TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 475

RECENT RESEARCHES ON THE AIR RESISTANCE OF SPHERES

By O. Flachsbart

From Physikalische Zeitschrift
Volume 28, 1927

Washington
August, 1928

TECHNICAL MEMORANDUM NO. 475.

RECENT RESEARCHES ON THE AIR RESISTANCE OF SPHERES.*

By O. Flachsbart.

The laws of mechanical similarity teach us that in two flows of an incompressible viscous fluid, along the course of which the relation between inertia and frictional forces, rather than gravity, plays the decisive role (geometrical similarity of the models employed being assumed), dynamic similarity exists only when in both cases the ratio of the inertia to the frictional force is the same, that is, when the Reynolds Number R.N. has the same value.** If the results of two investigations, for which the above condition is satisfied, are set down in a non-dimensional form, that is, for the measurement of a resistance, in the well-known expression:

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 S}$$

(where D = resistance or drag, ρ = fluid density, V = velocity of the fluid relative to the model, S = area of principal plane of model), then equal values of C_D must be expected for equal values of R.N.

*"Neue Untersuchungen über den Luftwiderstand von Kugeln," from Physikalische Zeitschrift, Volume 28, 1927, pp. 461-469.

**R.N. = Vl/ν; V = velocity of fluid relative to the model; l = characteristic linear dimension of the model, for spheres, the diameter D; ν = kinetic viscosity of the fluid = μ/ρ = $\frac{\text{coefficient of viscosity}}{\text{density}}$.

These fundamental principles of the mechanics of fluids have been confirmed by extensive experimental data. All the more striking, therefore, is the fact that one of the earliest objects of hydrodynamic research, the sphere, does not seem to obey the above laws. In fact, if we compare the available results of experiments with spheres, some of the more recent of which are grouped in Figure 1,* we find that they do not agree at all. The cause of this great discrepancy can only be that the stipulated geometric similarity of the models is not fulfilled, or the possibility that Reynolds' law of similitude does not embrace all the factors which affect the flow. Since, however, every perfect sphere is similar to every other, dissimilarity between sphere models can only be due to inaccuracies in the process of manufacture, which fact does not explain, however, the great variation in the results. It must be concluded therefore (at least in the case of the sphere) that Reynolds' law is a necessary condition for the dynamic similarity of the flow phenomena but not an adequate one. It is accordingly assumed that the geometric similarity applies only to the model itself and not to the model suspension nor to the turbulent struc-

*The individual curves have been taken from the following sources, the diameter D of each sphere investigated being given in a parenthesis:
Eiffel, Reference 1,** (D = 24.4 cm);
N.P.L. = Nat. Phys. Lab., B.A.C. Reports & Memoranda No. 190, (D not given); Reference 9;
Göttingen 1923, Reference 4, (D = 28.25 cm);
H. Krey, Reference 8, (D = 20 cm) slender towing rod;
Weiselsberger - Japan 1925, Reference 7 (D = 20 cm);
The three American measurements were taken from Reference 9, (D = 20 cm).

**For all References, see p. 15.

ture of the air stream. The researches herein reported on the cause of the discrepancies in the results of experiments with spheres clarify all these questions.

All the resistance curves of Figure 1, in spite of their differences, exhibit some typical points in common; first of all, large values of C_D; then, with increasing values of R.N., a more or less sudden drop to small values of C_D. Analogous to the critical Reynolds Number of a flow through tubes and over plates, there appears accordingly a corresponding region of critical value, below which the lower part of the C_D curve is clearly separated from the upper part. This typical behavior of the resistance of spheres, discovered by Eiffel, who was thereby led to infer that the value of C_D was considerably smaller at high air speed, was explained by Prandtl in 1914 (Reference 2). To the transition from the subcritical to the supercritical phase,* there corresponds a transition from the original laminar flow into a turbulent flow, retarded near the body in the so-called "boundary layer," and a traveling of the line of flow separation from a parallel of latitude a few degrees ahead of the equator to a parallel behind the equator.** This shifting of the separation line behind the equator necessitates a reduction of the dead region behind the sphere and a consequent reduction

*This use of the prefixes "sub" and "super" refers to position and not degree.
**The separation occurs not precisely on a circle of latitude, but deviates in a zigzag manner along a mean circle of separation.

of the resistance of the sphere (Fig. 2).

On the causal relation between separation-travel and variation in the flow structure in the boundary layer, it may be said that the outer flow drags the boundary layer along by virtue of the fluid friction due to the pressure increase. The greater the towing effect, the greater the deviation. Evidently this towing effect is smallest, when the boundary layer and outer flow are in a laminar state, and increases when the flow becomes turbulent. These extremes include all possible individual cases, particularly those in which the whole flow is affected by local disturbances produced by the suspension apparatus either in the outer layer or in the boundary layer, or in both, as in experiments with models. It is therefore necessary to investigate first the effect of disturbances in the boundary layer and then in the outer air stream. Such an analysis is also fundamental for the following experiments with spheres, which are intended to show the effect of the supports on the measured resistance of the model and also for comparison with the old Eiffel experiments (Reference 1) and the more recent ones by Bacon and Reid (Reference 9).

