Copy 92
RM E57K25a

NACA RM E57K25a

NACA

RESEARCH MEMORANDUM

A METHOD FOR DETERMINING TURBINE DESIGN CHARACTERISTICS FOR ROCKET TURBODRIVE APPLICATIONS

By Warner L. Stewart, David G. Evans, and Warren J. Whitney

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

May 2, 1958

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

A METHOD FOR DETERMINING TURBINE DESIGN CHARACTERISTICS FOR ROCKET TURBODRIVE APPLICATIONS

By Warner L. Stewart, David G. Evans, and Warren J. Whitney

SUMMARY

This report presents a method for making a rapid determination of the design characteristics of turbines for rocket turbodrive applications. Factors considered include relations between turbine flow and weight and rocket weight, as well as such turbine parameters as stage number, size, and pressure ratio. Independent variables used in the report include the amount of pump flow bled off for driving the turbine and the number of turbine stages. The design considerations presented herein are applied to an example rocket application using hydrogen and oxygen as the propellant to illustrate the type of results obtained through use of this method.

INTRODUCTION

The turbopump component of a rocket has the function of increasing the pressure level of the fuel and oxidizer from that of the tanks to the high pressure needed in the thrust chamber. The turbopump unit consists of the pumps which develop the pressure, the turbine that supplies the required power to the pumps, and the necessary gearing that connects the pumps and turbine.

The ultimate merit of a turbopump system for a given rocket is indicated by minimum rocket gross weight. A consideration of rocket gross weight as affected by the turbopump system includes the effect of the dry weight of the components as well as the effect of the propellants consumed by the turbine.

The optimization of the turbine component in this manner for any given application is complicated because a number of variables and limits must be considered. Because of this it is important to have a design method available that includes these effects. The method should permit a rapid screening of a large number of turbine designs to determine the configuration that best suits the application.

This report presents one such method. Independent variables considered include the ratio of turbine flow to pump flow and the number of turbine stages. The report first considers equations to relate the turbine flow and weight variations to over-all rocket weight characteristics. Since the turbine weight is related to its frontal area in addition to its stage number, a subsequent part of the report is devoted to the development of equations necessary to obtain this quantity. In selecting the range of stage number, consideration is given to its dependency on rotor blade speed and specific work output. The analysis results of reference 1 are used to do this. The necessary equations to obtain the turbine pressure ratio are included. The effect of turbine rotor disk and blade stress limitations on the selection of certain turbine parameters is also described.

In order to illustrate the method and type of results that can be obtained, an example application will be described. This example case is a two-stage rocket using hydrogen and oxygen as the propellant. These fluids were chosen as representing an advanced application problem. Also, in the example, the weight of all components other than the turbine is assumed constant and therefore independent of variations in turbine characteristics. Under this assumption a range of turbine designs yielding minimum rocket gross weight is obtained.

SYMBOLS

A	area, sq ft
$\mathcal{A}$	aspect ratio
c_p	specific heat at constant pressure, Btu/lb/$^{\circ}$R
e	disk taper parameter
F	thrust, lb
G	ratio of thrust to gross weight
g	acceleration due to gravity, 32.17 ft/sec^2
$\Delta H'$	total enthalpy requirement of turbine, Btu/sec
h	specific enthalpy, Btu/lb
I	specific impulse, sec
J	mechanical equivalent of heat, 778.2 ft-lb/Btu

K constant of proportionality in turbine weight equation (eq. (14))

n number of turbine stages

P payload, lb

p pressure, lb/sq ft

R gas constant, ft/$^{\circ}$R

r radius, ft

S structural parameter

$\bar{S}$ structural parameter excluding effect of turbine weight

s stress, lb/sq ft

T temperature, $^{\circ}$R

t burning time, sec

U turbine blade speed, ft/sec

V absolute velocity, ft/sec

W weight, lb

w weight-flow rate, lb/sec

y ratio of turbine flow to total pump flow

γ ratio of specific heats

ε ratio of specific impulse in exhausting turbine flow to that of the flow passing through the thrust chamber

η efficiency

η_s turbine efficiency based on static- to total-pressure ratio

λ work-speed parameter, $U_m^2/gJ\,\Delta h'$

ρ density, lb/cu ft

Φ rocket trajectory angle measured from vertical, deg

ω rotative speed, radians/sec

Subscripts:

an annulus

B burnout

b blade

cr critical

d disk

e empty

F frontal at turbine exit

f fuel

g gross

H horizontal

h hub

id ideal

l last stage

m mean

o oxidant

P pump or pumps

p propellant

ST stage

s static

T turbine

t tip

u tangential component

v vertical

x axial component

1 turbine inlet

2 turbine exit

Superscript:

' absolute total state

RELATION BETWEEN TURBINE AND ROCKET

As pointed out in the INTRODUCTION, this report is concerned with a method for screening turbine designs for a specified turbopump application with the objective of obtaining a turbine design that is optimum from a rocket weight standpoint. Thus, it is necessary to first determine equations that relate the two turbine parameters that affect rocket weight, turbine flow and weight, to the weight of the rocket. These equations are now described.

Relation Between Turbine Flow and Ratio of Rocket Gross Weight to Empty Weight

Figure 1 presents a schematic diagram of a typical turbopump system where certain percentages of the fuel and oxidant are bled off at the pump exits. These fluids are then burned in a gas generator, passed through the drive turbine, and thereafter are exhausted overboard.

If none of the pump flow was bled off for powering the turbine, the thrust that would be obtained for a given thrust-chamber specific impulse I would be

$$F = Iw_P \tag{1}$$

However, since a certain percentage of the flow must be diverted through the turbine, this equation must be modified to

$$F = I\left[w_P - (1 - \varepsilon)w_T\right]$$

or

$$F = Iw_P\left[1 - (1 - \varepsilon)y\right] \tag{2}$$

where $y = w_T/w_P$ and represents the percentage of the total pump flow bled off at the pump discharge and ε is the ratio of specific impulse achieved in exhausting the turbine flow overboard to that achieved in the thrust chamber.

The manner in which y affects the rocket weight can be determined through consideration of its trajectory and acceleration characteristics. Figure 2 presents a schematic diagram of a rocket in flight. Excluding the effects of aerodynamic drag and variation of acceleration of gravity with altitude, the following equations can be written relating the force acting on the rocket and the acceleration along the vertical and horizontal directions:

$$F \cos \Phi - W = \frac{W}{g} \frac{dV_V}{dt} \tag{3a}$$

$$F \sin \Phi = \frac{W}{g} \frac{dV_H}{dt} \tag{3b}$$

Then, since

$$W = W_g - w_P t \tag{4}$$

equations (2) and (4) can be substituted into equations (3) to yield

$$\frac{I w_P [1 - (1 - \varepsilon)y] \cos \Phi}{W_g - w_P t} - 1 = \frac{1}{g} \frac{dV_V}{dt} \tag{5a}$$

$$\frac{I w_P [1 - (1 - \varepsilon)y] \sin \Phi}{W_g - w_P t} = \frac{1}{g} \frac{dV_H}{dt} \tag{5b}$$

In order to integrate equations (5a) and (5b) I, ε, and Φ must be known as a function of time. For simplification in this report, however, let I and ε be assumed constant and two limiting trajectory paths be used. For the specifications of vertical flight, $\Phi = 0^\circ$. Thus, equation (5a) can be integrated to

$$\frac{\Delta V_B}{gI[1 - (1 - \varepsilon)y]} = \ln\left(\frac{W_g}{W_e}\right) - \frac{t_B}{I[1 - (1 - \varepsilon)y]} \tag{6a}$$

If the trajectory is horizontal, $\Phi = 90^\circ$ and equation (5b) can be integrated to

$$\frac{\Delta V_B}{gI[1 - (1 - \varepsilon)y]} = \ln\left(\frac{W_g}{W_e}\right) \tag{6b}$$

Acceleration characteristics permit another equation to be developed which can be used in conjunction with the trajectory equations. By defining a parameter G as the ratio of thrust to gross weight, then

$$G \equiv \frac{F}{W_g} = \frac{Iw_p[1 - (1 - \varepsilon)y]}{W_g}$$

Thus,

$$\frac{w_p}{W_g} = \frac{G}{I[1 - (1 - \varepsilon)y]} \tag{7}$$

Also,

$$W_g = W_e + w_p t_B$$

Dividing through by W_g gives

$$1 = \frac{W_e}{W_g} + \frac{w_p}{W_g} t_B \tag{8}$$

Substituting equation (7) into (8) then yields

$$1 = \frac{W_e}{W_g} + G \frac{t_B}{I[1 - (1 - \varepsilon)y]}$$

or solving for W_g/W_e gives

$$\frac{W_g}{W_e} = \left(1 - G\left\{\frac{t_B}{I[1 - (1 - \varepsilon)y]}\right\}\right)^{-1} \tag{9}$$

Equations (6) and (9) can be solved simultaneously for W_g/W_e and t_B for given rocket requirements and limits over a range of the first independent variable y. Thus, the effect of varying y on gross-to-empty weight can be obtained. An example solution of equations (6) and (9) is presented in figure 3 in dimensionless form for G = 1.4. The subsequent effect of variations in W_g/W_e on the over-all rocket weight for a given payload is now described in conjunction with turbine weight considerations.

