NACA

RESEARCH MEMORANDUM

METHOD FOR STRESS ANALYSIS OF A SWEPT PROPELLER

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Declassified 8-15-51

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON
September 27, 1948

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SUMMARY

The methods used to estimate and reduce the stresses in a swept propeller to be tested in the Langley 8-foot high-speed tunnel are presented. The incremental moments and forces produced by the centrifugal and air forces of each of the elements of the blade outboard of a given minimum section at the neutral point of that section have been added to obtain the total moments and forces at that point. The stresses at the section have then been determined using the usual beam formulas. At the points where the sweep changes abruptly, the formulas have been revised to account for the curvature of the line through the neutral points of the nonuniform minimum sections at these locations. The moments about major axes of the minimum sections have been eliminated by means of the proper orientation of the blade elements.

INTRODUCTION

To determine the effectiveness of sweep in delaying and reducing the losses in efficiency of propellers at high forward Mach numbers due to the onset of strong shocks on the blades, the NACA is in the process of designing, building, and testing propellers with blades which are swept in a manner similar to that shown in figure 1. The propellers are designed with the outboard portion swept back and the inboard portion swept forward to reduce the moments at the root. The use of sweep completely alters the distribution of stresses in the blade, and the stress analysis of a swept blade is greatly different from and much more complex than that of an unswept blade. Special methods have been developed to determine and reduce the stresses and deflections in the blades of the swept propellers to be tested in the Langley 8-foot high-speed tunnel. A description of these methods is presented herein.

GENERAL ANALYSIS

Method for Determination of Stresses

The general procedure of the analysis is similar to that used in other like problems; the incremental moments and forces produced by the centrifugal and air forces of each of the elements of the blade outboard of a given minimum section at the neutral point of that section have been added to obtain the total moments and forces at that point. The stresses at the section have then been determined by the usual beam formulas. At the knee (fig. 1) and root, the formulas have been revised to account for the curvature of the line through the neutral points at these locations.

In designing the blade, the design sections were laid out perpendicular to the radii to the midpoints of the chord lines of these sections. Twist was applied in the planes of the sections about these midpoints, and sweep was based on the projection of the line through these midpoints in the planes which include the chord lines and the corresponding radii. To make possible the use of these same sweeps and twists in the computations, the stress analysis has been made using the line through these midpoints as the principal reference line.

The forces and moments produced by incremental centrifugal and air loads of a segment of the blade bounded by two design sections relatively close to each other have been assumed to act at the center of percussion of the segment. The center of percussion is assumed to be the midpoint of the chord line of the section equidistant from these two bounding sections. (The errors in the final results arising from this assumption are small.) These resultant centrifugal and air forces have been resolved into three components: one parallel to the axis of rotation, one parallel with the radius through the midpoint of a chord line of a design section inboard of the source of forces, and one perpendicular to the other two. (See fig. 2.) The moments of these force components about this midchord reference point have then been determined. The components of the forces and moments along and about the chord line of the minimum section passing through the reference point, the axis perpendicular to this section, and the axis perpendicular to these lines, all passing through the previously mentioned reference point (fig. 2), have been calculated. These axes will be referred to as the major, polar, and minor axes of the minimum section, respectively.

This process has been carried out for each of the segments of the blade outboard of the reference point and the total effects of the centrifugal and air loads for the several segments on the forces and moments along and about the axes through the reference point have been determined by adding the individual effects. These effects are not the total effects of the blade outboard of the minimum section since they include forces and moments of blade elements inboard of the minimum section and neglect forces and moments of elements outboard of this section, as indicated by the shaded regions in figure 1. Since the superfluous force produced by the inboard triangular blade segment is approximately equal to the neglected force produced by the outboard triangular segment, it may be assumed the actual forces along the axes of the minimum section produced by the total blade outboard of the minimum section are the same as those produced by the several segments bounded by design sections. It may also be assumed that the total moments about the major and polar axes of the minimum section produced by the blade outboard of that section are the same as those produced by these several segments. The total moment about the minor axis of the minimum section produced by blade may be significantly different from that produced by the segments, however, and corrections have been applied to account for this difference.

The moments about the three axes through the neutral points of the minimum section, parallel to the three previously defined axes of that section, have been determined in the usual manner. The maximum stresses at this minimum section are then determined using the forces and moments obtained. The deflections produced by the moments can also be determined. Using the total deflections of the blade, changes in the aerodynamic angle and orientation of the blade elements outboard of the section under consideration can be calculated, and the changes in the air and centrifugal loads on these elements produced by these changes can be estimated.

The equations used to determine the various forces and moments and the stresses produced by these factors are presented in the appendixes. The equations can also be used to determine the forces and moments at the pitch-changing mechanism.

Method for Reduction of Moments

An analysis of the geometry of a swept blade indicates that, if the elements of the blade are oriented in the proper manner, the bending moments about the major axes of all the minimum sections may be eliminated. Since the blade angle, sweep angle, and section are fixed for each position along the radius for a given design, it follows that the only variable that can be changed to obtain this proper orientation is the dihedral. (See fig. 3.) The proper dihedral for each radial position is obtained by a process of trial; that is, a dihedral distribution is assumed and the moments are then calculated. This process is continued until the moments are eliminated. The process is greatly simplified if the moments about sections near the tip are eliminated first and those about sections progressively farther inboard are then eliminated. In this process the geometry of the blade outboard of a given section is fixed, once the dihedral distribution to eliminate the moments about this section is applied. The moment about a section slightly farther inboard is then eliminated by merely changing the dihedral of the blade between these two sections. The moments may be eliminated for only one operating condition. However, this procedure generally reduces the moments for most other operating conditions.

The use of dihedral may produce adverse effects, however, which may be more important than its advantages. The dihedral required may be so large that the aerodynamic and structural characteristics of the blade would be altered. Such alterations might severely reduce the performance of the propeller. In the case of the propeller designed to be tested in the Langley 8-foot high-speed tunnel, the dihedral required to eliminate the moments about the major axis of the sections along the portions of the blade with relatively uniform sweep have been applied. However, the dihedrals required to eliminate the moments at the knee were fairly large. Since the effect of this large dihedral on the performance was unknown, it was reduced, and small stresses about the major axis at the knee were accepted. These stresses were not large enough to limit the final design of the propeller, however.

The moments about all axes perpendicular to the pitch-change axis at the center of the pitch-changing mechanism may also be eliminated for one condition by the proper orientation of this axis with respect to the blade. However, the moment about the pitch-change axis cannot be reduced below a minimum value, which is fairly great in comparison with the moment about the pitch-change axis for an unswept propeller.

After the dihedral distribution required to eliminate or reduce the moments about the major axes of the minimum sections and the position of the pitch-change axis required to eliminate the moments about the axes perpendicular to this axis are incorporated into the blade for a given design condition, the forces and moments and the stresses produced by these factors at the various minimum sections and at the pitch-changing mechanism for the design condition and conditions away from that of design are determined by the process described in the previous section.

