NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

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CLAMPED LONG RECTANGULAR PLATE UNDER COMBINED AXIAL LOAD AND NORMAL PRESSURE

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National Bureau of Standards

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SUMMARY

A solution is presented for the buckling load and load carried after buckling of a clamped rectangular plate having a width-length ratio of 1:4 under combined normal pressure and axial load. The calculations are carried out for two values of normal pressure and a range of axial loads considerably in excess of those required to buckle the plate.

The results indicate that normal pressure causes a smaller increase in the buckling load of plates with clamped edges than of plates with simply supported edges. They also indicate that neglecting the effect of lateral pressure on the sheet buckling load and on the load carried by the sheet after buckling is conservative design in the elastic range.

INTRODUCTION

The sheet in airplane wings, fuselages, and hull bottoms constructed of sheet metal reinforced by stringers frequently is subjected to normal pressure as well as forces in the plane of the sheet. It is important, therefore, to determine the effect of normal pressure on the ability of a long rectangular plate, which approximates the sheet between stringers, to withstand forces in its own plane.

Experimental results on the behavior of a reinforced flat sheet under combined normal pressure and axial load are given in reference 1. Theoretical results for the extreme case of a plate with simply supported edges are given in reference 2. The theoretical solution for the other extreme case of a plate with clamped edges is given in the present paper. The plate considered will have a ratio of width to length of 1:4. This ratio is the same as that chosen in reference 2 as typical of both hull-bottom plating and monocoque wings.

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SYMBOLS

The symbols have the following significance (see fig. 1):

a length of plate

$b = a/4$ width of plate

h thickness of plate

w deflection of plate

x,y coordinate axes with origin at corner of plate, x-axis in direction of longer side

E Young's modulus

$\mu = \sqrt{0.1} = 0.316$ Poisson's ratio

$D = Eh^3/12(1-\mu^2)$ flexural rigidity of plate

p uniform normal pressure on plate

e average compressive strain at edges $y = 0$ and b

e_{cr} critical strain for buckling

P axial load on plate

$w_{m,n}$ deflection coefficients

k_m, t_n edge moment coefficients

m_x, m_y edge moments per unit length along the longer and shorter sides, respectively

DEFLECTION EQUATIONS

An initially flat rectangular plate of uniform thickness will be considered. The edges of the plate are assumed to be clamped in such a manner that they remain straight under load and that there is no rotation of the plate in the clamps. The loading consists of a uniform normal pressure combined with axial loading in the direction of the longer side of the rectangle.

By use of the method outlined on page 2 of reference 3 and page 4 of reference 4, it can be shown that, if the lateral deflection of the plate is approximated by

$$w = \sum_{m,n\ \mathrm{odd}}^{\infty} w_{m,n} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{1}$$

and if the edge bending moments necessary to maintain zero slope at the edges are given by

$$\left.\begin{aligned} m_x &= \frac{4b^2 p}{\pi^3} \sum_{m=1,3\ldots}^{\infty} k_m \sin\frac{m\pi x}{a} \\ m_y &= \frac{4a^2 p}{\pi^3} \sum_{n=1,3\ldots}^{\infty} t_n \sin\frac{n\pi y}{b} \end{aligned}\right\} \tag{2}$$

where $w_{m,n}$, k_m, and t_n represent undetermined constants whose values are to be determined to satisfy the boundary conditions, then a series of equations holds, whose terms are given in table 1. For example, the first half of the second column of table 1 is equivalent to the equation:

$$0 = 16h^2 w_{1,1} - 9.562(256pb^4/\pi^6 Eh) - 9.562k_1(256pb^4/\pi^6 Eh)$$

$$- 9.562t_1(256pb^4/\pi^6 Eh) + 0.5980w_{1,1}\ (16Pb^2/\pi^2 E)$$

$$+ 9.600w_{1,1}{}^3 - 28.70w_{1,1}{}^2 w_{3,1} \ldots \quad (3)$$

The restriction in equation (1) to values of m and n which are only odd, causes some error in the results. This restriction was necessary in order to limit the number of variables in the equations to a number which could be treated in a reasonable length of time. It was estimated that this restriction would cause errors not greater than about 5 percent since the buckling load of a 1 ; 4 rectangular plate is relatively insensitive to an increase or decrease of one buckle in the buckle pattern. (See fig. 5 of reference 7.)

By use of the method outlined on page 8 of reference 4, the additional relations in table 2 were obtained. For example, the second column of table 2, is equivalent to the equation:

$$0 = -16.96(256pb^4/\pi^6 Eh) - 9.562k_1(256pb^4/\pi^6 Eh)$$

$$- 13.27k_3(256pb^4/\pi^6 Eh) - \ldots + 0.5980w_{1,1}(16Pb^2/\pi^2 E)$$

$$+ 7.470w_{3,1}(16Pb^2/\pi^2 E) + \ldots$$

$$- 3.67w_{1,1}{}^3 + 0.78_{1,1}{}^2 w_{3,1} - \ldots \quad (4)$$

In tables 1 and 2, only those cubic terms involving $w_{1,1}$, $w_{3,1}$, $w_{5,1}$, $w_{7,1}$ and $w_{9,1}$ were retained since the remaining deflection coefficients were considerably smaller than the largest of these and their cubic products were considered negligible.

EDGE STRAIN

The average compressive strain in the x-direction along the edges $y = 0$, $y = b$, was determined from equation (11) of reference 3 as:

$$e = \frac{P}{Ebh} + \frac{\pi^2}{128b^2} \sum_{m,n\ \mathrm{odd}}^{\infty} m^2 w_{m,n}^2 \qquad (5)$$

Equations (1) and (2) require the use of an infinite number of terms. For the finite number of terms considered in tables 1 and 2, errors are introduced into the solution. On the basis of work in reference 3, it is estimated that for plate deflections less than twice the plate thickness, these errors are probably less than 5 percent.

SOLUTION

The simultaneous equations in tables 1 and 2 were solved for the deflection coefficients $w_{m,n}$ as a function of axial load P and lateral pressure p, using the following steps:

1. The equations in table 2 were solved simultaneously for the values of k_1, $k_3 \ldots k_{27}$, t_1, $t_3 \ldots t_7$ as a function of the pressure ratio pb^4/Eh^4 and the deflection ratios $w_{m,n}/h$.

2. The values of k_1, $k_3 \ldots$, t_1, $t_3 \ldots$ obtained in step (1) were substituted in the equations in table 1, thus giving a new set of equations involving only the deflection coefficients $w_{m,n}$, the pressure p, and the axial load P.

3. The resulting equations were expanded in Taylor series, omitting terms involving derivatives of higher order than the first.

4. Values of the deflection coefficient ratios $w_{1,1}/h$ $w_{1,3}/h$, and so forth, were then estimated corresponding to chosen values of Pb/Eh^3 and pb^4/Eh^4.

5. These estimated values were substituted in the Taylor series obtained in step 3, and the resulting linear equations were solved for the difference between the estimated deflection coefficient ratios and their improved values. Crout's method was used (reference 5).

