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RESEARCH MEMORANDUM

INVESTIGATION OF TWO-STAGE AIR-COOLED TURBINE SUITABLE FOR FLIGHT AT MACH NUMBER OF 2.5

I - VELOCITY-DIAGRAM STUDY

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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SUMMARY

For a set of engine conditions suitable for flight at a Mach number of 2.5, a two-stage air-cooled turbine configuration and its velocity diagrams are evolved for use in the design of blade shapes. The method used to determine the velocity diagrams is one in which the turbine frontal area and the rotor hub inlet and outlet velocities are minimized by varying the hub-tip ratio, the turbine- to compressor-tip-diameter ratio, and the work split between the first and second stages.

The final two-stage turbine design has a turbine frontal area that is only 15 percent larger than that of the compressor and has both rotors operating at hub inlet and outlet relative critical velocity ratios of about 0.75. A comparison of the two-stage turbine with a one-stage turbine with and without a downstream stator showed that the frontal areas of the latter two configurations were considerably larger and their velocity levels were more critical. An investigation of a three-stage turbine showed that it was more conservative than desired, and it was not an improvement over the two-stage design considering the aerodynamic limits specified and the difference in the turbine-cooling requirements.

INTRODUCTION

At the NACA Lewis laboratory an investigation is being made of the problems that arise in engines suitable for high flight speeds. The basic problem is to obtain maximum thrust for minimum engine weight and drag with an engine which is suitable for takeoff and design flight speeds and which has a low specific fuel consumption at cruising conditions. One such powerplant configuration that has been analyzed is the subject axial-flow turbojet engine which is suitable for flight at a Mach number of 2.5.

In order to obtain a high thrust capacity in the engine, the design turbine-inlet total temperature was specified to be 2500° R, which seemed to be a practical limit for which excessive amounts of air would not be required for cooling the turbine blades. Use of such a high turbine-inlet temperature with the blade materials presently available requires that some effective means of cooling the turbine blades be employed. The method of turbine cooling that seems most promising is internal blade cooling with air ducted from the compressor discharge. In order to simplify this type of cooling configuration, the cooling air is exhausted from the blades into the mainstream of the turbine. Because the effect of this cooling air on the turbine aerodynamic performance is not known, a cold-air investigation of the turbine is being made.

This report presents a velocity-diagram study which was made in order to reduce the turbine frontal area without penalizing turbine performance. On the basis of the results of this analysis, final turbine hub and tip configurations and blade-section velocity diagrams are evolved for use in the design of blade shapes.

SYMBOLS

The following symbols are used in this report:

A	frontal area, sq ft
c_p	specific heat at constant pressure, Btu/(lb)(°F)
d	tip diameter, in.
e	parameter used in rating compressor and turbine, $\frac{wU_t^2}{A\delta\sqrt{\Theta_{cr,C}}}$
f	fuel-air ratio
g	acceleration due to gravity, 32.17 ft/sec^2
$\Delta h'$	specific work output, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
K	constant in eq. (4) representing $288g/\Gamma\psi U_{t,C}^2$, sq in./lb
p	absolute pressure, lb/sq ft
R	gas constant, ft/°R

r radius, ft

T absolute temperature, $^{\circ}R$

U rotor-blade speed, ft/sec

V absolute gas velocity, ft/sec

W relative gas velocity, ft/sec

w weight flow, lb/sec

Γ density of blade metal, lb/cu ft

γ ratio of specific heats

δ ratio of compressor-inlet-air total pressure to NACA standard sea-level pressure of 2116.22 lb/sq ft, $p'_{C,i}/p_{sl}$

η adiabatic efficiency

Θ_{cr} squared ratio of critical velocity at inlet of compressor or turbine (as indicated by subscripts) to critical velocity at NACA standard sea-level conditions, $(V_{cr}/V_{cr,sl})^2$

ρ gas density, lb/cu ft

τ rotor-blade hub stress, lb/sq in.

φ ratio of turbine cooling air to compressor-inlet weight flow

ψ ratio of tapered-blade to untapered-blade hub stress

Subscripts:

B burner

C compressor

cr conditions at Mach number of 1.0

h hub

i inlet

m mean

o outlet

sl	NACA standard sea-level conditions
T	turbine
t	tip
u	tangential
x	axial
0	station at turbine entrance
1	station at outlet of first-stage stator
2	station at outlet of first-stage rotor
3	station at outlet of second-stage stator
4	station at outlet of second-stage rotor

Superscript:

'	absolute total state

REQUIREMENTS, LIMITATIONS, AND ASSUMPTIONS

The scope of this investigation is limited to an investigation of the aerodynamic performance of the turbine; therefore, only engine requirements used in the turbine design, the limitations on the turbine aerodynamic design, and the turbine design assumptions are discussed subsequently.

Engine Design Requirements

The design requirements of the subject engine are based on the results of a detailed analysis. Because the method of determining the engine design requirements is beyond the scope of this investigation and was influenced by the characteristics desired in each of the major engine components, a complete history of the engine development is not given. However, a brief discussion of the engine requirements that directly affect the turbine is given subsequently.

On the basis of past experience with compressors, it was felt that a compressor equivalent weight flow per unit frontal area of 37.5 pounds per second per square foot would not penalize the compressor performance and would result in a low drag per unit frontal area. The value of parameter e (see ref. 1), which is the product of the equivalent weight flow per unit frontal area and the square of the equivalent blade tip speed, that was selected for the compressor was 44.1 million pounds per second^3, corresponding to a compressor-inlet relative Mach number of 1.2 and a compressor hub-tip ratio of 0.4 (see fig. 3(a) of ref. 1). The selection of the preceding two values then determines a value of the compressor design equivalent blade tip speed of 1085 feet per second.

The maximum mechanical blade tip speed for a turbine with a frontal area equal to that of the compressor and an allowable blade hub stress of 40,000 pounds per square inch was calculated to be 1146 feet per second, which corresponds to the compressor design equivalent blade tip speed of 1085 feet per second at a flight Mach number of 1.53 and an altitude of 35,330 feet. The engine was thus designed with the assumption that from sea-level takeoff to a flight Mach number of 1.53 at 35,330 feet, the compressor would operate at a design equivalent blade tip speed of 1085 feet per second and design equivalent weight flow per unit frontal area of 37.5 pounds per second per square foot. For higher flight Mach numbers, it was assumed that the compressor would operate at a constant mechanical blade tip speed of 1146 feet per second. By assuming that the turbine will operate at a constant inlet total temperature of 2500° R, then the constant compressor blade tip speed of 1146 feet per second determines a constant turbine equivalent blade tip speed for a given turbine tip diameter. From a preliminary analysis, it was determined that the turbine would operate at almost constant specific work over the range of flight Mach numbers from 1.53 to 2.5; therefore, the turbine design was based on conditions at a flight Mach number of 2.0 and a compressor blade tip speed of 1146 feet per second.

For the compressor selected, the compressor pressure ratio at the turbine design flight Mach number of 2.0 is 5.86 at about 90 percent of compressor design equivalent blade tip speed. At a flight Mach number of 2.5, this same compressor would develop a pressure ratio of about 4.5 at about 80 percent of the compressor design equivalent blade tip speed. Thus, at a flight Mach number of 2.5, the compressor would be well out of the stall region; and it should give good performance up to, and even above, this Mach number.

On the basis of the assumptions of reference 1, it is possible then to predict some of the characteristics of a turbine designed to operate at the preceding specified conditions. For the engine temperature ratio of about 3.5 at a flight Mach number of 2.0 and a compressor pressure ratio of 5.86, figure 4(d) of reference 1 indicates that a conservative one-spool two-stage turbine with an allowable blade hub stress of 40,000 pounds per square inch would have a turbine equivalent weight flow per unit turbine frontal area of 30 pounds per second per square foot. Since this value is almost equal to 32.8 pounds per second per square foot for the compressor at turbine design operating conditions, a two-stage turbine could be used which is slightly larger than the compressor with the turbine operating at turbine design hub relative Mach numbers between 0.6 and 0.8.

