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AERODYNAMIC CHARACTERISTICS OF THIN EMPIRICAL PROFILES AND THEIR APPLICATION TO THE TAIL SURFACES AND AILERONS OF AIRPLANES

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AERODYNAMIC CHARACTERISTICS OF THIN EMPIRICAL PROFILES AND THEIR APPLICATION TO THE TAIL SURFACES AND AILERONS OF AIRPLANES.*

By A. Toussaint and E. Carafoli.

The increasing use of airplane wings equipped with ailerons and the importance of knowing the aerodynamic characteristics of tail units (empennages) provided with movable parts (rudders and elevators) impart some interest to the so-called "empirical profiles." This term is applied to wing profiles which are not drawn according to any theoretical method. Generally these profiles are of no particular interest, since they can always be replaced by theoretical profiles which satisfy the same aerodynamic and structural conditions. For the above-mentioned applications, however, the profiles, modified by the deflection of the aileron, elevator or rudder, necessarily become empirical profiles, of which it is important to know the aerodynamic characteristics and, above all, to know how these characteristics are affected by the magnitude of the deflections.

For this purpose we thought best to employ the method proposed by Munk for the approximate theoretical study of thin, slightly curved profiles, assimilable, from an aerodynamic view-

*From L'Aérophile, June, 1928, pp. 179-183.

point, to their mean camber line.

Principle of the Method

Given a profile whose mean camber line ASB is moderately curved and whose chord AB = l.

It is well known that, in the conformal transformation

$$Z = \zeta + \frac{a^2}{\zeta} \qquad (1)$$

a circle C, of radius $a = \frac{l}{4}$ and centered at O, gives a straight line AB of length $l = 4a$.

On adopting the above transformation function (1), the line ASB, very close to the chord AB, will have for its antecedent in the ζ plane a curve S' which will lie very close to the circle C, and which will cut this circle at the points A' and B', antecedents of A and B.

The curve S' may be defined by the expression

$$\zeta' = a\,(1 + r)\,e^{i\theta} \qquad (2)$$

in which r is the radial segment, as a function of θ, measuring the small distance between the circle C and the curve S' along the radius vector of "argument" θ.

The mean line ASB will be obtained by the transformation

$$Z = \zeta' + \frac{a^2}{\zeta'} \qquad (1')$$

which, on neglecting the terms in r^2, gives

$$Z = 2\ a \cos\theta + 2\ i\ ar \sin\theta .$$

$$\left.\begin{array}{l}\text{For the abscissa } X = 2\ a \cos\theta \\ \text{the ordinate will be } Y = 2\ a\ r \sin\theta\end{array}\right\} \qquad (3)$$

To transform the circle C into the curve S' so that a point $\zeta' = a\,(1 + r)\,e^{i\theta}$ of the curve S' will correspond to a point $\zeta = a\,e^{i\varphi}$ of the circle C, let

$$\varphi = \theta + \beta \qquad (4)$$

Under these conditions, the trailing edge B of the profile ASB will correspond to the point B', $(\theta = \pi)$ of the curve S' and to the point B'_o $(\varphi = \pi + \beta_o)$ of the circle C. According to Joukowski's hypothesis, the point B'_o must be a point of zero velocity. Hence, the circulation will have the value

$$\Gamma = 4\,\pi\,a\,V \sin\,(\alpha + \beta_o) \qquad (5)$$

The transformation function under the general form

$$\zeta' = \zeta + a\,(A_o + i\,B_o) + a^2\,\frac{(A' + i\,B')}{\zeta} + \dots$$
$$+ a\,\frac{A_n + i\,B_n}{\zeta_n} \qquad (6)$$

will enable us to pass from the circle C to the curve S'. On replacing ζ' and ζ by their respective values,

$$\zeta' = a\,(1 + r)\,e^{i\theta} \quad \text{and} \quad \zeta = a\,e^{i\varphi}$$

and, since β and r are very small,

$$r \cong \Sigma A_n \cos (n+1) \theta + \Sigma B_n \sin (n+1) \theta$$

$$\beta \cong \Sigma A_n \sin (n+1) \theta - \Sigma B_n \cos (n+1) \theta$$

Since the profile ASB has no thickness, the ordinate

$$y = 2\, ar \sin \theta$$

takes the same value for $\pm\theta$. Hence, r changes sign with θ and

$$A_0 = A_1 = A_3 = \ldots\ldots = A_n = 0.$$

This gives

$$\left.\begin{aligned} r &\cong \Sigma B_n \sin (n+1) \theta \\ \beta &\cong -\Sigma B_n \cos (n+1) \theta \end{aligned}\right\} \tag{7}$$

