{e_1} - \Delta\delta_{e_2}\right) - C_1 \Delta\delta_{e_2} b\right]\left[e^{-\frac{a\tau}{2}}\left(\frac{2}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2} \tau - \zeta\right)\right] -$$

$$\left(C_3 \Delta\delta_{e_2}\right)\left[e^{-\frac{a\tau}{2}}\left(\frac{2}{\sqrt{b}\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2} \tau - Z\right)\right] -$$

$$\left(C_1 \Delta\delta_{e_1} b - C_2 \Delta\delta_{e_1}\right)\left[e^{-\frac{at}{2}}\left(\frac{2}{b\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2} t - \zeta\right)\right] -$$

$$\left(C_3 \Delta\delta_{e_1}\right)\left[e^{-\frac{at}{2}}\left(\frac{2}{\sqrt{b}\left|\sqrt{a^2 - 4b}\right|}\right)\cos\left(\frac{\left|\sqrt{a^2 - 4b}\right|}{2} t - Z\right)\right]$$

and

$$\gamma_c = -\left(C_2 \Delta\delta_{e_2}\right)\frac{\tau}{b}$$

The effect of center-of-gravity movement is to increase the various components but in a manner that they tend to compensate one another when added, with the result of only small actual differences in the response time.

As has been mentioned, the effect of center-of-gravity position on the response of airplane B as presented in figures 3 to 8 included the effect of more available up-elevator deflection for the rearward center-of-gravity position because of the different amounts of elevator required for trim. Calculations by use of equation (1) using the same control movements for both center-of-gravity positions are included in figure 10. The results show that with the same control movements the time lag becomes greater for the rearward center-of-gravity position than for the forward position. It may be reasoned that this may be the result of a somewhat larger value of γ_1 (at $\tau = 0$) for the rearward center-of-gravity position. Calculations for which the initial pushover control movement was reduced for the rearward center-of-gravity position so that γ_1 was the same for both center-of-gravity positions are also shown in figure 10. The results show only little difference in lag time and indicate that the difference in available elevator deflection is apparently as important to the motion as is the change in stability.

Calculations for airplane B with increased up elevator and $C_{L_{\delta_e}}$ equal to zero are compared with calculations for airplane A in figure 11. The results show an improvement in lag time for airplane B in the same manner as shown in figure 6.

A comparison of the variation of γ with time as calculated by equation (1) and by the analog computer is made in figure 12. The analog computer results generally show somewhat larger variations in γ and longer response times which would appear to be a direct result of an increase in velocity indicated in, for example, figure 3. Some differences in the analog computations also exist for airplane A and airplane B with the rearward center-of-gravity position because of a third control motion used to prevent the airplanes from stalling. This third movement was not used for the forward center-of-gravity position of airplane B. On any account, it would appear that equation (1) could be used to evaluate reasonably accurate response times as well as indicate approximate differences in response times between different configurations.

Additional calculations were made, by use of equation (1), of the time required to check an established rate of descent in a steady 10 to 1 glide (fig. 13). An initial glide-path angle of about $5\frac{3}{4}^{\circ}$ and the maximum available elevator deflection were used for these calculations. Airplane A responds more quickly than does airplane B. The effect of center-of-gravity position is more prominent on the response of airplane B than it was for a control induced disturbance, even when the same amount of elevator deflection was used. It would appear then that in a control induced disturbance the effects of stability changes tend to be compensated for during short time periods, whereas for responses to single control movements, as in checking a glide, reduced static stability tends to improve response characteristics somewhat.

Differences in the response characteristics are shown in figures 10 and 13 between the two center-of-gravity positions for airplane B when the same control deflections were used for both center-of-gravity positions and the same initial flight-path angles existed. In figure 10 for a control induced descent the response time is somewhat larger for the rearward center-of-gravity position, whereas in figure 13 when checking a steady glide the response time is somewhat shorter for the rearward center-of-gravity position than for the forward center-of-gravity position. This difference is primarily caused by the existence of derivatives of γ_1 in the control induced motion which are zero in the steady glide case. This reiterates the fact that the effects of changes in stability tend to compensate themselves in a control induced checked maneuver; whereas in a single control motion quicker response is possible.

CONCLUSIONS

The results of a limited analytical investigation using both three-degrees-of-freedom analog computations and two-degrees-of-freedom calculations indicated the following conclusions regarding the short-time response to longitudinal control of a swept-wing airplane of low aspect ratio having no horizontal tail during landing approaches:

1. Changes in longitudinal static stability by a practical movement of the center-of-gravity had relatively small influence on the short-time response characteristics of the airplane.

2. In a push-pull maneuver, that is, one in which a rate of descent is induced by control movement and then checked, the effects of changes in stability tended to be compensated and the response time to check the descent was relatively unaffected.

3. In a single control movement such as may be used in a flareout from a steady glide, the response time required to check the initial rate of descent was shortened somewhat by reduced static stability.

4. An increase in the amount of available up-elevator deflection resulting from trim changes due to a reduction of static stability, was as significant in changing the response characteristics as were changes in stability.

5. The response time, that is, the time from a final control movement until a rate of descent is stopped for short-time maneuvers, was adequately estimated by calculations based on an analytical solution of the equations of two degrees of freedom.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 21, 1954.

REFERENCE

1. Bihrle, William, Jr., and Stone, Ralph W., Jr.: Analytical Studies of the Response to Longitudinal Control of Three Airplane Configurations in Landing Approaches. NACA RM L53B10, 1953.

TABLE I.- AERODYNAMIC, MASS, AND DIMENSIONAL CHARACTERISTICS

[Aerodynamic characteristics are referred to stability axes; mass and aerodynamic characteristics given for landing configuration.]

Characteristics	Airplane A	Airplane B	
		c.g. at $0.14\bar{c}$	c.g. at $0.20\bar{c}$
Wing area, sq ft	400.0	535.3	535.3
Mean aerodynamic chord, ft . .	8.28	13.69	13.69
Weight, lb	19,642	22,862	22,862
Center of gravity, percent $\bar{c}$	25	14	20
Moment of inertia about Y axis, slug-ft^2	40,658	43,750	43,750
Radius of gyration about Y axis, ft	8.17	7.85	7.85
Airplane relative-density coefficient, μ	77.4	40.7	40.7
C_{m_q} per radian	-12.0	-1.5	-1.5
$C_{m_{\delta_e}}$ per deg	-0.0172	-0.0050	-0.0050
$C_{L_{\delta_e}}$ per deg	0.00600	0.01025	0.01025
$C_{D_{\delta_e}}$ per deg	0.00056	0.00090	0.00090
C_{m_α} per deg	-0.01034	-0.00675	-0.00362
C_{L_α} per deg	0.0842	0.0525	0.0525
C_m/C_L	-0.123	-0.129	-0.069
C_{m_o}	0.04550	0.1475	0.0669
C_{L_o}	0.814	-0.187	-0.083
$\delta_{e_{max}}$, deg	-18	-30	-30

TABLE II.- INITIAL TRIM VALUES FOR STEADY LEVEL

FLIGHT AT 110 KNOTS (185.8 FT/SEC)

Airplane	α, deg	γ, deg	θ, deg	T, lb	δ_e, deg
A	4.40	0	4.46	2,642	5.0 (1.5° up stabilizer)
B, c.g. at $0.14\bar{c}$	21.85	0	21.85	4,770	-20.0
B, c.g. at $0.20\bar{c}$	19.85	0	19.85	4,289	-8.4

[redacted]

TABLE III.- ELEVATOR DEFLECTIONS USED DURING PUSH-PULL MANEUVERS AND COMPARISON OF RESULTING PARAMETERS AFFECTING LONGITUDINAL MOTION

Airplane	Push-down $\Delta\delta_e$, deg (from trim deflection)	Pull-up $\Delta\delta_e$, deg (from push-down deflection)	$\frac{\Delta\delta_e}{(\Delta\delta_e)_A}$ (for pull-up)	$\frac{\frac{V^2 C_{m_{\delta_e}} \Delta\delta_e}{2\mu k_y^2} \frac{C_{L_\alpha}}{2\mu\bar{c}}}{\left(\frac{V^2 C_{m_{\delta_e}} \Delta\delta_e}{2\mu k_y^2} \frac{C_{L_\alpha}}{2\mu\bar{c}}\right)_A}$ (a)	$\frac{\frac{V C_{L_{\delta_e}} \Delta\delta_e}{2\mu\bar{c}}}{\left(\frac{V C_{L_{\delta_e}} \Delta\delta_e}{2\mu\bar{c}}\right)_A}$ (b)
Original elevator deflection					
A	1.86	-24.86	1.00	1.00	1.00
B, c.g. at $0.14\bar{c}$	[c]4.33	-14.33	0.58	0.25	1.14
B, c.g. at $0.20\bar{c}$	[c]4.33	-25.93	1.04	0.45	2.06
Increase up-elevator deflection					
B, c.g. at $0.14\bar{c}$	[c]4.33	[c]-57.81	2.33	1.00	4.59
B, c.g. at $0.20\bar{c}$	[c]4.33	[c]-57.81	2.33	1.00	4.59

[a]Parameter indicates total available elevator effectiveness in causing a rate of change of flight-path angle (neglecting change in lift due to elevator deflection).