The design requirements specified for the design of the turbine at a flight Mach number of 2.0 at 35,330 feet are:

Turbine-inlet total temperature, T_0', $^{\circ}R$	2500
Compressor-outlet to -inlet total-pressure ratio, $(p_o'/p_i')_C$ . . .	5.86
Burner-outlet to -inlet total-pressure ratio, $(p_o'/p_i')_B$.	0.95
Compressor adiabatic efficiency, η_C	0.85
Compressor equivalent weight flow, $\frac{w_C \sqrt{\Theta_{cr,C}}}{A_C \delta}$, lb/(sec)(sq ft) . .	32.80
Ratio of turbine cooling air to compressor-inlet weight flow, φ .	0.09
Fuel-air ratio, f	0.0204
Compressor tip diameter, d_C, in.	28.00
Compressor equivalent blade tip speed, $U_{t,C}/\sqrt{\Theta_{cr,C}}$, ft/sec . . .	981.5

In order to operate a turbine at the high inlet total temperature of 2500° R, presently known turbine blade materials must be cooled. In order that the required amount of coolant would be reduced, the maximum blade hub stress τ was specified to be 40,000 pounds per square inch. Throughout the subject report, this limit on τ will be assumed. For such a high value of τ, the turbine blades would have to be effectively cooled by some means such as internal air cooling. This type of cooling configuration is considerably simplified by allowing the cooling air for the blade rows downstream of the first-stage stator to flow out of the blades and mix with the mainstream of the turbine gas flow; therefore, this type of cooling-air configuration was adopted.

On the basis of preliminary calculations using the cooling-air configuration mentioned, the following estimates are given for the cooling-air requirements for the two-stage turbine discussed herein in terms of the ratio of turbine cooling air to compressor-inlet weight flow φ:

First-stage stator, φ_1	0.025 (exhausted overboard)
First-stage rotor, φ_2	0.020
Second-stage stator, φ_3	0.025
Second-stage rotor, φ_4.	0.020

Any small change in the cooling-air requirements for any blade row would have a small effect on the velocity diagrams; therefore, the cooling-air assumptions are satisfactory for the purpose of the subject investigation.

Aerodynamic Limitations

While it was desired that the turbine design should not be too conservative for the flight conditions specified, it was felt that certain limitations should be placed on the flow conditions within the turbine to ensure a good aerodynamic design. These limitations were based on values considered to be liberal but yet conservative enough to avoid imposing unnecessary penalties on the aerodynamic performance.

In order to avoid the possibility of the turbine operating in the limiting-loading range at design speed and work, the turbine-exit axial critical velocity ratio $(V_x/V_{cr})_4$ was limited to a maximum of 0.6, as compared with the limiting-loading condition of $(V_x/V_{cr})_4$ equal to approximately 0.7. The value of 0.6 was considered to be conservative enough to accommodate the large amount of cooling air required even though the effect of the cooling air is not known.

In order to avoid any unnecessary surface decelerations on the blade surface, it was prescribed that the hub inlet relative velocity should be approximately equal to that at the hub outlet. This stipulation on the hub velocities then resulted in reaction conditions at other radii for the free-vortex design used herein.

Because the size of the turbine is of great importance owing to drag at a flight Mach number of 2.5, a slight amount of turbine exit whirl was employed to provide a larger change in whirl through the second-stage rotor, which permits a reduction in turbine tip diameter for the impulse conditions specified at the rotor hub. This exit whirl was specified by a value of the tangential critical velocity ratio at the turbine-exit mean radius $(V_u/V_{cr})_{m,4}$ which was assumed to be -0.1, the negative sign indicating the direction opposite to that of the wheel rotation.

In the analysis, the work of each stage was adjusted until the velocity level in the first-stage rotor was approximately equal to that in the second-stage rotor in order to avoid unnecessarily high velocities in either stage and the additional losses associated with them.

During the analysis, the hub-tip ratio was not allowed to vary too greatly from the inlet to the outlet of a blade row in order to avoid difficult three-dimensional design problems.

Design Assumptions

In order to design a two-stage turbine, it is necessary to assume certain efficiencies that might be expected for each stage. Even though the effect of the cooling air on the mainstream flow through the turbine is not known, a fairly good adiabatic efficiency of 0.85 was assumed from the turbine inlet to the outlet of the first stage and from the inlet to the outlet of the turbine. This assumption results in a slightly lower stage efficiency for the second stage.

In order to calculate a total pressure at the outlet of each stator, the stator outlet total pressure is assumed equal to 0.97 of the stator inlet total pressure for each stage.

For the turbine-inlet total temperature of 2500° R and the assumed fuel-air ratio, the values assumed for the ratio of specific heats γ and the specific heat at constant pressure c_p are 1.3 and 0.2974, respectively.

The adopted cooling-air configuration may have many effects on the turbine aerodynamic performance; however, in the design it was assumed that the cooling air entering within a given blade row would only displace the mainstream and would not do any work within the blade row. Downstream of the exit of the blade row where the cooling air entered, the cooling air was assumed to have the same properties as the mainstream and was assumed to flow in the direction of the mainstream. In either a cold-air test of the turbine or in a test of an actual engine at design operating conditions, the total temperature and total pressure of the cooling air would probably not be too far different from those of the mainstream of the turbine. Therefore, the preceding assumption regarding the cooling air is considered to be reasonable.

ANALYSIS OF TURBINE VELOCITY DIAGRAMS

During the analysis, the requirements, limitations, and assumptions given in the previous section will be adhered to in determining the best possible configuration. In the preliminary steps of the analysis, a constant turbine tip diameter is assumed. This assumption affects the turbine size and velocity level to only a small degree; but it, of course, penalizes the hub contour by necessitating large changes in the hub-tip ratio from station to station. After the preliminary analysis of the turbine is completed, subsequent variations of the tip diameter from station to station are made to alleviate any hub or tip contour problem.

Preliminary Two-Stage Turbine Design

The first step in the investigation is to assume a work split between the two stages. As the work of the first stage is increased, the velocity level in the first-stage rotor increases and the reaction of the second-stage stator decreases. Therefore, calculations were made to determine the best configuration for each value of work split chosen, and then a work split was selected which resulted in approximately equal hub velocities in the first- and second-stage rotors and satisfactory reaction across the second-stage stator. The ratio of first- to second-stage work that seemed most satisfactory was 60:40, and a discussion of the determination of the best configuration for this work split is given subsequently. The method used for the 60:40 work split is similar to that used for other work splits which were compared; therefore, only the calculations based on the 60:40 work split are outlined.

It should be noted throughout the report that the rotor-blade hub stress is a maximum at the turbine exit, where the annular area is a maximum. Therefore, once the turbine-exit geometry is determined, no further consideration is given to the problem of rotor-blade hub stress at other stations in the turbine. The equation for the rotor-blade hub stress τ given by equation (8) of reference 2 is

$$\tau = \frac{\Gamma\psi U_{t,T}^2}{288g}\left[1 - \left(\frac{r_h}{r_t}\right)^2\right] \tag{1}$$

In this equation the following quantities are assumed to be constant:

Density of blade metal, Γ, lb/cu ft 494
Ratio of tapered-blade to untapered-blade hub stress, ψ 0.738

Because the compressor blade tip speed $U_{t,C}$ is specified to be 1146 feet per second, the turbine blade tip speed can be related to $U_{t,C}$ by the turbine- to compressor-tip-diameter ratio as follows:

$$U_{t,T} = U_{t,C}\left(\frac{d_T}{d_C}\right) \tag{2}$$

Equation (1) can then be expressed as

$$\tau = \frac{\Gamma\psi}{288g}\left[U_{t,C}^2\left(\frac{d_T}{d_C}\right)^2\right]\left[1 - \left(\frac{r_h}{r_t}\right)^2\right] \tag{3}$$

In this analysis, the maximum allowable rotor-blade hub stress τ was previously specified to be 40,000 pounds per square inch. Therefore, equation (3) can be rearranged in such a way that the minimum hub-tip ratio can be determined for given values of turbine- to -compressor-tip-diameter ratio d_T/d_C and rotor-blade hub stress τ where all other factors in equation (3) are considered constant and are presented by the term K as follows:

$$\frac{r_h}{r_t} = \left[1 - \frac{K\tau}{\left(\frac{d_T}{d_C}\right)^2}\right]^{1/2} \tag{4}$$

From equation (4) it can be seen that for each value of d_T/d_C there is a corresponding value of the hub-tip ratio for a given τ.

