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RESEARCH MEMORANDUM

THE EFFECTS OF TARGET AND MISSILE CHARACTERISTICS ON THEORETICAL MINIMUM MISS DISTANCE FOR A BEAM-RIDER GUIDANCE SYSTEM IN THE PRESENCE OF NOISE

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SUMMARY

A study has been made to determine the relative importance of those factors which place an inherent limitation on the minimum obtainable miss distance for a beam-rider navigation system operating in the presence of glint noise and target evasive maneuver. Target and missile motions are assumed to be coplanar. The factors considered are the missile natural frequencies and damping ratios, missile steady-state acceleration capabilities, target evasive maneuver characteristics, and angular scintillation noise characteristics.

By means of a modified orthogonal-square analysis, a simple correlation equation has been derived which expresses the theoretical minimum miss distance as a function of the above factors. It is shown that: (1) The three most important parameters that affect minimum miss distance are target acceleration, glint noise, and missile acceleration capability. (2) For realistic values, the switching period of target acceleration has negligible effect on minimum obtainable miss distance. (3) The ideal missile dynamics are those with infinite natural frequencies and zero damping ratios; any other dynamic factors will have a deleterious effect on the miss distance, although for realistic dynamics the effect is small.

Examples are given utilizing the correlation equation to indicate possibilities for improvement of existing systems, to indicate the points of diminishing returns beyond which relatively small benefits can be gained by improvements in missile dynamics and acceleration capability, and to evaluate the effects of altitude and Mach number on optimum system performance.

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INTRODUCTION

The noise signals which occur in a missile guidance system can impose a serious limitation on the effectiveness of the system. This is because the noise signals are often indistinguishable from the true target signal. Consequently, the missile responds to these unwanted signals and the miss distance is thereby increased. By careful design most sources of noise can be largely reduced or eliminated. An exception to this is glint noise which has its physical origin at the target and cannot be eliminated in systems utilizing radar detection. The guidance system should therefore be designed to minimize the errors resulting from this particular source of noise. Since the noise is random, a statistical approach is indicated; theoretical methods of the type devised by Wiener are especially appropriate.

In reference 1, the application of Wiener filter theory to minimize the effects of glint noise in a beam-rider guidance system was considered. The study established both the optimum system characteristics and the miss distances which would result if such a system could be built. However, the required acceleration capabilities of the missile were larger than available in practice; hence the indicated minimum miss distances were not physically attainable. The effect on the minimum miss distance of placing a restriction on missile maneuverability was considered in a subsequent study (ref. 2) by means of Newton's modification of the Wiener filter theory (ref. 3). This study showed that for the case considered, filtering could be chosen to place the desired restriction on missile maneuverability with little accompanying increase in minimum obtainable miss distance. Thus a practical approach to the design of the beam-rider guidance system was demonstrated.

The previous study showed that there are several factors which place an inherent limitation on the minimum obtainable miss distance. The factors were shown to be the maneuvering capabilities of the target and missile, the glint noise, and the missile dynamic characteristics. In reference 2 equations were developed which related these factors to the miss distance, but the equations are complicated and have not been solved in explicit terms. For this reason the theory can only be used to evaluate numerically the optimum performance for specific cases. It is clear that this lack of an explicit solution makes it difficult to draw general conclusions as to the effects of the above factors. The purpose of the present report will be to determine a simple approximate relationship between minimum miss distance and the factors which determine this minimum value, and to use the result to assess the relative effects of each factor.

SYMBOLS

a_M maximum steady-state acceleration capability of the missile perpendicular to the beam, g's

a_T target acceleration perpendicular to the beam, g's

f_a approximately $\frac{1}{2\pi}\left(\frac{-M_\alpha}{I_Y}\right)^{1/2}$, undamped natural frequency in the denominator of the missile transfer function, cps

f_b $\frac{1}{2\pi}\left(\frac{L_\alpha M_\delta - L_\delta M_\alpha}{L_\delta I_Y}\right)^{1/2} = \frac{f_a}{\sqrt{R}}$, undamped natural frequency in the numerator of the missile transfer function, cps

H_{co} optimum transfer function of the compensating network

H_f aerodynamic transfer function of the missile

I_Y moment of inertia about the pitch axis of the missile, slug-ft^2

L lift, lb

M moment, ft-lb

m mass of missile, slugs

N noise magnitude or zero frequency spectral density, ft^2/radian/sec

R $\frac{-L_\delta M_\alpha}{L_\alpha M_\delta - L_\delta M_\alpha}$, ratio of lift developed by movable control to total lift

s variable in the Laplace transform

$\overline{T}$ average switching period of the target acceleration, sec

$T_s{}^2$ reciprocal of the aerodynamic gain, radians/ft/sec^2

t time, sec

V missile velocity, ft/sec

Y_o over-all optimum transfer function of the system

y_M missile displacement from a reference line, ft

y_N apparent target displacement from true target center due to noise, ft

y_T target displacement from a reference line, ft

α angle of attack, radians

δ control-surface deflection, radians

ϵ error between target and missile position, $y_T - y_M$, ft

ζ_a $\dfrac{(L_\alpha/mV) - (M_{\dot\theta}+M_{\dot\alpha})/I_Y}{4\pi f_a}$, damping ratio of the denominator of the missile aerodynamic transfer function

ζ_b $-\dfrac{M_{\dot\theta}+M_{\dot\alpha}}{4\pi f_b I_Y}$, damping ratio of the numerator of missile aerodynamic transfer function

θ angle of pitch, radians

Φ_N spectral density of noise displacement y_N, ft^2/radian/sec

Φ_T spectral density of target displacement y_T, ft^2/radian/sec

ω angular frequency, radians/sec

Note: First and second derivaties with respect to time are indicated by $(\dot{\ })$ and $(\ddot{\ })$ respectively. The symbols L_α, $M_{\dot\theta}$ represent $\dfrac{\partial L}{\partial \alpha}$, $\dfrac{\partial M}{\partial \dot\theta}$, etc.

Subscripts

a denominator of missile transfer function

b numerator of missile transfer function

Superscripts

+ the part of a function having poles and zeros in the upper half plane

- the part of a function having poles and zeros in the lower half plane

GENERAL CONSIDERATIONS

The problem of beam-rider guidance in the presence of glint noise is illustrated in figure 1(a) for the case wherein the target and missile move in the same plane.[1] It can be seen here that displacements are referred to a line fixed in space. The true displacement of the target due to evasive maneuver is indicated as y_T. Superimposed on this signal and indistinguishable from it is the glint noise, y_N, indicated as a displacement from the true target center. It is the sum of these two signals that is detected by the radar, and an attempt is made then to make the missile position, y_M, coincide with that of the true target position. The amount by which the missile fails to follow the target, y_T-y_M, is indicated by ϵ which obviously should be minimized in some sense. The corresponding block diagram representation of the problem is shown in figure 1(b).

The problem has been studied in references 1 and 2 where the inputs y_T and y_M were treated as statistical quantities. Since use will be made of the previous results, it will be necessary to review and summarize this work briefly. For more detail than given here the reader is referred to these works.

From the previous work it was found that in the realistic optimization problem it is necessary to consider the effects of limiting. In particular, limiting of the control-surface deflection was found to be the critical factor. In reference 2, this problem was considered and an approach was used wherein the system was optimized so as to minimize the miss distance with a restriction on the available control motion. The restriction is imposed so that the probability of the control surfaces hitting physical stops is small and, hence, the system operates essentially as a linear one.

[1] Although the head-on approach is shown in figure 1(a), the results presented herein include coplanar attacks for all aspects when it is assumed that the beam does not rotate in space. This condition is achieved when the launcher is flying a collision course and very nearly achieved when the launcher is sufficiently far from the target.

The essentials of the problem are illustrated by figure 2 where H_f represents the transfer function of the given aerodynamics, δ is the control deflection at the input to H_f, and H_{co} represents the transfer function of the compensating network which is to be determined. This network is chosen so as to minimize the rms miss distance, $\sqrt{\overline{\epsilon^2}}$, with a restriction on the rms control deflection, $\sqrt{\overline{\delta^2}}$. As previously indicated, the solution to the problem depends only on the characteristics of the noise, target maneuver, and the missile aerodynamic transfer function. The following representations of these factors were used:

1. The noise was represented by a flat spectral density of magnitude N, rather than by an actual spectrum, since it was shown in reference 1 that this assumption reduces the mathematical complexity and produces essentially the same result.

2. The target acceleration, a_T, was defined as the component of acceleration perpendicular to the beam. The target maneuver was then represented by an alternate switching of this acceleration in opposite directions with random duration. The spectral density of target displacement corresponding to this type of maneuver was defined (as in ref. 2) by

$$\Phi_T = \frac{k a_T^2}{\pi\omega^4(\omega^2+k^2)}$$

where $k/2$ is the average switching rate of target acceleration. For the present study it is convenient to use the average switching period $\overline{T} = 2/k$.