Relation Between Turbine Weight and Rocket Gross Weight

The variations in rocket gross weight are studied in terms of the ratio of gross weight to payload W_g/P using a structural parameter S. By definition,

$$S \equiv \frac{W_e - P}{W_g - W_e}$$

that is, the ratio of structure weight to propellant weight. Dividing through by W_e gives

$$S = \frac{1 - \frac{P}{W_g}\frac{W_g}{W_e}}{\frac{W_g}{W_e} - 1}$$

Then solving for W_g/P yields

$$\frac{W_g}{P} = \frac{\frac{W_g}{W_e}}{1 - S\left(\frac{W_g}{W_e} - 1\right)} \tag{10}$$

Figure 4 presents equation (10) in graphical form where W_g/P is presented as a function of W_g/W_e for varying S where the effects of varying these parameters on gross weight can be determined.

The derivations presented thus far are for one rocket stage. The effect of multistaging the rocket can be seen through consideration of a two-stage rocket as an example. In this case, the gross weight of the second stage is the payload of the first stage. Since primary interest is in the ratio of over-all rocket gross weight to second-stage payload $(W_g/P)_{over\text{-}all}$, this parameter can be obtained from the equation

$$\left(\frac{W_g}{P}\right)_{over\text{-}all} = \left(\frac{W_g}{P}\right)_{first\ stage}\left(\frac{W_g}{P}\right)_{second\ stage} \tag{11}$$

In the method of analysis presented herein, the effect of variations in turbine weight on the rocket weight is considered through variations in the structural parameter S. The parameter is comprised of the contribution of all component weights. That is,

$$S = (\Delta S_{tank} + \Delta S_P + \Delta S_{gearing} + \Delta S_{others}) + \Delta S_T$$

$$= \bar{S} + \Delta S_T \qquad (12)$$

This report is concerned with equations for obtaining ΔS_T. In an actual study, however, all the components must be considered to evolve an optimum compatible combination since, in general, the tanks, pumps, gearing, and turbine are interdependent.

The equation for ΔS_T is, by definition,

$$\Delta S_T \equiv \frac{W_T}{W_p} = \frac{W_T}{w_P t_B} \qquad (13)$$

An equation for use in obtaining turbine weight was formulated from both published (ref. 2) and unpublished data and is expressed in terms of turbine frontal area representable by that at the turbine exit A_F and stage number n. The equation, as used herein, is

$$W_T = Kn^{1/2}A_F \qquad (14)$$

From single-stage turbine weight studies presented in reference 2, turbine weight was found to vary approximately as A_F for a value of K of approximately 70. Additional unpublished turbine weight studies verified this value for K in addition to fixing the staging effect on turbine weight at approximately $n^{1/2}$.

Substituting equation (14) into (13) then yields

$$\Delta S_T = \frac{Kn^{1/2}}{t_B} \frac{A_F}{w_P} \qquad (15)$$

Since t_B is obtained from the rocket requirements and n is given as the second independent variable, the only other quantity needed to complete the calculation of ΔS_T is the turbine frontal area parameter A_F/w_P. The method used for obtaining A_F/w_P is presented in the section DETERMINATION OF TURBINE FRONTAL AREA PARAMETER. The computed value of ΔS_T can then be used to obtain S (eq. (12)). This quantity can finally be used together with W_g/W_e obtained from the rocket trajectory and acceleration equations to compute the ratio of gross weight to payload (eqs. (10) and (11)).

RELATION BETWEEN TURBINE AND PUMP

Two turbine characteristics are related to the pump, namely power and rotative speed. The turbine power requirement can be obtained through consideration of the pump pressure requirement.

The power required for the fuel pump is

$$J\ \Delta H_f' = w_f \frac{\Delta p_f'}{\rho_f \eta_{P,f}}$$

and that for the oxidizer pump is

$$J\ \Delta H_o' = w_o \frac{\Delta p_o'}{\rho_o \eta_{P,o}}$$

The total power that the turbine must deliver is then the sum of these two powers; that is,

$$J\ \Delta H' = w_f \frac{\Delta p_f'}{\rho_f \eta_{P,f}} + w_o \frac{\Delta p_o'}{\rho_o \eta_{P,o}}$$

Dividing through by $w_P = w_f + w_o$ gives

$$\frac{J\ \Delta H'}{w_P} = \frac{w_f}{w_f + w_o} \frac{\Delta p_f'}{\rho_f \eta_{P,f}} + \frac{w_o}{w_f + w_o} \frac{\Delta p_o'}{\rho_o \eta_{P,o}}$$

or

$$\frac{\Delta H'}{w_P} = \frac{1}{1 + \frac{w_o}{w_f}} \frac{\Delta p_f'}{J\rho_f \eta_{P,f}} + \frac{\frac{w_o}{w_f}}{1 + \frac{w_o}{w_f}} \frac{\Delta p_o'}{J\rho_o \eta_{P,o}} \tag{16}$$

Thus, from equation (16) the total turbine power requirement can be obtained in terms of unit pump flow. This requirement is one of the quantities needed to determine the turbine characteristics.

The other characteristic linking the turbine to the pumps is the rotative speed if no gearing is used. Variation in turbine rotative speed can be considered in terms of variations in the turbine frontal area parameter A_F/w_P. Multiplying numerator and denominator of the parameter by ω^2 yields

$$\frac{A_F}{w_P} = \frac{\pi r_t^2 \omega^2}{w_P \omega^2} = \frac{\pi U_t^2}{w_P \omega^2} \tag{17}$$

Now,

$$rpm = 30\omega/\pi \tag{18}$$

Solving equation (17) for ω and substituting into equation (18) give

$$rpm = 30U_t\left(\pi w_P \frac{A_F}{w_P}\right)^{-1/2} \tag{19}$$

Equation (19) shows that for given turbine parameters (U_t and A_F/w_P) an actual pump flow is required to obtain the turbine rotative speed. If direct drive is used between the turbine and either or both of the pumps, equation (19) must be used to determine A_F/w_P as a function of U_t. If gearing is used, however, the value of A_F/w_P can be selected primarily from turbine considerations.

TURBINE STAGE NUMBER CONSIDERATIONS

As pointed out in the INTRODUCTION, turbine stage number is being considered an independent variable. The range of stage number as well as the associated efficiency is, however, dependent upon the particular value of turbine flow y selected. The manner in which this range of n as well as the associated required static- to total-pressure ratio across the turbine can be obtained is now presented.

Reference 1 presents a method for obtaining the turbine efficiency based on the static- to total-pressure ratio for given turbine stage number, power, and speed requirements. Figure 5, a replot of figure 5 of reference 1, shows the variation in this efficiency with a work-speed parameter λ, defined as the ratio of the turbine mean section rotor blade speed squared to the required specific work output. That is,

$$\lambda = \frac{U_m^2}{gJ\ \Delta h'} \tag{20}$$

The parameter shown in the figure is the stage number n. If λ is known, the associated range of n can be obtained. The upper limit on n (where limiting efficiency occurs) is obtained from the equation

$$n = 1/\lambda \tag{21}$$

which represents a stage λ of unity (see ref. 1).

Turbine Required Work-Speed Parameter

The determination of the work-speed parameter λ involves first the calculation of the turbine mean section blade speed U_m and the specific work output $\Delta h'$. In obtaining the equation for U_m it is assumed that this speed is constant through the turbine and is equal to that at the exit (station 2). This mean section blade speed is related to the turbine-exit hub-tip radius ratio and tip speed by the equation

$$U_m = \frac{1 + \left(\frac{r_h}{r_t}\right)_2}{2} U_{t,2} \tag{22}$$

As will be discussed in the section HUB-TIP RATIO AND TIP SPEED SELECTION, the values of $(r_h/r_t)_2$ and $U_{t,2}$ must be compatible with rotor disk and blade stress limitations.