Next, consider the weight flow parameter at the mean radius which for free-vortex velocity diagrams is closely approximated by

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_m = \frac{w_T}{\rho' V_{cr} A_T\left[1 - \left(\frac{r_h}{r_t}\right)^2\right]} \tag{5}$$

where

$$\rho' = \frac{p'}{RT'}$$

$$V_{cr} = \sqrt{\frac{2\gamma}{\gamma + 1} gRT'}$$

Because the compressor frontal area A_C is considered constant, and known, it is convenient to express the turbine frontal area as

$$A_T = A_C\left(\frac{d_T}{d_C}\right)^2$$

Then equation (5) can be rewritten as

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_m = \frac{w_T/A_C}{\rho' V_{cr}\left(\frac{d_T}{d_C}\right)^2\left[1 - \left(\frac{r_h}{r_t}\right)^2\right]} \tag{6}$$

Now, by substituting equation (4) into equation (6), the weight flow parameter is given by

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_m = \frac{w_T/A_C}{\rho' V_{cr} K\tau} \tag{7}$$

Therefore, from equation (7) it can be seen that for a constant rotor-blade hub stress τ the value of the weight flow parameter is independent of changes in the turbine- to compressor-tip-diameter ratio.

For constant values of the weight flow parameter and the rotor exit whirl at the mean section, an increase in the turbine tip diameter increases the rotor-blade speed U, which results in an increase in the relative velocities at the rotor exit. Thus, the value of the rotor hub outlet relative velocity $W_{h,4}$ increases with diameter, as shown by figure 1(a).

In order to determine the effect of changes in the turbine tip diameter and rotor inlet hub-tip ratio on the second-stage rotor inlet velocities, first consider the Euler work equation

$$\Delta h'_{3,4} = \frac{(UV_u)_{m,3} - (UV_u)_{m,4}}{gJ} \tag{8}$$

Rearranging the equation gives

$$V_{u,m,3} = \frac{gJ\,\Delta h'_{3,4} + (UV_u)_{m,4}}{U_{m,3}} \tag{8a}$$

Now, with $\Delta h'_{3,4}$ and $V_{u,m,4}$ both specified as constant, and $U_{m,4}$ given by a combination of equations (2) and (4) to be equal to

$$U_{m,4} = \frac{1}{2}\left\{1 + \left[1 - \frac{K\tau}{\left(\frac{d_T}{d_C}\right)^2}\right]^{1/2}\right\}U_{t,C}\left(\frac{d_T}{d_C}\right)$$

then for each value of d_T/d_C and $(r_h/r_t)_3$ a value of $V_{u,m,3}$ can be obtained from equation (8a). Using values of second-stage rotor inlet weight flow, total temperature, and total pressure calculated on the basis of assumptions previously discussed, a weight flow parameter at the second-stage rotor inlet mean radius $(\rho V_x/\rho' V_{cr})_{m,3}$ can be calculated by equation (6) for selected values of hub-tip ratio r_h/r_t and the turbine- to compressor-tip-diameter ratio d_T/d_C. For each set of values of $V_{u,m,3}$ and $(\rho V_x/\rho' V_{cr})_{m,3}$, the rotor inlet velocity diagram at any radius can be obtained assuming a free-vortex design.

The effects of increasing the value of d_T/d_C while holding r_h/r_t constant are a decrease in the rotor inlet weight flow parameter

$(\rho V_x/\rho' V_{cr})_{m,3}$ (see eq. (6)) and a decrease in the required change in whirl with increase in the mean wheel speed U_m (see eq. (8)). The combination of the decrease in the weight flow parameter and whirl at the rotor inlet results in a decrease in the rotor inlet axial velocity $V_{x,3}$ and the rotor hub inlet relative velocity $W_{h,3}$, as shown in figure 2(a).

The effects of increasing the value of r_h/r_t while holding d_T/d_C constant are an increase in the weight flow parameter and the axial velocity $V_{x,3}$ and a decrease in the required change in whirl with increase in the mean wheel speed U_m (see eq. (8)). The combination of these two effects results in the change in the rotor hub inlet relative velocity $W_{h,3}$ shown in figure 2(b), which is based on a d_T/d_C of 1.00. It should be noted in both figures 2(b) and (c) that $W_{h,3}$ reaches a minimum as r_h/r_t varies; thus an attempt will be made to utilize a hub-tip ratio that corresponds to a value of $(W/W_{cr})_{h,3}$ that is near the minimum.

In order to determine a turbine- to compressor-tip-diameter ratio and second-stage rotor inlet and outlet hub-tip ratios that will result in a turbine of minimum size and good hub contouring, the curves of figures 1(b) and 2(c) are replotted in figure 3. It can be seen that for a d_T/d_C of 1.00 the values of $(W/W_{cr})_{h,3}$ would exceed $(W/W_{cr})_{h,4}$ by about 0.1 in any case, and thus large surface diffusions would result and the design conditions could not be met. For a value of d_T/d_C of 1.10, the values of $(W/W_{cr})_{h,3}$ are less than for $(W/W_{cr})_{h,4}$; therefore, little surface diffusion would be required, but the turbine tip diameter is larger than that required by the impulse condition limit specified for the rotor hub. In view of this, an intermediate value of d_T/d_C of 1.04 at both stations 3 and 4 was chosen which meets the conditions of equal hub inlet and outlet relative critical velocity ratios, reasonable change in the hub-tip ratio, and a minimum turbine size for these requirements. It should be noted that the value chosen for d_T/d_C of 1.04 results in a turbine frontal area which is about 8 percent greater than that of the compressor.

With a stator between the two rotors, the first-stage rotor exit whirl is not limited as it was for the second stage. Therefore, a selection of a first-stage rotor exit whirl must be made by an iteration process based on finding rotor hub inlet and outlet relative velocities that are equal at values of r_h/r_t that give a satisfactory hub contour. Instead of assuming values of rotor exit whirl, however, a range of values of the specific work output contributed by the first-stage rotor exit whirl was selected because the specific work output is independent of changes in the hub-tip ratio. The effect of thus varying the first-stage rotor exit whirl on the rotor hub inlet velocity diagram is indicated by figures 4(a) and (c), which show that the major effect is a change in the inlet whirl velocity with a corresponding change in the hub inlet relative velocity. Likewise, the effect of varying the rotor hub exit whirl on the hub outlet velocity diagram (figs. 4(b) and (c)) is, of course, a change in hub outlet relative velocity primarily as a result of the change in exit whirl.

The values of first-stage rotor exit whirl and r_h/r_t that were selected are indicated in figure 4(c). These values were chosen at a relative critical velocity ratio of 0.740, which is approximately the same as that chosen for the second-stage rotor (see fig. 3). This equality of the hub relative velocities in the two rotors indicates that the selected work split and turbine- to compressor-tip-diameter ratio are satisfactory. A side view of the two-stage turbine for the hub-tip ratios selected is shown in figure 5.