3. The missile aerodynamic transfer function from control deflection to displacement (without feedback) was assumed to be of the following form:

$$H_f(s) = \frac{1}{Ts^2}\,\frac{T_b^2 s^2+2\zeta_b T_b s+1}{s^2(T_a^2 s^2+2\zeta_a T_a s+1)}$$

Again, for the present study it is more convenient to consider the natural frequencies, f_a and f_b, defined by $f_a = 1/2\pi T_a$ and $f_b = 1/2\pi T_b$.

If the magnitudes of these factors are known the optimum solution can be obtained from the following series of equations:

(a) The optimum compensating network, H_{co}:

$$\left.\begin{aligned} H_{co}(i\omega,\rho) &= \frac{1}{2\pi\Lambda^{+}(\omega)}\int_{0}^{\infty} e^{-i\omega t}\int_{-\infty}^{\infty}\frac{\overline{H_f(i\alpha)}\Phi_T(\alpha)e^{i\alpha t}}{\Lambda^{-}(\alpha)}\,d\alpha\,dt \\ \text{where} & \\ \Lambda(\omega) &= \left[H_f(i\omega)\overline{H_f(i\omega)} + \rho\right]\cdot\left[\Phi_T(\omega) + \Phi_N(\omega)\right] = \Lambda^{+}(\omega)\Lambda^{-}(\omega) \end{aligned}\right\} \quad (1)$$

Here $\overline{H_f(i\omega)}$ is the complex conjugate of $H_f(i\omega)$ and ρ is the Lagrangian multiplier.

(b) The called-for mean-square control deflection:

$$\overline{\delta^2} = \int_{-\infty}^{\infty}\left|H_{co}(i\omega,\rho)\right|^2\left[\Phi_T(\omega) + \Phi_N(\omega)\right]d\omega \quad (2)$$

(c) The optimum over-all transfer function, Y_o:

$$Y_o(i\omega) = H_{co}(i\omega)H_f(i\omega) \quad (3)$$

(d) The minimum mean-square miss distance, $\overline{\epsilon^2}$:

$$\overline{\epsilon^2} = \int_{-\infty}^{\infty}\left|1 - Y_o(i\omega)\right|^2\Phi_T(\omega)d\omega + \int_{-\infty}^{\infty}\left|Y_o(i\omega)\right|^2\Phi_N(\omega)d\omega \quad (4)$$

These equations comprise the solution, but they have not been solved explicitly for minimum miss distance as a function of the variables involved. An iterative numerical solution has been used in which the value of ρ is varied until the resultant H_{co} from equation (1) gives the desired value of $\overline{\delta^2}$ from equation (2).

The difficulty in obtaining an explicit solution greatly hampers a fuller understanding of the filter problem. There are a great many important questions which are difficult to answer, such as, "How much missile acceleration is necessary to keep rms miss distance within a specified limit when attacking a target of known acceleration capabilities?" or "How important is it to increase the missile natural frequency?" These and similar questions cannot readily be answered because of the lack of an expression for the miss distance in terms of factors which place an inherent limitation on miss distance. The specific factors involved, which are considered herein, are the following:

(1) Target maneuver characteristics

(a) The acceleration, a_T, of the target

(b) The average switching period, $\overline{T}$, of the target acceleration

(2) Scintillation noise characteristics; spectral density N

(3) Missile aerodynamic characteristics

(a) The missile dynamic terms, T_a and T_b (or equivalently, natural frequencies f_a and f_b) and the associated damping ratios ζ_a and ζ_b

(b) The rms of the called-for control motion, $\sqrt{\overline{\delta^2}}$, or equivalently the missile steady-state acceleration capabilities,[2] a_M.

Because of the desirability of evaluating the effects of these factors on the minimum obtainable miss distance, the remainder of the report will be devoted to the development and application of one method of evaluation and to a discussion of the results obtained by this method.

ANALYSIS

It is desired to formulate a simple functional relationship between miss distance and the factors listed above. It is clear that by having a sufficient number of specific solutions, it is possible to formulate such a relationship empirically; however, the programming of the required tests deserves careful consideration. This is especially true if the number of independent variables is large. Consider an example where n independent variables are involved. In the traditional method a standard (or reference) level is chosen for each of the n variables. Tests are programmed so that in the first set only the first parameter is varied

[2]For the purpose of this report it will be more convenient to place the restriction on this parameter rather than the control motion. Since limiting of the control motion is the critical factor, the restricted value of a_M must be chosen to correspond to the desired control motion restriction. For the control motion restricted to one half of the maximum available, a restriction on a_M is related to a restriction on δ by the aerodynamic gain, $1/T_s^2$, by the equation $a_M = 2\sqrt{\overline{\delta^2}}/32.2T_s^2$.

through its desired range while the others are held at the reference level. The same procedure is repeated for the other variables. Next, interaction between variables must be investigated. This requires a series of tests wherein various combinations of two, three, or more of the independent variables are considered. Such a program obviously will very quickly reach a practical limit. Some other method involving fewer tests and less computational work is needed.

One such method is that of orthogonal squares. A complete description of this method is beyond the scope of this report and the reader is referred to references 4, 5, 6, and 7. This discussion will be limited to a particular orthogonal square which is related to the problem at hand.

Figure 3 is a representation of the square to be considered. Each block represents a single experiment. Since the square is 5×5, it prescribes 25 individual experiments. Each block contains letters A, B, C, etc., which represent the independent variables. The subscripts of these letters denote the level of the variable. A 5×5 square accommodates up to six variables and permits each variable to assume five values (i.e., levels). A specific range is selected for each variable on the basis of the requirements of the problem. The range is divided into four increments which are usually (but not necessarily) of equal, or nearly equal, size. The values which define the boundaries of these increments will be termed the "levels" that the variable will assume in the experiments. Figure 3 gives the arrangement for the various levels in the orthogonal square.

That the orthogonal square requires fewer experiments than the traditional method can be seen from the following considerations. The particular arrangement of variables prescribes experiments from which the effect of any one parameter can be isolated. For example, notice that in the first column the variable A is held fixed at the level A_1, while the other variables assume each of the five assigned levels once. In the second column A is held at A_2 while the other variables range through their five levels. The same ordering is true for the remaining columns. The average results of each column fairly well represent the influence of A, not for fixed standard levels of the other variables but for an average of conditions throughout the whole range. Examination of the orthogonal square will show that the same is true of each of the remaining variables. Thus it is clear that the effect of each variable can be found from the same 25 experiments. In contrast, the traditional method requires a separate set of experiments for the effect of each variable and also for the effects of interactions among variables.

The value of the dependent quantity is now experimentally determined for each of the 25 combinations of variable levels prescribed by the square. It is desired to write the dependent variable in terms of A, B, C, D, E, and F. To do this it is necessary to assume a form for the functional relationship and assign unknown constant coefficients to each term. A least square fitting of these coefficients to the experimental data is then performed. The procedure is repeated until a correlation equation of acceptable form and accuracy is obtained.

In application of the method to the present missile problem, the eight factors, f_a, f_b, ζ_a, ζ_b, a_M, a_T, $\overline{T}$, and N, can be considered to be the independent variables corresponding to A, B, C, etc. The choice of range and level of these parameters prescribes each test. For each test the miss distance as given by equation (4) can be considered to correspond to the dependent variable. Thus, for a 5×5 orthogonal square, 25 tests are performed, and the 25 corresponding values for the miss distance are obtained. It is this data which must then be fitted by the least-square curve-fitting method.

For this particular problem two conditions have been imposed on the choice of certain of the independent variables. One of these,[3] that $a_M/a_T \geq 4$, is made since this is the region of interest for the short range missile; the other, that $f_a \leq f_b$, arises from limiting the study to positive lift-ratio missiles (canard or variable-incidence, for example), as can be deduced from the definition of these terms given in the symbols. In order to incorporate these restrictions into the analysis it was necessary to modify the orthogonal-square technique. The variables a_M, f_a, N, $\overline{T}$, ζ_a, and ζ_b were placed in the orthogonal square in the conventional manner. For the remaining two variables, f_b and a_T, the selection of the level values were modified to satisfy the above restrictions while the ordering of these various levels in the square remained the same.

In order to prescribe the orthogonal-square program it was necessary to assign ranges to each of the independent variables under study. In general, the ranges of the variables have been chosen sufficiently wide to include most air-to-air target-missile intercept problems of interest. The parameters and corresponding ranges are tabulated below:

f_a, f_b	0.5 to ∞ cps	a_T	0.5 to 3 g's
ζ_a, ζ_b	0 to 0.5	$\overline{T}$	3.33 to 10 sec
a_M	4 to 20 g's	N	7.5 to 30 ft^2/radian/sec

with the additional conditions $f_a \leq f_b$, $a_M/a_T \geq 4$.

[3]This restriction is not so severe as it might appear, since a_T is only the coplanar component of target acceleration perpendicular to the beam.

The determination of the size of the orthogonal square which must be used is ordinarily dictated by the number of independent variables required. To include all of the eight parameters previously discussed in a single orthogonal square at least a 7×7 square would be required. However, to obtain a better understanding of the relative importance of the various factors the problem was divided into two phases.

