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AIRPLANE DRAG

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A I R P L A N E D R A G.*

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Induced Drag

The development of aviation during the last decade has been principally characterized by the commercial airplane, which has shown much improvement in its aerodynamic characteristics as compared with the old military airplanes. This is manifested, above all, by the fact that the importance of the "aspect ratio," $b^2 : F$ (b = span, F = wing area) has been universally recognized. At the same time Prandtl's conception of the "induced drag" has been introduced into the practice of airplane designing.

Hitherto it has been less well understood that the induced drag (or, better said, the undesired increase in the induced drag as compared with the theoretical minimum calculated by Prandtl) plays a decisive role in the process of taking off and therefore in the requisite engine power. Likewise any undesired increase in the induced drag of an airplane greatly increases the speed of vertical descent, thereby limiting the maximum allowable landing speed and carrying capacity.

The mathematical definition says but little concerning this effect of the induced drag. It seems opportune, therefore, to

*"Der Luftwiderstand am Flugzeug" from Die Luftwacht, September, 1929, pp. 410-415.

explain more clearly the conception of the induced drag in connection with the technical questions under consideration without the necessity of going into the deductions of mathematical aerodynamics.

The term "induced drag" comes from a comparison of the vortex pattern formed on the airfoil with electric phenomena. The induced drag is a hypothetical quantity and synonymous with the drag in a frictionless flow, which produces a certain lift on the airfoil.

Let us imagine the conditions as they exist in a wind tunnel. That is, the airfoil is stationary and the air strikes it at a certain angle of attack (Fig. 1). If the air stream produces a lift A on an airfoil, then an equal reactionary force is exerted by the airfoil on the air, deflecting the flow from its original direction by a small angle Δ. If we disregard the friction of the air within itself and on the surface of the airfoil, the resultant air force R can have no component in the direction of the flow, but is perpendicular to the flow at the airfoil. Since, however, according to the above statement, the flow at the airfoil is deflected downward by a small angle Δ from its original direction, R is inclined aft with respect to A, since the lift A, according to definition, is perpendicular to the direction of flight and therefore perpendicular to the original direction of the wind. The component of R which coincides with the original wind direction is called the induced drag.

As already stated, the force generated by the airfoil in a frictionless flow is perpendicular to the direction of flow. The fact that we can nevertheless speak of a drag depends first of all on the choice of a system of coordinates according to which we resolve the resultant air force into two components A and W at right angles to one another. In the case of an airplane it does not depend on the direction of flow at the wing, but on the direction of motion of the center of gravity of the airplane. This is identical with the original wind direction, as existing in the wind tunnel at some distance from the airfoil. In other words, in the horizontal flight of an airplane, a wing finds itself in a downward air flow generated by itself, against which it must continuously climb in order to maintain horizontal flight. This necessitates the consumption of energy which is represented by the product of the flight speed and the induced drag.

The overcoming of gravity by an airplane depends on the continuous downward deflection of a certain amount of air per second. With the downward acceleration of the air a certain amount of kinetic energy is lost, which appears on the airplane itself as engine power for overcoming the induced drag.

The theory of the induced drag only says that, even in frictionless flow, a certain engine power is required to maintain horizontal flight. The lift, as such, therefore necessarily involves a certain drag which represents the maximum limit attainable by the elimination of structural drag. The calculation of this ideal drag is valuable for judging a wing section and also the airplane as a whole. From Figure 1 we obtain $W_i = \tan\Delta \times A$.

It is obvious that the angle of deflection Δ is proportional to the deflecting force A. If w denotes the downward velocity imparted to the air and m the quantity of air flowing past the wing per second, then, according to the momentum theory, the lift $A = m\ w$. For the downward velocity according to Figure 1, we have $w = v \tan\Delta$ and hence $A = m \tan\Delta \times v$ and $\tan\Delta = \frac{A}{mv}$.

Hence a given lift can be produced in various ways. If the wing acts on a large quantity of air per second, then, for a given lift, according to the above formula, the angle A will be smaller than when the wing acts on a smaller quantity of air. The amount of air encompassed per second by an airplane wing increases on the one hand with the speed of the airplane and on the other hand with the span b of the wing.