The turbine specific enthalpy required is

$$\Delta h' = \frac{\Delta H'}{w_T}$$

or, if y is introduced,

$$\Delta h' = \frac{\frac{\Delta H'}{w_P}}{y} \tag{23}$$

The parameter $\Delta H'/w_P$ is obtained from equation (16), and y is one of the two independent variables. Equations (22) and (23) can be substituted into equation (20) to give

$$\lambda = \frac{\left[\frac{1 + \left(\frac{r_h}{r_t}\right)_2}{2}\right]^2 U_{t,2}^2 y}{gJ \frac{\Delta H'}{w_P}} \tag{24}$$

For the range of y selected, values of λ can be obtained from equation (24) for use in determining the range of n from figure 5.

Turbine Required Pressure Ratio

Figure 5 is used not only to obtain the variation in n but also to obtain the associated turbine static-to-total efficiency η_s. The required turbine static- to total-pressure ratio can then be obtained using the following definition of η_s:

$$\eta_s = \frac{\Delta h'}{\Delta h'_{id,s}} \tag{25}$$

where $\Delta h'$ is the actual enthalpy drop (eq. (23)) and $\Delta h'_{id,s}$ is the ideal specific enthalpy drop corresponding to the static- to total-pressure ratio across the turbine. Since $\Delta h'_{id,s}$ is related to this pressure ratio by the equation

$$\Delta h'_{id,s} = c_p T'_1 \left[1 - \left(\frac{p_2}{p'_1}\right)^{(\gamma-1)/\gamma}\right] \tag{26}$$

equations (23) and (26) can be substituted into equation (25) with the resultant equation solved for p_2/p'_1 to yield

$$\frac{p_2}{p'_1} = \left(1 - \frac{\frac{\Delta H'}{w_P}}{y c_p T'_1 \eta_s}\right)^{\gamma/(\gamma-1)} \tag{27}$$

It might be noted that, if, for a given application, a lower limit on p_2/p'_1 is specified (see example), equation (27) can be used to determine the resultant variation in η_s with y (and, hence, λ). In this manner a lower limit on n can also be obtained when these limiting calculation results are drawn on figure 5.

DETERMINATION OF TURBINE FRONTAL AREA PARAMETER

As pointed out in the section entitled Relation Between Turbine Weight and Rocket Gross Weight, the turbine frontal area parameter A_F/w_P must be known or obtained before the rocket weight calculations can proceed. If the turbine is given flexibility of speed, the determination of this parameter is made through consideration of the required turbine pressure ratio and continuity at the turbine exit (station 2). By assuming uniform flow conditions, this continuity equation can be written as

$$w_T = (\rho V_x)_2 A_{an,2}$$

$$= \left(\frac{\rho V_x}{\rho' V_{cr}}\right)_2 A_{an,2}(\rho' V_{cr})_2$$

$$= \left(\frac{p}{p'}\right)_2^{1/\gamma}\left(\frac{V_x}{V_{cr}}\right)_2 A_{an,2}\,\frac{(\rho' V_{cr})_2}{(\rho' V_{cr})_1}\,(\rho' V_{cr})_1$$

$$= \left(\frac{p}{p'}\right)_2^{1/\gamma}\left(\frac{V_x}{V_{cr}}\right)_2 A_{an,2}\,\frac{\dfrac{p_2'}{p_1'}}{\left(\dfrac{T_2'}{T_1'}\right)^{1/2}}\,\frac{p_1'}{RT_1'}\sqrt{\frac{2\gamma}{\gamma+1}\,gRT_1'}$$

or

$$w_T = \left(\frac{p}{p'}\right)_2^{1/\gamma}\left(\frac{V_x}{V_{cr}}\right)_2 A_{an,2}\,\frac{\dfrac{p_2'}{p_2}\,\dfrac{p_2}{p_1'}\,p_1'}{\left(\dfrac{T_2'}{T_1'}\right)^{1/2}\sqrt{T_1'}}\sqrt{\frac{2\gamma}{\gamma+1}\,\frac{g}{R}} \qquad (28)$$

Now,

$$\left(\frac{p}{p'}\right)_2^{1/\gamma}\left(\frac{p}{p'}\right)_2^{-1} = \left(\frac{p}{p'}\right)_2^{-(\gamma-1)/\gamma} = \left(\frac{T'}{T}\right)_2 \qquad (29)$$

Also,

$$A_{an,2} = \left(\frac{A_{an}}{A_F}\right)_2 A_F$$

$$= A_F\left[1 - \left(\frac{r_h}{r_t}\right)_2^2\right] \qquad (30)$$

In addition,

$$\Delta h' = c_p(T_1' - T_2')$$

$$= c_p T_1'\left(1 - \frac{T_2'}{T_1'}\right)$$

Solving for T_2'/T_1' and introducing equation (23) yield

$$\frac{T_2'}{T_1'} = 1 - \frac{\frac{\Delta H'}{w_P}}{y c_p T_1'} \tag{31}$$

Substituting equations (29), (30), and (31) into equation (28) gives

$$w_T = \frac{\left(\frac{T'}{T}\right)_2 \left(\frac{V_x}{V_{cr}}\right)_2 A_F \left[1 - \left(\frac{r_h}{r_t}\right)_2^2\right] \frac{p_2}{p_1'} \frac{p_1'}{\sqrt{T_1'}} \sqrt{\frac{2\gamma}{\gamma + 1} \frac{g}{R}}}{\left(1 - \frac{\frac{\Delta H'}{w_P}}{y c_p T_1'}\right)^{1/2}}$$

Dividing both sides by w_P and solving for A_F/w_P result in

$$\frac{A_F}{w_P} = \frac{y \frac{\sqrt{T_1'}}{p_1'} \left(1 - \frac{\frac{\Delta H'}{w_P}}{y c_p T_1'}\right)^{1/2} \left(\frac{T}{T'}\right)_2}{\sqrt{\frac{2\gamma}{\gamma + 1} \frac{g}{R}} \left(\frac{V_x}{V_{cr}}\right)_2 \left[1 - \left(\frac{r_h}{r_t}\right)_2^2\right] \frac{p_2}{p_1'}} \tag{32}$$

One quantity in equation (32) requiring further derivation in this section is the temperature ratio $(T/T')_2$. From conservation of energy,

$$\left(\frac{T}{T'}\right)_2 = 1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2$$

$$= 1 - \frac{\gamma - 1}{\gamma + 1}\left[\left(\frac{V_x}{V_{cr}}\right)_2^2 + \left(\frac{V_u}{V_{cr}}\right)_2^2\right] \qquad (33)$$

As will be discussed, the axial component of critical velocity ratio $(V_x/V_{cr})_2$ is selected through limiting-loading considerations. The tangential component $(V_u/V_{cr})_2$ is considered as follows:

$$\left(\frac{V_u}{V_{cr}}\right)_2 = \frac{V_{u,2}}{\Delta V_{u,l}} \frac{\Delta V_{u,l}}{U_m} \frac{U_m}{V_{cr,2}} \qquad (34)$$

By using equation (22) and the equation for V_{cr}, an equation for the last term in equation (34) $U_m/V_{cr,2}$ can be written as

$$\frac{U_m}{V_{cr,2}} = \frac{\left[1 + \left(\frac{r_h}{r_t}\right)_2\right] U_{t,2}}{2\left(\frac{2\gamma}{\gamma + 1} gRT_2'\right)^{1/2}}$$

or, by using equation (31),

$$\frac{U_m}{V_{cr,2}} = \frac{\left[1 + \left(\frac{r_h}{r_t}\right)_2\right] U_{t,2}}{\left[\frac{8\gamma}{\gamma + 1} gRT_1'\left(1 - \frac{\frac{\Delta H'}{w_p}}{yc_pT_1'}\right)\right]^{1/2}} \qquad (35)$$

and can be thus obtained.

The parameter $\Delta V_{u,l}/U_m$ is by definition equal to the reciprocal of the stage work-speed parameter. Thus, using equal stage work as was done in reference 1 gives

$$\frac{\Delta V_{u,l}}{U_m} = \frac{1}{\lambda_{ST}} = \frac{1}{n\lambda} \qquad (36)$$

Finally, the parameter $V_{u,2}/\Delta V_{u,l}$ is obtained from the blade velocity diagram considerations. From reference 1 the following conditions are assumed to prevail:

For $0 < \lambda_{ST} < 0.5$,

$$\frac{V_{u,2}}{\Delta V_{u,l}} = \frac{1}{2} - \lambda_{ST} = \frac{1}{2} - n\lambda \tag{37a}$$

For $0.5 < \lambda_{ST} < 1$,

$$\frac{V_{u,2}}{\Delta V_{u,l}} = 0 \tag{37b}$$

Thus, by using equations (34) to (37) the temperature ratio $(T/T')_2$ can be obtained by equation (33).

