TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 529

METAL CONSTRUCTION DEVELOPMENT

By H. J. Pollard

PART IV

Moments of Inertia of Thin Corrugated Sections

From Flight, April 25, 1929

Washington
September, 1929

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Moments of Inertia of Thin Corrugated Sections.

One of the difficulties of the individual who has not learned a little very elementary calculus, and who wishes to be able to design beams, etc., from thin metal, lies in not being able to compute rapidly the constants of corrugated sections. For ordinary box or girder sections of timber there is no difficulty; the draftsman can remember the necessary formulas, or easily turn them up in almost any handbook bearing on engineering, but formulas for corrugated shapes are not as yet published in the usual handbooks, and it is fitting that we should discuss some of these. In the first instance, then, we will take a simple case and establish a formula for the moment of inertia of a circular arc or, more correctly, circular annulus, which is usually referred to as an arc, about any axis, which in this instance is typified by the line SS as shown in Figure 1.

Let AB (Fig. 1) then be such a circular annulus of thickness t which is small compared with r, the radius of the arc.

*From Flight, April 25, 1929. (For Parts I, II, and III, see N.A.C.A. Technical Memorandums Nos. 526, 527 and 528.)

The center of the arc is taken as origin of coordinates, and the radii oA and oB drawn from the origin through A and B makes angles α and β respectively, with oy the axis of y. Any radius typified by oP makes an angle θ with oy.

The length of an element is $r\,d\,\theta$, its width t, and its distance from the axis SS is $l + r\cos\theta$, consequently, its moment of inertia about SS is $r\,t\,(l + r\cos\theta)^2\,d\,\theta$, and we have for the whole annulus.

$$\frac{I_{SS}}{t} = r\int_\alpha^\beta (l + r\cos\theta)^2\,d\,\theta = r\int_\alpha^\beta (l^2 + 2lr\cos\theta + r^2\cos^2\theta)\,d\theta$$

$$= r\int_\alpha^\beta \left\{(l^2 + 2lr\cos\theta + \frac{r^2}{2}(1 + \cos 2\theta)\right\}d\theta$$

$$= r\left[\left(l^2 + \frac{r^2}{2}\right)\theta + 2lr\sin\theta + \frac{r^2}{4}\sin 2\theta\right]_\alpha^\beta$$

$$= r\left[\left(l^2 + \frac{r^2}{2}\right)(\beta - \alpha) + 2lr\sin(\beta - \alpha) + \frac{r^2}{4}(\sin 2\beta - \sin 2\alpha)\right]$$

In what follows we are concerned with the particular case:

$$\alpha = 0$$
$$l = 0$$
$$\beta = \beta$$

That is to say, with the moment of inertia about ox of an annulus beginning on oy struck from center o and subtending an angle β at o.

Then

$$\frac{I_{ox'}}{t} = \frac{r^3}{2}\left[\beta + \frac{\sin 2\beta}{2}\right] \qquad (1)$$

As we are not considering how variations of t affect the matter we may write

$$I_{ox}' = I_{ox} \times t$$

where I_{ox} is the moment of inertia of an arc without breadth. We then get

$$I_{ox} = \frac{r^3}{2}\left(\beta + \frac{\sin 2\beta}{2}\right) \tag{1a}$$

Again, to find the vertical distance designated by $\bar{y}$ of the center of gravity of the arc above SS, we have in the general case, the area of the element being as before

$$\bar{y} = \frac{r\int_\alpha^\beta (l + r\cos\theta)\,d\theta}{r\,(\beta - \alpha)} = \frac{(l\theta + r\sin\theta)_\alpha^\beta}{\beta - \alpha}$$

$$= \frac{l\,(\beta - \alpha) + r\,(\sin\beta - \sin\alpha)}{\beta - \alpha}$$

We are concerned with the particular case in which

$$\alpha = 0$$
$$l = 0$$
$$\beta = \beta$$

Then $\bar{y} = \dfrac{r\sin\beta}{\beta}$ (2)

The relationship between $\bar{y}$ and β in this equation is shown plotted in Figure 4.

Now by using the theorem of parallel axes we are able to calculate the moment of inertia of the arc AB (Fig. 1) about the axis GG, which passes through its center of gravity.