In the first place we need to devise a system of measurements which will be as free from objections as possible. The object investigated was a smoothly polished sphere of linden wood, 24.2 cm (9.53 in.) outside diameter and having walls about 5 cm (2 in.) thick. The experiments were performed in the small wind tunnel of the

Göttingen Aerodynamic Institute, the free diameter of which is 1.2 m (3.94 ft.), and in which a maximum air speed of about 30 m/s (98.4 ft./sec.) can be obtained. The sphere was fastened to the front end of an iron rod 2 cm (.7874 in.) in diameter and about 30 cm (11.811 in.) long, to which all the stay wires were attached, so that the sphere itself, aside from the iron rod in the dead space, presented a perfectly smooth surface. The supports therefore caused no disturbance in front of the sphere (Fig. 3). The resistance curve of the sphere plotted in Figure 3 was determined and checked several times in the series of experiments with this suspension apparatus, the resistance of which was found by detaching the sphere from the rod and holding it in place as a dummy by means of a separate suspension. From Figure 1, to which the above curve was transferred, we see, by comparison with the former measurements, that it has a smaller subcritical value of C_D, and that, together with new experiments by Wieselsberger in Japan, to which alone the new Göttingen experiments are closely related (and to an American result obtained by towing a sphere with an airplane, which is probably not very reliable), it exhibits the highest critical value of Reynolds Number thus far obtained, namely, R.N. 300,000.

These new Göttingen tests with spheres do not agree very well with those conducted at the same place several years ago (Fig. 3) and with which the comparison is made. The latter is much more significant for the author, since the conditions under which each measurement was made and, above all, the peculiarities of the model suspension are exactly known to him. As shown in Figure 3, a suspension was employed at that time, parts of which were attached in front of the sphere, to the rear, and also at points in the vicinity of the equator.

In spite of the fineness of all these parts, whose effect on the resistance of the sphere (aside from their own resistance, for which allowance is made in the evaluation) was assumed to be negligibly small, it can be conclusively stated that the difference in these results is due alone to this form of suspension. Further experiments, which have confirmed this conclusion, were therefore undertaken in this special case. They consisted in investigating the effect of the individual parts of the earlier suspensions on the sphere, supported so as not to disturb the air stream. These were supplemented by investigating the effect of screw eyes often employed in the suspension of models and of a small rod projecting laterally at different points on the sphere, so as to imitate the rod fastening of the model in an air stream, as well as when towed in a fluid. Local disturbances in the boundary layer and in the outer stream were also investigated. The effect of changes in the character of both flows on the resistance of the sphere was tested last.

The essential results are represented in Figures 4 to 7, from which the following conclusions are drawn.

1. Disturbances in front of the sphere and even single

fine wires greatly affect the critical Reynolds Number. They lower the $R.N._{crit}$.*

2. Disturbances around the sphere increased the drag of the sphere without materially affecting the value of $R.N._{crit}$ (Fig. 5). Two fine wire ends attached at or in front of the equator, increased the C_D nearly 100 per cent in the supercritical region. A single screw eye, correspondingly attached, likewise brought about an increase of nearly 130 per cent. A screw eye at the front point of full dynamic pressure had no effect.** Of finer local disturbances, only those in front of the sphere affected the $R.N._{crit}$, as shown very clearly by Figure 6.*** The round rod noticeably changes the value of $R.N._{crit}$ first in its position d at the forward point of full dynamic pressure, since, in this position, the end of the rod facing the flow produces some turbulence in the latter, which soon comes in contact with the sphere.

*Along with the effect of the 0.5 mm (.02 in.) wire b with screw eye (Fig. 4), the effect of this wire was also investigated without the screw eye, and later also the effect of a single wire 0.3 mm (.12 in.) in diameter, the results differing but little from one another.

**Corresponding results were obtained by Bacon and Reid (Ref. 9) with a similar arrangement. Compare also Eiffel (Ref. 1, p.90, footnote 1).

***The curve plotted for comparison in the lower portion of Fig. 6 was taken from Ref. 9. It was obtained by rotating the sphere in a horizontal plane about the vertical axis of the balance fastening. Since we do not know for which supercritical R.N. it was obtained, we have plotted it in both the lower diagrams of Fig. 6.

3. Great disturbances of the boundary layer of the sphere likewise change $R.N._{crit}$. A fine wire ring set in front of the equator lowers the value of $R.N._{crit}$ so much that it cannot generally be obtained experimentally.* If the air stream is, however, made very turbulent (as by passing it through screens), this condition is changed immediately. A critical R.N. is obtained of the approximate magnitude produced by an equally turbulent stream without a wire ring. The principal effect of the wire ring is then only to increase the drag.

4. Turbulence of the approaching air stream (produced by a wire screen , Fig. 7), as already learned from earlier experiments, lowers the critical R.N. and, under similar circumstances, as determined in the confirmation of the former Göttingen experiments (Reference 5) and of the more recent American investigation (Reference 9) also lowers the supercritical C_D.** The supercritical C_D depends on the fineness of the screen mesh and also on the pressure drop caused by the network, for

*This experiment is known as the classic experiment (experimentum crucis) of Prandtl and Wieselsberger, explaining the peculiarity of the C_D curve (a change of boundary layer from the laminar to the turbulent condition). The wire ring makes the boundary layer artificially turbulent and thus produces a premature reduction of the sphere drag. Instead of the former ring of 1 mm wire, this time a ring of only 0.2 mm (.008 in.) wire was used.

