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No. 479

THE SPAN AS A FUNDAMENTAL FACTOR IN AIRPLANE DESIGN

By G. Lachmann

From Zeitschrift für Flugtechnik und Motorluftschiffahrt
May 14, 1928

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Washington
September, 1928

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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THE SPAN AS A FUNDAMENTAL FACTOR IN AIRPLANE DESIGN.*

By G. Lachmann.

I. Effect of the Span on the Climbing Ability of Biplanes

1. Introduction

Among various requirements of airplane construction, aerodynamic and static conditions naturally call for the production of light and strong airplanes with minimum drag. These two conditions are generally contradictory, but there is a point of best conformity. In designing new airplane types or altering existing ones, both points of view should be considered jointly and simultaneously.

If particular stress is laid on speed, and if the climbing ability can be largely neglected, it is almost always better to develop the aerodynamic side and to accept the accompanying weight increments into the bargain, since the induced drag is small compared with the other drags. However, this case is rare (racing airplanes). The fact as to whether the reduction in parasite resistance resulting from the increase in weight justifies the simultaneous increase in the induced drag, is

*"Die Spannweite als grundlegendes Bestimmungsstück des Flugzeugentwurfs," in Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 14, 1928, pp. 198-208.

determined in each case by a simple comparison. Baumann was the first to publish this statement.* Vogt carried out monoplane tests dealing with the rather complicated relations between weight, aspect ratio, and ceiling.** The same problem applied to biplanes or sesquiplanes is tackled by the following method. However, instead of the aspect ratio, which is unpractical and leads to confusion, the span is adopted as the independent variable, and the wing weight, the climbing speed or the ceiling as the dependent variable.

2. Variation of the Wing Weight in Terms of the Span and of the Wing Loading

The span is one of the most important factors in airplane design. It is a decisive determinant of the wing weight, of the induced drag and of the maneuverability. Variations in the span do not affect these airplane characteristics in the same way. Increased span increases the weight, but reduces the induced drag and the maneuverability. This is how the problem presents itself to the designer of combat airplanes, who must keep all three conditions in mind, whereas maneuverability can be largely disregarded in designing commercial airplanes.

*A. Baumann, "Zusammenhang zwischen Widerstandsverminderung und Gewichtszunahme" ("Relation between Drag Reduction and Weight Increment"), Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1924, p. 10.

**R. Vogt, "Das gunstigste Seitenverhaltnis" ("The Most Favorable Aspect Ratio"), Z. F. & M., 1925, p. 167.

The endeavor to obtain maximum maneuverability leads us to multiplanes. The further development of our investigations refers to biplanes only and no heed is taken of maneuverability. The well-known Fokker wing cell with torsion strut is taken as an example, its simple static structure being best suited for our investigation. It is sought to determine to what extent the climbing speed and the ceiling are affected by variations in the span, when the wing loading (landing speed) remains constant.

The following simplifying assumptions are made:

That the upper and lower wings have the same profile and breaking load per square meter;

That the front and rear spars always occupy the same position in the wing section;

That the upper and lower wings have a similar plan form;

That the front and rear spars have the same thickness (Fig.1).

The following symbols are used (Fig. 2):

$2b$ = wing span,

b_o = half the effective span of the upper wing,

b_u = half the effective span of the lower wing,

t_o = chord of upper wing at the root,

t_u = chord of lower wing at the root,

t_{eo} = chord of upper wing at the tip,

t_{eu} = chord of lower wing at the tip,

F_o = area of upper wing,

F_u = area of lower wing,

h = distance between centers of gravity of spar flanges,

φ_1 = cross section of spar flange at the root,

φ_2 = cross section of spar flange at the wing tip,

α = share of breaking load on the front spar,

β = share of breaking load on the rear spar,

D = wing loading,

n = load factor for case A,

g = breaking load per square meter = $D\,n$,

x = half the span of central portion of wing (cabane),

y = reduction factor for the area of (trapezoidal) wings,

k = idem, reduction factor for the bending moment,

s = distance between wing root and the c.g. of wing area.

$$\frac{b_u}{b_o} = C_1; \qquad \frac{F_u}{F_o} = C_2; \qquad \frac{h}{t} = C_3.$$

I. Moment of Front Spar

The maximum moment for the front spar of the upper wing is:*

$$M_o = \frac{t_o + t_{e_o}}{2}\, b_o\, s_o\, \alpha\, g$$

or

$$M_o = \frac{t_o\, b_o^2}{2}\, \alpha\, g\, k$$

where

$$k = \frac{t_o + t_{e_o}}{2\, t_o}\, \frac{2\, s_o}{b_o}$$

*Neglecting the decrease in lift at the wing tip.

Or when

$$F_o = b_o\, t_o\, y = \frac{t_o + te_o}{2}\, b_o$$

$$y = \frac{t_o + te_o}{2\, t_o}$$

$$M_o = \frac{1}{2}\, F_o\, b_o\, \alpha\, g\, \frac{k}{y}.$$

Correspondingly, for the lower wing:

$$M_u = \frac{t_u\, b_u^2}{2}\, \alpha\, g\, k$$

or

$$M_u = \frac{1}{2}\, F_o\, C_2\, C_1\, b_o\, \alpha\, g\, \frac{k}{y}$$

II. Tensile and Compressive Stresses in Upper and Lower Flanges

$$S_o = \frac{M_o}{h_o} : h_o = C_3\, t_o = C_3\, \frac{F_o}{b_o\, y}$$

$$S_o = \frac{b_o^2}{2\, C_3}\, \alpha\, g\, k$$

$$S_u = \frac{M_u}{h_u} = \frac{b_o^2\, C_1^2}{2\, C_3}\, \alpha\, g\, k.$$

III. Weight of the Spar Flanges

Under the approximate assumption of a rectilinear decrease of the thickness of the flanges from the root to the wing tip, we can write:

$$G_g = \frac{1}{3}\, b_o,\, u\, (\varphi_1 + \varphi_2 + \sqrt{\varphi_1\, \varphi_2})\gamma$$

where

γ = the specific gravity of the material of the flange,

φ_1 = cross section of flange at the root,

φ_2 = cross section of flange at the tip (Fig. 3).

Let

$$\varphi_2 = m\,\varphi_1$$

Then

$$G_g = \gamma\,\frac{1 + m + \sqrt{m}}{3}\,b\,\varphi_1$$

or,

$$G_g = b\,\varphi_1\,\kappa\,,$$

where

$$\kappa = \gamma\,\frac{1 + m + \sqrt{m}}{3}\,.$$

Then

$$\varphi_1 = \frac{S}{\sigma},$$

(σ = either tensile or compressive stress)

S can be considered numerically equal to either the tensile or compressive stress. Experience shows that in most cases the cross sections of both flanges must be nearly the same, since, on account of the high load factor now adopted for combat airplanes, the downward load in case C is equal to or only slightly different from the upward load in case A. Thus the weight of the front-spar flanges of half the wing cell is

$$G_{g_{v(o,u)}} = (S_o\,b_o + S_u\,b_u)\,\frac{\kappa}{\sigma}$$

or

$$G_{g_{v(o,u)}} = \frac{b_o^{\,3}}{2\,C_3}\,(1 + C_1^{\,3})\,\frac{\kappa}{\sigma}\,\alpha\,g\,k.$$

Since $\alpha + \beta = 1$ the total weight of the spar flanges of half a wing cell is

$$G_g = \frac{b_o^3}{2\ C_3} (1 + C_1^3) \frac{\kappa}{\sigma} g\ k.$$

Discussion of this formula.

a) b_o and Constant Area

The weight of the spar flanges is minimum when $C_1 = b_u = 0$, i.e., in the case of a monoplane. This case, however, is excluded from our further consideration, since we would otherwise be deprived of the binding effect of the strut, which is particularly effective in case C. In any case it appears that, on account of the weight, the lower wing should be kept as small as possible.

The weight of the flanges is maximum when $C_1 = 1$, i.e., when both wings have the same span. In this case the weight of the flanges is

$$G_g = \frac{b_o^3}{C_3} \frac{\kappa}{\sigma} g\ k\ .$$

This fact is evidenced by very simple considerations.

b) $\frac{t}{b} y = \lambda$ and Constant Area

$$G_g = C' (b_o^3 + b_u^3)$$

$$C' = \frac{\kappa\ g\ k}{2\ C_3\ \sigma} = \text{constant}$$

$$y \frac{t_o}{b_o} = y \frac{t_u}{b_u} = \lambda = \text{constant}$$

$$\lambda (b_o^2 + b_u^2) = F_o + F_u = F \text{ (area of half a cell)}$$

$$b_u = \sqrt{\frac{F}{\lambda} - b_o^2}$$

$$G_g = C' \left[b_o^3 + \left(\frac{F}{\lambda} - b_o^2 \right)^{3/2} \right]$$

After differentiating and putting the expression equal to zero, a minimum weight of the flanges is obtained, when

$$b_o = \sqrt{\frac{1}{2} \frac{F}{\lambda}}, \quad \text{or when} \quad C_1 = \frac{b_u}{b_o} = \frac{\sqrt{\frac{F}{\lambda} - \frac{F}{2\lambda}}}{\sqrt{\frac{F}{2\lambda}}} = 1$$

and a maximum, when

$$b_o = 0, \; C_1 = \infty .$$

IV. Total Weight of the Wings

The total wing weight comprises the following components:

$$G_F = 2 \; (G_g + G_s + G_r)$$

G_g = weight of the spar flanges of half a cell,

G_s = weight of the spar webs of half a cell,

G_r = weight of ribs, veneering, fabric, fittings, shims, etc., of half a cell.