The selection of a value of $(V_x/V_{cr})_2$ must be made with consideration of the turbine limiting-loading characteristics. If a turbine is operated at a specified speed, decreases in static- to total-pressure ratio result in increases in specific work output. This occurs until a certain pressure ratio is reached where maximum work output is obtained. Further decreases in pressure ratio then reduce the efficiency without increasing the work output.

The pressure ratio where maximum work is first obtained is termed the limiting-loading point and is a result of choking in the plane of the rotor trailing edge at the turbine exit. With no trailing-edge blockage the point would occur at the same time as choking with the exit annulus at station 2. However, as a result of this blockage in the actual case, this point is obtained at values of exit axial Mach number on the order of 0.7. Since the usual practice is to design the turbine with some margin with respect to limiting-loading, axial Mach numbers less than 0.7 are usually prescribed. These considerations must then be made in selecting a value of $(V_x/V_{cr})_2$.

HUB-TIP RATIO AND TIP SPEED SELECTION

From equations (24), (32), and (35) it is evident that values of hub-to-tip radius ratio and tip speed at station 2, $(r_h/r_t)_2$ and $U_{t,2}$, must be selected before the aerodynamic aspects of the turbine design study can be considered. The selection of these two parameters must be made with consideration to disk taper and rotor blade hub centrifugal stress limitations. These considerations will now be made.

Turbine Disk Limitations

The turbine disk limitations are considered herein through consideration of the taper of the disk from the rim to the disk center. This taper is illustrated in figure 6. Reference 3 describes this problem in detail and develops an equation relating the disk taper to the turbine hub-tip radius ratio. This equation, which was obtained for constant disk stress and no center hole, can be written in terms of the symbols used in this report as

$$e = \exp\left[\frac{\rho_d s_b}{\rho_b s_d}\,\frac{\left(\dfrac{1+\dfrac{r_h}{r_t}}{1-\dfrac{r_h}{r_t}} - 1 - \dfrac{2}{\mathscr{A}}\right)^2}{4\left(\dfrac{1+\dfrac{r_h}{r_t}}{1-\dfrac{r_h}{r_t}}\right)}\right] \tag{38}$$

Figure 6 presents equation (38) in graphical form. The disk taper parameter e is shown on the left as a function of the hub-tip radius ratio for nontapered blades and stress-density parameters $\rho_d s_b/\rho_b s_d$ of 1 and 2/3. The plot is for an aspect ratio of 2. These curves can be used to determine whether or not the selected hub-tip radius ratio at the turbine exit lies within prescribed disk taper and stress limitations.

Turbine Rotor Blade Stress Considerations

Reference 3 also presents the development of an equation relating the turbine centrifugal stress at the rotor blade hub to the blade tip speed and hub-tip radius ratio. This equation (eq. (8) in ref. 2) can be rewritten in terms of this report's symbols for the last turbine stage as

$$\frac{\rho_b U_t^2}{s_b} = \frac{2g}{1-\left(\dfrac{r_h}{r_t}\right)^2} \tag{39}$$

where s_b is the rotor hub centrifugal stress using a nontapered blade. Tapering the blade would reduce the stress somewhat.

Equation (39) is also presented in graphical form in figure 6. The tip speed parameter $(\rho_b/s_b)U_t^2$ is shown at the right as a function of the hub-tip radius ratio r_h/r_t. From this curve it can be determined if the selected values of tip speed and hub-tup radius ratio at the turbine exit are within the rotor blade stress limitations.

APPLICATION TO EXAMPLE ROCKET

To illustrate the type of results obtained when applying the equations described in this report, an example application will be described. It will be assumed for this application that variations in the turbine characteristics do not affect the weight of other components, thereby permitting a constant value of structural parameter, excluding the turbine, to be used. Furthermore, the turbine will be given the flexibility of rotative speed and will use a tip speed and hub-tip radius ratio determined from limiting rotor disk and blade characteristics. Under these assumptions and conditions the rocket gross weight characteristic will be presented over a range of y and A_F/w_P to indicate the area of design that yields the lowest rocket gross weight.

The rocket will be assumed to have the following specifications:

(1) Rocket requirements and assumptions

- (a) Vertical flight
- (b) Two stages
- (c) Final burnout velocity, 25,000 ft/sec
- (d) Change in velocity per stage $\Delta V_{B,ST}$, 12,500 ft/sec
- (e) Thrust to gross weight, G = 1.4
- (f) Zero aerodynamic drag

(2) Structural parameter without turbine weight, $\overline{S} = 0.08$ (both stages)

(3) Propellant considerations (both stages)

- (a) Hydrogen fuel ($\rho_f = 4.42$ lb/cu ft)
- (b) Oxygen oxidant ($\rho_o = 71.2$ lb/cu ft)

(c) $w_o/w_f = 3.2$

(d) Specific impulse, $I = 360$ sec

(4) Pump

(a) $\Delta p_P' = 700$ lb/sq in. = 100,800 lb/sq ft

(b) $\eta_P = 0.60$

(5) Turbine (using fuel rich mixture)

(a) $U_{t,2} = 1400$ ft/sec

(b) $(r_h/r_t)_2 = 0.80$

From figure 6 these quantities are compatible with

$s_b = 40{,}000$ psi

$\rho_b = 526$ lb/cu ft

$e = 4$

$\rho_d s_b/\rho_b s_d = 1$

(c) $c_p = 2.12$

(d) $\gamma = 1.36$

(e) $R = 437$

(f) $T_1' = 1860^\circ$ R

(g) $p_1' = 0.90\ \Delta p_P' = 90{,}720$ lb/sq ft

(h) $(V_x/V_{cr})_2 = 0.40$

(i) Turbine weight constant, $K = 70$

(j) $(p_2/p_1')_{minimum} = 0.05$

(k) $\varepsilon = 0$

Determination of Turbine Frontal Area Parameter

Before calculation of the rocket weight characteristics can be made, variation in the turbine frontal area parameter A_F/w_P over the range of selected flow y and stage number n must be determined. The procedure used to obtain this is outlined in table I together with typical calculation results obtained for the example case. Columns 1 to 9 are the quantities either specified or computed by the indicated equations and represent input constants for the turbine aerodynamic phase of the study.

Column 10 represents the first independent variable being considered, y. Column 11 is the computed value of λ required before entering figure 5 to obtain the range of stage number n. Column 12 is then the other independent variable n, with the upper limit being $1/\lambda$ and the lower limit occurring at the limiting turbine pressure ratio. Figure 5 is also used to obtain the turbine required static-to-total efficiency η_s shown in column 13. Columns 14 and 15 finally represent the calculations necessary to obtain the turbine frontal area parameter A_F/w_P (column 15). The calculations necessary to obtain each column are indicated at the bottom of table I where the quantities needed and equations used are specified.

The actual plot used to obtain n and η_s is presented in figure 7. The circles shown in this figure represent the points at which the calculations were made.

Determination of Rocket Weight

Once the turbine frontal area parameter is known, the rocket weight calculations can commence. Columns 16 to 21 of table I itemize the additional required rocket and turbine constants. Then columns 22 to 26 present the necessary calculations to obtain the ratio of over-all rocket gross weight to payload (column 26).

The results of the calculations made for the example case are presented in figure 8. This figure shows a composite of the rocket and turbine primary parameters over the range of turbine flow y considered. The turbine parameters include the frontal area parameter A_F/w_P, presented as the ordinate, with the stage number n shown as solid lines and the static- to total-pressure ratio shown as dot-dashed lines. The upper crosshatched area represents the specified lower limit on this pressure ratio. The lower crosshatched area represents the limitation imposed by the attainable efficiency (fig. 7). Any point within this bounded area represents a turbine design that will meet the pump power requirements.