[b]Parameter indicates change in lift due to elevator deflection.

[c]The $\Delta\delta_e$ values result in the initial rate of change of γ being approximately the same as for airplane A (neglecting change in lift due to elevator deflection).

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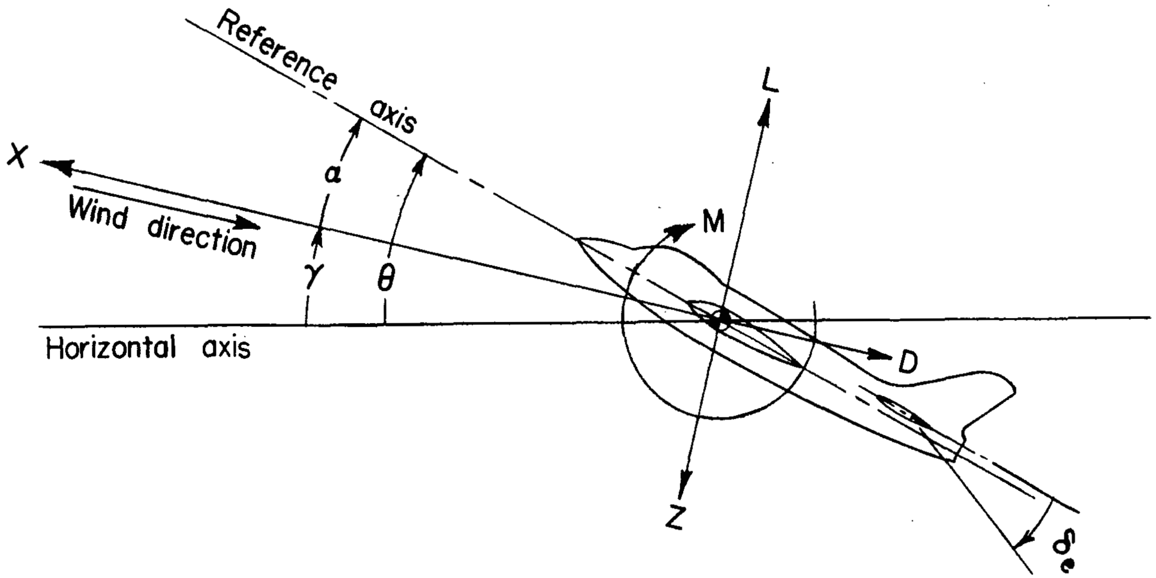

Figure 1.- Sketch showing stability axes. Arrows indicate positive direction of forces, moment, and angles.

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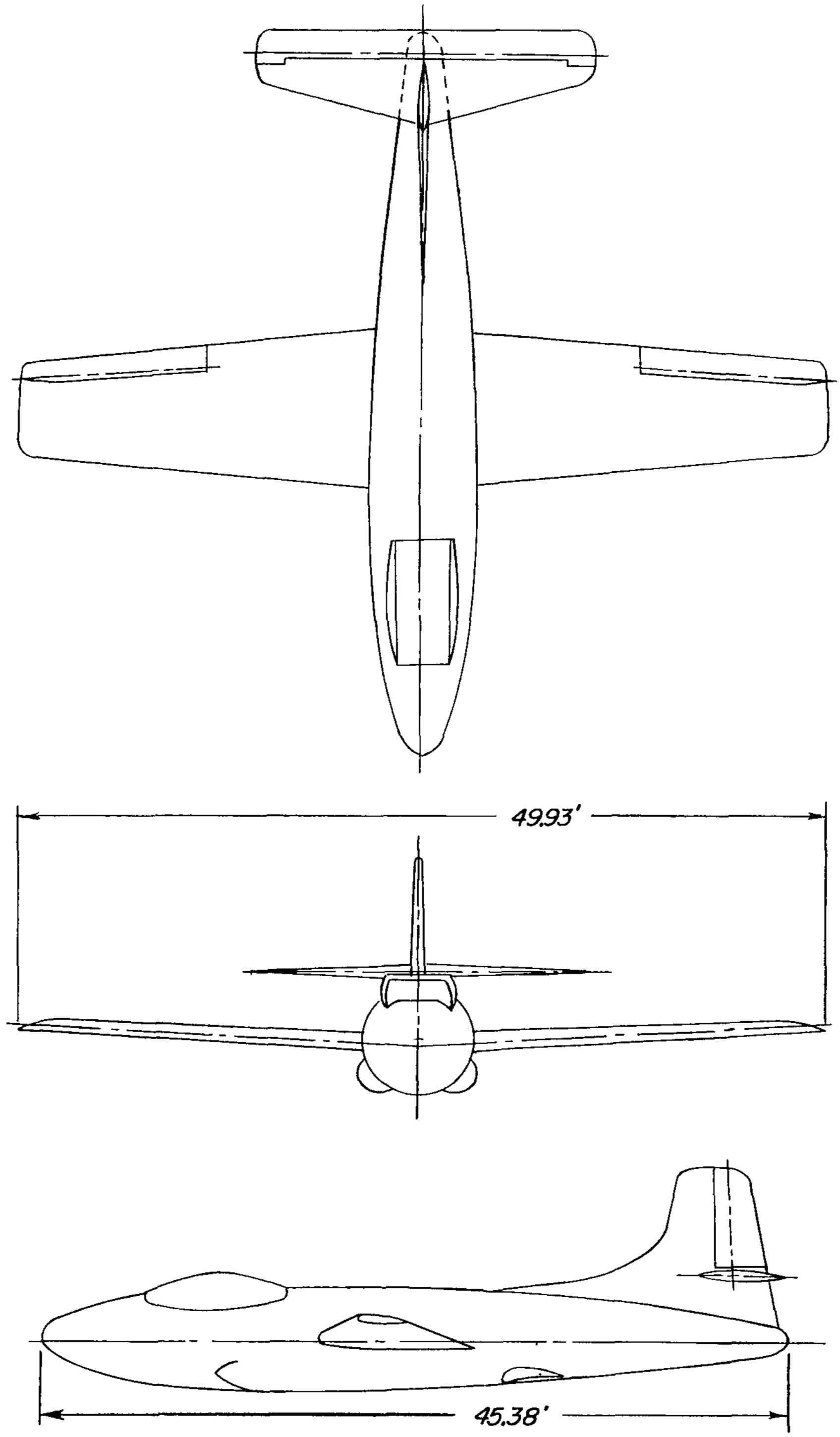
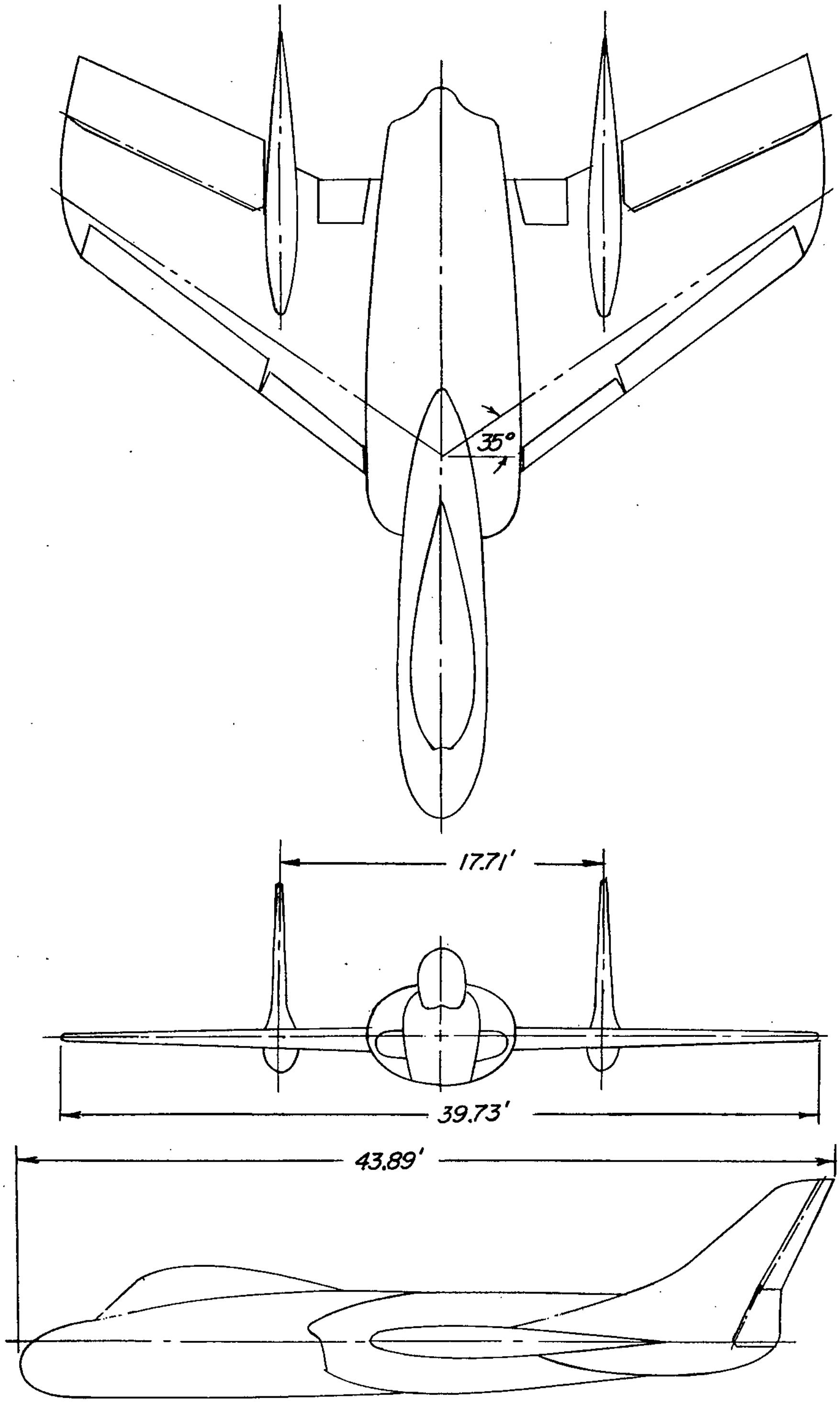


(a) Airplane A.