We are taught by experience that the variation of the amounts denoted by G_s and G_r is negligible, when the area and the breaking load per square meter remain constant.

Hence, if G and G are expressed by the constant B,

$$B = G_s + G_r,$$

the wing weight can be written

$$\frac{1}{2} G_F + B + b_o^3 \frac{(1 + C_1^3) \frac{\kappa}{\sigma} g k}{2 C_3}$$

Let H denote the expression $\frac{(1 + C_1^3) \frac{\kappa}{\sigma} g k}{2 C_3}$.

Then

$$G_F = 2 B + 2 H b_o^3.$$

Lastly, if half the effective span b and half the width of the cabane x are substituted for b_o, we obtain

$$G_F = 2 B + 2 H (b - x)^3.$$

That is, the weight of the wing consists essentially of two components, one of which can be assumed to be practically constant, while the other varies as the third power of the effective span. The values B and H are best determined from the data available for the example to be investigated.

V. Effect of the Torsion Strut

For the calculation of the spar moment, the assumption was made that, for a given area and breaking load per square meter, this moment increases as the span:

$$M_o = \frac{1}{2} F_o b_o \alpha g k.$$

As a matter of fact the moment of the statically indeterminate force x_i, exerted by the torsion strut on the wing, must also be taken into account, since it increases or reduces the moment of the cantilever wing (without struts) according to whether the force is directed downward or upward. Therefore, the value of the resultant moment, which determines the dimensions of the spar flanges, is

$$M_r = M_o \pm X_i\, b' \quad \text{(Fig. 4)},$$

M_o being the moment of the cantilever wing.

The value which, in a numerical example, must be introduced for the weight of the flanges, must correspond to their cross sections as necessitated by the moment M_r. The cross sections of the flanges, and hence the resultant moment, are assumed to be given, since we start from an existing structure. Thus, provided all the other assumptions hold good, the fact that M_r and M_o vary as the span has still to be demonstrated.

If the same material is used for the spars, and if a possible variation of the shape of the cross section of the spar is disregarded, the ultimate bending stress kb is constant. According to our assumption, the resultant breaking moment must increase to ΔM_r and hence the drag moment W of the cross section to ΔW, if the span b is increased to Δb. At the same time, however, the thickness of the spar decreases in the same proportion, since, according to our assumption, the wing section remains constant. Consequently, the moment of inertia of the spar section varies as $\frac{1}{\Delta}$.

The statically indeterminate force is a quotient of two deflections

$$X_i = \frac{\partial_{o_i}}{\partial_{i_i}}$$

where ∂_{o_i} = the deflection of the spar at the point i of the zero system under the influence of the distributed load;

and ∂_{i_i} = the deflection under the influence of a load 1 acting at point i in the direction of the strut.

In the particular case considered, both deflections ∂_{o_i} and ∂_{i_i} are proportional to Δ for a uniform increase of all the lengths and proportional to $b^3 \Delta^4$ for a constant load, X_i remaining constant. Accordingly, the moment exerted by the strut on the wing, and hence also the resultant breaking moment M_r vary as the span.

3. Effect of the Span on the Climbing Speed*

The following equation of the climbing speed has been derived from the thrust diagram in Figure 5:

$$w = \frac{(S - W)\ v}{G}$$

All the airplanes compared are assumed to climb with the

*The basic idea of the following method for the calculation of climbing performances independent of the polar is already contained in my book "Leichtflugzeugbau" ("Light Airplane Construction"), published in 1925. The obtained results, equivalent to Schrenk's forms, are perhaps simpler to derive and enable easy repetitions even when no books or tables are at hand.

same lift coefficient $c_a = 1$. This assumption seems to be sufficiently justified by practical experience.* The corresponding impact pressure is then $q = \frac{G}{F} = D =$ constant. The total drag W is composed of the induced drag W_i and of the residual drag W_r:

$$W = W_i + W_r$$

$$W_i = \frac{G^2}{\pi\ 4\ b^2}\ \kappa\ \frac{1}{D}$$

κ = the reduction coefficient for biplanes (according to Prandtl).

Or, on summing up

$$\frac{G^2 \kappa}{4\ \pi\ b^2} = \theta; \qquad W_i = \frac{\theta}{D}.$$

Furthermore,

$$W_r = f_{w_r}\ D; \qquad f_{w_r} = \text{residual-drag area.}$$

Hence,

$$W = \frac{\theta}{D} + f_{w_r}\ D.$$

The propeller thrust S is obtained from

$$S = \frac{N_o\ \nu\ 75\ \eta}{\sqrt{\frac{2g}{\gamma}}\ \sqrt{D}}$$

ν = reduction factor for the engine power,

γ = air density.

Let

$$N_o\ 75\ \eta = A$$

*M. Schrenk, "Zur Berechnung der Flugleistungen ohne Zuhilfenahme der Polare," 1927 Yearbook of the D.V.L., pp. 104-106 (N.A.C.A. Technical Memorandum No. 456, 1928).

then

$$S = \frac{A\,\nu}{\sqrt{D}\,\sqrt{\frac{2g}{\gamma}}} = \frac{A\,\nu}{D^{1/2}\,\gamma^{-1/2}\;4.43}$$

Thus the following value is obtained for the climbing speed:

$$w = \frac{A\,\nu - \left(\frac{\theta}{D} + f w_r\,D\right) D^{1/2}\,\gamma^{-1/2}\,4.43}{G}$$

The total airplane weight G comprises a residual weight G_R and the weight of the wings G_F

$$G = G_R + G_F = G_R + 2\,B + 2\,H\,(b - x)^3$$

$$G_R + 2\,B = Z$$

hence

$$G = Z + 2\,H\,(b - x)^3$$

and

$$w = \frac{\nu\,A - \left(\frac{\theta}{D} + f w_r\,D\right) D^{1/2}\,\gamma^{-1/2}\,4.43}{Z + 2\,H\,(b - x)^3}$$

An expression is thus obtained which contains only one dependent value b. All the other values are constant, save f which to a certain degree depends on G. However, as far as our investigations are concerned, this variation is negligible. The expression for w may be differentiated with respect to b in view of determining the optimum span. However, the solution of the resultant equation is difficult. Besides, the graphical study of w as a function of b would certainly be more instructive. Near the ground $\nu = 1$ and $\sqrt{\frac{2g}{\gamma}} \sim 4$, whence

$$w = \frac{A - \left(\frac{\theta}{D} + f w_r\,D\right) 4\,D^{1/2}}{Z + 2\,H\,(b - x)^3}$$

4. Effect of the Span on the Ceiling

At the ceiling

$$s = W$$

that is,

$$\frac{A\ \nu}{\sqrt{\frac{2g}{\gamma}\sqrt{D}}} = \frac{\theta}{D} + f w_r\ D$$

or

$$\frac{A\ \nu}{4.43\ \gamma^{1/2}\ D^{1/2}} = \frac{\theta}{D} + f w_r\ D$$

or else

$$\nu\ \gamma^{1/2} = \frac{\left(\frac{\theta}{D} + f w_r\ D\right) 4.43\ D^{1/2}}{A}$$

When an empirical curve of the decrease of power of the considered engine is given for an air density γ, the ceiling or the density at the ceiling are derived from this curve.

On introducing the law given by Schrenk* for the decrease in power

$$\nu = \left(\frac{\gamma_x}{\gamma_o}\right)^{1/4},$$

the ceiling density is directly derived from the following expression

$$\gamma_{min} = \left[\frac{\frac{\theta}{D} + f w_r\ D\quad 6.05\ D^{1/2}}{A}\right]^{0.526}$$

$$Z_g = 20.9 \log \frac{\gamma_o}{\gamma_{min}}$$

*Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1926, p. 161 (N.A.C.A. Technical Memorandum No. 456, 1928).

5. Numerical Example

The numerical calculation was carried out for a known type, the weight of its components and its other data having been supplied to the writer.