6. The process was repeated until the estimated error was less than 0.2 percent. One or two trials usually were sufficient to give a satisfactory answer.

7. In the neighborhood of the buckling load, the load remained nearly constant while the deflection changed rapidly. For such points, one of the deflection coefficient ratios was taken as an independent variable in place of the load ratio Pb/Eh^3.

8. As the load ratio Pb/Eh^3 was increased, it was observed that the deflection coefficients $w_{m,3}$ became nearly a constant proportion of $w_{m,1}$ and that the deflection coefficients $w_{11,1}$, $w_{13,1}$ became nearly a constant proportion of $w_{1,1}$. This result was taken advantage of, to reduce the number of unknowns and thus simplify the solution. It was also observed that deflection coefficients $w_{m,n}$ for which $m > 13$ had negligible effect on the other variables. These deflection coefficients were accordingly dropped subsequently.

Deflection coefficients determined by this procedure are given for $p = 15.02Eh^4/b^4$ in table 3 and for $p = 37.55Eh^4/b^4$ in table 4. The average compressive strain e at the edges computed from equation (5) is also given in tables 3 and 4.

Cubic equations like those in tables 1 and 2 frequently have more than one real solution. The single solutions given in tables 3 and 4 correspond to a continuous change in the buckle pattern from zero axial load to the maximum axial load considered.

The lateral deflection was computed from the deflection coefficients $w_{m,n}$ in tables 3 and 4 and equation (1) to show the development of the buckle pattern. The results are shown in figures 2 and 3 for pressures $p = 15.02Eh^4/b^4$ and $37.55Eh^4/b^4$, respectively. It is seen that the deflection of the plate at the axial center line is a single long bulge for low axial force P and gradually builds up to a regular buckle pattern at larger values of P. The shifting of the buckle pattern is not accompanied by a drop in axial load. It is significant to note that the initial general downward deflection of the sheet due to normal pressure, tends to disappear at high axial loads.

The axial load P in tables 3 and 4 is plotted against the average edge compressive strain e in figure 4. Curve A, corresponding to the lower normal pressure $p = 15.02Eh^4/b^4$, shows a continuous increase in axial load P with edge strain e, together with changes in the direction of the curve at $P = 6.8Eh^3/b$ and at $P = 13.2Eh^3/b$. These loads corresponded to changes in the buckle pattern. Curve B, corresponding to the higher normal pressure $p = 37.55Eh^4/b^4$, changes direction at $P = 8.3Eh^8/b$. There is only one change in buckle pattern at the higher pressure. The axial load for a long clamped plate without normal pressure (reference 6) is shown as curve C in figure 4. Comparison of curves A, B, and C in figure 4 indicates that the effect of normal pressure on the axial load for a given edge strain is negligible.

The axial load at which buckling first occurs is $P = 6.4Eh^3/b$ when $p = 0$ (reference 7), $P = 6.8Eh^3/b$ when $p = 15.02Eh^4/b^4$, and $P = 8.3Eh^3/b$ when $p = 37.55Eh^4/b^4$. The buckling load at the highest normal pressure is 1.3 times the buckling load with no normal pressure. The critical buckling strain is plotted against pressure in figure 5, together with a similar curve from reference 2, for plates with simply supported edges. It is evident from figure 5 that normal pressure causes a much greater proportionate increase in the buckling load of plates with simply-supported edges (curves A) than of plates with clamped edges (curve B). Curves C and D were experimentally determined (reference 1) for plates riveted to stringers which provided a support somewhere between the extreme conditions of simple support and rigid clamping. As might be expected, curve D, for thick sheet, is closer to curves A in slope, while curve C, for thin sheet, is closer to curve B in slope.

CONCLUSIONS

At low axial loads, the deflection of the plate is a single long bulge due to the normal pressure. At high axial loads, the deflection shows a regular buckle pattern, and the initial general downward deflection tends to disappear.

The effect of normal pressure on the axial load for a given edge strain is negligible.

The buckling load at the highest normal pressure studied is 1.3 times the buckling load with no normal pressure. Normal pressure causes a much smaller proportionate increase in the buckling load of plates with clamped edges than of plates with simply supported edges. Experimentally determined curves for the effect of normal pressure on buckling strain show slopes intermediate between those obtained in the present paper for plates with clamped edges and earlier theoretical results for plates with simply supported edges.

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Washington, D. C., October 19, 1945.

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Table I.- Equations for deflection coefficients. (for example of use of this table see eq.(3))