**The separation point continually recedes toward the rear center of dynamic pressure, that is, the mean-flow diagram gradually approaches that of a pure potential flow. The air, by virtue of the great apparent viscosity produced by the strong turbulence behaves like one of the fluids of great viscosity, for which it is known that the flow phenomena in them furnish a close similarity to the potential flow. (See Hele-Shaw's experiments on the "Demonstration of Potential Flow," Trans. Inst. Nav. Arch. 1898, Vol. II.)

the explanation of which further special research work is required. It is nevertheless interesting to learn from the present experimental results that, with the simple method of turbulence production in the approaching stream by means of a screen, the separation point of the air flow recedes far toward the rear center of dynamic pressure and in the super-critical range, the C_D of the sphere is reduced to a value which has hitherto been obtained only by removing the boundary layer by suction (Reference 10).

The explanation of the observations made, in so far as they concern the structural changes in the outer and boundary layers has already been given in connection with the mechanism of the flow about a sphere and the reciprocal interference of the boundary and outer layers. No adequate explanation of the striking effects of small local disturbances, in front of and on the sphere can yet be given. The effect of a fine wire in front of the sphere is partially comprehensible by analogy, since it is known that the turbulence behind a cylinder is noticeably stronger and more unstable in the subcritical condition, than behind a cylinder in the supercritical condition. Such indeed, is the effect of the fine wire (Compare pressure distribution behind a cylinder in Reference 4, p.73 ff.). The fact, however, that a screw eye, or two fine wires on the sphere, which by virtue of their own simple drag would increase the value of C_D by about 0.5 per cent, increase it instead by nearly 130 and 100 per cent, respectively, is for the time being not explainable, because the nature of the boundary-layer flow is not sufficiently understood. For this reason a knowledge of the experimental facts is all the more essential.

Among the deductions from the experimental results, the explanation of the deviation of the results of the Göttingen experiments with spheres in 1923 from the present ones is naturally of the first importance. The old suspension, in spite of its fineness, was not perfect. The wires in front of the model must have caused the deviation in the $R.N._{crit}$, while the wires on the model must have caused the deviation in the C_D. However, it can be further inferred that the, in part, extraordinary deviations of the remaining results of the experiments with spheres (with one restriction yet to be discussed relative to the $R.N._{crit}$) are attributable to differences in the suspensions.* In experimenting with spheres and, theoretically, with all bodies, which do not present well-defined separation points due to sharp edges, it is necessary to provide model mountings which minimize the disturbances by means of very fine suspension members ahead of and on the models (excepting at the rear center of dynamic pressure, where a larger support is necessary and where

*An indirect proof of this is, to a certain degree, furnished by Wieselsberger's Japanese experiments (Fig. 1), in which the sphere was mounted, like ours, on a rod attached to its rear.

its presence is not harmful.* By observing this precaution, we may expect results from the most differently equipped laboratories, which will agree as regards their typical character and their values of C_D. By virtue of the different degree of turbulence in the different wind tunnels, deviations in $R.N._{crit}$, however, remain unavoidable. Reynolds' law of similitude covers only the laminar and not the actual viscosity of the turbulent air stream, which, as here experimentally verified, decidedly affects the $R.N._{crit}$. Reynolds' law for turbulent flow (theoretical in every case, but practical only in the case of flow around bodies without sharp lines of separation) is not, therefore, a sufficient condition of similarity. Obviously, the law must fail in such cases, as long as there is no possibility of a quantitative determination of the degree of turbulence. However, since, aside from the effect of the supports, the turbulence of the air stream is the only thing that causes the results to differ from one another, a previous discovery by Prandtl (Refere-

*This renders it necessary in the case of models towed in water, for example, to avoid the use of large towing rods which would obstruct the flow in the plane of the main cross section of the sphere normal to the stream. Krey's measurements (Fig. 1) were conducted in this manner. The striking feature of these experiments, namely, that the C_D drops in two stages, is in fact traceable to the possibility that the rod itself may pass through a critical value of R.N. In the case of models supported on balances according to Eiffel's method, or according to that of the N.P.L., in which a horizontal rod fastened to the back of a sphere and a vertical rod, normal to the air stream, joins the horizontal one to the balance, there had to be tested in each case as to how far ahead the flow around the sphere was affected by the vertical arm. Bacon and Reid employed such a support for their sphere tests (Reference 9).

ence 3) may now be taken up with a prospect of success, namely, that of employing the air-resistance data of spheres and, in particular, the critical R.N. as a turbulence characteristic of wind tunnels.*

The single meaning of the above characteristics assumes, however, that the stipulation of geometric similarity at the models also includes the similarity of their surface roughness. In practice, it may be considered sufficient to require the models to be as smooth as possible. Lastly, in order to guarantee the full utility of such data concerning wire resistance, on the certainty of which the sphere resistance is quite dependent, an agreement must be reached regarding the auxiliary fastening of the sphere. In Göttingen this was accomplished in the following manner. Three small screw eyes were distributed on a circle of latitude close behind the equator, from which three stay wires extended obliquely forward, and one wire extended obliquely downward toward the rear. The diameter of the wires was 0.3 mm (.12 in.). It may be assumed that in this manner, the measurement of the wire drag is very little or not at all impaired.**

*From sphere tests made in the N.A.C.A. variable density tunnel one can conclude, on the basis of their low $R.N._{crit}$ and of their small supercritical value of C_D, that the air stream of this tunnel is very turbulent. This conclusion is confirmed by the statements of Bacon and Reid (Reference 9).