Relative Magnitude of Factors

The stress analysis of the swept propeller blades to be tested in the Langley 8-foot high-speed tunnel indicates that for a blade with a relatively large amount of sweep the greatest stresses are produced by the moments about the minor axes of the minimum sections. The stresses produced by the moments about the major axes of minimum sections with the dihedral used are considerably less than the moments about the minor axes and therefore do not limit the design. The tension stresses produced by the force components parallel to the polar axes of the minimum sections are much less than those produced by the moments about the minor axes of the sections. They are added directly to these primary stresses, however, and therefore affect to a secondary extent the design of the blade. The shear stresses produced by the shearing loads and the moments about the polar axis of the minimum section are well below the maximum shear strength of aluminum alloy when the tension stresses are less than the maximum tension strength for the alloy. Therefore, they also do not affect the design of the blade.

The deflections which change the aerodynamic angles of the blade elements and the orientation of the blade elements are produced primarily by the moments about the major axes of the minimum sections. Since the moments about these axes are reduced to very small values by the use of dihedral for the design condition, the deflections for this condition are small and may usually be neglected. The deflections for conditions away from the design condition may be significant. It should be pointed out, however, that the deflections will be such as to reduce the moments producing the deflections and no divergent stress variations may be expected. The deflections produced by the moments about the minor and polar axes of the minimum section change the aerodynamic angle and orientations to a secondary extent and should be considered only in a detailed calculation of the changes of these angles and orientations.

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APPENDIX I

FORMULAS FOR THE DETERMINATION OF FORCES AND MOMENTS

Definitions

β Blade angle of a section perpendicular to the radius through the midpoint of the chord line of the section measured from the plane of rotation to the chord line (fig. 3) for the design condition

Λ Sweep angle of a line through the midpoints of the chord lines of sections perpendicular to the radii through the midpoints, as measured from the radius of a given section in the plane through the radius and the chord line of the section (fig. 3) for the design condition

D Dihedral angle of a line through the midpoints of the chord lines of sections perpendicular to the radii through the midpoints, as measured from the radius of a given section in a plane perpendicular to that in which the sweep was measured (fig. 3) for the design condition

γ Angle between the chord line of a section and the projection of the line of the midpoints of the chord lines in a plane perpendicular to the radius to the midpoint of the chord line (fig. 3) for the design condition angle, and is defined as follows:

$$\gamma = \tan^{-1}\left(\frac{\tan D}{\tan \Lambda}\right)$$

μ Angle between the chord line of a section and the projection of the chord line of the assumed minimum section through the midpoint of the chord line in a plane perpendicular to the radius to the midpoint of the chord line (fig. 3) for the design condition

$$\mu = \tan^{-1}\left(\tan D \cos D \tan \Lambda\right)$$

The chord line of the minimum section is assumed to be perpendicular to the line of midpoints of the chord lines in the plane of the chord lines (fig. 3).

r — Distance along a radius from the axis of rotation to the midchord point of a section (fig. 3) for the design condition

XX — Axis parallel to the axis of rotation of the propeller through the reference point N, the point for which calculations are made; forces and moment vectors along axis in the direction of flight considered positive (fig. 2)

ZZ — Axis along radius to reference point; forces and moment vectors along axis in direction away from axis of rotation considered positive (fig. 2)

YY — Axis perpendicular to XX- and ZZ-axes through reference point N; forces and moment vectors along axis in direction of blade motion considered positive (fig. 2)

x — Distance parallel to the axis of rotation from the center of percussion of a segment of the blade to the YZ-plane (fig. 2) for the design condition; distances from YZ-plane rearward considered positive

θ — The angle between the radius to the center of percussion of a segment and the ZZ-axis in a plane normal to the axis of rotation (fig. 2) for the design condition

N — Reference point, or the point for which calculations are made; on the blade, these points were taken along the line through the midpoints of the chord lines of sections perpendicular to the radii to these midpoints and at the hub, they were taken along the center line of the hub

ΔC_n — The centrifugal force produced by a segment of the blade in pounds

$$\Delta C_n = \frac{W}{g} r_n w^2$$

where

W — weight of segment

g — gravitational constant

r_n — radius to center of percussion of segment

w — rotational velocity in radians per second

ΔA_n The aerodynamic force produced by a segment of the blade in pounds

$$\Delta A_n = C_L A q$$

where

C_L section lift coefficient for mean section of segment

A plan area of segment

q dynamic pressure at mean chord of segment

XX_r Axis of rotation of propeller

ZZ_r Pitch-change axis

YY_r Axis perpendicular to XX_r and ZZ_r axes through the intersection of these axes

x_r Distance parallel to axis of rotation from ZY_r plane to center of percussion of segment or reference point N depending on whether subscript is n or N, respectively; distances rearward considered positive

z_r Distance parallel to ZZ_r axis from XY_r plane to center of percussion of segment or reference point; distances away from axis of rotation considered positive

y_r Distance parallel to YY_r axis from XZ_r plane to center of percussion of segment or reference point; distances opposite to direction of rotation considered positive

θ_r Angle between XZ_r plane to radius of the center of percussion of a segment or a reference point; angles opposite to direction of rotation considered positive

Subscripts and superscripts:

n Refers to conditions at the midpoints of the segments

N Refers to conditions at the point for which calculations are made, the reference point

Refers to conditions away from those of design

Geometry of Blade

To determine the forces and moments produced at one point by the centrifugal and air loads on a segment farther outboard, the orientation of the segment with respect to the point is required.

The orientation of the center of percussion of the segment with respect to the point of reference has been defined in terms of the distance between the two points parallel to the axis of rotation x, the radii to the two points r_n and r_N, and the angle θ between the radii to two points. The total values x and θ are determined by adding incremental values Δx and $\Delta\theta$ for constant increments of the radii. These increments are functions of the sweep, blade angle, and dihedral for the incremental section. They are defined by the following expressions:

$$\Delta x = -\Delta r \frac{\tan \Lambda_n \sin (\beta - \gamma)}{\cos \gamma}$$

(See fig. 3.)

$$\Delta\theta = \frac{\Delta y}{r} = -\frac{\Delta r}{r} \frac{\tan \Lambda_n \cos (\beta - \gamma)}{\cos \gamma}$$

In the design of the blade to be tested in the Langley 8-foot high-speed tunnel, Δr was made equal to the width of a segment, which was a constant of 1.0 inch. Then the displacement of the center of segment to the point of reference is:

$$x = \frac{\Delta x_n}{2} + \Delta x_{n-1} + \cdot \cdot \cdot + \Delta x_N$$

$$\theta = \frac{\Delta\theta_n}{2} + \Delta\theta_{n-1} + \cdot \cdot \cdot + \Delta\theta_N$$

where Δx_N and $\Delta\theta_N$ are the increments just outboard of the reference point.

Forces and Moments in Planes of Reference

The components of the incremental centrifugal loads in the X-, Y-, and Z-directions at N due to an outboard segment are given by the following expressions:

Force in Y-direction, $C_Y = -\Delta C_n \sin \theta$

Force in Z-direction, $C_Z = \Delta C_n \cos \theta$

The components of the incremental air loads in the X-, Y-, and Z-directions at N due to an outboard segment are given approximately by the following expressions:

Force in X-direction, $A_X = \Delta A_n \cos \beta_n$

Force in Y-direction, $A_Y = -\Delta A_n \sin \beta_n \cos \theta$

Force in Z-direction, $A_Z = -\Delta A_n \sin \beta_n \sin \theta$

The moments of incremental centrifugal loads about the X-, Y-, and Z-axes at N due to an outboard segment are given by the following expressions:

Moment about X-axis, $M_X = \sum_t^N \Delta C_n r_N \sin \theta$

Moment about Y-axis, $M_Y = \sum_t^N -\Delta C_n x \cos \theta$

Moment about Z-axis, $M_Z = \sum_t^N \Delta C_n x \sin \theta$

where the limit t refers to the tip and N refers to the point for which the moments are being calculated.

