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IMPACT WAVES AND DETONATION

By R. Becker

PART I

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P R E F A C E

In making available in English the present article by R. Becker on Impact Waves and Detonation, there is here presented with complete bibliography not only an excellent critical resume of previous experimental and theoretical investigations of the Berthelot Explosive Wave but also the most notable recent contribution that has been made to the subject.

Among the numerous thermodynamic and kinetic problems that have arisen in the application of the gaseous explosive reaction as a source of power in the internal combustion engine, the problem of the mode or way by which the transformation proceeds and the rate at which the heat energy is delivered to the working fluid became very early in the engine's development a problem of prime importance. It was Nernst who first made it clear in an address entitled "Physico-Chemical Considerations Concerning the Process of Combustion in Gas Engines," given before the General Conference of German Engineers held at Magdeburg in 1905, that the thermodynamics of the gas engine did not rest, as is assumed in the case of the steam engine, upon the thermodynamics of a gaseous working fluid of constant composition; but that a thermodynamic reference cycle of maximum work applicable to internal combustion engines must be referred to the thermodynamic cycle of the chemical transformation taking place within the cylinder, viz.,

$$A = R T \ln K.$$

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At the same time he also pointed out that the rapid energy release within the cylinder raised other important problems besides that of the thermodynamics of the chemical reaction. These questions had to do with the hydrodynamics of the fluid, the profound effect of impact waves and their propagation through the burned and unburned gases. He showed how many of the phenomena connected with the combustion of the explosive gases observed by Berthelot, Dixon, and others found adequate explanation in hydrodynamic laws of fluids. This important phase of explosion phenomena was made the subject of extended investigation by Jouguet and Crussard with results that have been very generally accepted. The work of Becker here given is a notable extension of these earlier investigations, because it covers the entire range of the explosive reaction in gases - normal detonation and normal burning.

The successful practical working of the gas engine depends upon an explosive range usually designated as normal burning. The National Advisory Committee for Aeronautics has supported investigations into this phase of the reaction and would here call attention to some of the results of this work that seem to supplement in some measure the analysis left incomplete in the work of Becker. Reports of this work on the kinetics of the gaseous explosive reaction at constant pressure may be found in the Committee's Technical Reports* Nos. 176, 280 and 305.

Of particular interest in this connection as indicating a relation between the two known modes of explosive transformation - normal detonation and normal burning - is the experimental work of Wendlandt, "Experimental Investigations of the Limits of Detonation in Mixtures of Explosive Gases," Z. f. physik. chem. 110, 637 (1924). Also, bearing directly on the subject of combustion may be mentioned "Velocity of Reaction and Thermodynamics," by M. E. Jouguet, Ann. de Physique 5, 5, (1926). Also, "Thermal Equilibrium from the Standpoint of Chemical Kinetics and Photochemistry," by Werner Kuhn, J. de Chim. physique 23, 369 (1926).

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* "A Constant Pressure Bomb," by F. W. Stevens. N.A.C.A. Technical Report No. 176. (1923)

"The Gaseous Explosive Reaction - The Effect of Inert Gas," by F. W. Stevens. N.A.C.A. Technical Report No. 280. (1927)

"The Gaseous Explosive Reaction - A Study of the Kinetics of Composite Fuels," by F. W, Stevens. N.A.C.A. Technical Report No. 305. (1929)

C O N T E N T S

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 505.

IMPACT WAVES AND DETONATION.*

By R. Becker.

PART I.

Introduction

As Riemann (Uber die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite, Gött. Ges. d. Wiss. 8 (1860), und Riemann's ges. Werke, 2 Aufl. S. 156. Vergl. auch Riemann-Weber, Partielle differentialgleichungen, 5 Aufl. Bd. II, S. 507) was carrying out the integration of partial differential equations for a one-dimensional flow of an ideal gas, he made the discovery that a state of flow marked by constant distribution of density and velocity could pass over to a state of flow in which certain surfaces would form within the gases at which the constant magnitudes - density and velocity - mentioned above would vary within finite limits. A discussion concerning the further course of these disturbances can only follow after the differential equations have been affected by such conditions as will satisfy the equations of state for the gas on both sides of the unstable surface. These conditions lead to the statement that the laws of the conservation of mass and of energy as well as the impact law must not be violated by the passage of the gas through the unstable surface.

*From Zeitschrift für Physik, Volume 8, p.321 et seq. (1922).

Riemann in his treatment of the subject made the error of considering the energy equation unnecessary and introduced in its stead the assumption that the changes of state suffered by the gases in passing the unstable surface was adiabatic. In consequence, as Lord Rayleigh (Theory of Sound, vol. II, p. 41) has pointed out, his equations do not satisfy the energy laws. Later, Hugoniot (Journ. de l'ecole polytech., Paris, 57, 58, (1887), (1889)) without knowledge of the work of Riemann, gave an extended mathematical analysis of one-dimensional air movement in which the relationship with the energy laws was clearly brought out. His treatment of the unstable surfaces (which hereafter will be designated "impact waves" or concentration impulse) revealed the fact that by taking into account the energy laws the changes of state suffered by the gases in passing the surfaces of instability did not follow the law of (static) adiabatics but another law which he called "dynamic adiabatics" and which will be referred to in what follows as the "Hugoniot=equation."

Later an extended treatment of the mathematical side of our problem will be given, following the work of Hadamard (Propagation des ondes. Paris, (1903) and of Zemplen (Unstetege Bewegungen in Flussigkeiten, (Enzykl. d. math. Wissen. Bd. IV, 2 Teil, 1 Halfte). In the mathematical nomenclature we shall refer to a surface whose two sides differ in density and velocity by finite amounts, as unstable surfaces of the "first order." Unstable surfaces of the second, third, etc., orders are those whose first, second, etc. derivatives of those magnitudes are instable in reference to space and time. Our impulse wave is therefore an instability of the first order.

An important deduction of the theory is the consequence that concentration waves of finite over-pressure spontaneously pass into steep compression impulses (sound waves) whose rate of propagation is the normal rate of sound propagation in the gases only for the limiting case of infinitely small compression; but with increasing intensity the velocity of propagation may increase indefinitely. The fact that sound waves may travel with velocity greater than the ordinary speed of sound, was first demonstrated by Mach (Wiener Ber. 72 (1875) 75 (1877) 77 (1878), and his co-workers. He produced the sound waves studied either by an electrical spark or by a fulminate. Martin (Z. f. d. ges. Schiess. u. Sprengstoffwesen, 12, 39 (1917)) likewise worked with a number of explosives for the production of the sound waves studied by him. He succeeded in establishing a quantitative relation between the brisanz of the explosive and the velocity of propagation of its sound wave. Further, we have Wolff (Ann. d. Phys. 69, 329 (1899)) to thank for extensive measurements of sound waves generated by heavy explosions. All of these measurements have to do with the case of the free, spatial propagation of sound waves whose theoretical treatment has so far been unsuccessful. With the view of testing out the theory of one-dimensional movement in gases, Vielle (Memorial des poudres et salpetres, 10, 177, (1899-1900)) carried out a great number of experiments. He prevented the spatial expansion of the sound waves by producing the sound within a steel tube. By this means he was able to observe the increasing "steepness" of the wave front and to increase its velocity of propagation threefold above the normal velocity of sound.

Technical practice has presented us with two groups of phenomena whose relationship to the theory of compressional impulses has only become known and made clear after long and arduous experimental effort. The first group is concerned with the flow of gases and vapors from openings of different forms and is of special importance for the construction of steam turbines. Extended analyses of these processes and the problems they present will be found by Stodola (Die Dampfturbinen, Berlin, 1905), Prandtl (Handworlerbuch d. Naturwissenschaften, Bd. 4, Jene, 1913), Schroter and Prandtl (Enzykl. d. math. Wissen. Bd. V, Teil 1 Heft 2).

The second group of phenomena connected with the theory of compressional impulses arises from the rapid chemical transformations of explosive material. That the effect of such an explosive transformation on the surrounding air is to produce a disturbance of the nature of a sound wave, has already been referred to. But the spatial propagation of the area of explosive transformation within the explosive gases (the detonation wave) is in itself only a special case of a compressional impulse.

The "detonation wave" was first observed and measured by Berthelot (Sur la force des matieres explosives, Paris, (1883) C.R. 93, 18, (1881)). Its close relationship with Riemann's theory of compressional impact was recognized by Schuster (Philos. Trans. London (1893) p. 152); while Chapman (Phil. Mag. 47, 90 (1899)) was the first to deduce from the principles enumerated by Riemann the complete fundamental equations leading to the determination of the rate of propagation of the "detonation wave." An extended analysis and discussion of these equations accompanied by numerical experimental values was later carried out by Jouguet (Jour. d. Math. 1, 347 (1905) 2, 5, (1906)) and by Crussard (Bull. de la soc. d l'ind. minerale, Saint-Etienne, 6, 109 (1907)). Their results showed satisfactory and far-reaching agreement between the experimental values obtained by Dixon (Phil. Trans. London (1893) and (1903)) and the values calculated by them. An investigation carried out by Taffenel and Dautrische (C.R. 155, 1221 (1912)) in which they sought to demonstrate the theory of compressional impulses numerically as applied to solid explosives, came to grief through their error in using an approximated form of van der Waal's equation of state as an expression representing the real condition of gases at any concentration. In a short communication (Becker, Z. f. Elektrochem. 23, 40 (1917), Z. f. Physik 4, 393 (1921) I brought together a few considerations which in the simplest way and without any assumptions concerning the state of the reacting components led directly to the equations for detonation. I was able to show by the use of an equation of state based on the experimental values obtained by Amagat (Becker, l.c) that these equations led to reasonable values for the rate of propagation of the detonation wave even in the case of solid explosives.

