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RESEARCH MEMORANDUM

ANALYTICAL INVESTIGATION OF OFF-DESIGN PERFORMANCE OF A TRANSONIC TURBINE

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RESEARCH MEMORANDUM

ANALYTICAL INVESTIGATION OF OFF-DESIGN PERFORMANCE OF A TRANSONIC TURBINE

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SUMMARY

The off-design performance and a breakdown of the losses of a transonic turbine were determined by an analytical method that was previously developed for turbines of more conservative design. The analytically obtained performance map is compared with the performance map obtained from an experimental investigation of the turbine. The rotor hub conditions of incidence angle, relative Mach number, and reaction calculated from the analytical results are compared with those calculated from experimental data.

The loss breakdown obtained for the transonic turbine did not differ substantially from that previously obtained from a turbine of more conservative design, except that large stator-exit shock losses were predicted for the transonic turbine at low speeds. The trends of the rotor hub incidence angle, relative Mach number, and reaction calculated from the analytical results agreed well with those calculated from the experimental data over the performance range. These trends indicate that, compared with a turbine of more conservative design, the transonic turbine operated over a much smaller range of incidence angle, a much wider range of rotor relative Mach number, and at a considerably lower level of reaction. Good over-all agreement was obtained between the analytically predicted performance and the experimental performance, except at 40-percent design speed, where in the analysis the stator reached limiting loading before the rotor choked. Since this discrepancy resulted from errors in the simplifying assumptions used in the analysis, it is regarded as a limitation in the analytical method as applied to a transonic turbine.

INTRODUCTION

Recent research on turbines operating with relative rotor velocities in the transonic range has made this type of design appear feasible as a jet-engine component (e.g., ref. 1). It is therefore desirable to be able to determine the off-design performance for this type of turbine as well as the trends of the losses at off-design operation. An analytical method of predicting off-design turbine performance and losses was developed and is presented in reference 2. The entire turbine performance map can be predicted by this method from either the known or estimated design-point data. In reference 3 the method was applied to nine turbine designs, and a comparison between the predicted and the experimental performance showed the method to be fairly reliable in predicting off-design performance for turbines of conservative design. Reference 4 presents an analytical method of predicting off-design turbine performance and losses similar to that of reference 2, 3248 except that this method includes certain refinements in ascertaining the losses and the flow conditions throughout the turbine. The method of reference 4 was applied to a high-speed, high-specific-mass-flow turbine, and very close agreement was obtained between the analytically predicted off-design performance and that obtained experimentally.

An analysis was therefore made of the transonic turbine of reference 1 by the method of analysis of reference 4. (The term "method of analysis" used hereafter refers to the method of ref. 4.) In addition to the over-all performance and a breakdown of the losses, the rotor hub incidence angle, relative entrance Mach number, and reaction were also obtained in the analytical procedure. The purpose of this report is to present the results of the analysis and to compare them with the experimental results of the transonic turbine. The variation of the losses obtained analytically for the transonic turbine is compared with that previously obtained from a similar analysis of a subsonic turbine. Any limitations encountered in the analytical procedure are also discussed. The trends of the aforementioned rotor hub conditions for the transonic turbine are also compared with those obtained previously for the subsonic turbine to ascertain whether the variation of trends for the transonic turbine differed substantially from that of the subsonic turbine.

The investigation was conducted at the NACA Lewis laboratory.

SYMBOLS

The following symbols are used in this report:

c_p	specific heat at constant pressure, Btu/(lb)(°F)
c_v	specific heat at constant volume, Btu/(lb)(°F)
g	acceleration due to gravity, 32.174 ft/sec^2
$\Delta h'$	equivalent specific work based on total state, Btu/lb
K	blade effective loss parameter (ref. 4)

N	rotational speed, rpm
p	pressure, lb/sq ft
$p'_{6,x}$	turbine-outlet total pressure containing only the axial component of outlet velocity, defined as follows: $p'_{6,x} = p_6 \left[1 + \frac{(\gamma - 1)V_{x,6}^2}{\gamma 2gRT_6}\right]^{\frac{\gamma}{\gamma - 1}}$, lb/sq ft
R	gas constant, ft/°F
T	absolute temperature, °F abs
V	absolute gas velocity, ft/sec
w	weight-flow rate, lb/sec
γ	ratio of specific heats, c_p/c_v
δ	ratio of inlet-air pressure to NACA standard sea-level pressure, p'_0/p^*
ε	function of γ, $\frac{\gamma^*}{\gamma}\left[\frac{\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma - 1}}}{\left(\frac{\gamma^* + 1}{2}\right)^{\frac{\gamma^*}{\gamma^* - 1}}}\right]$
η	efficiency $= \frac{\Delta h'}{c_p T'_0\left[1 - \left(\frac{p'_6}{p'_0}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$
η_x	rating efficiency $= \frac{\Delta h'}{c_p T'_0\left[1 - \left(\frac{p'_{6,x}}{p'_0}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$

θ_{cr}	squared ratio of critical velocity at turbine inlet to critical velocity at NACA standard sea-level temperature, $(V_{cr}/V_{cr}^{*})^2$

Subscripts:

cr	conditions at Mach number of 1.0
x	axial component
0	stator inlet (fig. 1)
1	stator throat
2	station just inside stator trailing edge
3	stator outlet, rotor inlet
4	rotor throat
5	station just inside rotor trailing edge
6	rotor outlet, turbine outlet

Superscripts:

'	absolute total state
*	NACA standard conditions

CALCULATIONS

Analytical Performance and Losses

The over-all analytical performance and loss breakdown were calculated in the same manner as described in reference 4, except that the stator- and rotor-exit shock losses were based on the free-stream velocities (stations 3 and 6, fig. 1) rather than on those at stations 2 and 5. However, the difference in shock loss resulting from this assumption is felt to be negligible, since the velocities at stations 2 and 3 are nearly equal, as are those at stations 5 and 6. Because it was developed for subsonic turbines, the method assumes that the rotor passage is convergent and the minimum area is at the exit of the passage. For the transonic turbine of reference 1, the minimum area is about midway between blade entrance and blade exit, the difference in flow area, however, between the minimum-area station and the blade exit being only of the order of 0.1 of 1 percent. The value 0.45 of the loss parameter K was used to correlate the experimental and analytical performance at design point.

Rotor Hub Conditions

Calculated from analysis. - The variation of the flow conditions relative to the rotor was obtained over the performance range for the hub section, because these conditions are considered to be most critical at this section. The procedure of reference 4 used in the analysis of the transonic turbine is one-dimensional in that the average area at the various stations through the stator and rotor blading and the blade mean-section flow angles are used to obtain average flow conditions. The average flow conditions are substantially equal to the blade mean-section flow conditions for a free-vortex design. The total pressure was assumed the same for the hub section as for the blade mean-section at stations 2 and 5. At station 3 the hub flow conditions were calculated from the blade mean-section flow conditions with the assumption of free-vortex velocity distribution and simple radial equilibrium. At station 4 the hub flow conditions were obtained from the mean-section flow conditions by integrating equation (3) of reference 5 and assuming that simple radial equilibrium existed and that the flow followed the blade angle at station 4. The hub conditions at station 5 were calculated from those at station 4 with continuity and constant angular momentum between the two stations assumed. Although it was necessary to use simplifying assumptions to ascertain the hub conditions, the accuracy of the procedure was considered sufficient to indicate the desired trends. When the hub flow conditions were known, it was possible to calculate static-pressure ratio across the rotor hub (reaction), hub relative inlet Mach number, and hub rotor incidence angle.