The moments of incremental air loads about the X-, Y-, and Z-axes at N due to an outboard segment are given by the following expressions:

$$\text{Moment about X-axis, } M_X = \sum_t^N \Delta A_n \left(-r_n \sin \beta_n + r_N \sin \beta_n \cos \theta\right)$$

Moment about Y-axis, $M_Y = \sum_{t}^{N} \Delta A_n \left[-\cos \beta_n (r_n \cos \theta - r_N) + x \sin \beta_n \sin \theta\right]$

Moment about Z-axis, $M_Z = \sum_{t}^{N} \Delta A_n \left(-r_n \cos \beta_n \sin \theta - x \sin \beta_n \cos \theta\right)$

Forces and Moments with Reference to the Minimum Section

The force perpendicular to the minimum section is given approximately as:

$$F_T = \sum_{t}^{N} \Delta C_n \left[\sin \theta \cos (\beta - \gamma)_N \sin \Lambda_N + \cos \theta \cos \Lambda_N\right]$$

$$+ \sum_{t}^{N} \Delta A_n \left[-\cos \beta_n \sin (\beta - \gamma)_N \sin \Lambda_N + \sin \beta_n \cos \theta \cos (\beta - \gamma)_N \sin \Lambda_N - \sin \beta_n \sin \theta \cos \Lambda_N\right]$$

The forces in the plane of the minimum section are given approximately as follows:

<u>Along minor axis</u>

$$F_S = \sum_{t}^{N} \Delta C_n \left(\sin \theta \cos \beta_N + \cos \theta \sin D_N\right)$$

$$+ \sum_{t}^{N} \Delta A_n \left(-\cos \beta_n \cos \beta_N - \sin \beta_n \cos \theta \sin \beta_N - \sin \beta_n \sin \theta \sin D_N\right)$$

Along major axis

$$F_S = \sum_{t}^{N} \Delta C_n \left[\cos\theta \sin\Lambda_N - \sin\theta \cos(\beta + M)N \cos\Lambda_N\right]$$

$$+ \sum_{t}^{N} \Delta A_n \left[\cos\beta_n \sin(\beta + M)N \cos\Lambda_N\right.$$

$$- \sin\beta_n \cos\theta \cos(\beta + M)N \cos\Lambda_N$$

$$\left. - \sin\beta_n \sin\theta \sin\Lambda_N\right]$$

The total shearing force across the minimum section is the square root of the sum of the squares of the forces along the major and minor axes.

The moment about the polar axis of the minimum section as defined is given approximately by the following expression:

$$M_P = \sum_{t}^{N} \Delta C_n \left\{-\left(r_N \sin\theta\right)\left[\sin(\beta - \gamma)N \sin\Lambda_N\right]\right.$$

$$\left. + (x \cos\theta)\left[\cos(\beta - \gamma)N \sin\Lambda_N\right] + (x \sin\theta)\left(\cos\Lambda_N\right)\right\}$$

$$+ \sum_{t}^{N} A_n \left\{-\left(-r_n \sin\beta_n + r_N \sin\beta_n \cos\theta\right)\left[\sin(\beta - \gamma)N \sin\Lambda_N\right]\right.$$

$$- \left(-r_n \cos\beta_n \cos\theta + r_N \cos\beta_n\right.$$

$$\left. + x \sin\beta_n \sin\theta\right)\left[\cos(\beta - \gamma)N \sin\Lambda_N\right]$$

$$\left. + \left(-r_n \cos\beta_n \sin\theta - x \sin\beta_n \cos\theta\right)\left(\cos\Lambda_N\right)\right\}$$

The moment about the center of gravity of the minimum section is very nearly the same as that about the defined polar axis.

The moment about the major axis of the minimum section as defined is given approximately by the following expression:

$$M_M = \sum_t^N \Delta C_n \Big\{ \left(r_N \sin\theta\right) \left[\sin(\beta + M)_N \cos\Lambda_N\right]$$

$$- \left(x \cos\theta\right) \left[\cos(\beta + M)_N \cos\Lambda_N\right] + \left(x \sin\theta\right)\left(\sin\Lambda_N\right) \Big\}$$

$$+ \sum_t^N \Delta A_n \Big\{ + \Big(-r_n \sin\beta_n$$

$$+ r_N \sin\beta_n \cos\theta\Big) \left[\sin(\beta + M)_N \cos\Lambda_N\right]$$

$$+ \Big(-r_n \cos\beta_n \cos\theta + r_N \cos\beta_n$$

$$+ x \sin\beta_n \sin\theta\Big) \left[\cos(\beta + M)_N \cos\Lambda_N\right]$$

$$+ \left(-r_n \cos\beta_n \sin\theta x \sin\beta_n \cos\theta\right) \left(\sin\Lambda_N\right) \Big\}$$

The moment about the major neutral axis of the minimum section is very nearly the same as that about the defined major axis.

The uncorrected moment about the minor axis of the minimum section as defined is given approximately as follows:

$$M_{KU} = \sum_t^N \Delta C_n \Big(-r_N \sin\theta \cos\beta_N \cos D_N$$

$$- x \cos\theta \sin\beta_N \cos D_N + x \sin\theta \sin D_N\Big)$$

$$+ \sum_t^N \Delta A_n \Big[-\left(-r_n \sin\beta_n + r_N \sin\beta_n \cos\theta\right) \left(\cos\beta_N \cos D_N\right)$$

$$+ \left(-r_n \cos\beta_n \cos\theta + r_N \cos\beta_n - x \sin\beta_n \sin\theta\right) \left(\sin\beta_N \cos D_N\right)$$

$$+ \left(r_N \cos\beta_n \sin\theta + x \sin\beta_n \cos\theta\right) \left(\sin D_N\right) \Big]$$

The uncorrected moment about the minor neutral axis of the minimum section is given by the following expression:

$$M_{LU} = M_{KU} + F_T R$$

where R is the distance between the centers of the moments M_{KU} and M_{LU} along the minimum section.

The correction to the moment about the minor axis of the minimum section to account for the fact that the incremental sections for which the loads are computed do not include the elements of the blade outboard of the minimum section exclusively (fig. 1) is defined approximately as follows:

$$\Delta M_L = -\frac{1}{12} \sin \Lambda \cos \Lambda (1 + \cos \Lambda) c_N{}^3 \left(\frac{\Delta C_n}{\Delta A}\right)$$

where c_N is the chord of the design section passing through the reference point.

