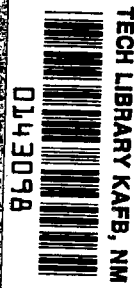# NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT 984

METHOD FOR DETERMINING THE FREQUENCY-RESPONSE CHARACTERISTICS OF AN ELEMENT OR SYSTEM FROM THE SYSTEM TRANSIENT OUTPUT RESPONSE TO A KNOWN INPUT FUNCTION

By HOWARD J. CURFMAN, Jr. and ROBERT A. GARDINER

1950

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

| | Symbol | Metric | | English | |
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	{kilometers per hour	kph	miles per hour	mph
		{meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight $=mg$

g Standard acceleration of gravity $=9.80665$ m/s² or 32.1740 ft/sec²

m Mass $=\dfrac{W}{g}$

I Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m⁻⁴-s² at 15° C and 760 mm; or 0.002378 lb-ft⁻⁴ sec²

Specific weight of "standard" air, 1.2255 kg/m³ or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\dfrac{b^2}{S}$

V True air speed

q Dynamic pressure, $\dfrac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\dfrac{L}{qS}$

D Drag, absolute coefficient $C_D=\dfrac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\dfrac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\dfrac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\dfrac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\dfrac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\dfrac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

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By Howard J. Curfman, Jr. and Robert A. Gardiner

SUMMARY

A method is presented for the determination of the frequency-response characteristics of an element or system by utilizing the transient output response to a known but arbitrary input to the system. Since the application of special inputs, such as step functions or sinusoids, is often imperfect or impractical, a method for utilizing arbitrary inputs is desirable. Simple flight-test data may be reduced by this method to give the frequency response of an aircraft. Examples are given as determinations of aircraft frequency responses; however, the method can be applied to any type of dynamic system, such as automatic-control components, vibration-absorption equipment, and many types of instruments. The method requires that the arbitrary input function tend to a finite value after a finite time and that the system or element output be measured as a representative quantity having a static sensitivity.

INTRODUCTION

Among the essential elements in the study of the problem of automatic stabilization and control of an aircraft are the frequency-response characteristics of the aircraft in the mode of motion under investigation. For example, if attitude stabilization is under consideration and is to be maintained by elevator control, the pitch response of the aircraft to sinusoidal inputs of control deflections at various forcing frequencies, expressed as an amplitude ratio and a phase relationship, is required. Heretofore, the determination of this important factor was extremely difficult if it was to be found or checked by flight tests. Such measurements have been made for a piloted airplane (reference 1) by the laborious time-consuming method of applying sinusoidal control-surface inputs. For a pilotless aircraft in which automatic stabilization and control are needed, the problem of successfully determining these characteristics in flight tests is more complex. The primary purpose of this report is to present and to illustrate a method by which simple flight-test results can be used to evaluate the aircraft frequency-response curves desired. The method shows how the frequency-response characteristics can be found if the output response is known for any known arbitrary input function.

This problem of determining desired frequency responses has been considered with regard to special types of inputs, such as the step-function input (reference 2) and the sinusoidal input (references 1 and 3). In the actual testing of various systems, factors such as time limitations and limiting accelerations may dictate the type of input that gives applicable data. In pilotless-aircraft studies the step-function input technique appears extremely valuable; however, the application of a true step function is, in itself, a problem. A method of handling inputs which are not too restricted and which are easily realized is, therefore, desirable in the determination of frequency-response characteristics.

Throughout the field of automatic control a knowledge of the frequency response, which relates the output of an element to its input, is desirable for an analysis and synthesis of a control system composed of a group of elements. Since the application of sinusoidal inputs of varying frequencies and the measurement of the output of an element are often extremely impractical, as in a hydraulic or pneumatic servomotor, the present method affords a means of evaluating the response characteristics needed for effective understanding and design of a system.

The present method is an extension of one presented in reference 2 in which the derivation of the frequency response (performance operator in reference 2) is shown for a known transient output response to a step-function input. Other methods, of course, have been developed to perform this operation. Reference 4 gives a method whereby the output transient to a step input is used with Duhamel's integral to produce the frequency-response curves. A discussion of a Fourier integral method is given in reference 5. The Fourier method is used in reference 2, and the approach therein was used to derive the present method for finding the frequency response of an element if its output response to an arbitrary input is known. Although the possibility exists, no attempt has been made to employ the line of reasoning of this extension to any of the other methods. The present extension is illustrated by three examples. The method, however, is only approximate in that a finite number of terms in a series expansion are used to determine the

response at each value of forcing frequency and that a linear differential equation is necessarily implied for the system under consideration.

SYMBOLS

ω angular forcing frequency, radians per second

$j=\sqrt{-1}$

φ phase angle, positive when output leads input, degrees

t time, seconds

D differential operator $\left(\dfrac{d}{dt}\right)$

$X,\ \eta$ illustrative variables

α angle of attack, positive when nose is above relative wind vector, radians (except as noted)

δ aircraft elevator deflection angle, positive when trailing edge is down, radians (except as noted)

I_y pitching moment of inertia, slug-feet square

q dynamic pressure, pounds per square foot

S wing area, square feet

c mean aerodynamic chord, feet

m mass of aircraft, slugs

V aircraft velocity, feet per second

C_L lift coefficient (Lift/qS)

C_m pitching-moment-coefficient (Moment/qSc)

C_{L_α} lift-curve slope ($\partial C_L/\partial \alpha$)

C_{m_α} pitching-moment-curve slope ($\partial C_m/\partial \alpha$)

C_{L_δ} rate of change of lift coefficient with elevator deflection ($\partial C_L/\partial \delta$)

C_{m_δ} rate of change of pitching-moment coefficient with elevator deflection ($\partial C_m/\partial \delta$)

$$C_{m_q}=\frac{\partial C_m}{\partial \dfrac{qc}{2V}}$$

q pitching angular velocity, radians per second

$$C_{m_{\dot\alpha}}=\frac{\partial C_m}{\partial \dfrac{\dot\alpha c}{2V}}$$

$\dot\alpha$ rate of change of angle of attack, radians per second

ANALYSIS

The analysis which presents the method for obtaining the frequency-response data from the transient response to an arbitrary input to an element or system is considered in two sections. The first section is a review of the results presented in reference 2 and discusses the determination of

the frequency response when the transient output for a step-function input is known. This procedure is herein called the "step-function input technique." In the second section the basic technique presented in the first section is extended, and the resulting procedure is termed the "arbitrary-input technique." The method requires that:

(1) The element or system is describable by linear differential equations.

(2) The arbitrary input function tends to a fixed value after a finite time.

(3) The representative output of the system tends to a fixed value. The ratio of this value to the final fixed input is the static sensitivity.