The coefficients B_n are therefore defined by the formula

$$B_n = \frac{1}{\pi} \int_0^{2\pi} r \sin (n+1)\, \theta\, d\, \theta \tag{8}$$

The aerodynamic characteristics of the profile ASB are then given by the following formulas:

1. Lift $P = \rho V \Gamma = 4 \pi a \rho V^2 \sin (\alpha + \beta)$

whence $C_z \cong 2\pi (\alpha + \beta_0)$. (9)

2. Moment $M_A = M_0 - P\, 2\, a \cos \alpha$.

Lét $M_A \cong 4 \pi \rho V^2 |X_1| (\alpha + \gamma) - 8 \pi a^2 \rho V^2 (\alpha + \beta_0)$

whence $C_{m(A)} \cong \frac{\pi}{2} \frac{|X_1|}{a^2} (\alpha + \gamma) - \pi (\alpha + \beta_0)$ (10)

$X_1 = C^2\, \iota^{2i\gamma}$ being the term in $\frac{1}{\zeta}$ of the transformation function (6). Thus, we have

$$X_1 = C^2 \; e^{2i\gamma} = a^2 \; (1 + i \; B_1) \tag{11}$$

The computation of C_z and $C_{m(A)}$ leads therefore to the calculation of β_o, $|X_1|$ and of γ.

Calculation of β_o

This angle corresponds to the point $B_o{}^3$, that is, to $\theta = \pi$. Under these conditions the formulas (7) give

$$\beta_o = - \Sigma B_n \cos (n + 1) \pi = B_o - B_1 + B_2 - B_3 + \ldots$$

Replacing B_o, B_1, B_2 ... etc., by their values derived from formula (8), we obtain

$$\beta_o = \frac{1}{\pi} \int_\theta^{2\pi} r \; (\sin \theta - \sin 2\theta + \sin 3\theta \ldots) \; d\theta$$

or

$$\beta_o = \text{p.im.} \; \frac{1}{\pi} \int_\theta^{2\pi} \frac{r \; e^{i\theta}}{1 + e^{i\theta}} \; d\theta$$

which reduces to

$$\beta_o = \frac{1}{\pi} \int_\theta^{2\pi} \frac{r \sin \theta}{21 + \cos \theta} \; d\theta = \frac{1}{2\pi a} \int_\theta^{\pi} y \; \frac{d\theta}{1 + \cos \theta} \tag{12}$$

In practice the profile abscissas and ordinates are referred to the axes $x' A y'$ and the relative coordinates

$$\begin{cases} \xi = \dfrac{x'}{4a} = \dfrac{1}{2} (1 - \cos \theta) \\ \eta = \dfrac{y'}{4a} = \dfrac{y}{4a} \end{cases}$$

are preferably considered.

Thus equation (12) gives for β_0 the value

$$\beta_0 = \frac{1}{\pi}\int_0^1 \eta \frac{d\,\xi}{(1-\xi)\sqrt{\xi\,(1-\xi)}} = \int_0^1 \eta\, f_1\,(\xi)\,d\,\xi \qquad (13)$$

by letting

$$f_1\,(\xi) = \frac{1}{\pi\,(1-\xi)\sqrt{\xi\,(1-\xi)}} \qquad (13')$$

When the mean line ASB of the profile is known, equation (13) can be integrated by employing computed values of $f_1\,(\xi)$.