Calculated from experimental performance. - The aforementioned hub conditions were also calculated from the experimental data as follows: The static-pressure ratio across the rotor hub was obtained from an average static pressure measured by taps located upstream and downstream of the rotor on the inner wall. The inner taps were located on the stator inner shroud (downstream of the stator) and were positioned circumferentially in the center of the projected stator passage. The free-stream velocity out of the stator at the inner wall was calculated by assuming an isentropic expansion from the turbine-inlet total state to the stator-outlet static pressure measured at the inner wall. The stator-outlet free-stream flow angle at the inner wall was calculated as described in reference 4 with the assumptions that no change occurred in angular momentum between the stator exit (station 2) and the free-stream condition (station 3) and no change occurred in the radial streamline height between the two stations. Thus, from the calculated stator-outlet free-stream conditions, the rotor hub incidence angle and relative Mach number were calculated.

RESULTS AND DISCUSSION

Over-All Performance

The over-all turbine performance map obtained by the analysis is presented in figure 2(a), where equivalent specific work $\Delta h'/\theta_{cr}$ is plotted as a function of the weight-flow - speed parameter $\varepsilon wN/\delta$, with contours of rating total-pressure ratio $p_1'/p_{6,x}'$, speed, and rating efficiency η_x superimposed. The comparable performance map obtained experimentally is shown in figure 2(b). These figures show that good over-all agreement was obtained between the calculated and experimental performance, except at 40-percent design speed. At this speed it was indicated in the analysis that stator limiting loading occurred before the rotor choked, and no analytical solution was obtained above the pressure ratio corresponding to this condition. Possible reasons for this discrepancy are discussed later.

It may also be noted from figure 2 that the specific work at limiting loading agreed within 2 percent over the range of speeds. This represents a very good check, as previous analyses (ref. 4, e.g.) have indicated that it is difficult to predict accurately the limiting-loading specific work.

Loss Breakdown

Curves of the breakdown of the various losses analytically predicted for this turbine are presented in figure 3. The losses are represented as a part of the isentropic enthalpy drop, and the bottom line is the efficiency based on axial-component outlet total pressure $p_{6,x}'$. Curves of two efficiencies, one based on outlet total pressure and one on axial-component outlet total pressure, which were obtained from a cross plot of the experimental data of reference 1, are included on the figures for comparison. The difference between these two efficiencies is an experimental evaluation of the exit-whirl loss. The magnitude of this loss as analytically predicted is about the same as that obtained experimentally over the range investigated. A comparison of the efficiency (based on outlet axial-component total pressure) from the analysis with that obtained experimentally shows good agreement except at the two lower speeds. The loss breakdown for the transonic turbine is similar to that obtained for the more conservative turbine of reference 4, with one exception. The stator-exit shock losses are quite substantial for the transonic turbine at the two lower speeds; whereas, for the turbine of reference 4, the stator was not choked and no stator-exit shock losses were predicted.

Although the stator viscous losses, as presented, might seem quite high, it should be pointed out that in this form they represent a percentage of the isentropic work available to the turbine that has become unavailable because of total-pressure losses in the stator. The flow coefficient for the stator (corresponding to these stator losses) varied between 0.96 and 0.97 over the range of performance. This value of flow coefficient is representative of that obtained experimentally and yields the correct weight flow, although it results in analytically determined stator viscous losses that are considered somewhat high.

A value of the loss parameter K was determined that would better correlate the data at 40-percent design speed. It was found that K would have to be increased from 0.45 to 0.715, and the efficiency for this value of K is also shown in figure 3(e). Although the higher value of K correlated the efficiencies at 40-percent design speed, the resulting weight-flow predicted by the analysis was 2 percent low, whereas the original value of K yielded correct weight flows. Thus, it is felt that the original value of K was correct for the stator and that the value of K for the stator was invariant over the performance range. For the rotor, however, it is believed that K probably is affected by incidence angle and reaction. Since the largest positive incidence angles, lowest reaction, and highest blade entrance Mach numbers were encountered at 40-percent design speed, as is shown in a later section, it is quite possible that these conditions induced separation in the blade passage, with accompanying substantial total-pressure losses. These additional total-pressure losses, which were not considered in the analysis, could in turn cause the rotor to choke and reach limiting loading. Although the assumption, that K for the rotor remained invariant over the performance range, was good for a more conservative turbine design, any factors affecting K or the incidence loss assumption would cause greater discrepancies in a turbine design of this type where the Mach number level (and the resulting flow losses) is considerably higher. This effect would be greatest at the lower speeds where the rotor flow conditions are most critical with respect to relative Mach number, incidence angle, and reaction, as is shown in the following section. However, since there is no provision to evaluate these effects or additional losses in the analytical method, the discrepancy must be regarded as a limitation of the method as applied to a transonic turbine.

Rotor Hub Conditions

The trends of the rotor hub conditions of incidence angle, relative Mach number, and reaction were calculated from the analytical results as well as from the experimental data to determine whether the variation of these trends over the performance range differs markedly from that of a more conservative turbine design such as that of reference 4. The rotor hub incidence angle and relative Mach number calculated from the analytical results are plotted on an outline of the performance map in figure 4(a).

The trends of incidence angle and Mach number calculated from the experimental results are shown in figure 4(b). The over-all trends are in close agreement on the two curves; one point of difference, however, can be noted. Greater negative incidence angles were predicted from the analytical results than were calculated from the experimental results in the region of high speed and low pressure ratio. The transonic turbine operates over a small range of incidence angle, from 8° to -14°, and for the major portion of the performance range the incidence angle varies between ±4°. The narrow range of incidence angle could explain why the efficiency of the transonic turbine did not decrease sharply at off-design conditions. A rotor blade with a sharp leading edge operating at high relative Mach number would not ordinarily be expected to operate over a wide range of conditions with good efficiency, because of its sensitivity to incidence angle. For the transonic turbine, however, only a small range of incidence angle was imposed on the rotor blade, over the range of performance, and this effect was not so important. For the more conservative turbine of reference 4, the range of incidence angle encountered at the mean section was from 30° to -40°. The transonic turbine also operates over a comparatively wide range in relative Mach number from 0.6 to 1.6, whereas the corresponding variation at the mean section for the turbine of reference 4 was approximately from 0.45 to 0.8.

For convenience, the variations of incidence angle and relative Mach number at the mean section of the turbine of reference 4 are used herein, since these quantities are presented in the reference. At the hub section, the incidence-angle variation would be somewhat smaller and the Mach number variation somewhat greater. This difference is small, however, compared to the difference in trends between the conservative turbine and the transonic turbine. In figures 4(a) and (b) it can be seen that the line of 40-percent speed is in the region of the highest relative Mach number and the largest positive incidence angles.

It may also be noted that the incidence-angle contours are somewhat symmetrical about the line for Mach number of 1.0 in figure 4(b), where the parameters were calculated on a one-dimensional basis from experimental data. This occurs because the flow area where the Mach number is 1.0 represents a minimum area. Therefore, subcritical and supercritical Mach numbers must occur only at an increase in flow area, corresponding to a reduction in flow angle, which accounts for the decrease in incidence angle from that at the line of Mach number 1.0. Because this line passes through the middle of the map, small variations in incidence angle were found to occur over the range of performance. In figure 4(a), a similar pattern is seen, except that it occurs at a slightly increased Mach number level. This difference is probably a result of the approximate nature of the assumptions used in the calculation of the hub conditions from the analysis, as well as from the experimental data.

Similar plots showing the hub reaction trends are presented in figure 5. Reaction is defined herein as the static-pressure ratio across the rotor hub. Good agreement can be noted between the reaction calculated in the analysis and that obtained experimentally. The transonic turbine operates over a range of reaction from 0.5 to 1.2 (fig. 5). The corresponding reaction variation obtained from the data of the more conservative turbine of reference 4 was from 0.9 to 1.6. Thus, the transonic turbine operates at a much lower level of reaction than the more conservatively designed turbine. The line of 40-percent speed is in the region of the lowest reaction.

Good over-all agreement was achieved in ascertaining the hub conditions from the analytical method and from the experimental results. The minor differences in magnitudes and shapes of the curves may be attributed in part to the approximate nature of the assumptions that were used in calculating these conditions.