The phase I square was constructed to study only the effects of missile aerodynamic parameters on miss distance. Accordingly, the target parameters a_T, $\overline{T}$, and N, were held fixed at values between the extremes listed previously. These values were $a_T = 1g$, $\overline{T} = 5$ sec, and N = 15 ft^2/radian/sec. The five missile aerodynamic parameters, f_a, f_b, ζ_a, ζ_b, and a_M, involved in this phase were placed in a 5×5 square constructed as outlined previously.

The phase II square was designed to consider the combined effects of missile and target maneuver and noise characteristics on the miss distance. The parameters considered were the missile parameters, f_a, f_b, and a_M, and and the target parameters, a_T, $\overline{T}$, and N. These parameters were also accommodated by a 5×5 square.

The miss-distance values corresponding to each "test" within each square were obtained from a digital computer on which the pertinent equations (1) through (4) were programmed. Since in phase II the general problem is considered, the values of the variables for each run and the resulting miss distances are tabulated in table I for the phase II square.

MISS-DISTANCE EQUATIONS

To obtain the correlation equations which express minimum miss distance as a function of the variables under consideration, it is necessary to assume some form for the functional relationship. The reasoning in choosing the functional form is largely heuristic. There are, however, several aids which can be used. First, several computer runs were made where only one parameter at a time was permitted to vary. This yielded information on the form and magnitude of the effect of each variable on miss distance. It serves only as a guide, however, because it yields no information regarding cross-product terms. Second, the rigorous equations were examined for indications of possible cross-product terms which might be expected to exist if the exact equations had been solved. Third, the orthogonal-square results were scrutinized in order to detect possible trends. From such information a reasonable form of equation was constructed with unknown coefficients. A least square fitting (ref. 8) of the coefficients to the experimental data was then performed, followed by a simple analysis to determine which terms were important and which could be discarded. On the basis of the root-mean-square criterion, if the fit of the equation so obtained was not satisfactory, new combinations (interaction terms) or higher powers of the basic variables were added as new terms. This procedure was repeated many times until correlation equations were obtained which satisfactorily represented the minimum obtainable miss distance for both phase I and phase II. For this particular problem, an rms deviation of a few feet was considered satisfactory.

In review, it will be remembered that in phase I only the effects of missile aerodynamic parameters on the miss distance are considered. The target maneuver and noise characteristics were held fixed at reasonable mid-range values between the expected extremes as given below. In phase II, the more general problem of the combined effects of missile aerodynamic parameters, target maneuver, and noise characteristics are considered. The following equations were obtained as a result of these studies:

Phase I:

$$\sqrt{\overline{\epsilon^2}} = 10.50 + \frac{116.13}{a_M} + 3.60\left(\frac{1}{f_a} - \frac{1}{f_b}\right) + 7.12\frac{\zeta_a}{f_a} - 5.86\frac{\zeta_b}{f_b} \qquad (5)$$

for

$$a_T = 1g, \ \overline{T} = 5 \text{ sec}, \ N = 15 \text{ ft}^2/\text{radian/sec}$$

Phase II:

$$\sqrt{\overline{\epsilon^2}} = 9.20 + 5.02\frac{Na_T}{a_M} + 1.34a_T^2 + \frac{7.94}{\overline{T}} + 6.22\left(\frac{1}{f_a} - \frac{1}{f_b}\right) \qquad (6)$$

for

$$\zeta_a, \zeta_b \ll 1$$

It should be pointed out that these equations can be used for any combination of numerical parameters as long as the values of all parameters lie within the ranges selected for this study. The accuracy of the equations in many cases rapidly deteriorates outside these ranges. Likewise the forms of the equations are not valid when extended beyond the test ranges. As for the accuracy of these equations, it has been found that both equations (5) and (6) fit the original tests of the orthogonal square with an rms deviation of 2.4 feet, and have twenty degrees of freedom.[4] The large number of degrees of freedom tend to insure that these equations will also satisfactorily represent the results obtained from

[4]Degrees of freedom can be defined as the difference between the number of data points and the number of unknown coefficients in the correlation equation (refs. 8 and 9). Eight to ten degrees of freedom are usually considered necessary for obtaining a good statistical fit.

equation (4) for any combinations of parameters which were not part of the orthogonal-square tests but were within the prescribed ranges. Many such combinations were tried, and the corresponding miss distances were in accord with the rms deviation.

Equations (5) and (6) can be readily used to reach certain conclusions as to the relative importance of the various factors which contribute to the miss distance. The effect of these factors will be discussed in the following sections. It should be noted that equation (5) can only be used to evaluate the effects of missile dynamics on miss distance. Equation (6) is, therefore, a more general and useful expression.

For purposes of later discussion in which the effects of individual parameters are illustrated by means of perturbations, it will be convenient at this point to introduce the term "reference level." This term will be used to denote a particular mid-range set of values of the independent variables of equation (6). They are as follows:

$$a_T = 1g, \quad \overline{T} = 5 \text{ sec}, \quad N = 15 \text{ ft}^2/\text{radian/sec}$$

$$f_a = 2.05 \text{ cps}, \quad f_b = 2.88 \text{ cps}, \quad a_M = 10 \text{ g's}$$

RESULTS AND DISCUSSION

Effects of Missile Parameters

The five missile parameters considered in this section are the aerodynamic natural frequencies, f_a and f_b, the aerodynamic damping ratios, ζ_a and ζ_b, and the missile steady-state acceleration capability, a_M. It will be necessary to make use of both equations (5) and (6) in order to examine more fully and understand the effects of these factors.

Consider first equation (5) rewritten in the following form:

$$\sqrt{\overline{\epsilon^2}} = 10.50 + 3.60\left(\frac{1}{f_a} - \frac{1}{f_b}\right) + 5.86\left(\frac{\zeta_a}{f_a} - \frac{\zeta_b}{f_b}\right) + 1.26\,\frac{\zeta_a}{f_a} + \frac{116.13}{a_M}$$

From the existence of negative terms in this equation it may appear that the missile's dynamic factors could be adjusted so that their net effect would be to reduce the miss distance. However, from the definitions given in the symbols, it can be shown that the restriction $f_a \leq f_b$ inherently implies that $\zeta_a \geq \zeta_b$ and, consequently, that $\zeta_a/f_a \geq \zeta_b/f_b$. From the viewpoint of achieving minimum miss distance, the ideal dynamics are those with infinite natural frequencies and zero damping ratios; any other set of dynamics will have a detrimental effect on minimum miss distance. It will be desirable to examine the quantitative effect of these parameters on miss distance.

Consider the effects of the damping ratios on minimum miss distance. The main point that can be made from equation (5) is that for realistic ranges of the missile parameters, the quantitative effects of the damping ratios are negligible. For example, the damping ratios of current missiles rarely exceed 0.3 and are usually much smaller. Thus, equation (5) clearly shows that the damping ratios have an effect which is small compared to the rms miss distance. For this reason, the effects of the damping ratios were not considered in phase II (eq. (6)) which will be discussed presently.

As for the natural frequencies, it is clear from equation (5) that from the standpoint of achieving minimum miss distance, the ideal missile would have infinitely fast acceleration response, that is, infinite natural frequencies. In the practical case such dynamics can only be approached by making the natural frequencies high. How high to make these frequencies can be discussed more comprehensively from equation (6) wherein the target maneuver and noise characteristics are also considered. This equation shows, first of all, that no important interrelation between missile dynamics and target characteristics or noise are present. It also shows that the variations of miss distance with natural frequencies are essentially similar to that found in equation (5), that is, miss distance varies linearly with $(1/f_a - 1/f_b)$. The quantitative effect on miss distance due to these natural frequencies is plotted in figure 4(a) as a function of the natural frequency ratio f_a/f_b for several values of f_a. From the definitions given in the symbols it is seen that the ratio f_a/f_b can also be interpreted in terms of the missile lift ratio, R, since $f_a/f_b = \sqrt{R}$. It is clear from this figure that the least adverse effect on miss distance occurs when $f_a = f_b$ (which is the limit on realizable missiles). In terms of lift ratio this means that variable-incidence configurations (lift ratios approaching unity) are the most desirable. However, if $f_a \neq f_b$ it is apparent from the figure that decreasing f_a or increasing f_b will increase the miss distance. It therefore follows that for configurations having a fixed low f_a/f_b ratio (i.e., low lift ratio missiles such as the canard type), it becomes more important to increase the natural frequency f_a in order to avoid increased miss distance. It is also apparent from figure 4(a) that the maximum effect the natural frequency can have (within the range of validity of the equations) occurs when $f_a = 0.5$ and $f_b = \infty$. For this condition the increase in miss distance is 12.4 feet. Further reductions in the natural frequency f_a would result in a further increase in miss distance although the equations would not be quantitatively accurate. In most situations, however, the natural frequencies would not have this great an effect because they would be closer than in the case just cited. For example, for the typical missile used in reference 2 in which $f_a = 2.05$ and $f_b = 2.88$, the value of $(1/f_a - 1/f_b)$ is 0.14. The increase in miss distance in this case is only about one foot. For this reason emphasis on achieving high natural frequencies is seldom warranted. The figure presented can be used to show the point of diminishing returns, that is, the frequency at which relatively small benefits can be gained by increasing natural frequencies.