Calculation of (X_1) and of γ

Since B_1 is small, formula (11) gives

$$C^2 = |X_1| = a^2\,(1 + B_1{}^2) \cong a^2$$

Similarly

$$\tan 2\,\gamma \cong 2\,\gamma = B_1$$

The value of B_1 is found by equation (8), which gives

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} r \sin 2\,\theta\, d\,\theta = \frac{2}{\pi a}\int_0^{\pi} y \cos\theta\, d\,\theta$$

Letting $\mu_0 = \frac{\pi}{4} B_1$ and employing the relative coordinates ξ and η, we obtain

$$\mu_0 = 2\int_0^1 \eta\,\frac{(1-2\,\xi)\,d\,\xi}{\sqrt{\xi\,(1-\xi)}} = 2\int_0^1 \eta\, f_2\,(\xi)\,d\,\xi \qquad (14)$$

with

$$f_2\,(\xi) = \frac{(1-2\,\xi)}{\sqrt{\xi\,(1-\xi)}} \qquad (14')$$

As before, the values of $f_2\ (\xi)$ can be calculated for different abscissas ξ, and the integration of equation (14) will be accomplished according to the values of η obtained on the mean camber line ASB. Expression (10) for $C_{m(A)}$ then becomes

$$C_{m(A)} = \frac{\pi}{2}\left(\alpha + \frac{B_1}{2}\right) - \pi\ (\alpha + \beta_0) = -\ 0.25\ C_z + C_{m_0} \qquad (10')$$

$$C_{m_0} = \frac{\pi}{4}\ B_1 - \frac{\pi}{2}\ \beta_0 \qquad (10'')$$

In short, the aerodynamic characteristics of empirical profiles, assimilable to their mean camber line, due to their relative thickness and curvature, can be calculated from the formulas

$$\begin{cases} C_z \cong 2\ \pi\ (\alpha + \beta_0) = 2\pi\left[\alpha + \int_0^1 \eta\ f_1\ (\xi)\ d\ \xi\right] & (9') \\ C_{m_A} \cong -\ 0.25\ C_z + C_{m_0} = -\ 0.25\ C_z & \\ \qquad + 2\int_0^1 \eta\ f_2\ (\xi)\ d\ \xi - 2\int_0^1 \eta\ f_1\ (\xi)\ d\ \xi & (10''') \end{cases}$$

for which the function $f_1\ (\xi)$ and $f_2\ (\xi)$ are given in the table below as functions of ξ.

ξ	0.025	0.05	0.10	0.20	0.30	0.40	0.50
$f_1\ (\xi)$	2.09	1.54	1.18	1.00	0.99	1.08	1.27
$f_2\ (\xi)$	6.10	4.13	2.67	1.50	0.87	0.41	0
ξ	0.60	0.70	0.80	0.90	0.95		
$f_1\ (\xi)$	1.62	2.31	3.98	10.6	29.2		
$f_2\ (\xi)$	-0.41	-0.87	-1.5	- 2.67	- 4.13		

Profiles Having Any Mean Camber Line

If the mean camber line of the profile cannot be represented by a simple relation between η and ξ, the integrations

$$\int_0^1 \eta \, f_1 \, (\xi) \, d\,\xi \quad \text{and} \quad \int_0^1 \eta \, f_2 \, (\xi) \, d\,\xi$$

can be made either graphically or mechanically, by making the products ηf_1 and ηf_2 according to the values of η derived from the mean camber line. In general, the value of ηf_1 in this calculation increases indefinitely toward the trailing edge (i.e., for $\xi = 1$). We can then evaluate the integral $_{o}\eta \, f_1 \, d\,\xi$, from $\xi = 0$ to $\xi = 0.95$, and estimate the portion of the integral from $\xi = 0.95$ to $\xi = 1.0$. Assuming that this portion of the mean camber line is rectilinear, we find that the additional contribution amounts to $2.9\ \eta'$, η' being the relative ordinate for $\xi = 0.95$ (according to Glauert).

Profiles with Particular Mean Camber Lines

When the form of the mean camber line can be expressed by a simple ratio between η and ξ, the values of β_o and μ_o can be calculated by direct integration. For example, the equation

$$\eta = b \, \xi \, (1 - \xi) \, (c - \xi)$$

represents a profile whose mean camber line is of double curvature when c varies between $\frac{1}{2}$ and 1. Direct integration gives

$$\beta_o = \frac{b}{8}(4c - 3)$$

and

$$\mu_o = \frac{\pi b}{32},$$

from which

$$C_{m_o} = \frac{\pi b}{32}(7 - 8c) \quad \text{is derived.}$$

For $c = \frac{7}{8}$ we get $C_{m_o} = 0$.