STEP-FUNCTION INPUT TECHNIQUE

In reference 2 the complete analysis has been given for determining the frequency-response characteristics of an element or system when the transient response to a step-function input is known. The method therein involves initially a representation of the output by a series of step functions of various amplitudes delayed by equal time increments from the time origin of the step input to the system. Since the output may also be considered as the product of a step-function input and the frequency response of the element or system, these expressions are equated. The required amplitude and phase for the system at the desired frequency are found from the solution of this equation. Since this feature is basic and is only extended in the present analysis, the procedure is briefly discussed.

Let it be assumed that the response to a step-function input is known. This response is a function of time, and the time scale is divided into equal time increments in the manner shown in figure 1. Since no general rule concerning the number of time increments necessary to give adequate results exists, cases with two different increments may be
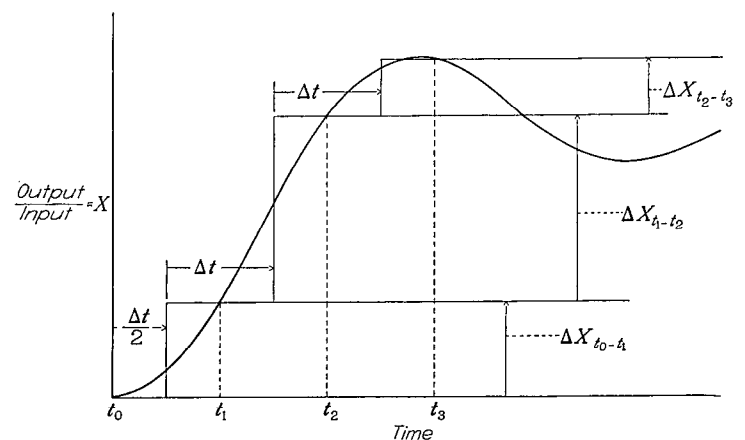

FIGURE 1.—Illustration showing incremental division of the transient response for application of the present method.

determined and then compared for differences. In general, for highly oscillatory responses more increments seem to be required to approximate the curve and the area beneath it. The change in output from the transient-response curve during each of the time increments must be measured. These changes are noted in figure 1 by increments of X and the time interval is noted in the subscript, for example, $\Delta X_{t_0-t_1}$. Some of these changes may be negative; however, their sum gives the output steady-state value (static sensitivity). As shown in reference 2, the amplitude ratio $|X|$ and the phase angle φ may be expressed as a function of the arbitrary forcing frequency ω by

$$X(j\omega) = |X|e^{j\varphi}$$
$$= \Delta X_{t_0-t_1}e^{-j\omega(\Delta t/2)} + \Delta X_{t_1-t_2}e^{-j\omega 3(\Delta t/2)} +$$
$$\Delta X_{t_2-t_3}e^{-j\omega 5(\Delta t/2)} + \Delta X_{t_3-t_4}e^{-j\omega 7(\Delta t/2)} + \ldots \quad (1)$$

The addition of these vectors results in a vector defining the response characteristics of the element or system at the forcing frequency used. This procedure is required for as many values of ω as are desired.

Another method of performing this operation is given in reference 6 in which the output transient is approximated by step functions not delayed by constant time intervals. This method, however, requires the definition of a fundamental frequency and therefore limits computation to the responses at the odd harmonics of this frequency. This method was not used in the analysis herein because the present calculations had been completed before the existence of reference 6 was noted. The use of the method of reference 2 is therefore not to be construed as meriting any particular preference.

ARBITRARY-INPUT TECHNIQUE

The determination of the frequency-response characteristics of an element when the transient response to an arbitrary input is known is an extension of the method given in the previous discussion of the step-function input technique. The data required are the time variations of the input and the corresponding time record of the output caused by this input.

In order to clarify the discussion of this method, figure 2 has been prepared. The solid lines in the block diagram indicate the element or system for which the frequency response is desired and for which the input and output time variations are known. This frequency response is expressed as a frequency-dependent vector

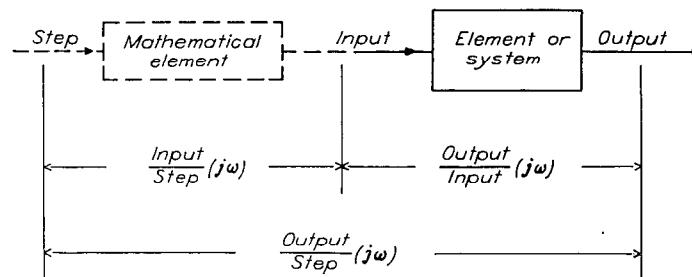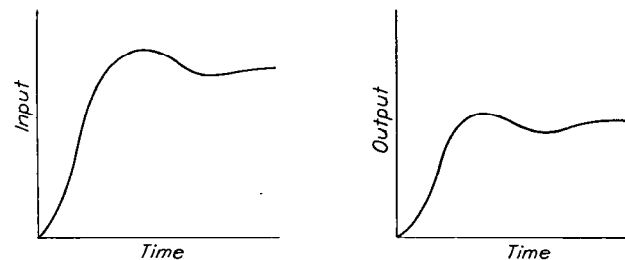$$\frac{\text{Output}}{\text{Input}}(j\omega)$$

FIGURE 2.—Block diagram showing method for determining frequency response when input is arbitrary.

Since the input is an arbitrary function of time, for example, as shown in the upper part of figure 2, the step-function method is not applicable. For the determination of the required frequency response, three steps are required.

The first step is the introduction of a mathematical element in series with the original one, as shown in figure 2. It is then assumed that a step-function input is applied to the mathematical element and that its result is the time variation of the input to the real element. After these assumptions are applied, the response for the mathematical element as a frequency-dependent vector can be written

$$\frac{\text{Input}}{\text{Step}}(j\omega) \quad (2)$$

This procedure is the same as the method outlined for the step-function input technique.

The second step is the consideration of the two elements or the over-all response characteristics from the step-function input to the mathematical element to the output of the element under investigation. In this step, the output-time variation found for the element in question is assumed to be the output response to the step-function input to the over-all (two-block) system. If the same method presented for the step-function analysis is used, the result is an over-all response represented by the frequency-dependent vector

$$\frac{\text{Output}}{\text{Step}}(j\omega) \quad (3)$$

The third step is the combination of these two frequency-dependent vectors to produce the frequency response of the subject element. This operation is the division of the second vector by the first vector at the same frequency. Since the division of two vectors is another vector, the response of the element in question is also a frequency-dependent vector expressed as

$$\frac{\text{Output}}{\text{Input}}\,(j\omega) = \frac{\dfrac{\text{Output}}{\text{Step}}\,(j\omega)}{\dfrac{\text{Input}}{\text{Step}}\,(j\omega)} \qquad (4)$$

The algebraic operation of these vectors is that of linear analyses and is valid since the method involved in determining the responses requires the assumption that the elements are represented by linear equations. Since the given input and output time responses are known for the element and since equations (2) and (3) are the characteristics required to give these responses, the vector operation indicated in equation (4) represents the required frequency response of the element.