The theory of compressional impulses therefore seems to rests upon a well established mathematical basis which is further supported by extensive experimental results. But in spite of this, from a purely physical standpoint, its present form is unsatisfactory. The initial given conditions required for an expression of state (density, pressure, velocity) existing on both sides of the surface of instability are indeed sufficient for a thorough macroscopic description of the phenomena; nevertheless they give us no insight into the actual processes involved in the transformation. It is for instance not made clear why in a detonation wave the compression no longer remains adiabatic but follows the Hugoniot equation instead. In order to arrive at a purely physical theory some insight is required of the macroscopic structure of the wave front. In what follows I shall show in Section 1 by simple means and by figures, in Section 2 by mathematical treatment of the same processes how the surfaces of instability originate if it is assumed that the fluid is free from friction and heat conduction. When, however, it is recognized and taken into account (Section 3) that no substance exists free from friction and heat conduction it must follow that a sharply defined surface of instability cannot arise. The impact wave must have a finite thickness. This statement was first made by Prandtl (Z. f. d. ges. Turbinenwesen 3, 241, (1906)). If the differential equations for one-dimensional movement are affected with terms expressing the effect of friction and heat conduction (Section 4), there is obtained by integration without particular difficulty not only the Riemann-Hugoniot equations for the macroscopic characteristics of impulse waves (Section 5), but the equations also lend some insight into their microscopic structure (Section 6). The computation of the thickness of impulse waves will be illustrated by numerical examples.

A knowledge of the processes taking place within the wave front is also a necessary preliminary to a real knowledge of the detonation wave; by carrying out the consequences of the theory of instability one is led by compelling and unmistakable ways to values of detonation velocity (Section 8 - See N.A.C.A. Technical Memorandum No. 506, which is a continuation of this report), and detonation pressure (Section 9 - T.M. 506); yet it remains entirely unexplained how the initial components against the wave front are brought to a condition of activation.

By application of the knowledge won concerning compressional impulses an understanding of this process is somewhat assisted although much yet remains to be satisfactorily explained (Section 10 - T.M. No. 506).

A. The Formation of Compression Impulses

1. A simple method of treatment.- In order to represent in a simple way how compression impulses may be formed, imagine the device represented in Figure 1 - a long tube closed at the left by a piston a, and filled with air. A small velocity dw, is imparted to the piston. This movement produces in the gases a weak compression wave that travels from left to right with the velocity of sound $c = \sqrt{\gamma RT}$. At a given instant (Fig. 1, b), the gas to the right of the wave front remains unchanged and at rest, while the air between the wave front and the piston is adiabatically compressed by an amount $d\rho$, and has the velocity dw. The velocity of the piston is now increased by the amount dw whereby a second compression wave is produced in the gas and is propagated along the tube behind the first (Fig. 1, c). By repeating this process the velocity of the piston is finally brought to the velocity w. There is thus produced within the mass of gas in the tube a terrassed form of wave whose particles to the left move with the velocity w. What is the further history and fate of this wave? In the first place it is plain that the stratum of the terrass to the

left has a greater velocity relative to the tube than the strata to the right. Besides, the temperature and hence the sound velocity is greater in the strata to the left than to the right. As a consequence the strata draw together and the wave front becomes steeper, (Figure 1,e and 1,f). It must not be overlooked what will happen when the steepness of the wave front becomes infinite (a condition to be considered in Section 2).

If, on the contrary, the piston is given a velocity to the left a rarefaction wave will be produced in the tube as may be easily realized from analogy to what has been stated. The rarefaction wave will, contrary to the compression wave, become ever flatter and flatter the further it advances in the tube.

In conventional expositions of the subject (for example, that of Riemann-Weber, vol. 2) as also in Section 9 (T.M. No. 506) of this "Arbeit," a consideration of rarefaction waves will be excluded because they involve a loss of entropy and because from the second law of thermodynamics they are impossible of propagation. It will be shown here that from the standpoint of pure mechanics they cannot develop. At the end of the next paragraph, also in Section 9 (T.M. No. 506), it will be shown that both conditions (the thermodynamic and mechanic) are really identical.

2. A mathematical treatment of the same processes.- Anticipating applications to be made later, the differential equations describing the unidimensional gas movement will be so written as to include the effect of friction and heat conductivity.

ξ represents the very small thickness of any cross section of the tube; x the spatial coordinate measured along the length of the tube; t the time; u the velocity; ρ the density; p the pressure.* Then, as is customary, the change in a characteristic G of a material particle with time may be written

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + u \frac{\partial G}{\partial x} \tag{1}$$

also

$$\frac{d\xi}{dt} = \xi \frac{\partial u}{\partial x} \tag{1a}$$

The mass of the cross section layer ξ is $\rho \xi$. The momentum $\rho \xi \quad E + \frac{u^2}{2}$ p_{11} is the effective pressure in the direction of the axis of the tube and perpendicular to the surface of the layer ξ; λ the heat conductivity; μ a friction coefficient. Then, from elementary laws,

$$\frac{d}{dt} (\rho \xi) = 0,$$

$$\frac{d}{dt} (u \rho \xi) = - \frac{\partial p_{11}}{\partial x} \xi$$

*All computations to follow refer to a column of cross section unity.

$$\frac{d}{dt}\,\rho\,\xi\,(E + \frac{u^2}{2}) = [-\frac{\partial\,(p_{11}\,u)}{\partial x} + \frac{\partial}{\partial x}\,(\lambda\frac{\partial T}{\partial x})]\,\xi$$

in which

$$p_{11} = p - \mu\,\frac{\partial u}{\partial x}\,,$$

where μ is related to viscosity, η as indicated by the equation

$$\mu = \frac{4}{3}\,\eta^{*} \tag{2}$$

which follows from the symmetry characteristics of pressure tensors p_{ik}. The three equations may then be written

$$\frac{d\rho}{dt} = -\,\rho\,\frac{\partial u}{\partial x}\,, \tag{3a}$$

$$\frac{du}{dt} = -\,\frac{1}{\rho}\,\frac{\partial}{\partial x}\left(p - \mu\,\frac{\partial u}{\partial x}\right) \tag{3b}$$

$$\frac{dE}{dt} = \left(p - \mu\,\frac{\partial u}{\partial x}\right)\frac{1}{\rho^2}\,\frac{d\rho}{dt} + \frac{1}{\rho}\,\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right). \tag{3c}$$

Introducing the entropy S, by the relation

$$TdS = dE - p\,\frac{d\rho}{\rho^2}$$

(3c) may be written

$$\rho\,T\,\frac{dS}{dt} = \mu\,\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)\,, \tag{3c'}$$

in which the change of entropy with time is given as affected by friction and heat conductivity.

But for the present we will neglect the effect of friction and heat conduction. Equation (3c') will then read simply

$$S = \text{const.}$$

*See Weber and Gans, Report d. Phys. I, 1, p.349.

That is, compression in the waves takes place adiabatically and for the case of an ideal gas,

$$p = a^2 \quad \rho^k \tag{4}$$

where a^2 is a constant and $k = \frac{c_p}{c_v}$ the ratio of specific heats. With reference to equations (1) and (4) and with $\mu = 0$ and $\lambda = 0$,

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \tag{5a}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{dp}{d\rho} \frac{\partial \rho}{\partial x} = 0 \tag{5b}$$

The integrals $u\ (x,t)$ and $\rho\ (x,t)$ corresponding to the simple treatment of the process carried out in Section 1, permit of a much simpler derivation, with the aid of the theory of characteristics, than that given by Riemann, Hadamard. To this end consider a linear element (dx, dt) drawn in the plane x,t (Fig. 2). Its direction is indicated by the equation

$$dx = \varphi \ dt$$

Any function whatever as $G\ (x,t)$ changes along this line by the value $dG = \left(\frac{\partial G}{\partial x} \varphi + \frac{\partial G}{\partial t}\right) dt$. From the expressions for u and ρ in equations (5) we will select as function of G, $u = f\ (\rho)$ where f, primarily an undetermined function of ρ, gives f'. Then,

$$d\,[u + f\ (\rho)] = \left(\frac{\partial u}{\partial x} \varphi + f' \frac{\partial \rho}{\partial x} \varphi + \frac{\partial u}{\partial t} + f' \frac{\partial \rho}{\partial t}\right) dt.$$

By addition and subtraction of the expression

$$u \frac{\partial u}{\partial x} + f' \, u \frac{\partial \rho}{\partial x},$$

the expression within the parentheses becomes

$$d\,(u + f) = \left[\left\{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial \rho}{\partial x} f' \, (\varphi - u)\right\}\right.$$

$$\left. + f' \left\{\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \frac{\partial u}{\partial x} \frac{\varphi - u}{f'}\right\}\right] dt.$$