The missile factor which has by far the largest effect on miss distance is the steady-state acceleration capability. Figure 4(b) illustrates the importance of this factor as it is varied through its range while all other parameters are held fixed at the reference level. Although it can be seen that this parameter is quite important, it should be noted that the dependence of miss distance on missile acceleration capability is not as simple as indicated in the figure. A complication arises in that a strong interaction exists between target acceleration, noise, and missile acceleration. The interaction effect will be discussed in a subsequent section devoted to this problem alone.

Effects of Target Maneuver and Noise

The parameters considered here are the target acceleration normal to the beam a_T, the average switching period of this acceleration $\overline{T}$, and the glint noise spectral density N. The quantitative effect of these parameters on the minimum miss distance as determined by equation (6) has been plotted in figures 5 and 6.

From figure 5(a), it can be seen that the average switching period has a small effect on the miss distance over the range considered. Periods shorter than those shown would eventually cause the curve to rise sharply, but this rise is of little significance since such short periods are not encountered. For the longer periods, the minimum miss distance becomes smaller and also relatively independent of the period. For example, for $\overline{T}$ = 10 sec, the contribution to the miss distance due to this term is, from equation (6), less than one foot. This is also illustrated by the asymptote shown in the figure.

As for the effects of target acceleration and glint noise, it is apparent from figures 5(b) and 6 that both parameters are very important and may cause serious deterioration of the minimum miss distance. Because they occur as a product, they will be discussed together in the next section.

Combined Effects of Target and Missile Acceleration and Noise

In this section will be considered the interaction between the three factors having the greatest effect on the minimum miss distance: target acceleration, missile acceleration, and the noise magnitude. The quantitative effects of these factors can be obtained from equation (6). To isolate these effects assume both infinitely fast missile acceleration response and infinite switching period of the target acceleration. Terms in equation (6) involving natural frequencies and switching period are then zero. With these assumptions, then, the miss distance is plotted in figure 7 as a function of missile acceleration capability for various values of target acceleration and noise. The curves have been drawn to include only the valid ranges for the variables. The figure illustrates the predominant effect which target acceleration has on minimum miss distance. Also from the lines of constant a_M/a_T ratio which have been superimposed on this plot, the importance of maintaining a sufficiently high a_M/a_T ratio is apparent. The large a_M/a_T ratio which is necessary in order to operate on the flatter portions of the curves is not so stringent a requirement as might appear, since a_T refers only to the component of target acceleration normal to the radar beam. Thus, for attacks other than tail and head-on approaches, a_T will be less than the actual target acceleration.

Effect of Type of Target Maneuver

It is well known that the design of a system normally depends on the input to which it is expected to be subjected. For this reason it is appropriate at this point to discuss briefly two important aspects: (1) the choice of target maneuver for which the system should be optimized, and (2) the effect on the miss distance of inputs for which the system was not specifically designed.

The type of target maneuver upon which to base the system design can never be determined with certainty, since the target quite obviously may maneuver in many different ways. First, it might be assumed that the target pilot possesses unlimited knowledge about the attacking missile and can therefore always maneuver in the optimum manner to avoid being hit. Such a concept is possibly somewhat unreasonable because of the difficulty in obtaining and properly utilizing all the information necessary to execute such a maneuver. A more reasonable assumption is that the target pilot knows only that he is being fired at and therefore executes some evasive maneuver. Although there are a great many maneuvers which could be made, one possibility is a step acceleration evasive maneuver initiated at some arbitrary time during the attack (ref. 10). Although a missile system can be optimized for such a maneuver, the resulting system is apt to have quite unusual characteristics. The reason is that the use of such an input inherently implies that the target is not capable of turning again, and this is quite different than the target not being likely to turn again. Another difficulty is that the use of such a maneuver implies that the target pilot will know when the missile is launched.

One of the most useful concepts in system design and the one which is used in this report is to picture the target evasive maneuver as a stationary random process in which the target turns at its maximum possible rate alternately in opposite directions without regard to what the attacking missile is doing. There are several important virtues and comments to be made concerning this input. First of all, a statistical description of the target maneuver process is generally acknowledged to be a desirable one, since target motions cannot be described as unique functions of time. Secondly, it is clear that the maneuver is a severe one and puts the system to a good test; it is often found that systems designed according to theories based on either no maneuver or very weak maneuvers are likely to be in trouble if the target happens to maneuver more severely. Another consideration not generally realized is that the stationary process described above is also applicable to certain important nonstationary processes. In any real problem it is apparent that the inputs are distinctly nonstationary. For instance they are nonstationary because the target motion and noise do not exist for an infinitely long time into the past. However, the nonstationary character of the input is due to the strict mathematical definition. It is clear that in the practical case it makes little difference to the missile so far as miss distance is concerned whether a process persists over an infinite or a finite period so long as the process begins before the end of the attack by an amount equal to or greater than the missile response time. (Of course, the process may terminate any time after the attack is over without affecting the results.) In other words, an infinite period is, for practical purposes, simply one which is longer than the system response time. Thus when the system response times are short, results obtained by means of the stationary input apply directly to an important class of nonstationary problems. The results presented herein are in this category.

Since the systems used in this report have been based on the random maneuver previously described, it is of considerable interest to examine the miss-distance performance for other specific target maneuvers which might be made. It has been pointed out that the random maneuver used herein is a severe one; as a result other less severe maneuvers would be expected to result in smaller miss distances. In order to illustrate this point, the miss distances against several alternative types of maneuvers have been determined, using the transfer functions as optimized for the random maneuver. The results are shown in figure 8. It will be observed in the first place that for input B (a step acceleration varying from -1g to +1g) the miss distances are essentially the same as for the random maneuver. This is as would be expected in view of the discussion in the previous paragraph; since such a single step maneuver can be obtained from the random process by extracting a finite interval of the process, it is equally severe and therefore results in the same miss. At the opposite extreme where the target fails to maneuver at all, figure 8 shows the miss distance to be considerably less since, in this case, the miss is due to noise alone. Other maneuvers will lie between these two extremes. For example the miss distance for input A (a step acceleration varying from 0 to +1g) is seen to be between the no maneuver and the random maneuver case. The results shown in figure 8, however, are not general since they apply to only one operating condition. The figure is intended only to illustrate that less severe maneuvers than used in this report will most certainly result in smaller miss distances, depending on the specific maneuver used.

Applications and Example

The three major uses of equation (6) which are considered in this report are as follows:

First, equation (6) may be used to evaluate the theoretical minimum miss distance for any specific case where missile parameters, target maneuvers, and noise are quantitatively known. For example, for the reference set of parameters, equation (6) gives a miss distance of 20.5 feet (the solution of the exact equations (1) through (4) gives 21.9 feet). This value establishes the theoretical minimum miss distance that could be achieved for these conditions. This result might then be compared to the miss distance of any other system to indicate possibilities for improvement.

Second, the equation may be used in preliminary design to evaluate the relative importance of each of the factors which influence minimum miss distance. Such evaluations are useful in determining those design changes which would be worthwhile in attaining smaller miss distances.

Third, the equation may be used to investigate the effect of parameters which can be expressed as some function of the independent variables given in the equation. An example of this, which will be considered here, is the study of the effects of altitude and Mach number on miss distance for a specific missile and target. For this example, a tail chase is considered.

The target is assumed to maneuver with full acceleration capabilities in the random manner previously described, where the variation of acceleration capability is assumed linear with altitude. Furthermore, the glint noise, which is independent of Mach number and altitude, is again represented by a constant spectral density magnitude. The values of the target maneuver and noise were chosen to represent a medium bomber and are as follows:

$$a_T = 3.5 - 2.5 \text{ altitude}/50{,}000$$

$$N = 15 \text{ ft}^2/\text{radian/sec}$$

$$\overline{T} = 5 \text{ sec}$$

The variation of target acceleration, a_T, with altitude is plotted in figure 9.

The variations of missile characteristics with altitude and Mach number were prescribed to be those of a typical missile as described in reference 11. These variations (acceleration capabilities and natural frequency f_a) are plotted in figure 9. An additional assumption is that the natural frequency $f_b = 1.4f_a$ over the entire range of altitude and Mach number; this is a reasonable assumption for supersonic flight.

With the above information, the effect of altitude and Mach number on minimum miss distance may be readily obtained from equation (6). This computation was made for several Mach numbers over the altitude range of 10,000 to 50,000 feet and the resulting curves are given in figure 10.

Although the example is for a specific case, there are several interesting features of the curves shown in figure 10. First, it can be seen that the miss distance decreases with increasing altitude. Second, an increase in Mach number causes a decrease in miss distance at high altitudes but has little effect at low altitude. The reason for these unusual effects is that at the lower altitude the missile's acceleration capability is fixed by the structural limit of the missile. Since the target acceleration capability continues to increase at lower altitudes, the a_M/a_T ratio is reduced and hence the miss distance is increased. Increasing Mach number at the lower altitude also does little to reduce the miss distance for the same reasons since the missile is operating at, or near, its structural limit.