Application to Tail Units with Movable Parts

For a stabilizer having a symmetrical biconvex profile, the deflection of the elevator HB by a certain moderate angle β, forms a profile whose mean camber line is composed of two straight lines, AH and HB.

Let σ be the ratio of the movable part HB to the total surface area AH + HB, then:

$$\sigma = \frac{H'B}{A\ B}; \quad \xi_{(H)} \cong \frac{AH'}{A\ B} = 1 - \sigma \quad \text{and} \quad h = \frac{H'H}{A\ B} = \sigma(1 - \sigma)\beta$$

For ξ varying from 0 to $(1 - \sigma)$ we get

$$\eta = \frac{h}{1 - \sigma}\xi$$

and for ξ varying from $(1 - \sigma)$ to 1 we get

$$\eta = \frac{h}{\sigma}(1 - \xi).$$

Under these conditions the values of β_o and μ_o are calculable by direct integration of formulas (13) and (14).

Thus we get

$$\beta_0 = \frac{1}{\pi}\int_0^{1-\sigma} \frac{h}{1-\sigma}\,\xi\,\frac{d\,\xi}{(1-\xi)\sqrt{\xi\,(1-\xi)}}$$

$$+\frac{1}{\pi}\int_{1-\sigma}^{1} \frac{h}{\sigma}\,(1-\xi)\,\frac{d\,\xi}{(1-\xi)\sqrt{\xi\,(1-\xi)}}$$

and

$$\mu_0 = 2\int_0^{1-\sigma} \frac{h}{1-\sigma}\,\xi\,\frac{(1-2\,\xi)\,d\,\xi}{\sqrt{\xi\,(1-\xi)}}$$

$$+\,2\int_{1-\sigma}^{1} (1-\xi)\,\frac{h}{}\,\frac{(1-2\,\xi)\,d\,\xi}{\xi\,(1-\xi)}$$

and finally

$$\beta_0 = \frac{2h}{\pi}\left[\frac{1}{\sqrt{\sigma\,(1-\sigma)}} + \frac{\pi}{2\sigma} - \frac{\text{arc cos}\sqrt{\sigma}}{\sigma\,(1-\sigma)}\right] \tag{15}$$

and

$$\mu_0 = h\left\{\frac{2\sigma-1}{\sqrt{\sigma\,(1-\sigma)}} + \frac{\pi}{2\sigma} - \frac{\text{arc cos}\sqrt{\sigma}}{\sigma\,(1-\sigma)}\right\} \tag{16}$$

Aerodynamic Characteristics

Here we have:

$$C_z = 2\pi\,(\alpha + \beta_0).$$

In practice, the lift coefficient is expressed as a function of the deflection β of the elevator and of the angle of attack of the relative wind on the stabilizer. AH. Therefore, we may write

$$C_z = 2\pi\,(\alpha_1 + m_0\,\beta) \tag{17}$$

Since $h \cong \sigma (1 - \sigma)$, and β and $\alpha = \alpha_1 + \gamma_1 = \alpha_1 + \sigma\beta$, we finally obtain

$$C_z = 2\pi \left[\alpha_1 + \left(1 + \frac{2}{\pi}\sqrt{\sigma (1 - \sigma)} - \frac{2}{\pi} \text{arc cos}\sqrt{\sigma}\right)\beta \right]$$

hence

$$m_o = 1 - \frac{2}{\pi} \left[\text{arc cos}\sqrt{\sigma} - \sqrt{\sigma (1 - \sigma)}\right] \qquad (18)$$

Figure 3 represents the variation of m_o with respect to σ according to formula (18). The experimental results of different aerodynamic laboratories, for deflections β varying between $\pm 5^\circ$ and $\pm 15^\circ$, are here shown. The locations of these representative points with reference to the theoretical curve show that the value of the coefficient m, applicable in the formula

$$C_z = 2\pi (\alpha + m \beta),$$

diminishes somewhat with the magnitude of the deflection β. As regards this analysis, we need only to call attention to the fact that the experimental points very close to the theoretical curve apply to elevators extending over the whole tail span without any central cut-out. When there is such a cut-out, the experimental values of m are somewhat smaller than the theoretical values, and the empirical expression

$$m = \sqrt{\sigma} \qquad (19)$$

very closely represents the mean values applicable to moderate deflections ($\beta < 10^\circ$).