ILLUSTRATIVE EXAMPLES AND DISCUSSION

In the first two examples, cases were chosen in which the actual time responses could be analytically determined. The method of this report was then applied and the results compared with the theoretical frequency-response curves. The final example is a case in which an aircraft frequency response is determined from experimental flight-test data. These results are compared with the theoretical values found by using the stability derivatives obtained from the same flight-test data.

EXAMPLE I

The block diagram for example I is as follows:
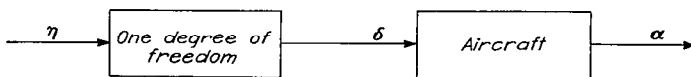
The first block is taken as a single-degree-of-freedom system having the same characteristics as a spring-mass-viscous-damping system two-tenths critically damped with an undamped natural frequency of 50 radians per second. The differential equation is

$$(D^2 + 20D + 2500)\delta = K\eta \qquad (5)$$

In this case K is a constant and is chosen as 2,500 to give a static sensitivity of unity for this system; that is, a unit η produces a unit δ at steady state.

The second block represents the transfer function α/δ for an aircraft having the characteristics given in table I. If two degrees of freedom are considered longitudinally, the force equation along the longitudinal axis is neglected, and the velocity is considered as constant, the response in terms of the differential operator D is

These results were reduced from the equations of motions adapted from reference 3.

The time variations of δ and α were computed for a step-function input η of 10°. These computations resulted in the transients shown in figure 3. For the problem under consideration, the δ was the input variation that caused the output variation α. The airplane frequency response is the quantity of interest in this report.

If the method described in this report is used, three steps are taken:

(1) The step-function-input technique is applied to the transient δ variation and the vector frequency response δ/η is determined for various forcing frequencies ω.

(2) The step-function-input technique is applied to the transient α variation and the vector frequency response α/η is determined for the same forcing frequencies.

(3) The frequency response α/δ is determined by the vector division of α/η by δ/η at the same forcing frequencies.

In step (1) a total transient time of 0.48 second was used and a time increment Δt of 0.015 second was chosen. These conditions gave a total of 32 increments defining the δ/η variation.

In step (2) a total transient time of 1.75 seconds was used and a time increment Δt of 0.05 second was chosen. These conditions gave a total of 35 increments defining the α/η variation.

The results of steps (1) and (2) are presented in figure 4. In figure 4(a) the δ/η response from the step-function-input technique is shown by the test points and the dashed curve faired through these points. The theoretical frequency response of δ/η was found from equation (5) by letting $D = j\omega$ and solving the resultant expression for various values of ω. The theoretical curve is shown as the solid line. The corresponding curves for the α/η response are shown in figure 4(b).

TABLE I

AIRCRAFT PARAMETERS USED IN ILLUSTRATIVE EXAMPLES

Aircraft parameter	Example	
	I and II (canard)	III (delta-wing configuration)
Mach number	1.8	1.2
I_y, slug-ft^2	30	17.10
S, ft^2	2.52	6.25
c, ft	1.4	2.19
m, slugs	4.66	5.72
q, lb/ft^2	4,270	1,920
V, ft/sec	1,963	1,320
C_{L_α}, per radian	3.01	2.705
C_{m_α}, per radian	−2.22	−0.77
C_{L_δ}, per radian	−0.218	0.585
C_{m_δ}, per radian	1.58	−0.564
$C_{m_q}+C_{m_{\dot\alpha}}$, per radian	−19.48	−2.65
C_{m_q}, per radian	−19.48	−1.59

