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EFFECT OF STREAMLINE CURVATURE ON LIFT OF BIPLANES

By L. Prandtl

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EFFECT OF STREAMLINE CURVATURE ON LIFT OF BIPLANES.*

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The biplane experiments described in Chapter IV, Section 4, of Report II, demonstrated that the conversion from the monoplane to the biplane, for the determination of the induced drag, must be made with another value of κ than for the determination of the angle of attack.** If the latter value is designated by κ', with the subscript E for the monoplane and D for the biplane, we then have

$$c_{w_D} = c_{w_E} + \frac{c_a^2}{\pi}\left(\frac{\kappa F_D}{b_D^2} - \frac{F_E}{b_E^2}\right) \quad (1)$$

$$\alpha_D = \alpha_E + \frac{c_a}{\pi}\left(\frac{\kappa' F_D}{b_D^2} - \frac{F_E}{b_E^2}\right)^{***} \quad (2)$$

According to the theory of the supporting lines, we should have $\kappa' = \kappa$. Table 45, on page 39, of Report II, shows great discrepancies between the values of κ and κ' obtained from the experiments, κ' being throughout greater than κ. The opinion was expressed that the curvature of the streamlines, which the one wing causes in the vicinity of the other, would be found to be responsible for this discrepancy. In a footnote, added during the proof reading, it was possible to state

*"Ueber den Einfluss der Stromlinienkrummung auf den Auftrieb von Doppeldeckern." From "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Report III, 1927, pp. 9-13.

**The same determination was previously made by Max Munk. See his article in "Technische Berichte," II, p.187 (Bibliography of Report I, C. No. 24).

***In this article, all angles are given in circular measure.

that the mathematical proof of this had been successfully developed in the interim. This refers to the dissertation of N. K. Bose (No. 64 of the bibliography of Report II) which, for the time being, is accessible only in the typewritten copies in the government library in Berlin and in the library of the University of Göttingen, an abstract of which will probably appear, however, in the "Zeitschrift für angewandte Mathematik und Mechanik." In what follows, the principal results of this work will be given, in so far as they are of practical importance.

The problem is divided into two parts: first, to determine the lift of a wing which is situated in a curvilinear flow; and second, to calculate the curvature which one wing of a biplane produces in the vicinity of the other. The combination of these two relations leads to the desired result. For the first part, a superposition is possible, just as is customary in the wing theory. If the "element" of a wing* is situated in a descending flow, the effective angle of attack is then equal to the geometrical angle of attack minus the angle of inclination of the flow. Similarly, it can be assumed that, if a wing element is situated in a flow of the curvature $1/R'$, the effective curvature of the section is then equal to the geometrical curvature of the section minus $1/R'$. The correctness of this relation is obvious for the case when the

* "Element" here means a portion of the wing of full-length chord but very narrow in the direction of the span.

geometrical curvature of the section and the flow curvature coincide. In this case, the curved section in the curvilinear flow will be situated just the same as a corresponding flat plate (or the corresponding symmetrical profile) in a parallel flow. If the angle of attack is zero, there will be no lift. The above relation will be at least an approximately correct formula for moderate curvatures of the flow and section.

The dependence of the lift on the curvature of the airfoil section, at a constant angle of attack, can be deduced from systematic tests of such sections. However, the approximate formula

$$c_a = 2\pi \left(\alpha + \frac{\beta}{4}\right) \qquad (3)$$

derived according to Kutta's method for infinitely long thin circularly curved plates in a frictionless fluid will suffice here. In this formula, α is the angle of attack of the airfoil-section chord and β the angle at the center subtended by the section arc. Then, $\sin \frac{\beta}{2} = \frac{t/2}{R}$ (Fig. 1), and hence with the approximation, which makes the sine equal to the arc, $\beta = t/R$. Therefore

$$c_a = 2\pi \left(\alpha + \frac{t}{4R}\right) \qquad (4)$$

According to the earlier statement for a wing in a curvilinear flow, $1/R - 1/R'$ must be substituted for $1/R$. It is also assumed that α is known, for which, in the case of a wing of finite length, the true angle of attack is to be substituted. In the determination of α, however, as can be seen, the curvature again enters. Because of the curvature, the inclination of the flow at the different points of the section is different. It is allowable to assume the lift of the inducing wing (equation 2), hence of the other wing, which disturbs the field of motion in the vicinity of the wing considered (equation 1), as concentrated on a line. Then, in the vertical plane through this line, the induced velocity w_{21} is present, which is furnished by the previous multiplane theory (Report II, Chapter III, 1). Since the angles of attack in equations (3) and (4) are referred to the chord of the arc, the correction of the angle of attack, for the presence of the second wing, is given by the inclination of the chord of the streamline coming from the second wing at the position of the first wing. In Fig. 2, D denotes the center of pressure of the second wing, at which point the whole lift is considered concentrated. SS is the streamline, which is produced at the position of the first wing by the lift acting at D, when the first wing is not there. It touches the straight line drawn at the inclination w/V vertically above D. A is the position of the leading edge, B of the trailing edge of the first wing, and the dotted line is the chord of the streamline. The correctness of the above can easily be demonstrated by considering a thin section which coincides exactly with the streamline SS between