SUMMARY OF RESULTS

An analytical method of predicting off-design turbine performance, which was previously developed and was found to work well for conservative turbines, was applied to a transonic turbine. In addition, the rotor hub conditions of incidence angle, relative Mach number, and reaction as calculated from the analytical results were compared with those calculated from the experimental data. The pertinent results are as follows:

1. The loss breakdown analytically obtained for the transonic turbine did not differ significantly from that obtained for a more conservative turbine design, except that high stator-exit shock losses were predicted for the transonic turbine at the lower speeds, whereas, for the more conservative turbine the stator was not choked and no stator-exit shock losses were predicted.

2. The trends of the rotor hub conditions of incidence angle, relative Mach number, and reaction calculated from the analysis generally agreed well with those calculated from the experimental data.

3. From the trends of the rotor hub conditions it was shown that, compared with a turbine of more conservative design, the subject transonic turbine operated over a very wide range of rotor hub relative Mach number, a very small range of rotor hub incidence angle, and a much lower level of rotor hub reaction.

4. The agreement between the analytically predicted performance map and the experimental performance map was good, except at 40-percent design

speed. At this speed it was indicated in the analysis that stator limiting loading occurred before the rotor choked, and no analytical solution was obtained above the pressure ratio corresponding to this condition. Because this discrepancy occurred as a result of errors in the simplifying assumptions used in the analysis, it must be regarded as a limitation in the analytical method as applied to a transonic turbine.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, March 30, 1954

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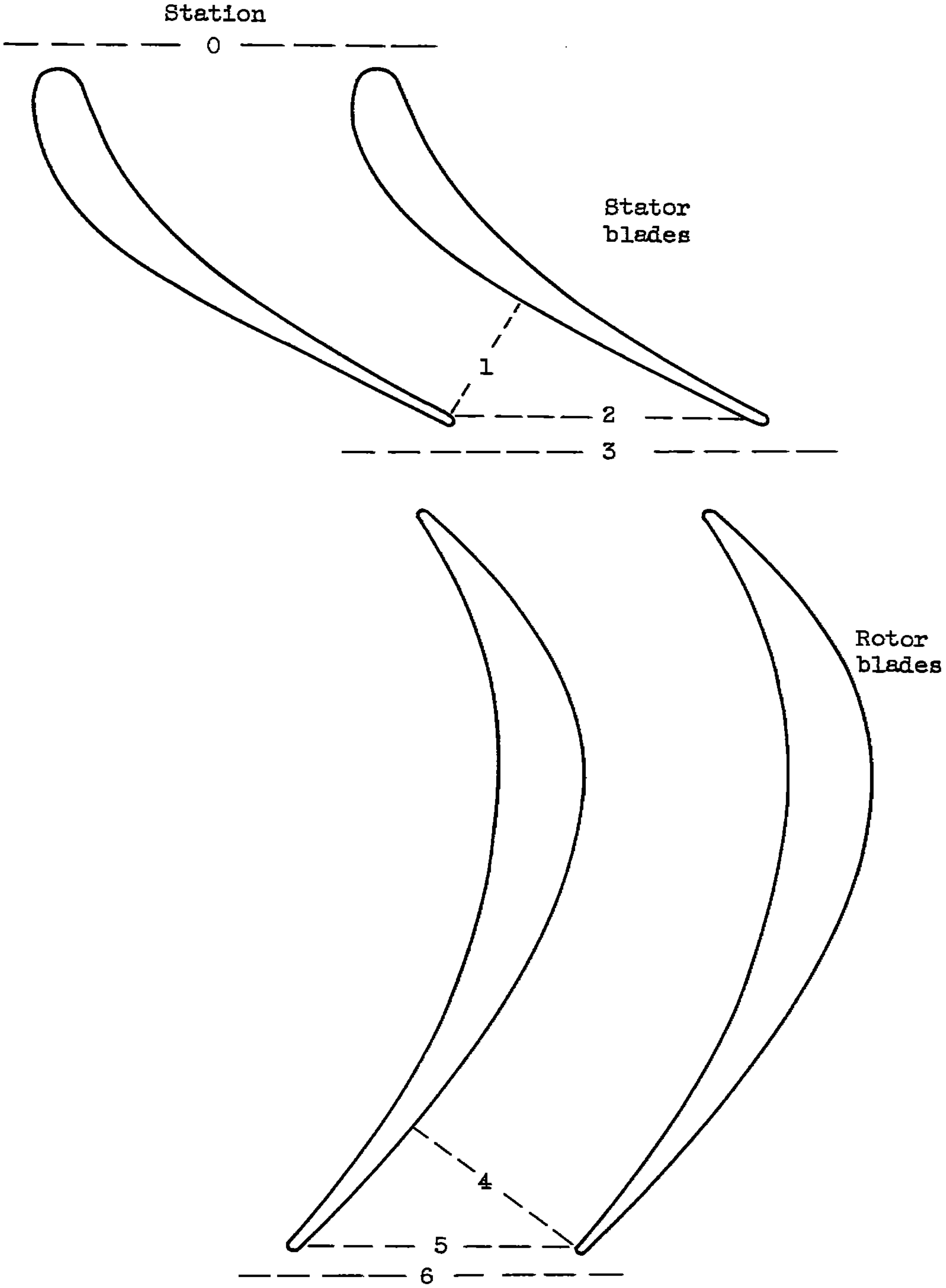

Figure 1. - Sketch of stator and rotor blading illustrating stations used in analysis.

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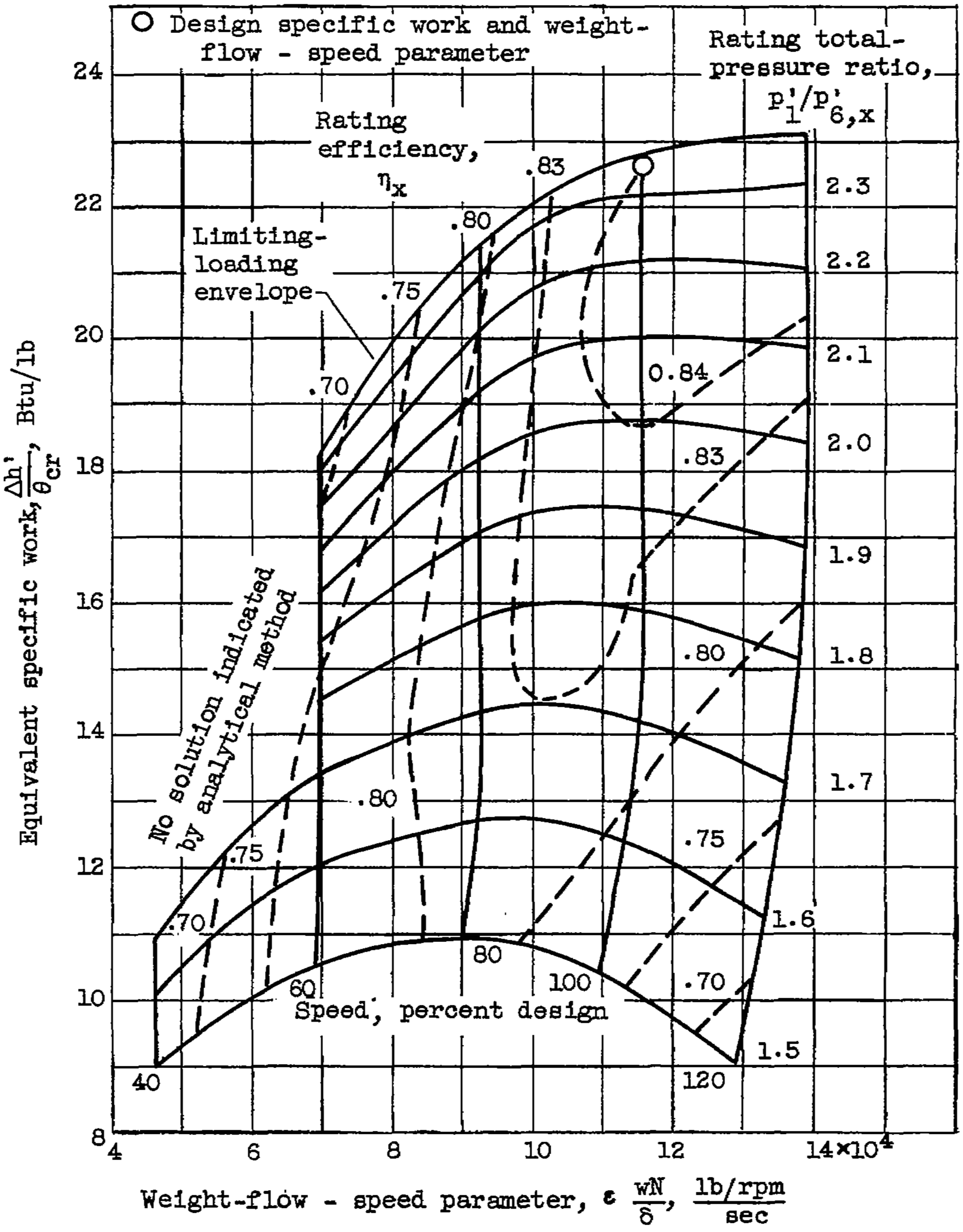

