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# RESEARCH MEMORANDUM

## METHODS OF PREDICTING HELICOPTER STABILITY

By Robert J. Tapscott and F. B. Gustafson

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON
November 2, 1954

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

# METHODS OF PREDICTING HELICOPTER STABILITY

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## SUMMARY

Some of the methods of predicting rotor stability derivatives have been reviewed. The methods by which these rotor derivatives are employed to estimate helicopter stability characteristics have been summarized.

It is concluded that, although these methods are not always feasible for predicting absolute values of the stability of the helicopter, the effects on stability of changes in individual derivatives can generally be estimated satisfactorily.

## INTRODUCTION

In order to predict helicopter stability - for example, to estimate theoretically whether a prospective helicopter will meet the flying-qualities requirements - both the applicable equations of motion and the necessary stability derivatives must be determined.

The processes for handling equations of motion have been well established in conjunction with airplanes, and the modification of these procedures for helicopter use has been found a secondary problem in comparison with the provision of values of stability derivatives.

The prediction of stability derivatives, in general, requires a knowledge of the contributions of both the rotor and the fuselage. Fuselage characteristics are not open to as specific an analysis as rotor characteristics, and preliminary estimates can be handled on the basis of data from previous designs and from wind-tunnel model tests. The fuselage seems to be subject to greater modification up to the time production starts; consequently, this paper is confined primarily to the rotor contributions to the derivatives. In the past few years, considerable information has been published which permits the most pertinent rotor derivatives to be predicted with fairly good accuracy where no stall is present. A general picture of this work is presented herein.



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## SYMBOLS

$W$ gross weight of helicopter, lb

$m$ mass of helicopter, slugs

$R$ blade radius, ft

$c$ blade-section chord, ft

$c_e$ equivalent blade chord (on thrust basis), $\dfrac{\int_0^R cr^2 dr}{\int_0^R r^2 dr}$, ft

$\sigma$ rotor solidity, $bc_e/\pi R$

$\Theta$ instantaneous blade-section pitch angle; angle between line of zero lift of blade section and plane perpendicular to rotor shaft, $\theta - A_1 \cos \psi - B_1 \sin \psi$, radians

$\theta$ collective pitch; average value of $\Theta$ around azimuth, radians

$\rho$ mass density of air, slugs/cu ft

$\gamma$ mass constant of rotor blade, expresses ratio of air forces to mass forces, $\rho caR^4/I_1$; also, climb angle, radians

$I_X$ helicopter rolling moment of inertia about center of gravity, slug-ft$^2$

$I_Y$ helicopter pitching moment of inertia about center of gravity, slug-ft$^2$

$I_Z$ helicopter yawing moment of inertia about center of gravity, slug-ft$^2$

$\Delta\theta$ difference in collective-pitch angle of front and rear rotors, positive when rear rotor is greater, radians; also increment of $\theta$

$\Delta T$ — difference in thrust of front and rear rotors, positive when thrust of rear rotor is greater, lb

$\Delta R, \Delta(\Omega R), \Delta\sigma$ — definitions analogous to that for $\Delta\theta$

$\Delta\alpha_d$ — difference in angle of attack of front and rear rotors due to longitudinal swashplate tilt, positive when rear-rotor angle of attack is greater

$A_1, B_1$ — coefficients of $-\cos\psi$ and $-\sin\psi$, respectively, in expression for $\Theta$; therefore lateral and longitudinal cyclic pitch, respectively, radians

$\delta$ — control motion, inches from trim

$V$ — true airspeed of helicopter along flight path, ft/sec

$v$ — sideslip velocity of helicopter in Y-direction, ft/sec

$\alpha$ — rotor angle of attack; angle between flight path and plane perpendicular to axis of no feathering, positive when axis is inclined rearward, radians

$\Omega$ — rotor angular velocity, radians/sec

$\mu$ — tip-speed ratio, $\frac{V \cos\alpha}{\Omega R}$ assumed equal to $\frac{V}{\Omega R}$

$U_T$ — component at blade element of resultant velocity perpendicular to blade-span axis and to axis of no feathering, ft/sec