A and B. Its geometrical angle of attack would be represented by the dotted chord. If the inclination of the chord is subtracted, the effective angle of attack will then be zero and, since the curvature correction will also make the effective curvature zero, the lift will be zero, as it must be, when there is introduced into a flow a surface which exactly follows the streamlines present without it. Since the streamline (with the here necessary accuracy) can be regarded as a parabolic curve, the inclination of the chord can be replaced by the inclination of the tangent to the streamline at the middle of the profile, at M. If D is situated at a horizontal distance of e in front of M, then the angle e/R' (R' being the radius of curvature of the streamline) must also be added to w_{21}/V. Moreover, since the downward velocity w_{11}, produced by the first wing, is also added, with $w_{21} + w_{11} = w_1$ and with α_1 = geometrical angle of attack of wing 1, the effective angle of attack $\alpha_1 - w_1/V - e/R'$ is obtained. With the curvature correction, however, we have

$$c_{a_1} = 2\pi\left(\alpha_1 + \frac{t}{4R} - \frac{w_1}{V} - \frac{\frac{t}{4} + e}{R'}\right) \tag{5}$$

This relation holds good for wing 1. A perfectly analogous relation, with somewhat modified values of the individual quantities, holds good for wing 2 of the biplane. The calculation is here continued only for the unstaggered biplane with two equal wings, in which it will also be assumed that the lift is divided equally between the two wings. In this case (within the degree of accuracy here desired), all the quantities of equation (5) can be assumed as equal for the upper and lower wings, and we can also put $e = \frac{t}{2} - s$, where s denotes the distance of the center of pressure D from the leading edge A (Fig. 2). As the final formula for this special case, we accordingly obtain

$$c_a = 2\pi\left(\alpha + \frac{t}{4R} - \frac{w}{V} - \frac{\frac{3}{4}t - s}{R'}\right) \qquad (6)$$

The first three terms in the brackets correspond to the previous theory, while the fourth term is newly added. Since s approximately equals $\frac{c_m}{c_a}t$, this can also be written $\frac{t}{R'}\left(\frac{3}{4} - \frac{c_m}{c_a}\right)$.

There still remains the second part of the problem, the determination of the curvature $1/R'$ of the air flow due to the second wing. If $w\ (x, y)$ denotes the downward velocity produced by this wing, at the height h above (or below) it, and if the X-axis has the direction of the wing span and the Y-axis the longitudinal direction, then the curvature $1/R''$ at any point (a small inclination of the streamline being assumed) is the differential quotient of the inclination according to y. If the inclination is w/V, then $\frac{1}{R''} = \frac{1}{V} \times \frac{\partial w}{\partial y}$. This value was calculated by Bose for a series of points on the vertical plane through the supporting line (i.e., through D in Fig. 2), so that it applies to biplanes of various height ratios h/b and to all values of x. The assumption seems justified, that the curvature, thus calculated, is approximately correct, even for the remaining points of the unstaggered "first wing." The quite difficult calculations show that $1/R''$ falls nearly to zero at the wing tips. Since our discussion refers only to the total result of all the curvature effects, a mean distribution above the wing has also been assumed, so that, if a is the lift intensity at the point x, we have

$$\frac{1}{R'} = \frac{\int a \, d \frac{x}{R''}}{\int a \, d x}$$

The lift intensity for this mean distribution is assumed to be elliptically distributed. The results have been tabulated for the different values of h/b. It is manifest that, within the values of h/b usually found in biplanes, the approximate formula

$$\frac{1}{R'} = 0.0875 \, c_a \frac{t}{h^2} \qquad (7)$$

can be used. Bose also investigated the effect of the circumstance that in reality the lift of the second wing is not concentrated on a line, but is distributed over the whole chord. The curvature is thus somewhat reduced. We will not proceed further with this refinement, however. With equation (7) equation (6) becomes

$$c_a = 2\pi\left(\alpha + \frac{t}{4R} - \frac{w}{V} - 0.0875\,\frac{t^2}{h^2}\left(\frac{3}{4}\,c_a - c_m\right)\right) \qquad (8)$$

If we pass from this formula to a transformation formula of the type of equation (2) (i.e., if we put $c_{a_D} = c_{a_E}$), then, because the curvature term $t/4R$ occurs in the same way for both monoplane and biplane and is therefore eliminated, and because, moreover, $\frac{c_a}{\pi} \times \frac{\kappa F_D}{b_D^2}$ can be written for $\frac{w}{V} = \frac{w_{21} + w_{11}}{V}$, we have

$$\alpha_D = \alpha_E + \frac{c_a}{\pi}\left(\frac{\kappa F_D}{b_D^2} - \frac{F_E}{b_E^2}\right) + 0.0875\,\frac{t^2}{h^2}\left(\frac{3}{4}\,c_a - c_m\right) \qquad (9)$$