Final Two-Stage Turbine Design

The preceding results show that the two-stage turbine is satisfactory in all respects except for the three-dimensional channel design problems resulting from using a tip contour of constant radius and a steep hub contour. Therefore, an investigation was made as to the possibility of relieving the steepness at the hub and at the same time designing for a constant mean radius throughout the turbine.

In order for the mean radius to be constant, the divergence at the hub must be equal to that at the tip. For this type of wall divergence, the tip diameter at the turbine exit had to be increased slightly in order that the rotor hub velocities would be about the same as those previously obtained for the two-stage turbine with a constant tip diameter. For the first trial the turbine- to compressor-tip-diameter ratio d_T/d_C at the turbine exit was increased from 1.04 to 1.071 (corresponding to a d_T of 30 in.). This increase in d_T/d_C resulted in a change in the minimum hub-tip ratio from 0.530 to 0.570 for the maximum value of rotor-blade hub stress τ specified (see eq. (4)). The curve in figure 1(b) representing the variation in rotor-blade hub outlet relative critical velocity ratio $(W/W_{cr})_{h,4}$ for a constant value of τ but with changes in the tip diameter indicates that $(W/W_{cr})_{h,4}$ increased slightly from 0.733 to 0.751 for the corresponding changes in hub-tip ratio.

For impulse conditions at the rotor hub and the specified turbine exit-whirl conditions, the work capacity of the second-stage rotor increases with an increase in the turbine-exit tip diameter because of the resulting increase in the mean blade speed. For this reason, the work output of the second stage was increased from 40 to 42 percent of the total turbine work output in order that the size of the first stage could be smaller than that of the second stage with approximately equal hub relative velocities in both stages. If the work split between stage had not been altered, the velocity level in the first stage would have exceeded that in the second stage for the desired type of divergent hub and tip contour previously discussed.

For the given values of second-stage work output and rotor outlet hub-tip ratio, rotor hub inlet relative velocities were calculated for a range of turbine- to compressor-tip-diameter ratios and hub-tip ratios at the rotor inlet. The hub-tip ratio for a particular tip-diameter ratio was calculated on the basis of a constant mean radius equal to that at the turbine exit. In figure 6 the value of d_T/d_C at the rotor hub inlet corresponding to the rotor hub outlet relative critical velocity ratio of 0.751 is between 1.040 and 1.060. Also, it can be seen that a wide range of hub-tip ratios in this region of d_T/d_C correspond to a very small change in the rotor inlet relative velocity for the conditions specified. Therefore, the tip diameter at the second-stage rotor inlet can be varied to some extent and may be chosen on the basis of the velocity level desired in the first stage. For this reason, calculations were made for the first stage over a range of tip diameters at the first-stage rotor inlet and outlet which correspond to a constant divergence from station 1 to station 4 with equally spaced intermediate stations (see fig. 7). By letting the tip diameter at station 1 vary over a range from 27 inches to 30 inches, it was found that a value of 28 inches resulted in rotor hub relative critical velocity ratios of 0.748, 0.747, 0.752, and 0.751 at stations 1, 2, 3, and 4, respectively. Thus, noting that the velocity levels in the two stages are about equal and that hub and tip divergence is constant and not too great for the usual blade channel design procedures, the final turbine was specified to vary from a tip diameter of 28 inches at station 1 to 30 inches at station 4 with a corresponding change in the hub, as shown in figure 7. The velocity diagrams at the hub, mean, and tip for these values are shown in figure 8.

Discussion of Final Design Velocity Diagrams

In the previous sections considerable attention is paid to the velocity levels at the rotor hub and also the contours at the hub and tip, but there are other items which have to be considered in the final design. For instance, it was necessary to provide enough camber in each blade row so that there would be sufficient internal area at each horizontal cross section of the blade for the proposed internal blade cooling arrangement. For this reason, the most critical section for the subject turbine was the tip of the second rotor, as indicated by the inlet and outlet relative flow angles for that section shown in figure 8. If, in laying out the blades, the cross-sectional area at the second-stage rotor tip had been found to be too small to satisfy the cooling requirements, the blade profile would have had to have been changed. In order to obtain a satisfactory profile, it is possible that the velocity diagrams would also require changing. However, a preliminary layout showed that sufficient area at the tip would be possible without deviating from present channel design procedures.

Another result which was examined during the velocity-diagram study was the small amount of reaction at the tip of the second-stage stator, as shown in figure 8. Here the critical velocity ratio at the stator inlet $(V/V_{cr})_{t,2}$ is 0.447 and that at the stator outlet $(V/V_{cr})_{t,3}$ is 0.724. This represents a change in the static- to total-pressure ratio from 0.892 to 0.736, which indicates low reaction. Just how low the reaction in the second-stage stator can be without substantially affecting the turbine performance is subject to speculation; however, it was felt that the low reaction indicated was not unsatisfactory for the second-stage stator.

COMPARISON OF TWO-STAGE TURBINE WITH OTHER CONFIGURATIONS

Although the final two-stage turbine configuration meets all the requirements specified herein, it is interesting to compare the two-stage turbine with a one-stage turbine with and without downstream stator blades and with a three-stage turbine that would satisfy these same requirements except for size and velocity level.

A velocity-diagram study similar to that of the two-stage turbine was made of a one-stage turbine without a downstream stator, and the results showed that its turbine frontal area would be 82 percent greater than that of the compressor. A one-stage turbine would also have rotor hub inlet and outlet relative Mach numbers of about 0.87 as compared with about 0.73 for the two-stage turbine. Some of the good features of a one-stage turbine would be a possible reduction in cooling-air requirements and simplicity of design.

The next configuration investigated was a one-stage turbine with a downstream stator which would require a slightly more complex blade cooling arrangement than that of a one-stage turbine without this additional stator. The addition of a downstream stator permits an increase in the rotor exit whirl and a reduction in the turbine tip diameter to that value corresponding to impulse conditions at the rotor hub. In considering an increase in the rotor exit whirl, it should be noted that the weight flow parameter is limited by the rotor-blade hub stress τ (see eq. (7)); therefore, for a given stress level the weight flow parameter will remain constant. Thus, an increase in rotor exit whirl must be accompanied by an increase in the rotor outlet axial velocity. For example, a change in the rotor outlet tangential critical velocity ratio at the mean section from -0.1 to -0.34 for the subject turbine design corresponds to an increase in the rotor outlet axial critical velocity ratio from 0.58 to 0.62. This increase in the axial velocity component means that such a one-stage turbine with a downstream stator would operate closer to the limiting-loading condition than either a one-stage or the two-stage turbines previously discussed, and this axial velocity component of 0.62 would exceed the limit of 0.6 previously specified. However, counterbalancing the criticalness of the aerodynamic design is a reduction in the turbine- to compressor-frontal-area ratio from 1.82 for a one-stage turbine to 1.42 for the example given for a one-stage turbine with a downstream stator. Even with this reduction in frontal area, the size of a one-stage turbine with a downstream stator was considered to be too large and its velocities too critical; therefore, this configuration was eliminated as a solution to the design problem.

The analysis of a three-stage turbine was limited to a design which had a tip diameter equal to that of the compressor, because it was felt that there would be no advantage in having a turbine smaller than the compressor. In order that the velocity level in the first two stages would be as low as possible, the work in the third stage was increased until impulse conditions existed at the third-stage rotor hub for a hub-tip ratio at the rotor inlet which was 0.05 greater than that at the rotor outlet. Under these conditions, the required specific work output that would be performed by the third stage was 31 percent of the total. The remaining 69 percent was divided between the first and second stages in the ratio of 36:33. For impulse conditions at the rotor hub for all three stages, the rotor hub relative critical velocity ratios were 0.51, 0.58, and 0.72 for the first, second, and third stages, respectively. Thus, the velocity levels of the first two stages would be in a much more conservative range than those required in the two-stage turbine. Counteracting the advantage of a tip diameter equal to that of the compressor and the reduced velocity levels compared with the two-stage turbine is the disadvantage of a more complicated blade cooling arrangement and the possibility of an increase in the required amount of cooling air if all three stages had to be cooled.