Similarly the moment coefficient $C_{m(A)}$ is finally obtained as

$$C_{m(A)} = -\,0.25\,C_z - 2\sqrt{\sigma\,(1-\sigma)^3}\,\beta \qquad (20)$$

The experimental data available for verifying this theoretical formula are, however, very few.

Application to Wings Provided with Ailerons

The preceding results may be extended to the case of supporting wings equipped with ailerons for the purpose of increasing the maximum lift coefficient.

Let ASB represent the mean camber line of a wing profile, the rear portion of which HB forms an aileron rotating about the hinge H (Fig. 4). The aerodynamic characteristics of the undisturbed mean camber line are given by formulas (9') and (10'), as previously demonstrated.

Let β represent the angle of deflection, such that the mean camber line becomes AHB. As previously shown, the approximate theory is applicable to moderate values of deflection β, so that the chord AB' of the modified profile is very nearly equal to the chord AB of the original profile.

The aerodynamic characteristics of the profile ASB' will be calculable by the application of formulas (13) and (14) to the equation of the line ASB' with respect to the axes ox' and oy', γ being the angle which ox makes with ox'. Let

$\sigma = \frac{HB}{AB}$ then $\gamma \cong \sigma \beta$. For the portion AH, characterized by the relative abscissa $\xi = (1 - \sigma)$ we then get:

$$\begin{cases} \xi' = \xi \cos\gamma - \eta \sin\gamma \cong \xi \\ \eta' = \eta \cos\gamma + \xi \sin\gamma \cong \eta + \xi \sigma \beta \end{cases}$$

In an analogous manner for the portion HB' we get:

$$\xi' = \xi$$

$$\eta' \cong \eta + (1 - \sigma) \beta (1 - \xi)$$

The calculation of β_0' and μ_0' is in this case made by formulas (13) and (14) applied between 0 and $(1 - \sigma)$ for the portion AH and between $(1 - \sigma)$ and 1 for the portion HB'. We thus find

$$\beta_0' = \int_0^1 \eta \, f_1 (\xi) \, d\xi + \sigma \beta \int_0^{1-\sigma} \xi \, f_1 (\xi) \, d\xi + (1 - \sigma) \beta \int_{1-\sigma}^1 (1 - \xi) \, f_1 (\xi) \, d\xi \qquad (13')$$

and

$$\mu_0 = 2 \int_0^1 \eta \, f_2 (\xi) \, d\xi + 2 \left[\sigma \beta \int_0^{1-\sigma} \xi \, f_2 (\xi) \, d\xi + (1 - \sigma) \beta \int_{1-\sigma}^1 (1 - \xi) \, f_2 (\xi) \, d\xi\right] \qquad (14')$$

The first integral in β_0' represents the angle of zero lift β_0 of the profile ASB. The other two integrals represent the angle of zero lift of the deflection β of the elevator. This angle being $\Delta \beta_0$, we have

$$c_{z(\beta)} = 2 \pi (\alpha + \beta_0 + \Delta \beta_0)$$

β_0 is calculable from the equation $\eta = f(\xi)$ of the initial mean camber line ASB.