$$\frac{\alpha}{\delta}(D) = \frac{C_{L_\delta}\dfrac{I_y}{qSc}\,D - C_{L_\delta}C_{m_q}\dfrac{c}{2V} - C_{m_\delta}\dfrac{mV}{qS}}{-\dfrac{I_y}{qSc}\dfrac{mV}{qS}\,D^2 + \left[\dfrac{mV}{qS}\left(C_{m_q}+C_{m_{\dot\alpha}}\right)\dfrac{c}{2V} - C_{L_\alpha}\dfrac{I_y}{qSc}\right]D + C_{m_\alpha}\dfrac{mV}{qS} + C_{L_\alpha}C_{m_q}\dfrac{c}{2V}} \qquad (6)$$
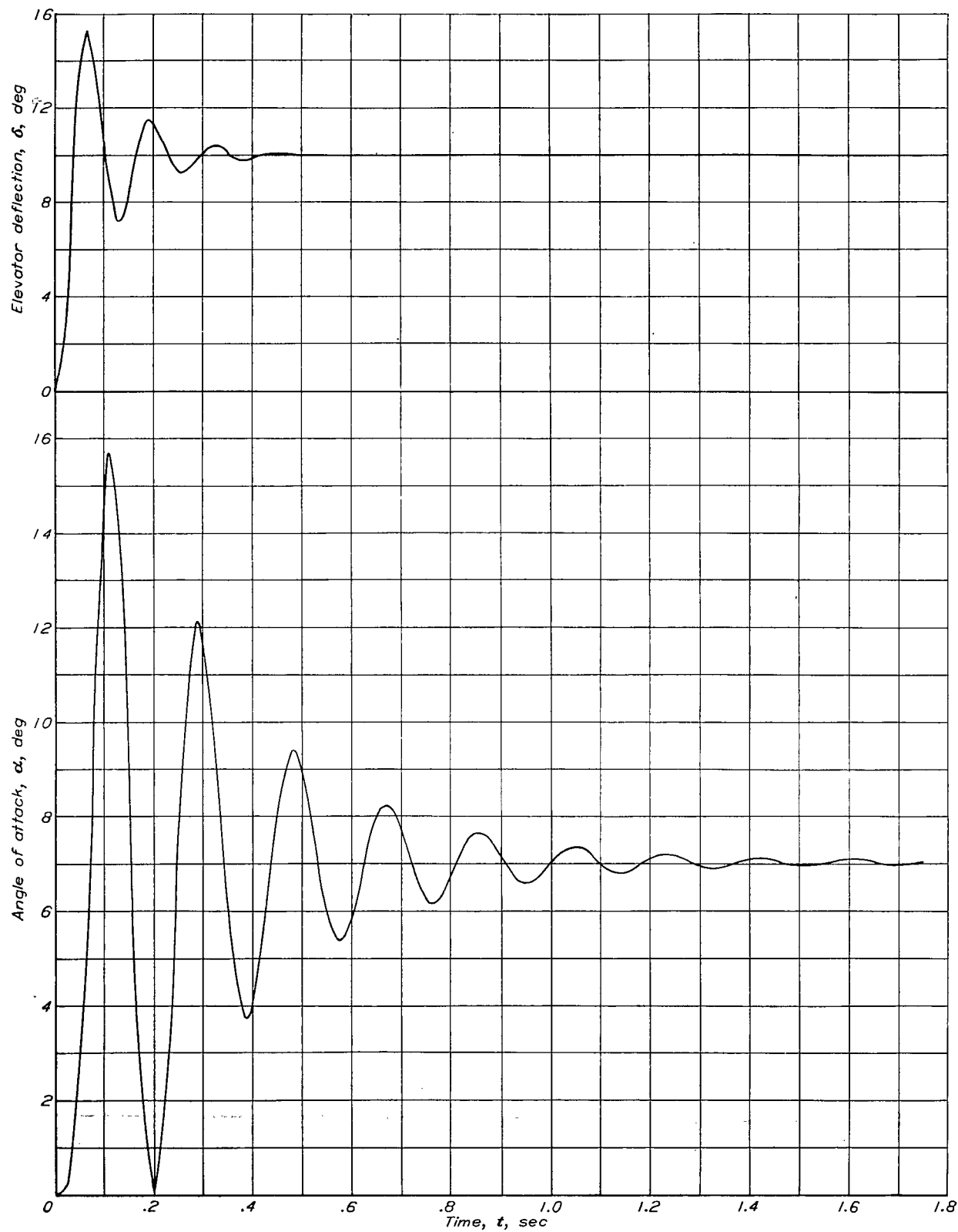
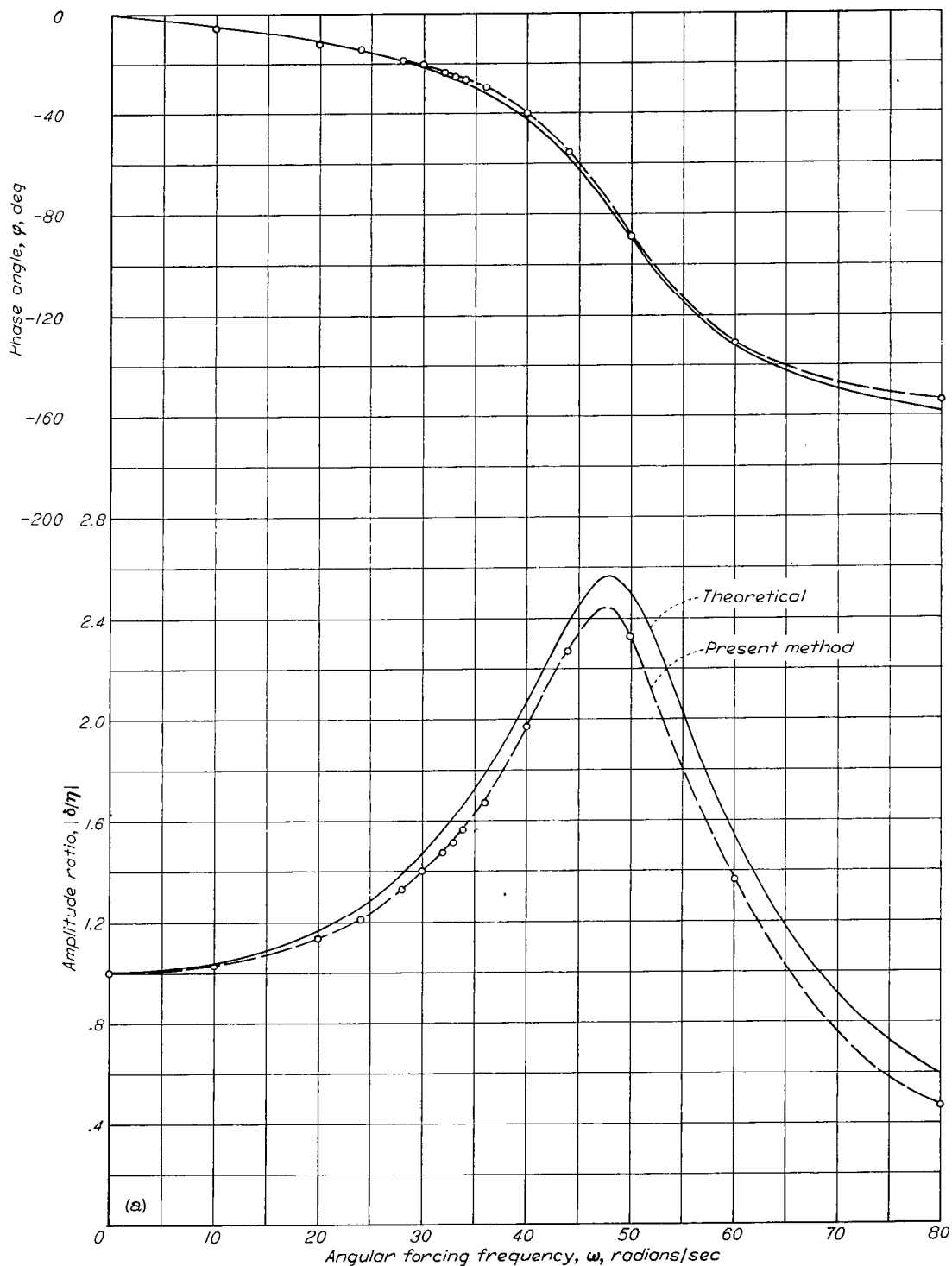FIGURE 3.—Calculated input δ and output α transients.

(a) One-degree-of-freedom frequency response, δ/η.

FIGURE 4.—Comparison of theoretical frequency responses and computed frequency responses determined from the transients of figure 3.