SUMMARY OF ANALYSIS

For a set of engine conditions suitable for flight at a Mach number of 2.5, a two-stage turbine configuration and its velocity diagrams were evolved for use in the design of blade shapes. The method used to determine the velocity diagrams was one in which the turbine frontal areas and the rotor hub inlet and outlet velocities were minimized by varying the hub-tip ratio, the turbine- to compressor-tip-diameter ratio, and the work split between the first and second stages.

The final two-stage turbine design had a turbine frontal area that was only 15 percent larger than that of the compressor and has both rotors operating at hub inlet and outlet relative critical velocity ratios of about 0.75. A comparison of the two-stage turbine with other turbine configurations showed that the frontal area of a one-stage turbine and a one-stage turbine with a downstream stator was larger than that of the compressor by 82 and 42 percent, respectively, and the velocity level was more critical. An investigation of a three-stage turbine showed that it was more conservative than desired and it was not an improvement over the two-stage design considering the aerodynamic limits specified and the difference in the turbine cooling requirements.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 15, 1956

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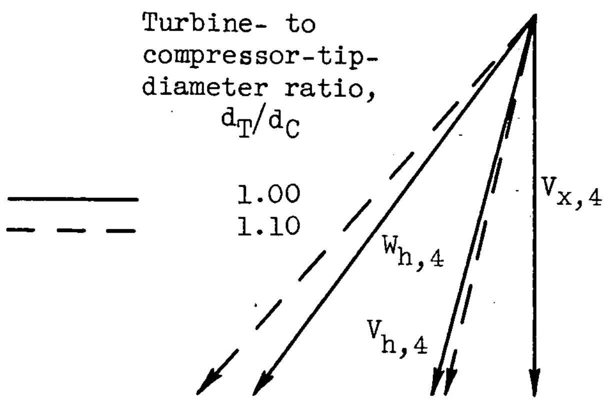
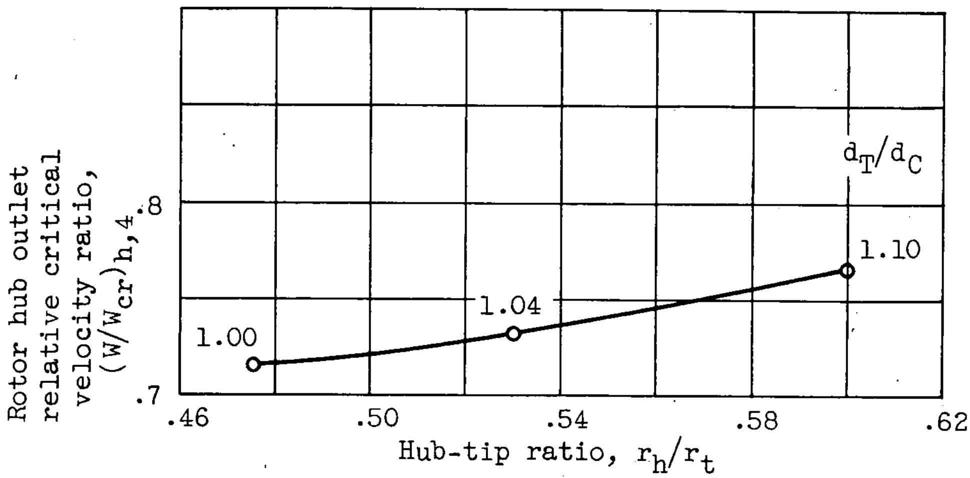
2. LaValle, Vincent L., and Huppert, Merle C.: Effects of Several Design Variables on Turbine-Wheel Weight. NACA TN 1814, 1949.

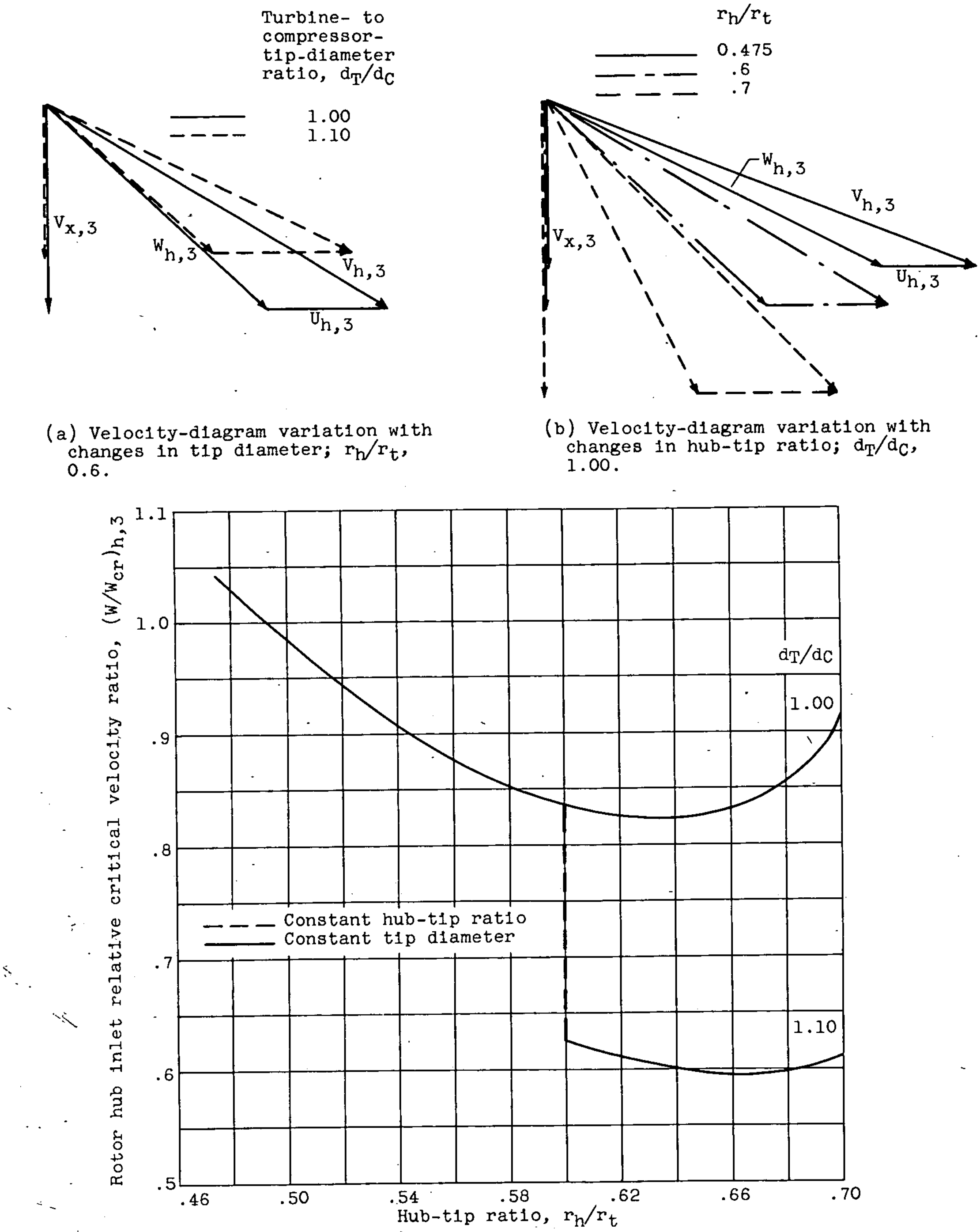


(a) Velocity-diagram variations with changes in tip diameter.



(b) Variation of velocity and hub-tip ratio with changes in tip diameter.

Figure 1. - Variation of second-stage rotor hub outlet velocity and hub-tip ratio with changes in turbine tip diameter for constant values of rotor exit whirl at mean section, blade hub stress, and compressor tip speed.



