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# RESEARCH MEMORANDUM 

SOME PROPERTIES OF WING AND HALF-BODY ARRANGEMENTS AT SUPERSONIC SPEEDS By Eugene Migotsky and Gaynor J. Adams

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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS <br> WASHINGTON

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# SOME PROPERTIES OF WIING AND HALF-BODY ARRAIVGEMEINTS 

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SUMMARY

The efficiency of certain wing-body combinations as measured by the lift-drag ratio is analyzed in this paper for two arrangements: (a) a half-cone mounted under a triangular wing and, (b) a half Newtonian ogive similarly mounted. The range of Mach numbers treated is approximately 1.5 to 5.0. The comparison of the lift-drag ratio for these bodies with others in which the body (of identical volume and length) is symmetrically arranged with respect to the wing shows that the half-body configurations have slightly higher maximum lift-drag ratios at the lower supersonic Mach numbers and for small body volumes. At the higher supersonic Mach numbers and larger volumes the maximum lift-drag ratio for the half-body configurations are larger by about 8 percent. Also included, in the appendix, is an approximate slender-body theory analysis which shows that the lift coefficient induced by a slender half body of revolution mounted below a sonic-leading-edge triangular wing depends essentially on the length and base area of the body, and is independent of the shape of the body and the Mach number.

## INTIRODUCTION

In the design of an airplane or missile for flight at supersonic speeds more volume is required to enclose such items as the crew or guidance equipment, fuel, powerplant, payload, etc., than is available in the wings of the airframe. This additional volume is generally provided by a slender body whose prime purpose is to enclose the required items with a minimum penalty in drag, little attempt being made, in general, to develop a significant amount of lift from such an enclosure.

It is possible, however, to arrange this required volume in such a way that it will induce lift on the wing (ref. l). One such arrangement is to enclose the volume in a half-cone under the wing (ref. 2) ; in this case the cone induces pressures on the wing which are greater than the
free-stream pressure, and therefore produces a lift on the wing. The drag of a half-cone enclosing a given volume is greater than that of a full cone enclosing the same volume; however, the additional lift induced by the half-cone results in a higher maximum lift-drag ratio for the flat-topped configuration. Eggers and Syvertson (ref. 3) have investigated the liftdrag ratios of such configurations at high supersonic speeds, and found experimentally that maximum lift-drag ratios of the order of 6.5 can be obtained at a Mach number of 5; the configurations tested consisted of half-cones mounted under arrow wings having leading edges coinciding with the shock from the apex of the cone at a Mach number of 5. Syvertson, Wong, and Gloria (ref. 4) have made an experimental investigation of the effects of various shape variables on the lifting efficiency, and static longitudinal stability, of flat-topped configurations. Several of the configurations tested had convex ogive bodies.

In this paper the effectiveness of such an arrangement for utilizing the pressure field around a body of revolution is analyzed for two types of configurations: (a) half-cones mounted under a thin sonic-leading-edge triangular wing, and (b) half Newtonian ogives similarly mounted; the Mach number range considered is from 1.5 to 5. These configurations are compared, on the basis of maximum lift-drag ratios, with configurations in which the body (of fixed volume and length) is symmetrically arranged with respect to the wing.

The lift induced on the wings by the half-cone is calculated, at zero angle of attack, by means of the Kopal tables (ref. 5). The same tables are used to obtain an estimate of the lift induced by the half-ogives, by replacing the latter with a half-cone of the same base area and length. The thickness drag of the wings is taken to be that predicted by linear theory, and the interference effects between wing and body when the configuration is at an angle of attack are estimated from linear theory.