Also shown in this figure are contours of computed over-all rocket gross weight to payload $(W_g/P)_{over-all}$. Contours of 70, 68, and 67 as well as a line indicating the minimum value of 66.5 are included. Inspection of this figure shows that, for this example case, the turbine flow has the predominate effect on the change in $(W_g/P)_{over-all}$ with the effect of A_F/w_P being of secondary importance. This occurs because, for a given y, as A_F/w_P is varied the stage number n also varies in such a manner as to yield relatively small changes in turbine weight. It can also be seen that when consideration is given to the rocket weight characteristics a fairly well defined turbine design range emerges. One such turbine design could have the following characteristics:

$$y = 0.01$$

$$A_F/w_P = 17\times10^{-4}$$

$$n = 4 \text{ stages}$$

$$p_2/p_1' = 0.05$$

and from figure 7

$$\eta_s = 0.65$$

CONCLUDING REMARKS

This report has described a method of screening turbine designs for a rocket-pump drive application. In the example case minimum rocket gross weight was used as a criterion, with all rocket components except the turbine assumed to be fixed or constant and therefore not affected by turbine operational characteristics. Rocket gross weight was used in this manner to illustrate the combined effects of turbine weight and turbine propellant consumption on over-all rocket performance. In order to optimize an entire rocket design for a given application, the weights of the other major components such as propellant tanks, propellant pumps, and turbopump gearing (if any) must be considered, as well as their interdependent variation with one another. These additional factors must then be considered in an actual application to achieve an optimum compatible combination, which involves a study beyond the scope of this report.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, January 30, 1958

REFERENCES

1. Stewart, Warner L.: Analytical Investigation of Multistage-Turbine Efficiency Characteristics in Terms of Work and Speed Requirements. NACA RM E57K22b, 1958.

2. Colvin, D., Fyffe, R. J., and Sackson, M.: The Effects of Selected Parameters on the Design of Rocket Engine Pumping Plants. SPD 230, M. W. Kellogg Co., May 15, 1949. (Contract W-33-038-ac-14221.)

3. LaValle, Vincent L., and Huppert, Merle C.: Effects of Several Design Variables on Turbine-Wheel Weight. NACA TN 1814, 1949.

TABLE I. - EXAMPLE CALCULATIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\frac{\Delta H'}{w_P}$	c_p	γ	R	T_1', $^{\circ}R$	p_1'	$\left(\frac{V_x}{V_{cr}}\right)_2$	$\left(\frac{r_h}{r_t}\right)_2$	$U_{t,2}$	y	λ	n	η_s	$\frac{p_2}{p_1'}$	$\frac{A_F}{w_P}$
13.93	2.12	1.36	437	1860	90,720	0.40	0.80	1400	0.01	0.0455	3.9 4 6 8 12 16 22.0	0.646 .653 .738 .785 .823 .855 .870	0.050 .053 .085 .105 .120 .134 .140	1.70×10^{-3} 1.62 1.03 .85 .74 .67 .63
Where obtained														
Eq. (16)	Given	Given	Given	Given	Given	Given	Given	Given	Range selected	Column 1 Column 8 Column 9 Column 10 Eq. (24)	Column 11 Fig. 5 Range obtained	Column 11 Column 12 Fig. 5	Column 1 Column 2 Column 3 Column 5 Column 10 Column 13 Eq. (27)	Columns 1 to 12 Column 14 Eqs. (32) to (37)

16	17	18	19	20	21	22	23	24	25	26
$\Delta V_{B,ST}$	G	$\overline{S}$	I	ϵ	K	$\left(\frac{W_g}{W_e}\right)_{ST}$	$t_{B,ST}$	ΔS_T	S	$\left(\frac{W_g}{P}\right)_{over\text{-}all}$
12,500	1.4	0.08	360	0	70	5.31	206.7	0.00113 .00109 .00086 .00081 .00087 .00090 .00101	0.08113 .08109 .08086 .08081 .08087 .08090 .08101	66.78 66.74 66.53 66.49 66.54 66.57 66.66
Where obtained										
Given	Given	Given	Given	Given	Given	Column 10 Column 16 Column 17 Column 19 Column 20 Eq. (6a) Eq. (9)	Column 10 Column 16 Column 17 Column 19 Column 20 Eq. (6a) Eq. (9)	Column 12 Column 15 Column 21 Column 23 Eq. (15)	Column 18 Column 24 Eq. (12)	Column 22 Column 25 Eq. (10) Eq. (11)

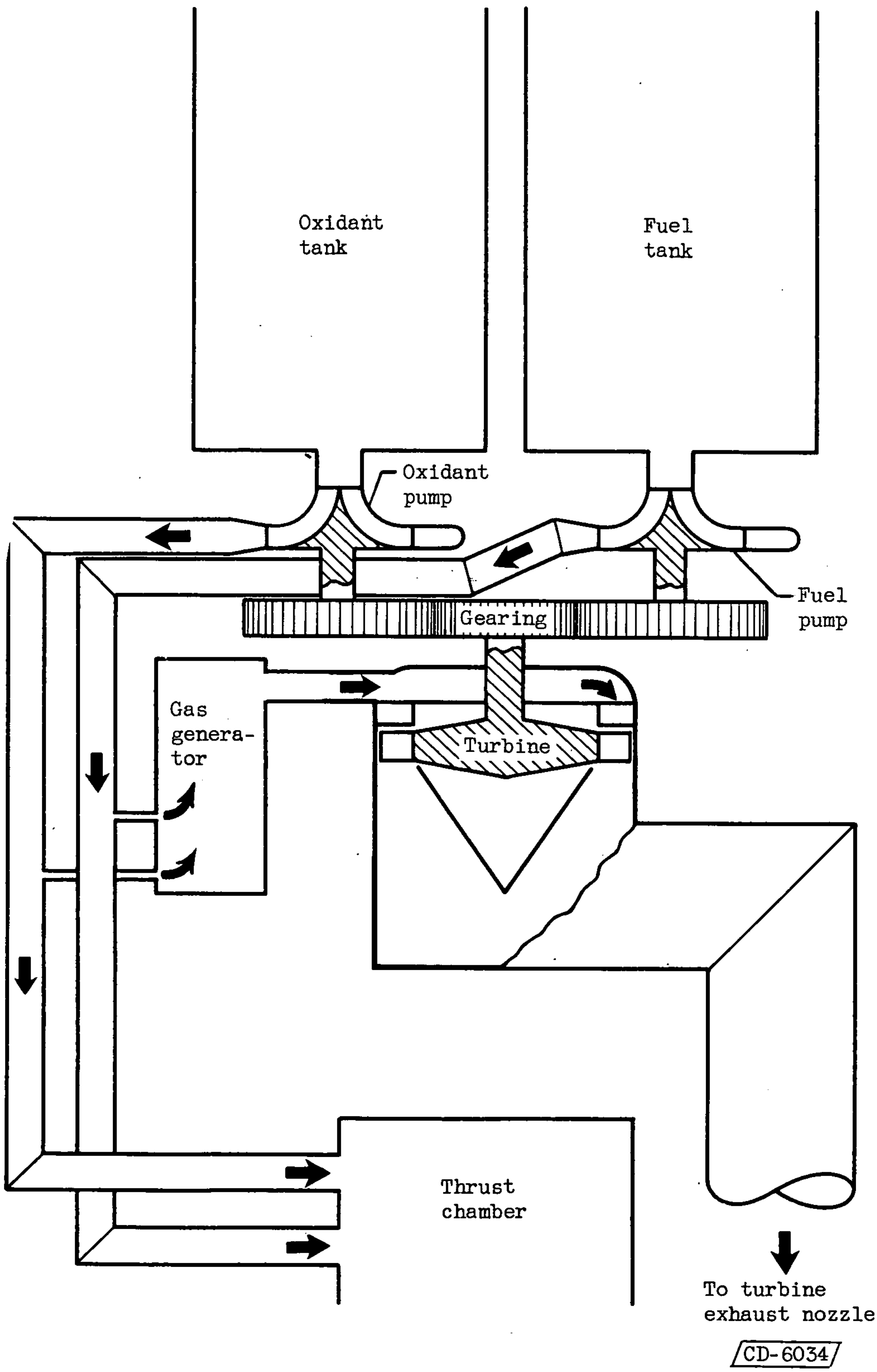

Figure 1. - Schematic diagram of typical turbopump system.