Basic Data

N_o = 450 HP. 2 B = 148 kg

η = 0.6 (mean propeller efficiency) 2 H = 0.7

D = 70 kg/m² x = 1.1 m

n = 10 fw_r = 0.76 m²

G_R = 1260 kg k = 0.96

Result of Calculation

a) Climbing speed near the ground

b m	G_F kg	G kg	θ kg²/m²	A mkg/s	fw_r m²	w m/s
4	165	1425	9700	20250	0.76	9.7
5	190	1450	6405	"	"	10.6
6	230	1490	4713	"	"	10.9
7	292	1552	3750	"	"	10.7
8	378	1638	3200	"	"	10.3

The curves obtained for G and w are plotted in Figures 6-7. The maximum of the w-curve is flat for a total span of 2b = 12 m. It would therefore be useless, in the present case, to lengthen the span in order to increase the climbing speed near the ground.

b) Ceiling

b m	γ_{min} kg/m³	$\frac{\gamma_o}{\gamma_{min}}$	Zg km
4	0.677	1.86	5.6
5	0.573	2.13	6.9
6	0.531	2.35	7.8
7	0.498	2.51	8.3
8	0.481	2.60	8.7

The resulting curve is shown in Figure 8. While, for the climbing speed, the static and aerodynamic influences in the neighborhood of $2b \sim 12$ fairly offset one another, the static influences, as regards the ceiling, are still slightly exceeded by the aerodynamic influences, so that the maximum is shifted toward the region of longer spans. However, this gain is not large enough to justify the loss in maneuverability.

6. Conclusion

It is obvious that a similar relation may also exist between the wing weight and the span of braced biplanes and strutted monoplanes, i.e., that they follow a law consisting of a practically constant part and a member varying as the third power of b. Accordingly, we can assume a similar climbing ability to what we found in our special example, that is, we may assume that the span of each airplane type has a "reasonable" limiting value, which it is useless to exceed for the sake of climbing speed, and which it would be scarcely worth while exceeding for

for the sake of the ceiling.

The nondimensional expression

$$C = \frac{2\ b}{\sqrt{F\ k}}$$

is derived as a comparative value from

$$\lambda = \frac{4\ b^2}{F\ \kappa},$$

k being equal to 1 for monoplanes, or in each case the value of the "reasonable" span may be derived from

$$2\ b = C\sqrt{F\ k}$$

The value C was calculated for a certain number of German and foreign airplanes of different makes (mostly braced biplanes or strutted monoplanes) and recognized good flying ability and was then plotted as a function of $\frac{G}{N}\sqrt{\frac{G}{F}}$ (Fig. 9).

Figure 9

No.	Make	Airplane type
1	Curtiss Hawk P_1 a	Pursuit
2	Spad 51	"
3	Fokker D XIII	"
4	Nieuport-Delage 42 C I	"
5	Armstrong Siskin V	"
6	Gloster Gamecock	"
7	Bristol Fighter	"
8	Fokker C V E	Observation
9	Breguet XIX	"
10	Potez 25 A 2	"
11	Albatros L 68	Sport
12	De Havilland "Moth"	"
13	Raab-Katzenstein "Schwalbe"	"
14	Udet U.12	"
15	Bellanka (monoplane)	Commercial
16	Dornier Merkur (monoplane)	"
17	" Wal "	"
18	Ryan	"
19	Albatros L 73 (biplane)	"

The mean values obtained for the various types are: for a single-seat or two-seat pursuit airplane $C = 2.12$; for observation airplanes $C = 2.32$; for commercial airplanes $C = 2.47$; and for sport types $C = 2.19$. Monoplanes have a slightly higher value than biplanes. This, however, may be connected with the problem of longitudinal stability.

Pursuit airplanes are near the lower limit, which is easily explained by the fact that they are comparatively little affected by the span or by the induced drag, on account of their low power loading and because of the important part played by maneuverability of this type. Naturally C increases slightly for greater lower loadings, since the influence of the span is then relatively greater.

It is particularly remarkable that two airplanes of extraordinary efficiency, of so fundamentally different types as the Dornier "Wal" and the De Havilland "Moth," have a similar, comparatively small value of C (Wal, $C = 2.285$); Moth, $C = 2.185$).

Hopes regarding the improvement of the climbing ability by increasing the span, awakened by the now generally recognized aerodynamic importance of the span, prove illusory in the light of a static-aerodynamic investigation. One is tempted to believe that monoplanes are better suited than biplanes for altitude record flights.

The result discourages aerodynamic research and encourages static research.

II. Maneuverability and Span

1. Definition of Maneuverability

"Maneuverability" means that property of an airplane which enables the pilot to change its direction of flight in the shortest possible time. This property is of particular importance for combat airplanes, especially for single-seat fighters, two-seat fighters, and observation airplanes. Lateral and rolling maneuverability are of paramount importance in fighting while flying in curves.

Judging the maneuverability is generally left entirely to the pilot. The designer (so long as he does not fly) depends chiefly on the pilot's judgment. It is very important, however, for the designer of combat airplanes to know exactly how maneuverability is affected by the various structural features and, accordingly, to be able to develop the most favorable form from the beginning, or to make the requisite modifications of already existing types.

2. Analytical Study of Maneuverability

During the war the behavior of the airplane in steady curvilinear flight was used in Germany for the analytical study of maneuverability.* Kann's investigations are based on compara-

*Kann, "Der wagerechte Kurvenflug" (Horizontal Curvilinear Flight), Technische Berichte III, No. 7. In the same number, "Der Kurvenflug eines Flugzeugs" (The Curvilinear Flight of an Airplane) by E. Salkowski.

tively simple assumptions, and his formulas are consequently handy. Salkowski tackles the problem on general principles. The calculation is therefore more complicated and less lucid.

The results of Kann's work are briefly indicated below. Figure 10 is an airplane in steady curvilinear flight.

Z = centrifugal force, A = lift, R = resultant force.

Conditions of equilibrium:

$$R = c_a\, F\, q$$

$$\frac{75\, N\, \eta}{v} = c_w\, F\, q$$

The moment $A\, e$ must be offset by a corresponding deflection of the ailerons.

If a linear law is substituted for the decrease in engine power with air density (Kann assumes that

$$N = N_o \frac{\gamma}{\gamma_o} .$$

$$N_i = N_{io} \frac{\gamma}{\gamma_o}$$

would be more correct, N_i being the indicated horsepower), the following simple relation is obtained:

$$\frac{R}{G} = \frac{\gamma}{\gamma_\alpha} \quad \text{or} \quad R = G\, \frac{\gamma}{\gamma_\alpha} .$$

Thus the steady curvilinear flight, in air of density γ, is carried out with the same angle of attack γ or c_a-value as a horizontal flight in air of a lower density γ_α.

Since

$$Z = \sqrt{R^2 - G^2} = G\sqrt{\left(\frac{\gamma}{\gamma_\alpha}\right)^2 - 1},$$

the radius of the curve is

$$\rho = \frac{G}{g}\,\frac{v^2}{Z} = \frac{v^2}{g\sqrt{\left(\frac{\gamma}{\gamma_\alpha}\right)^2 - 1}}$$

and the time required to fly a circle is

$$T = \frac{2\,\pi\,\rho}{v} = \frac{2\,\pi\,v}{g\sqrt{\left(\frac{\gamma}{\gamma_\alpha}\right)^2 - 1}}$$

The requisite bank of the airplane is

$$\tan\mu = \frac{Z}{G} = \sqrt{\left(\frac{\gamma}{\gamma_\alpha}\right)^2 - 1}\ .$$

Kann then shows that it is possible to calculate without any great error the shortest time required to fly a circle at the angle of attack corresponding to the c_a-value at which the airplane flies straight forward at the ceiling. Thus the following expressions are obtained for the curve which can be most easily flown:

$$\rho_{min} = \frac{v^2}{g\sqrt{\left(\frac{\gamma}{\gamma_g}\right)^2 - 1}} \qquad \tan\mu = \sqrt{\left(\frac{\gamma}{\gamma_g}\right)^2 - 1}$$

$$T_{min} = \frac{2\,\pi\,v}{g\sqrt{\left(\frac{\gamma}{\gamma_g}\right)^2 - 1}}$$

American Formula for the Determination of Maneuverability

In America the following expression is used for the analytical estimation of maneuverability:

$$A = \frac{RV}{r}$$

where

R = climbing speed in feet per minute and

r = radius of smallest curve at sea level.

Hence the value A has the dimension of an acceleration. The minimum value of A in the specifications for single-seaters is 7.

It is obvious that both Kann's formula and the American expression are more or less a criterion of the climbing ability of the airplane.

Hence the maneuverability must be affected in the same way by all structural features which raise the ceiling and likewise improve the climbing speed. Thus, for instance, the weight and the speed being constant, an increase in span effects a decrease in the induced drag and hence an improvement in the climbing ability. Theoretically, this should also improve the maneuverability. This, however, is contrary to the sense of flight and to practical experience.

This apparent contradiction between theory and practice is due to the fact that the above-mentioned German and American considerations apply only to steady curvilinear flight, whereas, in practice, nonsteady flight, i.e., the transition from rectilinear to curvilinear flight is of much greater importance. In the first case the bank μ is taken for granted, while in the second case particular importance is attached to the time required to reach this banked position.