## SYMBOLS

A twice the cross-sectional area of the half-body, $\pi r^{2}$
$C_{D} \quad$ drag coefficient, $\frac{D}{q S}$
$\mathrm{C}_{\mathrm{D}_{\mathrm{O}}} \quad$ drag coefficient at zero angle of attack
$C_{D_{B}}{ }^{\prime} \quad$ drag coefficient, $\frac{D}{q_{\mathrm{S}}}$
$C_{F} \quad$ skin-friction coefficient, $\frac{D_{F}}{q W_{S}}$

```
    C L lift coefficient, }\frac{L}{qS
    C}\mp@subsup{L}{\mp@subsup{L}{0}{}}{}\quadlift coefficient at zero angle of attack
    Cp pressure coefficient, }\frac{\mp@subsup{p}{l}{}-p}{q
    C
    D drag
    G parameter defined by equation (7)
    K similarity parameter, }\frac{2\textrm{Mr}}{2
    l length of body and root chord of wing
    L lift
    LW lift induced on the wing by the cone pressure field, at zero
    angle of attack
    M free-stream Mach number
    p
    p free-stream static pressure
    q free-stream dynamic pressure, }\frac{1}{2}\rho\mp@subsup{V}{}{2
    r radius of body
    S area of wing
    u. perturbation velocity in free-stream direction
    V free-stream velocity
    WS wetted area
    x,y,z Cartesian coordinates in streamwise, cross stream, and vertical
    directions, respectively
        (The origin is at the cone apex.)
```

| Z | wing surface ordinate |
| :--- | :--- |
| $\alpha$ | angle of attack of flat lower surface of the wing |
| $\beta$ | $\sqrt{M^{2}-1}$ <br> $\theta$ |
| angle between free-stream direction and an arbitrary ray from <br> the apex of the cone |  |
| $\theta_{\text {S }}$ | semiapex angle of cone |
| $\theta_{\text {W }}$ | semiapex angle of shock wave <br> angle of sweepback of wing leading edge |
| free-stream density |  |$\quad$| base of body |
| :--- |
| B |

## AIVALYSIS

Half-Cone Mounted Below a Triangular Wing



Sketch (a)

The first arrangement considered is the half-cone mounted below a thin triangular wing (sketch (a)). The leading edge of the wing is taken to be coincident with the conical shock wave, that is, the sweep angle $\Lambda$ is equal to $90^{\circ}-\theta_{\mathrm{W}}$. The wing is flat on the lower surface and double wedge on the upper surface with a maximum thickness of 2 percent at the half-chord.

Zero angle of attack. - For the case of zero angle of attack, that is, the lower surface of the wing aligned with the supersonic free stream, the drag of the cone (assuming zero base drag) and the lift induced by the body on the wing are readily computed from the Kopal tables (ref. 5). The lift on the half-cone is given by

$$
\begin{equation*}
\frac{L_{B}}{q}=\frac{r_{b}{ }^{2} C_{p}\left(\theta_{S}\right)}{\tan \theta_{S}} \tag{I}
\end{equation*}
$$

and the pressure drag of the body is

$$
\begin{equation*}
\frac{D_{B}}{q}=\frac{\pi r_{b}{ }^{2}}{2} C_{p}\left(\theta_{S}\right) \tag{2}
\end{equation*}
$$

The lift induced on the wing is easily shown to be given by the integral

$$
\begin{equation*}
\frac{L_{W}}{q}=\frac{r_{b}{ }^{2}}{\tan ^{2} \theta_{S}} \int_{\theta_{S}}^{\theta_{W}} \frac{c_{p}(\theta) d \theta}{\cos ^{2} \theta} \tag{3}
\end{equation*}
$$

which was evaluated numerically. (By linear theory, the wings considered herein have no lift due to camber.)

The wave drag of the wing was obtained from the linear-theory calculations of Puckett and Stewart (ref. 6). Since the 2-percent-thick wing used in the present analysis has a flat lower surface, its wave drag at zero angle of attack is one half the wave drag of the full double-wedge triangular wings of reference 6 with 4 -percent thickness ratio.