$16a^2$	$w_{1,1}$	$3w_{3,1}$	$5w_{5,1}$	$7w_{7,1}$	$9w_{9,1}$	$11w_{11,1}$	$13w_{13,1}$	$15w_{15,1}$	$17w_{17,1}$	$19w_{19,1}$	$21w_{21,1}$	$23w_{23,1}$	$25w_{25,1}$	$27w_{27,1}$
$256pb^4/\pi^6 m$	-9.562	-4.485	-1.644	-.654	-.294	-.147	-.081	-.048	-.030	-.019	-.013	-.0093	-.0067	-.0050
$256pb^4/\pi^6 m$	$-9.562k_1$	$-13.27k_3$	$-6.220k_5$	$-4.580k_7$	$-2.646k_9$	$-1.621k_{11}$	$-1.050k_{13}$	$-.714k_{15}$	$-.505k_{17}$	$-.370k_{19}$	$-.278k_{21}$	$-.2141k_{23}$	$-.1682k_{25}$	$-.1349k_{27}$
$256pb^4/\pi^6 m$	$-9.562t_1$	$-39.81t_1$	$-41.11t_1$	$-32.05t_1$	$-23.81t_1$	$-17.84t_1$	$-13.66t_1$	$-10.72t_1$	$-8.58t_1$	$-7.02t_1$	$-5.84t_1$	$-4.92t_1$	$-4.21t_1$	$-3.63t_1$
$16pb^2/\pi^2 m$	$0.5980w_{1,1}$	$7.470w_{3,1}$	$18.89w_{5,1}$	$14.03w_{7,1}$	$13.40w_{9,1}$	$12.26w_{11,1}$	$11.09w_{13,1}$	$10.04w_{15,1}$	$9.19w_{17,1}$	$8.33w_{19,1}$	$7.56w_{21,1}$	$7.08w_{23,1}$	$6.57w_{25,1}$	$6.13w_{27,1}$
$w_{1,1}^2$	$9.600w_{1,1}$	$-13.27w_{1,1}$	$-24.87w_{3,1}$	$-14.21w_{5,1}$	$-11.16w_{7,1}$	$-5.416w_{9,1}$	0	0	0	0	0	0	0	0
$w_{1,1}^2$	$-28.70w_{3,1}$	$54.39w_{3,1}$	$35.56w_{5,1}$	$21.42w_{7,1}$	$13.44w_{9,1}$	0	0	0	0	0	0	0	0	0
$w_{1,1}^2$	0	$-40.12w_{5,1}$	$-25.52w_{7,1}$	$-14.75w_{9,1}$	0	0	0	0	0	0	0	0	0	0
$w_{3,1}^2$	$39.16w_{1,1}$	$17.47w_{3,1}$	$26.51w_{1,1}$	$-14.02w_{11}$	$-2.046w_{3,1}$	0	$-3.31w_{7,1}$	$-2.334w_{9,1}$	0	0	0	0	0	0
$w_{3,1}^2$	$30.82w_{5,1}$	$-39.61w_{9,1}$	$47.42w_{5,1}$	$33.41w_{7,1}$	$23.66w_{9,1}$	$-4.94w_{5,1}$	0	0	0	0	0	0	0	0
$w_{3,1}^2$	$-29.30w_{7,1}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$w_{5,1}^2$	$41.39w_{1,1}$	$76.52w_{3,1}$	$28.30w_{5,1}$	$22.86w_{3,1}$	$10.86w_{1,1}$	$-5.53w_{1,1}$	0	0	$-1.54w_{7,1}$	$-1.160w_{9,1}$	0	0	0	0
$w_{5,1}^2$	$39.23w_{9,1}$	$66.29w_{7,1}$	0	$60.80w_{7,1}$	$47.42w_{9,1}$	0	$-3.23w_{3,1}$	$-.71w_{5,1}$	0	0	0	0	0	0
$w_{7,1}^2$	$44.79w_{1,1}$	$96.89w_{3,1}$	$109.1w_{5,1}$	$47.51w_{7,1}$	$89.08w_{5,1}$	$13.07w_{3,1}$	$5.84w_{1,1}$	$-2.90w_{1,1}$	$-1.69w_{3,1}$	$-1.13w_{5,1}$	$-.20w_{7,1}$	$-.6496w_{9,1}$	0	0
$w_{7,1}^2$	0	0	$90.19w_{9,1}$	0	$89.11w_{9,1}$	0	0	0	0	0	0	0	0	0
$w_{9,1}^2$	$47.47w_{1,1}$	$116.8w_{3,1}$	$147.3w_{5,1}$	$154.2w_{7,1}$	$70.46w_{9,1}$	$40.76w_{7,1}$	$18.71w_{5,1}$	$8.064w_{3,1}$	$3.599w_{1,1}$	$-1.800w_{1,1}$	$-1.042w_{3,1}$	$-.6942w_{5,1}$	$-.5122w_{7,1}$	$-.1345w_{9,1}$
$w_{1,1}w_{3,1}$	$-57.85w_{5,1}$	$65.59w_{5,1}$	$58.13w_{7,1}$	$32.38w_{5,1}$	$-16.86w_{5,1}$	$14.04w_{9,1}$	$-7.428w_{9,1}$	0	0	0	0	0	0	0
$w_{1,1}w_{3,1}$	0	$-81.88w_{7,1}$	$-52.42w_{9,1}$	$35.93w_{9,1}$	$20.88w_{7,1}$	$-10.89w_{7,1}$	0	0	0	0	0	0	0	0
$w_{1,1}w_{5,1}$	$-59.39w_{7,1}$	$93.85w_{7,1}$	$67.49w_{9,1}$	0	0	$15.36w_{7,1}$	$-7.74w_{7,1}$	$-5.664w_{9,1}$	0	0	0	0	0	0
$w_{1,1}w_{5,1}$	0	$-84.63w_{9,1}$	0	0	0	0	$11.27w_{9,1}$	0	0	0	0	0	0	0
$w_{3,1}w_{5,1}$	$67.65w_{7,1}$	0	$82.15w_{7,1}$	$57.79w_{9,1}$	$33.39w_{7,1}$	$24.70w_{9,1}$	0	$-4.56w_{7,1}$	$-3.369w_{9,1}$	0	0	0	0	0
$w_{3,1}w_{5,1}$	$-60.98w_{9,1}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$w_{7,1}w_{9,1}$	$-61.57w_{1,1}$	$104.1w_{1,1}$	$103.6w_{3,1}$	0	0	$46.01w_{5,1}$	$20.89w_{3,1}$	$9.03w_{1,1}$	$-4.479w_{1,1}$	$-2.606w_{3,1}$	$-1.79w_{5,1}$	0	0	0
$w_{7,1}w_{9,1}$	$75.02w_{3,1}$	$167.3w_{5,1}$	0	$100.5w_{5,1}$	0	0	0	0	0	0	0	0	0	0

Table I. (Continued).