Conditions Away from Design

When the blade is turned about the hub away from its design condition, the orientation of the center of percussion of a segment with respect to the point of reference N obviously changes. To calculate the moments and forces at N produced by the segment for these conditions, the new orientation of the segment with respect to N must be known. The orientation is defined by x, θ, r_n, and r_N; and, to define the new orientation, new values of these variables must be determined. This has been done by determining the values of x_r, y_r, z_r, and θ_r for both the segment and the point of reference with respect to the YZ_r, XZ_r, and XY_r planes; calculating new values of x_r, y_r, and z_r for the new blade setting away from design; determining new values of the angle θ_r with respect to the XZ_r and XY_r planes, using the new values of y_r and z_r with respect to these planes; and subtracting the new values of θ_r and x_r obtained for the point of reference from those obtained for the segment to get the new values of x and θ of the segment with respect to the reference point. The equations used are as follows:

$$x' = x_{r_n}' - x_{r_N}'$$

$$x_r' = x_r \csc\left[\tan^{-1}\left(\frac{r}{y_r}\right)\right] \sin\left[\tan^{-1}\left(\frac{x_r}{y_r}\right) + \Delta\beta_1\right]$$

For both n and N

$$\theta' = \theta_{r_n}' - \theta_{r_N}'$$

$$\theta_r' = \tan^{-1}\left\{\frac{x_r}{z_r} \csc\left[\tan^{-1}\left(\frac{x_r}{y_r}\right)\right] \cos\left[\tan^{-1}\left(\frac{x_r}{y_r}\right) + \beta_1\right]\right\}$$

For both n and N

$$r_n' = r_n \cos\theta_{r_n} \sec\theta_{r_n}'$$

$$r_N' = r_N \cos\theta_{r_N} \sec\theta_{r_N}'$$

where $\Delta\beta_1$ is the change in orientation of the blade about pitch-change axis.

The angles Λ', β', and γ' for conditions away from that of design may be determined from Λ, β, γ, θ_r, θ_r', and $\Delta\beta_1$, using the expressions that follow. These expressions were developed using the diagrams presented in figure 4. A triangular portion of the chord plane of the blade 1,2,3, as shown in figure 3, was rotated through an angle $\Delta\beta_1$ about an axis parallel to the hub axis. Using the new value of θ_r, θ_r' for point Z, a new triangular portion of the chord plane 1',2',3' was constructed and new values of Λ, β, and γ, and Λ', β', and γ' were obtained from this triangle.

The development is applicable only if the pitch-change axis passes through the center of rotation of the propeller. Equations for configurations with some other orientation of this axis would be much more complex and have not been developed in the present analysis since the pitch-change axis of the propeller to be tested in the Langley 8-foot high-speed tunnel was arbitrarily oriented to pass through the axis of rotation. Somewhat lower bending moments about the major axes of the various minimum sections at conditions away from that of design could probably be obtained by the use of some other orientation of the pitch-change axis. However, the probable reduction in moments produced by using such an orientation does not justify the extensive complex calculations required to find that particular orientation.

$$c = \Delta r \tan \Lambda$$

$$b = \Delta r \tan D$$

$$d = \frac{b}{\sin \gamma}$$

$$e = d \cos (\beta - \gamma)$$

$$f = d \sin (\beta - \gamma)$$

$$g = c \sin \beta$$

$$h = c \cos \beta$$

$$A = \tan^{-1} \left(\frac{\Delta r}{e}\right)$$

$$i = \Delta r \tan \theta_r$$

$$j = (e + i) \cos \theta_r$$

$$k = h \cos \theta_r$$

$$\beta_1 = \tan^{-1} \left(\frac{g}{k}\right)$$

$$C = \tan^{-1} \left(\frac{f}{j}\right)$$

$$l = \frac{g}{\sin \beta_1}$$

$$m = \frac{f}{\sin C}$$

$$n = l \sin(\beta_1 - C)$$

$$o = l \cos(\beta_1 - C)$$

$$B = \tan^{-1}\left(\frac{n}{m - o}\right)$$

$$q = l \cos(\beta_1 + \Delta\beta_1)$$

$$s = m \cos(C + \Delta\beta_1)$$

$$t = j \tan A - \theta_r$$

$$u = k \tan \theta_r$$

$$E = \tan^{-1}\left(\frac{t}{s}\right)$$

$$F = \tan^{-1}\left(\frac{u}{q}\right)$$

$$v = \frac{s}{\cos E}$$

$$w = \frac{q}{\cos F}$$

$$\Delta r' = v \sin\left|\theta_r' + E\right|$$

$$e' = v \cos\left|\theta_r' + E\right|$$

$$\epsilon = v \sin\left|F + E\right|$$

$$\zeta = v \cos\left|F + E\right|$$

$$G = \tan^{-1}\left(\frac{\epsilon}{\zeta - w}\right)$$

$$\delta = \frac{u \sin G}{\sin\left(\theta_r' - F + G\right)}$$

$$\kappa = \delta \cos \theta_r'$$

$$\eta = (s - \kappa)\left\{\tan\left[B - \left(C + \Delta\beta_1\right)\right]\right\}$$

$$f' = m \sin\left(C + \Delta\beta_1\right)$$

$$H = \tan^{-1}\left(\frac{f'}{e'}\right)$$

$$d' = \frac{e'}{\cos H}$$

$$c' = d' \cos \gamma'$$

$$\gamma' = \beta' - H$$

$$\beta' = \tan^{-1}\left(\frac{f' + \eta}{\delta}\right)$$

$$\Lambda' = \tan^{-1} \frac{c'}{\Delta r'}$$

Using the new blade and sweep angles for each of the sections as determined by these formulas and the stream angles as determined from the operating conditions, the aerodynamic loadings on the blade segments may be estimated. Using the new values of r, r', the new centrifugal loading for the segments may be estimated. From these loadings the forces and moments at the points of reference may be determined, using the new values of x and θ, and r, x', θ', and r'. The forces and moments along and about the axes of the minimum section are then determined using revised values of Λ_N, B_N, and D_N, Λ_N', B_N', and D_N' in the formulas given for calculating forces and moments at the design condition.

APPENDIX II

DETERMINATION OF STRESSES

The maximum stresses produced by the tension and shear forces at all minimum sections along the blade are determined by the usual relation: stress is equal to force divided by area. The maximum stresses produced by the moments about the major and polar axes of all the sections, and the moments about the minor axis of sections along the relatively straight portions of the blade, are determined using the well-known relation: stress is equal to the moment times the distance from the centroid to the extreme element divided by the moment of inertia of the section about the axis of the moment. This expression cannot be used alone, however, to determine the maximum stresses produced by the moments about the minor axis of the sections at the knee and root.

Expressions presented by Timoshenko in reference 1 indicate that, for the radii of curvature needed at the knee and root of blades with large amounts of sweep, the maximum stresses at these points are much greater than those calculated by the usual beam formulas. The expressions presented in the above reference are derived assuming uniform sections and radii of curvature, however, and therefore are not directly applicable to the accurate determination of the maximum stresses at the knee and root of the usual blade, since the sections and radii at these points are not uniform. The maximum stresses may be approximated rather closely, however, with a method which is based on these expressions.

