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SPINDLED AND HOLLOW SPARS

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SPINDLED AND HOLLOW SPARS.*

By J. D. Blyth.

The most usual method of arriving at the maximum amount of spindling or hollowing out permissible in the case of any particular spar section is by trial and error, a process which is apt to become laborious in the absence of good guessing - or luck. The following tables have been got out with the object of making it possible to arrive with certainty at a suitable section at the first attempt.

The following symbols are employed:

I = Moment of inertia.

Z = Section modulus.

M = Bending moment.

T = Torque.

S = Shear force.

f_c = Ultimate compressive strength of material.

f_t = " tensile " " "

f_s = " shear " " "

We will first consider a spar of rectangular section, laterally loaded only, which we wish to spindle to either I, ⊐ , or ▭ section.

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B = Width of section.

D = Depth " "

d = Thickness of flange.

t = " " web.

It should be noted that in the case of the hollow □ section, t is the combined thickness of the walls forming the web.

The most rapid method gives flange and web thicknesses slightly greater than are actually required, so the spar will be on the safe side. The procedure is as follows:

First find t, the thickness of the web. This is given by

$$t = \frac{3}{2\,D} \times \frac{S}{f_s} .$$

Next find the value of C, which is given by

$$C = \frac{M}{f_c\,Z}$$

where
$$Z = \frac{B\,D}{6} .$$

In Table I, values of λ are tabulated for values of C from 0 to 1.0.

$$d = \lambda D .$$

Table I. Values of λ for all Values of C. Rectangular Sections.

C	0	1	2	3	4	5	6	7	8	9
.0	0	.002	.004	.005	.007	.009	.010	.012	.014	.016
.1	.017	.019	.021	.023	.025	.027	.028	.030	.032	.034
.2	.036	.038	.040	.042	.044	.046	.048	.050	.052	.054
.3	.056	.058	.060	.063	.065	.067	.069	.072	.074	.076
.4	.079	.081	.083	.086	.088	.091	.093	.096	.098	.101
.5	.103	.106	.109	.111	.114	.117	.120	.123	.126	.129
.6	.132	.135	.138	.141	.145	.148	.151	.155	.158	.162
.7	.166	.169	.173	.177	.181	.185	.190	.194	.198	.203
.8	.208	.213	.218	.223	.229	.235	.241	.247	.254	.261
.9	.268	.276	.285	.294	.305	.316	.329	.345	.365	.393
1.0	.500	–	–	–	–	–	–	–	–	–

The method will be made clear by taking an example and work- it out.

Suppose we have a spar whose section is 2 in. wide and 4 in. deep,

M = 16,000 lb.-in.

S = 900 lb.

f_c = 5,500 lb. per square inch.

f_s = 800 " " "

The spar is to be spindled to I section.

We get $t = \frac{3}{8} \times \frac{900}{800} = 0.42$ in.

and $Z = 5.33$

whence $C = 0.545$.

From Table I we see that when $C = .54$, $\lambda = .114$, and when $C = .55$, $\lambda = .117$. Interpolating in these values we get a value in the present case,

$$\lambda = .116$$

$$d = \lambda D = .464 \text{ in.}$$

Remembering that the values found are on the high side, we will make the flanges 0.45 in. deep, and the web 0.4 in. thick.

Checking the section so obtained, we find that we have for bending a factor of safety of 1.15, and for shear a factor of safety of 1.14.

The section found in this way will be suitable in most cases. Occasions may arise, however, when it is desired to lighten the spar as much as possible, i.e., to spindle it to the limit. In this case the procedure is a little longer.

First find t as before.

Next find the value of $\frac{f_c t D^2}{6}$, and subtract the value so found from the known value of M. Calling the remainder M', find the value of C, which in this case is given by

$$C = \frac{M'}{f_c Z'}$$

where $Z' = \frac{(B - t)D^2}{6}$.

Look up the value of λ in Table I; and as before, $d = \lambda D$.

Taking as an example the spar already described.

As before, $t = 0.4$ in.