$u_T = U_T/\Omega R$

$\psi$ — blade azimuth angle (measured in direction of rotation from downwind position when flight-path axes are used, or measured in direction of rotation from position of blade when pointing rearward along longitudinal axis of helicopter when body axes are used), radians

$a$ — slope of curve of section lift coefficient against section angle of attack, per radian

$L$ — rotor lift, lb; also, rolling moment, ft-lb

$T$ — rotor thrust, component of rotor resultant force parallel to axis of no feathering, lb

$Q$ rotor-shaft torque, lb-ft

$C_T$ rotor thrust coefficient, $\frac{T}{\pi R^2 \rho (\Omega R)^2}$

$C_Q$ rotor-shaft torque coefficient, $\frac{Q}{\pi R^2 \rho (\Omega R)^2 R}$

$\beta$ blade flapping angle at particular azimuth position, radians; also sideslip angle, radians

$a_0$ constant term in Fourier series that expresses $\beta$; therefore, rotor coning angle

$a_1$ coefficient of $-\cos \psi$ in expression for $\beta$; therefore, longitudinal tilt of rotor cone

$b_1$ coefficient of $-\sin \psi$ in expression for $\beta$; therefore, lateral tilt of rotor cone

$a'$ projection of angle between rotor force vector and axis of no feathering in plane containing flight path and axis of no feathering

$p$ helicopter rolling velocity, radians/sec

$q$ helicopter pitching velocity, radians/sec

$r$ helicopter yawing velocity, radians/sec

$\phi$ angle of roll, radians

$N$ yawing moment, lb-ft

$M$ pitching moment, lb-ft

$h$ height of rotor hub above helicopter center of gravity, ft

$\eta$ angle between principal longitudinal axis of inertia of helicopter and flight path, positive nose up, radians

Dots over symbols indicate derivatives with respect to time.

## ROTOR STABILITY DERIVATIVES

In general, calculation of the helicopter stability derivatives needed for a study of helicopter characteristics depends on the knowledge of the individual rotor derivatives shown in the following table:

| | | | | |
|---|---|---|---|---|
| $\frac{\partial(C_T/\sigma)}{\partial\alpha}$ | $\frac{\partial(C_T/\sigma)}{\partial V}$ | $\frac{\partial(C_T/\sigma)}{\partial\Omega}$ | $\frac{\partial(C_T/\sigma)}{\partial\theta}$ | $\frac{\partial(C_T/\sigma)}{\partial q}$ |
| $\frac{\partial a'}{\partial\alpha}$ | $\frac{\partial a'}{\partial V}$ | $\frac{\partial a'}{\partial\Omega}$ | $\frac{\partial a'}{\partial\theta}$ | $\frac{\partial a'}{\partial q} = \frac{\partial b'}{\partial p}$ |
| $\frac{\partial(C_Q/\sigma)}{\partial\alpha}$ | $\frac{\partial(C_Q/\sigma)}{\partial V}$ | $\frac{\partial(C_Q/\sigma)}{\partial\Omega}$ | $\frac{\partial(C_Q/\sigma)}{\partial\theta}$ | $\frac{\partial(C_Q/\sigma)}{\partial q}$ |
| $\frac{\partial a_0}{\partial\alpha}$ | $\frac{\partial a_0}{\partial V}$ | $\frac{\partial a_0}{\partial\Omega}$ | $\frac{\partial a_0}{\partial\theta}$ | $\frac{\partial a_0}{\partial q}$ |
| $\frac{\partial a_1}{\partial\alpha}$ | $\frac{\partial a_1}{\partial V}$ | $\frac{\partial a_1}{\partial\Omega}$ | $\frac{\partial a_1}{\partial\theta}$ | $\frac{\partial a_1}{\partial q}$ |
| $\frac{\partial b_1}{\partial\alpha}$ | $\frac{\partial b_1}{\partial V}$ | $\frac{\partial b_1}{\partial\Omega}$ | $\frac{\partial b_1}{\partial\theta}$ | $\frac{\partial b_1}{\partial q}$ |