In the method developed by Timoshenko, it is assumed that the angle between two minimum cross sections of a uniformly curved bar $d\varphi$ changes by $\Delta\, d\varphi$ when a moment is applied to the bar. (See fig. 5(a).) The extension of any fiber at a distance y from the neutral surface is $y\, \Delta\, d\varphi$, and the corresponding unit elongation is

$$\epsilon = \frac{y\, \Delta\, d\varphi}{(r - y)\, d\varphi}$$

where r denotes the radius of the neutral surface. The bending stress at a distance y from the neutral axis is then

$$\delta_x = \frac{Ey\, \Delta\, d\varphi}{(r - y)\, d\varphi}$$

From the preceding expression

$$\delta_{max} = \frac{Mk_1}{Ae\left(r - k_1\right)}$$

where k_1 is the distance from the neutral axis to the most remote fiber and e, the displacement of the neutral axis from centroid of the section, is defined by

$$e = R\frac{m}{1 + m}$$

Assuming that the section is elliptical

$$m = \frac{1}{4}\left(\frac{k}{2R}\right)^2 + \frac{1}{8}\left(\frac{k}{2R}\right)^4 + \frac{5}{64}\left(\frac{k}{2R}\right)^6 \cdot \cdot \cdot$$

where $R = r + e$. The stress-concentration factor is then

$$F_{sc} = 4e\frac{\left(r - k_1\right)}{{k_1}^2}$$

It is apparent that, for the swept blade, the radius of curvature of the neutral axis r will vary from very large values along the relatively straight portions of the blade to relatively small values along the central portion of the knee and root bends. The corresponding displacement of the neutral surface from the centroids of the sections will vary from very small to relatively large values. It may be assumed that the transition from one condition to another occurs continuously. The orientation of the plane through the neutral surface along the blade, therefore, is similar to that shown in figure 5(b). To determine the radii of curvature of the neutral surface, the displacement of the neutral surface with respect to the centroids and, finally, the maximum stresses, it has been assumed that the previously stated expressions for curved beams are valid for each of the incremental elements between cross sections perpendicular to the plane through the neutral axis. (See fig. 5(b).) These sections are not necessarily minimum sections.

Because of the severe variations of blade width at the knee, the leading edge and trailing edges of the blade diverge considerably from perpendicular to the cross sections previously mentioned in this region.

The actual maximum tension and compression stresses along the trailing edge, therefore, will differ somewhat from the stresses calculated on the assumption that these stresses are perpendicular to the cross sections. The actual stress is probably greater than the calculated value by a factor of approximately the reciprocal of the cosine of the angle between the edge and a perpendicular to the section.

The determination of the quantities desired in this case is complicated by the fact that the various variables are interdependent. The displacement of the neutral surface at each element is a function not only of the radius of curvature of the neutral surface at that point and the length and shape of the element but is also a function of the conditions for adjacent elements. The radius of curvature of the neutral surface is a function of the local displacement of the neutral surface, the length of the element, and the conditions for adjacent elements. The lengths and shapes of the elements are functions of the orientation of the neutral surface which is a function of all other variables. These variables may be determined, however, using a method of trial based on the assumptions that neutral surface is continuous and tangent to the parallel surface through the centroids along the relatively straight portions of the blade.

By using a graphical method of solution, the trial process has been reduced to a relatively simple and rapid procedure. In this graphical method, an element, the neutral axis of which is assumed to fall on the correct continuous neutral surface, is constructed. (See fig. 5(c).) Adjacent elements with neutral surfaces continuous with that of the initial element and with the proper relationship between the variables are then constructed. This process is continued until the elements include the straight portions of the blade. If the neutral surface of the elements coincides with the plane through the centroids at these points, the assumptions for the original element were correct. Obviously, the chances of assuming the correct original element are small and several attempts are required to determine the correct dimensions of this element.

To speed the graphical process, the radius of curvatures of the neutral surface between two control elements relatively widely spaced in terms of the width of the elements (fig. 5(c)) has been assumed to be the same as the radius at the element for which calculations have been made. Such a process eliminates the calculations for all elements within the space and produces very little error in the final result, if the width of the space is held to proper limits.

In the actual graphical process, an element near the midpoint of the knee with a given displacement of the neutral surface with respect to the surface through the centroid is assumed, and the radius of curvature is determined, using a plot of the radius against displacement. With this radius an arc representing the neutral surface is constructed, and lines representing thin elements are drawn perpendicular to this arc at proper distances from the original element. The distance from the centroids of the new elements to the assumed neutral surface are measured; and, using these displacements, the radii of curvature of the neutral surface at these elements are similarly determined. This process is then continued until the elements include the straight portions of the blade.

In this analysis, it has been assumed that the major axes of all the elements of the blade fall in a plane. Actually, because of the twist of the blade, this is usually not the case. However, the usual amounts of twist have little effect on the results.

REFERENCE

1. Timoshenko, S.: Strength of Materials. Part II – Advanced Theory and Problems. D. Van Nostrand Co., Inc., 1930, p. 65.

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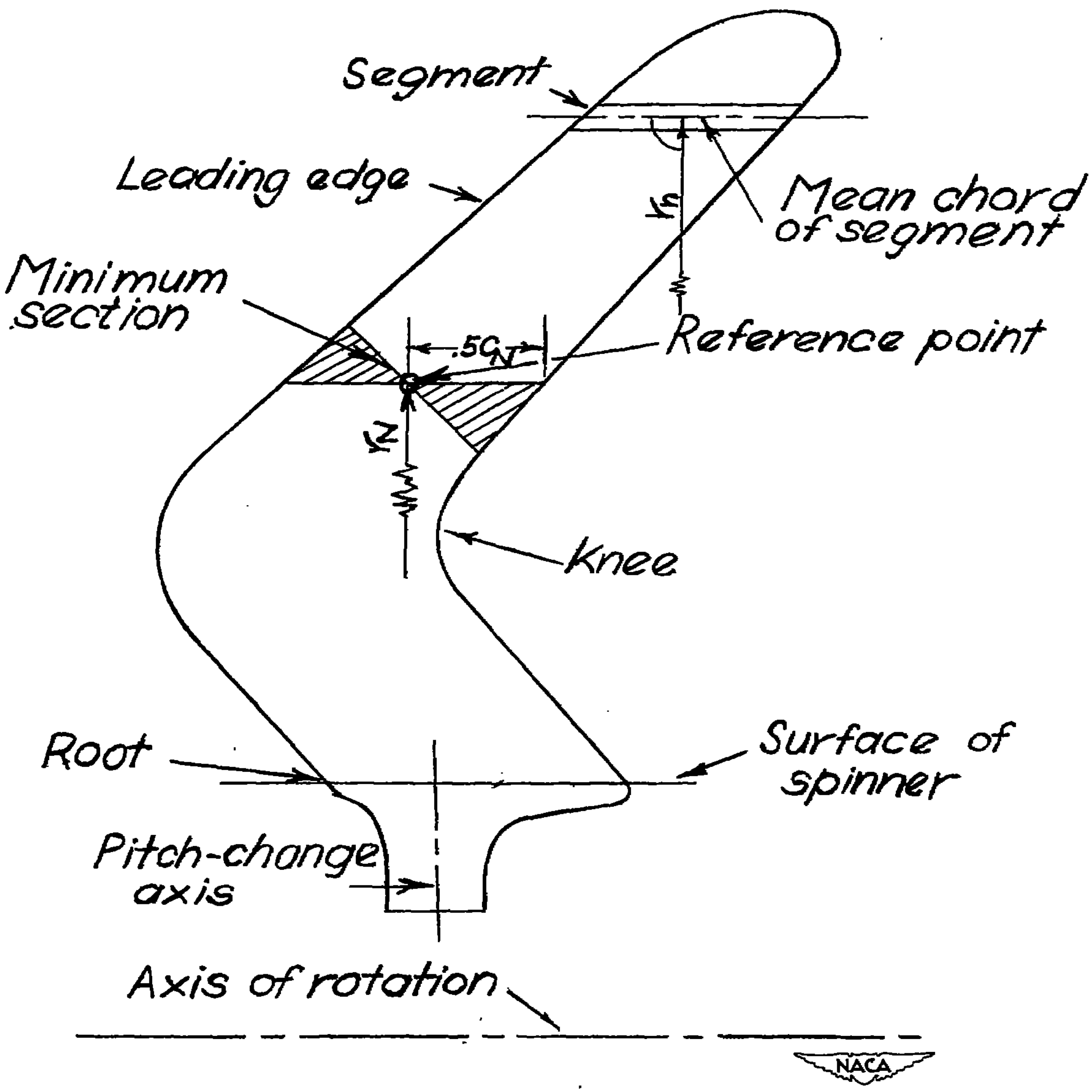