$\Delta \beta_0$ can be calculated as already described for the movable elevator, i.e., by the formula

$$\Delta \beta_0 = (m_0 - \sigma) \beta$$

with

$$m_0 = 1 - \frac{2}{\pi} [\operatorname{arc} \cos \sqrt{\sigma} - \sqrt{\sigma (1 - \sigma)}]$$

If the chord AB of the original profile is taken as the reference line, the angle of attack is then $\alpha_1 = \alpha - \gamma \cong \alpha - \sigma\beta$ and the result is

$$C_z = 2 \pi (\alpha_1 + \beta_0 + m \beta).$$

The first integral in the expression for μ_0' represents the value of μ_0 for the original profile. The other two integrals represent the correction $\Delta \mu_0$ due to the deflection of the aileron. $\Delta \mu_0$ will be calculated as in the case of the elevator, and we have

$$C_{m_0(\beta)} = C_{m_0(\beta=0)} - \left(\frac{\pi}{2} \Delta \beta_0 - \Delta \mu_0\right)$$

or

$$C_{m_0(\beta)} = C_{m_0(\beta=0)} - 2 \sqrt{\sigma} (1 - \sigma)^3 \beta$$

Experiments with wings equipped with ailerons verify quite well the theoretical values of $\Delta \beta_0$, in particular for positive moderate deflections β. In general, the values of

$$\Delta C_{m_0} = 2 \sqrt{\sigma (1 - \sigma)^3} \beta$$

under the same conditions, are somewhat larger than those found experimentally. The deviation is similar to the one found in the case of wing profiles having an appreciable C_{m_0}.

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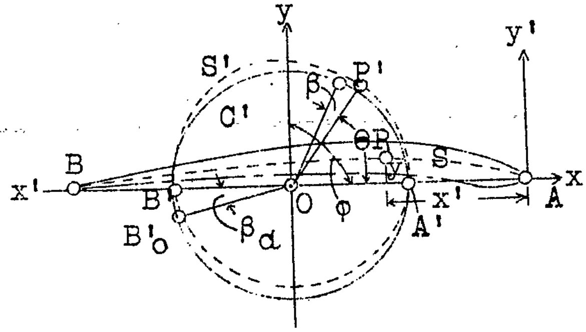

Fig.1

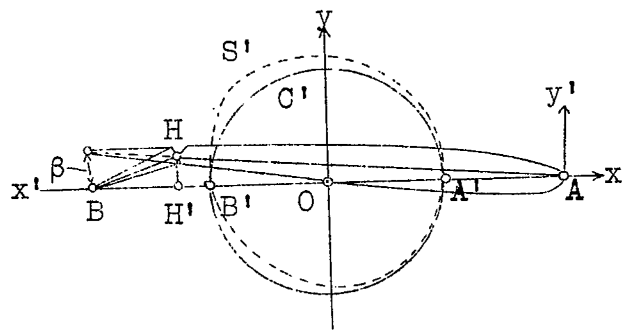

Fig.2

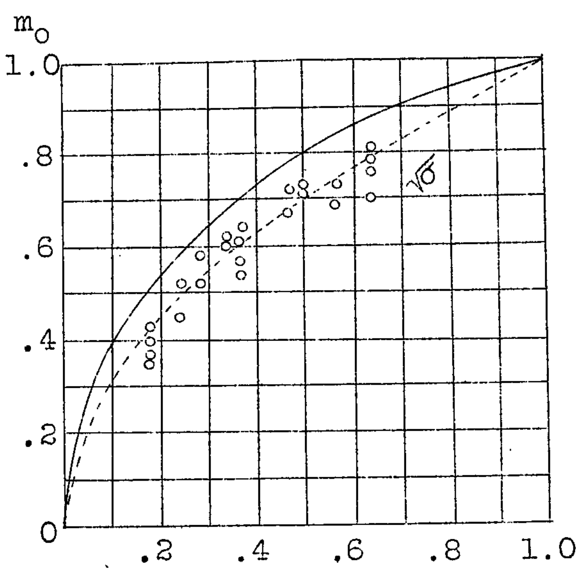

Fig.3

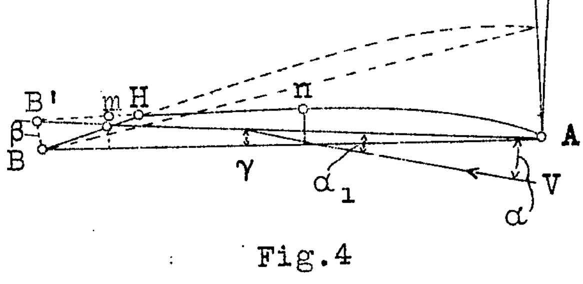

Fig.4