It is easily seen from this equation that it can not be put in the form of equation (2) on account of the occurrence of c_m. Hence, for making the comparative calculation, the following method will be used. The angle of attack of the biplane (i.e., the right side of equation (9) without the last term) as calculated by the former theory, may be designated by α_D'. Then the angular difference $\Delta\alpha = \alpha_D - \alpha_D'$, according to equation (9), is

$$\Delta\alpha = 0.0875\,\frac{t^2}{h^2}\left(\frac{3}{4}\,c_a - c_m\right) \qquad (10)$$

and, according to equation (2),

$$\Delta\alpha = (\kappa' - \kappa)\,\frac{c_a}{\pi}\,\frac{F_D}{b_D^2} \qquad (11)$$

Since these two equations for variable c_a cannot generally be combined, two definite values, c_{a_1} and c_{a_2} may be selected, and $\Delta\alpha_1$ and $\Delta\alpha_2$ may be calculated and, in analogy with the method according to which the values of κ' were determined from the experiments described in Report II, instead of equation (11), we may put

$$\Delta\alpha_1 - \Delta\alpha_2 = (\kappa' - \kappa)\ \frac{F_D}{\pi\ b_D^2}\ (c_{a_1} - c_{a_2}) \tag{12}$$

and from this $\kappa' - \kappa$ may be determined.

For the numerical computation, $c_{a_1} = 0.6$ and $c_{a_2} = 0$ have been assumed. From the monoplane test, the corresponding c_m values were found by interpolation to be 0.268 and 0.08. For the five biplanes with the same span for both wings, contained in Table 45 of Report II, the values were found which were given in the following table.

Table

DD. No.	h/t	b^2/F	$\Delta\alpha_1 - \Delta\alpha_2$	$(\kappa' - \kappa)$ theo.	κ theo.	κ' theo.	κ' exp.
1	0.8	3.0	0.0359	0.563	0.794	1.357	1.221
2	1.1	3.0	0.0190	0.298	0.754	1.042	1.049
3	1.4	3.0	0.01173	0.184	0.721	0.905	0.967
4	1.113	2.4	0.01857	0.234	0.722	0.956	0.949
5	1.113	1.44	0.01857	0.140	0.649	0.789	0.803

The theoretical value of $\kappa' - \kappa$, calculated from equation (12), is added to the theoretical value of κ and thus a theoretical value of κ' is found by purely mathematical means. The experimental value of κ' is placed beside it. If the first value is disregarded, the agreement is quite satisfactory in view of the numerous omissions. Had the distribution of the lift along the chord of the induced wing been taken into account, the values of $\kappa' - \kappa$ would have been somewhat smaller throughout, especially the first, since this correction makes the most difference at small h/t. Thereby this value would also have been brought nearer the experimental value. Moreover, as explained in Report II, the deviation of the lift distribution from the elliptical, as shown in a difference between $\kappa_{theo.}$ and $\kappa_{exp.}$, also affects the deviations of both the κ'. It may therefore be said that the cause of the discrepancy between the experimental values of κ and κ' in equations (1) and 2 is explained in principle by this investigation.

Dr. B. Eck of Aachen, has been able to explain these gaps in the former theory in an entirely different way. (See "Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1925, pp. 183 ff).

<u>Annotation</u>.- A slight curvature effect also occurs on a monoplane. This was investigated by Dr. Blenk (Bibliography B, No. 105) and helped to explain certain small discrepancies in the conversion formulas for monoplane experiments. The correc-

tions are important, however, only for very wide wings (with an aspect ratio of 1 : 2 and similar ones). Therefore, they do not need to be discussed here. Dr. Blenk has also investigated the curvature effect on a wing in the air stream of a wind tunnel, in an unpublished computation. The curvature corrections due to finite air flow diameter proved, however, in all cases to lie within the limits of the experimental errors. The curvature correction may, however, be of importance in experiments with large wings between walls (Cf. Report I, Chapter IV, 2). The curvature for this case has already been given in the Wing Theory II, No. 13, equation (58) (Bibliography of Report I, C.28).

Translation by Dwight M. Miner,
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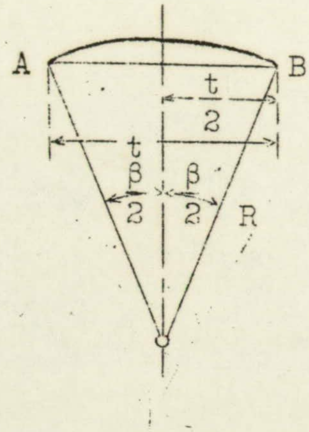

Fig.1

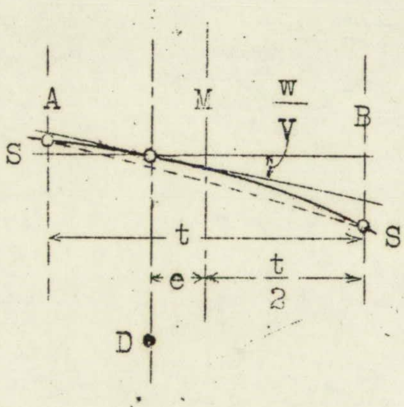

Fig.2