In order to estimate the skin-friction drag it was assumed that the boundary layer was turbulent over the entire configuration and the incompressible skin-friction coefficient $C_{F_{i}}$ was taken to be 0.002. This value of $C_{F_{i}}$ corresponds, according to the Kármán-Schoenherr theory (see ref. 7), to a Reynolds number of 100 million. To obtain the skinfriction coefficient at the supersonic Mach numbers, the chosen value of $C_{F_{i}}$ was multiplied by the ratio $C_{F_{M}} / C_{F_{i}}$ which was obtained from reference 7. If $W_{S}$ is the wetted area, the skin-friction drag is given by

$$
\begin{equation*}
\frac{D_{F}}{q}=0.002\left(\frac{C_{F M}}{C_{F_{i}}}\right) W_{S} \tag{4}
\end{equation*}
$$



Sketch (b)

The wing area decreases with increasing Mach number; the resulting variation of $C_{D_{f}}$ is shown for $\theta_{S}=5^{0}$ in sketch (b). (At a given Mach number in the range considered in this report, $C_{D_{f}}$ increases approximately 6 percent as $\theta_{\mathrm{S}}$ increases from $5^{\circ}$ to $10^{\circ}$.) The total drag at zero angle of attack is then obtained from the sum of the pressure drag of the body (eq. (2)), the wave drag of the wing, and the skin-friction drag (eq. (4)).

Angle-of-attack condition. - In order to estimate the effect of angle of attack, the assumptions were made that the pressures due to angle of attack were those obtained from linear theory for the wing alone and that these additional pressures could be superposed on the pressures obtained at zero angle of attack. With these assumptions and the approximation that $\alpha$ is sufficiently small that second and higher order terms in $\alpha$ may be neglected, the lift coefficient may be written

$$
\begin{equation*}
\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{L}_{0}}+\mathrm{C}_{\mathrm{I}_{\alpha}} \alpha \tag{5}
\end{equation*}
$$

where $C_{I_{\alpha}}=(4 / \beta)$ since the wings considered always have sonic leading edges. Similarly, the drag coefficient may be written

$$
\begin{equation*}
C_{D}=C_{I_{\alpha}} \alpha^{2}+C_{L_{0}} \alpha+G \alpha+C_{D_{0}} \tag{6}
\end{equation*}
$$

where the linear terms result from the zero-lift pressures acting on the slopes due to angle of attack and the angle-of-attack pressures acting on the zero-angle-of-attack slopes. The contribution to the latter term resulting from the flat plate angle-of-attack pressures acting on the upper surface of the wing was found to be negligible. The quantity $G$ therefore includes only the effect of the angle-of-attack pressure field of the wing acting on the surface of the cone, and was evaluated by assuming that the angle-of-attack pressure acting on the body was constant over the body and equal to the pressure at the root chord of the wing. Thus, $G$ may be written

$$
\begin{equation*}
G=\frac{2 r_{\mathrm{b}}^{2}}{S} \frac{\tan \theta_{\mathrm{W}}}{\sqrt{\beta^{2} \tan ^{2} \theta_{\mathrm{W}}-1}} \cos ^{-1}\left(\frac{1}{\beta \tan \theta_{\mathrm{W}}}\right) \tag{7}
\end{equation*}
$$

The lift-drag ratio then becomes

$$
\begin{equation*}
\frac{L}{D}=\frac{C_{L_{0}}+C_{L_{\alpha}} \alpha}{C_{D_{0}}+\left(C_{L_{0}}+G\right) \alpha+C_{I_{\alpha}} \alpha^{2}} \tag{8}
\end{equation*}
$$

Upon differentiating equation (8) with respect to $\alpha$ and setting the derivative equal to zero, the optimum value of $L / D$ is found to be

$$
\begin{equation*}
\left(\frac{L}{D}\right)_{\max }=\frac{ \pm C_{I_{\infty}}}{\sqrt[2]{-C_{L_{0}} G+C_{L_{\alpha}} C_{D_{0}}} \mp C_{L_{0}} \pm G} \tag{9}
\end{equation*}
$$

The upper signs correspond to the configuration considered in this paper; the lower signs correspond to the inverted arrangement. (Eggers and Syvertson (ref. 3) have shown that the flat-bottomed configuration has the lower maximum lift-drag ratio.)