Figure 1.- Sideview of blade.

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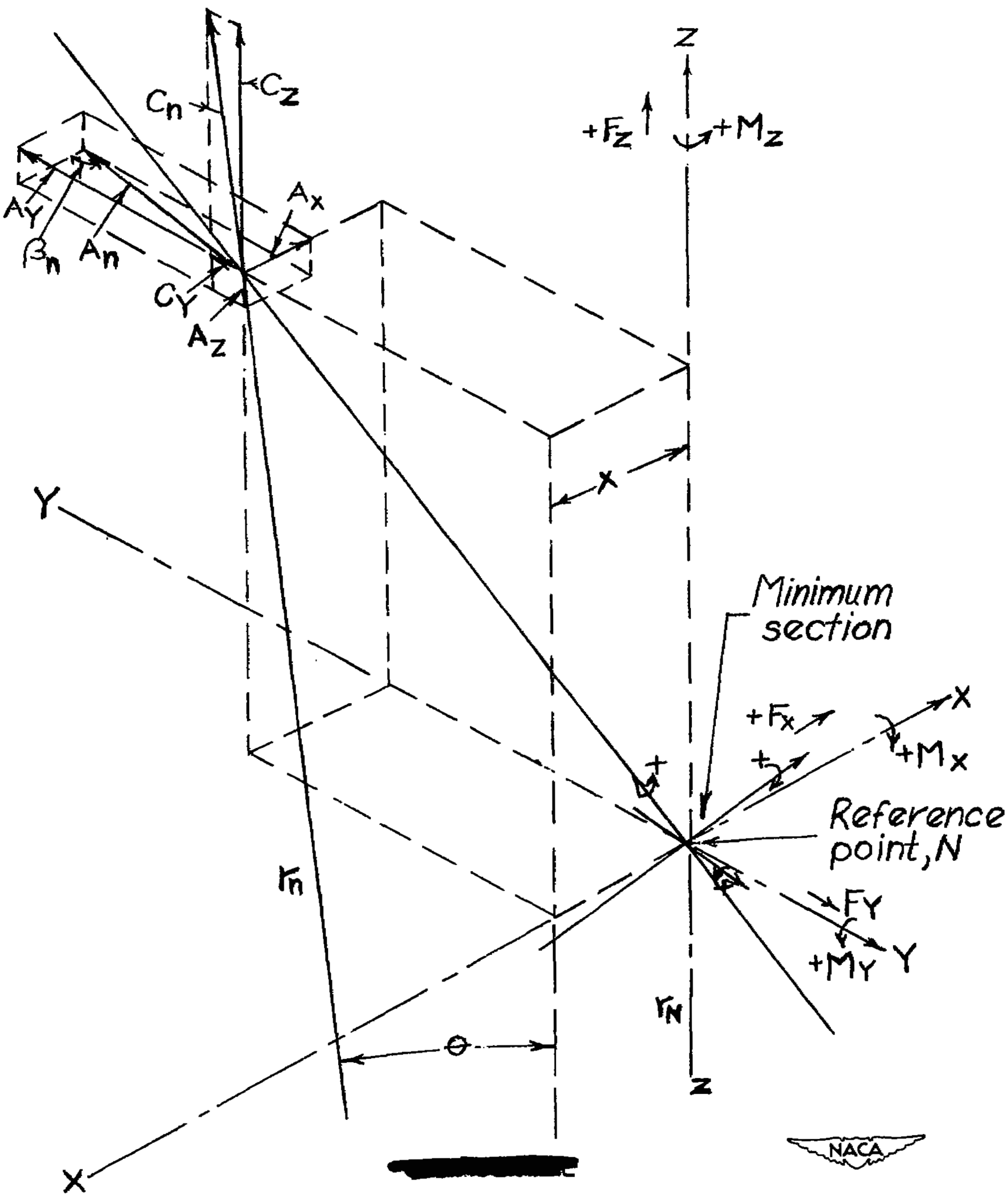

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Figure 2.- Forces and moments.

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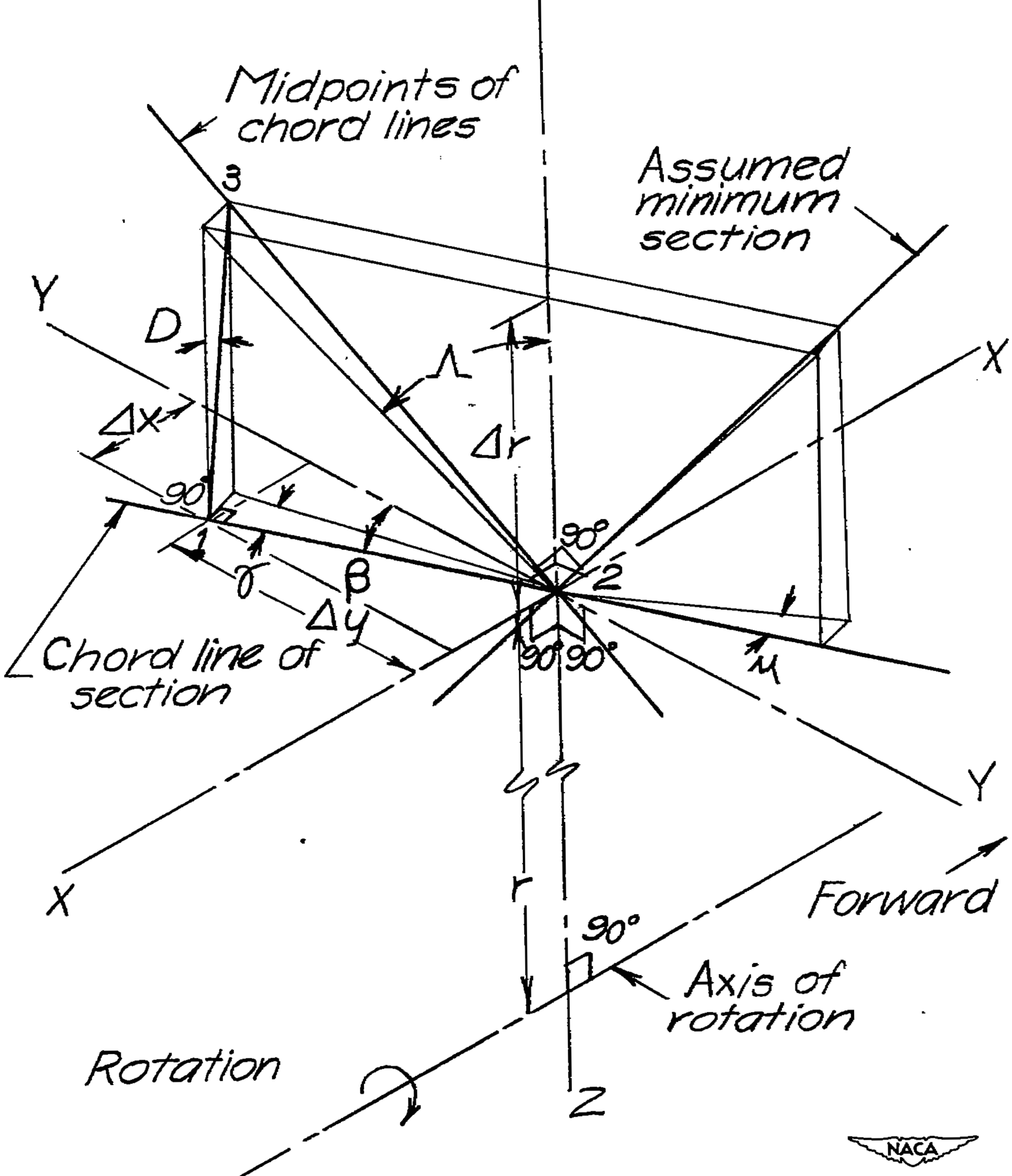