$16h^2$	$w_{1,3}$	$3w_{3,3}$	$5w_{5,3}$	$7w_{7,3}$	$9w_{9,3}$	$11w_{11,3}$	$13w_{13,3}$	$15w_{15,3}$	$17w_{17,3}$	$19w_{19,3}$	$21w_{21,3}$	$23w_{23,3}$	$25w_{25,3}$
$256pb^4/\pi^6 m$	$-.04385$	$-.03939$	$-.03285$	$-.02474$	$-.01820$	$-.01312$	$-.00940$	$-.00677$	$-.00491$	$-.00361$	$-.00269$	$-.00204$	$-.00156$
$256pb^4/\pi^6 m$	$-.3949k_1$	$-1.063k_3$	$-1.451k_5$	$-1.559k_7$	$-1.475k_9$	$-1.299k_{11}$	$-1.100k_{13}$	$-.914k_{15}$	$-.752k_{17}$	$-.618k_{19}$	$-.5090k_{21}$	$-.4212k_{23}$	$-.3507k_{25}$
$256pb^4/\pi^6 m$	$-.1316t_3$	$-1.063t_3$	$-2.420t_3$	$-3.637t_3$	$-4.425t_3$	$-4.765t_3$	$-4.767t_3$	$-4.570t_3$	$-4.260t_3$	$-3.912t_3$	$-3.563t_3$	$-3.229t_3$	$-2.922t_3$
$16Fb^2/\pi^2 m$	$.0062w_{1,3}$	$.1994w_{3,3}$	$.7565w_{5,3}$	$1.591w_{7,3}$	$2.49w_{9,3}$	$3.876w_{11,3}$	$3.87w_{13,3}$	$4.26w_{15,3}$	$4.53w_{17,3}$	$4.64w_{19,3}$	$4.676w_{21,3}$	$4.642w_{23,3}$	$4.57w_{25,3}$
$w_{1,1}^2$	$-.000514w_{1,1}$	$-.02043w_{3,1}$	$.00121w_{3,1}$	$.0133w_{5,1}$	$.0369w_{7,1}$	$.06599w_{9,1}$	0	0	0	0	0	0	0
$w_{1,1}^2$	$.00726w_{3,1}$	$.07175w_{5,1}$	$-.1077w_{5,1}$	$-.2666w_{7,1}$	$-.432w_{9,1}$	0	0	0	0	0	0	0	0
$w_{1,1}^2$	0	0	$.1982w_{7,1}$	$.3172w_{9,1}$	0	0	0	0	0	0	0	0	0
$w_{3,1}^2$	$-.0420w_{1,1}$	$-.1121w_{3,1}$	$-.0266w_{1,1}$	$.0013w_{1,1}$	$-1.532w_{9,1}$	$.0004w_{5,1}$	0	$.0091w_{9,1}$	0	0	0	0	0
$w_{3,1}^2$	$-.1169w_{5,1}$	$.7349w_{9,1}$	$-.5559w_{5,1}$	$-1.007w_{7,1}$	0	0	$.0035w_{7,1}$	0	0	0	0	0	0
$w_{3,1}^2$	$.2056w_{7,1}$	0	0	0	0	0	0	0	0	0	0	0	0
$w_{5,1}^2$	$-.2118w_{1,1}$	$-1.046w_{3,1}$	$-1.181w_{5,1}$	$-.4604w_{3,1}$	$-.0999w_{1,1}$	$.0111w_{1,1}$	$.0004w_{3,1}$	0	$.0001w_{7,1}$	$.0006w_{9,1}$	0	0	0
$w_{5,1}^2$	$-.7896w_{9,1}$	$-1.589w_{7,1}$	0	$-3.545w_{7,1}$	$-4.497w_{9,1}$	0	0	0	0	0	0	0	0
$w_{7,1}^2$	$-.902w_{1,1}$	$-2.529w_{3,1}$	$-6.21w_{5,1}$	$-4.874w_{7,1}$	$-8.202w_{5,1}$	$-.677w_{3,1}$	$-.1502w_{1,1}$	$.0241w_{1,1}$	$.0024w_{3,1}$	$.0001w_{5,1}$	0	$.00002w_{9,1}$	0
$w_{7,1}^2$	0	0	$-6.86w_{9,1}$	0	$-11.56w_{9,1}$	0	0	0	0	0	0	0	0
$w_{9,1}^2$	$-.8300w_{1,1}$	$-4.214w_{3,1}$	$-11.12w_{5,1}$	$-19.59w_{7,1}$	$-12.60w_{9,1}$	$-6.136w_{7,1}$	$-2.201w_{5,1}$	$-.6316w_{3,1}$	$-.1530w_{1,1}$	$.0314w_{1,1}$	$.0051w_{3,1}$	$.0005w_{5,1}$	$.00002w_{7,1}$
$w_{1,1}w_{3,1}$	$.1198w_{5,1}$	$-.2582w_{5,1}$	$-.6677w_{7,1}$	$-.1394w_{5,1}$	$.0154w_{5,1}$	$.0474w_{7,1}$	0	0	0	0	0	0	0
$w_{1,1}w_{3,1}$	0	$.5977w_{7,1}$	$1.171w_{9,1}$	$-1.155w_{9,1}$	$-.287w_{7,1}$	$-.4646w_{9,1}$	$.0694w_{9,1}$	0	0	0	0	0	0
$w_{1,1}w_{5,1}$	$.3651w_{7,1}$	$-.8980w_{7,1}$	0	0	0	$-.2959w_{7,1}$	$.0442w_{7,1}$	0	0	0	0	0	0
$w_{1,1}w_{5,1}$	0	$1.418w_{9,1}$	$-1.638w_{9,1}$	0	0	0	$-.4353w_{9,1}$	$.0806w_{9,1}$	0	0	0	0	0
$w_{3,1}w_{5,1}$	$-.6605w_{1,1}$	0	$-2.078w_{7,1}$	0	$-1.271w_{7,1}$	0	0	$.0037w_{7,1}$	0	0	0	0	0
$w_{3,1}w_{5,1}$	$.9097w_{9,1}$	0	0	$-3.15w_{9,1}$	0	$-1.675w_{9,1}$	0	0	$.01075w_{9,1}$	0	0	0	0
$w_{7,1}w_{5,1}$	$.7007w_{1,1}$	$-1.792w_{1,1}$	0	0	0	0	0	$-.3519w_{1,1}$	$.0665w_{1,1}$	$.0093w_{3,1}$	$.0006w_{5,1}$	0	0
$w_{7,1}w_{9,1}$	$-1.289w_{3,1}$	$-6.371w_{5,1}$	$-4.361w_{3,1}$	$-8.411w_{5,1}$	0	$-4.79w_{5,1}$	$-1.467w_{3,1}$	0	0	0	0	0	0

Table II.– Equations for moment coefficients. (for example of use of this table see eq.(4))

$256\rho b^4/\pi^6 m$	-16.96	-0.8095	-.08715	-.00707	-9.784	-1.522	-.3530	-.1069	-.04105	-.01856	-.00952	-.00540	-.00324	-.002051	-.001396	-.009705	-.0006952	-.0005107
"	$-9.562k_1$	$-.3949k_1$	$-.0860k_1$	$-.03140k_1$	$-9.562t_1$	$-13.87t_1$	$-8.220t_1$	$-4.580t_1$	$-2.646t_1$	$-1.621t_1$	$-1.050t_1$	$-.714t_1$	$-.505t_1$	$-.370t_1$	$-.278t_1$	$-.2141t_1$	$-.1681t_1$	$-.1345t_1$
"	$-13.87k_3$	$-1.063k_3$	$-.2480k_3$	$-.0928k_3$	$-.3947t_3$	$-1.063t_3$	$-1.451t_3$	$-1.559t_3$	$-1.475t_3$	$-1.299t_3$	$-1.100t_3$	$-.914t_3$	$-.752t_3$	$-.618t_3$	$-.5087t_3$	$-.4212t_3$	$-.3508t_3$	$-.2938t_3$
"	$-8.280k_5$	$-1.451k_5$	$-.3828k_5$	$-.1478k_5$	$-.0860t_5$	$-.2480t_5$	$-.3828t_5$	$-.4799t_5$	$-.5380t_5$	$-.5601t_5$	$-.5549t_5$	$-.5305t_5$	$-.4950t_5$	$-.4535t_5$	$-.4103t_5$	$-.3685t_5$	$-.3289t_5$	$-.2929t_5$
"	$-4.580k_7$	$-1.559k_7$	$-.4799k_7$	$-.1954k_7$	$-.0314t_7$	$-.0928t_7$	$-.1478t_7$	$-.1954t_7$	$-.2328t_7$	$-.2600t_7$	$-.2770t_7$	$-.2851t_7$	$-.2858t_7$	$-.2806t_7$	$-.2709t_7$	$-.2580t_7$	$-.2438t_7$	$-.2282t_7$
"	$-2.646k_9$	$-1.475k_9$	$-.5380k_9$	$-.2328k_9$	$-12.06k_1$	$-6.785k_3$	$-3.768k_5$	$-2.507k_7$	$-1.901k_9$	$-1.544k_{11}$	$-1.307k_{13}$	$-1.133k_{15}$	$-.997k_{17}$	$-.894k_{19}$	$-.808k_{21}$	$-.738k_{23}$	$-.678k_{25}$	$-.628k_{27}$
"	$-1.621k_{11}$	$-1.299k_{11}$	$-.5601k_{11}$	$-.2600k_{11}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-1.050k_{13}$	$-1.100k_{13}$	$-.5549k_{13}$	$-.2770k_{13}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.714k_{15}$	$-.914k_{15}$	$-.5305k_{15}$	$-.2851k_{15}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.505k_{17}$	$-.752k_{17}$	$-.4950k_{17}$	$-.2858k_{17}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.370k_{19}$	$-.618k_{19}$	$-.4535k_{19}$	$-.2806k_{19}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.278k_{21}$	$-.509k_{21}$	$-.4103k_{21}$	$-.2709k_{21}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.214k_{23}$	$-.421k_{23}$	$-.3685k_{23}$	$-.2580k_{23}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.168k_{25}$	$-.371k_{25}$	$-.3289k_{25}$	$-.2438k_{25}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-.134k_{27}$	$-.294k_{27}$	$-.2929k_{27}$	$-.2282k_{27}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$-871.8t_1$	$-90.50t_3$	$-54.3t_5$	$-36.8t_7$	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$16\rho b^2/\pi^2 k$	$.5980w_{1,1}$	$.008225w_{1,3}$	*	-	$.5980w_{1,1}$	$2.490w_{3,1}$	$2.570w_{5,1}$	$2.003w_{7,1}$	$1.469w_{9,1}$	$1.114w_{11,1}$	$.853w_{13,1}$	$.669w_{15,1}$	$.537w_{17,1}$	$.439w_{19,1}$	$.365w_{21,1}$	$.3077w_{23,1}$	$.2626w_{25,1}$	$.2270w_{27,1}$
"	$7.470w_{3,1}$	$.1994w_{3,3}$	-	-	$.0247w_{1,3}$	$.1994w_{3,3}$	$.454w_{5,3}$	$.688w_{7,3}$	$.830w_{9,3}$	$.893w_{11,3}$	$.893w_{13,3}$	$.856w_{15,3}$	$.800w_{17,3}$	$.733w_{19,3}$	$.6680w_{21,3}$	$.6055w_{23,3}$	$.5479w_{25,3}$	-
"	$12.85w_{5,1}$	$.7565w_{5,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$14.03w_{7,1}$	$1.591w_{7,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$13.40w_{9,1}$	$2.45w_{9,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$12.26w_{11,1}$	$3.276w_{11,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$11.09w_{13,1}$	$3.87w_{13,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$10.04w_{15,1}$	$4.28w_{15,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$9.13w_{17,1}$	$4.57w_{17,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$8.33w_{19,1}$	$4.64w_{19,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$7.56w_{21,1}$	$4.68w_{21,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$7.06w_{23,1}$	$4.64w_{23,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$6.57w_{25,1}$	$4.57w_{25,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
"	$6.13w_{27,1}$	$4.46w_{27,3}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