Full Cone Mounted Symmetrically on
a Triangular Wing

For the purpose of comparison with the half-cone configurations, the lift-drag ratios were determined for full cones, of the same volume and length as the half-cones, mounted on identical triangular wings. The drag at zero angle of attack is obtained in a manner similar to the previous case by adding the pressure drag of the full cone, the wave drag of the wing, and the skin-friction drag of the combination. The angle-of-attack calculations are considerably simplified since $C_{L_{0}}=G=0$ in equations (5), (6), and (8). Thus, the optimum lift-drag ratio reduces to

$$
\begin{equation*}
\left(\frac{L}{D}\right)_{\max }=\frac{1}{2} \sqrt{\frac{C_{L_{\alpha}}}{C_{D_{0}}}} \tag{10}
\end{equation*}
$$

or, since $C_{L_{\alpha}}=(4 / \beta)$, to

$$
\begin{equation*}
\left(\frac{L}{D}\right)_{\max }=\frac{1}{\sqrt{\beta C_{D_{0}}}} \tag{II}
\end{equation*}
$$

Half Newtonian Ogive Mounted Below a Triangular Wing

For cases in which the lift of the body is of small consequence, the use of a low-drag body, rather than a cone, to enclose a given volume is normally dictated by the saving in drag. For our purposes the lift induced by the half-body on the wing is of prime importance and the magnitude of this lift is not immediately obvious. It is shown in the appendix, however, by an approximation to linear theory, that a slender half-body of revolution mounted beneath a flat sonic-leading-edge triangular wing induces a lift which is proportional to the base area of the half-body and is essentially independent of the shape of the body and of the Mach number. From this result it immediately follows that the body which has minimum drag for a given ratio of base area to length squared also gives the optimum lift-drag ratio at zero angle of attack when mounted as a half-body below a flat plate. A Newtonian ogive (ref. 8) may be considered such a minimum drag body and, therefore, configurations with a half Newtonian ogive mounted under a triangular wing will be analyzed. The triangular wing is taken to be the same as in the preceding example.

Zero angle of attack.- The skin-friction drag coefficient and the wing wave-drag coefficient at zero angle of attack were taken to be the same as that obtained for a half-cone, of identical volume and length, mounted below the triangular wing. In determining the volume of the half Newtonian ogive the following approximate relation (see ref. 8) was used for the shape of the ogive.

$$
\begin{equation*}
\frac{r}{r_{b}}=\left(\frac{x}{2}\right)^{3 / 4} \tag{12}
\end{equation*}
$$

From equation (12) it is readily shown that the base radius of the half Newtonian ogive is related to the semiapex angle of a half-cone of the same volume by

$$
\begin{equation*}
\frac{r_{b}}{2}=\sqrt{\frac{5}{6}} \tan \theta_{S} \tag{13}
\end{equation*}
$$

In order to determine the pressure drag of the ogive, Jorgensen's calculations (ref. 9) for the drag of a fineness-ratio-3 Newtonian ogive at three Mach numbers were extended by means of the hypersonic similarity rule (see ref. 10). In particular, the drag coefficients computed in
reference 9 were put in the form $C_{D_{B}}{ }^{\prime}(q / p)$, where $C_{D_{B}}{ }^{\prime}$ is based on the base area, and plotted against the similarity parameter $K$ with slight extrapolations (see sketch (c)). The pressure drag of the body is then readily obtained for a given Mach number and fineness ratio.

The lift at zero angle of attack was determined by assuming, on the basis of the analysis of the appendix, that the lift induced by the body is the same as that induced by a half-cone with the same


Sketch (c) base area as the half Newtonian ogive. Instead of using the linear-theory lift derived in the appendix, the lift obtained for the half-cone configuration from the Kopal tables was used.

Angle-of-attack condition.- The effects of angle of attack for the half Newtonian ogive mounted under a triangular wing were estimated in the same manner as previously described for the case of the half-cone mounted under the wing. It should be noted, however, that in evaluating the quantity $G$ (eq. (7)) the value of $\theta_{\mathrm{W}}$ is the same as that for a half-cone of identical volume, whereas the radius of the base ( $r_{b}$ ) is that of the half ogive.

