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CALCULATION OF THE PRESSURES ON AIRCRAFT ENGINE BEARINGS By O. Steigenberger -

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 543.

CALCULATION OF THE PRESSURES ON AIRCRAFT ENGINE BEARINGS.* By O. Steigenberger.

The area of the bearing surfaces plays a decisive role in the calculation of the driving gear of aircraft engines. Aside from the torsional rigidity of the crank shaft and its vibrational characteristics, the utility and life of an aircraft engine depend largely on the bearings. An ample area of the bearing surfaces is chiefly opposed by considerations of weight and structural length, so that the bearing load has to be carried to its maximum limit.

For automobile engines we are generally limited to the determination of the maximum pressures exerted on the driving gear by the gas and by inertia. This suffices so long as only one piston works on one crank. The development of aircraft engines, however, produced engine types in which two or more pistons work on one and the same crank. This made it necessary to determine the resulting pressures more accurately and to give special attention to the combined effect of several forces acting simultaneously in different directions in the same plane.

We have to determine the maximum pressure, the span between the maximum and minimum pressure, and the pressure curve between "Beitrag zur Triebwerksberechnung von Flugmotoren," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, March 14, 1929, pp. 113-123.

the two pressures with respect to both time and location. This is necessary in order to determine the dimensions of the bearings, to locate the maximum local pressures, and to obtain data for the lubrication (determination of the points at which the lubricant can be applied).

Moreover, it is very important to know how the total pressure is affected by variations in the revolution speed. A method is needed which will enable the constructor to determine, without repeated tedious calculations, the course and variation of the pressure under different conditions of operation. For aircraft engines the three principal operating conditions are idling speed, cruising speed, and diving with engine stopped. It is also desirable to know the effect of the transition from one condition to another.

In what follows, we will discuss a method which affords a good idea of the course of the pressure for the above-mentioned operating conditions. The pressures produced in the driving gear are of three kinds, namely, the pressure due to the gases, the pressure due to the inertia of the rotating masses, and the pressure due to the inertia of the reciprocating masses.

It is assumed that only gas pressures corresponding to the normal indicator diagram occur at idling speed. For the investigation it is expedient to take them at their full value, though in reality they are somewhat smaller, due to the imperfect mixture. The inertia forces are disregarded because of their small-

ness and their favorable effect.

At cruising speed the full gas pressures and the inertia pressures work together corresponding to this revolution speed.

In diving only the inertia forces at the diving revolution speed, which can be assumed to be 20-25% above the cruising speed, are taken into consideration. The engine works like a pump, but the resulting pressures, opposed to the inertia pressures, are disregarded, because they are relatively small.

In the investigation, the gas pressures can therefore be assumed to be constant, as independent of the revolution speed. The inertia pressures, on the contrary, are dependent on the revolution speed and vary in the ratio of the squares of the same. In the determination of the bearing pressures, it is therefore expedient to consider the gas and inertia pressures apart from one another.

The method will first be explained for the simplest case, when only one cylinder works on one crank. The crank circle is divided into 24 equal parts of 15° each, starting from the upper dead center. The divisions are numbered from 1 to 24, the points for the third and fourth strokes being designated by 1' to 24'. With reference to the driving-gear constants, $\mathbf{r} = \text{crank radius}$, l = connecting-rod length, and $\lambda = \text{connecting-rod ratio}$, the tangential and normal components of the gas pressures on the crank pin are determined in the usual way from the indicator diagram. The normal components work in the direction of the crank

web, and the tangential components perpendicular to it (Fig. 1).

For the determination of the inertia pressures, the weights of the rotating and reciprocating masses and the weights per square centimeter of the piston area must first be determined. The inertia pressures from the rotating masses are given by the formula

$$p_u = \frac{\dot{m}}{f} r \omega^2$$

in which m represents the rotating masses and f the area of the piston. The pressures from the rotating masses act in a normal direction.

For the pressures from the reciprocating masses we have the formula

$$p_h = \frac{m}{f} r \omega^2 (\cos \alpha \pm \cos 2 \alpha).$$

Their diagram, as based on the piston stroke, is plotted in the usual way (Fig. 2) and resolved into normal and tangential components the same as for gas pressures (Fig. 3). The tangential forces, therefore, are the tangential components of the gas pressures, the normal components of the inertia pressures from the reciprocating masses, and the inertia pressures from the rotating masses.

In order to obtain a good idea of the effect of the separate pressures, it is expedient to refer all the pressures to the crank pin at rest and to plot them as vectors proceeding from the crank pin. With reference to the three above-mentioned operating

conditions, the separate pressures are plotted according to their origin.

