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RESEARCH MEMORANDUM

CALCULATED EFFECTS OF THE LATERAL ACCELERATION DERIVATIVES ON THE DYNAMIC LATERAL STABILITY OF A DELTA-WING AIRPLANE

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

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CALCULATED EFFECTS OF THE LATERAL ACCELERATION DERIVATIVES ON THE DYNAMIC LATERAL STABILITY OF A DELTA-WING AIRPLANE

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SUMMARY

Calculations have been made of the dynamic lateral stability of a 60° delta-wing interceptor airplane with the lateral acceleration derivatives $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ included and neglected. Calculations were made for angles of attack of 10°, 20°, and 30°, with the airplane flying at sea level and at an altitude of 50,000 feet. Including the lateral acceleration derivatives in the calculations caused changes in stability that were small at 10° angle of attack where the values of these derivatives were small, fairly large at 20° angle of attack where the derivatives were larger, and very large at 30° angle of attack where the derivatives were very large. These results indicate the necessity for including the lateral acceleration derivatives in calculations of dynamic lateral stability. In practically all cases, including these derivatives caused increases in the damping of the Dutch roll oscillation. The effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ varied greatly when the airplane was assumed to have different values of static directional stability $C_{n_{\beta}}$ and effective dihedral $C_{l_{\beta}}$. The effects on stability of varying the yawing derivatives C_{n_r} and C_{l_r} were different from the effects of varying $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$, especially at the high angles of attack.

INTRODUCTION

Recent NACA research with wind-tunnel oscillation testing equipment has shown that large values of the lateral acceleration derivatives $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ sometimes are obtained with highly swept wings at high angles of attack. (See refs. 1 and 2.) In the past, these derivatives have usually been neglected in making dynamic-lateral-stability calculations for airplanes because little information was available to permit reasonably accurate estimates of the derivatives. Now that experimental data on $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ have become available, a theoretical investigation of the effects of these derivatives on dynamic lateral stability has been undertaken. In this investigation, calculations are being made for a variety of configurations and flight conditions with the lateral acceleration derivatives included and neglected. Some preliminary results of this investigation, consisting of stability calculations for a delta-wing interceptor airplane, are presented in this report.

Since the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ appear to be appreciable only at moderate and high angles of attack, the calculations for the delta-wing interceptor were made only for angles of attack of 10°, 20°, and 30°. The calculations were made both for sea level and for an altitude of 50,000 feet. The airplane was assumed first to have values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ of zero and then to have values of these derivatives similar to those obtained from oscillation tests of a delta-wing model in the Langley free-flight tunnel.

SYMBOLS AND COEFFICIENTS

ϕ angle of bank, radians

ψ angle of azimuth, radians

β angle of sideslip, v/V, radians

V airspeed, ft/sec

v sideslip velocity along Y-axis, ft/sec

$\dot{v}$ sideslip acceleration along Y-axis, ft/sec^2

p rolling velocity, $d\phi/dt$, radians/sec

r yawing velocity, $d\psi/dt$, radians/sec

$\dot{\beta}$ rate of change of angle of sideslip, $\dot{v}/V$, radians/sec

$\omega b/2V$ reduced-frequency parameter

ω angular velocity, radians/sec

ρ mass density of air, slugs/cu ft

q dynamic pressure, $\frac{1}{2}\rho V^2$, lb/sq ft

b wing span, ft

S wing area, sq ft

W weight of airplane, lb

m mass of airplane, W/g, slugs

g acceleration due to gravity, ft/sec^2

μ relative-density factor, $m/\rho Sb$

η angle of attack of principal longitudinal axis of inertia, deg (see fig. 1)

ϵ angle between reference axis and principal longitudinal axis of inertia, deg (see fig. 1)

α angle of attack of reference axis, deg (see fig. 1)

γ angle of climb, deg (see fig. 1)

k_{X_o} radius of gyration in roll about principal longitudinal axis of inertia, ft

k_{Z_o} radius of gyration in yaw about principal normal axis of inertia, ft

K_{X_o} nondimensional radius of gyration in roll about principal longitudinal axis, k_{X_o}/b

K_{Z_o} nondimensional radius of gyration in yaw about principal vertical axis, k_{Z_o}/b

K_X nondimensional radius of gyration in roll about longitudinal stability axis, $\sqrt{{K_{X_o}}^2\cos^2\eta + {K_{Z_o}}^2\sin^2\eta}$

K_Z nondimensional radius of gyration in yaw about vertical stability axis, $\sqrt{{K_{Z_o}}^2\cos^2\eta + {K_{X_o}}^2\sin^2\eta}$

K_{XZ} nondimensional product-of-inertia factor, $\left(K_{X_0}^2 - K_{Z_0}^2\right)\cos\eta \sin\eta$ (Note that K_{XZ} is negative for positive values of η.)

C_L trim lift coefficient, Lift/qS

C_l rolling-moment coefficient, Rolling moment/qSb

C_n yawing-moment coefficient, Yawing moment/qSb

C_Y lateral-force coefficient, Lateral force/qS

C_{l_β} effective-dihedral derivative, rate of change of rolling-moment coefficient with angle of sideslip, $\partial C_l/\partial\beta$, per radian

C_{n_β} static directional-stability derivative, rate of change of yawing-moment coefficient with angle of sideslip, $\partial C_n/\partial\beta$, per radian

C_{Y_β} rate of change of lateral-force coefficient with angle of sideslip, $\partial C_Y/\partial\beta$, per radian

$C_{l_{\dot\beta}}$ rate of change of rolling-moment coefficient with lateral-acceleration factor, $\partial C_l \Big/ \partial\frac{\dot\beta b}{2V}$, per radian

$C_{n_{\dot\beta}}$ rate of change of yawing-moment coefficient with lateral-acceleration factor, $\partial C_n \Big/ \partial\frac{\dot\beta b}{2V}$, per radian

$C_{Y_{\dot\beta}}$ rate of change of lateral-force coefficient with lateral-acceleration factor, $\partial C_Y \Big/ \partial\frac{\dot\beta b}{2V}$, per radian

C_{n_r} damping-in-yaw derivative, rate of change of yawing-moment coefficient with yawing-angular-velocity factor, $\partial C_n \Big/ \partial\frac{rb}{2V}$, per radian

C_{n_p} rate of change of yawing-moment coefficient with rolling-angular-velocity factor, $\partial C_n \Big/ \partial\frac{pb}{2V}$, per radian

C_{l_p} rate of change of rolling-moment coefficient with rolling-angular-velocity factor, $\partial C_l \Big/ \partial\frac{pb}{2V}$, per radian