**From a conversation of the writer with Dr. Wieselsberger, he gathered that, in the new experiments of 1925, the sphere used as a dummy was held in the working section by means of a horizontal rod. The deviation of his result from ours is perhaps due to this variation in the method of determining the wire drag, in so far as it is not ascribable to any other disturbance of the air stream.

In estimating accuracy of the measurements, the following information may be of service. The suspension drag in the subcritical region amounted to about 15 per cent; in the supercritical region, to nearly 50 per cent of the measured total drag. The latter (if any impairment due to auxiliary wires is disregarded) could be measured in the supercritical region to ±0.5 per cent. The wire drag in this region is measurable to ±1 per cent. This means a maximum error of about ±3 per cent for the supercritical C_D. The accuracy in the subcritical region is less, due to the small and fluctuating forces involved. For the evaluation of the error in the wire drag due to the auxiliary support of the sphere, for which an actual determination is lacking, it may be stated that an error of ±10 per cent in the wire drag would mean a possible error of about ±8 per cent in the supercritical C_D.

The results obtained by weighing were checked by pressure-distribution measurements in both the supercritical and subcritical regions (Fig. 8).* The graphic integration of this pressure distribution gives the values

$$\text{Lower critical} \quad C_D{}^1 = 0.460$$

$$\text{Upper critical} \quad C_D{}^1 = 0.082$$

whereby the course of the pressure curve in the vicinity of the rear center of dynamic pressure was interpolated as shown on

*The independent results, likewise obtained on two great circles of a sphere normal to each other, are lower than the corresponding Eiffel values for the supercritical region (Reference 1, p.92).

Figure 8. Here it is a matter of pure pressure resistance. Comparing the upper critical values (because of the greater uncertainty of the subcritical C_D values obtained by weighing) and taking into consideration the fact that the frictional resistance of the sphere in this region amounts to only a few per cent of the pressure resistance, we find a satisfactory agreement between the force and pressure measurements. It can be assumed therefore, that the error limits of the force measurements, (disregarding the subcritical region) do not exceed ±5 per cent.

Translation by
National Advisory Committee
for Aeronautics.

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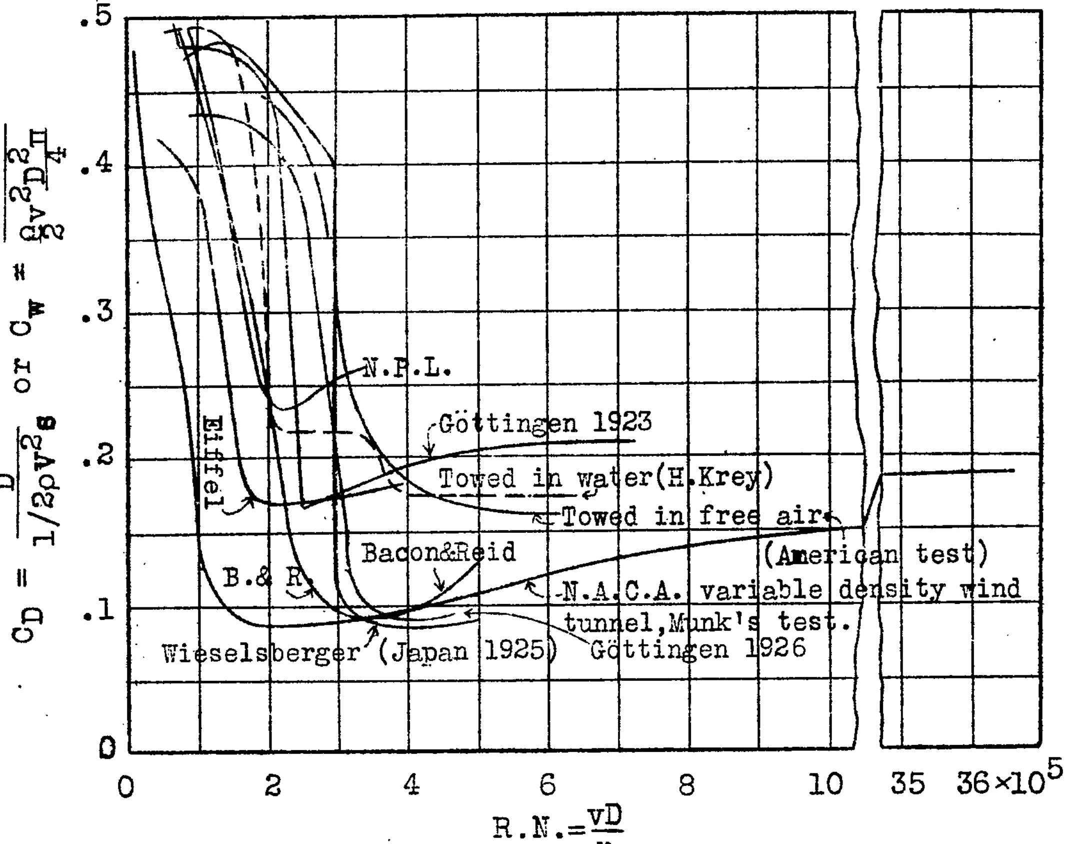
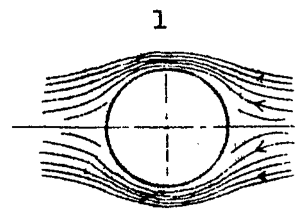


Fig.1 Various tests with spheres.