Figure 2.- Three-view drawings of airplanes investigated.



(b) Airplane B.

Figure 2.- Concluded.

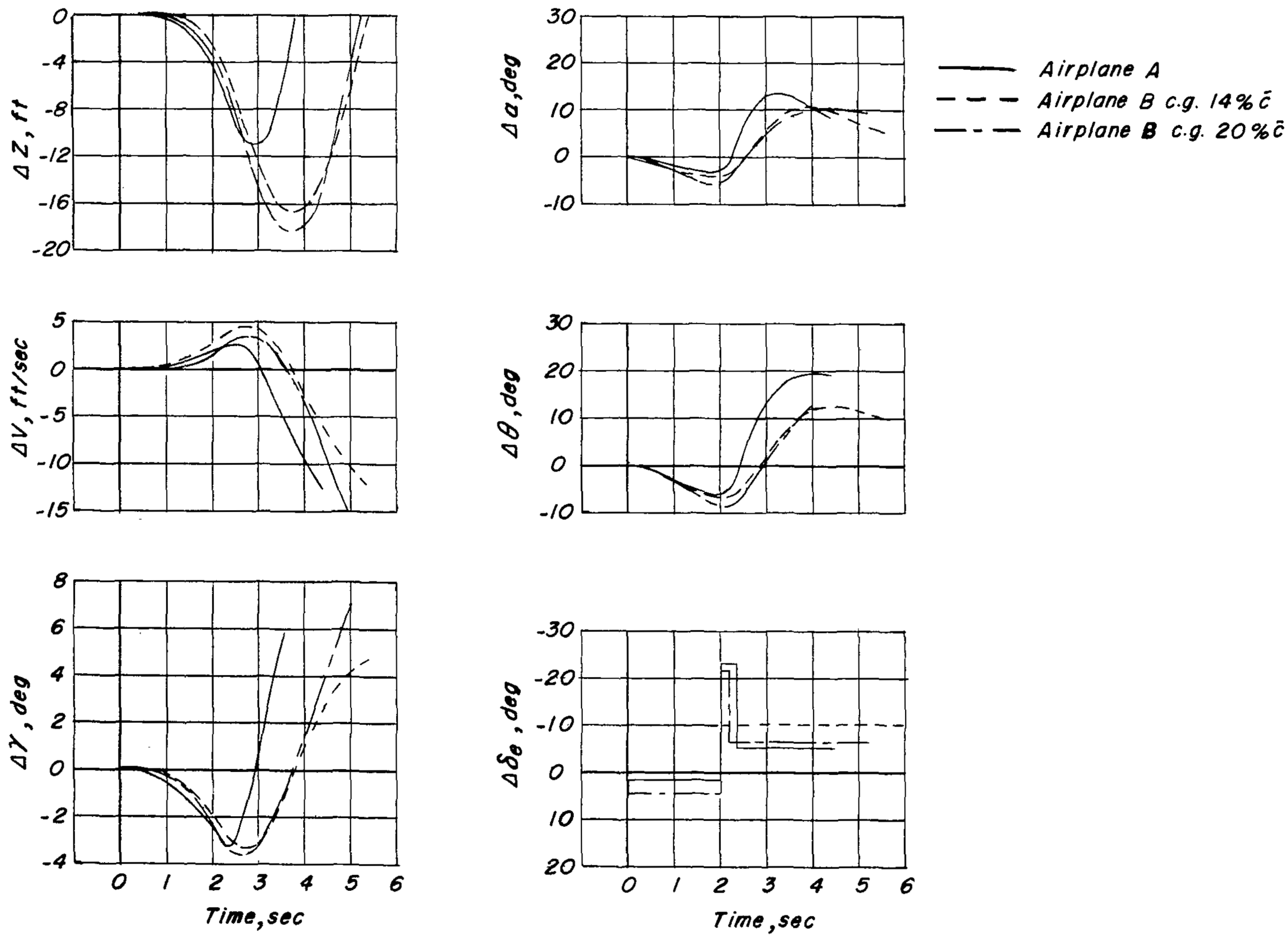


Figure 3.- Comparison of response to available longitudinal control on airplane A and airplane B with center of gravity at 0.20c̄ and 0.14c̄. Initial trim values given in table II.

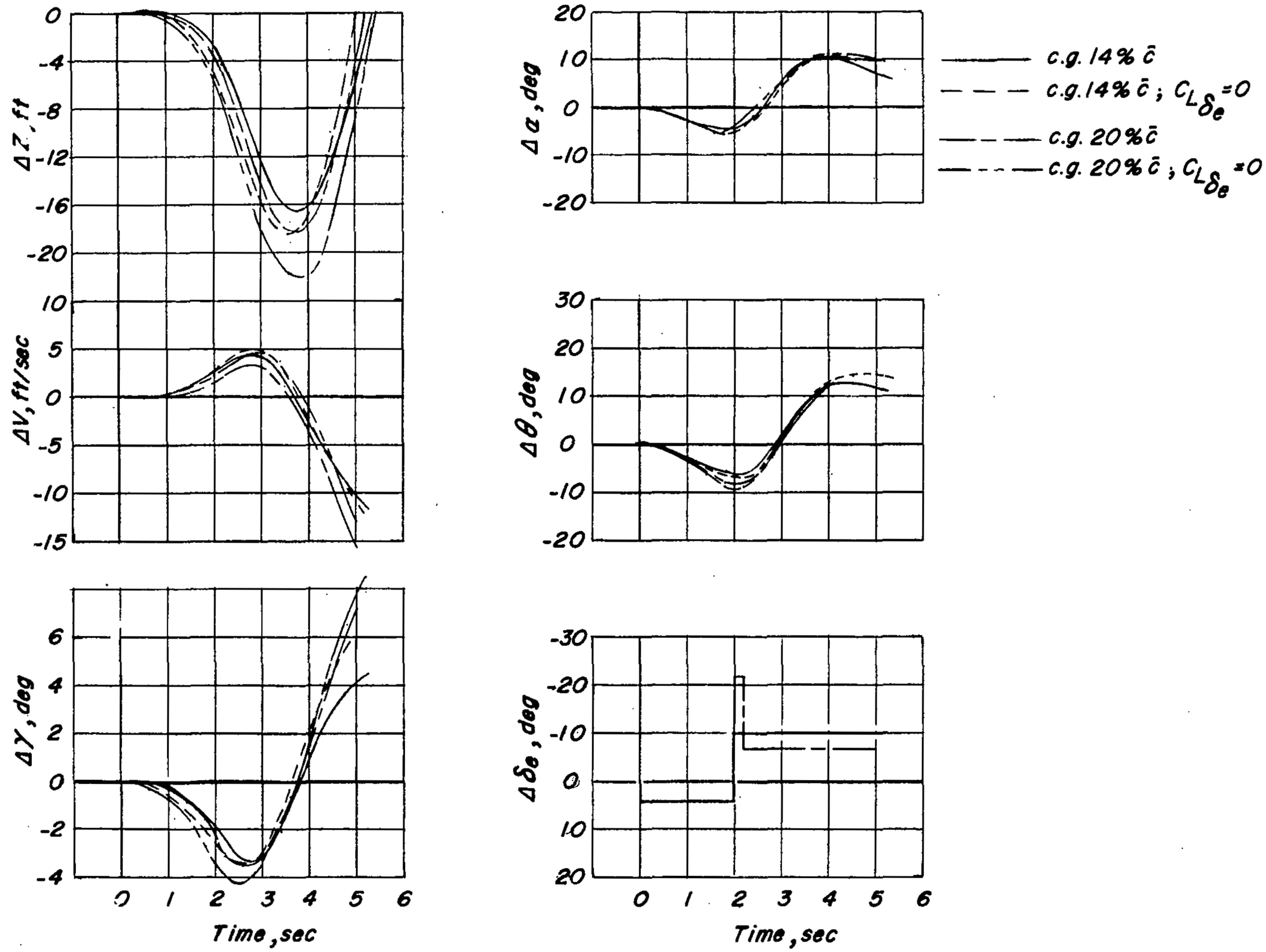


Figure 4.- Effect of eliminating the change in lift due to elevator deflection on the response of airplane B with the center of gravity at 0.20$\bar{c}$ and 0.14$\bar{c}$. Initial trim values given in table II.