C_{l_r} rate of change of rolling-moment coefficient with yawing-angular-velocity factor, $\partial C_l \Big/ \partial\frac{rb}{2V}$, per radian

C_{Y_p} rate of change of lateral-force coefficient with rolling-angular-velocity factor, $\partial C_Y \Big/ \partial\frac{pb}{2V}$, per radian

C_{Y_r} rate of change of lateral-force coefficient with yawing-angular-velocity factor, $\partial C_Y \Big/ \partial\frac{rb}{2V}$, per radian

t time, sec

s nondimensional time parameter based on span, Vt/b

D differential operator, d/ds

P period of oscillation, sec

$T_{1/2}$ time for amplitude of oscillation to change by a factor of 2 (positive value indicates a decrease to half-amplitude, negative value indicates an increase to double amplitude)

A,B,C,D,E coefficients of lateral-stability equation

EQUATIONS OF MOTION

The nondimensional linearized lateral equations of motion, referred to the stability axes, used to calculate the stability roots are as follows: (These equations are the same as those of reference 3 except for the addition of the $C_{n_{\dot\beta}}$, $C_{l_{\dot\beta}}$, and $C_{Y_{\dot\beta}}$ terms and a change in the sign of K_{XZ}.)

Rolling moment

$$2\mu\left(K_X{}^2D^2\phi - K_{XZ}D^2\psi\right) = C_{l_\beta}\beta + \frac{1}{2}C_{l_{\dot\beta}}D\beta + \frac{1}{2}C_{l_p}D\phi + \frac{1}{2}C_{l_r}D\psi$$

Yawing moment

$$2\mu\left(K_Z{}^2D^2\psi - K_{XZ}D^2\phi\right) = C_{n_\beta}\beta + \frac{1}{2}C_{n_{\dot{\beta}}}D\beta + \frac{1}{2}C_{n_p}D\phi + \frac{1}{2}C_{n_r}D\psi$$

Lateral force

$$2\mu\left(D\beta + D\psi\right) = C_{Y_\beta}\beta + \frac{1}{2}C_{Y_{\dot{\beta}}}D\beta + \frac{1}{2}C_{Y_p}D\phi + C_L\phi + \frac{1}{2}C_{Y_r}D\psi + \left(C_L \tan\gamma\right)\psi$$

When $\phi_o e^{\lambda s}$ is substituted for ϕ, $\psi_o e^{\lambda s}$ for ψ, $\beta_o e^{\lambda s}$ for β in the equations written in determinant form, λ must be a root of the stability equation

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

where

$$A = 8\mu^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right) - 2\mu^2\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)C_{Y_{\dot{\beta}}}$$

$$B = -2\mu\Big(2\mu K_X{}^2K_Z{}^2C_{Y_\beta} + \mu K_X{}^2C_{n_r} + \mu K_Z{}^2C_{l_p} - 2\mu K_{XZ}{}^2C_{Y_\beta} + \mu K_{XZ}C_{l_r} +$$

$$\mu K_{XZ}C_{n_p} - \frac{1}{4}K_X{}^2C_{n_r}C_{Y_{\dot{\beta}}} - \mu K_X{}^2C_{n_{\dot{\beta}}} - \frac{1}{4}K_Z{}^2C_{l_p}C_{Y_{\dot{\beta}}} - \frac{1}{4}K_{XZ}C_{l_r}C_{Y_{\dot{\beta}}} -$$

$$\mu K_{XZ}C_{l_{\dot{\beta}}} - \frac{1}{4}C_{n_p}K_{XZ}C_{Y_{\dot{\beta}}} + \frac{1}{4}K_{XZ}C_{Y_p}C_{n_{\dot{\beta}}} + \frac{1}{4}K_Z{}^2C_{Y_p}C_{l_{\dot{\beta}}} +$$

$$\frac{1}{4}K_X{}^2C_{Y_r}C_{n_{\dot{\beta}}} + \frac{1}{4}K_{XZ}C_{Y_r}C_{l_{\dot{\beta}}}\Big)$$

$$C = \mu K_X^2 C_{n_r} C_{Y_\beta} + 4\mu^2 K_X^2 C_{n_\beta} + \mu K_Z^2 C_{l_p} C_{Y_\beta} + \frac{1}{2}\mu C_{n_r} C_{l_p} + \mu K_{XZ} C_{l_r} C_{Y_\beta} +$$

$$4\mu^2 K_{XZ} C_{l_\beta} + \mu C_{n_p} K_{XZ} C_{Y_\beta} - \frac{1}{2}\mu C_{n_p} C_{l_r} - \mu K_{XZ} C_{n_\beta} C_{Y_p} -$$

$$\mu K_Z^2 C_{Y_p} C_{l_\beta} - \mu K_X^2 C_{Y_r} C_{n_\beta} - \mu K_{XZ} C_{Y_r} C_{l_\beta} - \frac{1}{8} C_{n_r} C_{l_p} C_{Y_{\dot\beta}} -$$

$$\frac{1}{2}\mu C_{l_p} C_{n_{\dot\beta}} + \frac{1}{8} C_{n_p} C_{l_r} C_{Y_{\dot\beta}} + \frac{1}{2}\mu C_{n_p} C_{l_{\dot\beta}} - \mu C_L K_{XZ} C_{n_{\dot\beta}} -$$

$$\mu C_L K_Z^2 C_{l_{\dot\beta}} - \mu K_X^2 C_{n_{\dot\beta}} C_L \tan\gamma - \mu K_{XZ} C_{l_{\dot\beta}} C_L \tan\gamma +$$

$$\frac{1}{8} C_{l_p} C_{Y_r} C_{n_{\dot\beta}} - \frac{1}{8} C_{n_p} C_{Y_r} C_{l_{\dot\beta}} - \frac{1}{8} C_{l_r} C_{Y_p} C_{n_{\dot\beta}} + \frac{1}{8} C_{n_r} C_{Y_p} C_{l_{\dot\beta}}$$

$$D = -\frac{1}{4} C_{n_r} C_{l_p} C_{Y_\beta} - \mu C_{l_p} C_{n_\beta} + \frac{1}{4} C_{n_p} C_{l_r} C_{Y_\beta} + \mu C_{n_p} C_{l_\beta} -$$

$$2\mu C_L K_{XZ} C_{n_\beta} - 2\mu C_L K_Z^2 C_{l_\beta} - 2\mu K_X^2 C_{n_\beta} C_L \tan\gamma -$$