Moreover, the ideal drag for a given lift A, according to the formula $W_i = A \tan\Delta$ derived from Figure 1, is smaller in proportion as the value of $\tan\Delta$ is smaller. Hence a large span and high speed are favorable. Correspondingly the formula for the induced drag, as developed from mathematical aerodynamics, is

$$W_i = \frac{A^2\ 2g}{\pi\ v^2\ \gamma\ b^2} \qquad (1)$$

In words this reads: For a given lift, the induced drag is smaller in proportion as the span and the air speed are greater.

This law brings the airplane into contrast with all other vehicles. While the drag or head resistance of a boat, airship, railway car or motor car steadily increases with the speed, the drag of an airplane (under the assumption of an ideal flow) decreases with the square of the speed. In fact the induced drag is only a portion of the total drag, since the flow about the wing and fuselage is not frictionless. This portion of the drag, which is called "harmful drag" (structural drag or parasite resistance) in the truest sense of the term, is of like nature and obeys the same law as the air resistance to other vehicles.

From the combination of the two drag components, one of which increases while the other decreases with the speed, follows the noteworthy fact that the drag of an airplane is the smallest at a medium speed. At the maximum and minimum speeds the drags are nearly equal and considerably greater than at the medium speed which is termed the "cruising speed."

The relation of the drag to the speed may therefore be expressed as follows. The maximum speed of an airplane is reached when the propeller thrust, with a fully loaded engine, equals the drag. Any further speed increase in rectilinear horizontal flight is then manifestly impossible. Maximum speed and maximum drag or resistance coincide, just as for any other vehicle. With decreasing speed, the drag decreases first, but only to the cruising speed, at which, as already stated, the total drag of the airplane is the smallest. If the speed is further reduced, as, for example, in climbing, the drag increases. In the case of an overloaded airplane it may even reach the same value as at maximum speed. When such is the case, the speed in horizontal flight can not be further reduced by increasing the angle of attack. In other words, the airplane can not maintain the same altitude at a lower speed.

This is especially important in the process of taking off. The airplane must first taxi until it attains the minimum speed at which horizontal flight is possible. The magnitude of this speed is important, since it determines the length of the take-off run. If an airplane takes off at the minimum speed, it immediately gains in speed in the air because any increase in speed means at first a corresponding decrease in drag.

The minimum speed in gliding flight, called "landing speed," is not determined, however, by the magnitude of the drag but by the maximum allowable angle of attack or corresponding maximum lift coefficient $C_{a\ max}$. Normally the minimum speed in horizontal flight should equal the landing speed. If this is not the case, it may be taken as an indication that the airplane is overloaded or, more correctly, that the power loading (weight per horsepower) is too great. In any case, the drag at the minimum speed is considerably greater than at cruising speed. Hence the induced drag is the deciding factor at all speeds below the cruising speed.

Equation (1) for the induced drag can be transformed by introducing a nondimensional coefficient C_{wi} corresponding to the coefficients for the lift and total drag,

$$W_i = \frac{C_{wi}}{100} F q$$

Near the ground

$$q = \frac{\gamma}{2g} v^2 = \frac{v^2}{16}$$

If $A = \frac{C_a}{100} F_q$ is introduced into equation (1), we have

$$\frac{C_{wi}}{100} F q = \frac{C_a^2}{100^2} F q \frac{1}{\pi q b^2}$$

$$C_{wi} = \frac{C_a^2}{100} \frac{1}{\pi} \frac{F}{b^2} . \qquad (2)$$

This equation demonstrates the importance of the previously mentioned aspect ratio $b^2 : F$, since C_{wi} is independent of the shape of the wing section and is determined only by the aspect ratio. The greater the span of an airplane wing in proportion to its area, the smaller the coefficient of the induced drag.

While the aspect ratio of commercial airplanes has hardly exceeded 7, gliders have gone as high as 18. The fact that gliders have a wing loading of about 14 kg/m² (2.87 lb./sq.ft.), as compared with 100 kg/m² (20.5 lb./sq.ft.) for large commercial airplanes, is no reason for this development, as claimed in a well-known book on aviation. Every airplane designer knows that the structural difficulties diminish with increasing wing loading, because the weight of the wing per unit area increases slower than the wing loading.