A statement of the theorem as applied to an arc is: If the length of an arc be L and the radius of gyration about any axis passing through its centroid be k, then the moment of inertia of the arc about any parallel axis distance C from the centroidal axis is $L (k^2 + C^2)$. This "theorem of parallel axes" applies equally to the moment of inertia of surfaces or to masses, consequently the moment of inertia of the arc AB about the axis GG through the center of gravity and parallel to ox is by (1a) and (2).

$$\frac{r^3}{2}\left(\beta + \frac{\sin 2\beta}{2}\right) - \frac{r\beta \times r^2 \sin^2 \beta}{\beta^2}$$

$$= \frac{r^3}{2}\left[\beta + \frac{\sin 2\beta}{2} - \frac{2 \sin^2 \beta}{\beta}\right] \tag{3}$$

Equation (3) is shown plotted on Figure 5

$$\left(\alpha + \frac{\sin 2\alpha}{2} - \frac{2 \sin^2 \alpha}{\alpha}\right) \text{ against } \alpha \text{ in degrees.}$$

The whole range taken cannot be shown on one ordinary size chart as a continuous curve, so that for the small angles radii the horizontal scale is a hundred times that for angles of 30° and over.

Figure 6 gives $\bar{y}$ for a number of values of r and similar curves should be plotted from equation (3), thus I_{cg} for a large range of arcs could be read off directly.

Now with these figures the moment of inertia of a contour consisting of arcs of circles and flats can be rapidly obtained.

The method will best be illustrated by means of an example.

Let us consider the flange only by a spar, say, 4 inches deep; this section is shown in Figure 2.

The various values are shown tabulated in Table I. The values of $\overline{y}$ are read off the curves in Figure 6. The values of c are obtained by simple addition or subtraction and I_{cg} is taken from Figure 5.

TABLE I.

Part of contour	Length	y	c	Ac^2	I_{cg}
a	0.0925	0.086	1.806	0.302	Negligible
b	0.389	0.365	1.945	1.47	0.0011
c	0.55	0.31	1.89	1.965	0.0050
d	0.1309	0.075	1.64	0.352	Negligible
e	0.2	0.0	1.615	0.522	0.0
				4.611	0.0061

Therefore, I of half flange about horizontal axis through C.G. of spar = 4.617 x t.

It is seen that the figures in the sixth column are practically ineffective and could be ignored; moreover, in such a case as this it is unnecessary to get the position of the centers of gravity of the small arcs from the graphs, the distance of the centers of radii from the inertia axis, taken directly from the drawing, giving all the accuracy required.

The following is the expression for the moment of inertia of the flange when estimated in the usual way:

$$\frac{I}{2t} = 0.1 \int_{0}^{53} (1.892 - 0.1 \cos\theta)^2 \, d\theta$$

$$+ 0.42 \int_{-53}^{75} (1.58 + 0.42 \cos\theta)^2 \, d\theta$$

$$+ 0.1 \int_{0}^{75} (1.715 - 0.1 \cos\theta)^2 \, d\theta + 0.2 \times 1.615^2$$

By using equation (1) the result is eventually arrived at, but the work is laborious and unless one is continually making such calculations the chances of error are considerable. By using the graphs the result is arrived at in a fraction of the time and to the necessary degree of accuracy.

We have considered a case of the moment of inertia of a flange about the central axis of the spar. The moment of inertia of the web about the same axis can be obtained in a similar manner, thus giving the total moment of inertia of the section and as the y and A are known, the fiber stresses in a spar when given loads come on to it can be determined, if the assumption that the ordinary methods of estimating stresses for thick walled "boxes," etc., applies equally well for thin-walled sections inside the elastic range is made. The stress at which elastic instability is likely to set in must be estimated for these flanges, webs, etc., and this estimation involves finding the moment of inertia or radius of gyration of these parts about various axes. Usually a determination of the k about an axis passing through the centroid of the flange parallel to the spar axis is sufficient.

The calculation may be made in an exactly similar manner to that described above, but it frequently happens that terms which may be ignored in calculating the constant for the whole spar are relatively too big to be ignored in calculating the constant for a component part.

As illustrating this matter, Figure 3 is identical with Figure 2, so far as radii and angles are concerned, but the horizontal centroidal axis is shown, and the centers of curvature are given in relation to this line.

The complete expression for the moment of inertia of the section about this axis is:

$$\frac{I}{2t} = 0.1 \int_0^{53} (0.1 \cos\theta - 0.036)^2 \, d\theta$$
$$+ 0.42 \int_{-53}^{75} (0.42 \cos\theta - 0.276)^2 \, d\theta$$
$$+ 0.1 \int_0^{75} (0.141 + 0.1 \cos\theta)^2 \, d\theta + 0.2 \times 0.241^2$$

By using our graphs we can quickly tabulate and obtain the result as indicated in Table II.

TABLE II.