$$2\mu K_{XZ} C_{l_\beta} C_L \tan\gamma + \frac{1}{4} C_{l_p} C_{n_\beta} C_{Y_r} - \frac{1}{4} C_{n_p} C_{l_\beta} C_{Y_r} -$$

$$\frac{1}{4} C_{l_r} C_{n_\beta} C_{Y_p} + \frac{1}{4} C_{n_r} C_{l_\beta} C_{Y_p} + \frac{1}{4} C_L C_{n_r} C_{l_{\dot\beta}} - \frac{1}{4} C_L C_{l_r} C_{n_{\dot\beta}} +$$

$$\frac{1}{4} C_L C_{l_p} C_{n_{\dot\beta}} \tan\gamma - \frac{1}{4} C_L C_{n_p} C_{l_{\dot\beta}} \tan\gamma$$

$$E = \frac{1}{2} C_L \left(C_{n_r} C_{l_\beta} - C_{l_r} C_{n_\beta}\right) + \frac{1}{2} C_L \tan\gamma \left(C_{l_p} C_{n_\beta} - C_{n_p} C_{l_\beta}\right)$$

CALCULATIONS

Calculations were made to determine the period and damping of the oscillatory mode and the damping of the aperiodic modes using the equations presented in the preceding section. The dimensional and mass characteristics assumed for the airplane are presented in table I and the flight conditions for which calculations were made are given in tables II and III. Calculations were made for angles of attack of 10°, 20°, and 30° for sea level and for an altitude of 50,000 feet.

Values of the stability derivatives used in the calculations are given in tables II and III and plots of variations with angles of attack of some of the derivatives are shown in figure 2. For 30° angle of attack, four different combinations of the important sideslip stability derivatives C_{n_β} and C_{l_β} were assumed as follows:

Combination	C_{n_β}	C_{l_β}
A	-0.0573	0
B	-.0573	-.0573
C	.0573	0
D	.0573	-.0573

Studies of force-test results for several delta-wing configurations have indicated that, depending upon the particular configuration, any one of these combinations of C_{n_β} and C_{l_β} might exist at 30° angle of attack. Values of the rolling derivatives C_{l_p} and C_{n_p} and the yawing derivatives C_{n_r} and C_{l_r} were estimated from experimental data obtained by the NACA on delta-wing configurations; and the derivatives C_{Y_p}, C_{Y_r}, and $C_{Y_{\dot{\beta}}}$ were assumed to be zero.

Values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ were estimated from the data presented in references 1, 2, and 4. In making the estimates of $C_{n_{\dot{\beta}}}$, values of C_{n_r} obtained from curved-flow tests (ref. 4) were subtracted from the values of $C_{n_r} - C_{n_{\dot{\beta}}}$ of reference 2. For estimating $C_{l_{\dot{\beta}}}$, values of $C_{n_r} - C_{n_{\dot{\beta}}}$, C_{l_p}, C_{n_p}, and C_{l_r} measured about the stability axes and values of $C_{n_r} - C_{n_{\dot{\beta}}} \cos \theta$ and $C_{l_p} + C_{l_{\dot{\beta}}} \sin \theta$ measured about the body axes (ref. 1) were substituted in the equations relating the stability-axis and body-axis damping derivatives. The equations were then solved for $C_{l_{\dot{\beta}}}$. The data of references 1 and 2 were obtained at values of the reduced-frequency parameter $\omega b/2V$ of about 0.21 to 0.25. Subsequent oscillation tests of a delta-wing model (results unpublished) have indicated large effects of frequency, particularly at angles of attack above about 15°. The effects of frequency are such that greater values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ are obtained at the smaller values of $\omega b/2V$. In addition to these results, some results (also unpublished) have recently been obtained in the Langley stability tunnel which show that the values of C_{n_r} and C_{l_r} for a 60° delta wing in an oscillation are actually much greater than the values obtained in the curved-flow tests of reference 4. The values of C_{n_r} and C_{l_r} used in the present calculations are therefore probably smaller than they should be. These data also indicate that the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ used in the present calculations are too large since they were obtained by subtracting the curved-flow-test values of C_{n_r} and C_{l_r} from the oscillation-test values of $C_{n_r} - C_{n_{\dot{\beta}}}$ and $C_{l_r} - C_{l_{\dot{\beta}}}$. The overall significance of these additional unpublished results in connection with the present calculations will be discussed in the "Results and Discussion" section.

Calculations were made for each condition with $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ neglected, with $C_{n_{\dot{\beta}}}$ included, and with both $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ included. In addition to these three calculations for each condition, two other calculations were made with $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ neglected - one in which C_{n_r} was increased by an amount which would correspond to $C_{n_r} - C_{n_{\dot{\beta}}}$ and the other in which C_{n_r} and C_{l_r} were increased to correspond to $C_{n_r} - C_{n_{\dot{\beta}}}$ and $C_{l_r} - C_{l_{\dot{\beta}}}$, respectively. These latter calculations were made to determine whether C_{n_r} and C_{l_r} might have the same general effects on stability as $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$. It might be reasoned that such could be the case, since it appears that most of the terms of primary importance containing these four derivatives in the coefficients of the lateral-stability equation can be combined into terms involving $C_{n_r} - C_{n_{\dot{\beta}}}$ and $C_{l_r} - C_{l_{\dot{\beta}}}$.

RESULTS AND DISCUSSION

The results of the calculations are presented in tables II and III. Some of the results showing the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ on the period and damping of the Dutch roll oscillation are presented in figures 3 and 4.

Effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$

The data of table II and figure 3(a) show that for 10° angle of attack the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ on either the Dutch roll oscillatory mode or the aperiodic modes were quite small for both the sea-level and 50,000-foot-altitude cases. The Dutch roll damping was increased slightly by including $C_{n_{\dot{\beta}}}$ and still further by including both $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$. The relatively small effect of the lateral acceleration derivatives at 10° angle of attack can be attributed to the fact that these derivatives are small at this angle of attack as shown by figure 2.

For 20° angle of attack, the data of table II and figure 3(b) show that substantial improvements in Dutch roll damping at sea level and at 50,000 feet were obtained by including $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ in the calculations. The changes in the damping of the aperiodic modes resulting from inclusion of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ at this angle of attack, however, were very small.