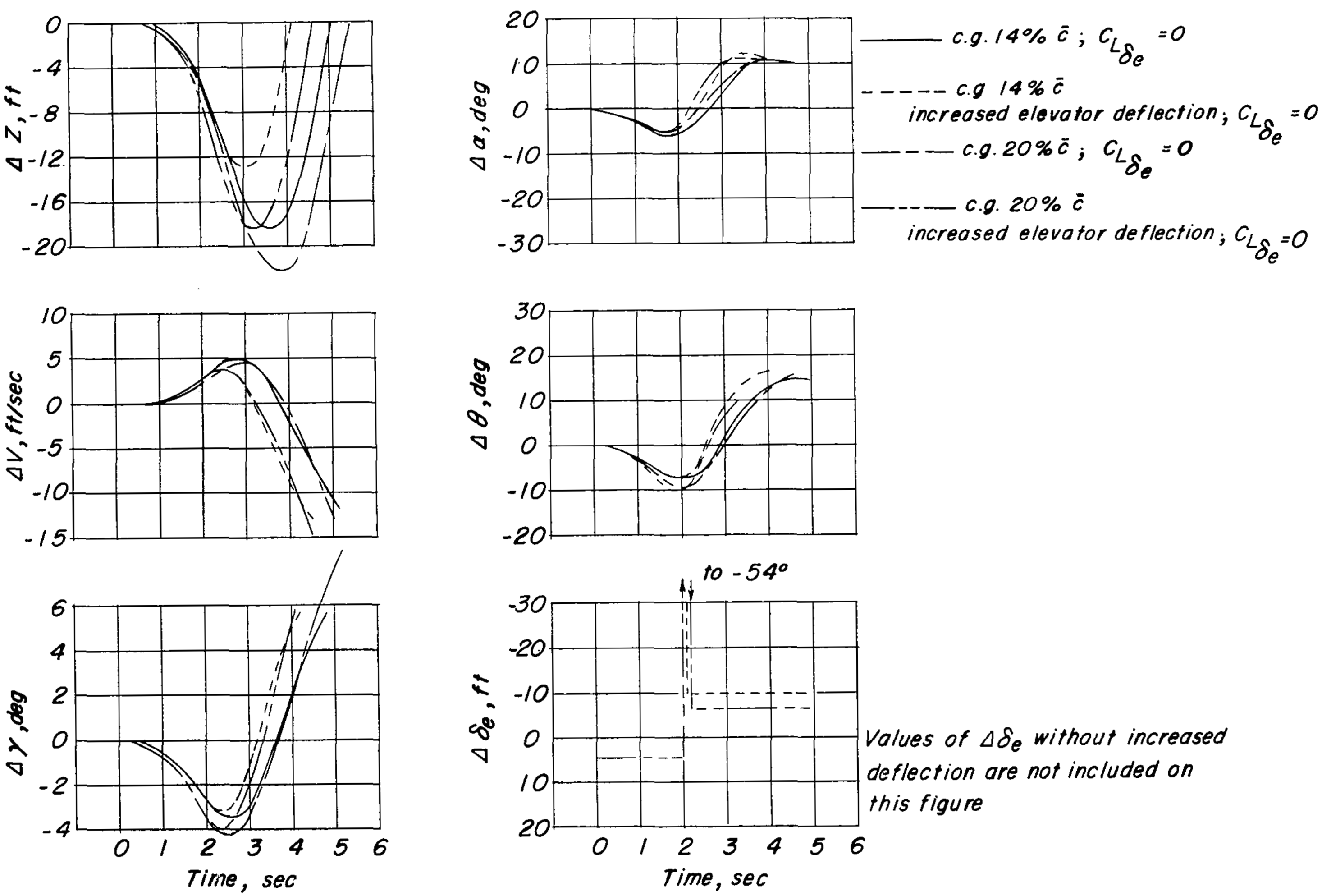


Figure 5.- Effect of increasing up-elevator deflection and of eliminating the change in lift due to elevator deflection on the response of airplane B with the center of gravity at 0.20c̄ and 0.14c̄. Initial trim values given in table II.

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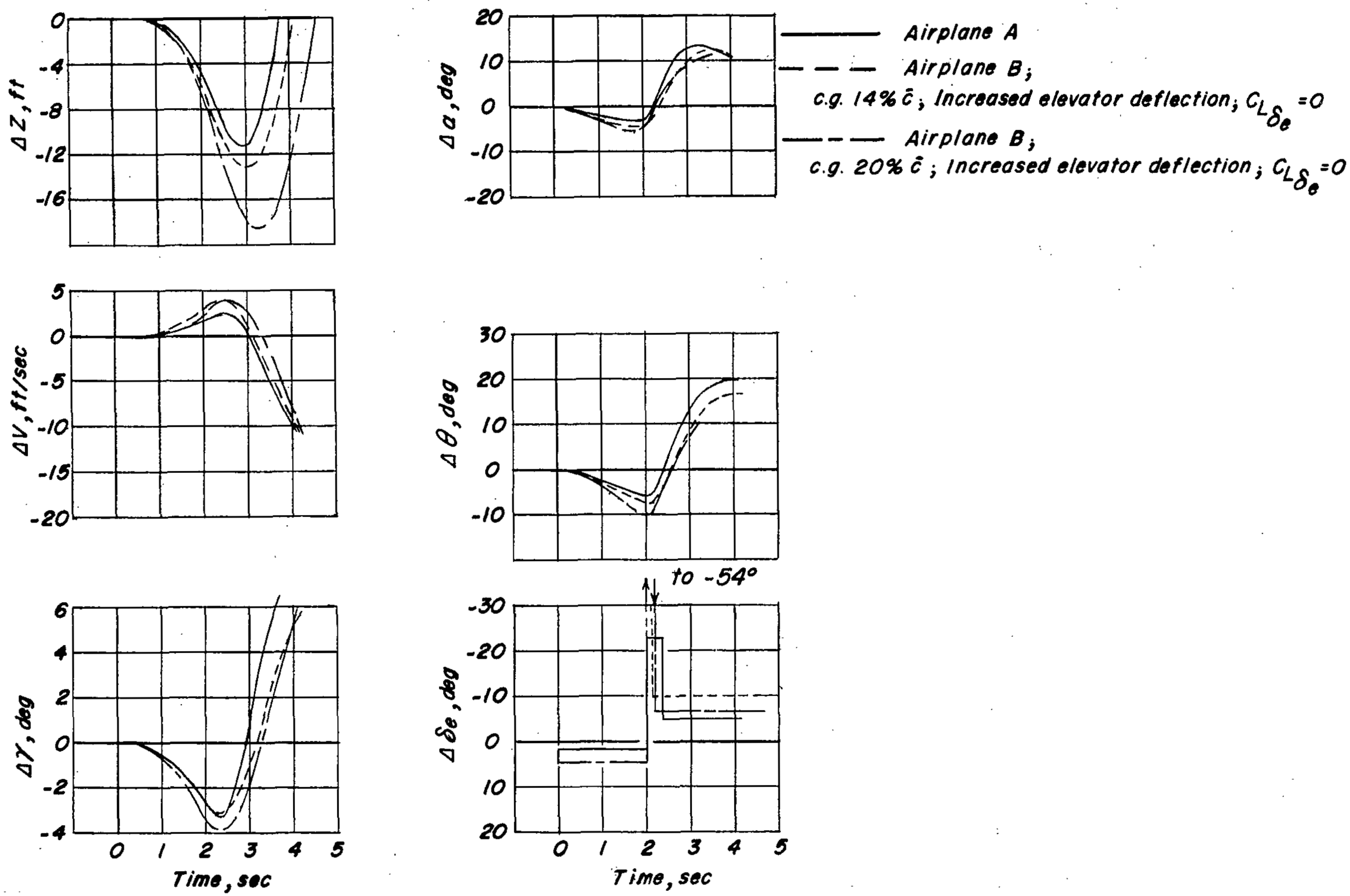

Figure 6.- Comparison of response to longitudinal control on airplane A and airplane B with center of gravity at 0.20c̄ and 0.14c̄. Airplane B for both center-of-gravity positions has increased up-elevator deflection and the change in lift due to elevator deflection eliminated. Initial trim values given in table II.