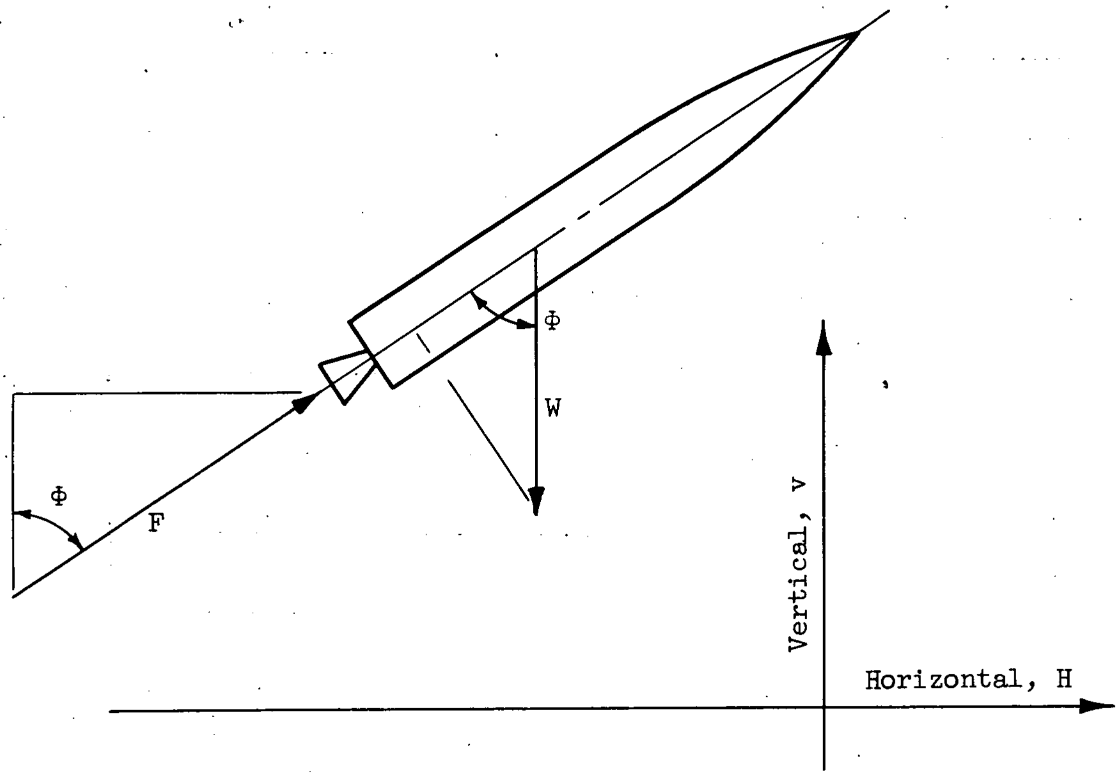

Figure 2. - Schematic diagram of rocket showing relation between thrust, weight, and attitude.

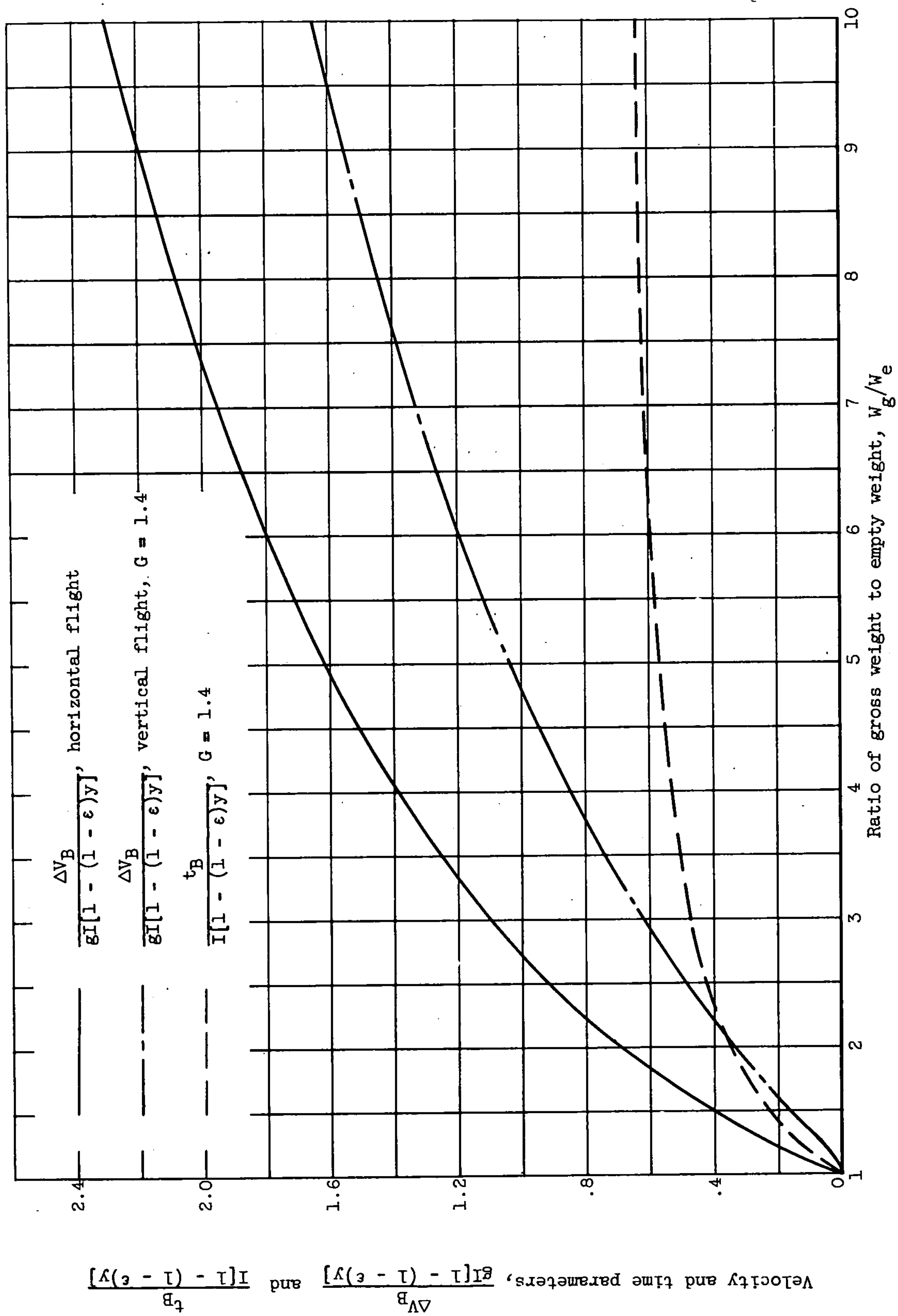

Figure 3. - Rocket velocity-weight characteristics. Zero aerodynamic drag, no variation in g with altitude.

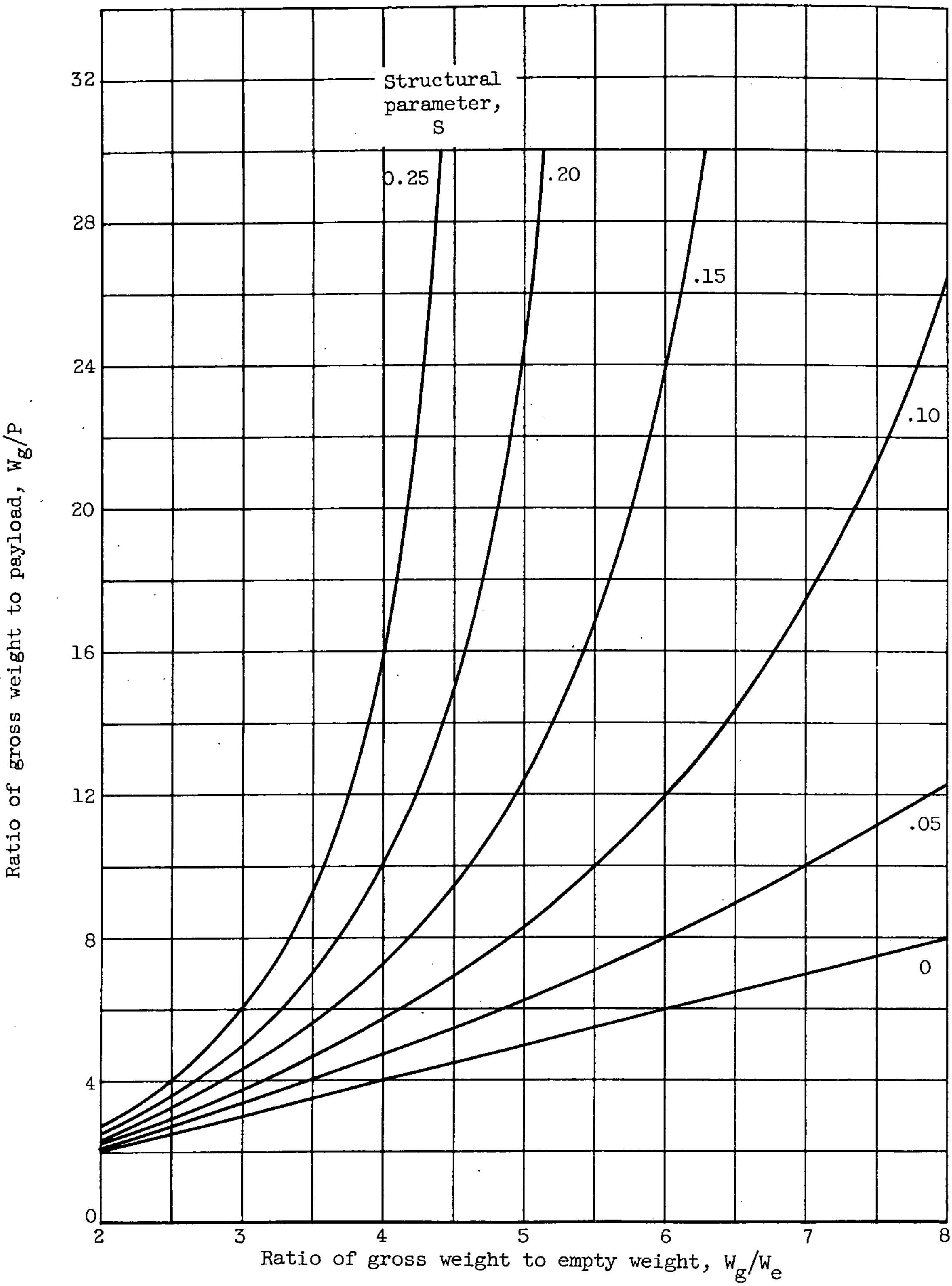

Figure 4. - Effect of structural parameter and ratio of gross weight to empty weight on ratio of gross weight to payload.