From (5a) and (5b) the right side of the above equation vanishes when

$$f' \, (\varphi - u) = \frac{1}{\rho} \frac{dp}{d\rho} \quad \text{and} \quad \frac{\varphi - u}{f'} = \rho,$$

that is, if

$$f' = \frac{1}{\rho} \sqrt{\frac{dp}{d\rho}} \quad \text{and} \quad \varphi = u + \sqrt{\frac{dp}{d\rho}}$$

or

$$f' = -\frac{1}{\rho} \sqrt{\frac{dp}{d\rho}} \quad \text{and} \quad \varphi = u - \sqrt{\frac{dp}{d\rho}}$$

But this means, in reference to the problem in hand, that the curve

$$\frac{dx}{dt} = u + \sqrt{\frac{dp}{d\rho}}$$

the expression

$$u + \int \frac{1}{\rho} \sqrt{\frac{dp}{d\rho}} \, d\rho = \text{const.} \tag{6a}$$

and along the curve

$$\frac{dx}{dt} = u - \sqrt{\frac{dp}{d\rho}}$$

the expression

$$u - \int \frac{1}{\rho} \sqrt{\frac{dp}{d\rho}} \, d\rho = \text{const.} \tag{6b}$$

The application of this result to the problem as simply discussed in Section 1 is self-evident: In the tube of infinite length, the position of the piston at $t = 0$ is $x = 0$ and it is at rest (Figure 2). Its position in succeeding intervals is indicated by the curve C in the x,t coordinate figure, as its velocity constantly changes between the instant $t = 0$ and $t = \tau$, and from then on it proceeds at a constant velocity u_1. If we indicate by the index s values referring to the piston, then, for

$$\left.\begin{array}{ll} 0 < t_s < \tau \;; & x_s = \frac{g}{2} t_s^2 \quad \text{and} \quad u_s = g\, t_s \\ \text{for} \quad t_s > \tau \;: & x_s = g\,\tau\, t_s - \frac{g}{2}\tau^2 \quad \text{and} \quad u_s = g\,\tau = u_1 \end{array}\right\} (7)$$

Further, throughout the tube, let $t = 0$, then $u = 0$ and $\rho = \rho_0$ and the curves constructed from (6b) fill the entire space between the x-axis and the curve C. Since, now, for $t = 0$, $u - \int \frac{1}{\rho}\sqrt{\frac{dp}{d\rho}}\, d\rho$ has the same value throughout the entire range,

$$u - \int \frac{1}{\rho}\sqrt{\frac{dp}{d\rho}}\, d\rho = \text{const.}$$

and besides, since for the curve (6a)

$$u + \int \frac{1}{\rho}\sqrt{\frac{dp}{d\rho}}\, d\rho = \text{const.},$$

so must it also follow that along the (6a) curve u and ρ remain constant. On the x-axis itself $u = 0$. Therefore, throughout the entire range the relationship between u and ρ will be

$$u = \int_{\rho_o}^{\rho} \frac{1}{\rho} \sqrt{\frac{dp}{d\rho}}\, d\rho \qquad (8)$$

At the piston and hence along the curve C, u_s (according t (7)) is given; and from (8) ρ_s may also be known. We can therefore draw through every point x_s, t_s the straight line

$$x - x_s = (t - t_s)\left[u_s + \left(\sqrt{\frac{dp}{d\rho}}\right)_s\right] \qquad (9)$$

along which u and ρ have constant values u_s and ρ_s.

In the case of the piston motion (7) the portion of the coordinate figure enclosed by the x-axis and the curve C will be divided into three parts by the two lines drawn according to (9) from the points 0 and τ. For the lower portion $u = 0$. The middle portion u varies between $u = 0$ and $u = u_1$. In the upper portion u is finally constant $= u_1$.

In gaseous media according to (4):

$$\sqrt{\frac{dp}{d\rho}} = a\sqrt{k}\,\rho^{\frac{k-1}{2}} \quad \text{and} \quad \int \frac{1}{\rho} \sqrt{\frac{dp}{d\rho}}\, d\rho = \frac{2\,a\sqrt{k}}{k-1}\,\rho^{\frac{k-1}{2}}$$

If the velocity of sound at initial conditions be given as

$$c_o = a\sqrt{k}\,\rho_o^{\frac{k-1}{2}}$$

Then according to (8)

$$u = \frac{2}{k-1}\left(\sqrt{\frac{dp}{d\rho}} - c_o\right)$$

or

$$\left(\frac{\rho}{\rho_o}\right)^{\frac{k-1}{2}} = \left(\frac{p}{p_o}\right)^{\frac{k-1}{2k}} = 1 + \frac{u}{c_o}\,\frac{k-1}{2} \qquad (10)$$

Finally the slope of the curves (6a) and (6b) is given by

$$\left.\begin{aligned} u + \sqrt{\frac{dp}{d\rho}} &= c_o + u\,\frac{k+1}{2} \\ u - \sqrt{\frac{dp}{d\rho}} &= -\left(c_o - u\,\frac{3-k}{2}\right) \end{aligned}\right\} \qquad (11)$$

This solution denies that u may possess at the instant of crossing of any two curves of the (9) group, two different values. The intersection of two curves of the (9) group is the complete analytical counterpart of the conditions referred to in Section 1, where one wave overtakes another. Position X and time T of this coincidence are given by the values of x and t calculated from (9) together with the equation obtained by differentiating with respect to t_s:

$$-\,g\,t_s = \tan\frac{k+1}{2} - t_s\,g\,(k+1) - c_o,$$

where, by the help of (11) and (7) the magnitudes x_s, u_s, $\left(\frac{dp}{d\rho}\right)_s$ are expressed as functions of t_s. In this way there is obtained

$$T = \frac{2}{k+1}\left(t_s\,k + \frac{c_o}{g}\right)$$

$$X = \frac{1}{2}\,k\,g\,t_s^2 + c_o\,T.$$

The first position of instability occurs from the coordinate point of reference, $t_s = 0$ at the instant

$T_o = \frac{c_o}{g}\;\frac{2}{k+1}$ and at the point $X_o = \frac{c_o^2}{g}\;\frac{2}{k+1}$.

If the piston in one-half second is moved from rest to a velocity of 100 m/s and then proceeds at that constant rate,

$$g = 200 \text{ m/s}^2$$

$$\rho_0 = 330 \text{ m/s}$$

$$k = 1.4$$

$$\tau = 0.5 \text{ sec}$$

$$u_1 = 100 \text{ m/s}$$

so that the time and place of the first surface of instability will be

$$x = 453 \text{ m}$$

$$T = 1.38 \text{ sec}$$

For this example the pressure increase calculated from (10)

$$\frac{p_1}{p_0} = 1.51$$

and the increase in density

$$\frac{\rho_1}{\rho_0} = 1.34$$

In Figure 3 the example just given is represented graphically. The course of the velocity u of the wave along the axis of the tube x, is drawn for the intervals 0.2, 0.6, 1.0, and 1.4 sec. The figure plainly shows the increasing steepness of the wave form.

The mechanical production of a compression impulse according to the above, depends upon the condition that within an adiabatic wave train those regions of greater density strive to become more dense at the expense of the less dense regions. That is the velocity expressed by (6a),

$$\frac{dx}{dt} = u + \left(\sqrt{\frac{dp}{d\rho}}\right)_{adiab.}$$

must increase with increasing density. If we substitute for u its value in (8) we have the condition

$$\frac{d}{d\rho}\left(\int \frac{1}{\rho}\sqrt{\frac{dp}{d\rho}}\, d\rho + \sqrt{\frac{dp}{d\rho}}\right) > 0$$

or

$$\frac{1}{\rho}\,\frac{d}{d\rho}\left(\rho\sqrt{\frac{dp}{d\rho}}\right) > 0$$

If we substitute for ρ, $\frac{1}{v}$ we obtain

$$v\,\frac{3d}{dv}\sqrt{-\frac{dp}{dv}} > 0$$

or, finally,

$$\left(\frac{d^2 p}{dv^2}\right)_{adiab.} > 0$$

It is possible then to make the following generalization: <u>In any given medium it is mechanically possible to produce only compression or rarefied impulses according as the value of $\left(\frac{d^2 p}{dv^2}\right)_{ad.}$ is positive or negative</u>. Exactly this same criterion will be met with (Section 9 - T.M. No. 506) in discussing the thermodynamic possibility of producing compressional impulses.

3. The necessity of taking into account the effect of friction and heat conductivity. The considerations set forth in Sections 1 and 2 gave a solution of the problem only to the instant at which instability in the gases appeared. A further consideration of the processes is made possible if there be added to the Riemann-Hugoniot line of analysis three equations involving the magnitudes u, and p on both sides of the instable surface. This extension of the analysis of the processes is made necessary if we are to secure the reasoning against any possible violation of the laws of the conservation of mass and of energy, also the impact law. These equations are identical with equations (14). They will later on receive extended consideration.

This procedure is free from objection - indeed, it seems the only possible one - in so far as equations (5) are axiomatically accepted as describing what actually takes place. But from the standpoint of physics, this objection may be made: Equations (5) hold only so long as friction and heat conductivity may be considered negligible. But since no substances are known to exist free from these characteristics, these equations must give results that are in error as soon as the temperature decrease or the rate of change of volume exceeds a certain limit. These values according to the above considerations would appear to be too significant to be neglected. The application of equations (5) are not admissible at this point.

If we refer for a moment to the simple exposition of the process as given in Section 1, we will be led to expect the following: When the wave front has reached a certain steepness, the counter forces of friction and heat conduction oppose the tendency to further compression. A condition will be reached where these two tendencies compensate each other and from this point on a quasi-stationary wave form will be propagated along the tube.