* Dashes correspond to higher order terms which can be neglected in using this table.

Table II. (Continued).

$w_{1,1}^3$	-3.67	-.000514	0	0	9.599	-4.483	0	0	0	0	0	0	0	0	0	0	0	0
$w_{1,1}^2w_{3,1}$	.78	-.01195	0	0	-28.68	18.10	-4.971	0	0	0	0	0	0	0	0	0	0	0
$w_{1,1}^2w_{5,1}$	-18.75	-.0226	0	0	0	-13.30	7.047	-8.084	0	0	0	0	0	0	0	0	0	0
$w_{1,1}^2w_{7,1}$	-15.26	-.0297	0	0	0	0	-4.986	2.947	-1.227	0	0	0	0	0	0	0	0	0
$w_{1,1}^2w_{9,1}$	-6.73	-.0488	0	0	0	0	0	-1.971	1.349	-.4746	0	0	0	0	0	0	0	0
$w_{3,1}^2w_{1,1}$	51.67	-.0675	0	0	39.05	0	5.284	-2.002	0	0	0	0	0	0	0	0	0	0
$w_{3,1}^3$	14.82	-.1121	0	0	0	5.708	0	0	-.8939	0	0	0	0	0	0	0	0	0
$w_{3,1}^2w_{5,1}$	73.30	-.6716	0	0	30.47	0	9.16	0	0	-.4492	0	0	0	0	0	0	0	0
$w_{3,1}^2w_{7,1}$	.80	-.7979	0	0	-28.68	0	0	4.339	0	0	-.2591	0	0	0	0	0	0	0
$w_{3,1}^2w_{9,1}$	-18.46	-.788	0	0	0	-12.54	0	0	2.121	0	0	-.1538	0	0	0	0	0	0
$w_{5,1}^2w_{1,1}$	46.72	-.3003	0	0	40.76	0	0	0	1.173	-.4994	0	0	0	0	0	0	0	0
$w_{5,1}^2w_{3,1}$	96.17	-1.506	0	0	0	24.47	0	3.071	0	0	-.2467	0	0	0	0	0	0	0
$w_{5,1}^3$	27.59	-1.161	0	0	0	0	4.952	0	0	0	0	-.0476	0	0	0	0	0	0
$w_{5,1}^2w_{7,1}$	125.51	-5.134	0	0	0	20.51	0	7.16	0	0	0	0	-.0905	0	0	0	0	0
$w_{5,1}^2w_{9,1}$	85.5	-5.29	0	0	0	0	0	0	3.768	0	0	0	0	-.0609	0	0	0	0
$w_{7,1}^2w_{1,1}$	47.73	-.628	0	0	43.28	0	0	0	0	0	.4143	-.1888	0	0	0	0	0	0
$w_{7,1}^2w_{3,1}$	108.83	-3.800	0	0	0	29.76	0	0	0	1.005	0	0	-.0990	0	0	0	0	0
$w_{7,1}^2w_{5,1}$	137.0	-6.42	0	0	0	0	18.09	0	2.490	0	0	0	0	-.0596	0	0	0	0
$w_{7,1}^3$	47.23	-4.872	0	0	0	0	0	4.696	0	0	0	0	0	0	-.0132	0	0	0
$w_{7,1}^2w_{9,1}$	178.7	-18.48	0	0	36.86	0	13.92	0	6.049	0	0	0	0	0	0	-.0282	0	0
$w_{9,1}^2w_{1,1}$	49.3	-.952	0	0	44.98	0	0	0	0	0	0	0	.1847	-.0897	0	0	0	0
$w_{9,1}^2w_{3,1}$	125.8	-4.84	0	0	0	35.38	0	0	0	0	0	.4113	0	0	-.0489	0	0	0
$w_{9,1}^2w_{5,1}$	165.3	-13.32	0	0	0	0	22.78	0	0	0	.9327	0	0	0	0	-.0301	0	0
$w_{9,1}^2w_{7,1}$	194.5	-25.7	0	0	0	0	0	13.64	0	2.032	0	0	0	0	0	0	-.0204	0
$w_{9,1}^3$	70.3	-12.60	0	0	0	0	0	0	3.630	0	0	0	0	0	0	0	0	-.0049
$w_{1,1}w_{3,1}w_{5,1}$	43.22	-.8630	0	0	-57.49	28.86	0	4.963	-1.866	0	0	0	0	0	0	0	0	0
$w_{1,1}w_{3,1}w_{7,1}$	-13.80	-.5096	0	0	0	-26.51	11.23	0	2.223	-.9776	0	0	0	0	0	0	0	0
$w_{1,1}w_{3,1}w_{9,1}$	-9.87	-.359	0	0	0	0	-9.761	4.638	0	1.150	-.5508	0	0	0	0	0	0	0
$w_{1,1}w_{5,1}w_{7,1}$	42.12	-.785	0	0	-58.25	30.38	15.19	0	3.879	1.315	-.5853	0	0	0	0	0	0	0
$w_{1,1}w_{5,1}w_{9,1}$	-11.57	-.774	0	0	0	-86.79	12.39	0	0	0	.7665	-.9616	0	0	0	0	0	0
$w_{1,1}w_{7,1}w_{9,1}$	47.1	-1.376	0	0	-59.47	38.91	0	0	0	0	0	.5318	-.2517	-.1357	0	0	0	0
$w_{3,1}w_{5,1}w_{7,1}$	178.59	-4.02	0	0	65.61	0	0	0	0	0	0	-.3035	0	0	0	0	0	0
$w_{3,1}w_{5,1}w_{9,1}$	18.11	-3.90	0	0	-58.25	0	0	6.900	0	1.789	0	0	-.1960	0	0	0	0	0
$w_{3,1}w_{7,1}w_{9,1}$	196.4	-7.10	0	0	71.16	0	18.11	0	0	0	1.282	0	0	0	0	0	0	0
$w_{5,1}w_{7,1}w_{9,1}$	312.1	-19.6	0	0	0	49.40	0	10.76	0	2.876	0	0	0	0	-.0832	0	0	0