The data of table III and figure 4 show that at 30° angle of attack the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ were very pronounced for both the Dutch roll and aperiodic modes of some of the combinations of C_{n_β} and C_{l_β}. In most cases the damping of all the modes was increased. An extreme change from a high degree of instability to a high degree of stability of the Dutch roll oscillation was obtained with combination D by including the acceleration derivatives. On the other hand, including $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ in the case of combination A caused only relatively minor changes in Dutch roll damping. These results indicate that the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ are greatly dependent on the values of the other stability parameters such as C_{n_β} and C_{l_β}. A detailed analysis of the results of this and other similar investigations might reveal the reasons for some of the differences in the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ for different cases but more than likely it will not be possible to make any generalizations regarding the effects of these derivatives. Previous studies of dynamic lateral stability have indicated that it is unwise to attempt to generalize regarding the effect of any given stability derivative because of the strong interdependence that exists between most of the derivatives.

As pointed out in the "Calculations" section, values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ used in the present study were estimated from oscillation test data obtained at values of $\omega b/2V$ from about 0.21 to 0.25. The results of the calculations, however, show that for the flight conditions considered in this investigation the values of $\omega b/2V$ are in most cases much smaller than 0.21. As stated previously, unpublished test results obtained with a delta-wing model subsequent to the tests of references 1 and 2, have indicated that the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ are much larger at the lower values of $\omega b/2V$, particularly for angles of attack above 15°. On this basis it would appear that the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ used in the present calculations for 20° and 30° angle of attack are too small. On the other hand, as pointed out in the "Calculations" section, the additional unpublished results on the derivatives C_{n_r} and C_{l_r} indicate that the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ used in the calculations are too large. A preliminary analysis of the limited amount of data available at present indicates that these factors are roughly compensating so that the values of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ used are at least of the right order of magnitude. In any event, it is unlikely that the changes in $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ which might be involved would alter the principal conclusions drawn from these calculations - that the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ are appreciable and that these derivatives should be considered in studies of dynamic lateral stability.

Effects of C_{n_r} and C_{l_r}

The results of calculations made to determine whether C_{n_r} and C_{l_r} have the same effects on stability as $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ are presented in tables II and III. In each group of five rows in these tables, the second and fourth rows afford a comparison of the effects of $C_{n_{\dot{\beta}}}$ and C_{n_r} while the third and fifth rows give the results when either $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ or C_{n_r} and C_{l_r} were changed.

It is apparent from the results presented in tables II and III that the effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ cannot always be properly simulated by making corresponding changes in C_{n_r} and C_{l_r}. The differences between the effects of the $\dot{\beta}$ and r derivatives were especially great at 30° angle of attack. Since, for the purpose of this report, this general result is the only one of interest in connection with the C_{n_r} and C_{l_r} calculations, no detailed discussion of these results will be given.

CONCLUDING REMARKS

The results of the dynamic-lateral-stability calculations for the 60° delta-wing interceptor airplane with the lateral acceleration derivatives $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ included and neglected can be summarized as follows:

1. Including $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ in the calculations caused changes in stability that were small at 10° angle of attack where the values of these derivatives were small, fairly large at 20° angle of attack where the derivatives were larger, and very large at 30° angle of attack where the derivatives were very large. These results indicate the necessity for including the lateral acceleration derivatives in calculations of dynamic lateral stability.

2. In practically all cases, including $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ caused increases in the damping of the Dutch roll oscillation.

3. The effects of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ varied greatly when the airplane was assumed to have different values of $C_{n_{\beta}}$ and $C_{l_{\beta}}$.

4. The effects on stability of varying C_{n_r} and C_{l_r} were different from the effects of varying $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$, especially at the high angles of attack.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 16, 1954.

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TABLE I

DIMENSIONAL AND MASS CHARACTERISTICS ASSUMED FOR AIRPLANE

Weight, lb	22,850
Wing loading, lb/sq ft	34.5
$(\mu)_{h=0}$	11.85
${K_{X_0}}^2$	0.0135
${K_{Z_0}}^2$	0.0844
ϵ, deg	1.2
Wing:	
Area, sq ft	662
Span, ft	38.1
Aspect ratio	2.2
Sweepback of leading edge, deg	60
Vertical tail:	
Area, sq ft	6.8
Span, ft	8.7
Aspect ratio	1.1
Sweepback of leading edge, deg	60

TABLE II

CALCULATION CONDITIONS AND RESULTS FOR ANGLES OF ATTACK OF 10° AND 20°

$\left[C_{Y_p} = 0;\ C_{Y_r} = 0;\ C_{Y_{\dot{\beta}}} = 0;\ \gamma = 0^{\circ}\right]$

Flight conditions				Mass parameters				Aerodynamic derivatives									Results			
																	Aperiodic modes		Oscillatory mode	
α, deg	C_L	η, deg	h, ft	μ	K_X^2	K_Z^2	K_{XZ}	C_{Y_β}	C_{n_β}	C_{l_β}	C_{n_p}	C_{l_p}	C_{n_r}	C_{l_r}	$C_{n_{\dot{\beta}}}$	$C_{l_{\dot{\beta}}}$	$T_{1/2}$, sec	$T_{1/2}$, sec	P, sec	$T_{1/2}$, sec
10	0.4	8.8	0	11.85	0.0151	0.0827	-0.0107	-0.570	0.0573	-0.0573	0	-0.160	-0.19	0.10	0	0	14.80	0.44	4.26	1.69
													-.19	.10	.04	0	14.74	.44	4.29	1.53
													-.19	.10	.04	-.15	14.99	.42	4.45	1.35
													-.23	.10	0	0	10.07	.45	4.31	1.55
													-.23	.25	0	0	-70.92	.39	4.46	1.41
10	.4	8.8	50,000	77.80	.0151	.0827	-.0107	-.570	.0573	-.0573	0	-.160	-.19	.10	0	0	37.33	1.31	3.87	3.44
													-.19	.10	.04	0	37.33	1.31	3.87	3.18
													-.19	.10	.04	-.15	37.42	1.30	3.90	2.70
													-.23	.10	0	0	25.41	1.33	3.88	3.19
													-.23	.25	0	0	-176.19	1.24	3.91	2.72
20	.8	18.8	0	11.85	.0210	.0770	-.0216	-.570	.0573	-.0690	-.045	-.130	-.20	0	0	0	5.22	.95	4.13	2.87
													-.20	0	.22	0	5.07	.98	4.29	1.50
													-.20	0	.22	-.45	5.33	.90	5.10	.78
													-.42	0	0	0	-------	----	4.42	1.54
																			37.52	1.60
													-.42	.45	0	0	26.84	.59	5.71	.95
20	.8	18.8	50,000	77.80	.0210	.0770	-.0216	-.570	.0573	-.0690	-.045	-.130	-.20	0	0	0	13.41	2.54	4.02	6.99
													-.20	0	.22	0	13.35	2.55	4.03	3.80
													-.20	0	.22	-.45	13.45	2.53	4.12	1.94
													-.42	0	0	0	-------	----	4.05	3.80
																			106.73	4.26
													-.42	.45	0	0	67.04	2.13	4.17	1.97