Before seeking in this sense an integration of the general equation (3) we shall attempt to show in a wholly qualitative way how the course of temperature change is influenced by heat conduction. Let the line ABCD represent the course of temperature change in the neighborhood of a compression wave.(Fig. 4). Assume the increase of pressure to be such that due to adiabatic compression, the absolute temperature is increased threefold; for example, from 300° to 900° absolute. The role of heat conductivity will be the most significant among the gas molecules at B and C - the positions of greatest change in the temperature gradient. The gases flowing from D may gain in temperature about 200° and at B be cooled by a like amount. At 500° they are affected by adiabatic compression that increases the temperature threefold, that is, to 1500°. By conduction they lose at B 200°, thus proceeding toward A at a temperature of 1300°. At first sight the paradoxical result would seem to be that in consequence of heat conduction an initial temperature difference of 600° has been increased to 1000°! But in truth, with the change in temperature difference there has followed a change in pressure and density difference which are in themselves a source of wave formation thrown back from the original wave front toward the piston.* In this way the actual processes in the formation of compression impulses are seen to be so complicated that at present a complete theoretical treatment of their formation seems out of the question. Only after the impulse wave has become quasi-stationary do we again find conditions more satisfactory for theoretical analysis.

From a consideration of the above roughly qualitative discussion it is not to be wondered at if we meet with surprising temperature differences in impact waves of high compression.

B. The Stationary Compression Impulse

4. Differential equations.- In this paragraph we shall investigate the characteristics of compressional impulses after they have assumed the form of a quasi-stationary wave. We shall imagine that the coordinate system of reference moves synchronously with the compression wave. In this way the wave may be treated as actually stationary. We shall therefore integrate equations (3) for the case that the partial derivatives vanish with the time. Accordingly, we substitute for

*These waves find their analogue in detonation in the "retonation waves" of Dixon and le Chatelier.

$\frac{d}{dt}$, $u \frac{\partial}{\partial x}$ and write

$$u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0$$

$$\rho u \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} \left(p - \mu \frac{\partial u}{\partial x}\right) = 0$$

$$\rho u \frac{\partial E}{\partial x} = \left(p - \mu \frac{\partial u}{\partial x}\right) \frac{u}{\rho} \frac{\partial \rho}{\partial x} + \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x}\right)$$

The first equation may be integrated at once and by that the second. If we substitute from the solution of the first and second equations ρu and $p - \frac{\partial u}{\partial x}$ in the third equation, it may also be integrated. By the aid of the three integration constants M, J, and F and by the substitution of the density ρ, the reciprocal specific volume $\frac{1}{v}$, there is obtained the differential equations for the stationary compression impulse.

$$u = Mv \tag{12a}$$

$$M^2 v + p - J = \mu M \frac{dv}{dx} \tag{12b}$$

$$E + Jv - \frac{1}{2} M^2 v^2 - F = \frac{\lambda}{M} \frac{dT}{dx} \tag{12c}$$

From these equations energy E and temperature T are seen to be given functions of pressure and volume. A second integration of these equations gives the desired continuous transfer of the magnitudes p_1, v_1, u_1 in front of the concentration impulse, to their magnitudes p_2, v_2, u_2 behind it. The relations that prevail between these six magnitudes are at once manifest by observing that only within the wave front itself do the expressions $\frac{dv}{dx}$ and $\frac{dT}{dx}$ differ appreciably from O. For any point outside the wave front we may therefore write

$$\frac{u}{v} = M$$

$$\frac{u^2}{v} + p = J \qquad (13)$$

$$E + \frac{u^2}{2} + pv = F$$

If we compare any two such positions with each other, we must have

$$\frac{u_1}{v_1} = \frac{u_2}{v_2} \qquad (14a)$$

$$\frac{u_1^2}{v_1} + p_1 = \frac{u_2^2}{v_2} + p_2 \qquad (14b)$$

$$E_1 + \frac{u_1^2}{2} + p_1 v_1 = E_2 + \frac{u_2^2}{2} + p_2 v_2 \qquad (14c)$$

These fundamental equations expressing the macroscopic characteristics of impulse waves are, as given, independent of the magnitude of friction μ, and of heat conductivity λ. They are identical with the stipulations made in the introductory treatment for the conditions on both sides of the layer of instability, and could, in fact, be directly written there if it is also specified that for the case of a stationary wave the transport per second of mass, impulse and energy through any two cross sections of the tube are the same.

5. The macroscopic characteristics of compressional impulses.- Before carrying out the integration of equations (12) we will gather some conception of the significance of equations (14). To this end we solve (14a) and (14b) for u_1 and u_2 and substitute the values in (14c). We then have

$$u_1^2 = v_1^2 \frac{p_2 - p_1}{v_1 - v_2} \tag{15a}$$

$$u_2^2 = v_2^2 \frac{p_2 - p_1}{v_1 - v_2} \tag{15b}$$

$$E_2 - E_1 = \frac{1}{2} (p_1 + p_2) (v_1 - v_2) \tag{15c}$$

Equation (15c) is the Hugoniot equation which in the case of impact waves - detonation - takes the place of the adiabatic relation, $dS = 0$.

For small differences $E_2 - E_1$ and $v_1 - v_2$, (15c) becomes $dE - p\,d\,v = 0$, - an expression identical with the adiabatic.

The velocity of propagation D, of the impact (detonation) wave in a medium at rest and the flow velocity W set up in the medium behind the detonation wave are expressed by

$$\left.\begin{aligned} D &= u_1 = v_1 \sqrt{\frac{p_2 - p_1}{v_1 - v_2}} \quad \ldots \ldots \\ W &= u_1 - u_2 = (v_1 - v_2)\sqrt{\frac{p_2 - p_1}{v_1 - v_2}} \end{aligned}\right\} \tag{16}$$

The impulse (detonation) wave is determined by the initial condition of the medium (p_1 and v_1) as well as the pressure p_2, within the wave. Further, it is desired to find the factors (D, W, T_2, etc.).

First, we shall carry out the calculation for a perfect gas where

$$p\,v = R\,T \tag{17a}$$

$$E_2 - E_1 = \overline{c_V}\,(T_2 - T_1), \tag{17b}$$

where $\overline{c_V}$ is the average specific heat between T_1 and T_2° absolute. Let

$$\zeta_1 = \frac{2\,\overline{c_V}}{R} + 1$$

and

$$\pi = \frac{p_2}{p_1} \tag{18}$$

Then

$$\frac{T_2}{T_1} = \pi\,\frac{\pi + \zeta_1}{\pi\,\zeta_1 + 1} \tag{19a}$$

$$\frac{v_1}{v_2} = \frac{\rho_2}{\rho_1} = \frac{\pi\,\zeta_1 + 1}{\pi + \zeta_1} \tag{19b}$$

hence

$$D^2 = p_1\,v_1\,\frac{\pi\,\zeta_1 + 1}{\zeta_1 - 1} \tag{19c}$$

$$W^2 = p_1 \ v_1 \ (\zeta_1 - 1) \frac{(\pi - 1)^2}{\pi \ \zeta_2 + 1} \ . \tag{19d}$$

If the dependence of temperature on c_V be neglected then $\zeta_1 = \frac{k + 1}{k - 1}$ = (for diatomic gases) 6. Hence, as soon as the value of π becomes large as compared to 6, temperature T becomes proportional to pressure p. It is therefore necessary that ζ_1 be taken as a function of T.* According to the results of Pier (Z. f. Elektrochem. 15, 536 (1909), also 16, 897 (1910)) and Siegel (Z. f. physik. Chem. 87, 641, (1914)) the specific heat of oxygen and nitrogen carried out experimentally to 3000° abs. is

$$\overline{c_V}^{273, T} = 4.78 + 0.45 \times 10^{-3} \ T \ \frac{\text{cal.}}{\text{mol. grad.}},$$

from which we find

$$\zeta_1 = 5.82 + 0.46 \times 10^{-3} \ T_2 .$$

Since the values given in the following table are carried out for temperatures much above 3000° abs., the results given can be taken as representing only the order of the magnitudes to be expected. With the value given above for ζ_1 (19a) becomes a quadratic equation for T. Using this calculated value the other equations under (19) give the numerical results sought for the fluid air.

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*Rudenberg, Artill. Monatshefte (1916), p.237, has carried through a computation assuming c_V constant.

Compression Impulse in Air

$\pi = \frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{v_1}{v_2}$	T_2 absolute	D m/s	W m/s	$\frac{i}{p_1} = (\pi - 1) \frac{v_1}{v_2}$	T^0 abs. adiabatic
2	1.23	1.63	336	452	175	1.63	330
5	1.76	2.84	482	698	452	11.14	426
10	2.58	3.88	705	978	725	34.9	515
50	8.28	6.04	2260	2150	1795	296	794
100	14.15	7.66	3860	3020	2590	699	950
500	44.80	11.15	12200	6570	5980	5560	1433
1000	70.00	14.30	19100	9210	8560	14300	1710
2000	106.20	18.80	29000	12900	12210	37600	2070
3000	134.40	22.30	36700	15750	15050	63900	2180

Concerning the values given in the above table, it should be stated that sound waves have been produced in air having a rate of propagation around 13000 m/s. These waves were produced in air by detonating substances. By enclosure in a tube the one-dimensional movement of the wave was observed.* A wave of this velocity should, according to the above table, heat the air within it to around 30,000°, that is, to a temperature of the order attributed to fixed stars. In the last column of the table there is given the temperature that should result from adiabatic compression alone and corresponding to a given pressure. These values are seen to be only about 10% of the temperature of the impact wave. The next to the last column in the table is of interest in estimating the effect of an impact wave as it strikes an obstacle (Rudenberg, l.c. p.254). This force

*A report of these experiments will shortly appear in Z. f. techn. Physik. 3, 152 (1922), also 3, 249 (1922)).