Table III - Deflection coefficients as a function of axial load P and edge strain e for lateral pressure $p = 15.02\ Eh^4/b^4$.

Pb/Eh^3	0.62	1.23	2.46	4.31	6.16	6.76	6.96	7.32	8.41	10.20	10.66	12.02	13.00	13.10	13.42	15.65	19.66
eb^2/h^2	.64	1.26	2.50	4.36	6.25	7.02	7.38	8.14	10.58	14.50	15.52	18.51	20.77	21.25	22.43	28.71	39.26
$w_{1,1}/h$	.448	.452	.459	.471	.482	.468	.450	.414	.345	.300	.295	.290	.285	.271	.254	.183	.119
$w_{3,1}/h$	.097	.103	.116	.142	.179	.191	.198	.217	.275	.336	.353	.396	.424	.417	.356	.046	-.077
$w_{5,1}/h$	.017	.019	.024	.039	.108	.246	.325	.459	.751	1.063	1.121	1.250	1.192	.868	.369	-.432	-.669
$w_{7,1}/h$	-.005	-.006	-.007	-.013	-.046	-.140	-.200	-.300	-.500	-.700	-.750	-.900	-1.100	-1.300	-1.500	-1.700	-2.042
$w_{9,1}/h$	-.008	-.009	-.011	-.017	-.034	-.042	-.043	-.049	-.081	-.140	-.152	-.184	-.187	-.107	.078	.506	.659
$w_{11,1}/h$	-.018	-.018	-.018	-.019	-.019	-.019	-.018	-.016	-.014	-.012	-.012	-.012	-.011	-.011	-.010	-.007	-.005
$w_{13,1}/h$	-.011	-.011	-.011	-.011	-.011	-.011	-.011	-.010	-.008	-.007	-.007	-.007	-.007	-.006	-.006	-.004	-.003
$w_{1,3}/h$	-.066	-.067	-.068	-.070	-.071	-.069	-.067	-.061	-.051	-.044	-.044	-.043	-.042	-.040	-.037	-.027	-.018
$w_{3,3}/h$	-.014	-.015	-.016	-.020	-.025	-.027	-.028	-.031	-.039	-.047	-.050	-.056	-.060	-.059	-.050	-.006	-.011
$w_{5,3}/h$	-.002	-.002	-.003	-.005	-.014	-.031	-.041	-.058	-.095	-.134	-.142	-.158	-.151	-.110	-.047	.054	.085
$w_{7,3}/h$	.001	.001	.002	.002	.007	.019	.027	.039	.061	.085	.091	.109	.133	.157	.181	.206	.247
$w_{9,3}/h$	.001	.001	.002	.003	.005	.006	.006	.007	.011	.019	.021	.025	.026	.015	-.011	-.070	-.091
$w_{11,3}/h$	.002	.002	.002	.002	.003	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001
$w_{13,3}/h$	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000

Table IV - Deflection coefficients as a function of axial load P and edge strain e for lateral pressure $p = 37.55\ Eh^4/b^4$

Pb/Eh^3	0	.62	1.85	3.08	4.93	6.22	7.40	8.33	8.43	8.46	8.60	8.67	9.90	11.12	12.78	15.12	18.06	19.54	25.48	30.04	34.42
eb^2/h^2	.16	.79	2.03	3.29	5.18	6.52	7.82	9.16	9.46	9.64	10.23	11.06	13.66	16.50	20.25	25.46	31.99	35.31	46.83	59.33	69.43
$w_{1,1}/h$	1.054	1.062	1.079	1.098	1.128	1.148	1.160	1.114	1.069	1.023	.922	.831	.675	.590	.532	.497	.485	.484	.483	.479	.475
$w_{3,1}/h$	.238	.248	.270	.297	.338	.371	.392	.358	.335	.337	.359	.376	.391	.404	.425	.453	.480	.487	.496	.501	.504
$w_{5,1}/h$	.051	.055	.066	.082	.118	.164	.248	.453	.540	.599	.707	.814	1.081	1.310	1.561	1.866	2.237	2.433	3.159	3.625	4.019
$w_{7,1}/h$	-.007	-.007	-.008	-.009	-.010	-.012	-.016	-.032	-.071	-.150	-.300	-.400	-.550	-.650	-.750	-.850	-.903	-.896	-.810	.763	-.731
$w_{9,1}/h$	-.020	-.022	-.027	-.034	-.040	-.057	-.093	-.180	-.197	-.178	-.140	-.129	-.159	-.199	-.247	-.303	-.360	-.380	-.416	-.422	-.423
$w_{11,1}/h$	-.042	-.042	-.043	-.044	-.045	-.046	-.046	-.044	-.042	-.041	-.037	-.033	-.027	-.023	-.021	-.020	-.019	-.019	-.019	-.019	-.019
$w_{13,1}/h$	-.013	-.014	-.016	-.018	-.027	-.027	-.027	-.026	-.025	-.024	-.022	-.020	-.016	-.014	-.013	-.012	-.011	-.011	-.011	-.011	-.011
$w_{15,1}/h$	-.010	-.010	-.011	-.012	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{17,1}/h$	-.007	-.007	-.008	-.009	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{19,1}/h$	-.005	-.006	-.006	-.007	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{21,1}/h$	-.004	-.004	-.005	-.005	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{1,3}/h$	-.155	-.156	-.159	-.162	-.167	-.170	-.171	-.165	-.158	-.151	-.136	-.123	-.100	-.087	-.079	-.073	-.072	-.072	-.071	-.071	-.070
$w_{3,3}/h$	-.034	-.035	-.038	-.042	-.046	-.052	-.055	-.050	-.047	-.047	-.051	-.053	-.055	-.057	-.060	-.064	-.068	-.069	-.070	-.071	-.071
$w_{5,3}/h$	-.004	-.005	-.007	-.009	-.015	-.021	-.031	-.057	-.068	-.076	-.089	-.103	-.137	-.166	-.197	-.236	-.283	-.308	-.400	-.459	-.508
$w_{7,3}/h$	.003	.003	.003	.003	.003	.004	.004	.006	.011	.021	.040	.050	.062	.073	.085	.096	.102	.101	.098	.086	.083
$w_{9,3}/h$	.003	.004	.004	.005	.006	.008	.012	.022	.023	.021	.017	.015	.019	.023	.029	.036	.042	.045	.049	.050	.050
$w_{11,3}/h$	.003	.003	.003	.004	.006	.006	.006	.006	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.003	.003	.002
$w_{13,3}/h$	.002	.002	.002	.002	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
$w_{15,3}/h$	.001	.001	.001	.002	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{17,3}/h$	.001	.001	.001	.001	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{19,3}/h$	.001	.001	.001	.001	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{1,5}/h$	-.038	-.038	-.038	-.039	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{3,5}/h$	-.009	-.009	-.010	-.010	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{1,7}/h$	-.014	-.014	-.014	-.015	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$w_{3,7}/h$	-.003	-.004	-.004	-.004	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