TABLE III

CALCULATION CONDITIONS AND RESULTS FOR 30° ANGLE OF ATTACK

$$\left[C_L = 1.0;\ \eta = 28.8^\circ;\ C_{Y_p} = 0;\ C_{Y_r} = 0;\ C_{Y_{\dot\beta}} = 0;\ \gamma = 0^\circ\right]$$

(a) h = 0 ft

Combination of C_{n_β} and C_{l_β}	Mass parameters				Aerodynamic derivatives									Results			
														Aperiodic modes		Oscillatory mode	
	μ	K_X^2	K_Z^2	K_{XZ}	C_{Y_β}	C_{n_β}	C_{l_β}	C_{n_p}	C_{l_p}	C_{n_r}	C_{l_r}	$C_{n_{\dot\beta}}$	$C_{l_{\dot\beta}}$	$T_{1/2}$, sec	$T_{1/2}$, sec	P, sec	$T_{1/2}$, sec
A	11.85	0.0300	0.0679	-0.0300	-0.286	-0.0573	0	-0.200	-0.020	-0.10 -.10 -.10 -1.10 -1.10	-0.10 -.10 -.10 -.10 .60	0 1.0 1.0 0 0	0 0 -.70 0 0	-0.47 -1.30 -3.14 -1.06 1.21	0.74 .28 .19 .27 .18	30.09 29.38 35.50 35.83 17.04	6.33 6.12 2.71 5.61 -2.58
B	11.85	.0300	.0679	-.0300	-.286	-.0573	-.0573	-.200	-.020	-.10 -.10 -.10 -1.10 -1.10	-.10 -.10 -.10 -.10 .60	0 1.0 1.0 0 0	0 0 -.70 0 0	∞ ∞ ∞ 1.29 .95	.72 .31 .21 .31 .19	7.75 8.58 11.46 8.52 9.27	-1.07 -44.17 2.38 -2.25 -3.36
C	11.85	.0300	.0679	-.0300	-.286	.0573	0	-.200	-.020	-.10 -.10 -.10 -1.10 -1.10	-.10 -.10 -.10 -.10 .60	0 1.0 1.0 0 0	0 0 -.70 0 0	----- ----- .60 ----- -2.31	---- ---- .26 ---- .22	5.88 30.86 12.87 29.40 45.09 224.50 23.78 14.00	-2.23 4.81 .74 4.58 8.69 .69 8.01 1.28
D	11.85	.0300	.0679	-.0300	-.286	.0573	-.0573	-.200	-.020	-.10 -.10 -.10 -1.10 -1.10	-.10 -.10 -.10 -.10 .60	0 1.0 1.0 0 0	0 0 -.70 0 0	6.35 6.96 6.56 ----- 1.06	1.42 .80 1.30 ---- .26	3.83 5.25 25.12 8.25 7.94 17.03	-1.49 1.15 .42 .76 3.91 2.38

TABLE III.- Concluded

CALCULATION CONDITIONS AND RESULTS FOR 30° ANGLE OF ATTACK

$\left[C_L = 1.0;\ \eta = 28.8^\circ;\ C_{Y_p} = 0;\ C_{Y_r} = 0;\ C_{Y_{\dot\beta}} = 0;\ \gamma = 0^\circ\right]$

(b) h = 50,000 ft

Combination of C_{n_β} and C_{l_β}	Mass parameters				Aerodynamic derivatives									Results			
	μ	K_X^2	K_Z^2	K_{XZ}	C_{Y_β}	C_{n_β}	C_{l_β}	C_{n_p}	C_{l_p}	C_{n_r}	C_{l_r}	$C_{n_{\dot\beta}}$	$C_{l_{\dot\beta}}$	Aperiodic modes: $T_{1/2}$, sec	Aperiodic modes: $T_{1/2}$, sec	Oscillatory mode: P, sec	Oscillatory mode: $T_{1/2}$, sec
A	77.80	0.0300	0.0679	-0.0300	-0.286	-0.0573	0	-0.200	-0.020	-0.10	-0.10	0	0	-0.55	0.67	76.98	14.36
										-.10	-.10	1.0	0	-.84	.44	76.92	14.34
										-.10	-.10	1.0	-.70	-1.24	.35	73.79	12.04
										-1.10	-.10	0	0	-.82	.43	79.42	14.02
										-1.10	.60	0	0	-1.34 -2.79	.34 2.57	-----	-----
B	77.80	.0300	.0679	-.0300	-.286	-.0573	-.0573	-.200	-.020	-.10	-.10	0	0	∞	.98	10.03	-1.65
										-.10	-.10	1.0	0	∞	.62	10.60	-5.15
										-.10	-.10	1.0	-.70	∞	.47	11.84	29.19
										-1.10	-.10	0	0	2.60	.64	10.73	-2.76
										-1.10	.60	0	0	1.76	.46	11.56	-3.63
C	77.80	.0300	.0679	-.0300	-.286	.0573	0	-.200	-.020	-.10	-.10	0	0	-----	----	5.61 77.26	-6.00 13.75
										-.10	-.10	1.0	0	-----	----	5.92 77.20	1.86 13.73
										-.10	-.10	1.0	-.70	-----	----	6.34 81.99	.96 16.03
										-1.10	-.10	0	0	-----	----	6.05 74.34	1.85 14.15
										-1.10	.60	0	0	-4.17	2.17	6.80	1.00
D	77.80	.0300	.0679	-.0300	-.286	.0573	-.0573	-.200	-.020	-.10	-.10	0	0	16.25	3.32	3.91	-3.63
										-.10	-.10	1.0	0	16.48	3.08	4.07	2.39
										-.10	-.10	1.0	-.70	16.31	3.25	4.26	1.09
										-1.10	-.10	0	0	-----	----	4.16 30.80	2.37 5.28
										-1.10	.60	0	0	-----	----	4.43 74.46	1.09 5.30