(a) Analytically obtained transonic-turbine performance map.

Figure 2. - Turbine performance maps.

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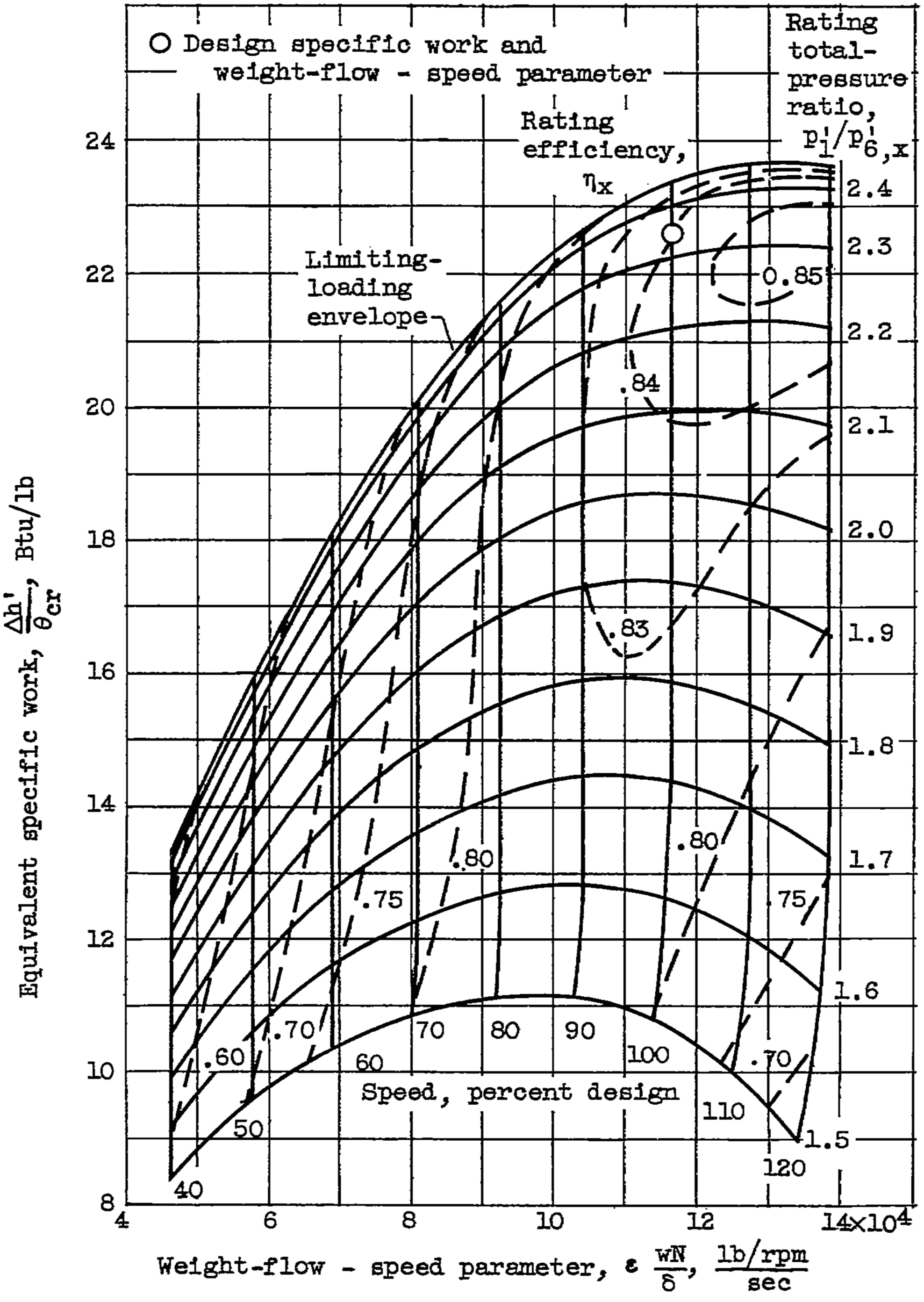

(b) Experimental performance map. (Data from ref. 1.)

Figure 2. - Concluded. Turbine performance maps.