Figure 3.- Definition of symbols.

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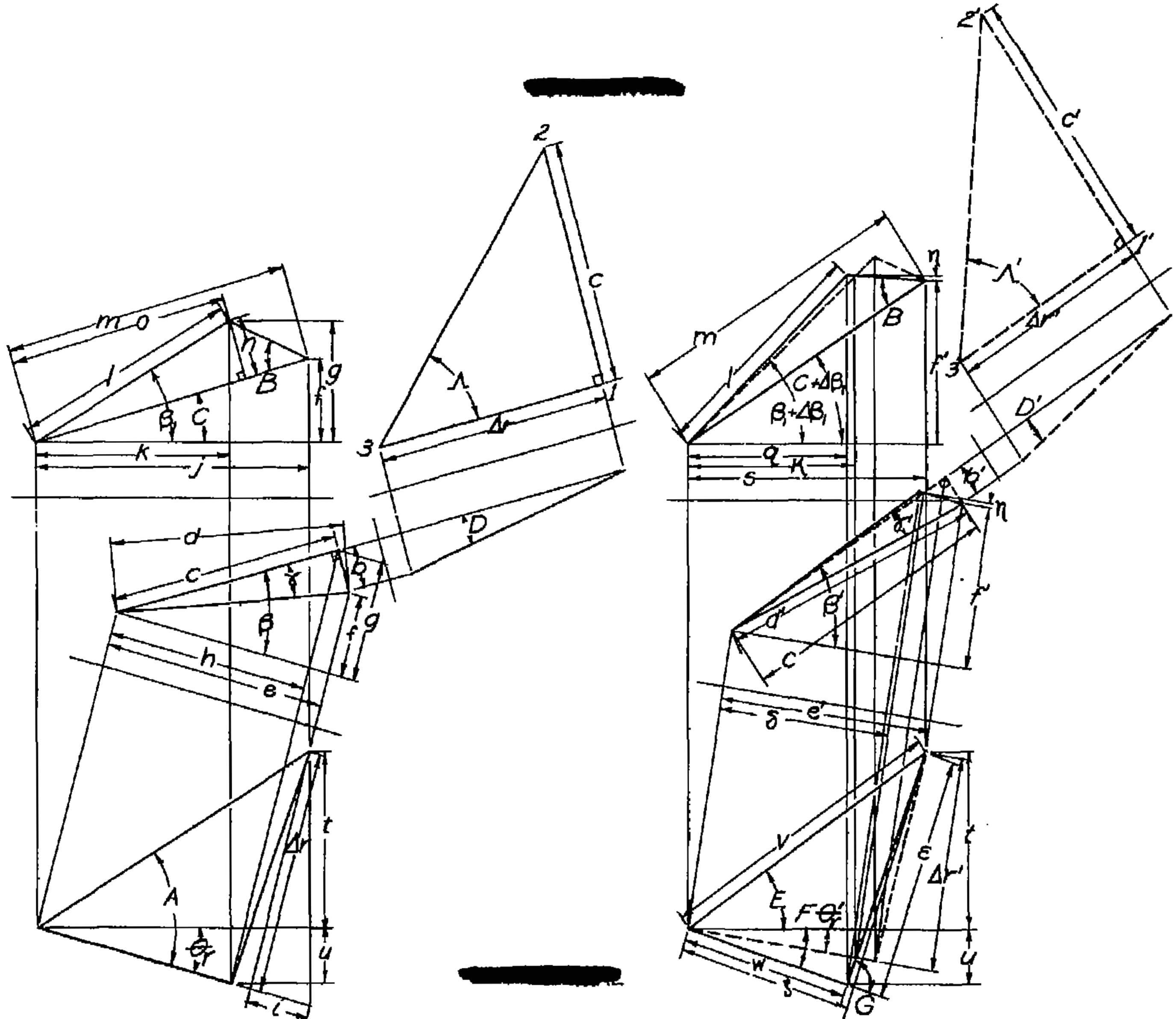

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Figure 4.- Development of conditions away from design.