As shown the rotor parameters considered are $C_T/\sigma$ and $a'$, the magnitude and tilt of the thrust vector, respectively; rotor torque $C_Q/\sigma$; blade coning angle $a_0$; and $b_1$ and $a_1$, the lateral and longitudinal flapping, respectively. These rotor parameters are functions of five independent variables: rotor angle of attack $\alpha$, forward speed $V$, rotational speed $\Omega$, collective pitch $\theta$, and pitching velocity $q$. The values of the derivatives are determined by the variations from trim of the rotor parameters with changes in each of the five variables. The total number of derivatives shown here is large; however, it is generally possible to reduce considerably the number needed for most applications. For instance, if the rotor under consideration has no flapping hinge offset, the flapping coefficients are not significant; thus, the number of derivatives to be considered is immediately reduced by about one-half. If necessary, however, these derivatives can be determined from the equations of references 1 and 2.

Of the remaining derivatives, the lift due to pitching and torque due to pitching generally can be neglected. For most cases, then, the derivatives needed have been reduced to those enclosed within the lines in the preceding table. All the $C_T/\sigma$, $C_Q/\sigma$, and $a'$ derivatives can be obtained from figures or equations of references 2 and 3, as will be shown in the following discussion.

The thrust due to angle of attack and collective pitch are presented as functions of tip-speed ratio as shown in figure 1. If the solidity and tip-speed ratio for a given case are known, these quantities can be read directly. The equations on which this figure is based and the processes by which they were derived are discussed in reference 3.

The change in thrust coefficient with tip-speed ratio is presented in reference 3 in the form of an equation from which, when the flight condition is known, the rate of change of thrust coefficient with forward speed and rotational speed can be computed.

The change in inclination of the rotor force vector due to steady pitching or rolling velocity has been derived in reference 2. This derivative is shown in figure 2 as a function of the parameter $\frac{\theta}{C_T/\sigma}$. As the effect of tip-speed ratio $\mu$ is small, the equation shown in figure 2 is fairly accurate below a tip-speed ratio of 0.5 for both roll and pitch. The rotor damping moment is determined simply by multiplying the quantity obtained from the figure by $18/\gamma\Omega$ and by the rotor thrust and rotor height above the helicopter center of gravity; that is,

$$\frac{\partial M}{\partial q} = \frac{\partial a'/\partial q}{18/\gamma\Omega} \times Th\,\frac{18}{\gamma\Omega}$$

For the rotor-vector angle and rotor torque derivatives, charts such as those shown in figure 3 have been derived and are published in reference 3. These charts are given for a range of collective-pitch angles from 0 to $14^{\circ}$ at $2^{\circ}$ increments; the sample shown herein is for a collective pitch of $8^{\circ}$. In these charts, the longitudinal rotor-vector angle $a'$ is plotted against thrust-coefficient—solidity ratio for specified values of tip-speed ratio. Lines of constant power parameter $C_Q/\mu C_T$ are cross-plotted on the charts. Combinations of these parameters which result in angles of attack of $12^{\circ}$ and $16^{\circ}$ on the retreating blade are indicated by dashed lines; these lines, in effect, serve as limit lines above which account must be made of stall. By using slopes or differences from these charts in conjunction with other derivatives and some simple equations, the remaining $a'$ and torque coefficient derivatives can be obtained. As an example, consider the derivative of $a'$ with respect to angle of attack. From this chart, the rate of change of $a'$ with thrust coefficient can be obtained at a given tip-speed ratio and thrust coefficient by scaling off the slope of the tip-speed-ratio line at the desired thrust coefficient. When this quantity is multiplied by the change in thrust coefficient with angle of attack, which has already been discussed, the result is the change in $a'$ with angle of attack. The other $a'$ and the torque coefficient derivatives can be obtained by similar procedures. These procedures are discussed in reference 3.

# PREDICTION OF HELICOPTER STABILITY

By using the rotor derivatives that have been discussed, it is believed that the rotor contribution to the essential helicopter derivatives can be predicted. The rotor derivatives discussed are applicable, in most cases, to a study of the stability characteristics of either a single- or tandem-rotor configuration. The difference arises in using the rotor derivatives to determine the helicopter derivatives for use in the equations of motion and in accounting for the effects of flow interference for a specific configuration. For most purposes, the lateral and longitudinal characteristics of the helicopter can be studied separately. Generally, equations of motion derived on the basis of constant forward speed are sufficient and are applicable to both single- and tandem-rotor configurations.