The α/δ response of the aircraft was formed by dividing the amplitude ratio of α/η by that of δ/η and by subtracting the phase angle of δ/η from the phase angle of α/η at corresponding values of the forcing frequency ω. Figure 5 is the desired final result, the dashed curve indicating the results computed from the method herein and the solid curve showing the theoretical values derived by letting $D=j\omega$ in equation (6).
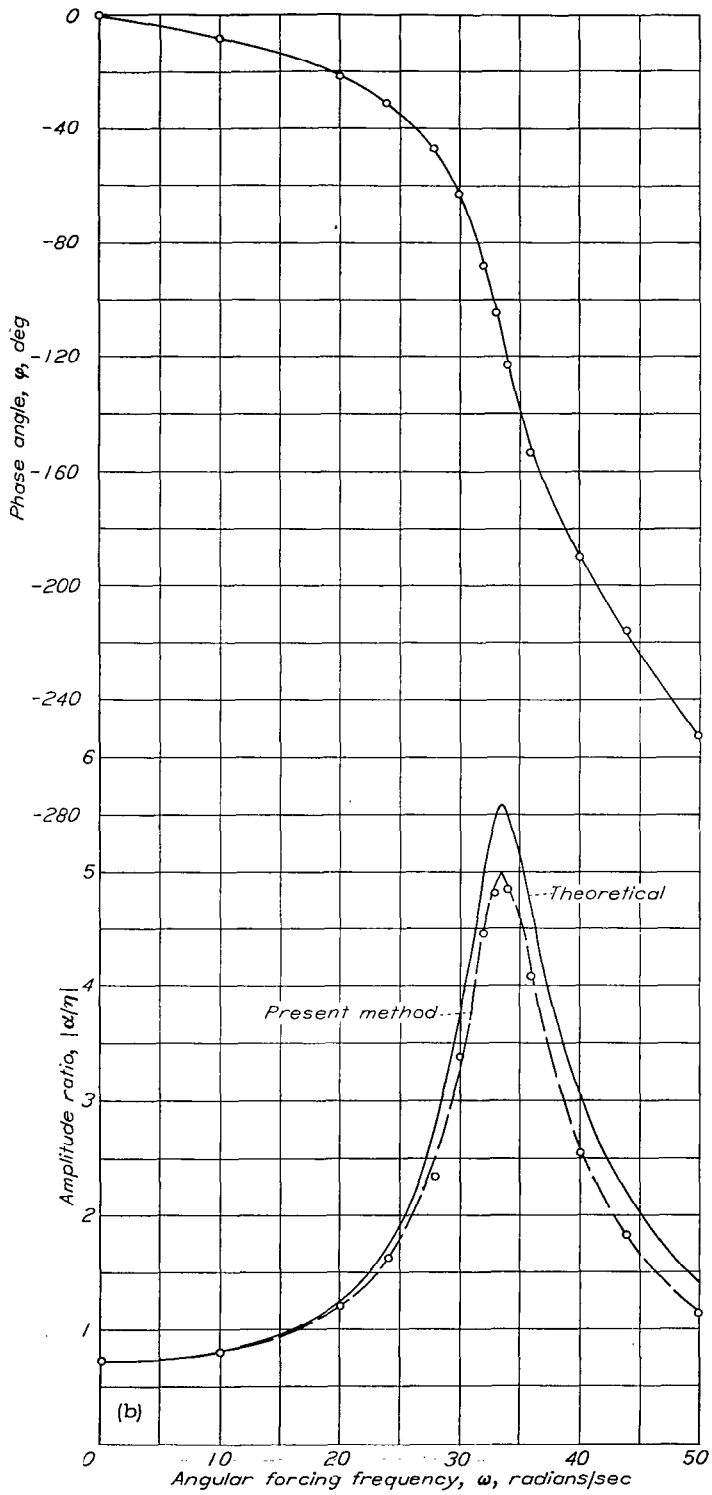
(b) Over-all frequency response, α/η.
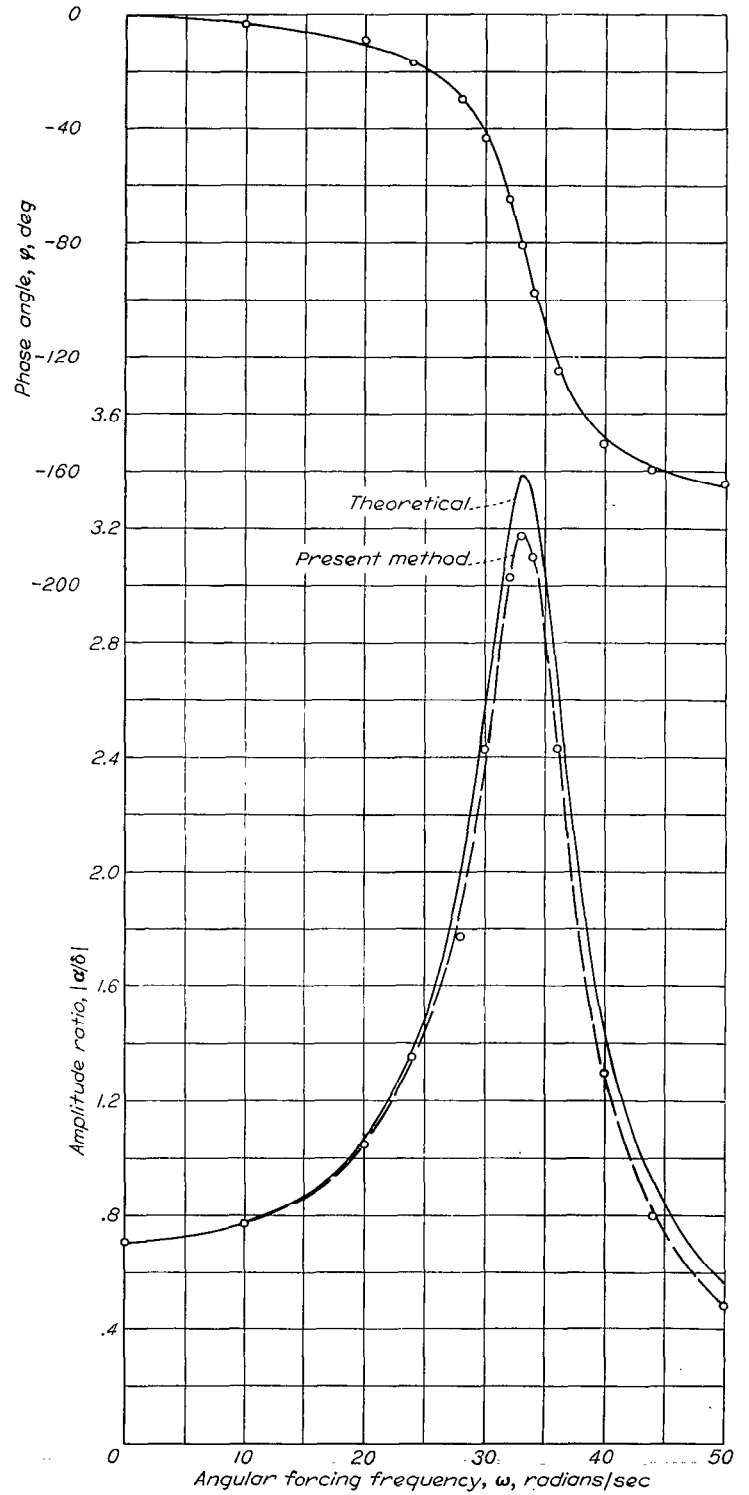Figure 4.—Concluded.