(a) Velocity-diagram variation with changes in tip diameter; r_h/r_t, 0.6.

(b) Velocity-diagram variation with changes in hub-tip ratio; d_T/d_C, 1.00.

(c) Variation of velocity with changes in hub-tip ratio and tip diameter.

Figure 2. - Variation of second-stage rotor hub inlet velocity with changes in tip diameter and hub-tip ratio for constant values of rotor exit whirl at mean section, blade hub stress, and compressor tip speed. Ratio of first- to second-stage work, 60:40.

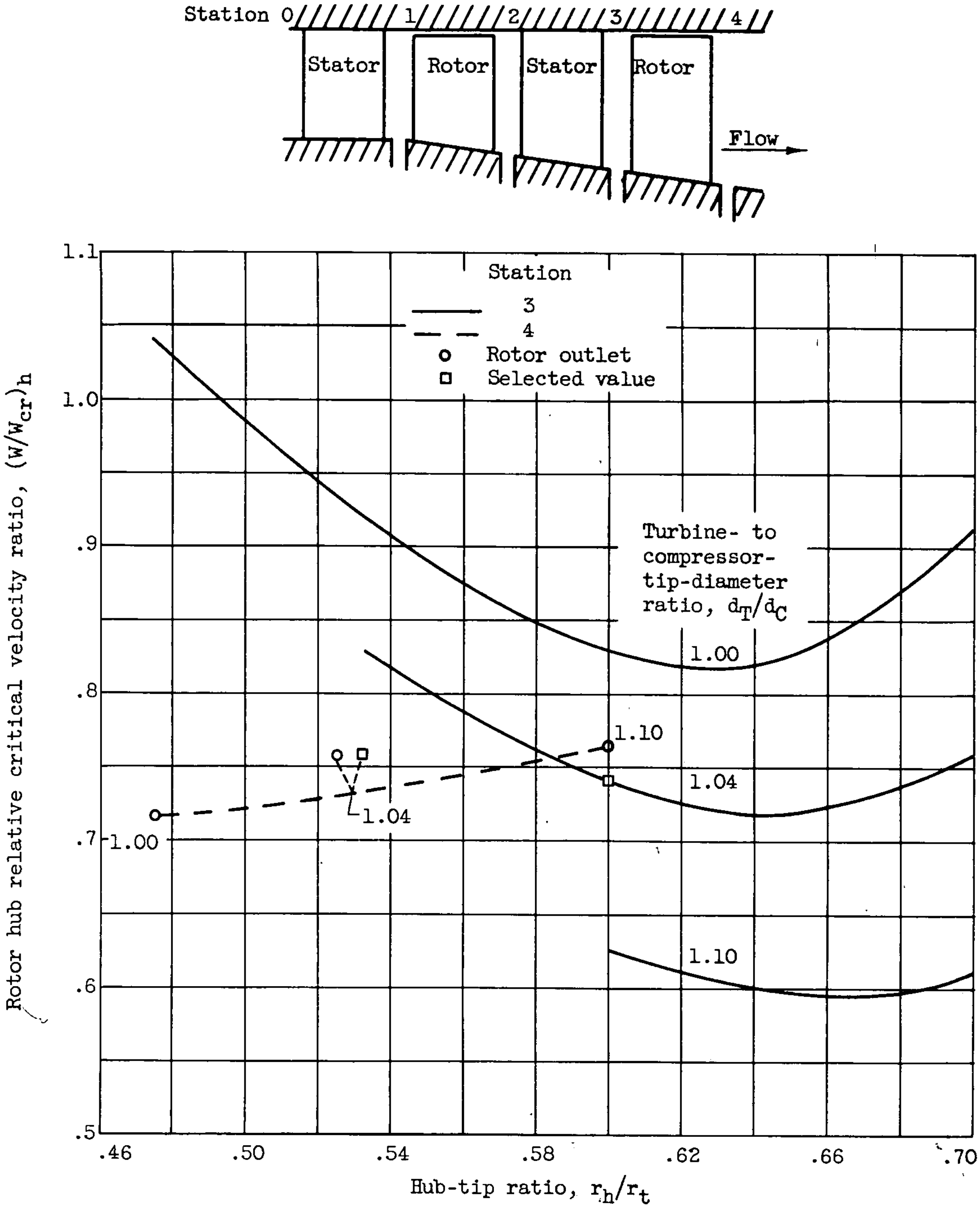
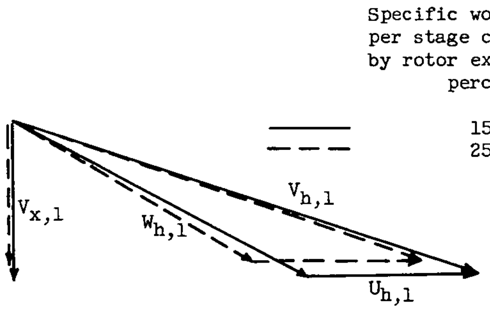
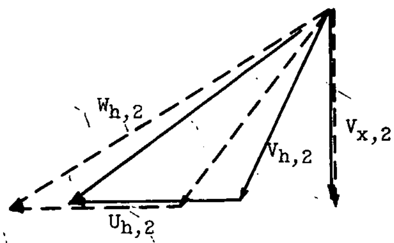


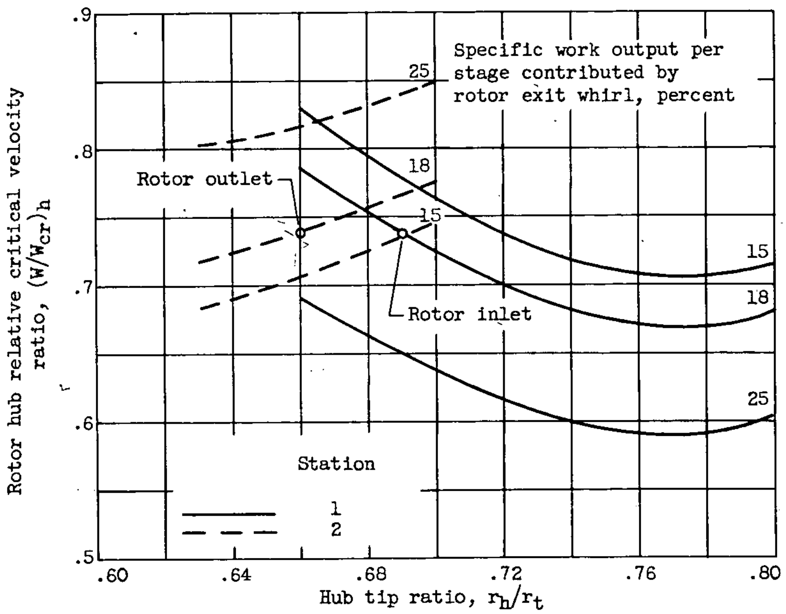
Figure 3. - Comparison of second-stage rotor hub inlet and outlet velocities for different turbine tip diameters and hub-tip radius ratios for constant values of exit whirl at mean section, blade hub stress, and compressor tip speed.



(a) Velocity-diagram variation at rotor inlet; r_h/r_t, 0.66.



(b) Velocity-diagram variation at rotor outlet; r_h/r_t, 0.66.



(c) Variation of rotor hub inlet and outlet velocities.

Figure 4. - Variation of first-stage rotor hub inlet and outlet velocities with changes in rotor exit whirl and hub-tip ratio. Turbine- to compressor-tip-diameter ratio, 1.04; ratio of first- to second-stage work, 60:40.