In a study of the longitudinal characteristics of the helicopter (refs. 4 and 5), an important criterion is that the time history of the normal acceleration shall be concave downward within two seconds after a step input to the longitudinal control; that is, the slope of the normal acceleration curve shall reach its maximum value and begin to decrease within two seconds. In order to assist in estimating theoretically whether a prospective helicopter will meet this criterion, the following equations of motion were devised:

$$\frac{\partial L}{\partial q} q + \frac{\partial L}{\partial \theta} \Delta\theta + \frac{\partial L}{\partial \alpha} \Delta\alpha - \frac{WV}{g} \dot{\gamma} = 0$$

$$\frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \theta} \Delta\theta + \frac{\partial M}{\partial \alpha} \Delta\alpha + \frac{\partial M}{\partial B_1} \Delta B_1 - I_Y \dot{q} = 0$$

$$\Delta\alpha = \int_0^t q\, dt - \Delta\gamma - \Delta B_1$$

These equations are based on flight-path axes and derived on the assumptions of constant forward speed, constant rotor speed, and constant stability derivatives. The assumption is also made that the dynamic maneuver can be represented by a series of static conditions; hence, the rotor parameters are always at their equilibrium values as determined by the instantaneous values of angle of attack, pitch angle, tip-speed ratio, and pitching velocity. The first and second equations shown represent, respectively, equilibrium normal to the flight path and equilibrium in pitch. The third equation simply relates the variables of the first two equations to permit a solution.

The form as shown applies specifically to the single-rotor configuration. For the tandem configuration, the pitching moment due to control motion results primarily from differential collective pitch on the two rotors rather than from cyclic pitch; thus, the term in the equations expressing pitching moment due to control motion must be modified to account for this difference.

In reference 5, these equations of motion have been solved for the climb angle $\gamma$, which, in turn, permits an expression to be written for the time history of the normal acceleration. Values of combinations of derivatives have been determined which, when substituted into the expression for the time history of normal acceleration, will result in a time history that is concave downward at 2 seconds. These values have been presented in the form of a chart, as shown in figure 4. The curve in figure 4, therefore, indicates a boundary line separating combinations of significant longitudinal stability derivatives which result in satisfactory characteristics or unsatisfactory characteristics according to the criterion previously mentioned. Shown along with the theoretical curve are data points corresponding to five helicopter configurations. The derivatives for the configurations corresponding to the points shown were measured in flight and the adjacent number is the approximate time for the corresponding normal-acceleration time history to become concave downward. The theoretical curve is indicated to be qualitatively correct for both single- and tandem-rotor helicopters for separating configurations which have satisfactory maneuver stability from those which have unsatisfactory maneuver stability according to the criterion. This indication can be seen by noting that the points for which the normal-acceleration time history becomes concave downward in less than 2 seconds fall in the satisfactory region whereas, for times of more than 2 seconds, the points fall in the unsatisfactory region.

Figure 4 shows an increase in the angle-of-attack stability or damping in pitch (that is, more negative $M_{\alpha}$ or $M_q$), or increases in the lift-curve slope $L_{\alpha}$ to be stabilizing. Also, it can be determined from this plot that an increase in pitching moment of inertia $I_Y$ can be destabilizing. Since the change in lift slope can be small only and is not expected to change sign, the primary changes in stability must be brought about by changes in the damping-in-pitch or angle-of-attack stability.

In the discussion of longitudinal stability so far, the forward speed has been assumed constant. For the tandem configuration, however, the downwash effects of the front rotor acting on the rear rotor cause an unstable variation in pitching moment with speed. A study of the speed stability of the tandem was made in reference 6. In that study, on the basis of the available derivatives discussed previously, an expression for the change of stick position with speed was derived. The equation, along with a plot of constants based on the rotor derivatives, is shown in figure 5. The constants for use in the equation are presented for several values of rotor solidities over a range of tip-speed ratios. After a flight condition is selected and the tip-speed ratio and thrust coefficient are thereby established, the slope of the stick-position variation with speed can be determined. This equation takes into account the effects on speed stability of center-of-gravity position, differential rotor speed, differential rotor radius and solidity, and "longitudinal dihedral." Effects of these parameters can be studied either together or separately.