1. In a non-viscous fluid (Potential flow)
2. In a viscous fluid:
   (a) Subcritical.
   (b) Supercritical.

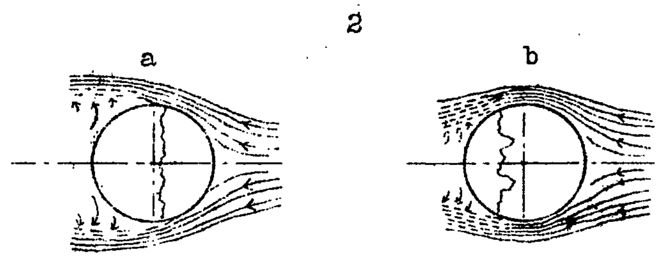


Fig.2 Flow about a sphere.

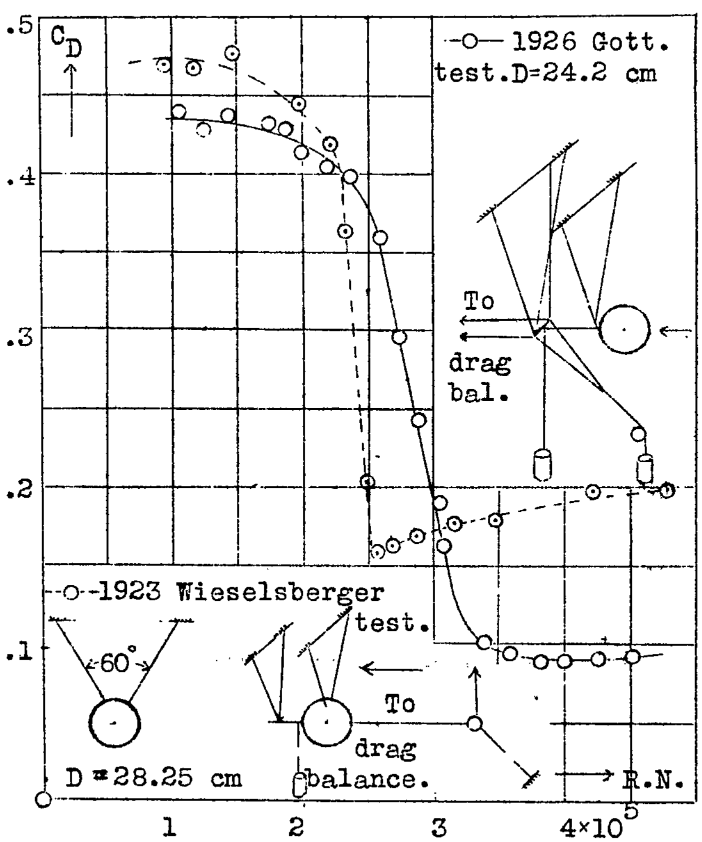


Fig.3

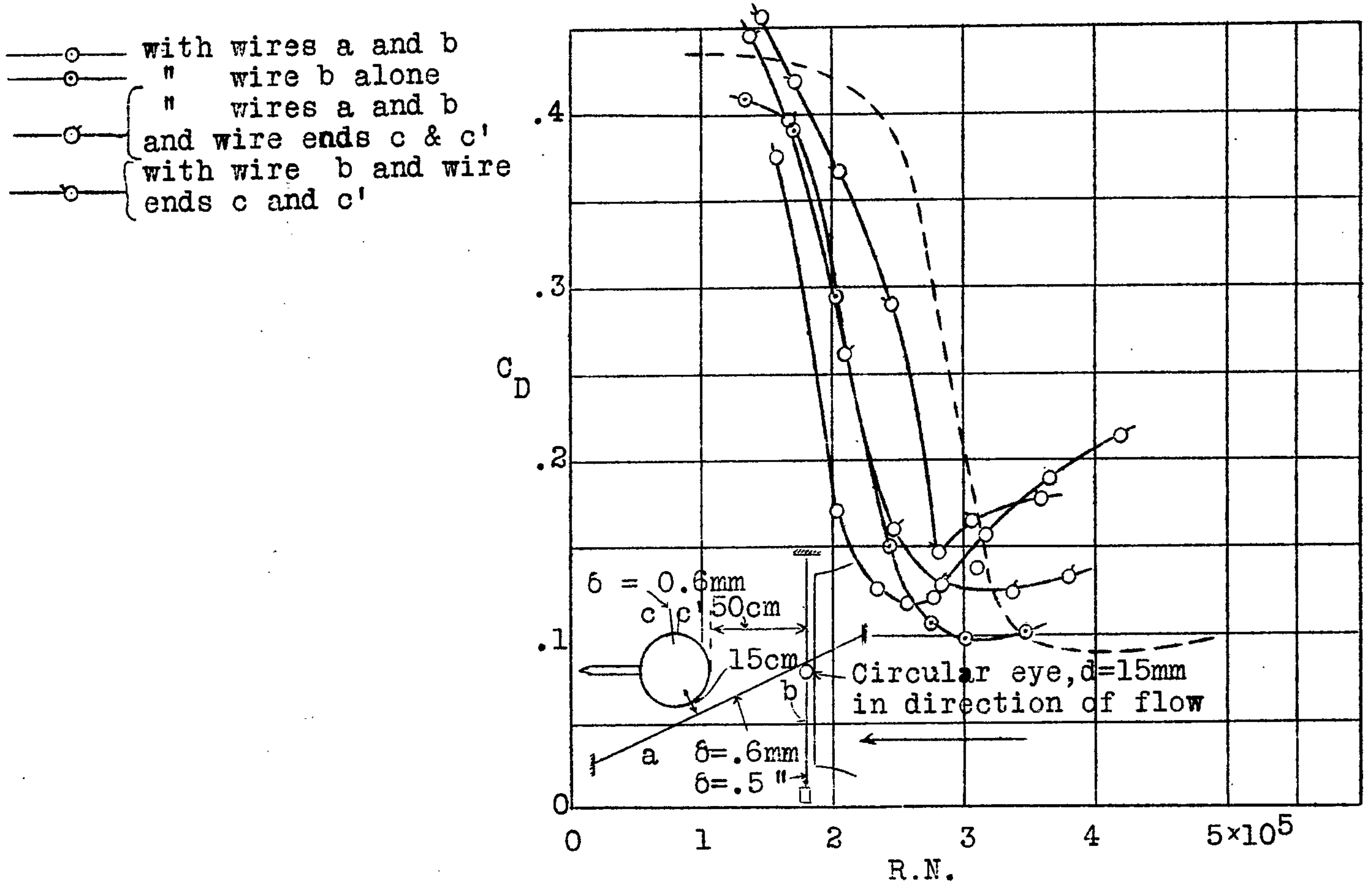