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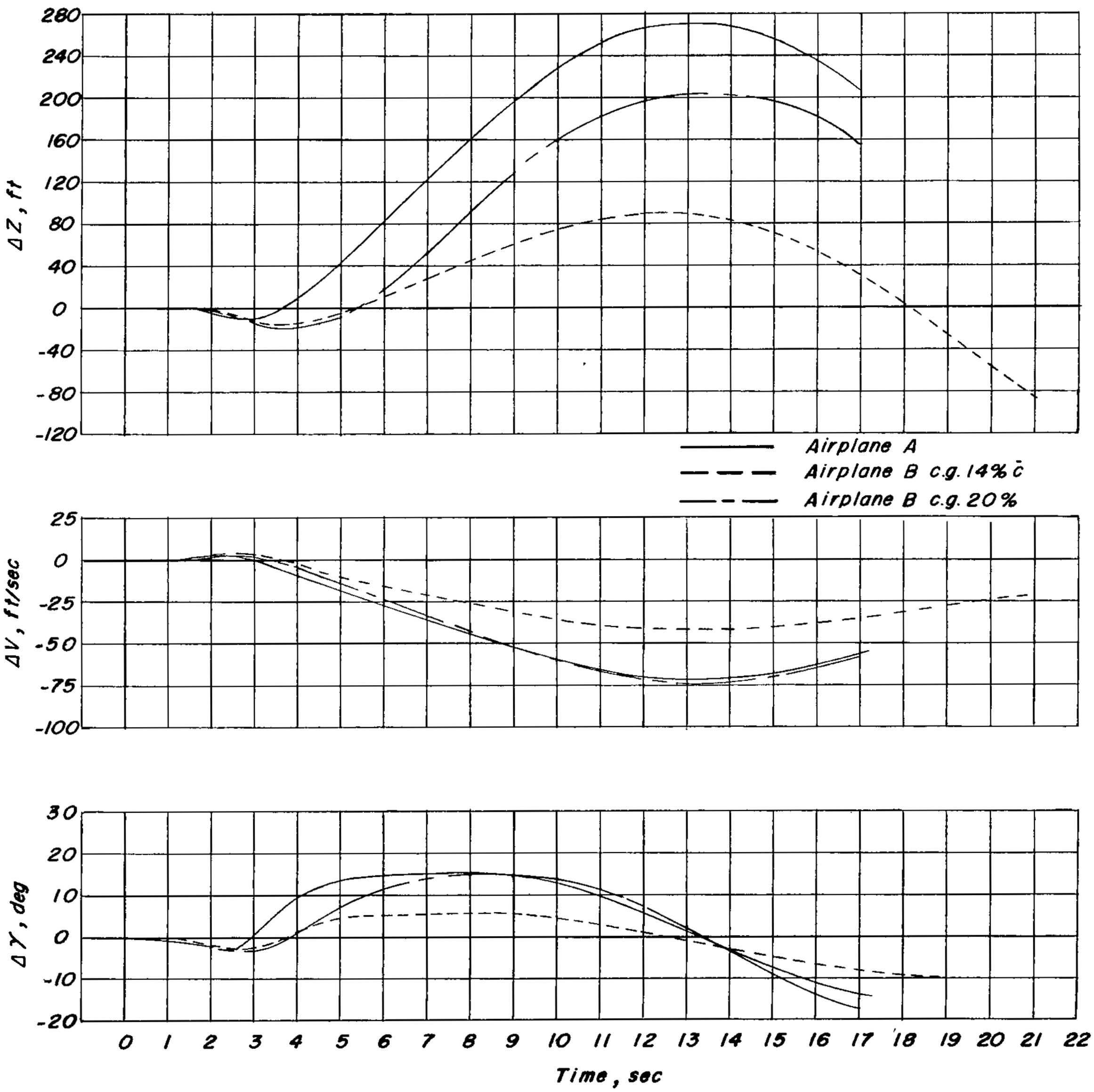

Figure 7.- Comparison of response to available longitudinal control on airplane A and airplane B with center of gravity at 0.20$\bar{c}$ and 0.14$\bar{c}$ for a long period of time. Initial trim values given in table II.

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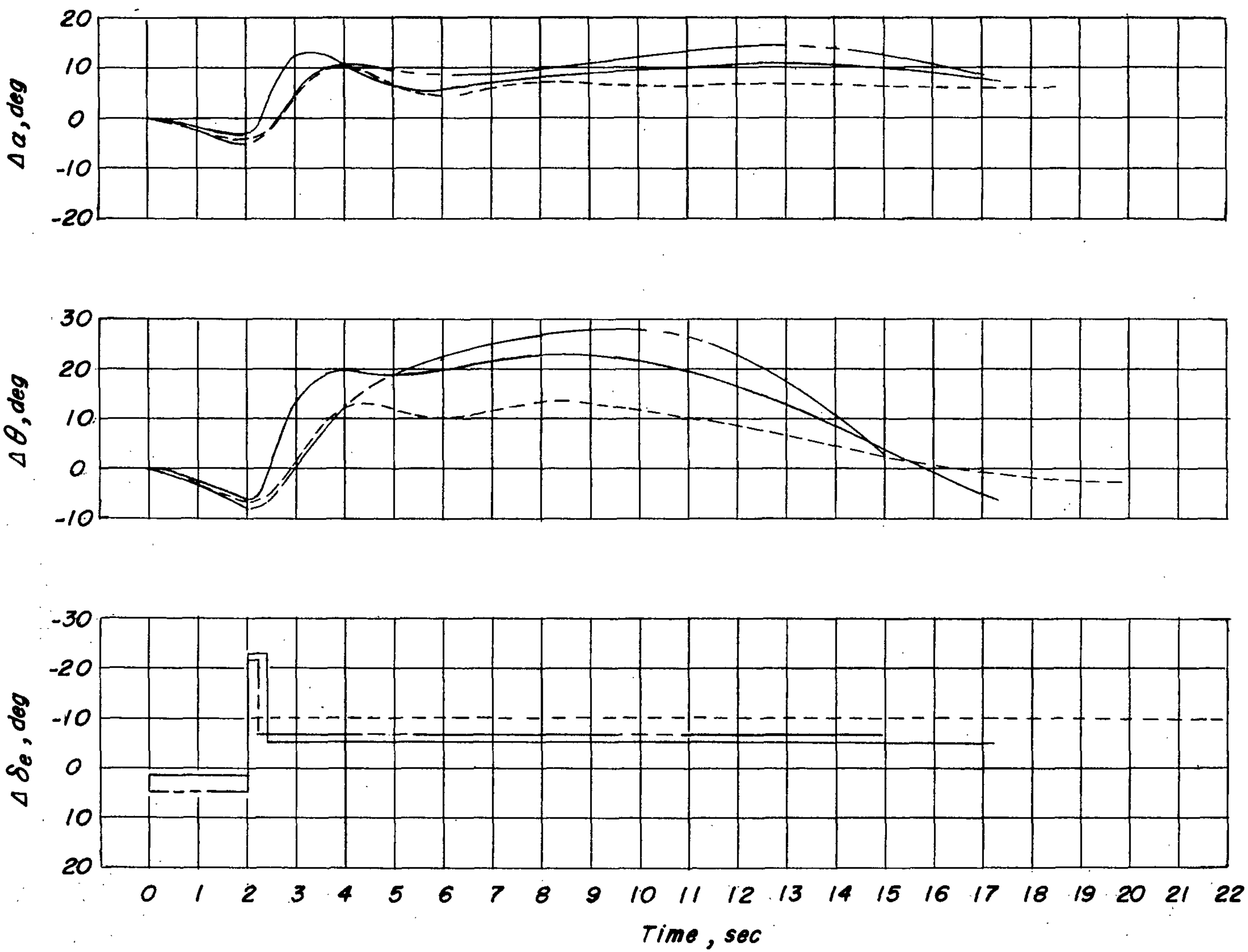


Figure 7.- Concluded.