CONCLUDING REMARKS

The primary objective of this study has been the evaluation of the effects of target and missile characteristics on the minimum miss distance. Consequently, from the designer's viewpoint the equations developed can be used in a preliminary fashion for the evaluation of the missile requirements to achieve a desired miss distance. However, this study is not intended to consider the design problem, that is, the determination of the system transfer functions, since this problem was the subject of reference 2.

The results of this study are intended to be applicable only to guidance systems of the beam-rider type. Nevertheless, unpublished studies indicate that these results may be applicable to guidance systems of the homing type. This problem, however, is beyond the scope of the present report.

Because this study was principally concerned with application to the short-range missile, limitations were placed on the ratio of missile-to-target acceleration. Since there are certain problems in which this ratio becomes quite small (such as might be encountered at very high altitudes or for larger missiles with lower structural limits), it would therefore appear desirable to extend this study to include lower ratios.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., June 26, 1957

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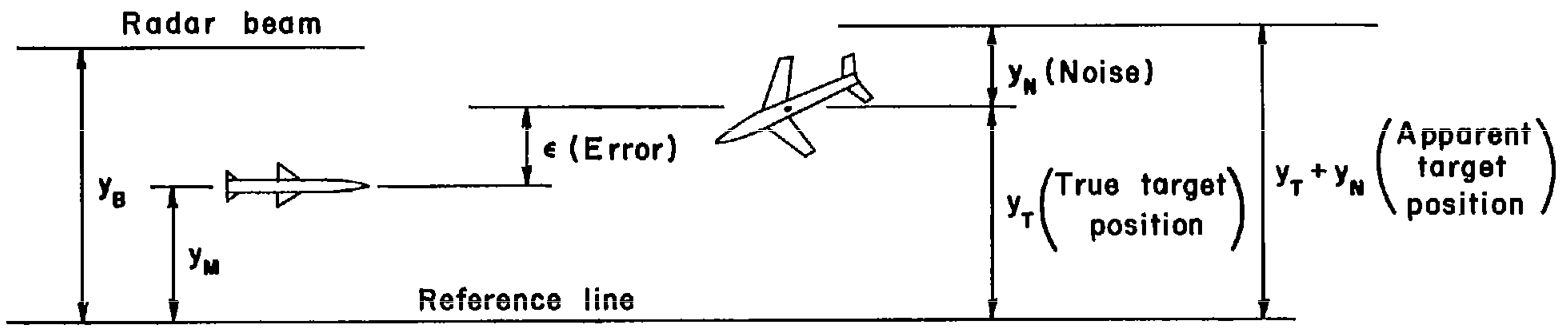
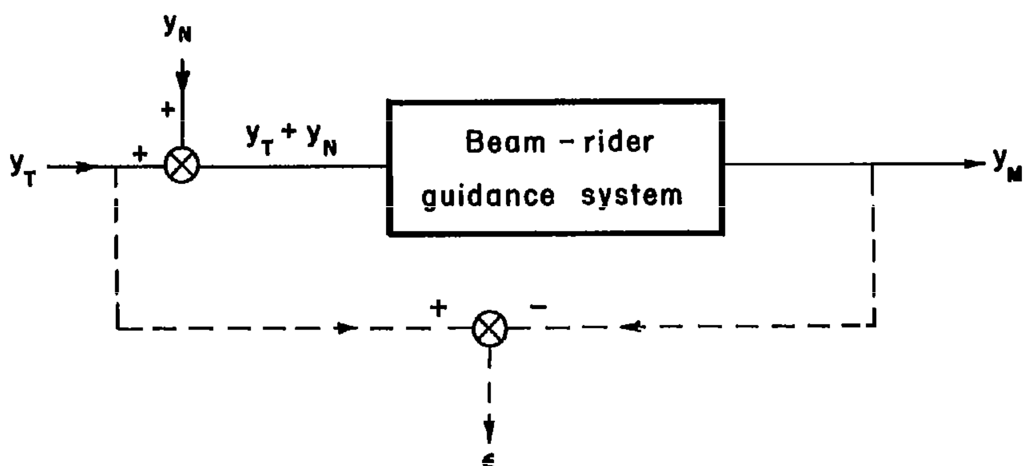
TABLE I.- PHASE II - ORTHOGONAL-SQUARE DATA

Run number	a_M, g units	a_T, g units	N, ft^2/radian/sec	$1/f_a$, 1/cps	$1/f_b$, 1/cps	$\overline{T}$, sec/switch	Miss distance, ft
1	8	2	7.5	1.5	0.375	10	33.8
2	12	.5	7.5	2	1	6.67	15.9
3	20	1.4	7.5	0	0	5	15.6
4	4	.8	7.5	.5	.5	4	22.1
5	5	1.1	7.5	1	0	3	28.0
6	5	.95	13.125	2	1.5	10	28.3
7	8	1.75	13.125	0	0	6.67	31.2
8	12	3.0	13.125	.5	0	5	41.4
9	20	.5	13.125	1	.25	4	16.9
10	4	.65	13.125	1.5	.75	3	26.5
11	4	.5	18.75	0	0	10	21.4
12	5	.75	18.75	.5	.125	6.67	26.0
13	8	1.4	18.75	1	.5	5	34.1
14	12	2.6	18.75	1.5	1.125	4	48.5
15	20	3.0	18.75	2	2	3	34.4
16	20	2.6	24.375	.5	.25	10	31.8
17	4	1	24.375	1	.75	6.67	44.0
18	5	.5	24.375	1.5	1.5	5	23.3
19	8	1	24.375	2	0	4	40.9
20	12	2.1	24.375	0	0	3	39.5
21	12	1.4	30	1	1	10	29.6
22	20	2.1	30	1.5	0	6.67	45.9
23	4	.9	30	2	.5	5	50.3
24	5	1.25	30	0	0	4	52.2
25	8	.5	30	.5	.375	3	22.7



(a) Physical description.



(b) Block diagram.

Figure 1.- Beam-rider guidance system.

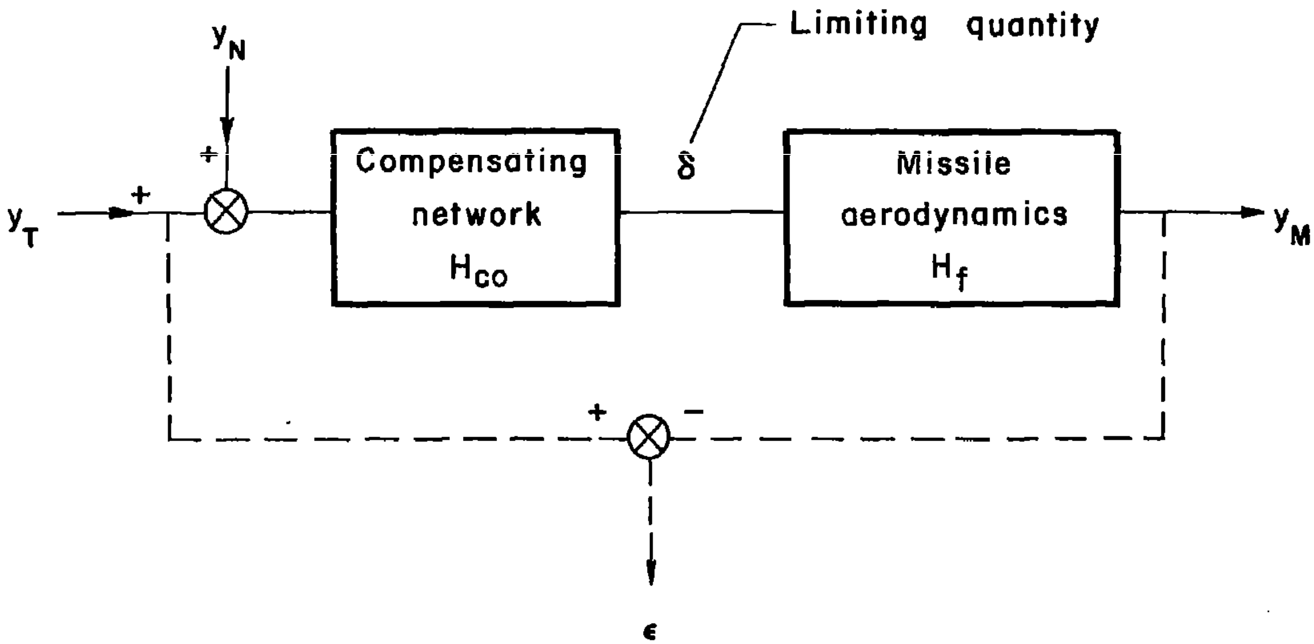
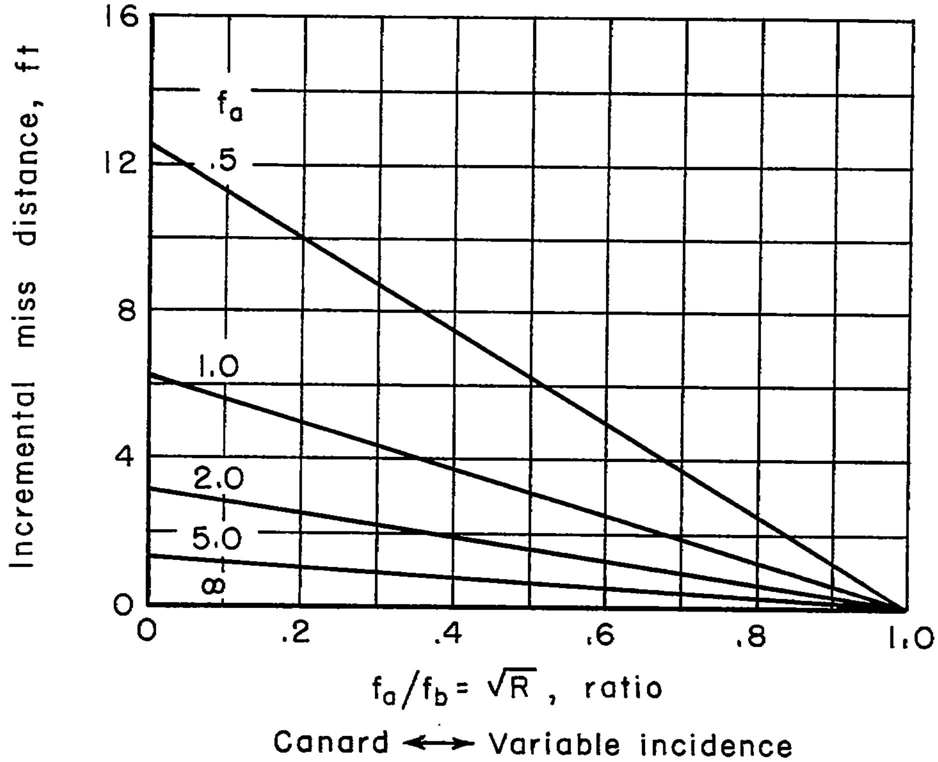