It is evident that the preceding method of estimating ( $L / D)_{\max }$ for ogive bodies mounted on triangular wings is not valid for the higher Mach numbers and lower slenderness ratios treated in this report, since the body will be outside the wing leading edge near the apex. However, it is believed that the extrapolation is useful as an approximate indication of the benefits to be obtained from the use of low-drag body profiles.

## Full Newtonian Ogive Mounted Symmetrically on Triangular Wing

Again, for the purpose of comparison, lift-drag ratios were estimated for full Newtonian ogives, having the same volume as the half ogives, mounted on identical wings. For these configurations equation (ll) still gives the maximum lift-drag ratio. The drag coefficient at zero angle of attack is determined in a manner similar to the preceding section, the only change being that the base radius $r_{b}$ ' of the full ogive is equal to $r_{\mathrm{b}} / \sqrt{2}$ where $r_{\mathrm{b}}$ is the base radius of the half Newtonian ogive.

## RESULTS AND DISCUSSION

In order to determine the effectiveness of locating a half-cone body under a thin triangular wing so that the body will induce lift on the wing, the lift-drag ratios of such arrangements were estimated for Mach numbers ranging from approximately 1.6 to 5 . These results are presented in figure l, in which the typical variation of $L / D$ with angle of attack is shown, and in figure 2, wherein the variation of maximum lift-drag ratios with Mach number for several volumes is presented. ${ }^{1}$ Also included in figure 2 are the maximum lift-drag ratios for configurations in which the body is a full cone (of identical volume) mounted symmetrically on the same wing.

It is clear, from the comparison in figure 2, that enclosing a given volume in a half-cone under the wing results in some improvement in (L/D) $\max$ over that obtained for a symmetrically mounted cone. These improvements are rather small at the lower supersonic Mach numbers (approximately 2) and the smaller volumes (corresponding to $\theta_{\mathrm{S}} \leqq 5^{\circ}$ ). At the higher Mach numbers and larger volumes, however, the increase in $(L / D)_{\max }$ is significant. For example, at a Mach number of 5 and $\theta_{S}=7.5^{\circ}$, the $(L / D)_{\max }$ increases from 5.7 to 6.2 (approximately 8.8 percent) when the volume is enclosed in the half-cone mounted under the triangular wing. For $\theta_{S}=5^{\circ}$, and friction drag coefficient $C_{D_{f}}=0.005$ (see ref. 3, p. 12) at $M=5$, the method of the present report yields a maximum lift-drag ratio of 5.75 , which is in good agreement with the experimental value of 6 obtained by Syvertson, Wong, and Gloria (ref. 4) for a similar configuration, with a l.75-percent-thick wedge wing, at $M=5.05$.

The calculated (L/D) max values presented in figure lof reference 3, for half-cone bodies mounted under zero-thickness arrow wings, show a rapid increase in this ratio with decreasing skin-friction drag coefficient $C_{D_{f}}$ and decreasing Mach number in the lower range of Mach numbers treated. Calculations made by the method of the present report (which include wingthickness drag) for variable $C_{D_{f}}$ show a smaller ( $\left.L / D\right)_{\max }$ increment, since $C_{D_{f}}$ is a smaller fraction of the total drag, if wing-thickness drag is included.

A comparison of the variation of maximum lift-drag ratio with Mach number for configurations with different bodies, all of which have the same volume as that of a $7.5^{\circ}$ semiapex-angle half-cone mounted under a triangular wing, is shown in figure 3. The improvement in (I/D) max

[^0]obtained by using a Newtonian ogive rather than a cone is clear. Further, an added increase in $(I / D)_{\max }$ is obtained by enclosing the volume in a

- half Newtonian ogive mounted under the wing as compared with the symmetrical Newtonian ogive arrangement. The improvement in maximum lift-drag ratio is again greatest at the higher Mach numbers and amounts to a 7.4percent increase at a Mach number of 5 for the half Newtonian ogive configuration as compared with the symmetric Newtonian ogive arrangement.