Figure 5.—Comparison of the theoretical response and the computed response determined from the computed frequency responses of figure 4.

These results obviously only approximate the theoretical values; this fact, of course, is expected since the individual response at each ω was calculated by a finite number of terms. In each of the responses, however, the comparison is considered entirely satisfactory. Furthermore, the extension presented in this report is shown to be reliable.

<div align="center">EXAMPLE II</div>

The second example is illustrated in figure 6 in which is considered the α/δ response of the same aircraft used in example I. The input δ is the ramp function as shown in the figure; the output transient response α is also presented.

In this case a mathematical element is considered as described in the analysis. An imaginary η as a step-function input is considered, and the output of the mathematical element is the ramp function δ. In this case no linear differential equation could reasonably be expected to give a ramp-function transient response to a step-function input. Therefore, the frequency response determined by the arbitrary-input technique and the theoretical frequency response may differ more than in the previous example.

The δ/η response is found by applying the step-function technique to the ramp function δ. Two cases were computed:

(1) The ramp-function transient time was 0.4 second. A time increment Δt of 0.05 second was used and resulted in eight terms in the summation.

(2) The transient time was 0.4 second, and a time increment Δt of 0.01 second, which gave 40 points, was used.

The step-function technique was applied to the output α response to give the α/η frequency response. In this case the transient time was chosen as 1.0 second, a time increment Δt of 0.025 second was used, and a total of 40 points resulted.

Figure 7 shows the results of combining the separate α/η and δ/η responses to give the desired α/δ frequency response. The cases for both δ/η results are shown. The solid curves are the theoretical frequency response and are the same as those used in example I.

In this example the discrepancy in the α/δ responses is evident. No sound explanation of these differences is known; however, conjectures arise. A simple explanation that may apply is that no linear differential equation could be expected to give a ramp-function transient response to a step-function input. Whatever may be said about this topic, the final results may be satisfactory in some cases. The general trend is revealed although the peak value is accentuated in this example.

<div align="center">EXAMPLE III</div>

The final example is the determination of the α/δ response of an aircraft from the experimental flight-test data of δ and α time responses. These data were obtained from the flight

of a rocket-powered aircraft model. Additional measurements of lift and other factors were made during this flight, and by using the period and rate of decay of the oscillations in angle of attack and normal acceleration, the longitudinal stability derivatives for the aircraft were found and presented in table I. From the flight-test data the factor $C_{m_q}+C_{m_{\dot\alpha}}$ was determined. Reference 7 was used to indicate that
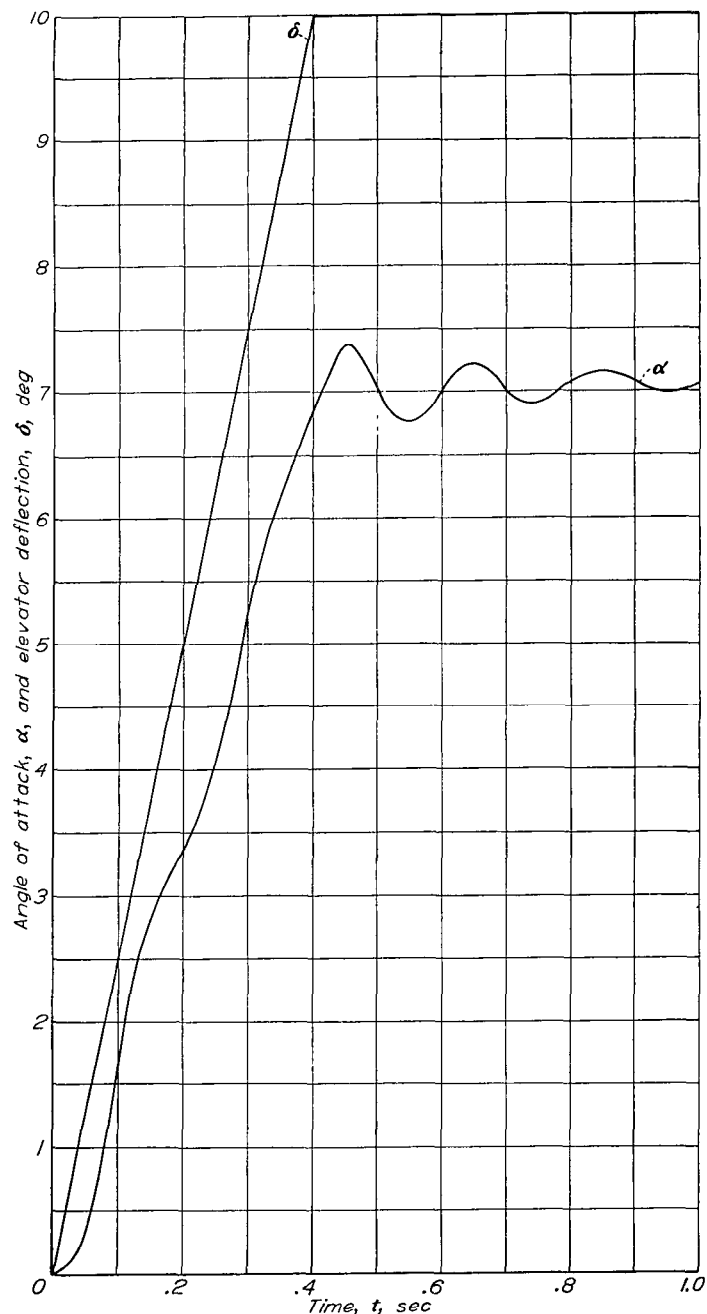