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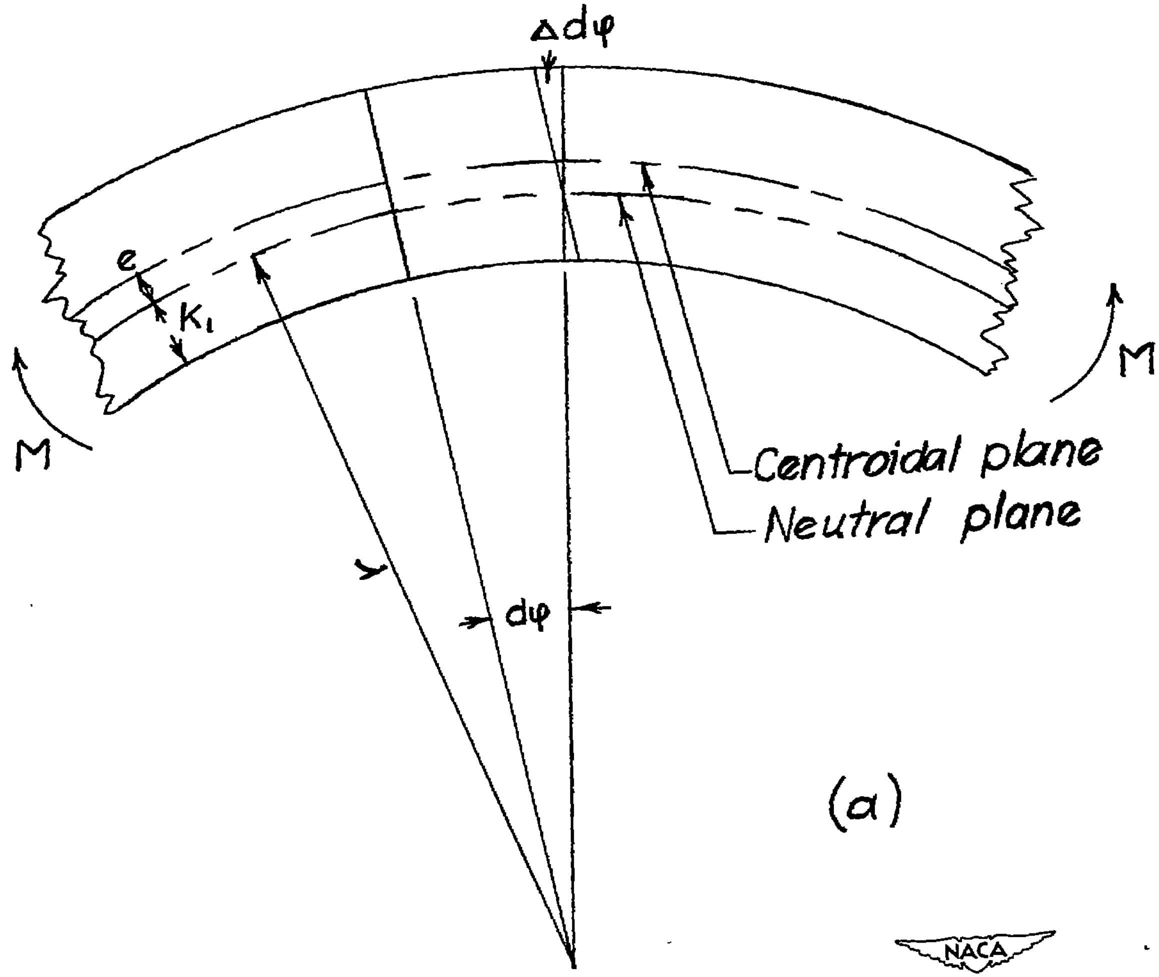

Figure 5.- Development of stress-concentration factor for knee.

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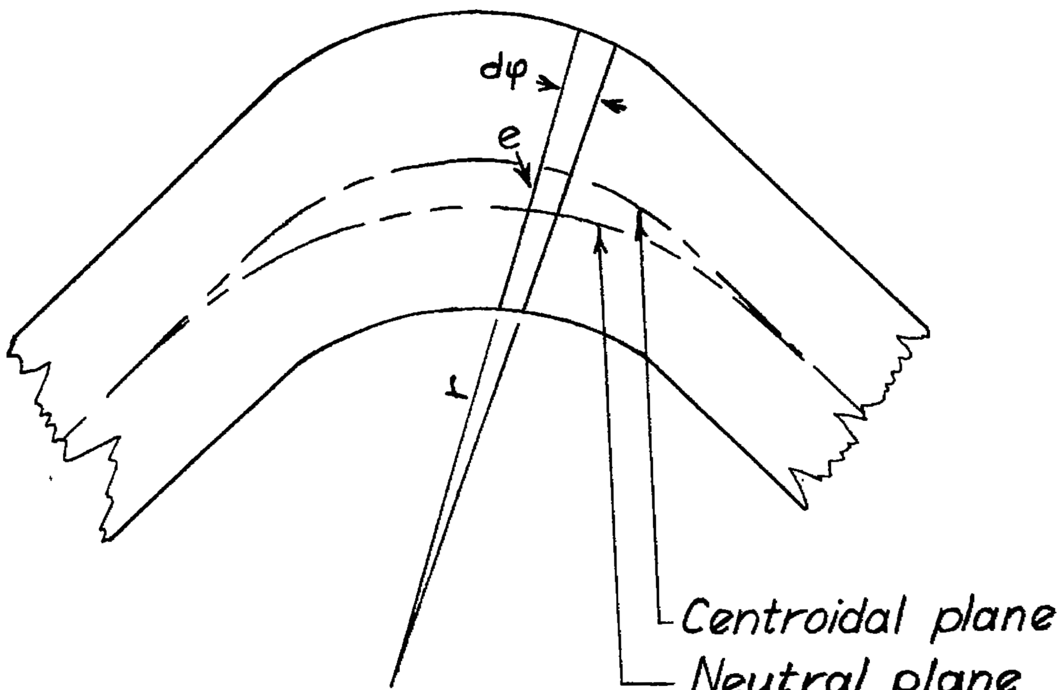

(b)

Figure 5.- Continued.

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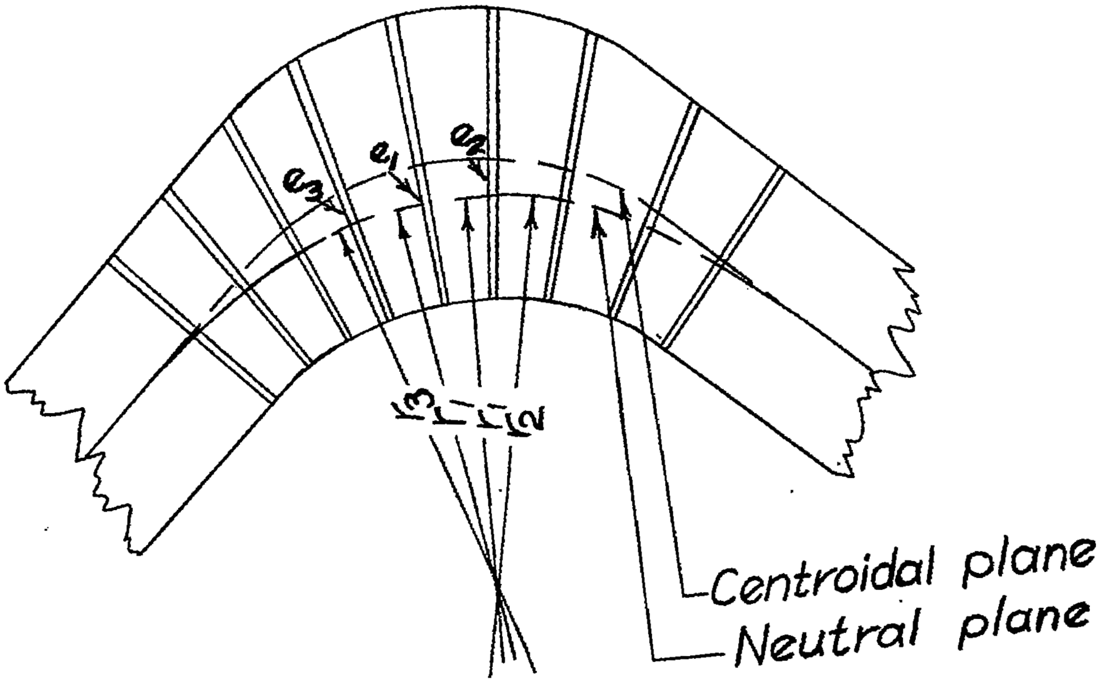

(c)

Figure 5.-Concluded.

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