In my opinion it would be more useful to substitute a dynamical study for the static investigation of the equilibrium and to introduce "rolling ability" as a criterion of maneuverability. I understand "rolling ability" to be represented by the angular velocity of the airplane about its X axis for a given impact or dynamic pressure and a given ailerom deflection. Stated practically, maneuverability is measured by the time required by an airplane in horizontal flight to complete a total or partial roll. It will be shown subsequently that the speed of turning in the horizontal plane is likewise decidedly affected by this "rolling speed."

The fact that the minimum values of the ceiling and of the climbing speed are generally defined by special requirements, affords a criterion of the maneuverability which is independent of the climbing performances.

3. Effect of Span on Rolling Ability

Let M_r denote rolling moment developed by the ailerons;

ω, angular speed of rotation;

J, moment of inertia of airplane about its X axis;

D, the damping moment.

Then,

$$M_r = J \frac{d\,\bar{\omega}}{d\,T} + D$$

where $$D = f(\omega).$$

The following simplifying assumptions are made for the rolling moment (Fig. 11):

b = the wing span;

t = the wing chord;

z = distance of c.g. of aileron surface from axis of rotation;

Δc_{nl} = variation in c_n-value of undisturbed lift distribution by deflection of left aileron;

Δc_{nr} = corresponding value for right aileron;

q = impact or dynamic pressure.

Then

$$M_r = q\ l\ t\ z\ (\Delta c_{nl} + \Delta e_{nr}).$$

The distribution of C_n along the span b is obtained as shown in Figure 13.

Let the total lift or the value of the integral

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}} c_n\, dz$$

remain unchanged.

This distribution, which cannot be adopted on account of its abrupt transition, is replaced by a trapezoidal distribution.

If we assume that

$$\Delta c_{n l} = \Delta c_{nr} ,$$

we thus obtain the distribution shown in Figure 13 which, as regards the developed moment, is equivalent to the stepwise distribution of Figure 12.

$$\Delta c_{n l'} = \Delta c_{nr'} = \frac{M_r \, 6}{b^2 \; t \; q} = 6 \; c_{mr}$$

when

$$\frac{M_r}{b^2 \; t \; q} = c_{mr},$$

c_{mr} being the nondimensional coefficient of the rolling moment.

When c_{mr} is based on wind-tunnel experiments* or reasonable estimates, the ideal distribution or its $\Delta c_n'$ value can thus be determined.

For each point of the span at a distance z from the axis of rotation the corresponding value of the ideal lift distribution becomes

$$c_n' = c_{no} + k \; z$$

$$k = \frac{12 \; c_{mn}}{b}$$

*See "Bibliography" at the end.

Resultant Moment

The resultant torque is a function of the lift or c_n distribution along the span and of the angular velocity ω. We assume the ailerons to be deflected in an infinitely short time, the ideal c_n distribution thus taking place suddenly. We also assume the wings to be set originally (i.e., in rectilinear flight) at an angle α. The rotation produces an additional flow component

$$w = z\,\omega$$

and an increment

$$\pm \Delta \alpha = \text{arc tan } \frac{z\omega}{v}$$

of the angle of attack (Fig. 14).

Owing to the normally small values of w as compared with v, we can assume that

$$\Delta \alpha \sim \frac{z\,\omega}{v} .$$

According to Figure 15, the resultant c_{nr} becomes

$$c_{nr} = c_n + \frac{d c_n}{d \alpha} \; \frac{z\,\omega}{v} .$$

Of course this relation holds good only for subcritical values of α.

The resultant moment M_r–D assumes the following value:

$$M_r - D = q\ t \int_{-\frac{b}{2}}^{+\frac{b}{2}} c_n \left\{ \alpha + \Delta\,\alpha \right\} z\ dz .$$

The bracket denotes the value of c_n belonging to the angle of attack $\alpha + \Delta\alpha$.

Or,

$$M_r - D = q\ t \int_{-\frac{b}{2}}^{+\frac{b}{2}} \left[(\alpha + kz) - \frac{d\ c_n}{d\ \alpha}\ \frac{zw}{v}\right] z\ dz\ .$$

This integral can be split into two component integrals:

$$M_r - D = q\ t \left[\int_{-\frac{b}{2}}^{+\frac{b}{2}} (\alpha + kz)\ z\ dz - \frac{dc_n}{d\ \alpha}\ \frac{w}{v} \int_{-\frac{b}{2}}^{+\frac{b}{2}} z^2\ dz^2\right] .$$

The first integral is an expression of the rolling moment and the second represents the corresponding value of the damping.

The calculated value is

$$M_r - D = q\ t\ \frac{b^3}{12}\ \left(k - \frac{dc_n}{d\ \alpha}\ \frac{w}{v}\right)$$

When this expression is introduced into the equation

$$M_r - D = J\ \frac{d\omega}{dT}\ ,$$

a linear differential equation is obtained in ω.

$$q\ t\ \frac{b^3}{12}\left(k - \frac{d\ c_n}{d\ \alpha}\ \frac{w}{v}\right) = J\ \frac{d\omega}{dT}$$

After introducing the abbreviations

$$A = q\ t\ \frac{b^3}{12}$$

$$B = \frac{d c_n}{d\ \alpha}\ \frac{1}{v}$$

$$A\ B = C$$

we obtain

$$A\ k - C\ \omega = J\ \frac{d\omega}{dT}\ .$$

(T is the symbol for time, to avoid confusion with the wing chord t.) After solving we have

$$-\frac{1}{C}\ ln\ (A\ k - C\ \omega) + Y = \frac{1}{J}\ T.$$

The constant Y results from the initial conditions:

$$T = 0$$

$$\omega = 0$$

$$Y = \frac{1}{C}\ ln\ A\ k\ .$$

Hence,

$$T = \frac{J}{C}\ ln\ \frac{A\ k}{Ak - C\omega}$$

or, when the values for A and B are introduced,

$$T = \frac{12\ J}{\frac{\gamma}{2g}\ t\ b^3\ v\ \frac{d\ c_n}{d\ \alpha}}\ ln\ \frac{1}{1 - \frac{\omega}{v}\ \frac{1}{k}\ \frac{d\ c_n}{d\ \alpha}}$$

The following expressions are obtained for a constant F (variable aspect ratio):

$$T = \frac{12\ J}{\frac{\gamma}{2g}\ b^2\ F\ \left(\frac{d\ c_n}{d\alpha}\right)\ v}\ \ln \frac{1}{1 - \frac{\omega}{v}\ \frac{1}{k}\left(\frac{d\ c_n}{d\alpha}\right)},$$

in which $\left(\frac{d\ c_n}{d\alpha}\right)$ denotes the value of $\frac{d\ c_n}{d\alpha}$ corresponding to each aspect ratio.

We can now plot the time-speed curve of the rotation. The corresponding integral curve $\int \omega dt$ indicated the value of the angle of rotation μ reached in a given time.

After effecting the graphic integration, ω is found to reach very quickly a value corresponding in practice to the state of equilibrium (uniform rotation at constant angular velocity). This means that the ω-curve, as a function of T, is asymptotic for $T = \infty$ (Figs. 16-18).

The task is to determine the variation of the angle of roll of a biplane when the span is increased from 13 m to 15 m. Both wings have ailerons of the same size. In both cases the upper and lower wings have the same span and chord. The aileron deflection is assumed to be the same in both cases. Likewise the flying speed v is assumed to be constant.

Airplane I	Airplane II
bo = bu = 13 m	bo = bu = 15 m
to = tu = 2 m	to = tu = 2 m

$$\frac{d\,c_n}{d\alpha} = 0.07 \times 57.3 = 4.01.$$

The slight variation in $\frac{d\,c_n}{d\alpha}$ for airplane II is neglected.

$$v = 50 \text{ m/s} = 180 \text{ km/h}$$

Let M_r (rolling moment) be 1030 mkg for airplane I and 1245 mkg for airplane II, corresponding to the increased length of the aileron lever arm.

$$\Delta c_n' = 0.117 \qquad \Delta c_n' = 0.105$$

$$k = 0.018 \qquad k = 0.0142$$

The moment of inertia comprises a share J_f of the wings and a remainder J_r formed by the shares of the landing gear, engine, fuselage, tanks and fuel, useful load and cabane. This remainder J_r is assumed to be constant for both airplanes. The following estimate is made for an airplane which has recently been completed:

$$J_k = 110 \text{ mkg s}^2.$$

The wing share was calculated according to the data available.

Airplane I	Airplane II
$J_{fI} = 435$ mkg s²	$J_{fII} = 501$ mkg s²

Thus, the total value of the moment of inertia is

$$J_I = 545 \text{ mkg s}^2 \qquad J_{II} = 611 \text{ mkg s}^2 .$$

The time-speed curves and the corresponding integral curves are plotted in Figures 16 and 17. Lastly, Figure 18 shows the integral curves plotted on a larger scale. The effect of the increase in span becomes more manifest when the motions of the two airplanes are represented in the form of motion pictures (Fig. 19).

Naturally these investigations apply to rectilinear flight only. Hence the expression "rolling ability" was adopted. When the airplane is simultaneously forced into a turn by the rudder, its rotational speed increases, because the air speed at the outer wing tips increases while that at the inner wing tips decreases.