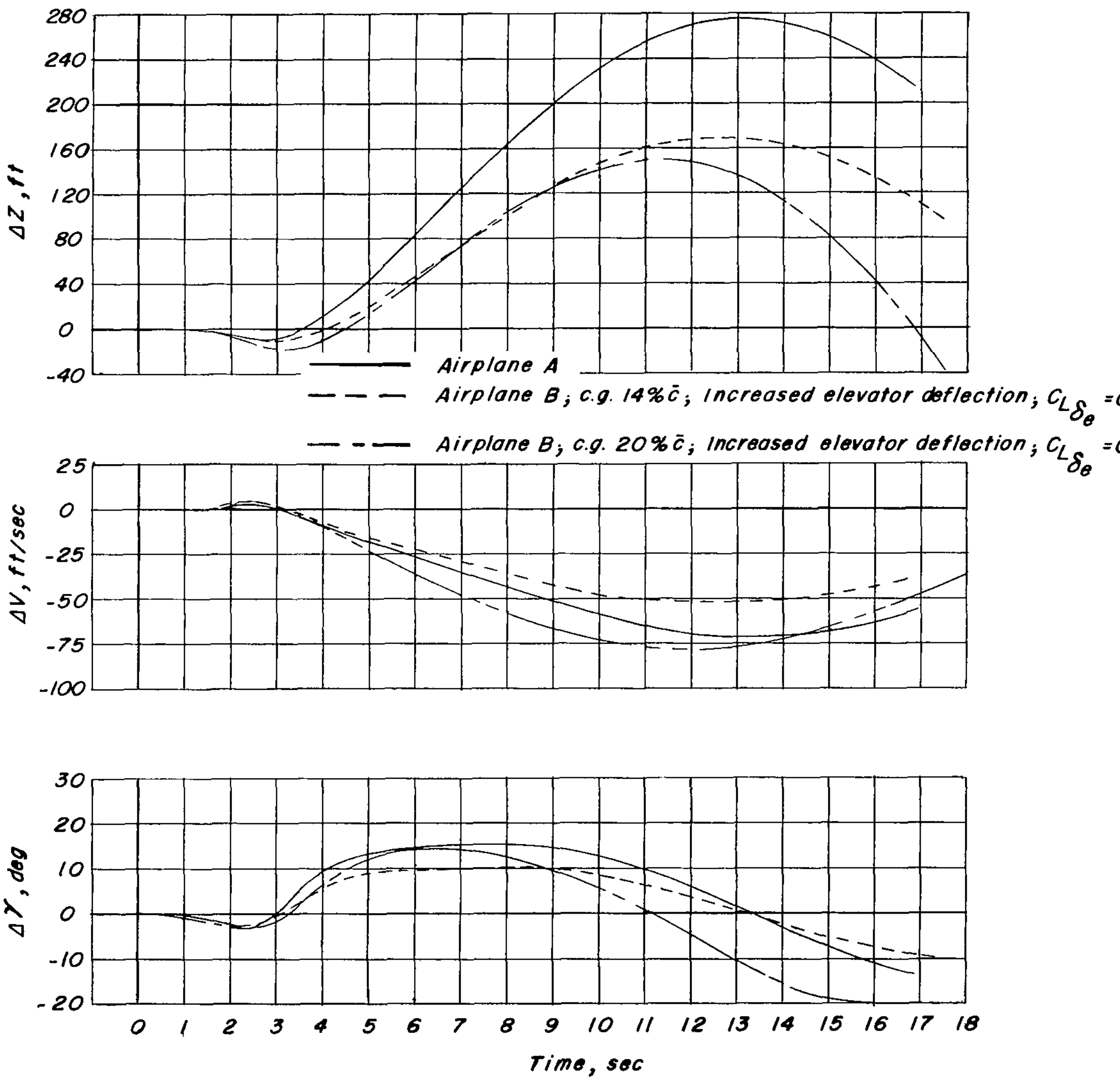


Figure 8.- Comparison of response to longitudinal control on airplane A and airplane B with center of gravity at 0.20c̄ and 0.14c̄ for a long period of time. Airplane B for both center-of-gravity positions has increased up-elevator deflection and the change in lift due to elevator deflection eliminated. Initial trim values are given in table II.

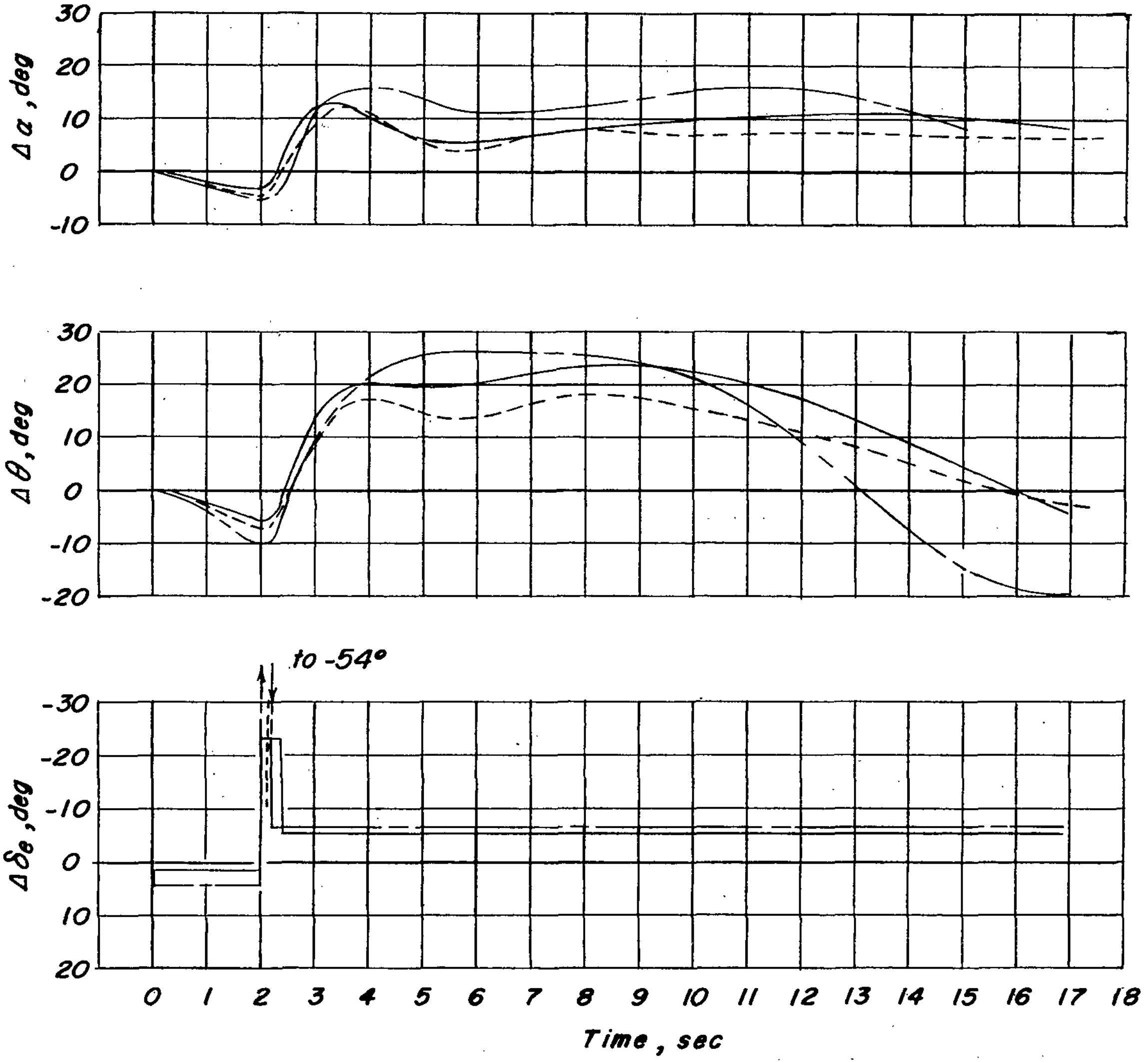


Figure 8.- Concluded.