Part	Length	y	c	Ac^2	I_{cg}
a	0.0975	0.086	0.05	0.000244	Negligible
b	0.3886	0.363	0.087	0.00299	0.0011
c	0.55	0.31	0.034	0.000636	0.005
d	0.1309	0.074	0.215	0.00605	Negligible
e	0.2	0.0	0.241	0.0116	0.0
				0.02147	0.0061

As before, y and I_{cg} are read from the graphs. I = 0.02757 x t. In this case then, two terms which were negligible in the over-all constant are of importance here. A computation such as the above is not a long job; nevertheless, the time can be shortened by a judicious choice of centers and the ignoring of terms which are obviously unimportant at the outset.

The graphs may, of course, be extended to cover a larger range of angles and radii, also, at the expense of some complication in plotting the large arc could be dealt with as one whole, instead of dividing it into portions b and c, but it is doubtful if this is worth while.

The reader will have noted that in obtaining formula (1), angles were measured from the vertical axis oy. If the values of the spar "constants" about the vertical axis oy are required, or if angles are measured from the horizontal axis, a length m replacing l, expression for integration is:

$$r \int_\alpha^\beta (m + r \sin^2 \theta) d\theta = r \int_\alpha^\beta (m^2 + 2\ mr \sin\theta + r^2 \sin^2 \theta)\ d\theta$$

$$= r \int_\alpha^\beta \left[m^2 + 2\ mr \sin \theta + \frac{r^2}{2} (1 - \cos 2\theta)\right] d\theta$$

Giving

$$r \left[\left(m^2 + \frac{r^2}{2}\right) \theta - 2\ mr \cos \theta - \frac{r^2}{4} \sin 2\theta\right]_\alpha^\beta$$

$$= r\left[\left(m^2 + \frac{r^2}{2}\right)(\beta - \alpha) - 2\ mr (\cos \beta - \cos \alpha) - \frac{r^2}{4} (\sin 2\beta - \sin 2\alpha)\right]$$

When

$$\alpha = 0$$
$$m = 0$$
$$\beta = \beta$$

the expression reduces to

$$\frac{I_{oy}}{r} = \frac{r^3}{2}\left(\beta - \frac{\sin 2\beta}{2}\right)$$

The distance of the center of gravity from oy is

$$\overline{x} = \frac{\int_\alpha^\beta (m + r \sin\theta)\, d\theta}{r\,(\beta - \alpha)} = \frac{m\,(\beta - \alpha) - r\,(\cos\beta - \cos\alpha)}{r\,(\beta - \alpha)}$$

when

$$m = 0$$
$$\alpha = 0$$
$$\beta = \beta$$

$$\overline{x} = \frac{r\,(1 - \cos\beta)}{\beta} \qquad (5)$$

Then

$$\frac{I_{cg} \text{ vertical}}{t} = \frac{r^3}{2}\left[\beta - \frac{\sin 2\beta}{2} - \frac{2\,(1 - \cos\beta)^2}{\beta}\right] \qquad (6)$$

From expressions (5) and (6) a set of graphs showing corresponding values of I_{cg}, r and β can be plotted. With such a set of charts at hand, the determining of the necessary constants for complicated sections made up of circular arcs and straight lines is easily and quickly carried out.