Approximate Solution

It can be inferred from the curves in Figure 18 that, for a rough approximation, the effect of the moment of inertia is negligible when the expression $\frac{M_r}{J}$ is not too small, which is actually the case for airplanes with good maneuverability (combat and sport airplanes).

The above consideration proves that the speed of rotation becomes uniform after an extremely short time. Therefore, let

$$\frac{d\,\omega}{d\,T} = 0,$$

and the following expression is obtained

$$\omega = \frac{v\ k}{\frac{d\ c_n}{d\alpha}} = \frac{12\ c_{mr}\ v}{\frac{d\ c_n}{d\alpha}}$$

The values of the angle of rotation μ after 4 seconds, as obtained by this formula in our example, are

$b = 13$ m, $\mu = 51.6^\circ$ $b = 15$ m, $\mu = 40.6^\circ$

Accurate calculation gave

51° 39.5°.

4. Time Required for a Reverse Turn

The elements of steady curvilinear flight hardly afford adequate means for making practical maneuverability measurements. No pilot pretends to judge the maneuverability of his airplane by flying uniformly banked circles. A criterion in better agreement with actual conditions of air combat is afforded by the time required for an airplane to turn through 180 degrees, starting from horizontal flight and returning to the same position (Fig. 20).

Let the airplane turn through an angle $d\beta$ in the time dT. The following relations then hold good:

$$ds = \rho\, d\beta = v\ dT$$

$$\rho = \frac{v^2}{g\ \tan\mu}$$

whence

$$dT = \frac{v}{g\ \tan\mu}\ d\beta$$

or

$$d\beta = \frac{g}{v}\ \tan\mu\ dT.$$

If we neglect the angular acceleration and the small variations of the rolling moment due to the speed difference between the inner and outer wing tip (The order of magnitude of the ratio, span to minimum radius of turn near the ground, is approximately 1/10. The effect is practically the same, when several airplanes with relatively small variations of the span are compared.) and put

$$\mu = \omega\, T,$$

we get

$$d\beta = \frac{g}{v} \tan\,(\omega T)\; dT,$$

or

$$\beta = \frac{g}{v} \int \tan\,(\omega T)\; dT.$$

After reaching a certain angle of bank μ in x seconds, the angle of rotation β_x becomes

$$\beta_x = \frac{g}{v} \int_{T=0}^{T=x} \tan\,(\omega T)\; dT$$

or, since $\omega x = \mu$,

$$\beta_x = -\frac{g}{v}\; \frac{\ln \cos \mu}{\omega}.$$

That is, the angle of turn of the airplane in the horizontal plane, to which a certain angle of bank μ in the vertical plane corresponds. The angle of bank μ is assumed to reach its maximum value after a turn of the flight path through $\beta = \frac{\pi}{2}$ and then to fall back in the same way to 0 (Fig. 21).

$$\mu = \text{arc} \cos e^{-\omega\beta \frac{v}{g}}$$

In order not to complicate the calculation unnecessarily, the speed is introduced as a constant mean value. If

$$\beta = \frac{\pi}{2} ,$$

then

$$n \cos (\omega T) = - \frac{\pi}{2} \omega \frac{v}{g},$$

or

$$\cos (\omega T) = e^{- \frac{\pi}{2}\omega\frac{v}{g}}$$

and, for $\beta = \pi$,

$$T' = \frac{2}{\omega} \text{ arc cos } e^{-\frac{\pi}{2}\omega\frac{v}{g}} .$$

E x a m p l e

v const. = 50 m/s = 180 km/h

b	ω rad./sec.	$\cos(\omega T)$	ωT rad.	T sec ($\beta = 90^\circ$)	$T' = 2T$ sec. ($\beta = 180^\circ$)
13	0.224	0.148	1.422	6.35	12.70
15	0.177	0.244	1.324	7.50	15.00
19	0.135	0.3396	1.225	9.05	18.12

After the completion of this work, an American report* dealing with the problem of maneuverability reached the writer. It contains a statement that the mean time required for turning through 180 degrees, as calculated from numerous test flights with the JN4h airplane, was 15 seconds. During these tests the slight effect of the lateral moment of inertia and of the flying speed was likewise established.

*N.A.C.A. Technical Report No. 153, 1922, "Controllability and Maneuverability of Airplanes," by F. H. Norton and W. G. Brown.

American Maneuverability Test Results

(All the flights were made by the same pilot)

Airplane	Air speed at start	Time for 180°
	A. Banked turn	
JN4h	96 km/h	13.1 seconds
"	112 "	13.4 "
"	128 "	15.1 "
"	96 "	16.8 "
"	112 "	15.5 "
"	128 "	14.6 "
S.E.5	104-136 "	8.0 "
VE-7	112 "	10.5 "
"	128 "	8.5 "
"	144 "	8.5 "
	B. Wing over	
JN4h	112 km/h	10.0 seconds
"	112 "	10.1 "
VE-7	128 "	9.1 "
S.E.5	128 "	7.5 "
	C. Immelmann turn	
JN4h	112 km/h	7.8 seconds
VE-7	129 "	9.2 "

The great effect of span on maneuverability has long been qualitatively recognized by pilots and designers of combat airplanes. One needs merely to recall the unparalleled maneuverability of triplanes during the war. The present investigation was conducted simply for the purpose of gaining a quantitative estimate of the effect of varying the span.*

The rapidity of the rolling motion about the X axis is of still greater importance to combat single-seaters than to large airplanes, since the modern single-seater does not accomplish its quickest 180° turn in the form of an ordinary turn but in the form of a so-called "Immelmann turn," which is a combination of a half-roll and a half-loop.

It follows from this investigation that, of airplanes with different spans and otherwise similar characteristics, the one with the smallest span has the best maneuverability. Thus, for a given wing area, a monoplane is inferior to a biplane as regards maneuverability. This is probably the chief reason for the important position the biplane occupies in international military airplane construction. A revival of the triplane does not seem impossible.

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*Of course the balancing of the ailerons is likewise very important. Optimum balancing, that is, instantaneous deflection of the ailerons was assumed in our calculation. The turning speed in the vertical and horizontal planes is obviously reduced by heavy ailerons. This can also be checked by calculation, which however, would afford no other information save that ailerons must be as easy to operate as possible without being overbalanced.

Summary

Previous theoretical investigations of steady curvilinear flight did not afford a suitable criterion of "maneuverability," which is very important for judging combat, sport and stunt-flying airplanes.

The idea of rolling ability, i.e., of the speed of rotation of the airplane about its X axis in rectilinear flight at constant speed and for a constant, suddenly produced deflection of the ailerons, is introduced and tested under simplified assumptions for the air-force distribution over the span. This leads to the following conclusions:

The effect of the moment of inertia about the X axis is negligibly small, since the speed of rotation very quickly reaches a uniform value.

The speed of rotation is directly proportional to the flying speed and to the nondimensional coefficient c_{mr} of the rolling moment, and is inversely proportional to the span and to the quotient $\frac{d\,c_n}{d\alpha}$ of the profile. That is, the speed of rotation is reduced by a "good" aspect ratio.

If two airplanes are "similarly" enlarged, i.e., without changing the aspect ratio of the wing and ailerons, c_{mr} and $\frac{d\,c_n}{d\,\alpha}$ remain constant. At the same speed and the same aileron deflection, the speed of rotation is inversely proportional to the span.

If a rotation in the horizontal flight path is attributed to each inclination in the vertical plane, the time required to make a complete turn of 180° can be easily determined.

As an example, a calculation is carried out for three airplanes differing in span only. It is found that the airplane with the smallest span requires the shortest time to accomplish a complete turn of 180°. The time required for such a turn is given by the formula

$$T = \frac{2}{\omega} \text{ arc cos } e^{-\frac{\pi}{2}\omega\frac{v}{g}},$$

where

$$\omega = \frac{12 \ c_{mr} \ v}{b \frac{d \ c_n}{d \ \alpha}} ,$$

provided the angle of attack of the wings is below its critical value.

The writer wishes to thank Mr. T. Fujimoto for his help in making the calculations.

Translation by W. L. Koporinde, Paris Office,
National Advisory Committee
for Aeronautics.