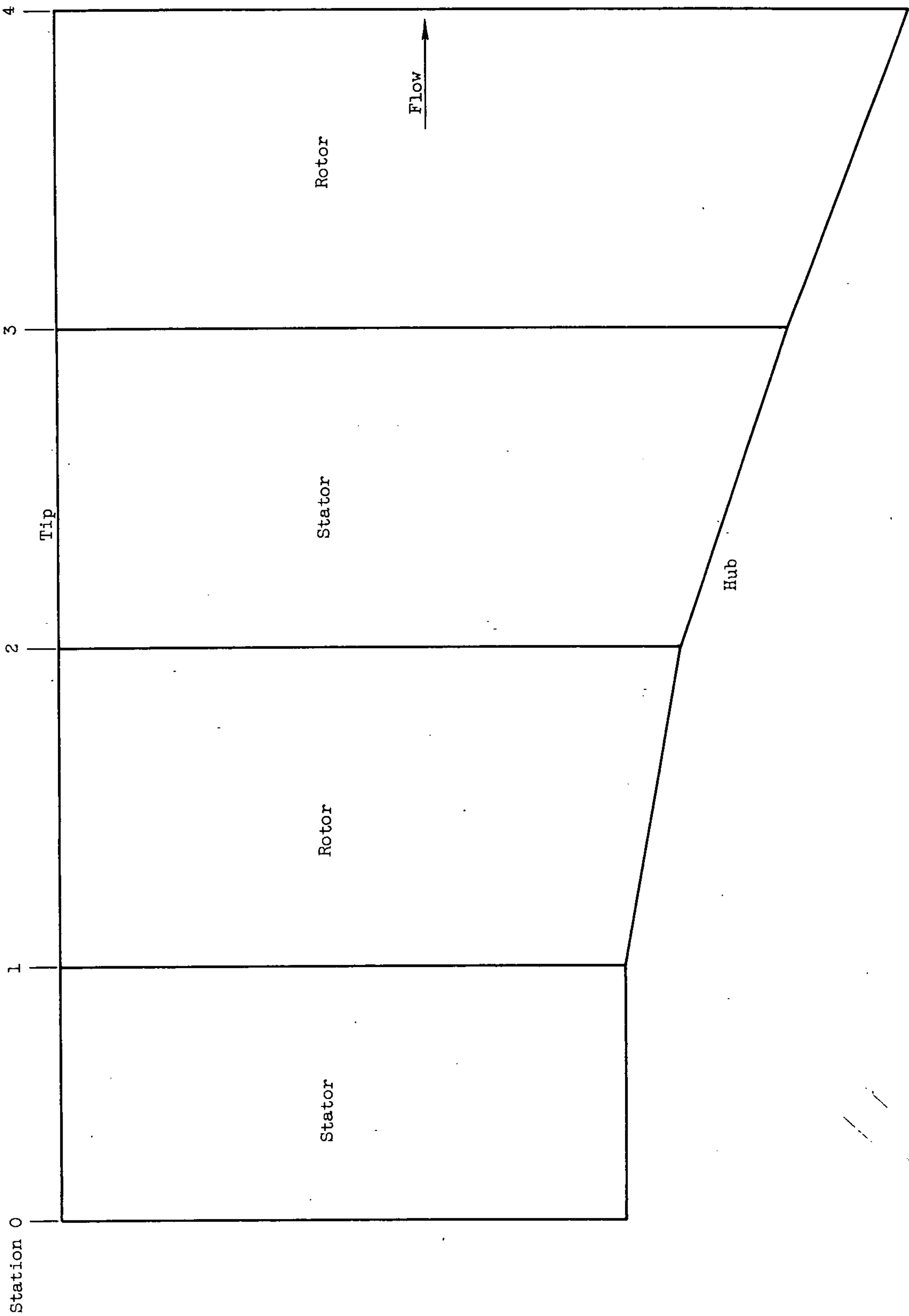


Figure 5. - Preliminary hub and tip configurations for two-stage turbine. (Constant tip diameter of 29.12 in.)

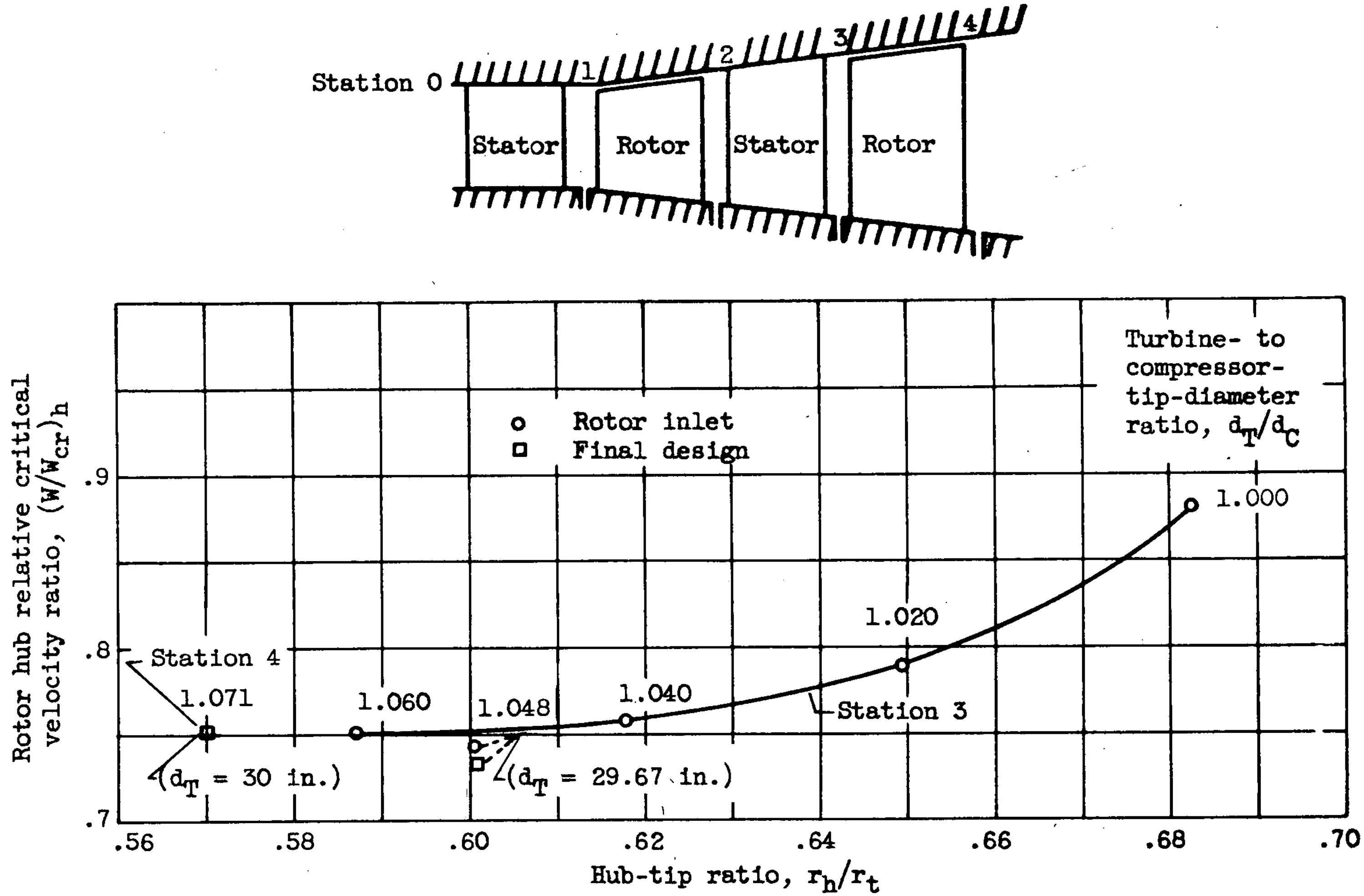


Figure 6. - Second-stage rotor hub inlet velocity variation for fixed rotor outlet conditions and varying inlet dimensions such that hub and tip are equally divergent. Ratio of first- to second-stage work, 58:42.