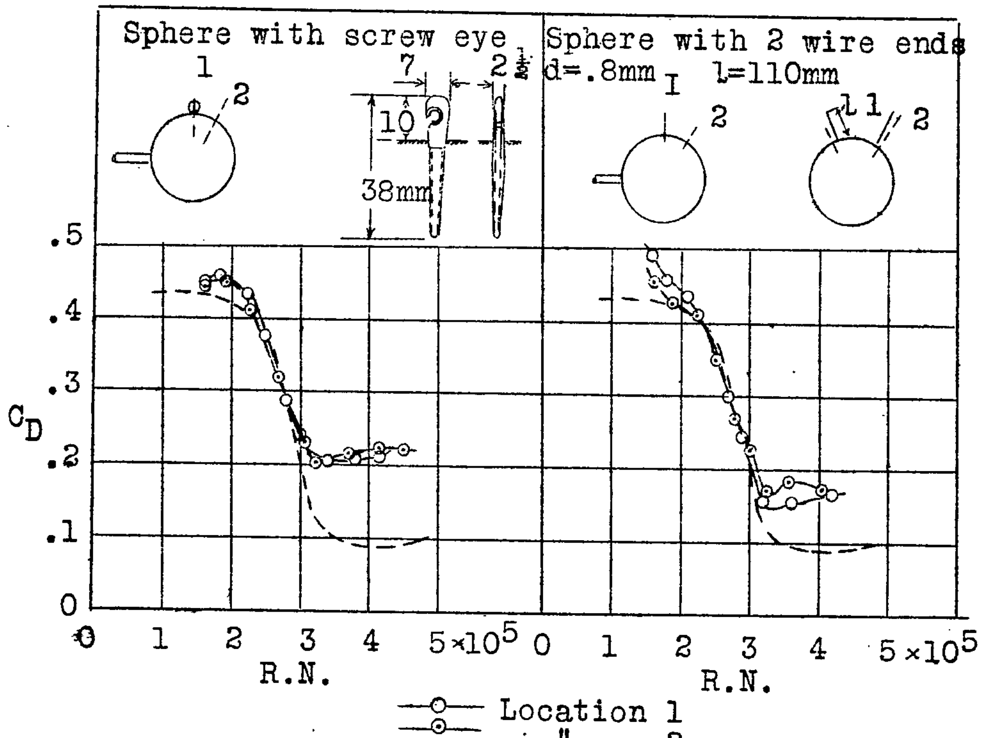


Fig.4 Effect of wires, in front of and on sphere, on drag of sphere.



Sphere with screw eye
1
2
7
2
10
38mm
Sphere with 2 wire ends
d=.8mm
l=110mm
I
2
1 1
2
C_D
.5
.4
.3
.2
.1
0
0 1 2 3 4 5×10^5
R.N.
0 1 2 3 4 5×10^5
R.N.
Location 1
" 2