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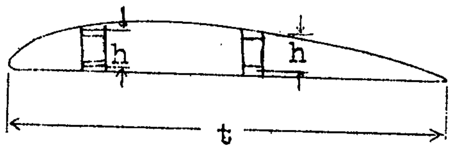


Fig.1

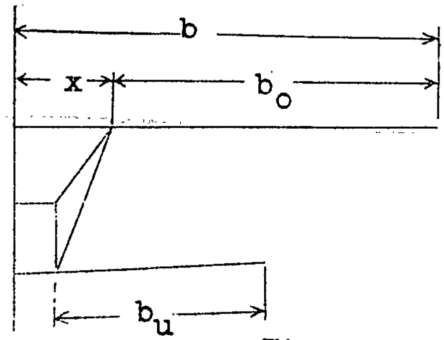


Fig.2

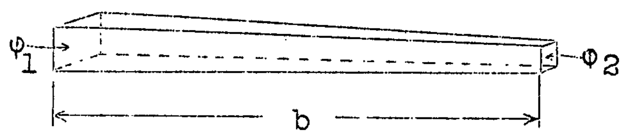


Fig.3

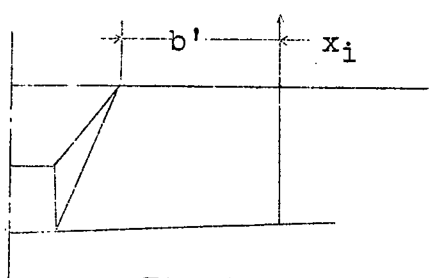


Fig.4

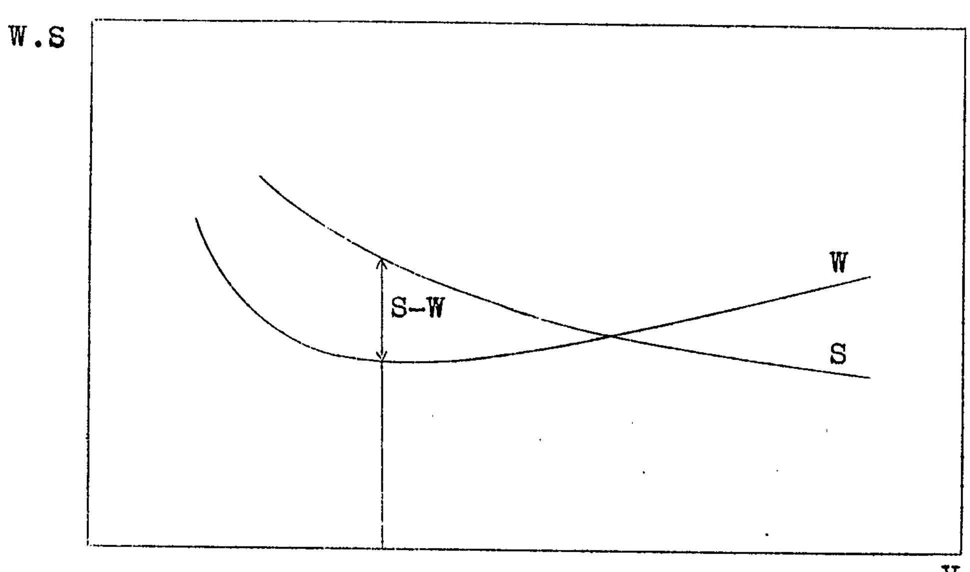
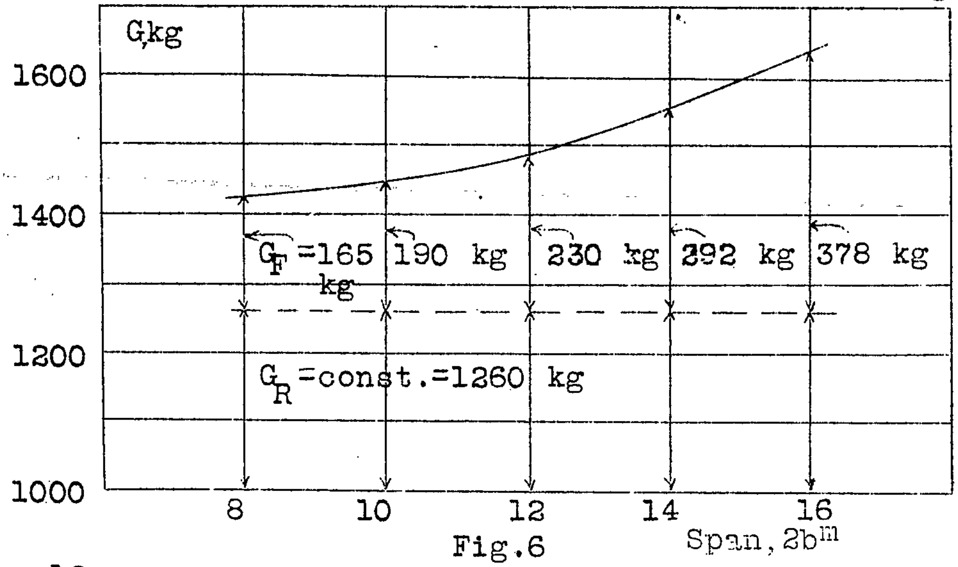


Fig.5



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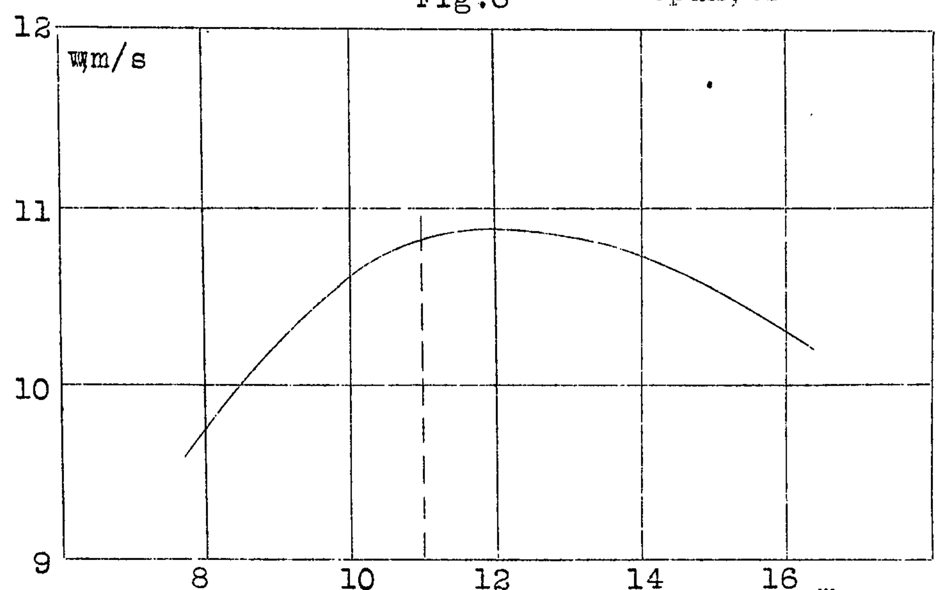


Fig.7

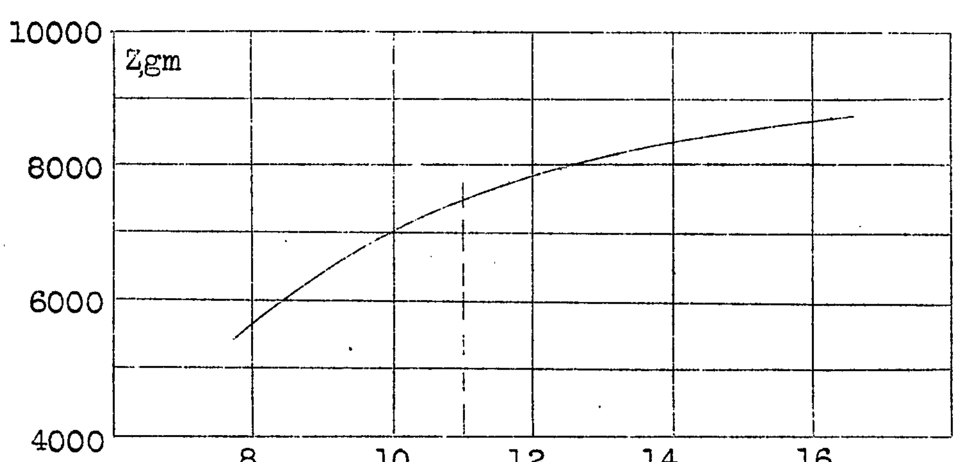
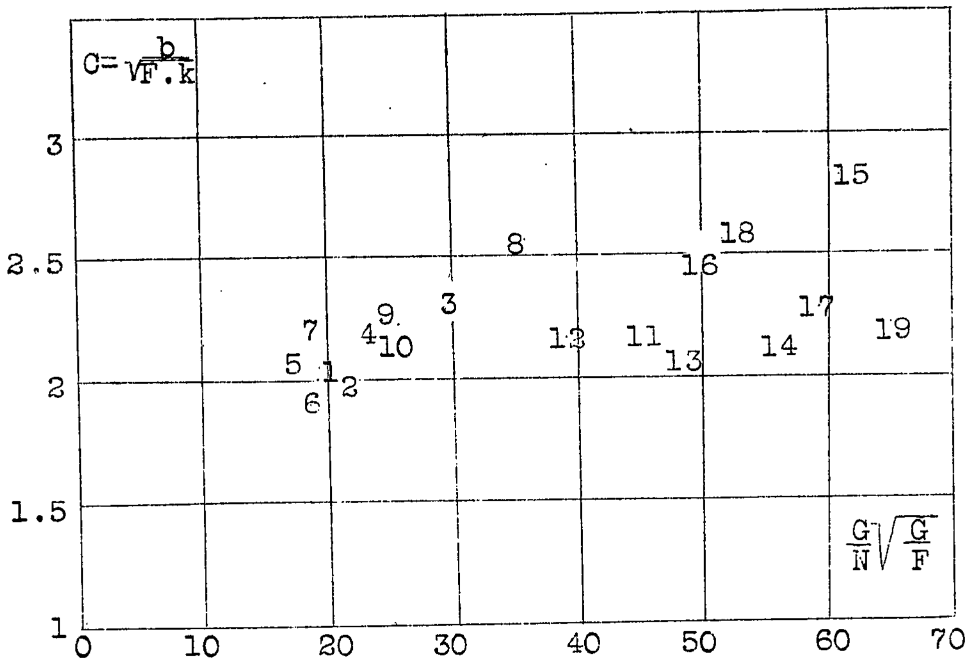