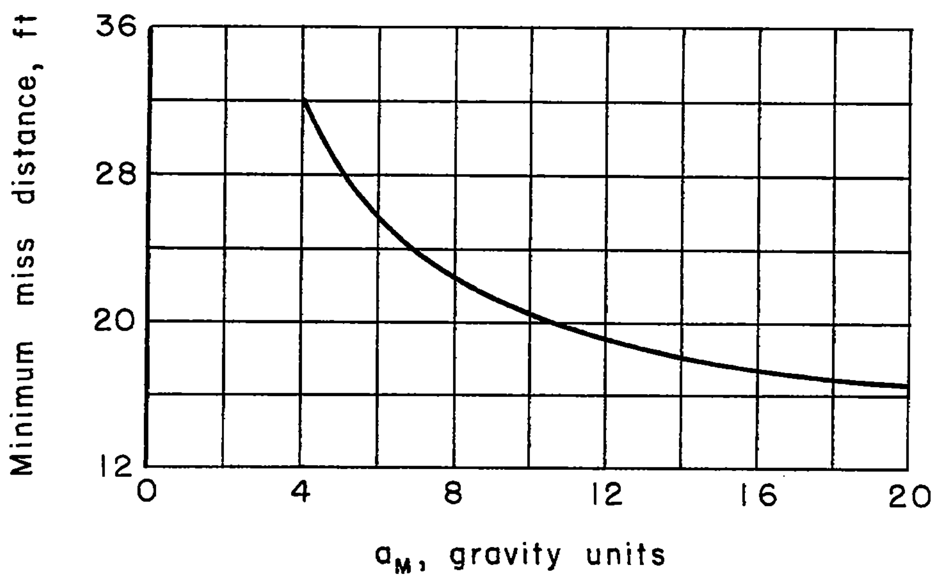
Figure 2.- Block diagram of beam-rider filter problem.

A_1 B_1 C_4 D_3 E_2 F_5	A_2 B_1 C_5 D_4 E_3 F_1	A_3 B_1 C_1 D_5 E_4 F_2	A_4 B_1 C_2 D_1 E_5 F_3	A_5 B_1 C_3 D_2 E_1 F_4
A_1 B_2 C_5 D_2 E_4 F_3	A_2 B_2 C_1 D_3 E_5 F_4	A_3 B_2 C_2 D_4 E_1 F_5	A_4 B_2 C_3 D_5 E_2 F_1	A_5 B_2 C_4 D_1 E_3 F_2
A_1 B_3 C_1 D_1 E_1 F_1	A_2 B_3 C_2 D_2 E_2 F_2	A_3 B_3 C_3 D_3 E_3 F_3	A_4 B_3 C_4 D_4 E_4 F_4	A_5 B_3 C_5 D_5 E_5 F_5
A_1 B_4 C_2 D_5 E_3 F_4	A_2 B_4 C_3 D_1 E_4 F_5	A_3 B_4 C_4 D_2 E_5 F_1	A_4 B_4 C_5 D_3 E_1 F_2	A_5 B_4 C_1 D_4 E_2 F_3
A_1 B_5 C_3 D_4 E_5 F_2	A_2 B_5 C_4 D_5 E_1 F_3	A_3 B_5 C_5 D_1 E_2 F_4	A_4 B_5 C_1 D_2 E_3 F_5	A_5 B_5 C_2 D_3 E_4 F_1

Figure 3.- Orthogonal-square plan.

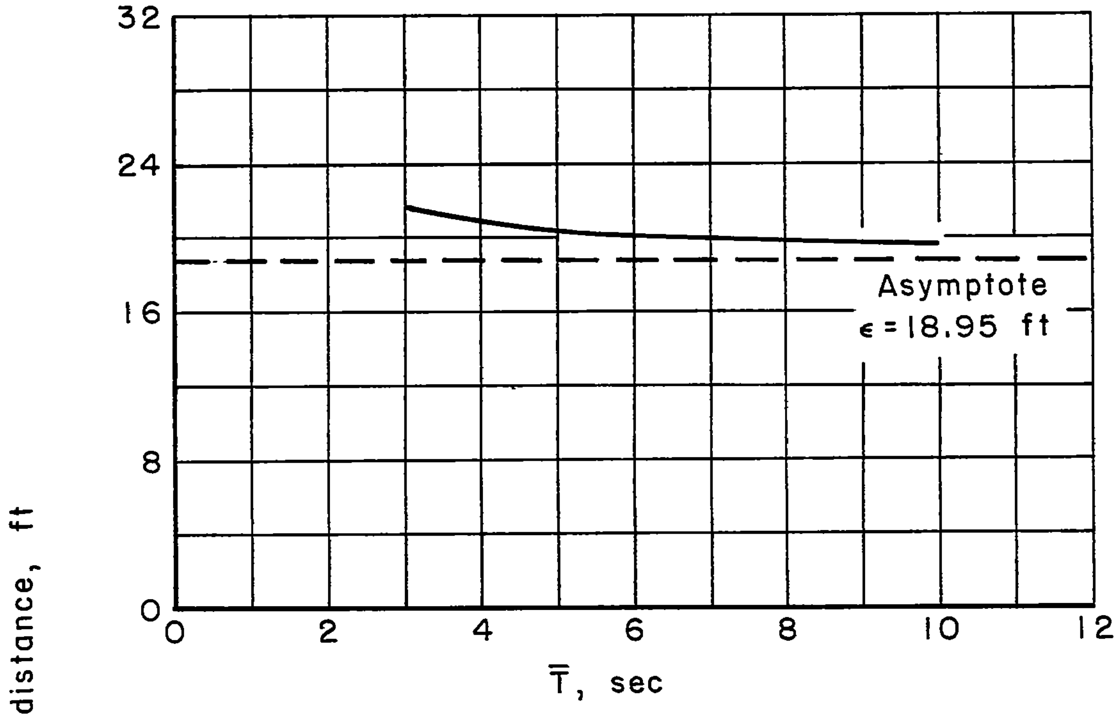


(a) Effect of natural frequencies.

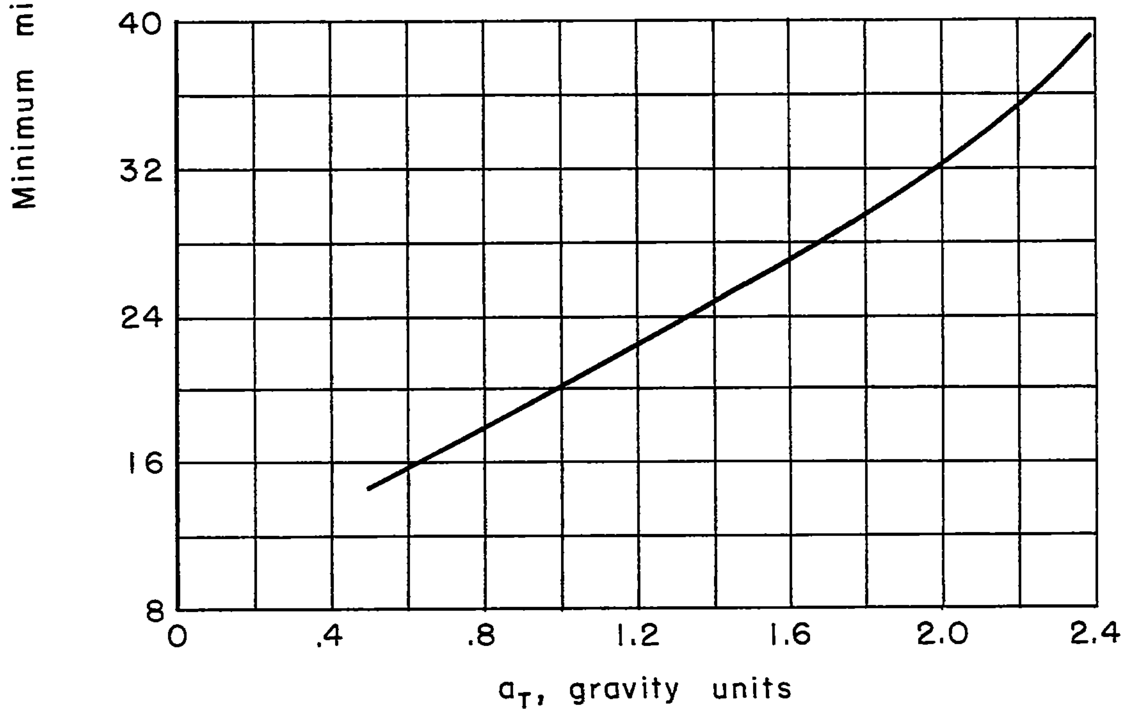


(b) Effect of missile acceleration, a_M.