The limitation of the aspect ratio in commercial and touring airplanes is determined rather by practical considerations, such as the cost of production, sheltering, transportation, cost of repairs, etc. The glider is a highly specialized record airplane in which economical considerations play no part. As in every record airplane, the value of a glider consists in determining the technical limits of a one-sided development for special purposes. That such one-sided development has contributed greatly to technical progress is demonstrated by the history of the development of racing automobiles.

The calculation of the induced drag according to equation (1) or (2) holds strictly good only for the so-called elliptical distribution as illustrated by Figure 2. The area of the semi-ellipse plotted over the span b represents the lift A produced by the wing. If the wing has geometrically similar sections throughout and if the angle of attack is uniform over the whole span (i.e., if the wing is not warped), then the coefficient of lift C_a is constant over the whole span. In this case elliptical distribution would exist on an elliptical wing. Such a wing would have the smallest possible induced drag. For any other lift distribution the induced drag would be greater than indicated by equation (1). The deviation from the elliptical distribution is so slight for the plan forms of wings actually used on airplanes (with the single exception of the pointed wing) that equations (1) and (2) are sufficiently accurate for all normal wings. These formulas also apply particularly to rectangular wings (See Report I of the Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, p.63 ff.).

A rectangular plan differs decidedly at the wing tips from an elliptical one. It may be surprising, therefore, that the lift distributions for these two wing shapes hardly differ from each other. Even in the case of a rectangular wing, the specific distribution (per linear unit of the span) sinks to zero at the wing tips, although the chord remains the same throughout the whole span. We must remember there is a positive pressure on the lower side of the wing and a negative pressure on the upper side which together produce the lift. At the tips the pressures are equalized by the flowing of the air from the lower to the upper side of the wing. This pressure equalization is, of course, not entirely restricted to the tips, but affects the pressure distribution over a considerable portion of the span, so as to produce, even on a rectangular wing, a lift distribution very similar to that shown in Figure 2.

That the lift distribution is physically related to the induced drag is indicated by the fact that it is determined by the circulation about the wing tips, as we have just seen (Fig. 2). This circulation consists of an upward flow of the air right and left from the wing, which is to be regarded as a flow of equalization to the downward motion of the air throughout the entire wing span.

The space traversed by the wing must be filled with air at every point. This is possible, however, only when the quantity of downward flowing air is counterbalanced by a like quantity of upward flowing air on both sides of the airplane. We recognized the kinetic energy contained in this counterflow as the physical cause of the induced drag, which is the immediate result of the lift even in frictionless flow. Some relation between the magnitude of the induced drag and the circulation about the wing tips follows directly from this consideration. On the other hand, since this circulation affects the lift distribution, it is obvious that there must also be a very definite relation between the induced drag and the lift distribution. This relation is of very great practical importance in airplane design and still receives much too little attention.

The induced drag of a wing of given span and area is the smallest in elliptical lift distribution. While the lift distribution is relatively little affected by the plan form of the wing, it is often surprisingly affected by the fuselage, engine nacelles (Fig. 3), wing cutaways, outside struts and brace wires, and always detrimentally. All these various effects on the lift distribution can be included in the term "circulation disturbance," but unfortunately they can not be calculated in advance.

How great this circulation disturbance may become is strikingly illustrated in Report III of the Berichte der Aerodynamischen Versuchsanstalt zu Göttingen, p. 124, in the testing of a model of a large Rohrbach seaplane. The apparent structural drag of the seaplane was tripled by the two lateral engines. I speak advisedly of the "apparent" structural drag, since the structural drag is generally understood in aviation as meaning the head resistance experienced by an object when exposed alone to the air flow. The head resistance of the two lateral engines is really small, however, and would increase the structural drag of the whole airplane by only a fraction. In this case, however, there was a decided circulation disturbance, i.e., a detrimental change in the lift distribution and a corresponding increase in the induced drag. Since, in the above-mentioned report, the induced drag was introduced into the polar for elliptical lift distribution (any other not being at all feasible), the really much more unfavorable induced drag was included in the structural drag.