The increase in maximum lift-drag ratio effected by using a half Newtonian ogive body is shown by the experimental results of Syvertson, Wong, and Gloria (ref. 4, fig. 14); the configuration, which had an arrow wing, showed a maximum lift-drag ratio of 6.6 at $M=5.05$, as compared with a value of 6.1 for a half-cone body on the same wing.

Ames Aeronautical Laboratory<br>National Advisory Committee for Aeronautics Moffett Field, Calif., May 15, 1957

## APPENDIX A

## AIV APPROXIMATE FORMULA FOR THE LIFT INDUCED BY A

SLENDER HALF-BODY OF REVOLUTION MOUNTED
UNDER A SONIC-LEADIIVG-EDGE

## TRIANGUTAR WIIVG

The problem considered here is the estimation, by means of slenderbody theory, of the lift induced by half of a slender body of revolution mounted on the lower side of a flat sonic-


Sketch (d) leading-edge triangular wing at zero angle of attack (see sketch (d)).

Since the upper and lower surfaces do not interact, the lift can be derived by integrating the axially symmetric pressure field of a complete body of revolution over the area in the $x y$ plane corresponding to the lower surface of the wing, and adding the lift due to the body surface pressure. In order to simplify the calculations, the lift integral is evaluated over the triangular region bounded by the wing leading and trailing edges, without regard for the presence of the body, and the second-order term in the formula for the pressure coefficient is neglected. The accuracy of the lift formula thus obtained will be checked by comparison with results obtained from numerical integration of the exact cone solutions given by Kopal (ref. 5).

The streamwise perturbation velocity $u$, in the plane $z=0$, for a slender body of revolution, may be written (see ref. 8)

$$
\begin{equation*}
u=-\frac{V}{2 \pi} \int_{0}^{x-\beta y} \frac{A^{\prime \prime}\left(x_{1}\right) d x_{1}}{\sqrt{\left(x-x_{1}\right)^{2}-\beta^{2} y^{2}}} \tag{Aㄱ}
\end{equation*}
$$

where $A(x)=\pi r^{2}$ and the double prime denotes the second derivative with respect to $x$. The pressure coefficient in the plane of the wing is, in this calculation, approximated by

$$
\begin{equation*}
C_{p}=-\frac{2 u}{V} \tag{A2}
\end{equation*}
$$

and the lift coefficient, based on the area of the wing ( $S=\tau^{2} / \beta$ ), is given by the integral

$$
\begin{equation*}
C_{I_{0}}=\frac{2 \beta}{z^{2}} \int_{0}^{z} d x \int_{0}^{x / \beta} C_{p}(x, y) d y \tag{A3}
\end{equation*}
$$

Substituting equations (A1) and (A2) into equation (A3) we obtain

$$
\begin{equation*}
C_{L_{0}}=\frac{2 \beta}{\pi r^{2}} \int_{0}^{\tau} d x \int_{0}^{x / \beta} d y \int_{0}^{x-\beta y} \frac{A^{\prime \prime}\left(x_{1}\right) d x_{1}}{\sqrt{\left(x-x_{1}\right)^{2}-\beta^{2} y^{2}}} \tag{A4}
\end{equation*}
$$

This integral is readily evaluated by interchanging the order of integration so that

$$
\begin{equation*}
C_{L_{0}}=\frac{2 \beta}{\pi r^{2}} \int_{0}^{2} A^{\prime \prime}\left(x_{1}\right) d x_{1} \int_{0}^{\frac{2-x_{1}}{\beta}} d y \int_{x_{1}+\beta y}^{2} \frac{d x}{\sqrt{\left(x-x_{1}\right)^{2}-\beta^{2} y^{2}}} \tag{A5}
\end{equation*}
$$