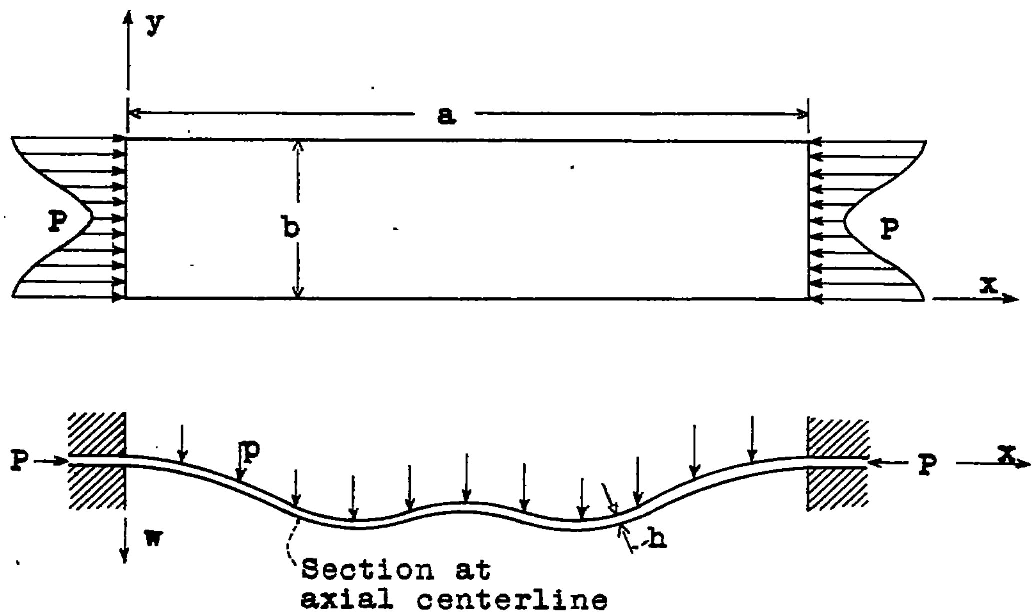

Figure 1.- Plate under axial load and normal pressure, a = 4b.

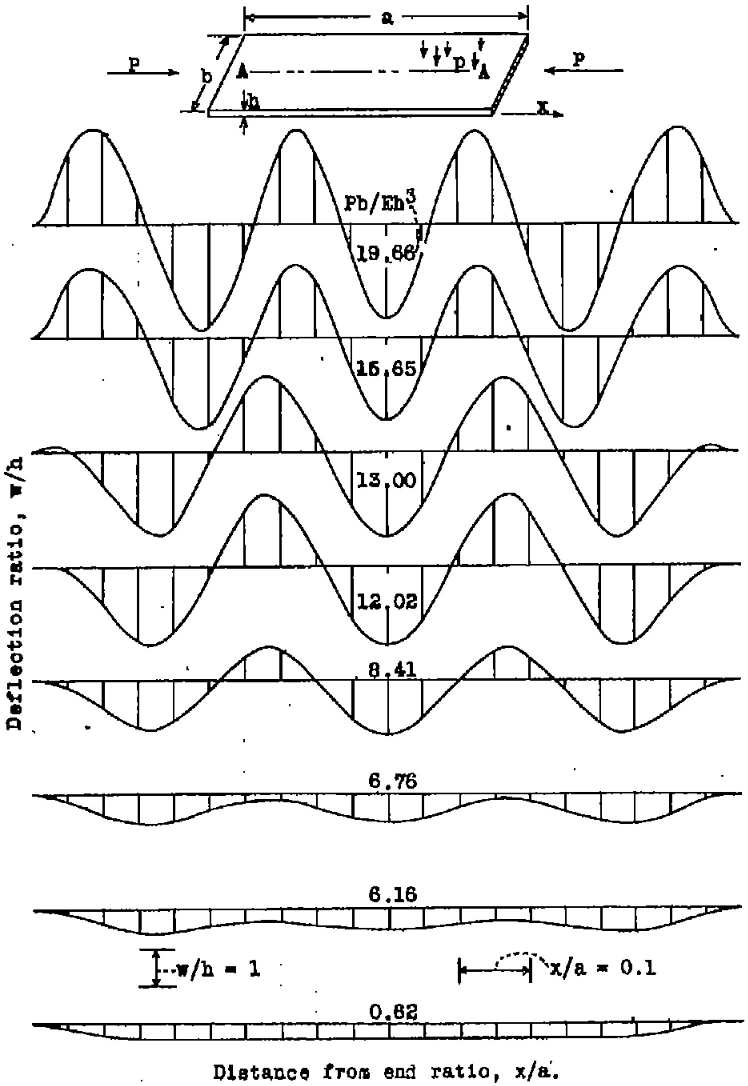

Figure 2.- Deflection at longitudinal centerline, A-A; $p = 15.02\ Eh^4/b^4$.