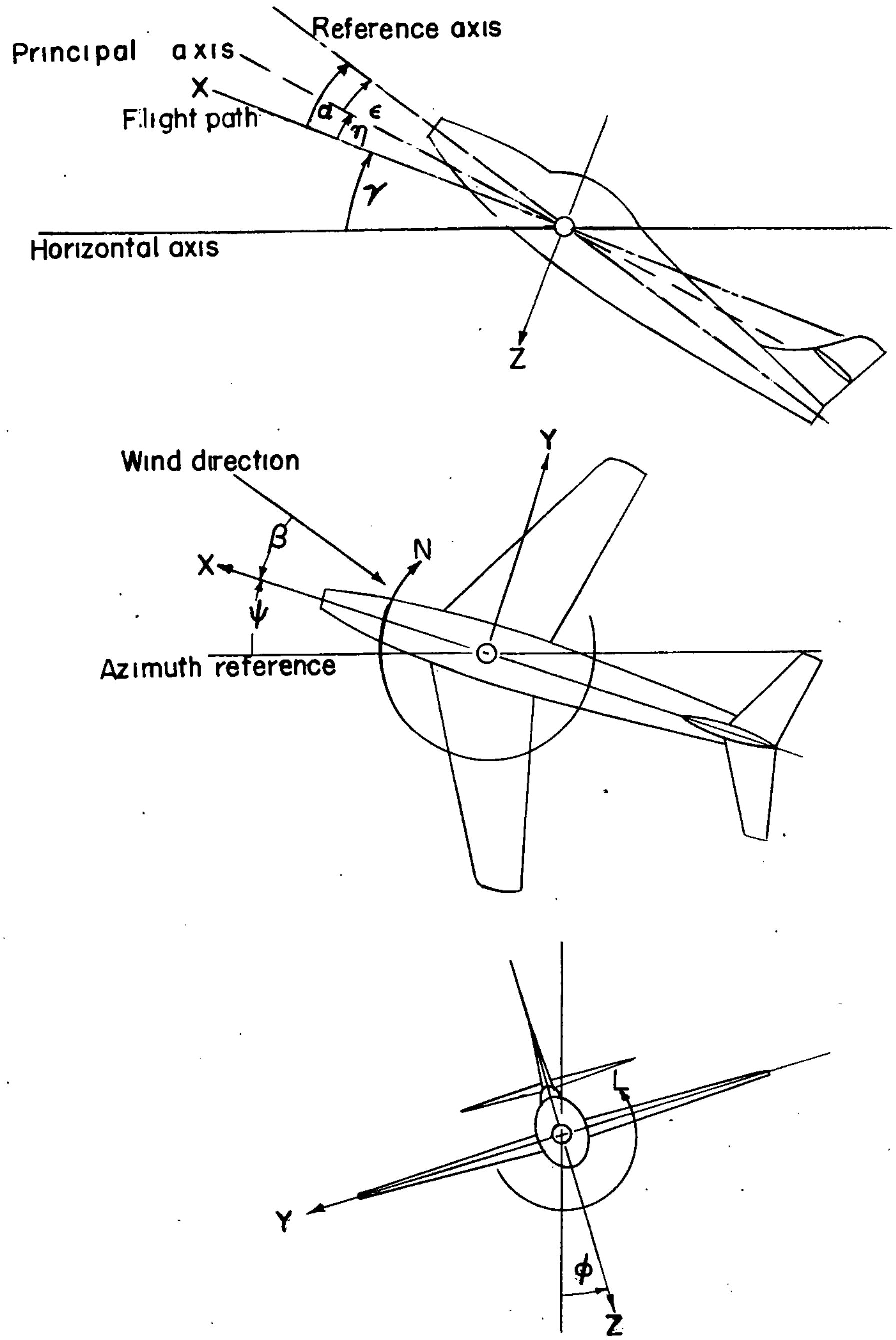


Figure 1.- The stability system of axes and angular relationships in flight. Arrows indicate positive directions of moments, forces, and angles. This system of axes is defined as an orthogonal system having the origin at the center of gravity and in which the Z-axis is in the plane of symmetry and perpendicular to the relative wind, the X-axis is in the plane of symmetry and perpendicular to the Z-axis, and the Y-axis is perpendicular to the plane of symmetry. At a constant angle of attack, these axes are fixed in the airplane.