Figure 4.- Effects of missile parameters on minimum miss distance.



(a) Effect of average switching period, $\bar{T}$.



(b) Effect of target acceleration, a_T.

Figure 5.- Effects of target manuever on minimum miss distance.

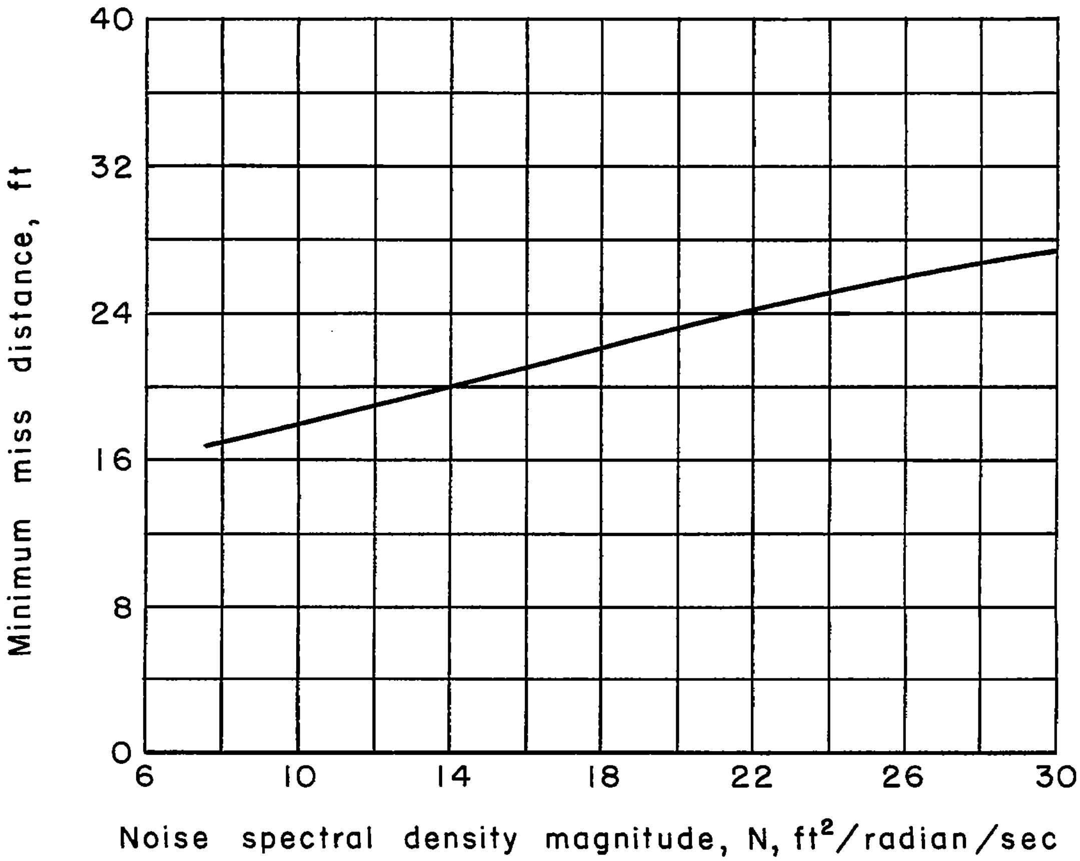
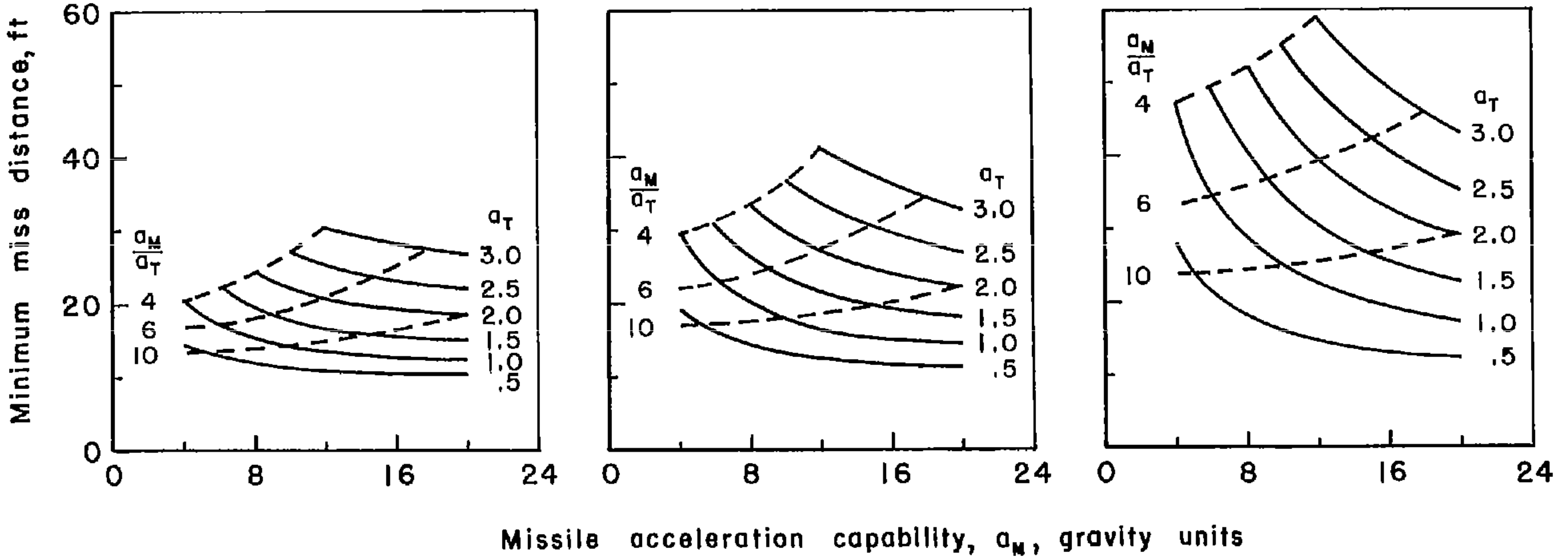


Figure 6.- Effect of noise on minimum miss distance.



(a) N = 7.5 ft^2/radian/sec (b) N = 15 ft^2/radian/sec (c) N = 30 ft^2/radian/sec

Figure 7.- Effects of target acceleration, missile acceleration, and noise on minimum miss distance.