The tangential and normal components of the gas pressures on the crank pin are first combined. The separate resultants (here equal to the connecting-rod forces) form vectors which vary in magnitude and direction. If their end points are connected, a closed curve is obtained as shown by the dash line in Figure 4.

In like manner the total inertia pressures are plotted from the rotating and reciprocating masses (Fig. 5). Here also a closed curve is obtained, which is naturally repeated in the four-stroke cycle. Since all inertia forces vary as the square of the revolution number, the direction of the vectors remains the same for all revolution speeds. Hence it is only necessary to enlarge or reduce each vector in the ratio of the squares of the revolution numbers in order to obtain the new pressure diagram. These pressure diagrams are similar for all revolution speeds.

While the gas-pressure curve alone gives an idea of the course of the pressure at idling speed, the curve of the inertia pressures shows the course of the pressure in diving. It still remains to plot the pressure diagram for the cruising revolution speed. This is accomplished very simply by adding geometrically the individual vectors of the inertia-pressure diagram to the vectors of the gas-pressure diagram. The combined resulting new vectors give the pressure diagram for the cruising revolution

speed. This combination is plotted in the upper part of Figure 4, where all the forces acting on the crank pin are taken into account.

It is expedient to plot the inertia-pressure vectors as lines, since on them lie the end points of the resultants of all the other cruising revolution speeds. Since the gas pressures may be assumed to be constant, it is only necessary to calculate the inertia pressures corresponding to the revolution speed as given above and plot them on the inertia-pressure vectors, in order to obtain the pressure diagram for any revolution speed. In most cases only a few values are necessary to obtain the desired information.

As was to be expected, at the revolution speed n = 3400r.p.m., the maximum pressures (points 24, 1' and 2') come from the inertia pressures at the end of the second and beginning of the third stroke.

The outermost tangents from the crank pin center on the curve enclose the angle within which the bearing pressures act on the crank pin. We can determine the location of the maximum pressures and therefore the points where the lubricant must be applied, which is led through the crank shaft to the connecting-rod bearings.

Moreover, if we wish to determine the load on the connectingrod bearings, it is only necessary to calculate the scale of the pressures. The conversion coefficient is obtained from the in-

verted ratio of the piston area to the area of the connecting-rod bearing. The points of equal load lie on circles around the crank pin center. These are plotted for a given case in Figure 4. The maximum mean pressure lies between 50 and 55 kg/cm², be-cause for a crank angle of 270° , the pressures lie in the field of these equidistants.

The above example refers to a case where only one connecting rod acts on one crank. It shows that the customary calculation method, which takes account of only the maximum explosion pressure or the maximum inertia pressure, is entirely adequate. The method here proposed acquires its importance first in the consideration of engines having more than one cylinder for one crank.

If it is assumed that the connecting rods of these engines are all alike and act directly on the crank pin, the method is quite simple. We determine the gas and inertia pressure components for one cylinder, find the crank positions from the ignition sequence and add the corresponding normal and tangential components of the individual cylinders algebraically, since they are all referred to the crank pin. The different pressure diagrams are then plotted just the same as for a one-cylinder crank.

In most recent engines with two or more cylinders working on one crank, only one connecting rod, called the "master connecting rod," acts directly on the crank pin, while the connecting rods of the other cylinders, called "articulated connecting rods," are hinged eccentrically to the master connecting rod. The points

of articulation do not therefore describe circles, but curves of a more or less elliptical shape. For these curves the known equations of crank drive no longer hold good, but must be replaced by others. Neither the strokes, piston speeds nor piston accelerations equal those of the master rod. It also follows that the angle between the ignition point and the dead center likewise differs for the different cylinders. No simple method for calculating the most favorable articulation has yet been devised.

In practice one seeks mathematically the smallest structurally allowable articulation radius and angle, which is generally not equal to the cylinder angle. The effect of the stroke difference on the compression ratio is offset by slight differences in the articulation radius, since it is preferred to have all the articulated connecting rods of the same length. As regards structural requirements, only compromises are possible, which generally necessitate different articulation radii and angles. We are interested in the effect of this eccentric articulation on the inertia pressures and will accordingly give a method for obtaining the equation for the piston acceleration with an eccentrically articulated connecting rod. We shall use the following notations:

r crank radius,

l length of master connecting rod,

ρ articulation radius of articulated connecting rod,

l' length of articulated connecting rod,

α crank angle

 β angle of deviation of the master connecting rod,

 β_x ". " " " articulated " " Y cylinder angle,

 δ articulation angle of the articulated connecting rod. The distance of the piston pin from the center of the crank shaft at the time is designated by x.