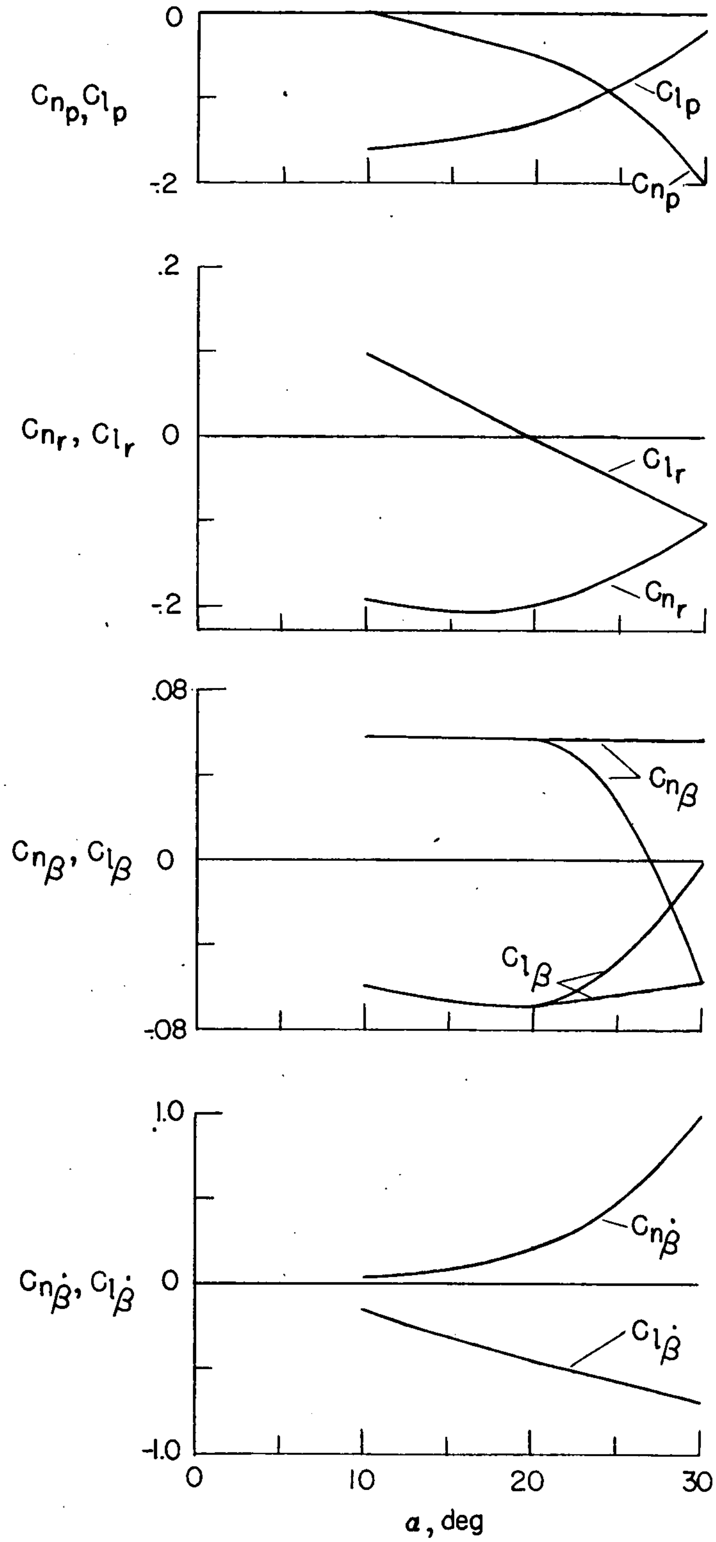
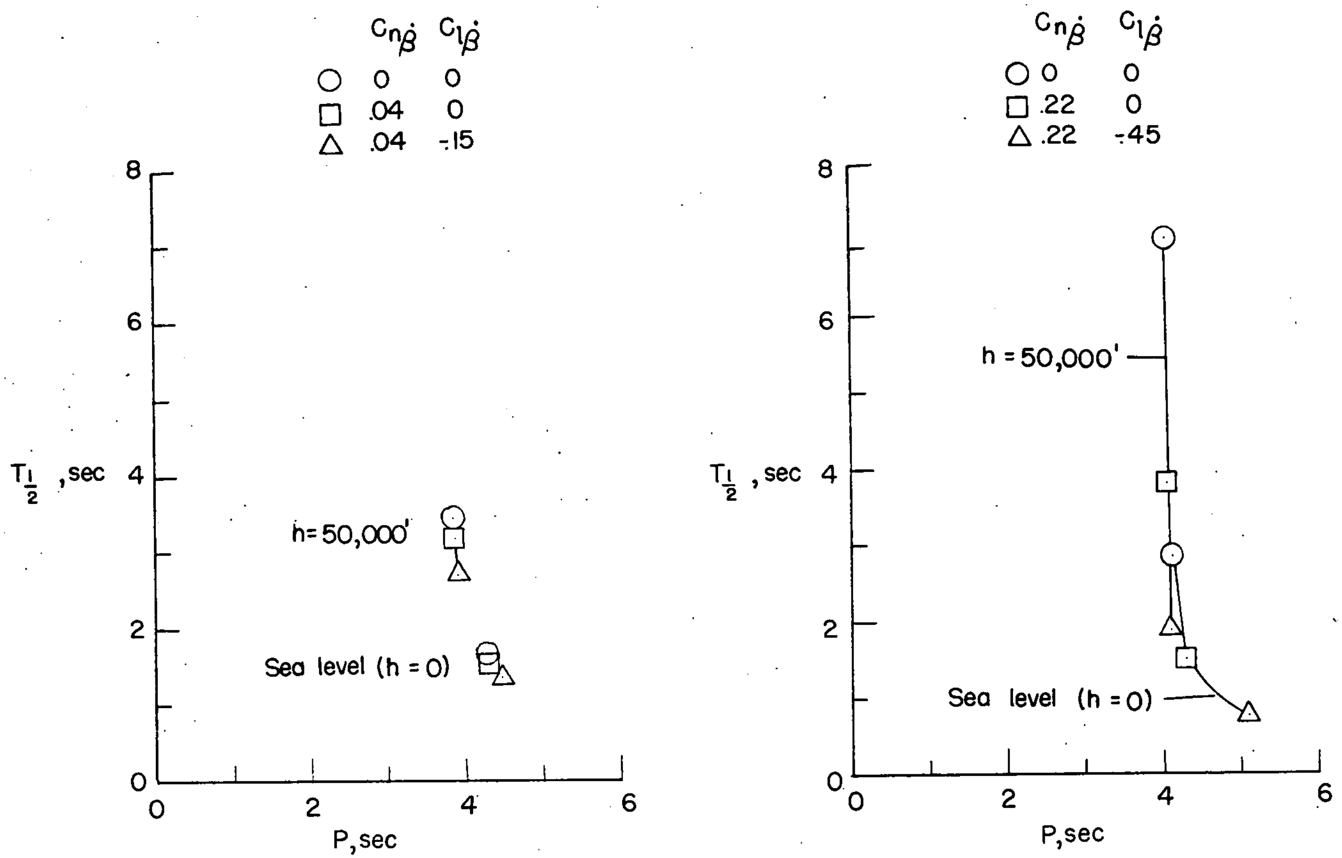