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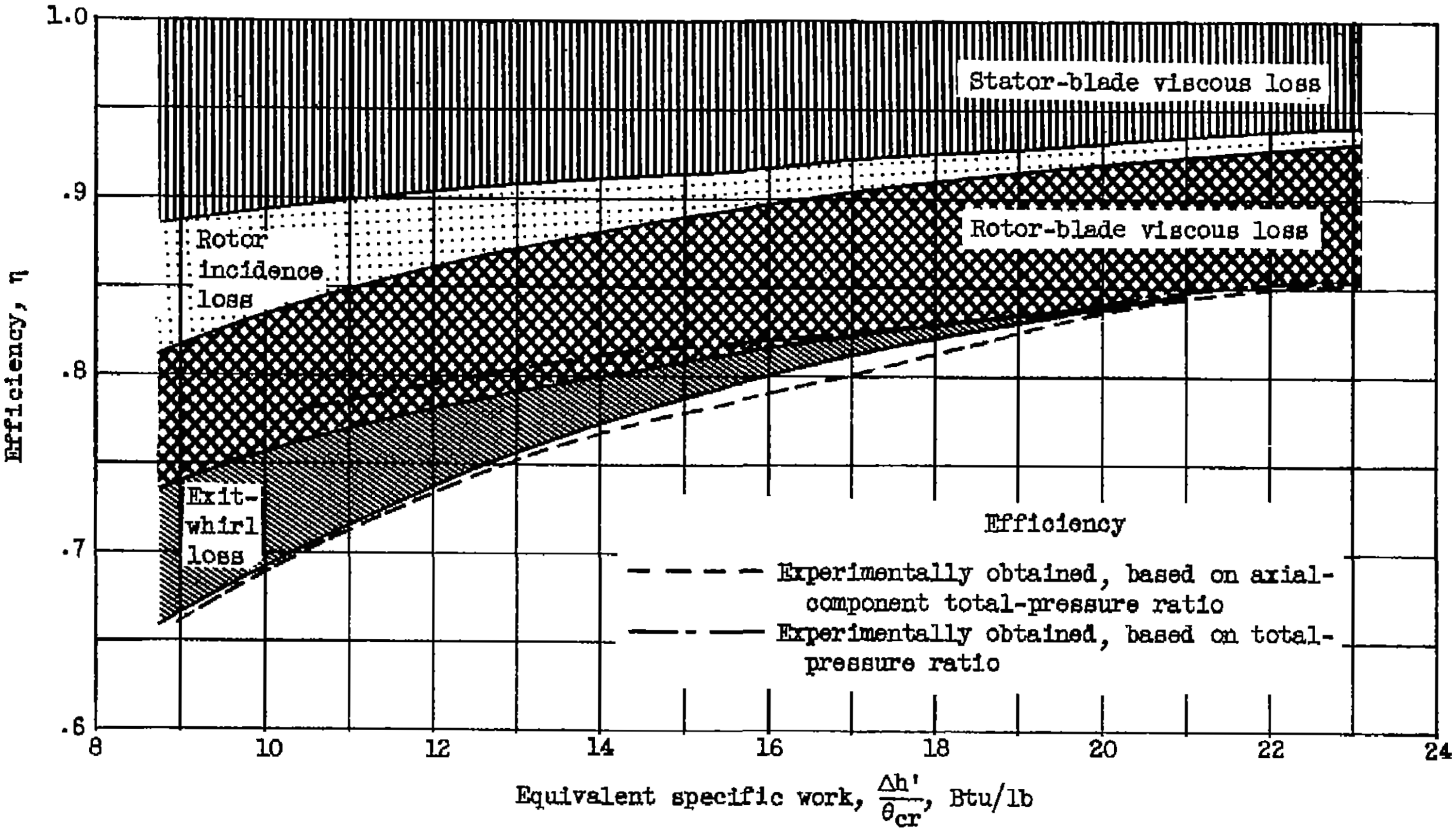
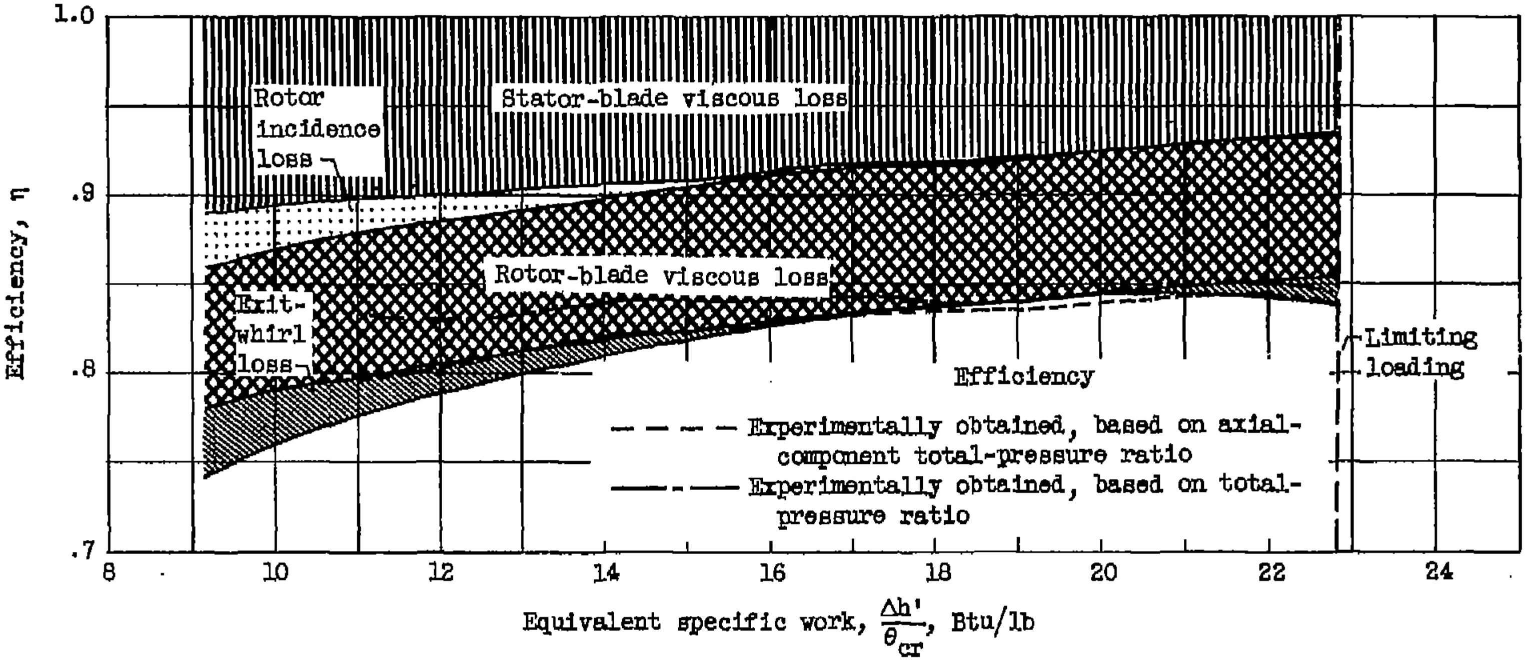


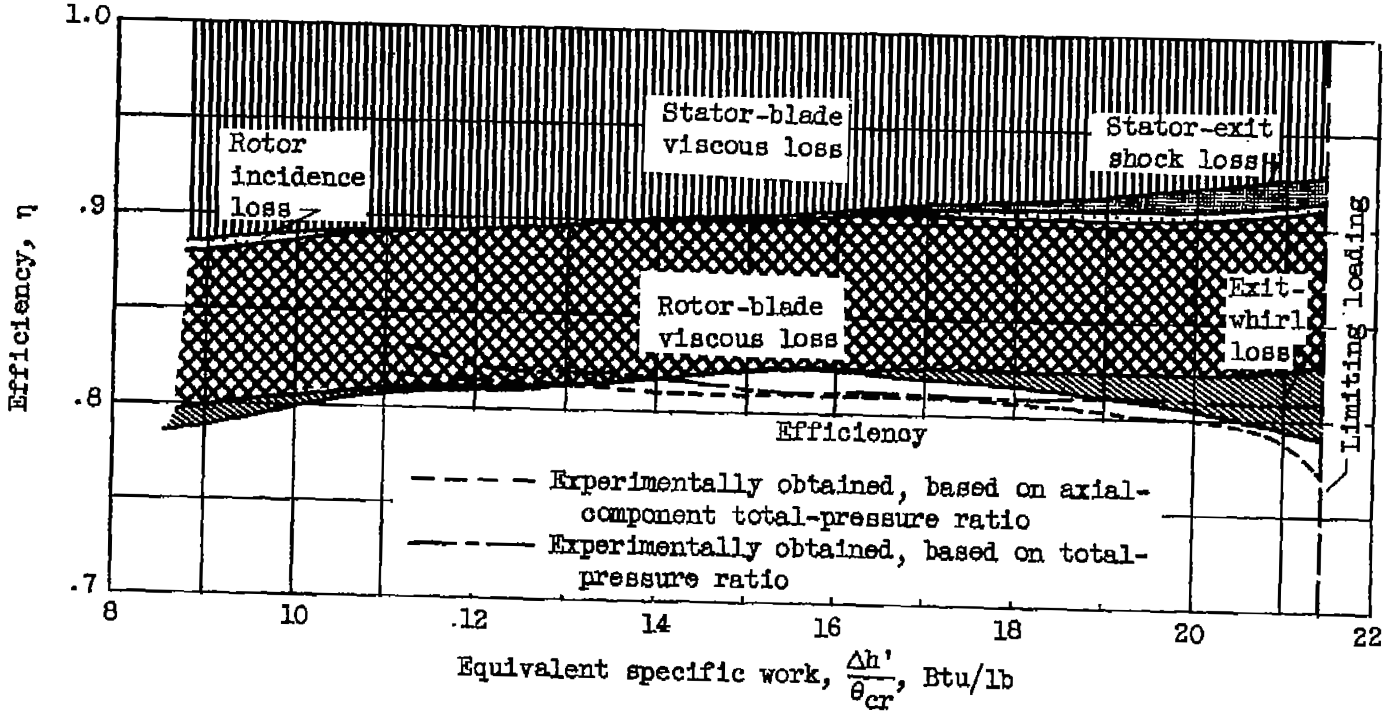
(a) Speed, 120-percent design.