 $x = r \cos (\gamma - \alpha) + \rho \cos (\gamma - \delta + \beta) + l' \cos \theta_x.$

If we remember that $\sin \beta = \lambda \sin \alpha$ and put $\gamma - \delta = \epsilon$ we then have, after several transformations for x according to the binomial theorem and disregarding the quadratic terms

 $x = r \cos \gamma \cos \alpha + r \sin \gamma \sin \alpha +$ $+ \rho \cos \epsilon - \frac{\rho}{2} \lambda^{2} \cos \epsilon \sin^{2} \alpha - \rho \lambda \sin \epsilon \sin \alpha +$ $+ l' - \frac{[r \sin \gamma \cos \alpha - r \cos \gamma \sin \alpha + \rho \sin \alpha - \frac{\rho}{2} \lambda^{2} \sin \epsilon \sin^{2} \alpha + \rho \lambda \cos \epsilon \sin \alpha]^{2}}{2 l'}$

For simplification, if we put

r	sin γ	= 8	ι	ρ	cos e	=	d
r	cos Y	= t	$\frac{\rho}{2}$	λ²	cos€	=	е
·ρ	sin ¢	= (;	ρλ	cos€	=	f

 $\frac{\rho}{2}\lambda^2 \sin \epsilon$ and $\rho\lambda \sin \epsilon$ can be disregarded, since they are very small. Then

$$x = b \cos \alpha + a \sin \alpha + d - e \sin^2 \alpha +$$
$$+ l' - \frac{[a \cos \alpha - b \sin \alpha + c + f \sin \alpha]^2}{2l'}$$
$$= -\left(\frac{(f - b)^2}{2l'} + e\right) \sin^2 \alpha + \left(a - \frac{c (f - b)}{l'}\right) \sin \alpha -$$
$$- \frac{a^2}{2l'} \cos^2 \alpha + \left(b - \frac{a \cdot c}{l'}\right) \cos \alpha -$$
$$- \frac{a (f - b)}{l'} \sin \alpha \cos \alpha + d + l' - \frac{c^2}{2l'}.$$

The first deduction from this equation gives

$$v = \frac{ds}{dt} = \frac{dx}{d\alpha} \frac{d\alpha}{dt} = \omega \frac{dx}{d\alpha}$$
.

If we now put

$$-\left(\frac{(f-b)^{2}}{2l^{*}}+e\right) = u; \qquad \left(b-\frac{a}{l^{*}}\right) = q; \\ \left(a-\frac{c}{l^{*}}(f-b)\right) = t; \qquad -\frac{a}{l^{*}}(f-b) = o; \\ -\frac{a^{2}}{2l^{*}} = s; \qquad d+l^{*}-\frac{c^{2}}{2l^{*}} = k^{*}; \end{cases}$$

then .

 $x = u \sin^2 \alpha + t \sin \alpha + s \cos^2 \alpha + q \cos \alpha +$

+ o sina cosa + k'

and

$$v = \omega (t \cos \alpha + o \cos 2\alpha - q \sin \alpha - (s - u) \sin 2\alpha).$$

The second deduction gives the piston acceleration:

$$t = \frac{dv}{dt} = \frac{dv}{d\alpha} \quad \frac{d\alpha}{dt} = \omega \frac{dv}{d\alpha} ;$$

$$b = \omega^{2}(-q \cos \alpha - 2 (s - u) \cos 2\alpha - t \sin \alpha - 20 \sin^{2} \alpha .$$

. .

The coefficients of the acceleration equation are

$$q = \frac{1}{l!} (l! r \cos\gamma - r \rho \sin\gamma \sin\epsilon);$$

$$2(s - u) = \frac{1}{l!} (r^{2} \cos 2\gamma + \rho^{2} \lambda^{2} \cos^{2}\epsilon + l! \rho \lambda^{2} \cos\epsilon - - 2 \rho \lambda r \csc \cos\gamma);$$

$$t = \frac{1}{l!} l! r \sin\gamma - \frac{\rho^{2} \lambda}{2} \sin 2\epsilon + \rho \lambda \sin\epsilon \cos\gamma;$$