This analysis of tandem speed stability is of value especially in the study of the relative effects of changes in the various parameters wherein it is not necessary to know accurately the value of downwash or the fuselage contribution. In order to predict the absolute value of speed stability, the downwash must be estimated for the configuration and flight condition under consideration and the variation of the fuselage moments must be known. The results of flow-field measurements presented in references 7 and 8 should be useful in estimating the magnitude of the downwash applicable to a specific condition.

The next important item in helicopter stability is the lateral-directional characteristics. If roll, yaw, and sideslip are considered as degrees of freedom, the equations of motion are as follows:

$$I_X'\ddot{\phi} + \frac{\partial L}{\partial p}\dot{\phi} + \frac{\partial L}{\partial v}v - \frac{\partial L}{\partial \delta}\delta = 0$$

$$I_Z'\ddot{\psi} + \frac{\partial N}{\partial r}\dot{\psi} + \frac{\partial N}{\partial v}v - \frac{\partial N}{\partial \delta}\delta = 0$$

$$\left(mV - \frac{\partial Y}{\partial r}\right)\dot{\psi} - \left(mV \sin \eta + \frac{\partial Y}{\partial p}\right)\dot{\phi} - T\phi - T\psi \tan \gamma + m\dot{v} - \frac{\partial Y}{\partial v}v - \frac{\partial Y}{\partial \delta}\delta = 0$$

For convenience in determining the derivatives, these equations are based on principal axes of inertia rather than on the relative wind. In general, the rotor contribution to the derivatives needed in these equations can also be determined from rotor theory. Fuselage contributions can be predicted from wind-tunnel data and previous experience or from flight data where available.

These equations have been found useful particularly in estimating the effects on helicopter characteristics of changes in the individual stability derivatives. As an example, figure 6 shows the theoretically predicted time histories of rolling velocity and sideslip angle of a tandem-rotor helicopter. The curves on the left-hand side of figure 6 represent the predicted time histories for the original helicopter. On the right-hand side of figure 6 are time histories of the same quantities when the effective-dihedral derivative is reduced 50 percent. From this figure, it is indicated that a 50-percent reduction in the effective dihedral would substantially improve the oscillatory characteristics of the helicopter. In an attempt to improve the characteristics, this means was tried experimentally. The results are shown in figure 7 wherein the experimentally measured time histories of rolling velocity and sideslip angle before and after the derivative change are compared. The comparison shows that, as predicted by the theory, a reduction in the effective dihedral improved the oscillation. Thus, in this case, the theory was employed successfully to indicate the course to be followed in making an improvement.

## CONCLUDING REMARKS

In general, on the basis of the studies discussed herein, it will frequently not be feasible to predict accurately absolute magnitudes for the stability of the complete helicopter particularly because of the difficulty of predetermining final, full-scale fuselage characteristics. However, a first approximation can be made, and by making some comparatively straightforward flight measurements of stability derivatives and re-employing the theory to show what modifications are needed, it appears feasible to handle at least those problems with which direct experience has been had. It appears likely that, in most cases, changes in several derivatives simultaneously would be necessary to achieve most efficiently the desirable stability characteristics, and the equations discussed herein should prove very useful in this type of study.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 21, 1954.

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VARIATION OF $C_T/\sigma$ DERIVATIVES WITH RESPECT TO $\mu$