(total impulse i) is made up of the static pressure difference $p_2 - p_1$, and the weight of the flow of the mass of gas behind the wave front $\rho_2 W^2$. With $\rho_2 = \frac{1}{v_2}$ and the value of W from (16)

$$i = (p_2 - p_1) + \rho_2 W^2 = (p_2 - p_1) \frac{v_1}{v_2}$$

The effect of the impact of the detonation wave is therefore greater than the pressure difference by the value of the concentration factor $\frac{v_1}{v_2}$.

A similar calculation may be carried out for the case of liquids. For this case the equation of state for exceedingly high pressures as stated by Tammann (Ann. d. Physik. 37, 975, (1912)) is applied:

$$p = \frac{C\ T}{v - b} - K,$$

where C, b, and K are constants.

The energy expression from the general equation is

$$dE = c_v\ d\ T + (p \frac{\partial p}{\partial T} - p)\ dv$$

$$E = c_v\ T + K\ v \qquad (20a)$$

writing

$$p' = p + K \quad \text{and} \quad v' = v - b \qquad (21)$$

then with (20a) and the Hugoniot equation (15c)

$$\overline{c_v}\ (T_2 - T_1) = \tfrac{1}{2}\ (p'_1 + p'_2)\ (v'_1 - v'_2)$$

and from (20)

$$p'\ v' = CT$$

These equations are in form identical with the gas equations above and their solution the same as given in (19). Hence if

$$\zeta_1 = \frac{2\ \overline{c_v}}{C} + 1 = \frac{2\ \overline{c_p}}{C} - 1$$

$$\pi' = \frac{p_2 + K}{p_1 + K}$$

$$\frac{T_2}{T_1} = \pi' \frac{\pi' + \zeta_1}{\pi' \zeta_1 + 1}$$

$$\frac{v_1 - b}{v_2 - b} = \frac{\pi' + \zeta_1}{\pi' \zeta_1 + 1} \qquad (22)$$

$$D^2 = v_1^2 \frac{p_1 + K}{v_1 - b} \frac{\pi' \zeta_1 + 1}{\zeta_1 - 1}$$

Using the following values: R. Becker (Z. f. Elektrochemie 23, 304 (1917)) $K = 2792$ atm. C, $0.1001 \frac{\text{cal.}}{\text{g. grad.}}$, $b = 0.94 \frac{\text{cm}^3}{\text{g}}$, $v_1 = 1.36 \frac{\text{cm}^3}{\text{g}}$ $c_p = 0.564 \frac{\text{cal.}}{\text{g. grad.}}$, the temperature increase shown in the following table was obtained for the case of ethyl ether.

p_2 atmospheres	$T_2 - T_1$ adiabatic	$T_2 - T_1$ impact	D m/s velocity
100	1.6	1.6	1260
1000	15.6	15.6	1445
10000	85	113	2680
20000	128	211	3000
60000	201	594	5010
100000	245	975	6430

In this case it is to be seen that the increase of temperature due to the impulse wave is, up to a pressure of some thousand atmospheres, not markedly different from what would be indicated by adiabatic compression. Only when very high pressures are reached does the difference become marked.

6. The structure of the compression impulse.- In order to gain some knowledge of the structure of the wave front it is necessary to carry through the integration of equations (12). Conceive first that the gas in the tube is such that its spe-

cific heat is independent of temperature. We introduce the constants

$$\frac{c_p}{c_v} = k \; (= 1.4); \quad \delta = \frac{R}{2c_v} = \frac{k-1}{2} \; (= 0.2); \quad \xi_1 = \frac{k+1}{k-1} = \frac{1+\delta}{\delta} = (=6) \tag{23}$$

The values given in parentheses refer to diatomic gases.

Further,

$$E = c_v \; T \quad \text{and} \quad p \; v = R \; T.$$

To make the notation of the equations as simple as possible (12b) is multiplied by $\frac{1}{J}$ and (12c) by $\frac{R \; M^2}{c_v \; J^2}$. In place of the unknown factors v, p, and T, we substitute for them proportional, dimensionless magnitudes,

$$\omega = v \frac{M^2}{J}, \quad \varphi = \frac{p}{J} \quad \theta = \frac{R \; T \; M^2}{J^2} \tag{24}$$

and further, let

$$\alpha + 1 = \frac{2 \; F \; M}{J^2}, \; \mu' = \frac{\mu}{M} \quad \lambda' = \frac{\lambda}{c_v \; M} \tag{25}$$

Then equations (12b) and (12c) take the form

$$\omega + \frac{\theta}{\omega} - 1 = \mu' \; \frac{d\omega}{dx}, \tag{26a}$$

$$\theta - \delta[(1 - \omega)^2 + \alpha] = \lambda' \; \frac{d\theta}{dx} \tag{26b}$$

$$\theta = \omega \; \varphi \tag{26c}$$

With the exception of the physical constants δ, μ', λ', the entire process is represented by the use of only one constant, α.

The magnitudes ω_1, θ_1, φ_1 and ω_2, θ_2, φ_2 which at both sides of the wave front, are obtained by solving the quadratic equations, which by placing the left side of equations (26a) and (26b) equal to 0, gives

$$\omega_{\frac{1}{2}} = \frac{1}{2\,(\delta + 1)} \left\{2\,\delta + 1 + \sqrt{1 - 4\,\delta\,(\delta + 1)\,\alpha}\right\} \quad \text{(27a)}$$

$$\theta_{\frac{1}{2}} = \frac{\delta}{2\,(\delta + 1)^2} \left\{1 + 2\,(\delta + 1)\,\alpha \mp \sqrt{1 - 4\,\delta\,(\delta + 1)\alpha}\right\} \quad \text{(27b)}$$

$$\varphi_{\frac{1}{2}} = \frac{1}{2\,(\delta + 1)} \left\{1 \mp \sqrt{1 - 4\,\delta\,(\delta + 1)\,\alpha}\right\} \quad \text{(27c)}$$

The relationship between the evident magnitudes $\pi = \frac{p_2}{p_1} = \frac{\varphi_2}{\varphi_1}$ and the constant α is, according to (27c)

$$\alpha = \frac{1}{\delta\,(\delta + 1)} \; \frac{\pi}{(\pi + 1)^2}$$

The values (27) are easily represented on a ω, θ-plane (Fig. 5), as intersection points of the two parabolas,

$$\theta = -\left(\frac{1}{2} - \omega\right)^2 + \frac{1}{4} \quad \text{(28a)}$$

and

$$\theta = \delta\,\left\{(1 - \omega)^2 + \alpha\right\} \quad \text{(28b)}$$

(28a) is a parabola independent of δ and α. With opening below and with maximum, $\omega = \frac{1}{2}$ $\left(\theta = \frac{1}{4}\right)$. (28b), on the other hand, has its opening above, its minimum, $\omega = 1$. The parabola (28b) is displaced downward (without change of form) with decreasing values of α. It is easy to recognize the following special cases: points of contact of the two parabolas for $\alpha = \frac{1}{4\,\delta\,(\delta + 1)} = 1.04$; $\pi = \frac{p_2}{p_1} = 1$ (limit of infinitely weak sound waves). (28b) intersects the peak of (28a) for $\alpha = \frac{1 - \delta}{4\,\delta} = 1.$ $\frac{p_2}{p_1} = 1.5$. (The limit of infinitely intense sound waves: $\alpha = 0$; $p = \infty$.)

The course of a single particle across the wave front as indicated by the ω, θ coordinate figure, would correspond to a curve whose differential equation as drawn from (26a) and (26b) would be

$$\frac{\lambda'}{\mu'} \frac{d\theta}{d\omega} = \omega \frac{\theta - \delta\left\{(1 - \omega)^2 + \alpha\right\}}{\omega^2 - \omega + \theta} \tag{29}$$

The curve of the integral of (29) should pass through the points of intersection of the parabolas, that is, through the common points of differential equation.

For three special cases the value of $\frac{\lambda'}{\mu'}$ the integration is easily followed through: In the first place, we see that for the extreme values $\mu' = 0$ or $\lambda' = 0$, the curve of the integral of (29) will be identical with the parabola (28a) or (28b).

The first of these cases, namely, $\mu' = 0$, is the case where the effect of friction is neglected. It gives in general no continuous course of ω through the wave front; from (26a) $\theta = -\omega^2 + \omega$; and hence from (26b),

$$\lambda' \frac{d\theta}{d\omega} = (1 + \delta)\ (\omega_1 - \omega)\ (\omega - \omega_2).$$

$\frac{d\theta}{d\omega}$ is therefore positive for all values of ω between ω_1 and ω_2. But when the gas particle moves along the curve (28a) from I to II, the value of θ, as we saw, at first increases with increasing compression impulses $\left(\frac{p_2}{p_1} > 1.5\right)$ and then again decreases. The only way to escape this apparent contradiction seems to be (following the suggestion of Professor Prandtl) to assume a continuous course from I only to that point II' of the parabola (28a) at which the temperature θ_2 is just attained (at $\omega = 1 - \omega_2$) and then that the volume from value $1 - \omega_2$ jumps to ω_2 (without change of temperature).