Evaluation of this integral gives

$$
\begin{equation*}
C_{L_{0}}=\frac{1}{z^{2}}\left[A(2)-A(0)-2 A^{\prime}(0)\right] \tag{A6}
\end{equation*}
$$

provided $A^{\prime}(o)$ is bounded. If, in addition, the body closes at the nose $(A(0)=0)$ and is not too blunt at the nose ( $\left.A^{\prime}(0)=0\right)$, then equation (A6) reduces to

$$
\begin{equation*}
C_{L_{0}}=\frac{A(\imath)}{r^{2}} \tag{A7}
\end{equation*}
$$

Thus, the lift coefficient induced by half of a slender body of revolution mounted under a flat sonic-leading-edge triangular wing at zero angle of attack is essentially independent of the body shape; it depends only upon the ratio of base area to the square of the body length.

The result given in equation (A7) can be shown to be equivalent to one obtained by Lagerstrom and Van Dyke (see p. 185, ref. II) for a planar
distribution of sources corresponding to a thin wing with no subsonic edges. If the trailing edge of the wing is unswept the lift coefficient of such a wing is given by

$$
\begin{equation*}
C_{L}=\frac{4 \bar{\alpha}}{\beta} \tag{A8}
\end{equation*}
$$

where $\bar{\alpha}$ is the average angle of attack of the mean camber surface and may be written

$$
\begin{equation*}
\bar{\alpha}=-\frac{1}{S} \iint_{S} \frac{\partial Z}{\partial x} d x d y \tag{A9}
\end{equation*}
$$

where $\partial z / \partial x$ is the streamwise slope of the mean camber surface. For a sonic-leading-edge triangular wing the lift coefficient becomes

$$
C_{L}=\frac{4}{\tau^{2}} \int_{0}^{\tau / \beta} d y \int_{\beta y}^{z} \frac{\partial z}{\partial x} d x
$$

or

$$
\begin{equation*}
C_{L}=\frac{4}{\tau^{2}} \int_{0}^{\tau / \beta}[z(\imath)-z(\beta y)] d y \tag{Al0}
\end{equation*}
$$

The integral in this equation represents the area of the projection on the yz plane of the wing edges. For the special case when the upper surface is aligned with the stream, this integral is equal to one half of the base area of the configuration (assuming a closed nose) and the lift coefficient may be written in the form

$$
\begin{equation*}
C_{L}=\frac{2}{2^{2}} \text { (Base area) } \tag{AI}
\end{equation*}
$$

which agrees with equation (A7), since $A(2)$ is precisely twice the base area of the half body mounted under the flat wing.

As mentioned previously in this section, a check of the accuracy of the simple result given in equation (A7) is easily made for the case of a half-cone mounted under the triangular wing. Upon the introduction of $\theta_{S}$, the semiapex angle of the half-cone, equation (A7) may be written

$$
\begin{equation*}
C_{L_{0}}=\pi \theta_{S}{ }^{2} \tag{A2}
\end{equation*}
$$

In sketch (e) are shown the lift coefficients at zero angle of attack as determined by equation (Al2), and by integrating numerically the exact theoretical solution for a cone as obtained from the Kopal tables (ref. 5). The agreement for slender cones is excellent; for the larger cones equation (Al2) is satisfactory only near a Mach number of unity. It should be noted that, for a given Mach number, the leading-edge sweep angles of the wings associated with equation (Al2) are greater than the sweep angles of the wings represented by the dashed lines of sketch (e).


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Figure l.- Estimated lift-drag ratios as a function of angle of attack for a wing and conical half-body combination; zero base drag.


Figure 2.- Maximum estimated lift-drag ratio as a function of Mach number for wing and conical body combinations; zero base drag.


Figure 3.- The effect of body shape on the maximum estimated lift-drag ratio for wing-body combinations, all body volumes equal to that of a half-cone having a $7.5^{\circ}$ semiapex angle; zero base drag.


[^0]:    ${ }^{1}$ It should be noted that the geometric aspect ratio of the wing decreases continuously with increasing Mach number.