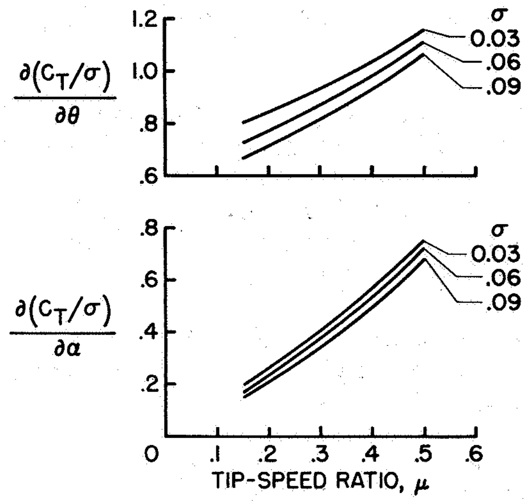



Figure 1

CHART FOR DETERMINING ROTOR DAMPING IN PITCH AND ROLL

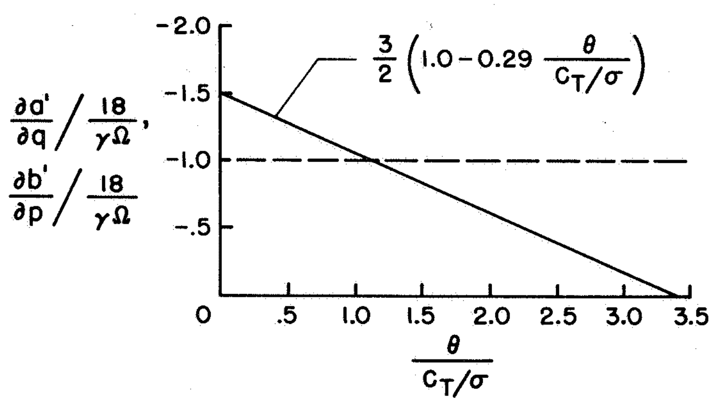



Figure 2

CHART FOR DETERMINING DERIVATIVES OF LONGITUDINAL TILT AND TORQUE-COEFF.—SOLIDITY RATIO

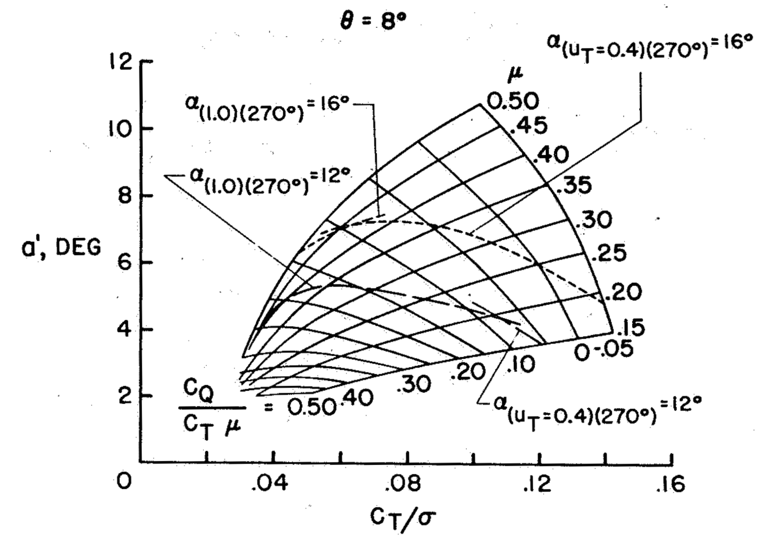
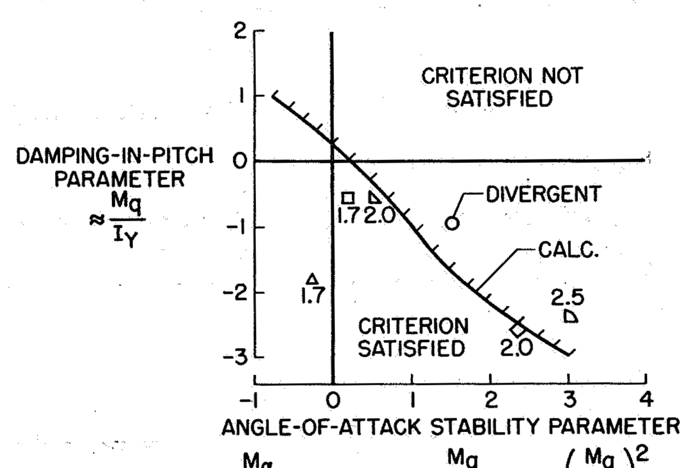



Figure 3

MANEUVER-STABILITY CHART FOR 2-SECOND CRITERION




$$\frac{\frac{M_\alpha}{I_Y} + 0.70 + 0.58\frac{M_q}{I_Y} + 0.12\left(\frac{M_q}{I_Y}\right)^2}{\frac{g}{WV} L_\alpha}$$