The second case $(\lambda' = 0)$ offers no such difficulty; for from (26b)

$$\theta = \delta [\omega^2 - 2\omega + 1 + \alpha]$$

and hence from (26a)

$$\mu' = \frac{d\omega}{dx} = (1 + \delta) \frac{(\omega - \omega_1)(\omega - \omega_2)}{\omega} \quad (30)$$

and hence, after integration,

$$\frac{x}{\mu'} = \frac{1}{1 + \delta} \frac{\omega_1 \ln (\omega_1 - \omega) - \omega_2 \ln (\omega - \omega_2)}{\omega_1 - \omega_2} \quad (30a)$$

The third case presents itself when we make the assumption by way of trial and write the integral of (29)

$$\theta = A\omega^2 + B\omega + C \quad (31)$$

The curve shall pass through the points I and II. If we introduce the value of θ in (31) into (29), the right side of the equation will consist of a polynomial of the second order in ω, which for ω_1 and ω_2 vanishes. Both polynomials have therefore up to one factor the value $(\omega - \omega_1)(\omega - \omega_2)$. Since this factor is identical with that of ω_2 it is clear to see that (29) by substitution of (31) becomes

$$\frac{\lambda'}{\mu'} (2A\omega + B) = \omega \frac{A - \delta}{A + 1}$$

which can only be so if

$$\frac{\lambda'}{\mu'} 2A = \frac{A - \delta}{A + 1} \quad \text{and} \quad B = 0 \tag{32}$$

on the other side, the points θ_1, ω_1 and θ_2, ω_2 must lie on (31) which also requires $B = 0$. Then

$$A = \frac{\theta_1 - \theta_2}{\omega_1^2 - \omega_2^2}; \qquad C = \frac{\theta_2 \omega_1^2 - \theta_1 \omega_2^2}{\omega_1^2 - \omega_2^2}$$

With the values (27),

$$A = - \frac{\delta}{\delta + 1}; \qquad C = \delta \frac{1 + \alpha}{2\delta + 1} \tag{33}$$

If this value of A is introduced in (32) the statement may be made: Equation (31) gives a solution of the problem only when

$$- \frac{\lambda'}{\mu'} \frac{2\delta}{2\delta + 1} = - \frac{\frac{\delta}{2\delta + 1} + \delta}{1 - \frac{\delta}{2\delta + 1}}$$

that is when

$$\frac{\lambda'}{\mu'} = 1 + 2\delta \tag{34}$$

This is the third special value for $\frac{\lambda'}{\mu'}$ for which the integration offers small difficulty. From (31) and (33)

$$\theta = \frac{\delta}{2\delta + 1} (1 + \alpha - \omega^2);$$

from (26a)

$$\mu' \frac{d\omega}{dx} = \frac{\omega^2 + \theta - \omega}{\omega}$$

becomes, since the numerator is to the right of the zero position of ω_1 and ω_2,

$$\mu' \frac{d\omega}{dx} = \frac{\delta + 1}{2\delta + 1} \frac{(\omega - \omega_1)(\omega - \omega_2)}{\omega} \tag{35}$$

or, by integration

$$\frac{x}{\mu'} = \frac{2\delta + 1}{\delta + 1} \frac{\omega_1 \ln(\omega_1 - \omega) - \omega_2 \ln(\omega - \omega_2)}{\omega_1 - \omega_2} \tag{35a}$$

This result differs from (30), where $\lambda' = 0$ was introduced, only by the factor $2\delta + 1$.

The physical application of this solution depends on how nearly equation (34) describes the process for real gases. From (25) and (23) we have the relations

$$\frac{\lambda}{\mu\, c_v} = \frac{c_p}{c_v} \quad \text{or} \quad \lambda = \frac{4}{3}\, \eta\, c_p$$

also

$$\frac{c_p}{c_v} = 1.4 \qquad \lambda = 1.86\, \eta\, c_v \tag{34a}$$

D. E. Meyer in his gas theory gives the value of

$$\lambda = 1.6\, \eta\, c_v$$

For air the observed values are ($\lambda = 0.56 \times 10^{-4}$, $\eta = 1.7 \times 10^{-4}$ and $c_v = 0.17$), $\frac{\lambda}{\eta c_v} = 1.94$. The value obtained by (34) is 1.86. It lies between the gas theory value and the observed values for air, 1.94. The solution given by (35) may be taken as satisfactory.

7. The thickness of the impulse wave.- We shall consider the value of ω as obtained by (35a) u function of x (Fig. 6), and draw a tangent at the point of steepest inclination to

x. The length l between the intersection of this tangent with the horizontal, ω_1 and ω_2 we define with Prandtl, the thickness l of the wave front. Then

$$l = (\omega_1 - \omega_2) : \left(\frac{d\omega}{dx}\right)_{max}.$$

According to (35), $\frac{d\omega}{dx}$ has its maximum for $\omega_{max} = \sqrt{\omega_1 \omega_2}$ with a value

$$\mu' \left(\frac{d\omega}{dx}\right)_{max} = - \frac{\delta + 1}{2\delta + 1} (\sqrt{\omega_1} - \sqrt{\omega_2})^2.$$

Hence

$$l = \frac{\mu}{M} \frac{2\delta + 1}{\delta + 1} \frac{\sqrt{\frac{\omega_1}{\omega_2}} + 1}{\sqrt{\frac{\omega_1}{\omega_2}} - 1} \tag{36}$$

If the increase in pressure $\pi = \frac{p_2}{p_1}$ given then according to (19)

$$\frac{\omega_1}{\omega_2} = \frac{v_1}{v_2} = \frac{\pi \zeta_1 + 1}{\pi + \zeta_1} \quad \text{and} \quad M^2 = \frac{u_1^2}{v_1^2} = \frac{p_1}{v_1} \frac{\pi \zeta_1 + 1}{\zeta - 1},$$

hence the wave thickness

$$l = \mu \frac{\zeta_1 + 1}{\zeta_1} \sqrt{\frac{v_1}{p_1}} \sqrt{\frac{\zeta_1 - 1}{\pi \zeta + 1}} \frac{\sqrt{\frac{\pi \zeta_1 + 1}{\pi + \zeta_1}} + 1}{\sqrt{\frac{\pi \zeta_1 + 1}{\pi + \zeta_1}} - 1} \tag{36a}$$

For air at atmospheric pressure and 0°C

$$\mu = \frac{4}{3} \eta = 2.3 \times 10^{-4} \frac{g}{cm/s}$$

$$v_1 = \frac{22400}{29} \frac{cm^3}{g}$$

$$p_1 = 1.013 \times 10^6 \frac{g}{cm/s^2}$$

$$\zeta_1 = 6$$

so that

$$\mu \frac{\zeta_1 + 1}{\zeta_1} \sqrt{\frac{v_1}{p_1}} = 74 \times 10^{-7} \text{ cm}$$

π, also the impulse pressure p_2, is expressed in atmospheres. For various values of p_2, the value of l from (36a) is

Impulse pressure p_2, in atm.	2	5	10	100	1000	2000	3000
Wave thickness, $l \times 10^{-7}$	447	117	66	16.5	5.2	3.6	2.9

The value of l is so small that it approaches molecular dimensions. According to the gas theory the average free path is 90×10^{-7}/cm and the average distance between two molecules is $\sqrt[3]{\frac{22400}{6.2 \times 10^{23}}} = 3.3 \times 10^{-7}$/cm. From these it is seen that the width of the wave front is for $p_2 = 8$ atm. already less than the average free path; and at something over 2000 atm. less than the average distance between two molecules.

The above consideration would indicate that the fundamental equations under (3) do not describe the actual processes taking place within the wave front. These equations, based on a physical continuum, have a real physical meaning only so long as the separate gas particles during a measurable change in T and v still represent a great number of impacts. As the results just given show such a condition cannot exist within an intense impact wave. The heating and compression is in reality much more the effect of single extremely small molecular impacts. A description of the compressional impact wave that shall tally with the actual process is only possible when based upon a consideration of these individual impacts instead of/on the concepts p, v, and T. These magnitudes within the area of the wave front can have little significance.

Under these circumstances a solution of the problem might be sought from the standpoint of the kinetic theory of gases, and as follows: A distribution function is defined

$$F\,(x,\ \xi,\ \eta,\ \zeta),$$

as having the meaning that at the point x of the tube the number of molecules (per unit volume) with velocities between ξ and $\xi + d\,\xi$, η and $\eta + d\eta$, ζ and $\zeta + d\zeta$ is given by

$$dN = F\,d\,\xi\,d\,\eta\,d\,\zeta = F\,d\,\omega.$$

F must be so specified that for $x = -\infty$ or $x = +\infty$ it passes into the Maxwell function

$$f_1 = n_1 \sqrt{\frac{h_1\,m}{\pi}}^{\,3} e^{-h_1 m[(\xi - u_1)^2 + \eta^2 + \zeta^2]}$$

$$f_2 = n_2 \sqrt{\frac{h_2\,m}{\pi}}^{\,3} e^{-h_2 m[(\xi - u_2)^2 + \eta^2 + \zeta^2]},$$

in which $m\,n = \rho$ = density; $\frac{1}{2hm} = RT = \frac{p}{\rho}$. And further, the transport of mass, momentum and energy in the x-direction, as well as the integrals

$$m\int_{-\infty}^{+\infty} \xi\,F d\omega \qquad m\int_{-\infty}^{+\infty} \xi^2\,F d\omega \qquad \frac{m}{2}\int_{-\infty}^{+\infty} \xi\,(\xi^2 + \eta^2 + \zeta^2)\,F d\omega$$

must be independent of x.