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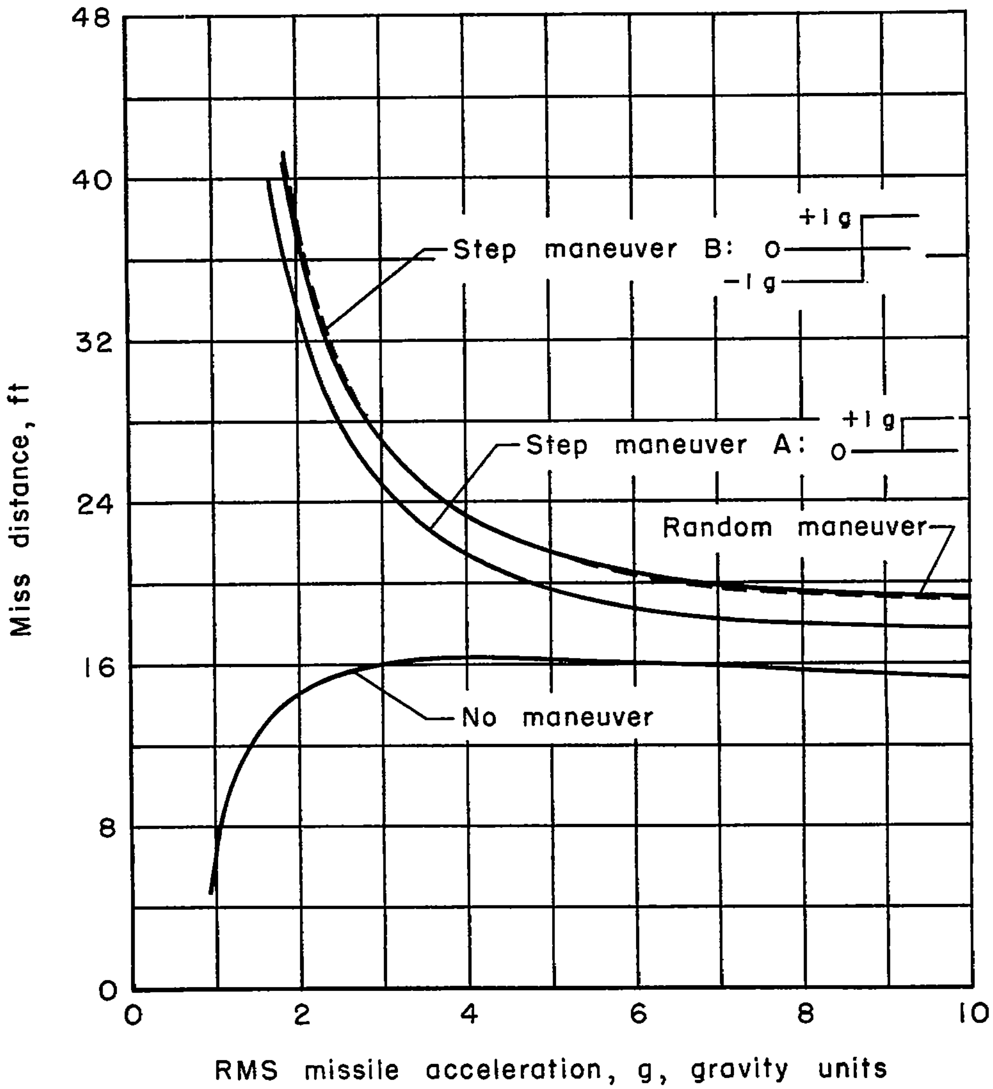

Figure 8.- Effect of type of target maneuver.

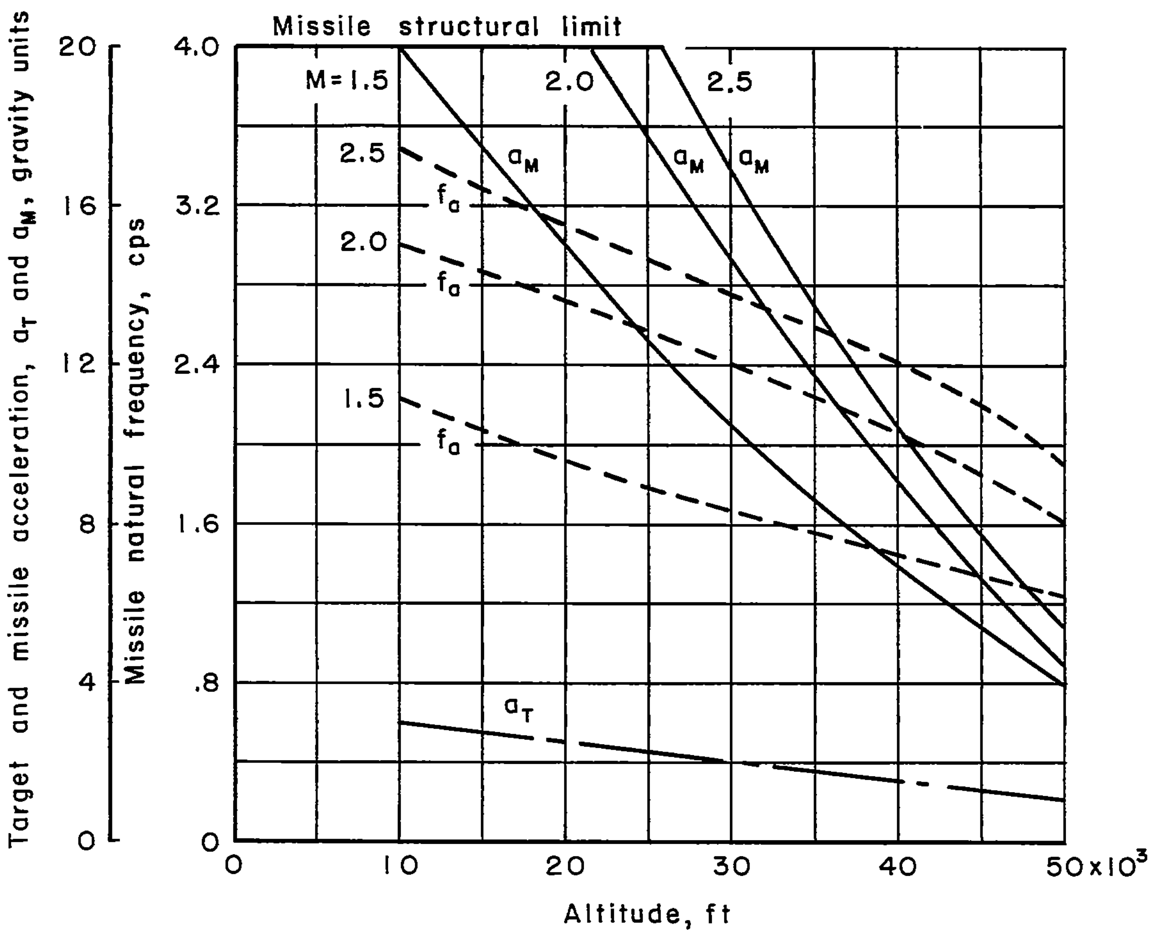

Figure 9.- Missile parameters and target acceleration for example problem.

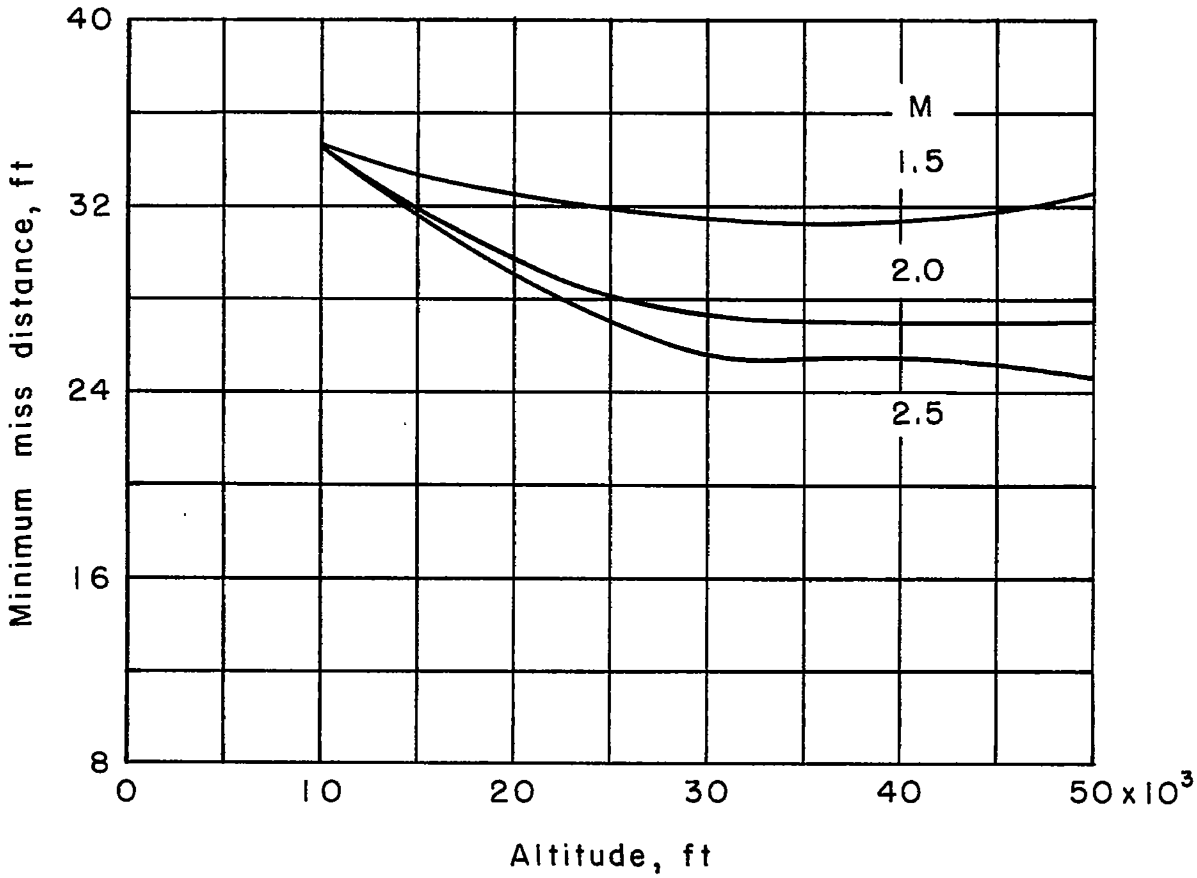

Figure 10.- Effects of altitude and Mach number on minimum miss distance for example problem.

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