Figure 4

[redacted]

TANDEM SPEED-STABILITY EQUATION

$$\frac{d(\Delta\theta)}{d\mu} = K_1\left(\frac{C_T}{\sigma}\right)_{AV}\left(\frac{\Delta T}{W} - \frac{\Delta R}{R_{AV}}\right) + K_2\left(\frac{C_T}{\sigma}\right)_{AV}\left[\frac{\Delta\sigma}{\sigma_{AV}} + 2\frac{\Delta(\Omega R)}{(\Omega R)_{AV}}\right] + K_3\Delta\alpha_d + K_4 C_T$$

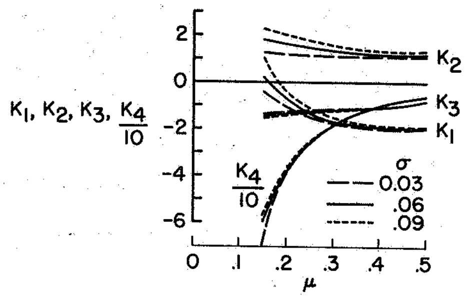



Figure 5

PREDICTED EFFECT OF REDUCTION IN EFFECTIVE DIHEDRAL ON LATERAL OSCILLATION

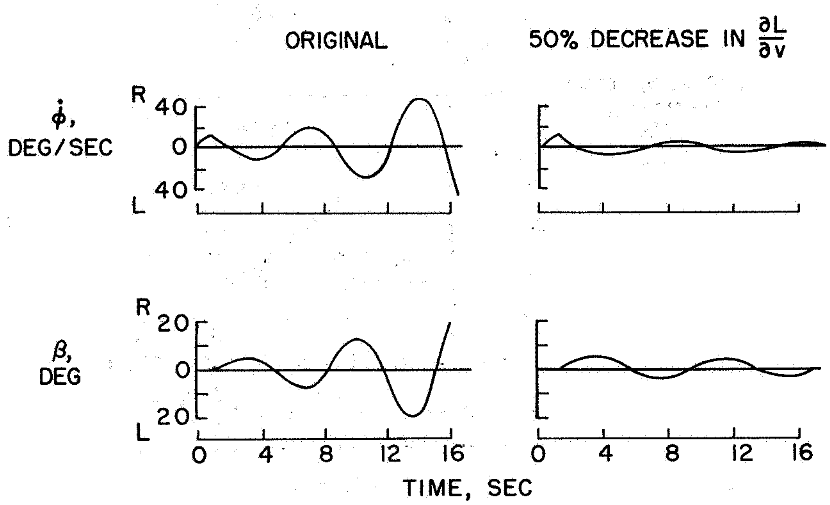



Figure 6

MEASURED EFFECT OF REDUCTION IN EFFECTIVE DIHEDRAL ON LATERAL OSCILLATION

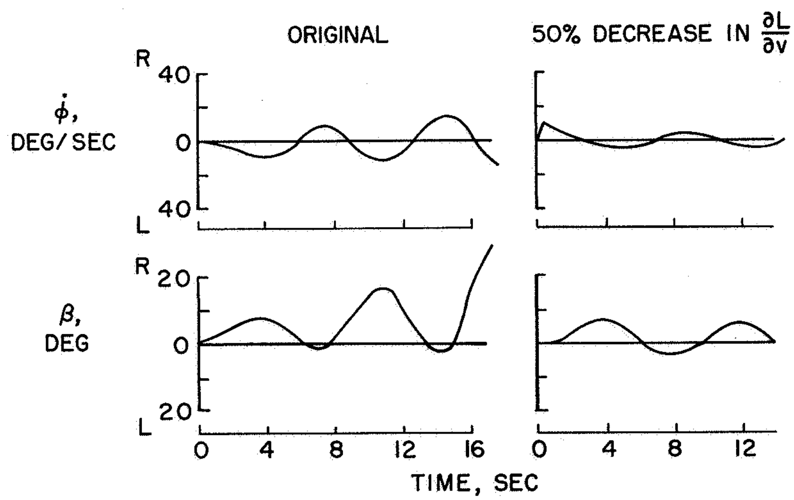



Figure 7