$$2o = \frac{1}{l!} (2 r \rho \lambda \sin\gamma \csc - r^{2} \sin 2\gamma);$$

If we put

$$\frac{\rho}{l'} = \mu$$
 and $l' = \frac{\rho}{\mu}$

then

$$q = \mu \left(\frac{r}{\mu}\cos\gamma - r\sin\gamma\sin\epsilon\right);$$

$$2 (s - u) = \mu \left(\frac{r^{2}}{\rho}\cos 2\gamma + \rho \lambda^{2}\cos^{2}\epsilon + \frac{\rho \lambda^{2}}{\mu}\cos\epsilon - 2\lambda r\cos\epsilon\cos\gamma\right);$$

$$t = \mu \left(\frac{r}{\mu}\sin\gamma - \frac{\rho \lambda}{2}\sin 2\epsilon + r\sin\epsilon\cos\gamma\right);$$

$$2o = \mu \left(2r \lambda \sin\gamma\cos\epsilon - \frac{r^{2}}{\rho}\sin 2\gamma\right);$$

Lastly we put

$$\frac{\mathbf{r}}{\mu} = \mathbf{g}; \qquad \rho \lambda^2 = \mathbf{i}; \qquad 2\lambda \mathbf{r} = \mathbf{j};$$
$$\frac{\mathbf{r}^2}{\rho} = \mathbf{h}; \qquad \frac{\rho \lambda^2}{\mu} = \mathbf{k}; \qquad \frac{\rho \lambda}{2} = \mathbf{n};$$

and the coefficients are simplified to

$$A = -\frac{1}{\mu}q = -(g \cos \gamma - r \sin \gamma \sin \epsilon);$$

$$B = -\frac{2}{\mu}(s - u) = -(h \cos 2\gamma + i \cos^{2} \epsilon + k \cos \epsilon - j \cos \epsilon \cos \gamma);$$

 $C = -\frac{1}{\mu}t = -(g \sin \gamma - n \sin^{2} \epsilon + r \sin \epsilon' \cos \gamma);$ $D = -\frac{2}{\mu}o = -(h \sin 2\gamma - j \sin \gamma \cos \epsilon)$

and the piston acceleration becomes

 $b = \omega^2 \mu (A \cos \alpha + B \cos 2\alpha + C \sin \alpha + D \sin 2\alpha);$ square... The accelerating and retarding forces per/centimeter of the

piston area are then determined from the equation

 $p = \frac{m}{f} \omega^2 \mu$ (A cos α + B cos 2 α + C sin α + D sin 2 α).

At first thought this equation for the more general case appears very troublesome on account of the coefficients. Such is not the case, however, since some of the coefficients drop out, and the calculation is further simplified by the use of tables. The values obtained with the aid of this equation give, when plotted against the piston stroke of the articulated connecting rod, the inertia-pressure diagram for eccentric articulation. Figs. 6 and 13 show these diagrams for a V engine and for an X engine. The resolution of these pressures into their tangential and normal components has to be done mathematically (Figs. 7 & 14).

Exactly the same method is followed with the pressures from

the gas forces whereby, for simplification, in order to be able to use the same indicator diagram, the piston stroke of the articulated connecting rod is reduced to that of the master connecting rod.

Lastly, all the pressures are represented in vector diagrams. Figures 8 and 15 are pressure diagrams for pure inertia pressures; Figures 9 and 16 for pure gas pressures (idling speed) and the total pressures at cruising speed.

It still remains to determine how great the effect of the eccentric articulation is in comparison with the central articulation. We will therefore investigate both cases for an X engine and for a nine-cylinder radial engine.

For the X engine (Figs. 13-17) it was found that the inertiapressure diagram suffered considerable distortion. This was the greatest for the normal components, even the absolute maximum pressures showing increases. For the gas forces the distortion is not so great. At the revolution speed, when all forces work together, the maximum pressures remain almost equal, and the variations in the normal and tangential directions are not very important.

The effect of the eccentric articulation is much greater on the radial engine (Figs. 18 and 19). While in the central articulation the tangential components of the inertia forces of the first and second order disappear and the normal components remain equal and in the same direction with reference to the crank shaft

so that they can be eliminated by a revolving counterweight, it is found in eccentric articulation that there are inertia forces which cannot be balanced by counterweights.