Fig.5

e, 45°
f, 90°
g, 135°
h, 180°
A, American test

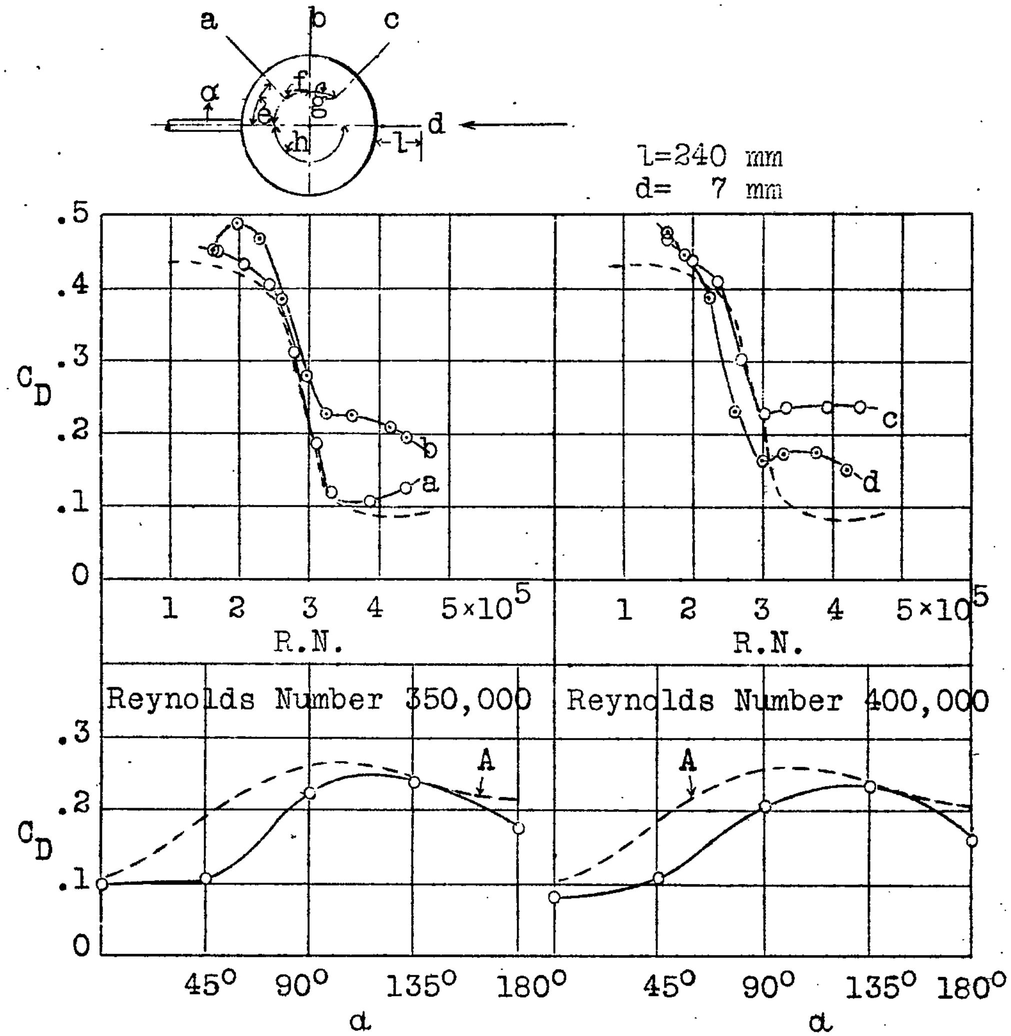


Fig.6 Sphere with round iron rod in different positions.