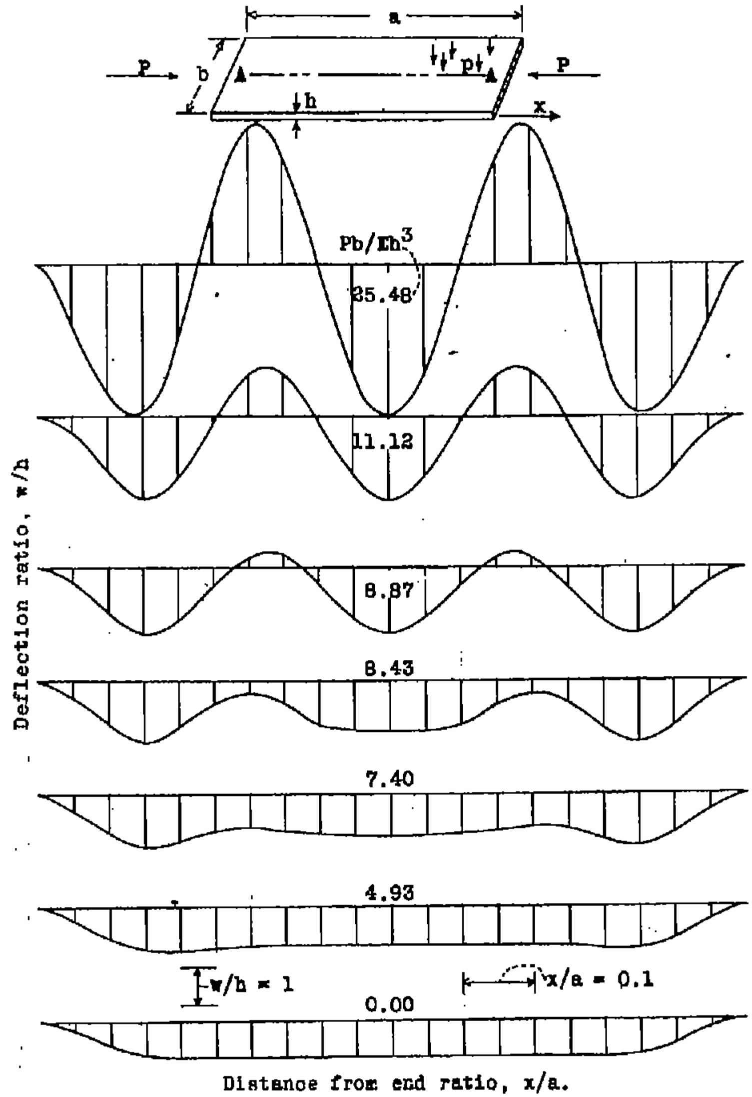

Figure 3.- Deflection at longitudinal centerline, A-A; $p = 37.55\ Eh^4/b^4$.

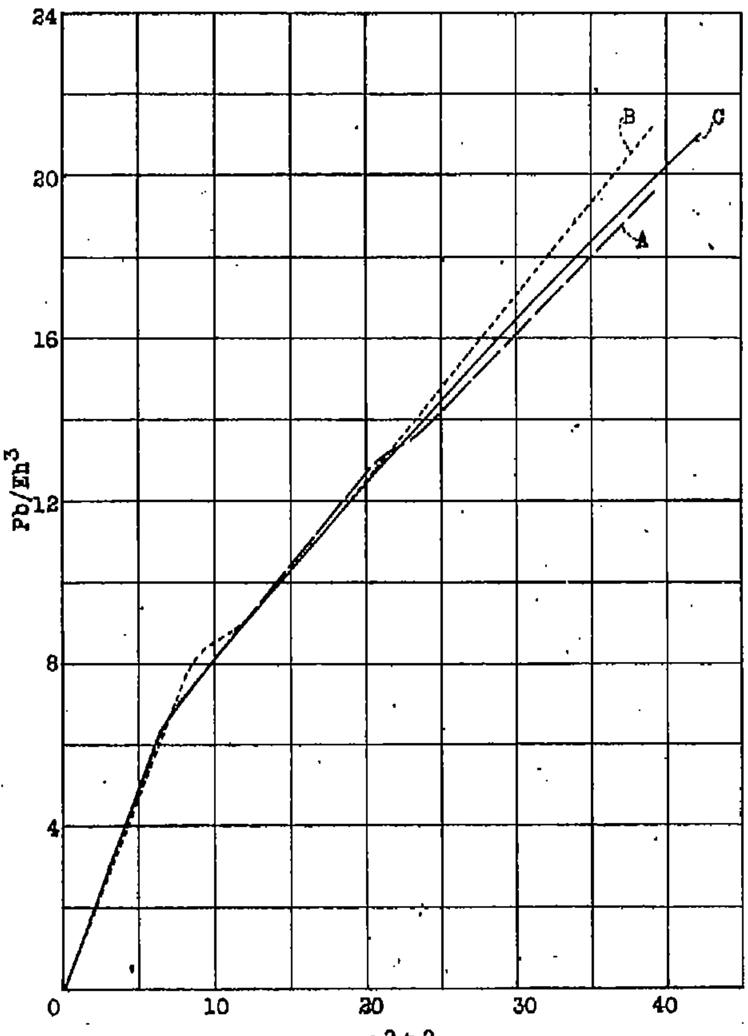

Figure 4.- Axial load P versus edge strain e; curve A, $p = 15.02\ Eh^4/b^4$; curve B, $p = 37.58\ Eh^4/b^4$; curve C, $p = 0$, long plate, reference 6.

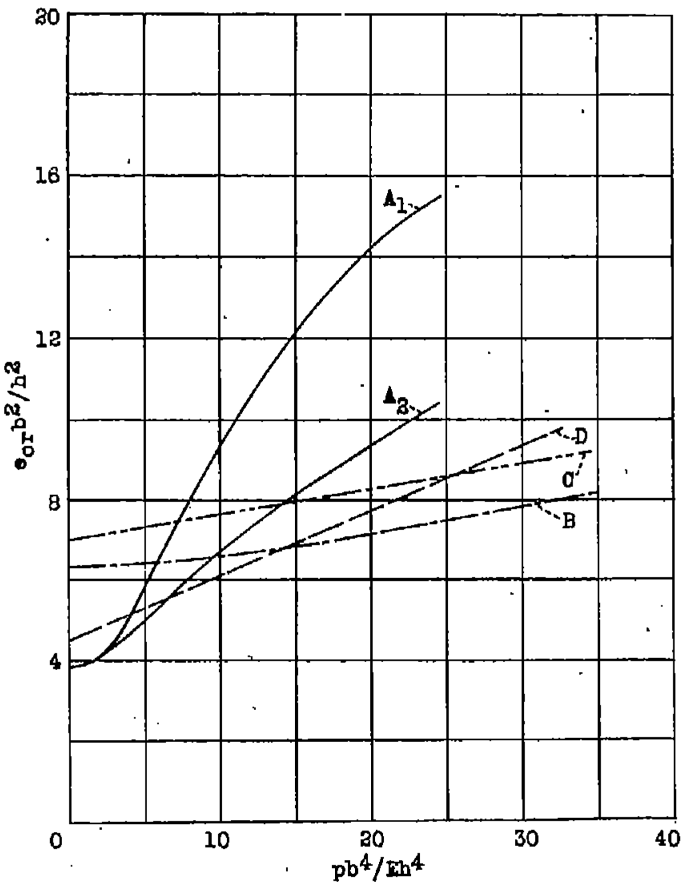

Figure 5.- Critical buckling strain e_{cr} versus normal pressure p. Curves A_1 and A_2, maximum and minimum respectively for simply-supported edges, reference 2; curve B, clamped edges, present paper; curves C and D, experimental for intermediate support, thin and thick sheet respectively, reference 1.