The course of the gas forces alone show considerable variations. The effect of the eccentric articulation is especially noticeable at the normal revolution speed. The course of the normal and tangential pressures shows much greater variations. Of especial intèrest here are the tangential pressures which determine the torsional curve. While, with central articulation, the tangential pressures vary between 6.75 and 14.3 kg/cm², i.e., by 7.55 kg/cm² = 64.5% of the mean pressure, the variation, with eccentric articulation, is $16.96 - 4.36 \text{ kg/cm}^2 = 12.6 \text{ kg/cm}^2 =$ 108% of the mean pressure. The torque is therefore considerably less uniform; the period of the first harmonic extends over 720°. instead of 80°, and consequently all the critical revolution speeds are considerably altered. Moreover, the lateral pressure of the pistons is increased, and the master connecting rod is stressed by additional bending moments, which also increase the lateral pressure in the main cylinder. Hence it follows that the effect of the eccentric articulation must be taken into account in radial engines.

In short, it may be said that the effect of the eccentric articulation can be disregarded for the first draft, in which it is only necessary to determine the order of magnitude of the pressures to be expected. A normal diagram for all engine types

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will suffice for the gas pressures, because the connecting-rod ratio is of subordinate importance so long as it remains within the usual limits. This greatly simplifies the investigation, since the pressure diagrams need to be plotted only once and can then be combined as desired.

In the above methods all the forces and pressures have thus far been referred to the crank pin at rest, for which, along with clearness, the viewpoint was also decisive that, for the connecting rod bearing, the lubrication was mostly effected through the crank pin. For certain cases, especially when the articulated connecting rods are hinged to one master connecting rod, it often appears important to know the direction and magnitude of the crank-pin pressures with respect to the connecting rod at rest. The pressure diagram can be easily obtained by considering the master rod stationary and by turning the vector diagram of the crank-pin pressures 15⁰ at a time about the crank pin. The relation of the corresponding points of the diagram to the connecting rod are thus determined. A new closed curve is thus obtained, which shows the course of the pressures with respect to the connecting rod. We can thus arrive at important conclusions for the location of the dividing line, the loading of the bearing cover, the direction of the pressure change, the delivery of the lubricant to the articulated connecting rod, etc. Figure 10 shows the pressure diagrams for a V engine for both rotational directions of the master rod.

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It still remains to determine the pressures on the journals. This is done by turning the corresponding vectors of the two vector diagrams of the crank-pin pressures of adjacent cranks by the amount of the crank angle in opposite directions and adding the vectors geometrically. The correspondence follows from the order of ignition. It should here be noticed that the pressure scale changes, e.g., 2:1 at n+1 journals for n cranks.

For the main bearing it is generally preferable to determine the pressure distribution with respect to the bearing itself or with respect to the crank case. For this purpose we use the same method as for the pressure determination of the connecting rod. We make the vector diagram of the journal pressures circle around the journal and plot the separate points of the curve in their relation to the bearing. This was done in Figure 11 for the front and rear bearings of a 12-cylinder V engine and in Figure 12 for the middle bearing of the same engine. Here also the gas pressures (idling speed) and inertia pressures were determined separately and then combined in the pressure diagram for the normal revolution speed. Moreover, the equidistants of the wing loading were plotted on both diagrams. It can be seen exactly where the lubricant must be applied.

In conclusion, attention is called to the fact that the method here presented for representing the pressures in the driving gear does not show simply the magnitude and location of the individual gas and inertia pressures on the piston pins and jour-

nals, but also furnishes an indication of the utilization of the driving gear, i.e., the ratio of the useful pressures (tangential) to the force-consuming pressures (normal pressures). Only the tangential pressures need to be plotted, in order to obtain the torque curve for the determination of the degree of irregularity.

Summary

We first described a method for determining the pressures developed in the driving gear of a crank-driven engine. This method gives a good idea of the gas and inertia pressures at various revolution speeds. Since two or more cylinders work on each crank in most aircraft engines, the equation for the piston speed and acceleration was developed for the case of an articulated connecting rod hinged to a master rod. The effect of the eccentric articulation was investigated on an X engine and on a 9-cylinder radial engine on the basis of the pressure diagrams for central and eccentric articulation. The crank-pin pressures, referred to the crank pin and to the reciprocating master connecting rod, were determined for a 12-cylinder V engine, and from these pressures the journal pressures were determined with respect to the journal and the main bearing.

Translation by Dwight M. Miner, National Advisory Committee for Aeronautics.

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Fig.1,2













Fig.8,10





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Weight of rotating masses=4.25 kg. Weight of reciprocating masses on master connecting rod=3.96kg. Weight of reciprocating masses on articulated connecting rod= 3.75 kg.

Fig.9



Fig. 11 Bearing pressure on crankshaft bearing between cranks I and IL.

Fig.ll













Fig.19 Crankpin forces of a 9 cylinder radial engine referred to the piston area at n=1800 r.p.m.