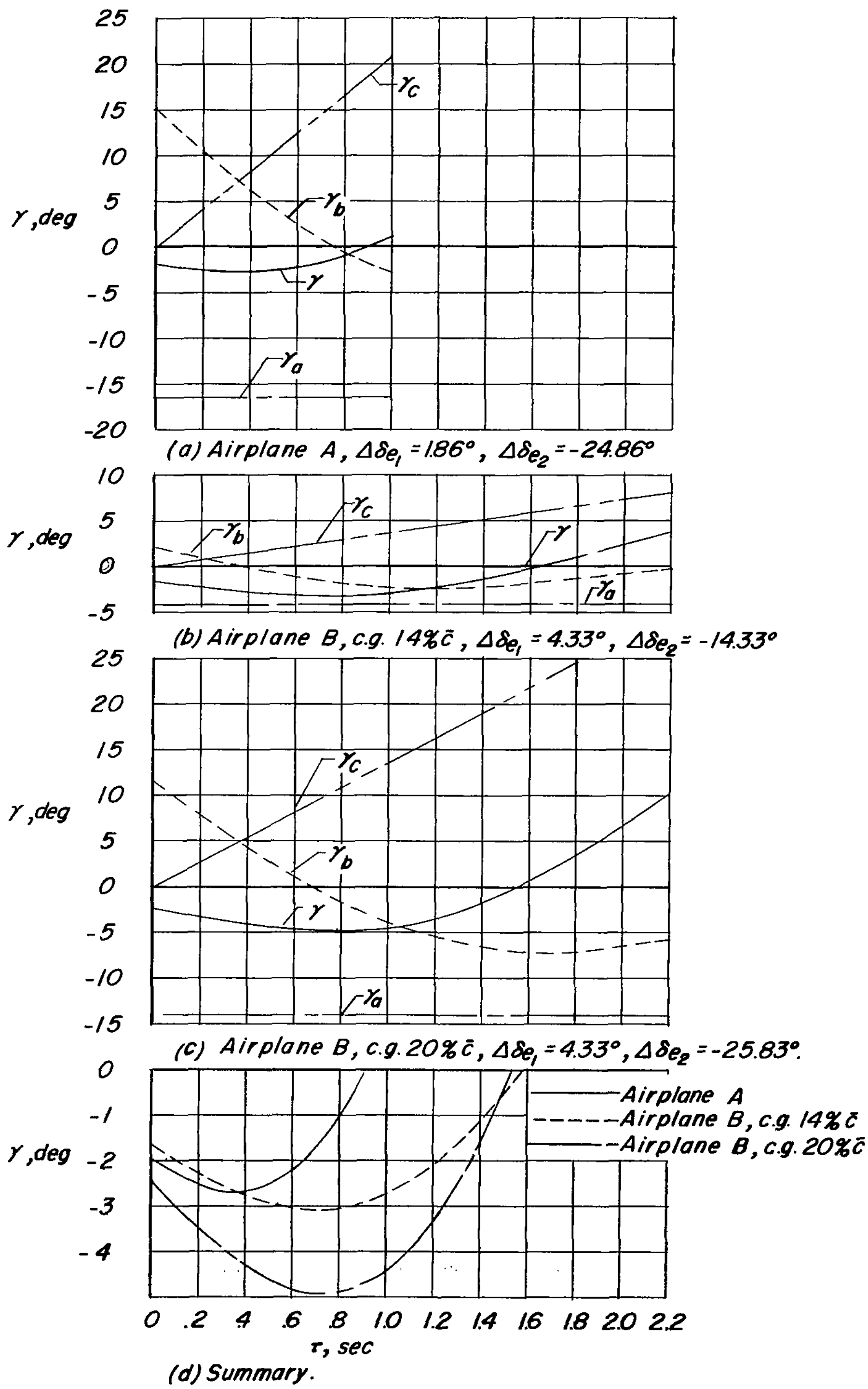
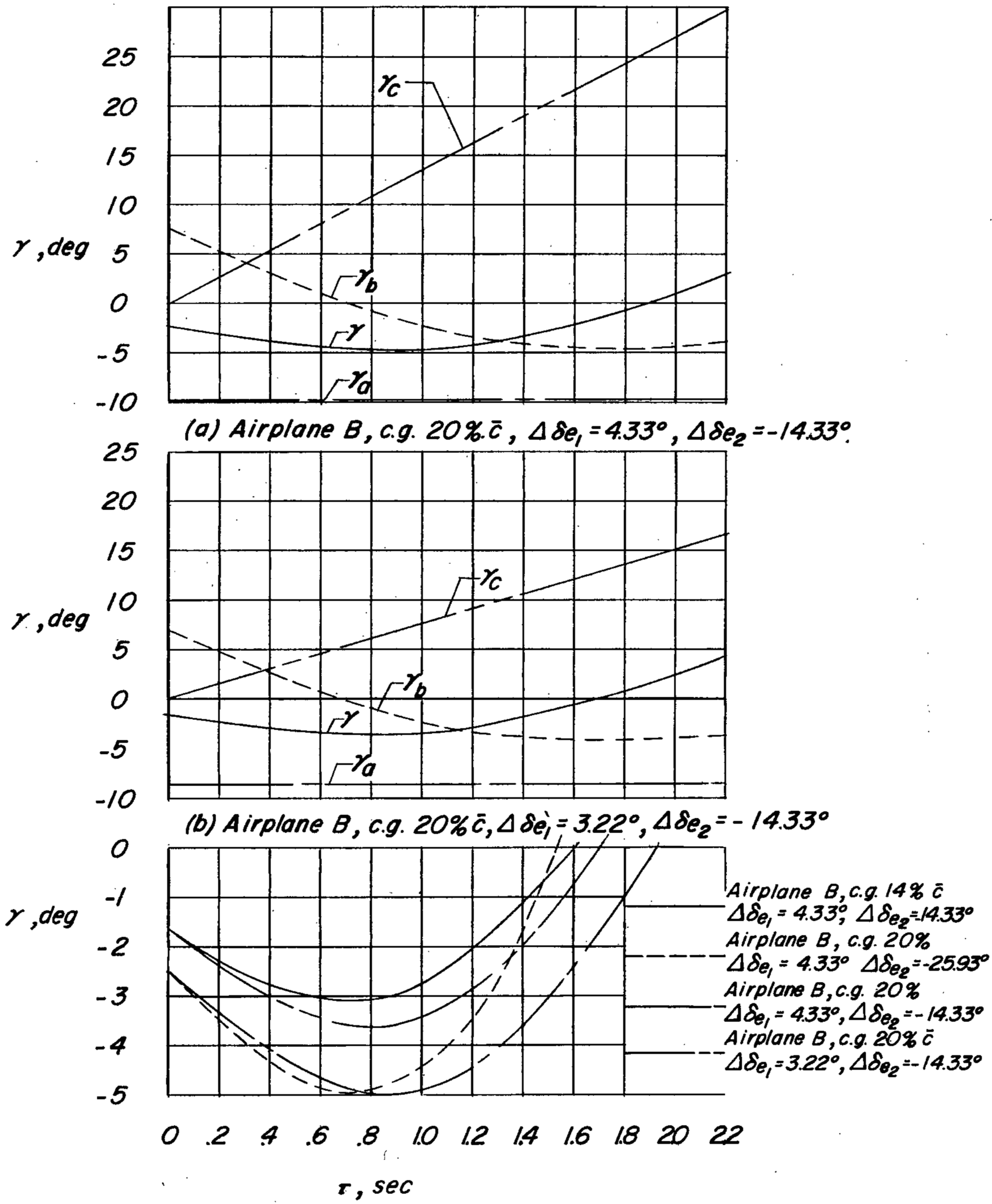


Figure 9.- Comparison of response to available longitudinal control on airplane A and airplane B with center of gravity at 0.20$\bar{c}$ and 0.14$\bar{c}$ from two-degrees-of-freedom calculations. Initial trim values given in table II.



(a) Airplane B, c.g. 20% $\bar{c}$, $\Delta\delta_{e_1} = 4.33°$, $\Delta\delta_{e_2} = -14.33°$.

(b) Airplane B, c.g. 20% $\bar{c}$, $\Delta\delta_{e_1} = 3.22°$, $\Delta\delta_{e_2} = -14.33°$

(c) Summary.

Figure 10.- Effect of use of various amounts of elevator deflection on the response of airplane B with the center of gravity at 0.20$\bar{c}$ as compared to that for airplane B with the center of gravity of 0.14$\bar{c}$ Based on two-degrees-of-freedom calculations. Initial trim values given in table II.