Figure 3. - Loss breakdown.



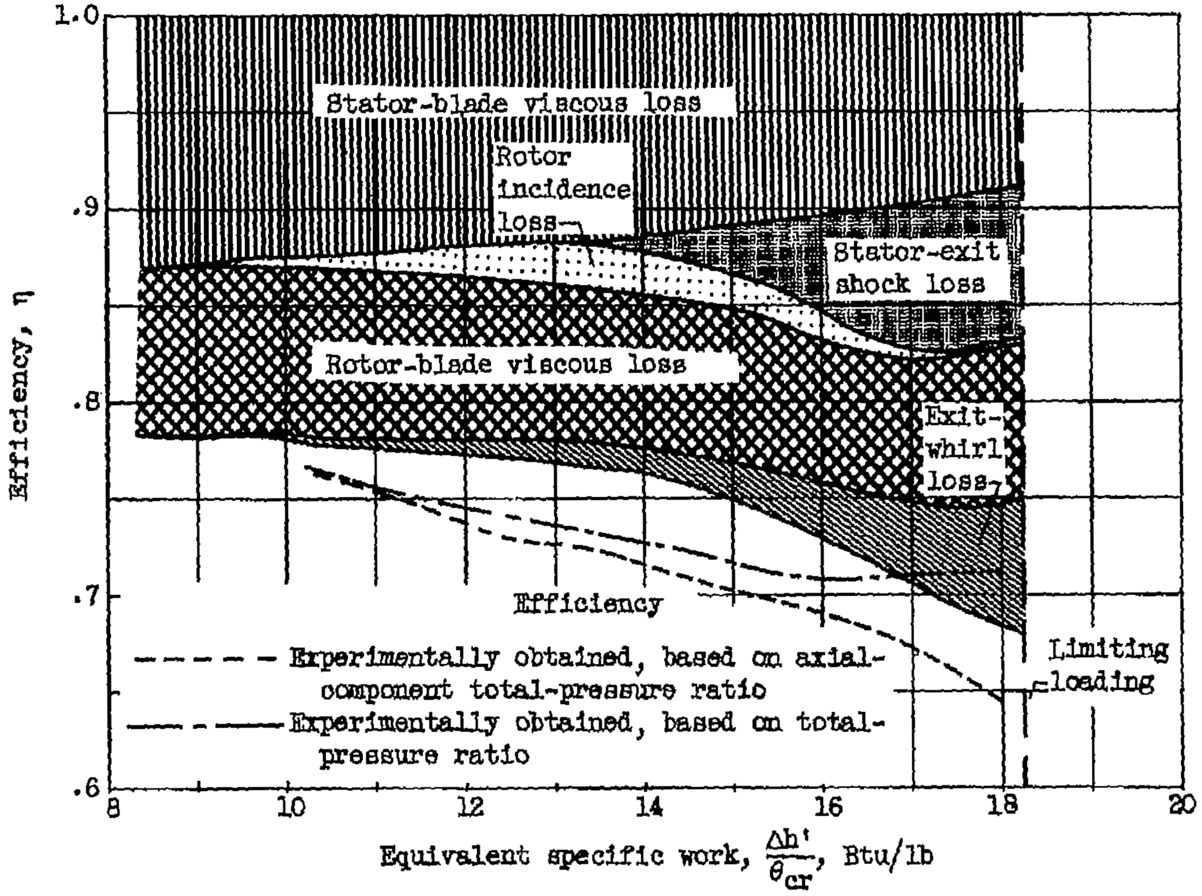
(b) Speed, 100-percent design.

Figure 3. - Continued. Loss breakdown.



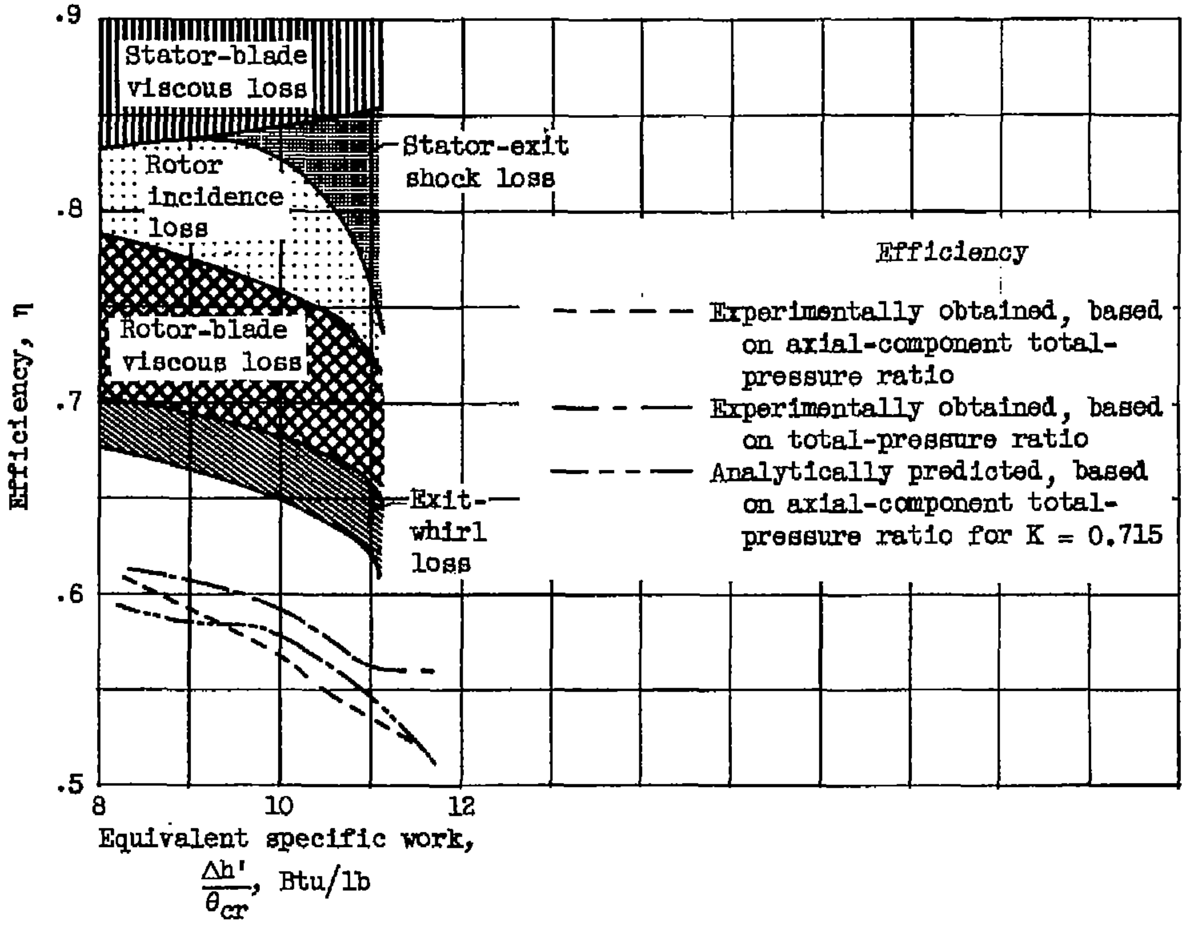
(c) Speed, 80-percent design.

Figure 3. - Continued. Loss breakdown.