The results of a systematic investigation of this phenomenon were published in Volume III of the Göttingen reports on page 115. It would require too much space to repeat them here in full. This valuable report can not be recommended too highly, however, to all airplane designers. Recently Horst Muttray published a comprehensive report of the Göttingen investigations in this technically important field.*

*"Untersuchungen über die Beeinflussing des Tragflugels eines Tiefdeckers durch den Rumpf," Luftfahrtforschung, June 11, 1928, pp. 33-39. For translation, see N.A.C.A. Technical Memorandum No. 517 ("Investigation of the Effect of the Fuselage on the Wing of a Low-Wing Monoplane").

Profile Drag and Structural Drag

The total drag of an airplane may be imagined as resolved into three components: first, drag in frictionless flow, called "induced drag"; second, drag due to the shape of the wing section or profile and to skin friction, called "profile drag"; third, drag due to the head resistance of all the nonlifting parts, called "structural drag."

In reality, however, this resolution of the drag can not be accomplished mathematically. The induced drag of a wing can be calculated according to equation (1). The induced drag of the whole airplane, however, can not be calculated, since we do not know the lift distribution which, due to the structural form of the airplane, especially of a multi-engine airplane, may differ considerably from the elliptical distribution as it would exist on the wing alone. Hence it is expedient to resolve the total drag of an airplane so that the induced drag will be calculated for elliptical distribution, since this represents the most favorable case attainable. The profile drag can be determined from the experimental results. The structural drag then appears as the remainder, after the pure wing drag has been deducted from the measured total drag of the airplane.

There can be no objection to this method, provided it is remembered that the structural drag, as thus determined, comprises not only the sum of the head resistances, but also that the increase in the induced drag, due to the altered lift distribution, appears as a component of the structural drag when the latter is considered as the remainder of the total drag.

These relations are represented by Figure 4. The plain curve represents the polar of the wing whose drag was measured without the fuselage and tail surfaces. The dash-dot curve gives the coefficient of the induced drag C_{wi} for elliptical lift distribution, as calculated according to equation (2). With C_{wp} and C_{ws} as the coefficients of profile and structural drag, we have the relation

$$C_w = C_{wi} + C_{wp} + C_{ws}.$$

C_{ws} is defined by the formula

$$W_s = \frac{C_{ws}}{100} F q$$

in which W_s is generally greater than the sum of all the component drags of the nonlifting parts.

The angle of glide φ is determined in the customary manner by a tangent from the origin of the system of coordinates to the airplane polar. $\cot an \varphi = (C_a : C_w)_{max}$ (Fig. 4). For this it is generally allowable, in the case of a single-engine airplane, to dispense with the measurement of the polar of the whole airplane and to calculate W_s as the sum of all the partial drags of the nonlifting parts without regarding the circulation disturbance. For brace wires, struts, fuselage, landing gear and the tail surfaces, we have reliable measurements according to which we can make such a calculation without new tests with models. The C_{ws}, thus found, is plotted to the left to the point O' (Fig. 4) from the origin of the wing polar, which can be taken from the Göttingen reports. From O' a tangent is drawn to the wing polar. In this manner the angle of glide can be determined with good approximation.

The importance of the angle of glide is generally overestimated. The $(C_a : C_w)_{max}$ is particularly important only when the fuel consumption plays a decisive role, i.e., for long nonstop flights. The speed of vertical descent and the climbing speed are much more important for ordinary commercial, sport, and military airplanes, because these determine the engine power and the maximum load. Not the ability to make long flights, but the take-off and landing characteristics are here the determining factors.

The take-off and landing are made at a lower speed than the cruising speed as defined by $(C_a : C_w)_{max}$, It is obvious from Figure 4 that the measured C_{ws} increases very rapidly in the region between the cruising speed and the landing speed, i.e., with increasing C_a. As already stated and as also follows from equation (1), the induced drag is great at low speed and represents the larger component of the total drag. The increase in the measured C_{ws} at low speed can be easily explained by the fact that, in the C_{ws}, that part of the induced drag appears which could not be covered by equations (1) and (2), namely, the additional induced drag due to the change in the lift distribution.