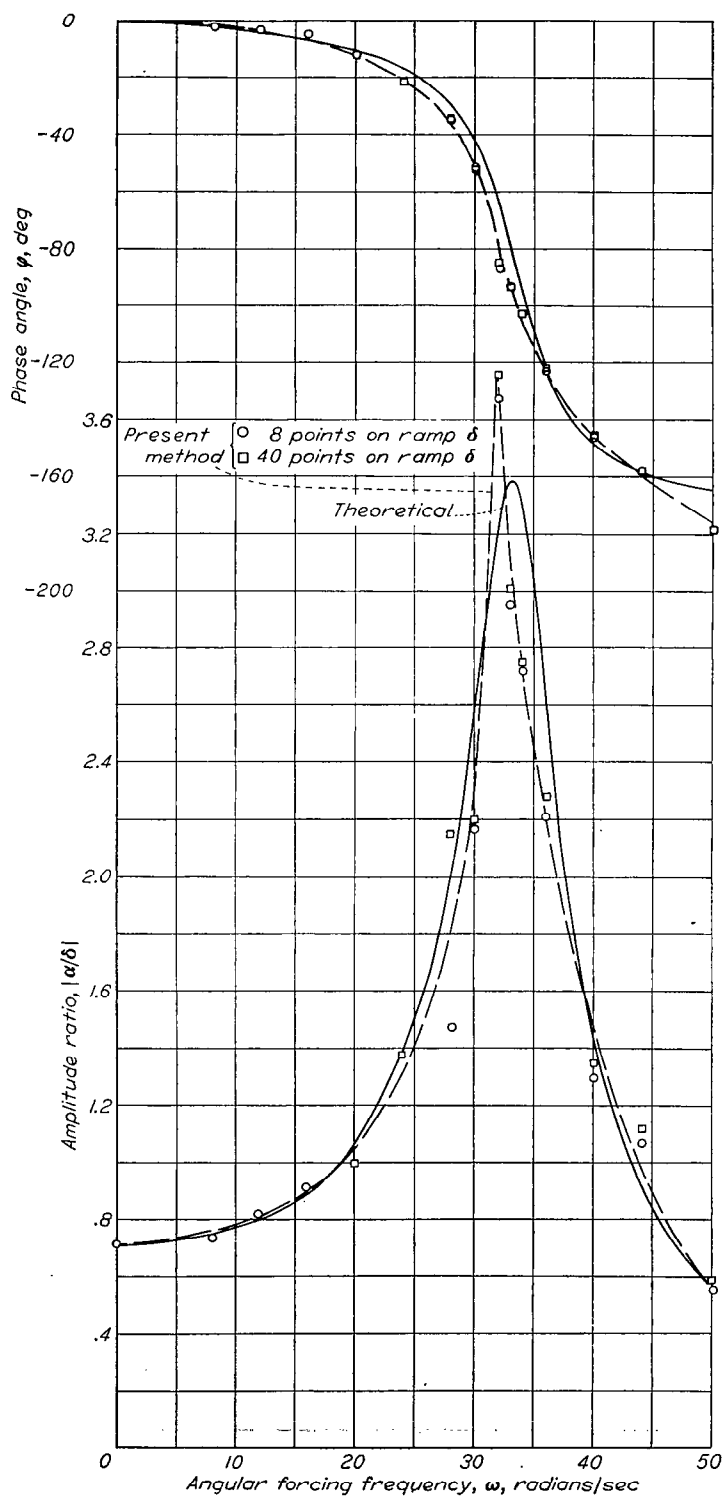FIGURE 6.—The input δ and output α transients for the ramp-function-input example.

FIGURE 7.—Comparison of theoretical and computed frequency responses for the ramp-function-input example.

C_{m_q} was 60 percent of this total factor, the value used in the theoretical calculations herein. For the present report the δ and α responses are presented in figure 8.

The step-function technique using a transient time of 0.55 second and a time increment Δt of 0.0275 second (20 points) was applied to both the δ and α transients.

The two responses were combined vectorially by the method herein to give the desired α/δ frequency response (fig. 9). The theoretical curves, shown as solid lines, were found by using equation (6) and the stability derivatives presented in table I. These curves are the amplitude-ratio and phase-angle curves and show satisfactory agreement. In this case the present method gives results comparable to those found by using the equations of motion and the stability derivatives. The response curves resulting from the present method may be even more reliable than the calculated curves since the equations of motion and the required coefficients may not be completely expressed.
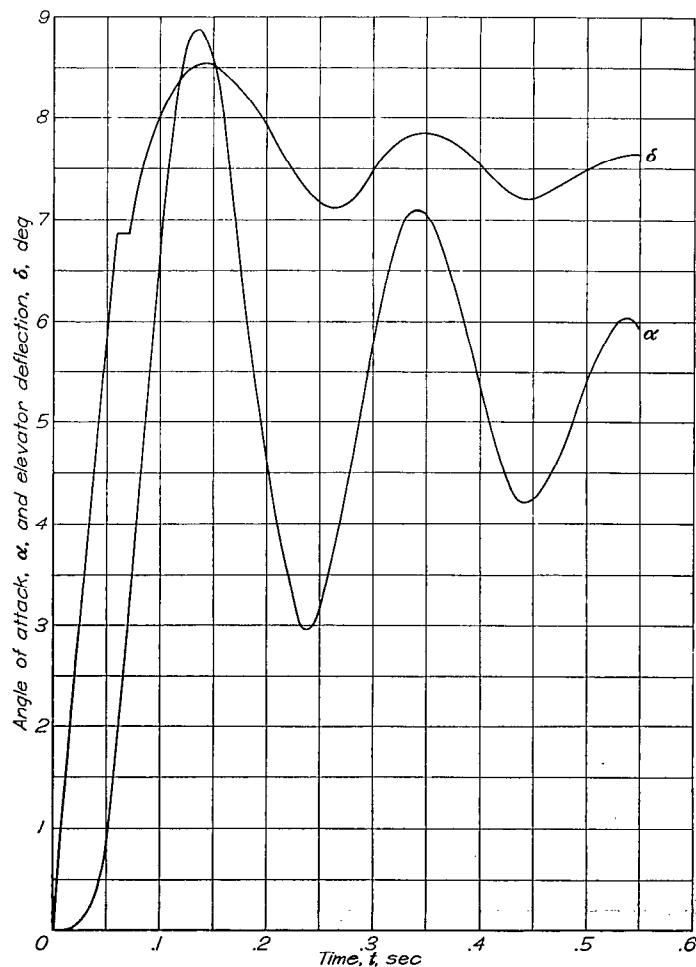