Fig.8



No.	Make	Airplane type
1	Curtiss Hawk P_1a	Pursuit
2	Spad 51	"
3	Fokker D XIII	"
4.	Nieuport-Delage 42 C I	"
5	Armstrong Siskin V	"
6	Gloster Gamecock	"
7	Bristol Fighter	"
8	Fokker C V E	Observation
9	Bréguet XIX	"
10	Potez 25 A 2	"
11	Albatros L 68	Sport
12	De Havilland "Moth"	"
13	Raab-Katzenstein "Schwalbe"	"
14	Udet U.12	"
15	Bellanca (monoplane)	Commercial
16	Dornier Mercury(monoplane)	"
17	" Wal "	"
18	" Ryan "	"
19	Alabatros L73 (Biplane)	"

Fig.9

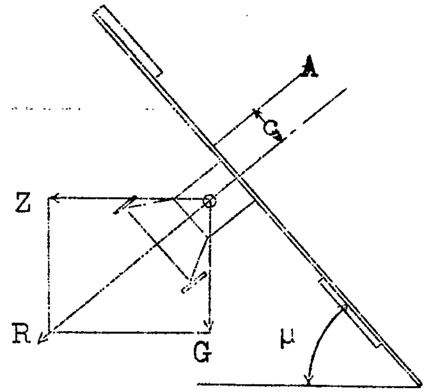


Fig.10

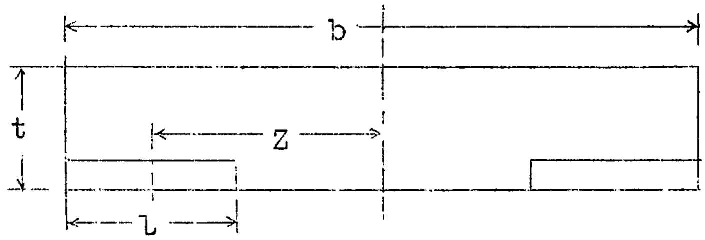


Fig.11

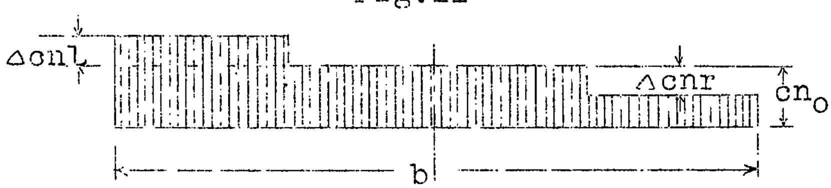
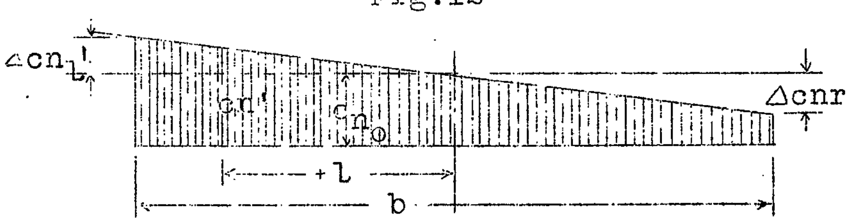


Fig.12



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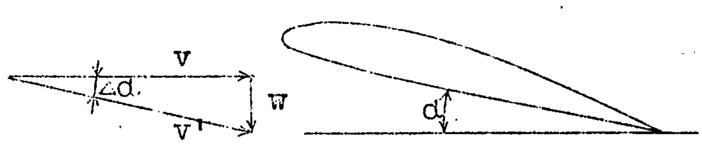