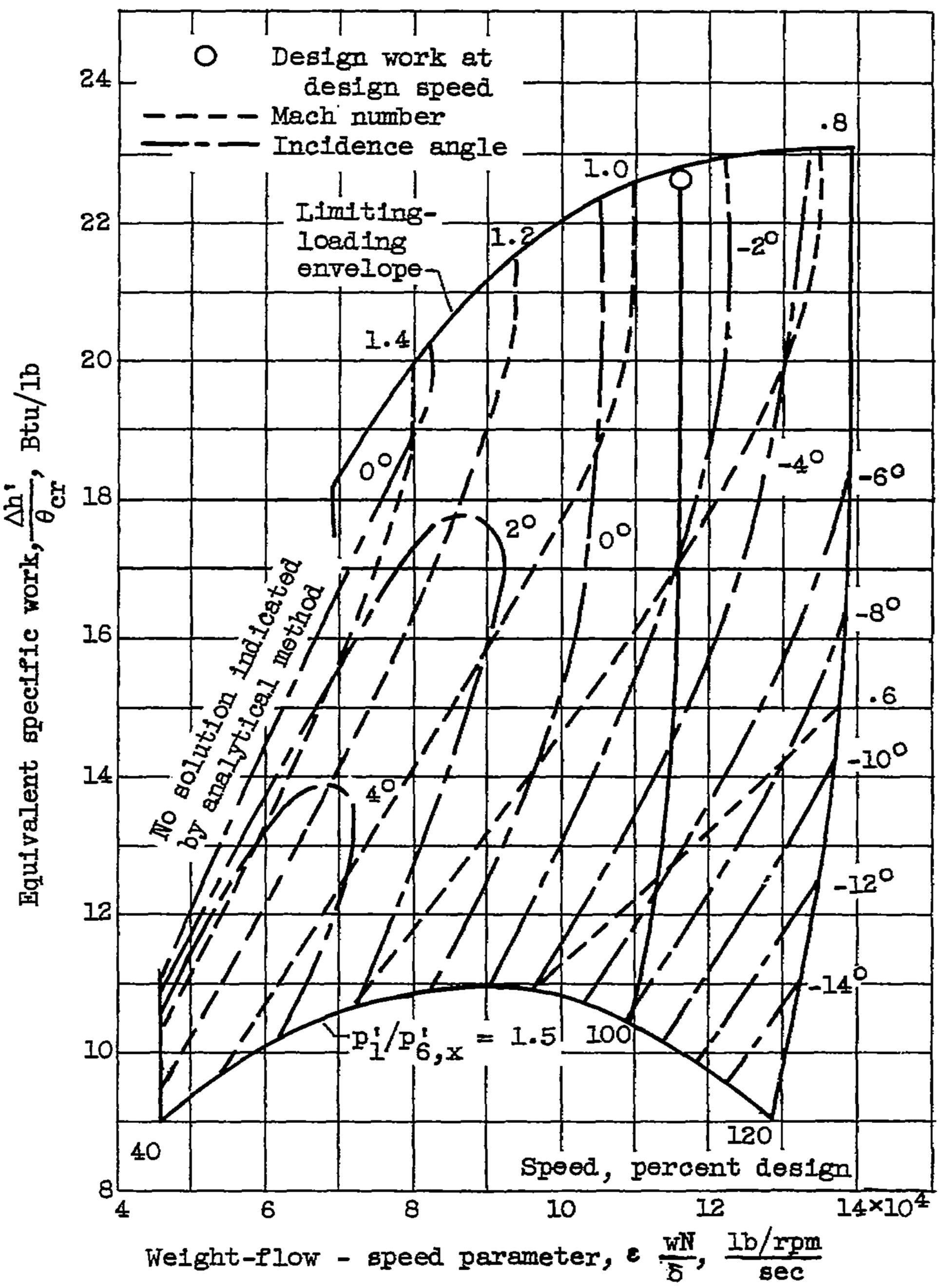
(d) Speed, 60-percent design.

Figure 3. - Continued. Loss breakdown.



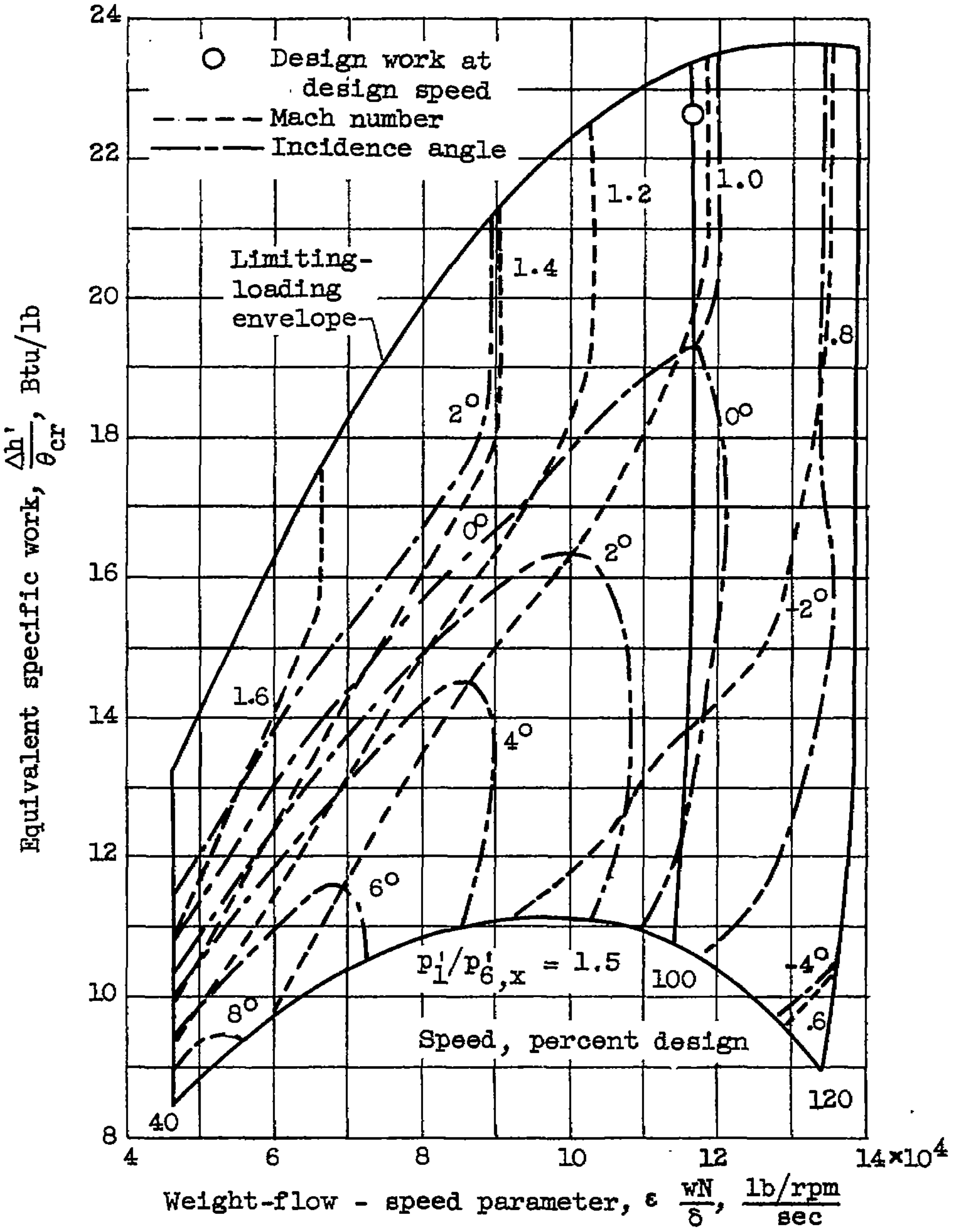
(e) Speed, 40-percent design.

Figure 3. - Concluded. Loss breakdown.