Conclusion

Our knowledge of the circulation disturbance or interference is still relatively very small. Professor Junkers was probably the first to make systematic investigations on the mutual effect of fuselage and wing. Our small previous knowledge concerning this phenomenon included the remarkable fact that the effects of the circulation disturbance on a braced biplane of small aspect ratio are much less apparent than on a cantilever monoplane. The greater the aspect ratio ($b^2 : F$) and the smaller the structural drag, just so much more sensitive is an airplane to any disturbance of the elliptical lift distribution by engine nacelles, etc. It is doubtless due largely to this circumstance that the circulation disturbance has only recently received much attention. It has been found especially troublesome in the recent three-engine airplanes.

In short, it may be said that the equilibrium polar, as measured on the model of a complete airplane in a wind tunnel and corrected with respect to the Reynolds Number by flight tests, can not be replaced by any drag calculation. If the designer of a new airplane type wishes to balance the desired aerodynamic advantages against the generally unavoidable structural difficulties and put them in the correct economical relation to one another, the measured equilibrium polar is the only practical criterion.

Translation by Dwight M. Miner,
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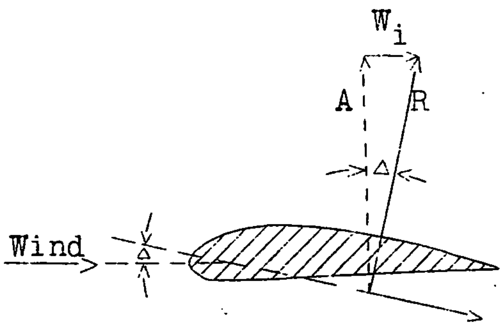

Fig.1 Induced drag

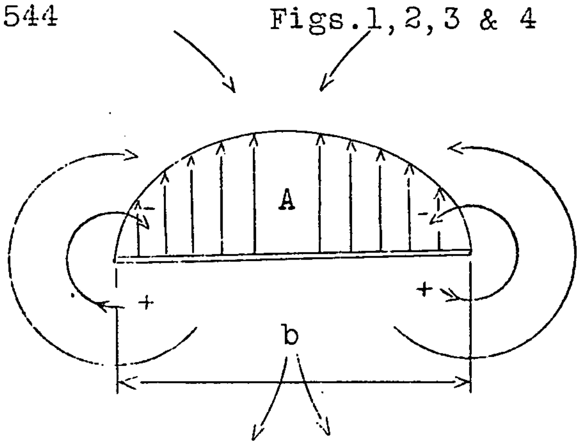

Fig.2 Elliptical lift distribution.

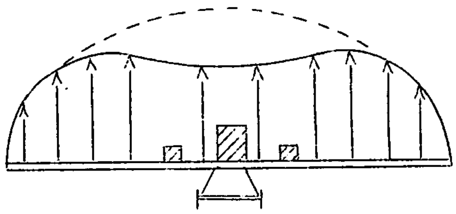

Fig.3 Disturbed lift distribution.

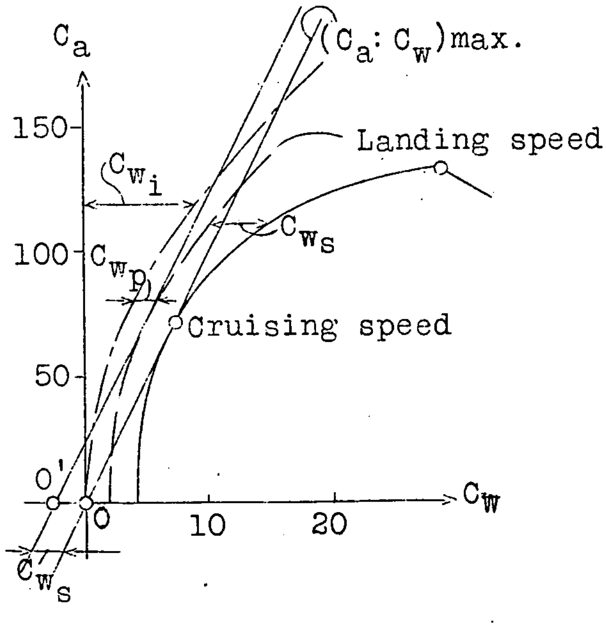

Fig.4 Airplane polars.