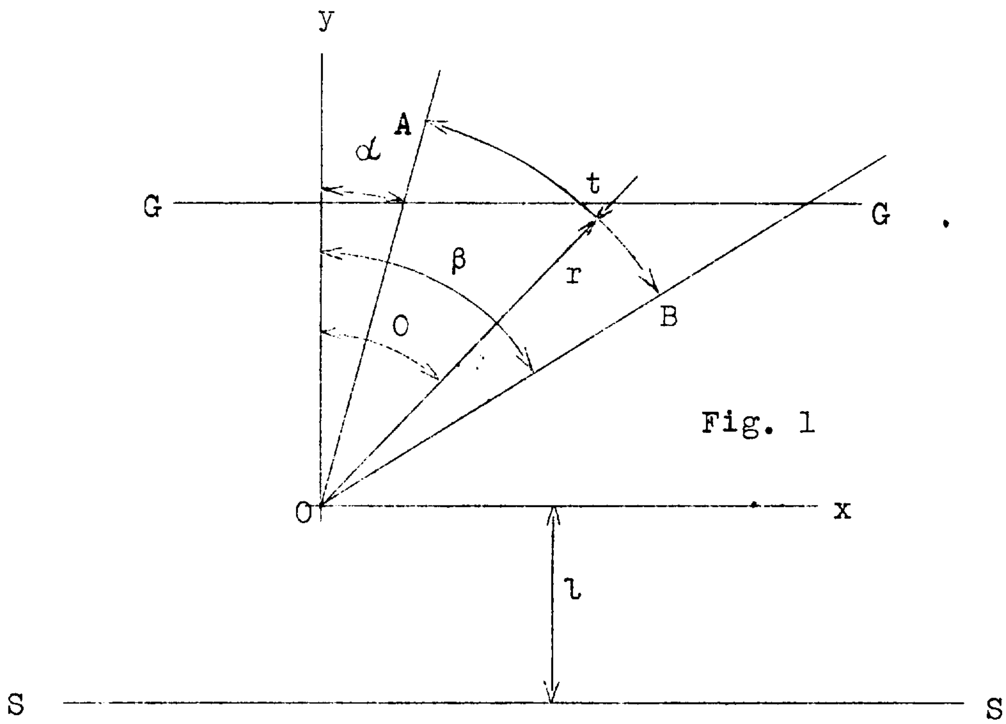

Fig. 1

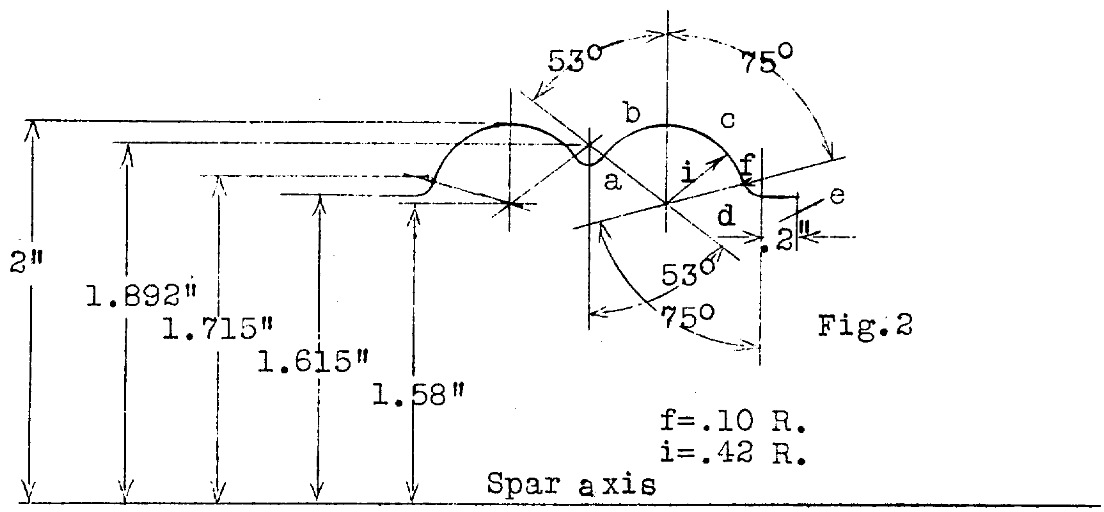

Fig.2

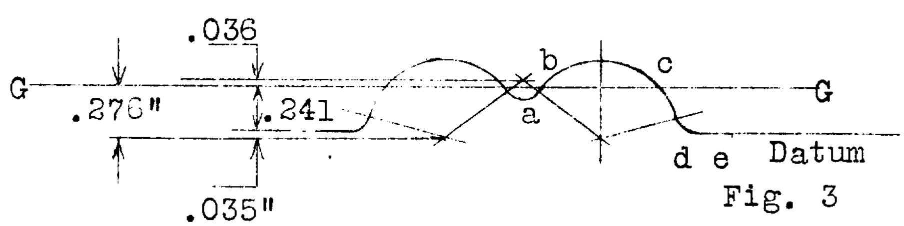

Fig. 3