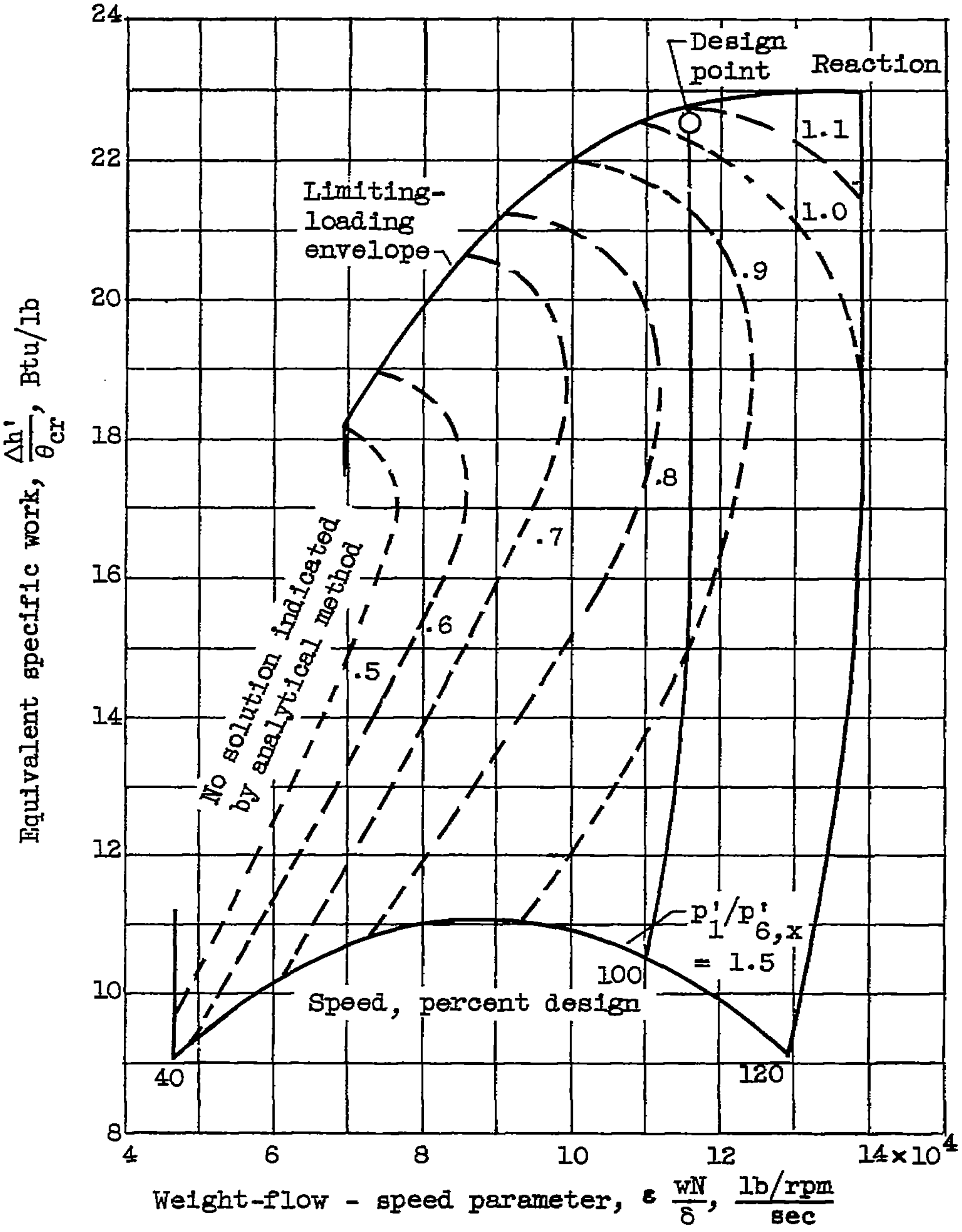
(a) Analytically obtained data.

Figure 4. - Variation of rotor hub incidence angle and entering Mach number over performance map.



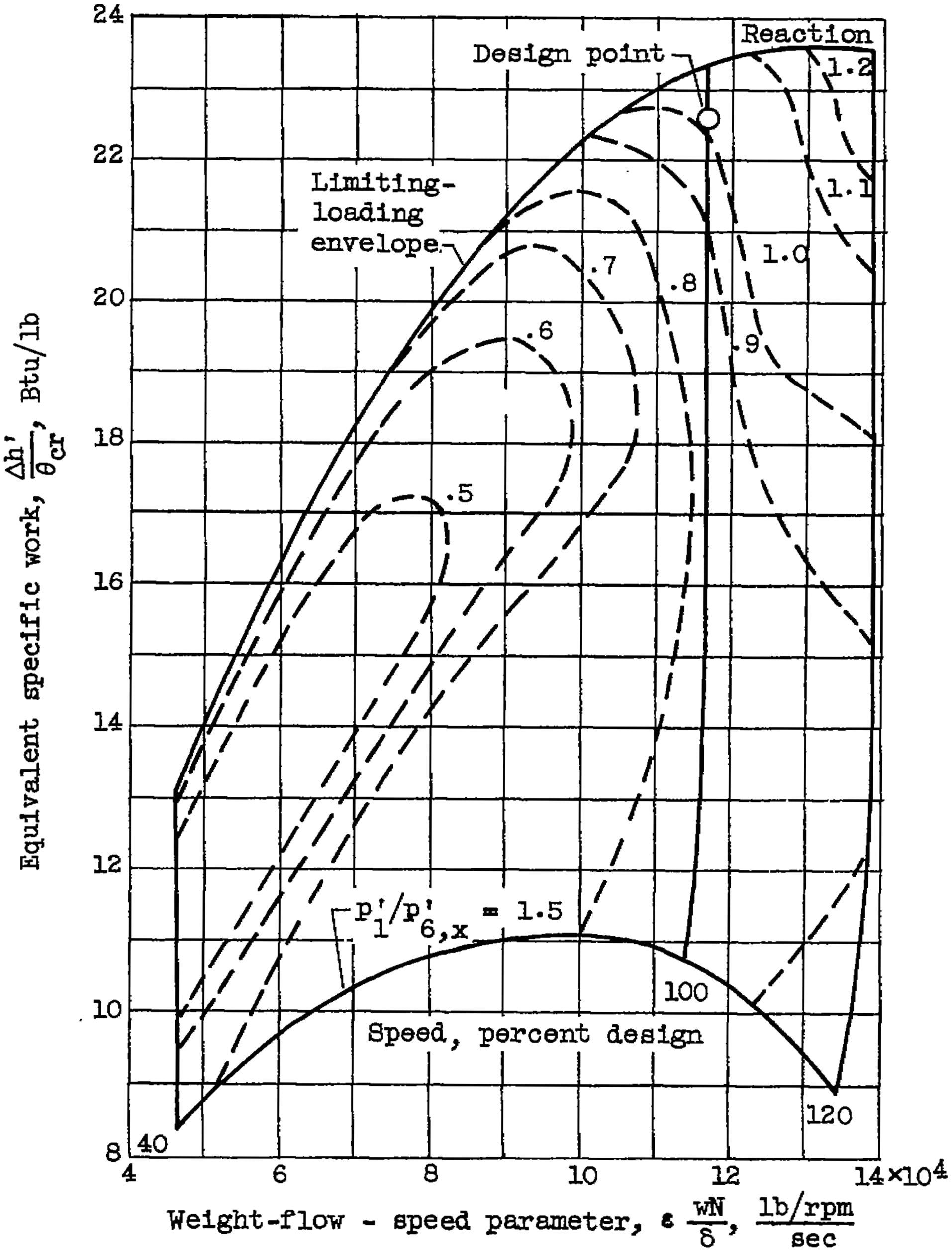
(b) Calculated from experimental data.

Figure 4. - Concluded. Variation of rotor hub incidence angle and entering Mach number over performance map.



(a) Analytically obtained data.

Figure 5. - Variation of reaction across rotor hub over performance map.



(b) Calculated from experimental data.

Figure 5. - Concluded. Variation of reaction across rotor hub over performance map.

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