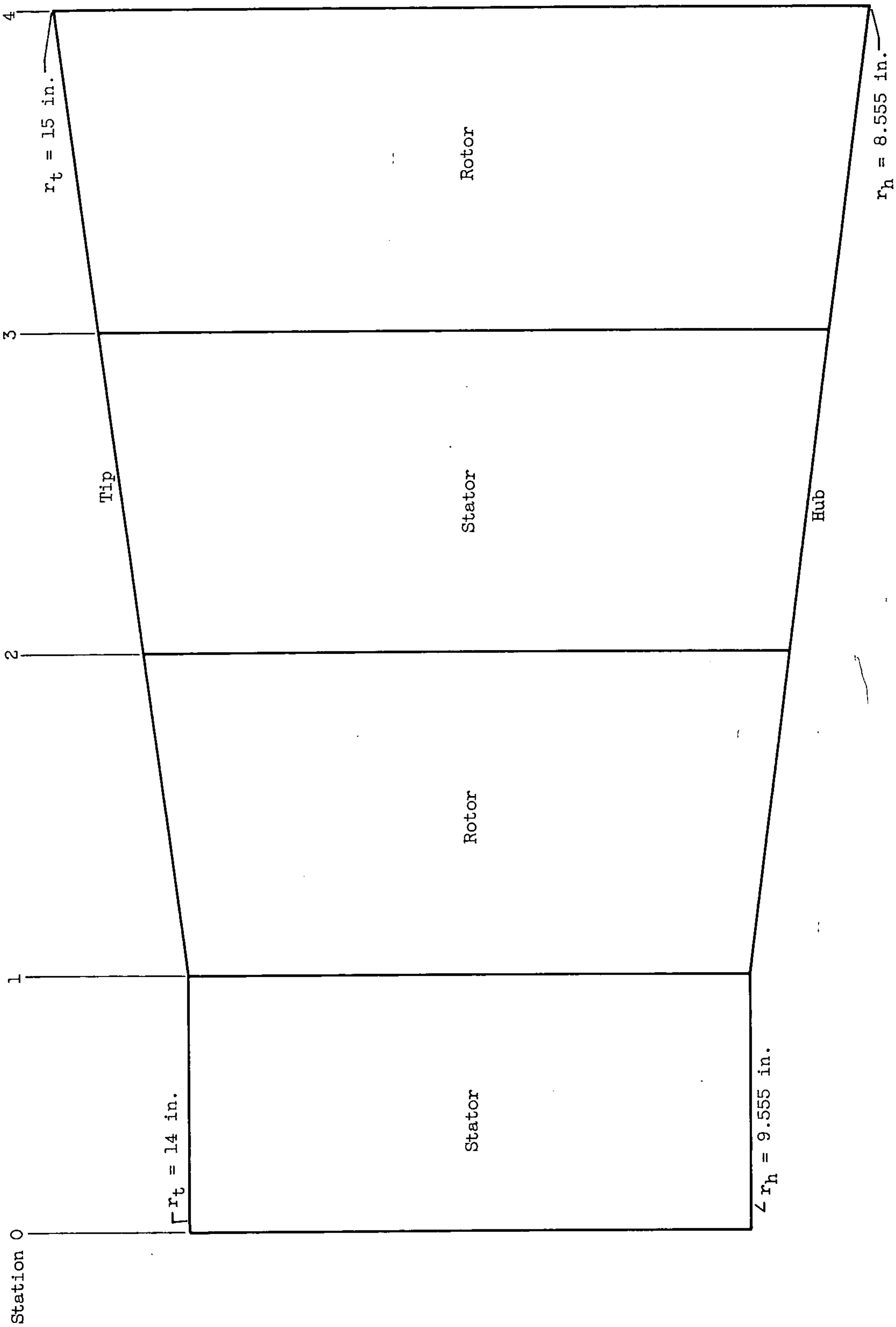


Figure 7. - Final hub and tip configurations for two-stage turbine.

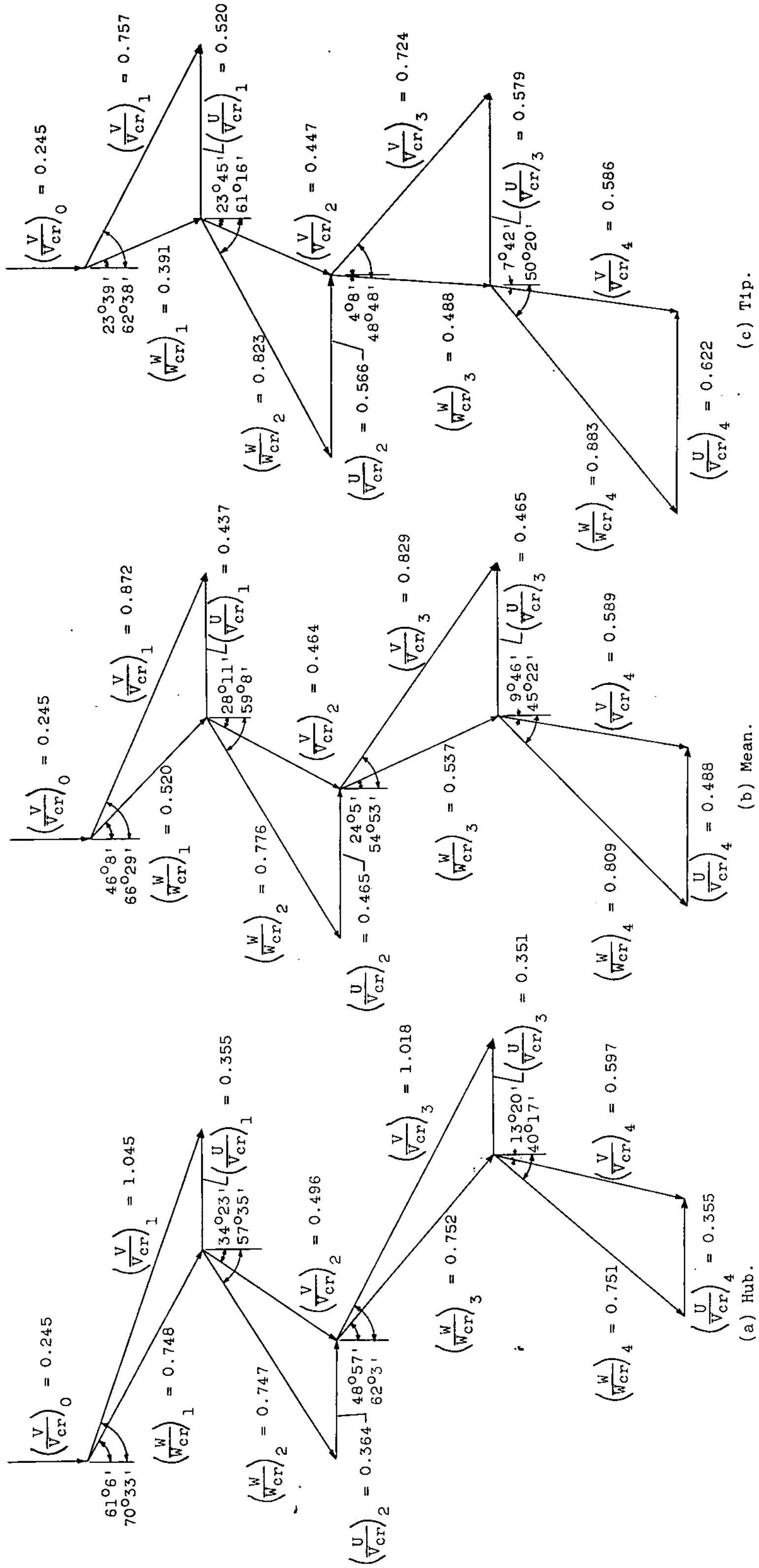


Figure 8. - Velocity diagram for final two-stage turbine configuration.