And, finally, the distribution, given by F, must be stationary as is the case according to Maxwell-Boltzmann (Boltzmann, Vorlesungen Bd. I, equation 114) if

$$\xi \frac{\partial F}{\partial x} = \int \int \int (F' \, F_1{}' - F \, F_1) \, g \, b \, d \, \omega_1 \, d \, b \, d \, \epsilon$$

in Boltzmann's notation.

The solution of the problem stated in this form would be, however, a very incomplete substitute for the treatment of single impacts which for intense concentrations would not maintain a constant distribution function.

The structure of the impact wave in liquids may be deduced exactly as in the case of gases (Section 5), for their macroscopic characteristics. By the use of Tammann's equation of state, the values (20, 20a, and 21) give the fundamental equations (12b) and (12c) for the stationary impact wave,

$$M^2 \, v' + \frac{CT}{v'} - (J + K - M^2 \, b) = \mu \, M \, \frac{dv'}{dx}$$

and

$$c_v T + v' \, (J + K - M^2 b) - \frac{1}{2} \, M^2 v'^2 - \left(F - b(J + K - \frac{M^2}{2} \, b)\right) = \frac{\lambda}{M} \, \frac{dT}{dx}$$

Let

$$J + K - M^2 b = J' \quad \text{and} \quad F - b \, (J + K - \frac{M^2}{2} \, b) = F'$$

Multiply the first equation by $\frac{1}{J'}$, the second by $\frac{CM^2}{c_v \, J'^2}$ and, as in the case of gases, let

$$\omega' = v' \, \frac{M^2}{J'}; \quad \theta' = \frac{CTM^2}{J'^2}; \quad \varphi' = \frac{p + K}{J}$$

and for the constants,

$$\delta' = \frac{C}{2 c_v}; \quad \alpha' + 1 = \frac{2F'M^2}{J'^2}; \quad \mu' = \frac{\mu}{M}; \quad \lambda' = \frac{\lambda}{c_v M},$$

Then we will have

$$\left.\begin{aligned} \omega' + \frac{\theta'}{\omega'} - 1 &= \mu' \frac{d\omega'}{dx} \\ \theta' - \delta'[(1 - \omega')^2 + \alpha'] &= \lambda' \frac{d\theta'}{dx} \\ \theta' &= \varphi' \omega'. \end{aligned}\right\} \quad (37)$$

These equations are in fact wholly analogous with (26) which have already been discussed. In order to determine which integral (30) or (35) is to be selected, we have the observed value of $\frac{\lambda'}{\mu'}$ which may be compared with the values in (34). For the "third case" we have

$$\frac{\lambda}{c_p \eta} = \frac{4}{3} = 1.33.$$

For the case of ethyl ether it has been observed,

$$\lambda = 0.00035 \frac{\text{cal}}{\text{cm s deg.}}; \quad c_p = 0.564 \frac{\text{cal}}{\text{g deg.}}; \quad \eta = 0.0028 \frac{\text{g}}{\text{cm s}};$$

hence

$\frac{\lambda}{c_p \eta} = 0.22.$ $\frac{\lambda}{\eta}$ is therefore about six times smaller than the value indicated in (34). We would come nearer the truth if we select the "second case," ($\lambda' = 0$), which according to (30) gives

$$\mu' \frac{d\omega'}{dx} = (1 + \delta') \frac{(\omega' - \omega'_1)(\omega' - \omega'_2)}{\omega'}$$

and the wave front thickness

$$l_{fl} = \frac{\mu}{M} \frac{1}{1 + \delta'} \frac{\sqrt{\frac{\omega'_1}{\omega'_2} + 1}}{\sqrt{\frac{\omega'_1}{\omega'_2} - 1}} \tag{38}$$

From equation (22) we obtain, as in the case of gases,

$$l_{fl} = \mu \frac{\zeta - 1}{\zeta} \sqrt{\frac{v_1 - b}{p_1 + K}} \sqrt{\frac{\zeta - 1}{\pi' \zeta + 1}} \frac{\sqrt{\frac{\pi' \zeta + 1}{\pi' + \zeta} + 1}}{\sqrt{\frac{\pi' \zeta + 1}{\pi' + \zeta} - 1}} \tag{39}$$

wherein, as in (23)

$$\zeta = \frac{1 + \delta'}{\delta'} \quad \text{and} \quad \pi' = \frac{p_2 + K}{p_1 + K}$$

By substituting the values given above, we obtain the values for the thickness of the impact wave as follows:

Impact pressure p_2, atm.	100	1000	10000	100000
Wave thickness $l \times 10^{-7}$cm	52	5.3	0.65	0.14

The thickness of the wave front for the case where the fluid is a liquid is seen to be of the same order as that of gases. There is met with again in this case calculated values for the thickness of impact waves of intense concentration, magnitudes that are smaller than the average distance between two molecules which for ether is calculated to be 0.55×10^{-7} cm. Continuum physics is in this case, as in gases, inadequate to describe processes occurring within impact waves.

(To be followed by Technical Memorandum No. 506, containing Part II of this article.)