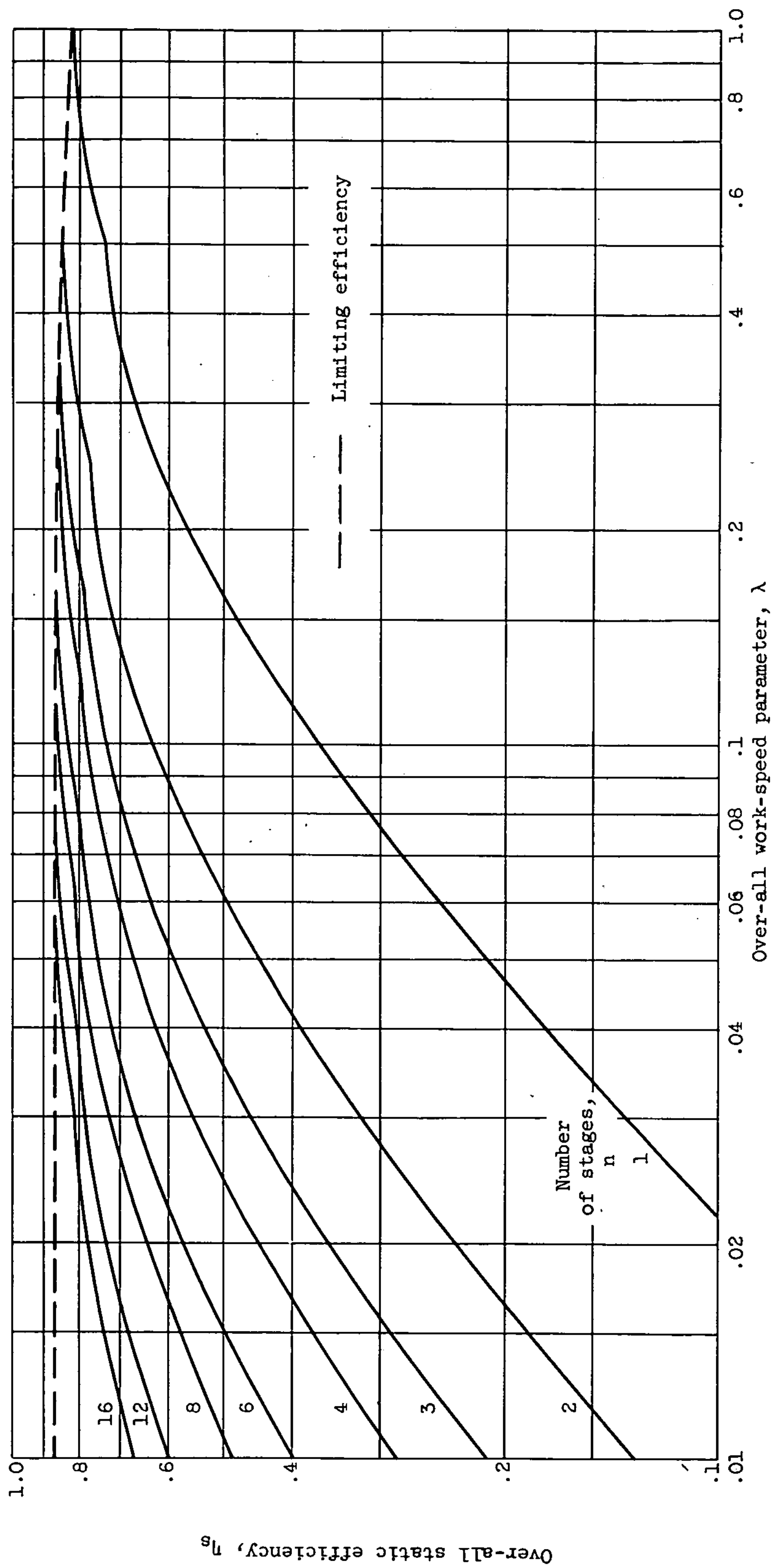

Figure 5. - Multistage turbine efficiency characteristics (ref. 1).

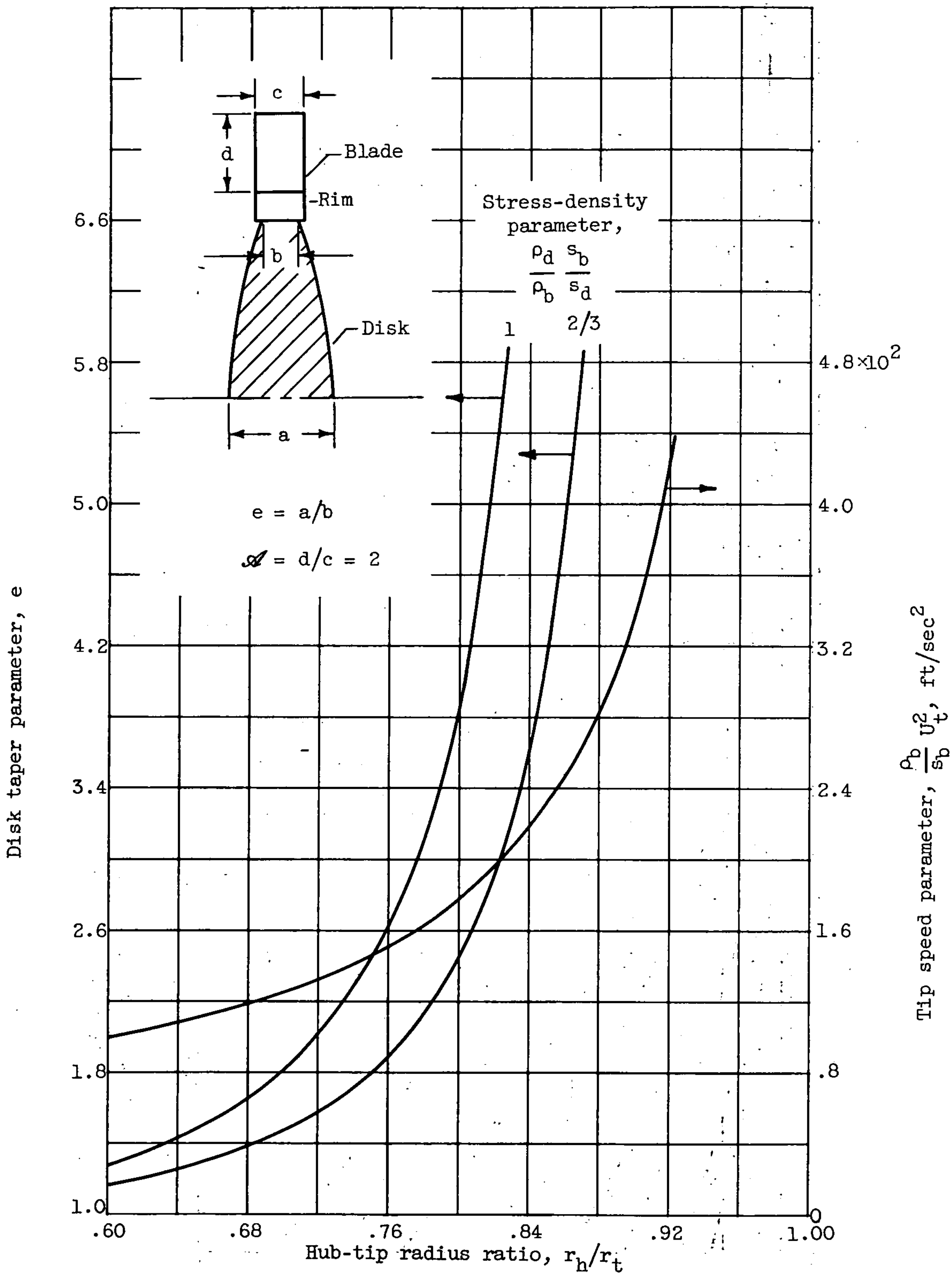

Figure 6. - Turbine disk taper and rotor stress considerations.