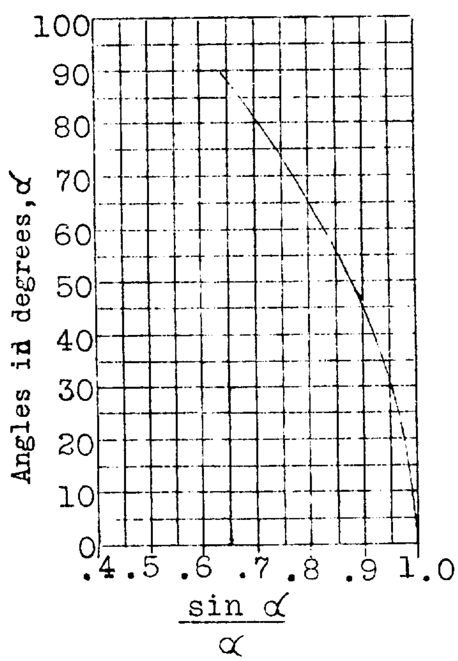

Fig. 4

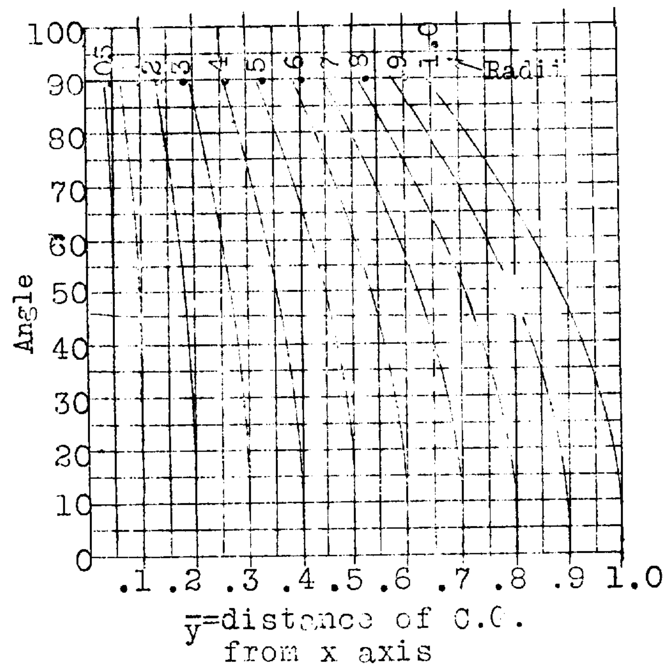

Fig. 6

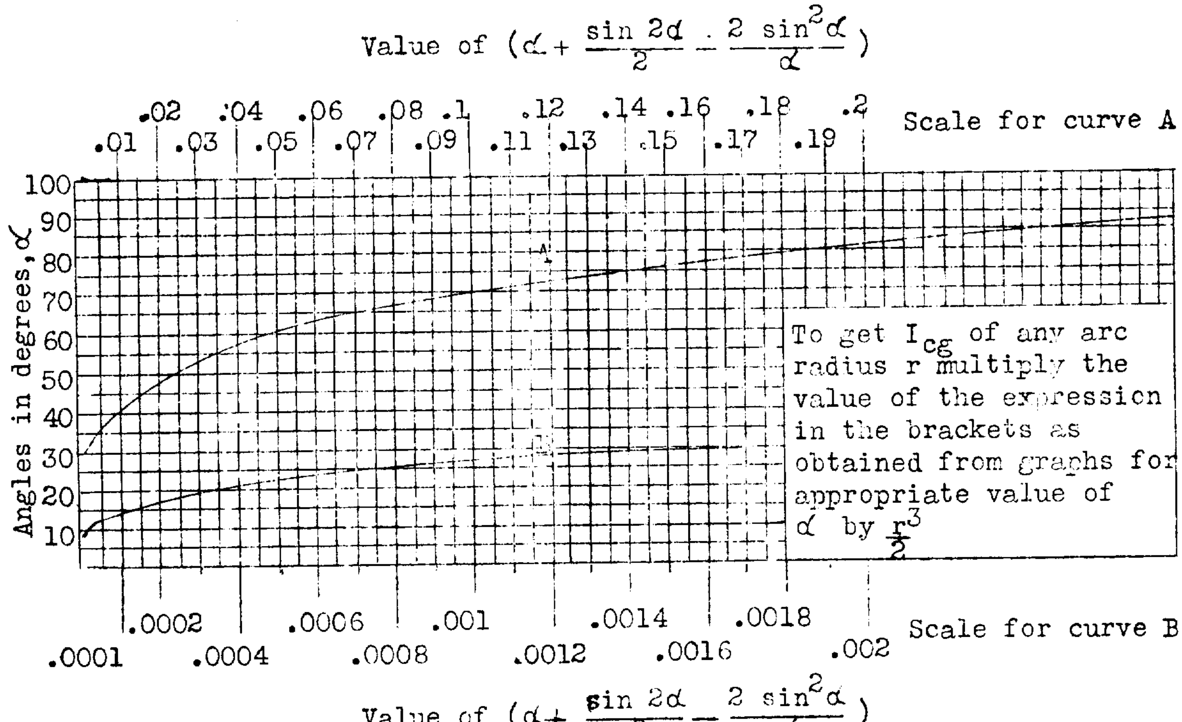

Fig. 5