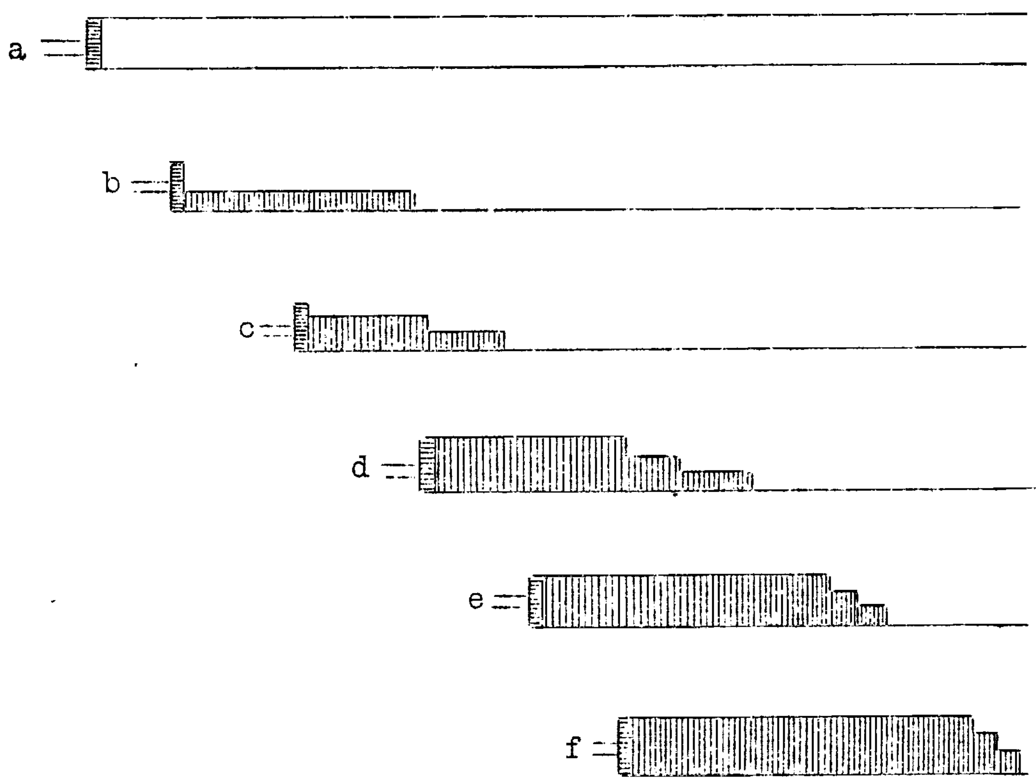

Fig.1

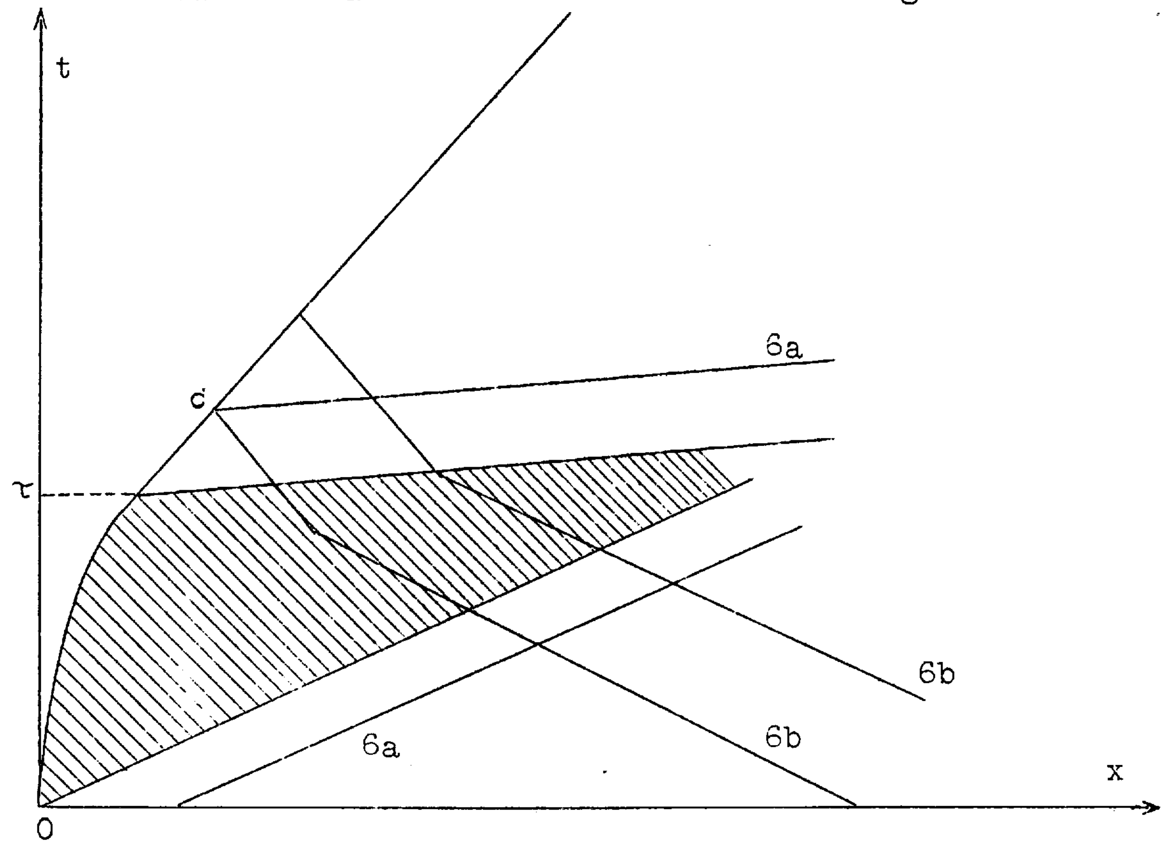
t
c
τ
6a
6a
6b
6b
x
0

Fig.2

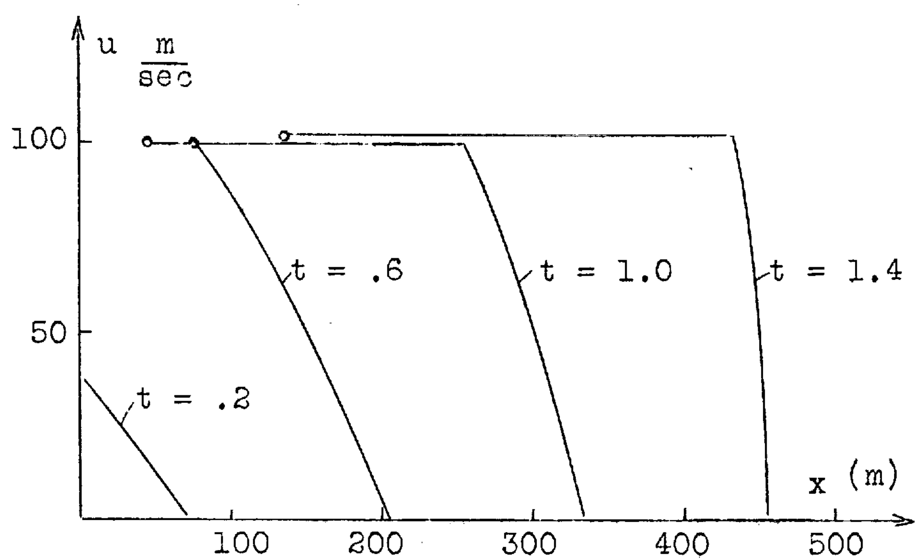
u m/sec
100
50
t = .2
t = .6
t = 1.0
t = 1.4
x (m)
100
200
300
400
500

Fig.3

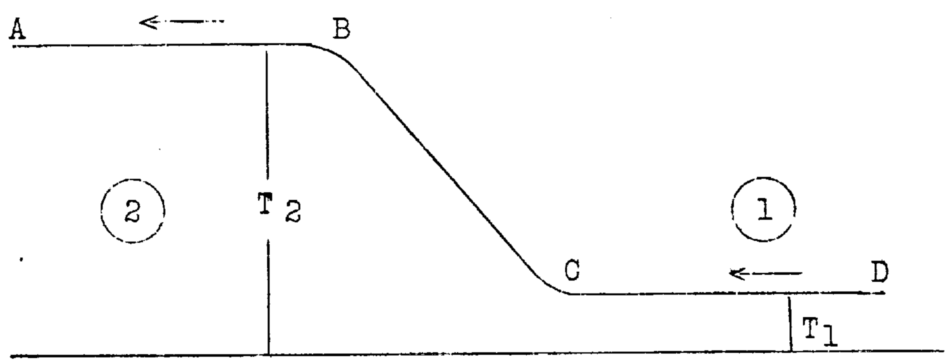

Fig.4

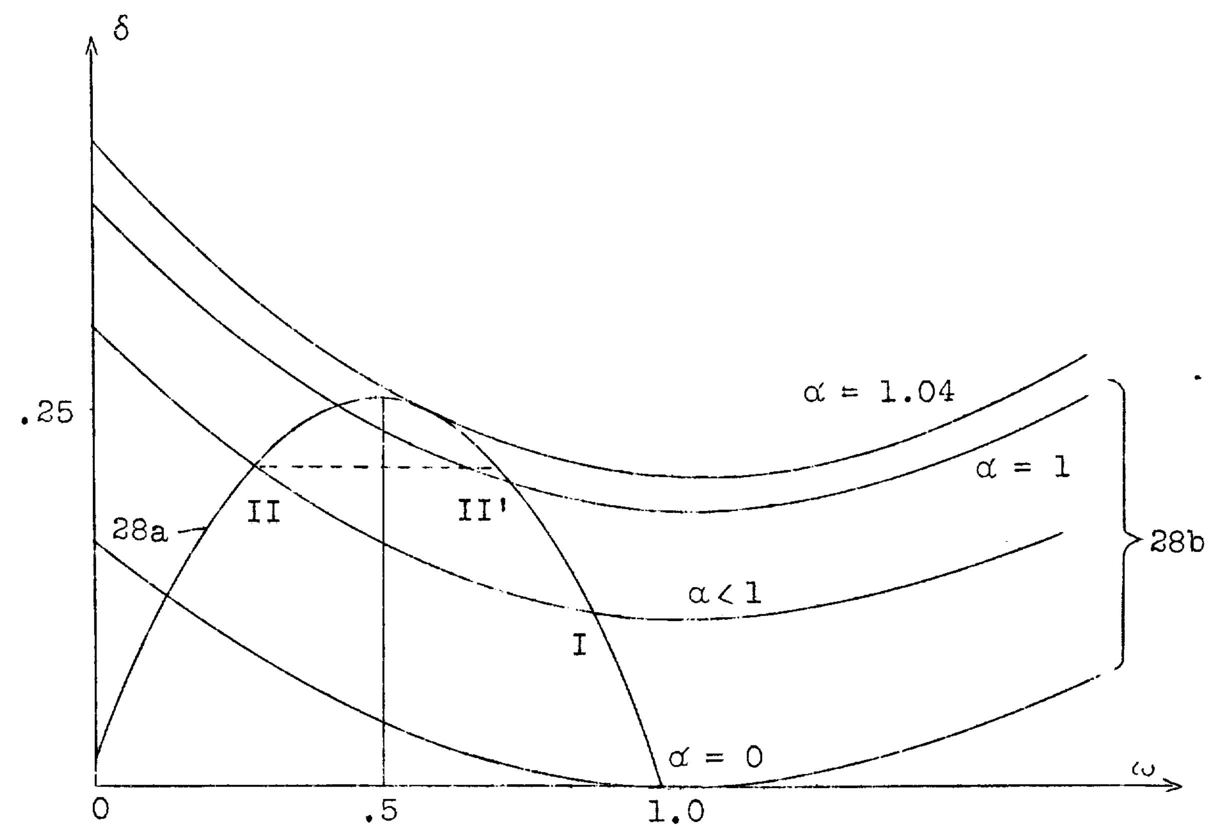

Fig.5

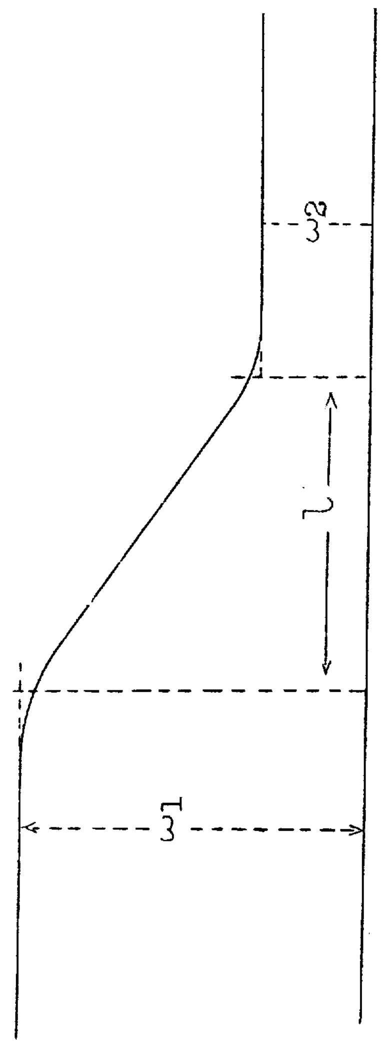

Fig.6