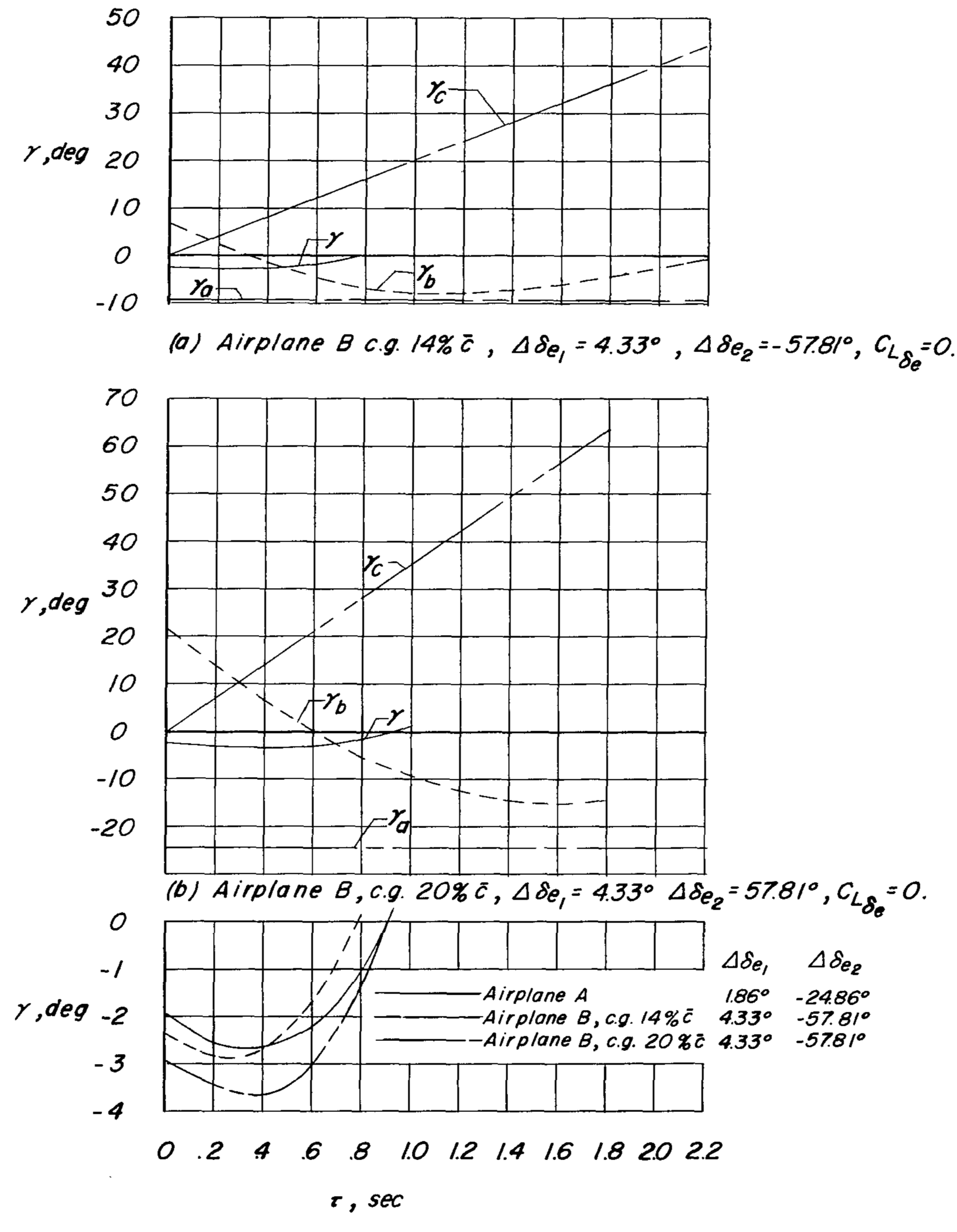
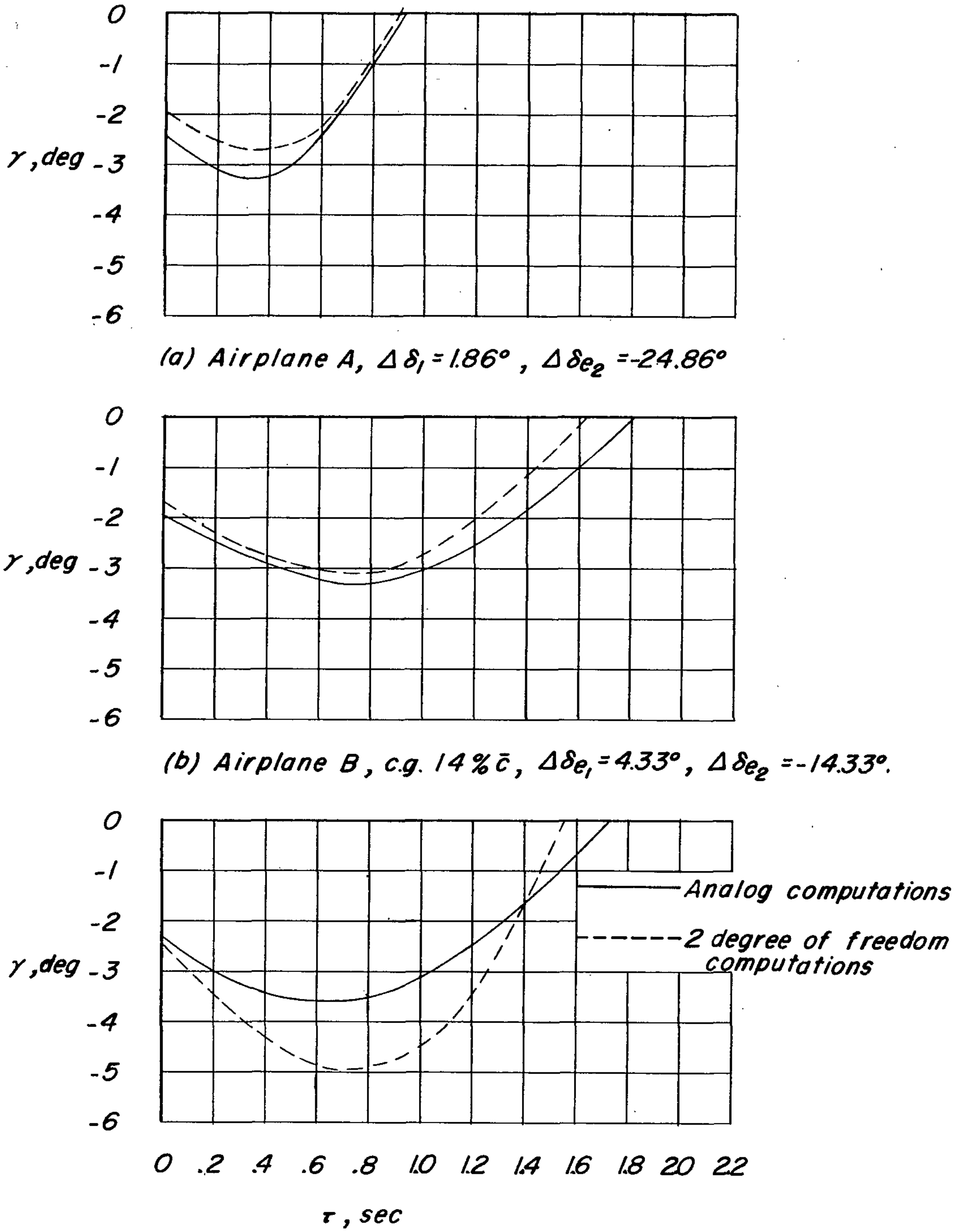


Figure 11.- Comparison of response to longitudinal control on airplane A and airplane B with center of gravity at 0.20c̄ and 0.14c̄. Airplane B for both center-of-gravity positions has increased up-elevator deflection and the change in lift due to elevator deflection eliminated. Based on two-degrees-of-freedom calculations. Initial trim values given in table II.



(a) Airplane A, $\Delta\delta_1 = 1.86°$, $\Delta\delta_{e_2} = -24.86°$

(b) Airplane B, c.g. 14% $\bar{c}$, $\Delta\delta_{e_1} = 4.33°$, $\Delta\delta_{e_2} = -14.33°$.

(c) Airplane B, c.g. 20% $\bar{c}$, $\Delta\delta_{e_1} = 4.33°$, $\Delta\delta_2 = -25.83°$.

Figure 12.- Comparison of the response to available elevator control of airplane A and airplane B with the center of gravity at 0.20$\bar{c}$ and 0.14$\bar{c}$ as calculated by analog and two-degrees-of-freedom calculations.

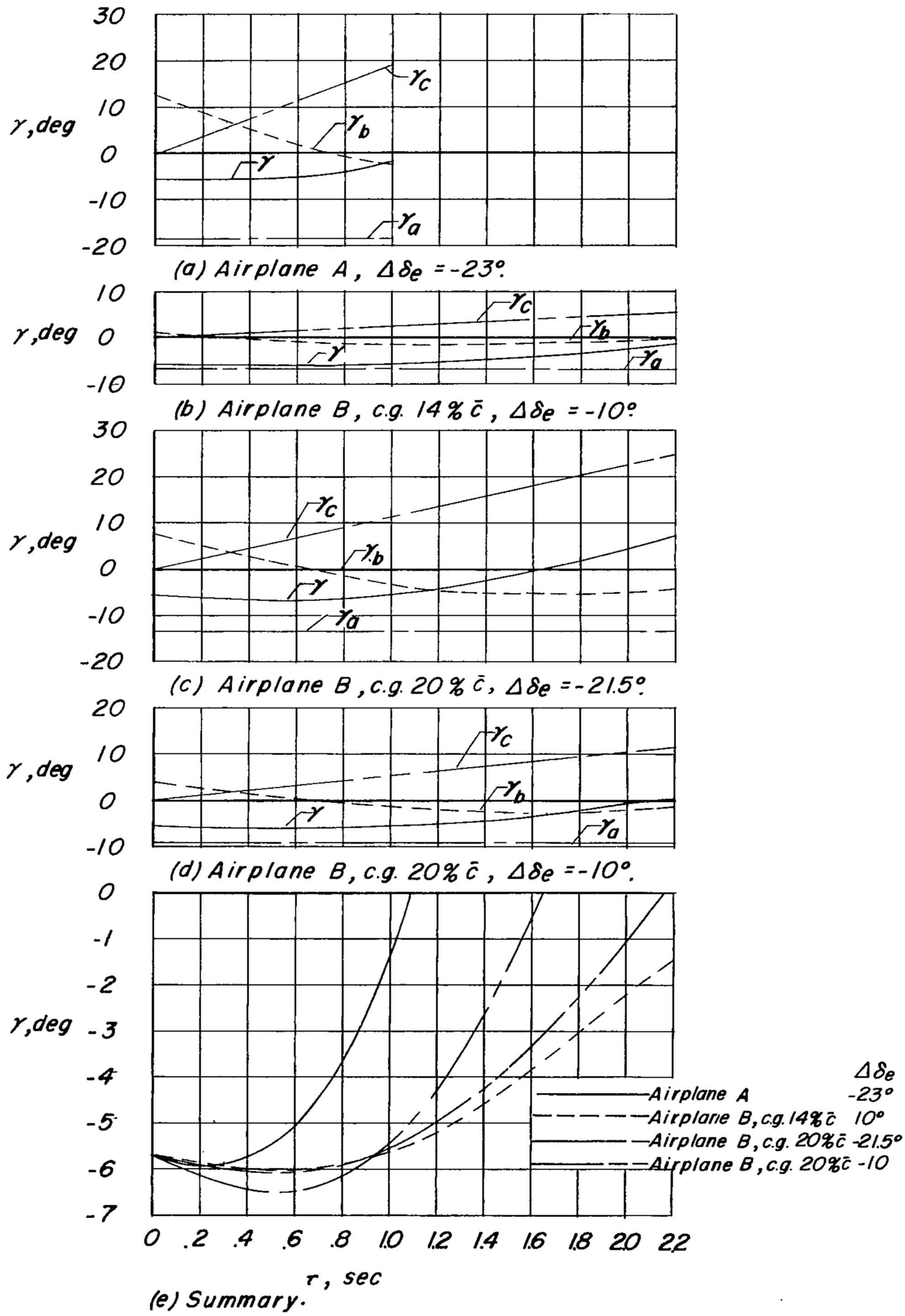


Figure 13.- Comparison of the response when checking a rate of descent in a steady glide of airplane A and airplane B with the center of gravity at 0.20c̄ and 0.14c̄. Based on two-degrees-of-freedom calculations.

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