Fig.14

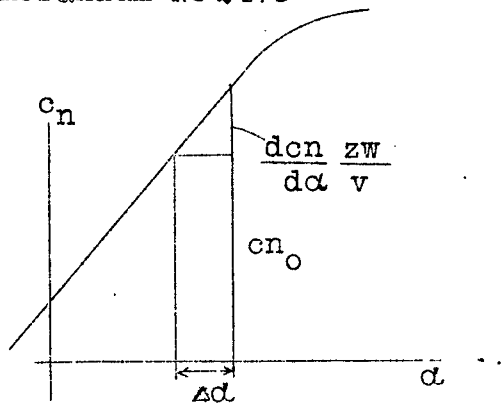

Fig.15

b=13

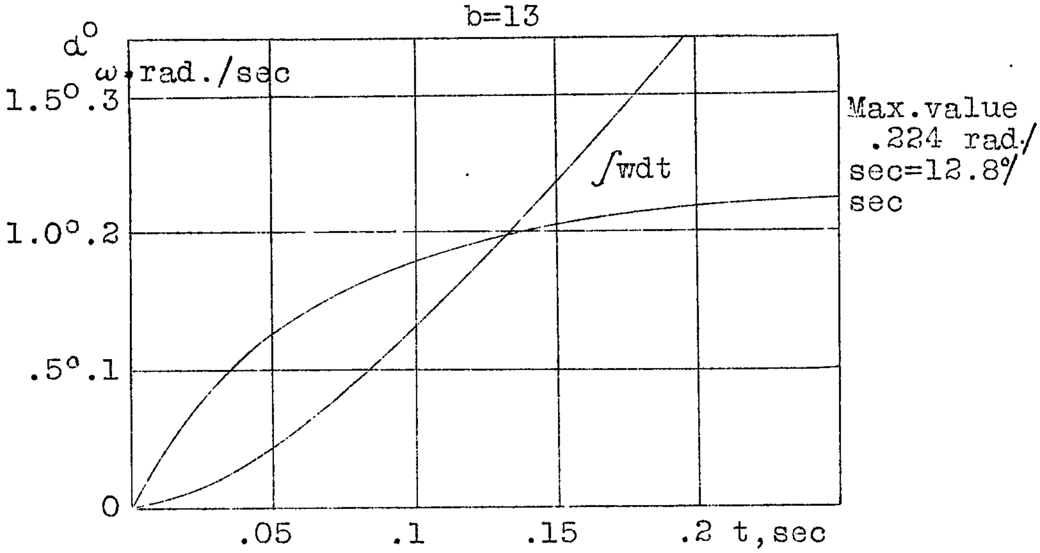

Fig.16

b=15

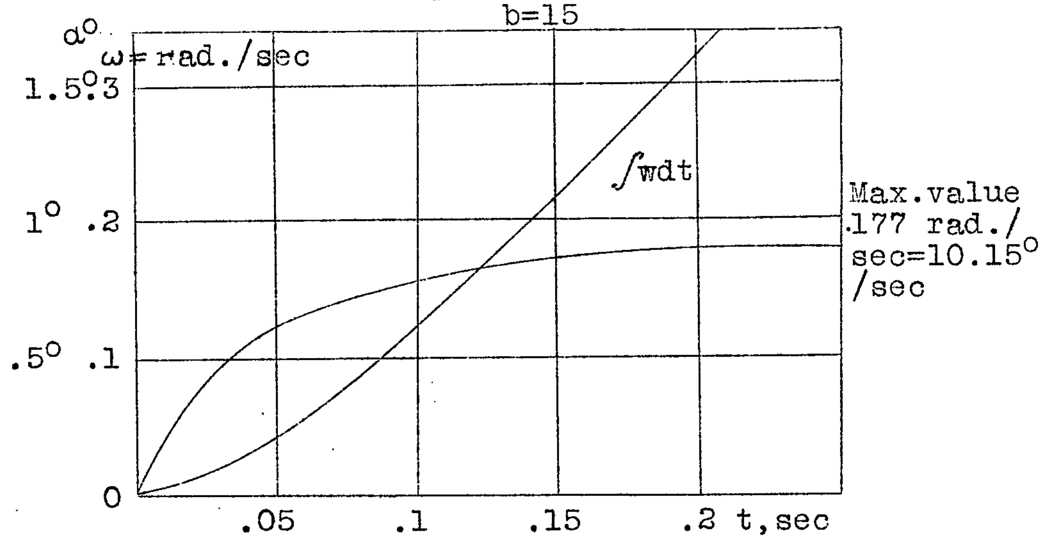

Fig.17

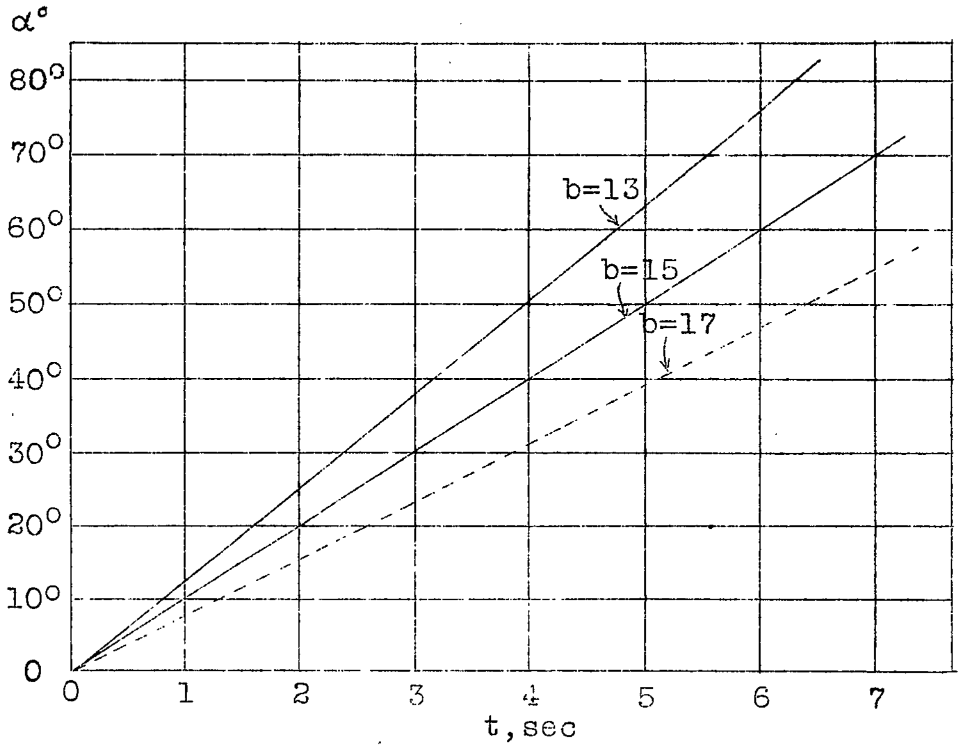

Fig.18

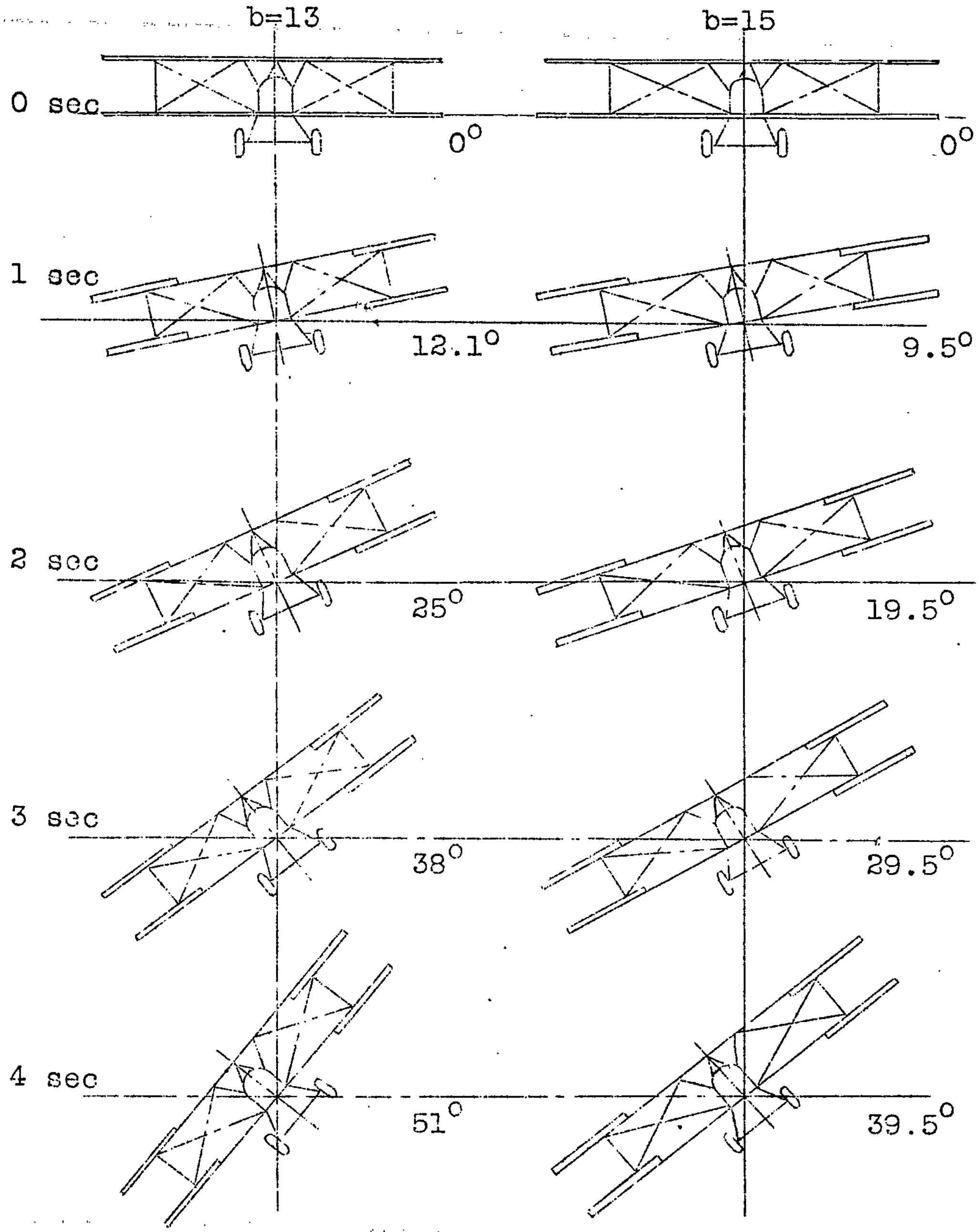
b=13
b=15
0 sec
0°
0°
1 sec
12.1°
9.5°
2 sec
25°
19.5°
3 sec
38°
29.5°
4 sec
51°
39.5°

Fig.19

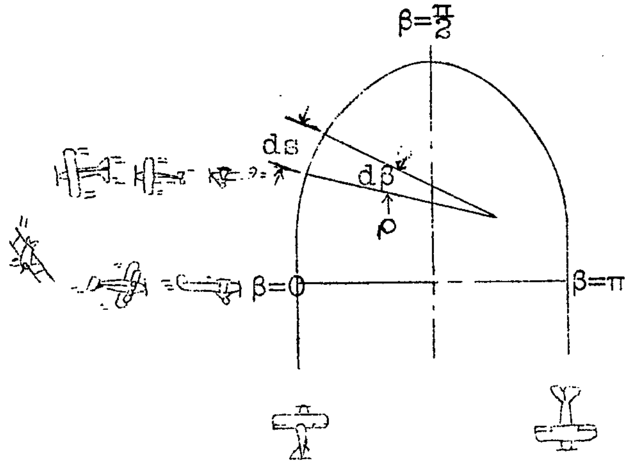

Fig.20

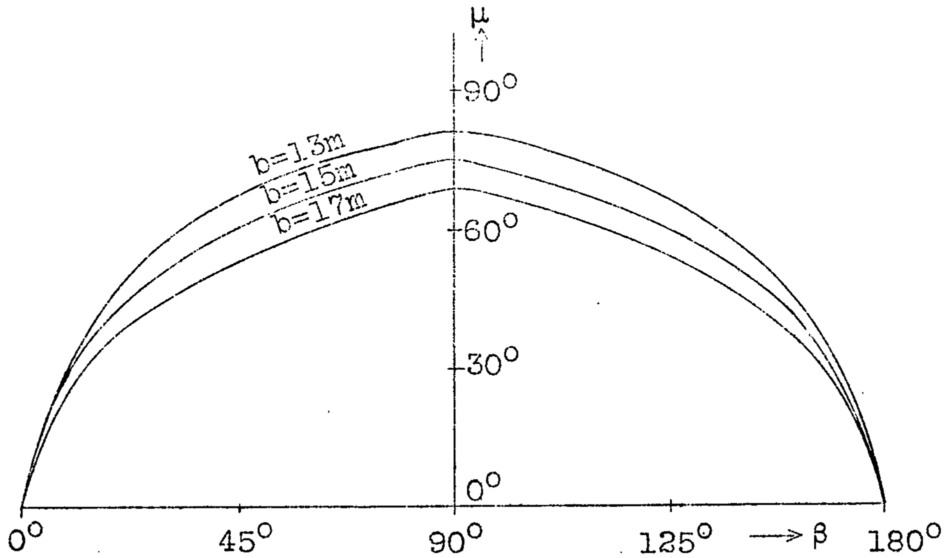

Fig.21