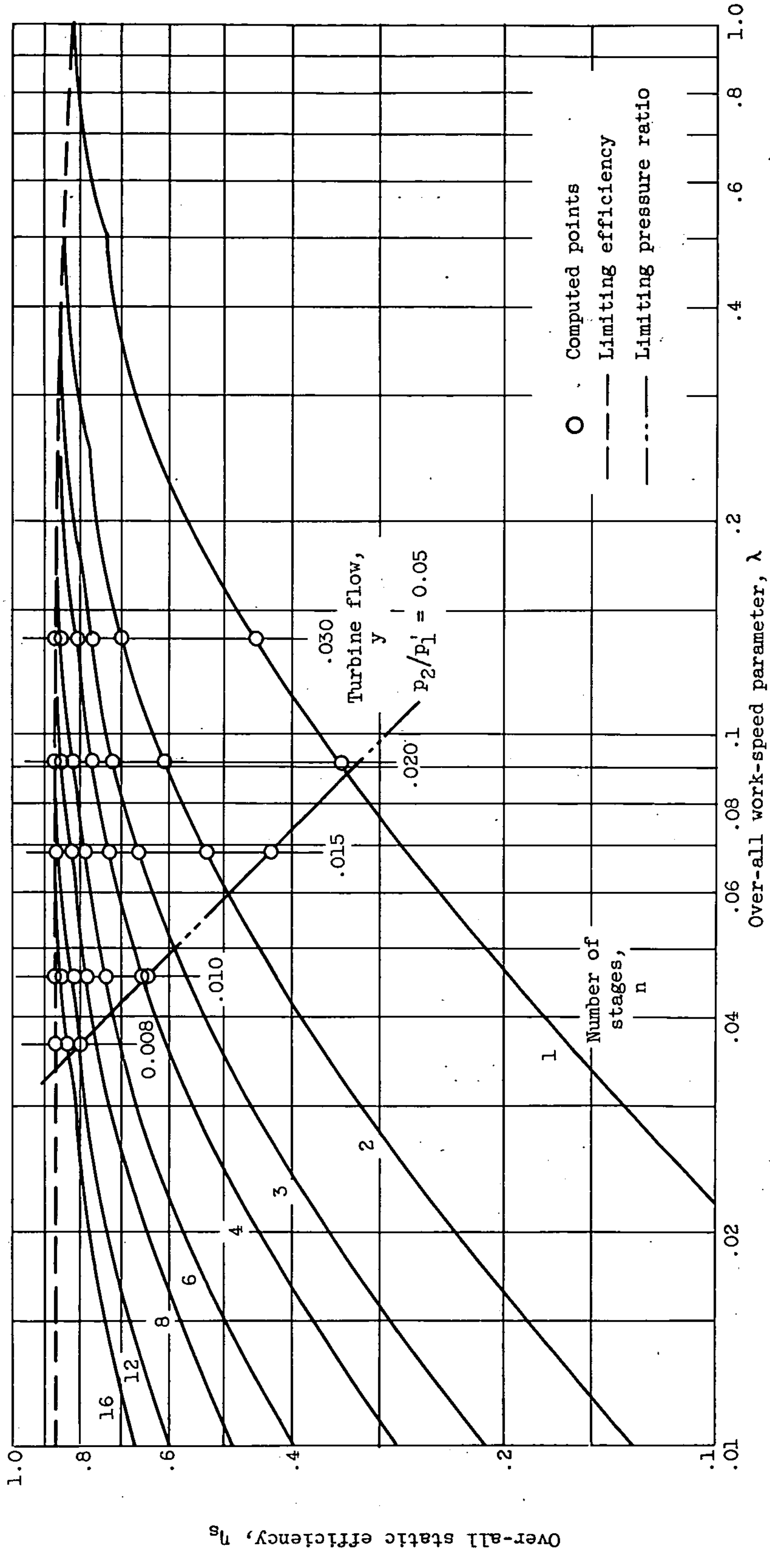

Figure 7. - Application of multistage turbine efficiency characteristics to example case.

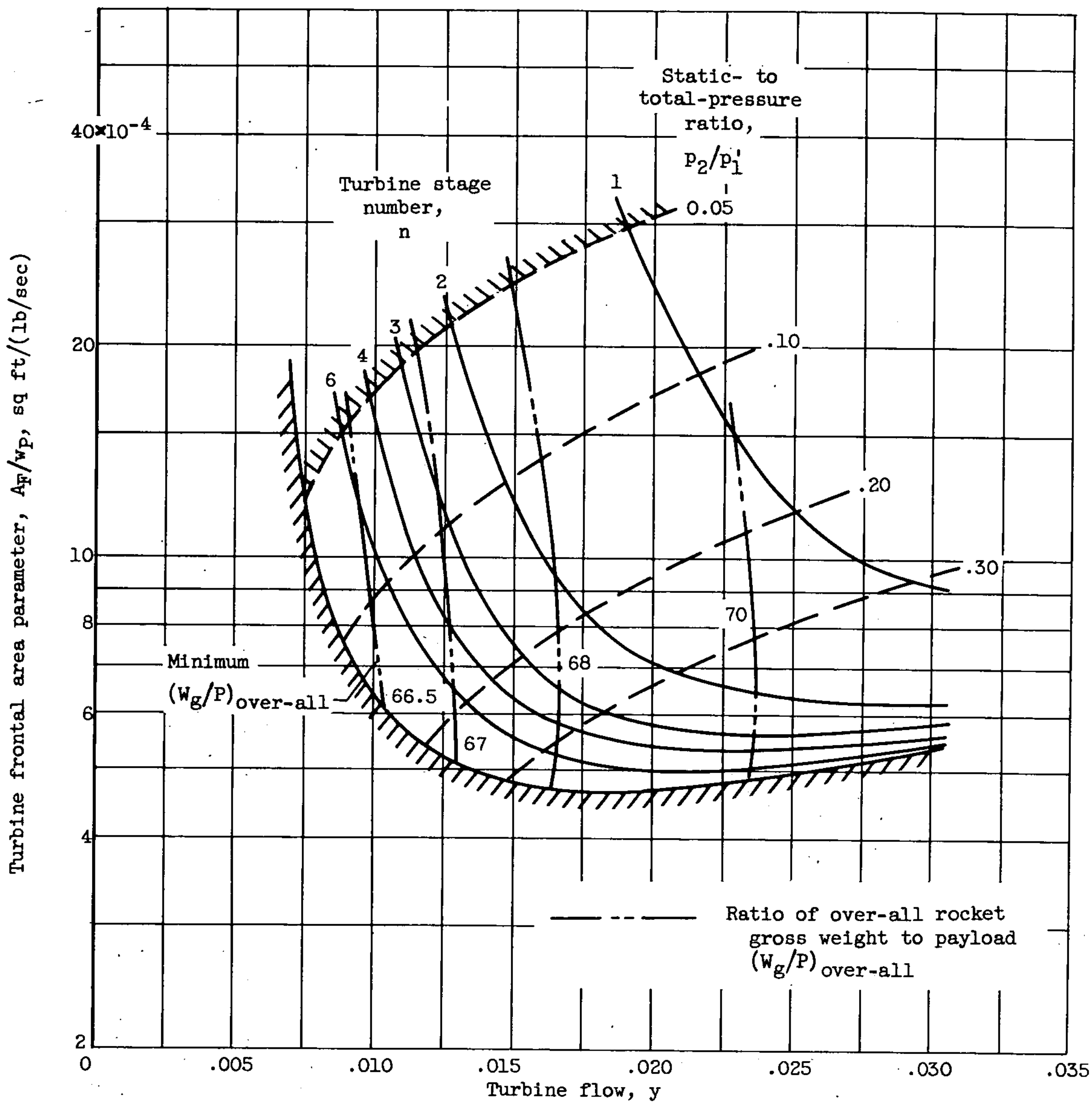

Figure 8. - Example turbine characteristics including effect on ratio of over-all rocket gross weight to payload.

CONFIDENTIAL