Then $M' = M - \frac{5,500 \times 0.4 \times 16}{6}$

$= 10,150$ lb.-in.

$Z' = 4.27$

$C = .433$

whence $\lambda = .087$, and $d = 0.35$ in.

This section has for bending a factor of safety of 1.01.

We will next consider a spar of rectangular section, laterally loaded as before, together with an end load P. In this case the procedure is divided into two steps, as follows:

First find the section neglecting end load, in the manner already described, and let A square inches be the area of the section so found. This enables us to find the value of P/A.

Repeat the process, using the value $f_c - \frac{P}{A}$ instead of f_c, or, where applicable, $f_t + \frac{P}{A}$ instead of f_t. It should be noted that for compressive end loads the numerical value of P is taken as positive, while if the end load is tensile the numerical value of P is negative.

Taking as an example the spar already worked out, and supposing that it is subjected to a compressive end load of 1000 lb.

Neglecting the end load, we have as before

$$t = 0.4 \text{ in.}$$

$$d = 0.45 \text{ in.}$$

We get $A = 3.04$ sq.in., and $\frac{P}{A} = 329$.

New value of $f_c = 5,500 - 329 = 5,171$.

Since the shear stress is unaffected by the end load, t remains as before.

$$C = \frac{16,000}{5,171 \times 5.35} = 0.58$$

$$\text{whence } \lambda = 0.126$$

$$\text{and } d = 0.5 \text{ in.}$$

This section has a factor of safety of 1.13.

If the Perry correction is to be applied the method is similar. In this case the section is found as before, and the value of $f_c - \frac{P}{A}$ derived, and also the value of P_e, the Euler load for the spar treated as a strut for the length of the portion between points of contraflexure. Now multiply M by $\frac{P_e}{P_e - P}$ and with the new values of M and f_c proceed as before.

We now turn to spars of circular section. These are most commonly subjected to a lateral load, combined with a torsional load; and this case we will consider, first taking the case of wooden spars, where the thickness t of the wall is not small compared with the diameter D (this being the external diameter).

The procedure is as follows:

First find T_e, the equivalent torque. This is given by

$$T_e = \sqrt{M^2 + T^2}$$

Next find the value of C, which is given by

$$C = \frac{16\ T_e}{\pi\ D^3\ f_s}$$

Turn to Table II, and find the value of λ corresponding to the value of C.

Then $t = \lambda D$.

Table II. Values of λ for all Values of C.

Circular Sections.

C	0	1	2	3	4	5	6	7	8	9
.0	0	.001	.003	.004	.005	.006	.008	.009	.010	.012
.1	.013	.014	.016	.017	.019	.020	.021	.023	.024	.026
.2	.027	.029	.030	.032	.033	.035	.036	.038	.040	.041
.3	.043	.045	.046	.048	.050	.051	.053	.055	.057	.058
.4	.060	.062	.064	.066	.068	.070	.072	.074	.076	.078
.5	.080	.082	.084	.086	.088	.090	.093	.095	.098	.100
.6	.102	.105	.108	.110	.113	.116	.118	.121	.124	.127
.7	.130	.133	.137	.140	.143	.147	.150	.154	.158	.162
.8	.166	.170	.175	.179	.184	.189	.194	.200	.206	.212
.9	.219	.226	.234	.243	.250	.264	.277	.292	.312	.342
1.0	.500	–	–	–	–	–	–	–	–	–

In employing this method it must be remembered that the wall thickness found is not greater than is actually required; and further, is only that required to resist the shear stress due to torsion, no account having been taken of the shear stress due to direct shear. In the case of such spars the shear stress due to direct shear is not usually high, and with a little practice it will be found easy to estimate the amount (which is very small) by which the computed value of t must be increased for safety.

In the case of circular metal spars, where the thickness of the wall is small compared with the outside diameter, the required thickness t is given at once by

$$t = \frac{2}{\pi D^2} \times \frac{T_e}{f_s} .$$

No examples have been given in the case of spars of circular section, as it is thought that the examples worked out in the cases of spars of rectangular section show the method sufficiently clearly.