FIGURE 8.—The experimental arbitrary elevator input and resulting angle-of-attack output from flight test of a rocket-powered aircraft model.
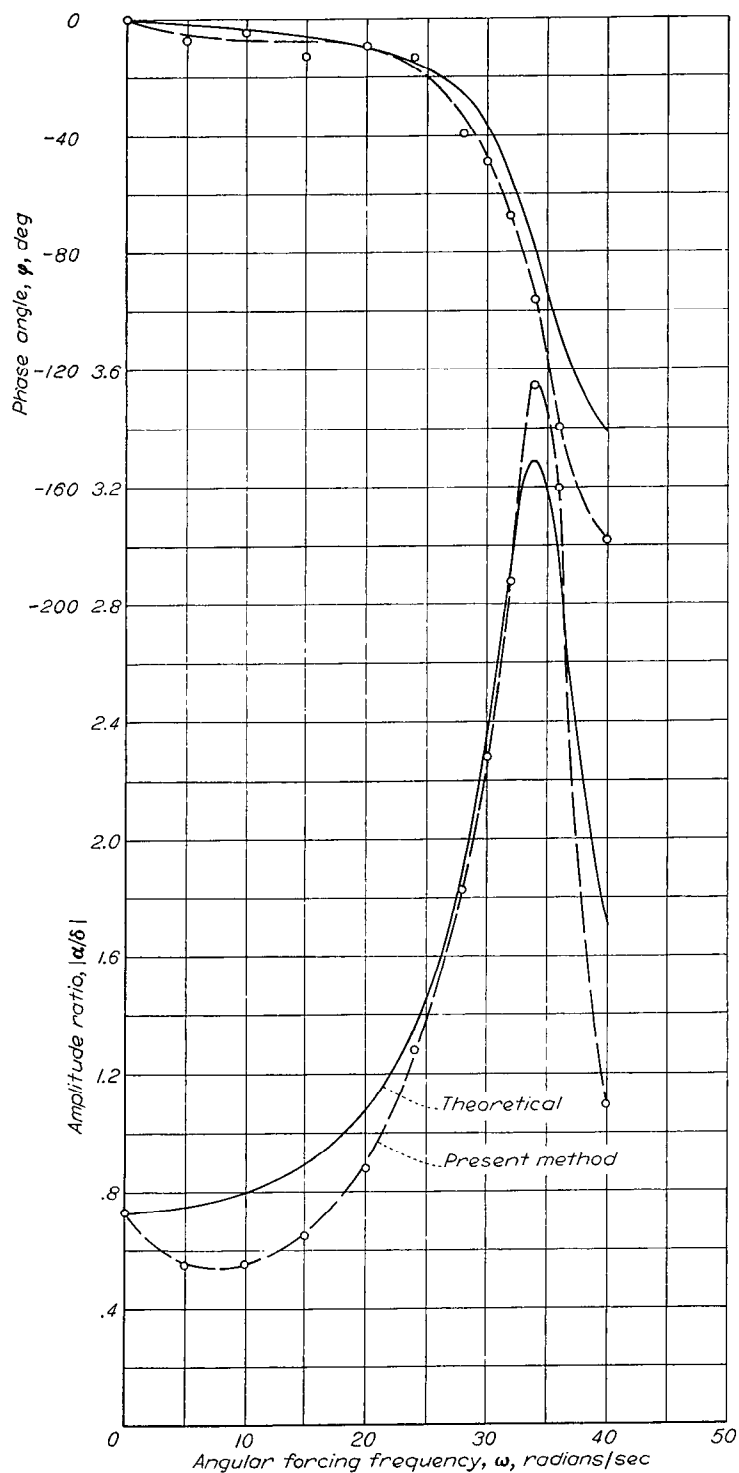
FIGURE 9.—Comparison of theoretical and computed aircraft angle-of-attack frequency responses as determined from flight-test data.

The phase angles resulting from the use of equation (6) with this configuration can be seen to have the incorrect signs for the true lag angles. These values have been corrected by subtracting 180° from the calculated value. This inconsistency arises since the NACA sign convention requires that a positive δ (input) to this airframe produce a negative α (output). The canard airframe of example 1, however, has a positive δ which produces a positive α; thus, equation (6) in that case gives the correct signs for the lagging phase angles.

CONCLUDING REMARKS

A method has been presented for determining the frequency response of an element or system when the transient output response to a known arbitrary input function is obtainable. This method has been derived by extending an analysis that permitted the determination of the frequency response when the transient resulting from a step-function input is known. This method has been illustrated by three examples, which include the determination of an aircraft angle-of-attack response from experimental flight-test data involving an arbitrary elevator input. The method is limited to inputs that tend to a fixed value after a finite time and to systems having an output that can be measured as a quantity having a static sensitivity.

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  NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
    LANGLEY AIR FORCE BASE, VA., *August 23, 1949.*

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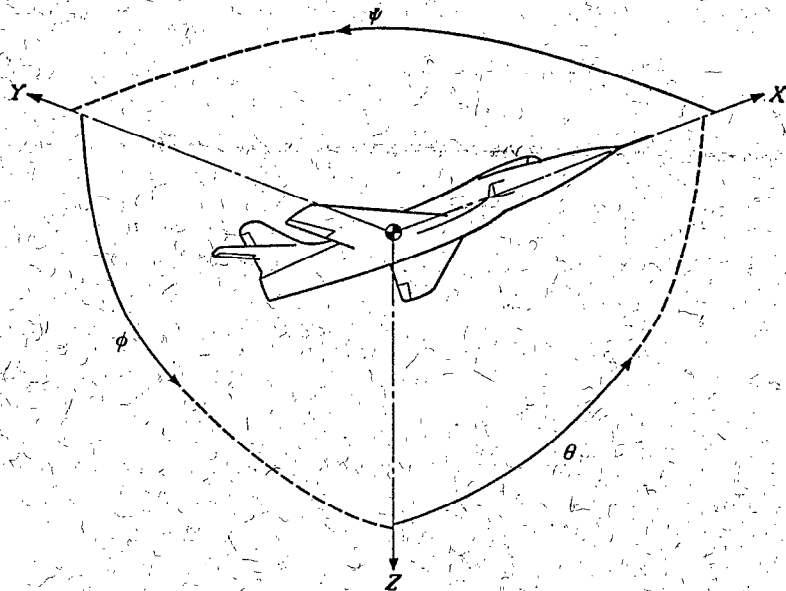
Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS} \qquad C_m = \frac{M}{qcS} \qquad C_n = \frac{N}{qbS}$$
$$\text{(rolling)} \qquad \text{(pitching)} \qquad \text{(yawing)}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D Diameter

p Geometric pitch

p/D Pitch ratio

V' Inflow velocity

V_s Slipstream velocity

T Thrust, absolute coefficient $C_T = \dfrac{T}{\rho n^2 D^4}$

Q Torque, absolute coefficient $C_Q = \dfrac{Q}{\rho n^2 D^5}$

P Power, absolute coefficient $C_P = \dfrac{P}{\rho n^3 D^5}$

C_s Speed-power coefficient $= \sqrt[5]{\dfrac{\rho V^5}{P n^2}}$

η Efficiency

n Revolutions per second, rps

Φ Effective helix angle $= \tan^{-1}\left(\dfrac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp=76.04 kg-m/s=550 ft-lb/sec

1 metric horsepower=0.9863 hp

1 mph=0.4470 mps

1 mps=2.2369 mph

1 lb=0.4536 kg

1 kg=2.2046 lb

1 mi=1,609.35 m=5,280 ft

1 m=3.2808 ft