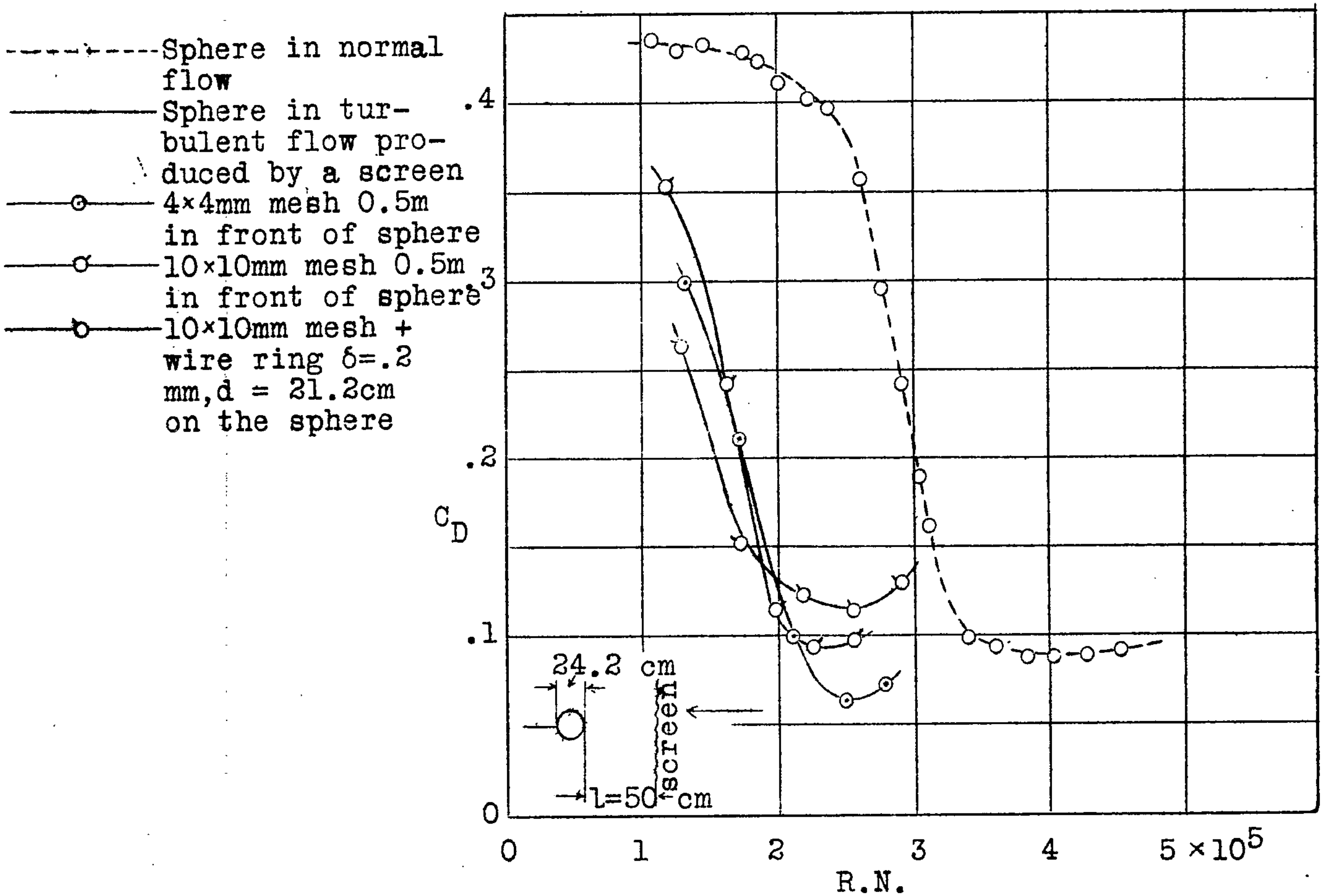
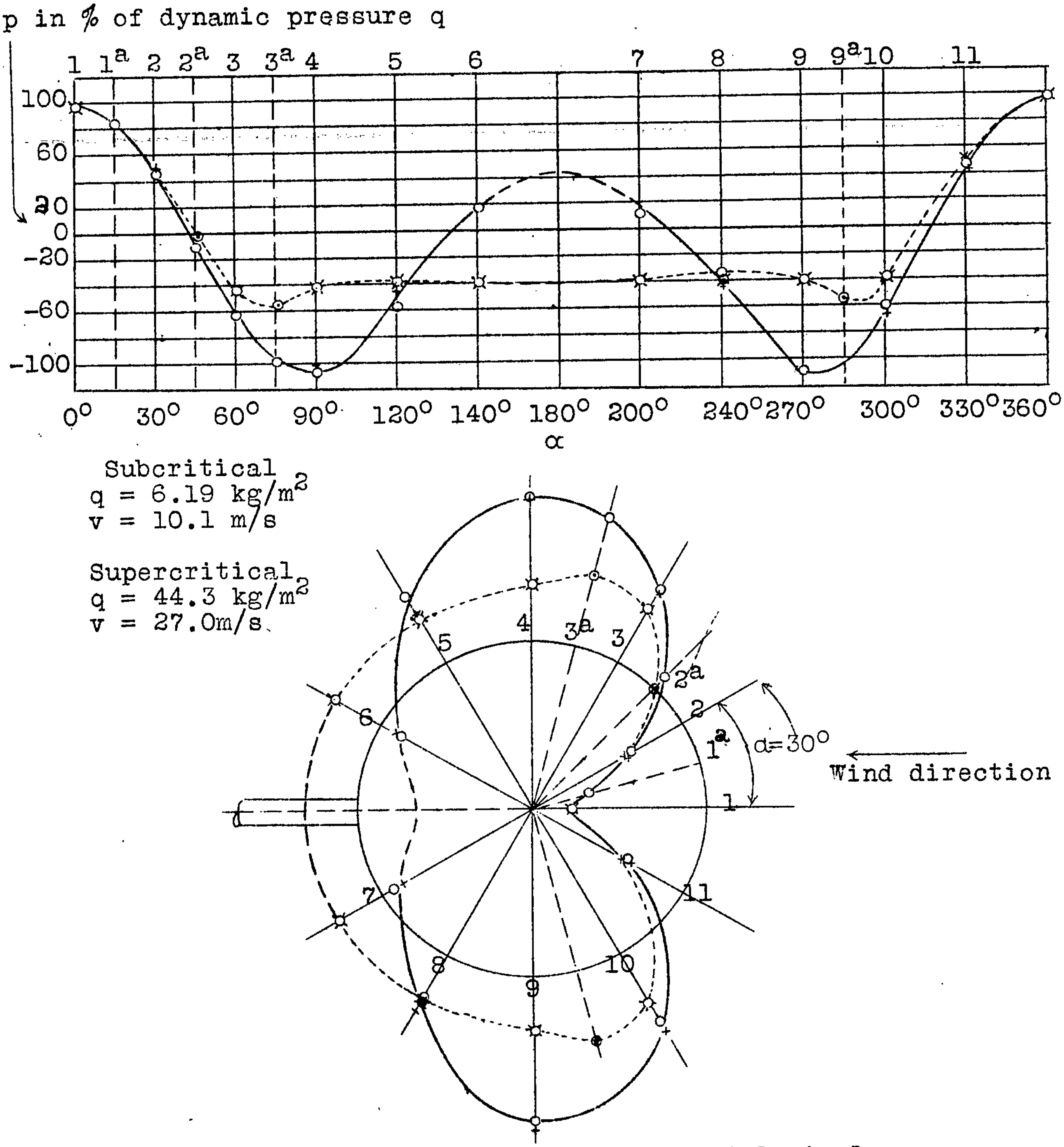

Sphere in normal flow
Sphere in turbulent flow produced by a screen
4×4mm mesh 0.5m in front of sphere
10×10mm mesh 0.5m in front of sphere
10×10mm mesh + wire ring δ=.2 mm,d = 21.2cm on the sphere
.4
.3
.2
.1
0
C_D
24.2 cm
screen
l=50 cm
0
1
2
3
4
5 × 10^5
R.N.

Fig.7



Subcritical { measurements on equatorial circle; " " meridian "
Supercritical { " " equatorial "; " " meridian "

Fig.8 Pressure distribution on a wood sphere. D = 24.2 cm