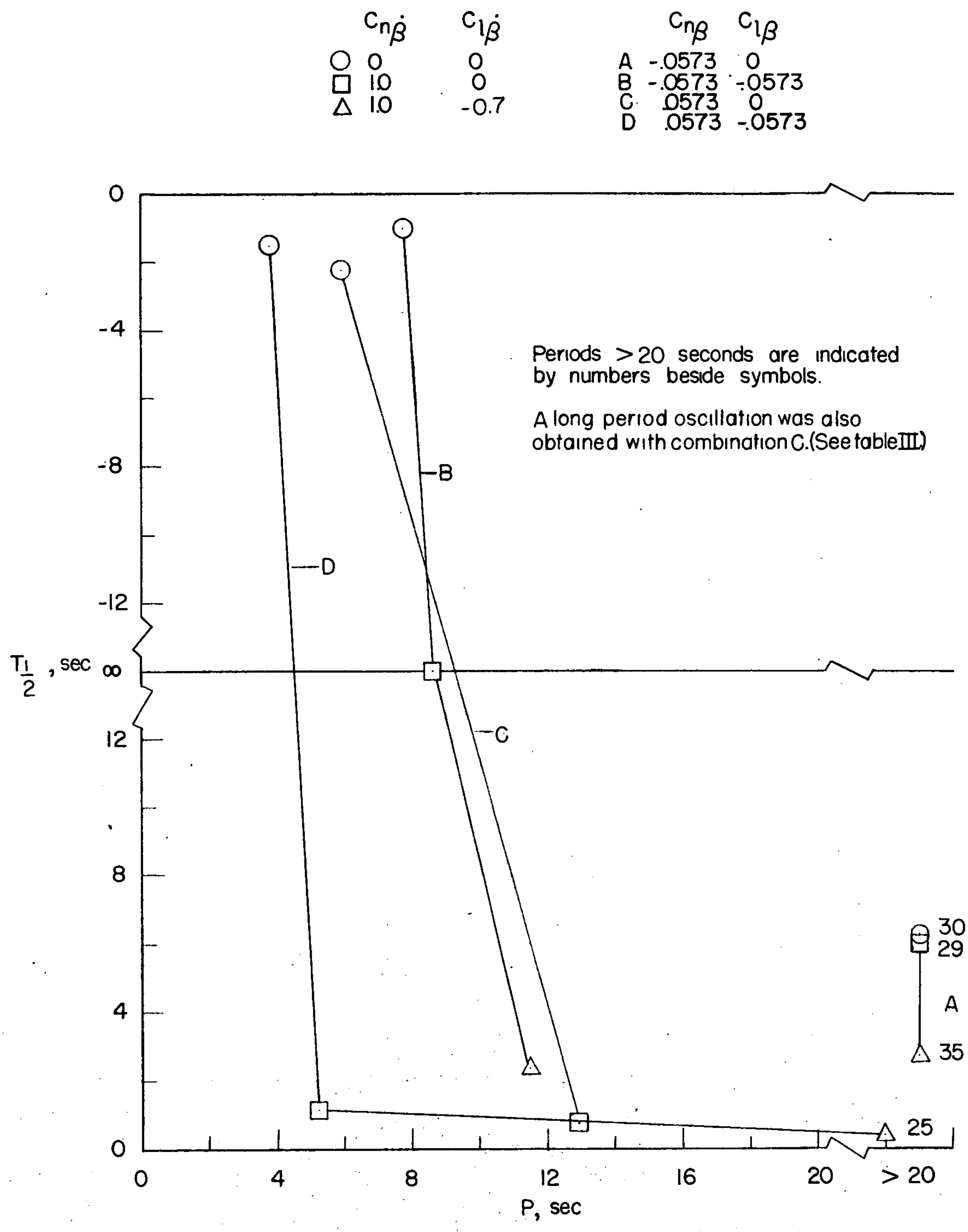
Figure 2.- Assumed variation of stability derivatives with angle of attack.



(a) $\alpha = 10^{\circ}$.

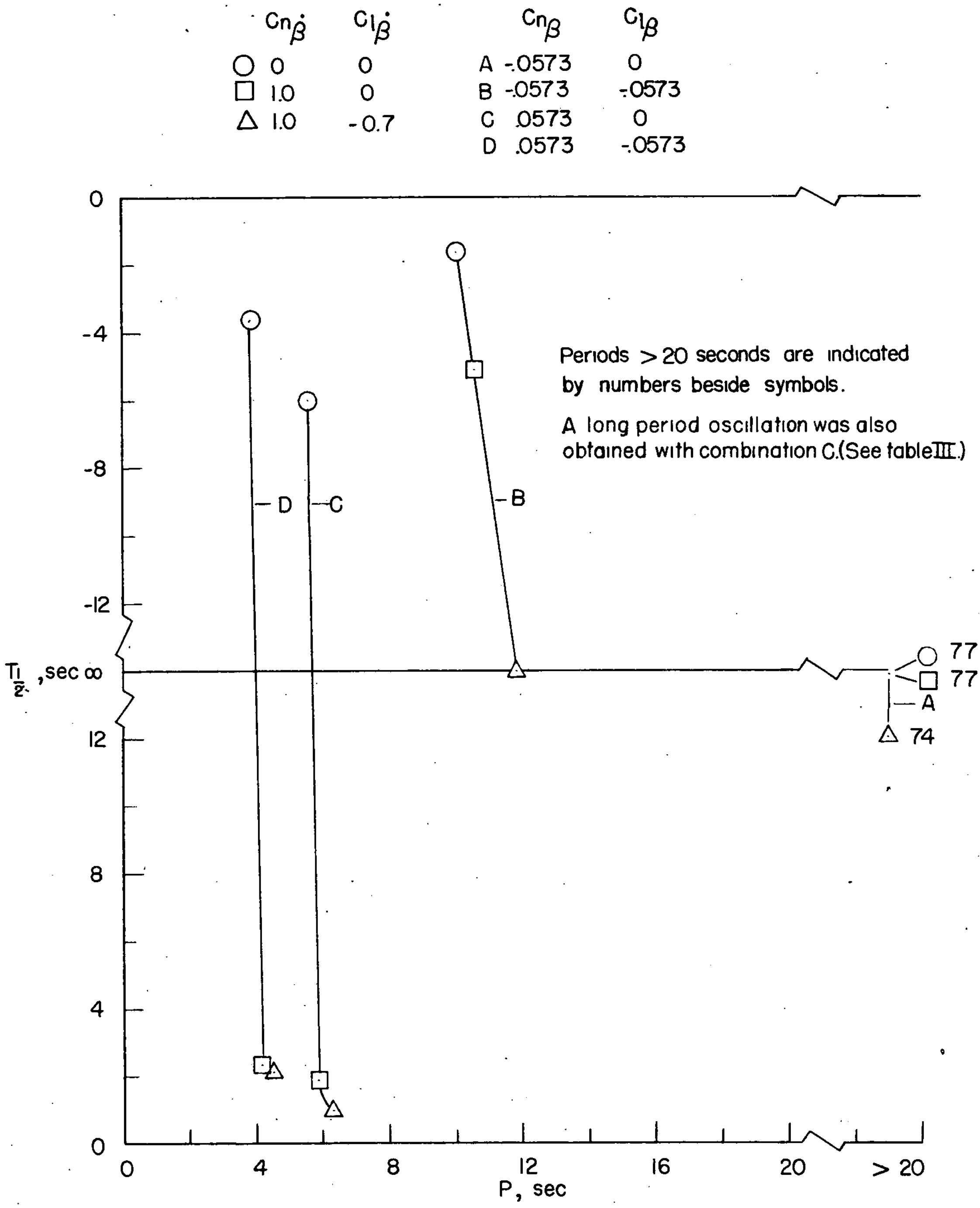
(b) $\alpha = 20^{\circ}$.

Figure 3.- Effect of $C_{n_{\dot{\beta}}}$ and $C_{l_{\dot{\beta}}}$ on period and damping at angles of attack of 10° and 20°.



(a) Sea level (h = 0).

Figure 4.- Effect of $C_{n_{\dot\beta}}$ and $C_{l_{\dot\beta}}$ on period and damping at 30° angle of attack for various combinations of C_{n_β} and C_{l_β}.



(b) h = 50,000 feet.

Figure 4.- Concluded.

[redacted]

[redacted]