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REPORT No. 69

A STUDY OF AIRPLANE RANGES AND USEFUL LOADS

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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A STUDY OF AIRPLANE RANGES AND USEFUL LOADS

By J. G. COFFIN
Director of Research, Curtiss Engineering Corporation

Part I.—NUMERICAL ANALYSIS
Part II.—THEORETICAL ANALYSIS
Part III.—EFFECT OF WIND ON RANGE

CONTENTS.

[1] Submitted Sept. 18, 1918.

[2] Submitted Sept. 1, 1919.

INTRODUCTION.

In airplane performance estimates the fuel load is given usually in terms of a quantity sufficient for a certain number of hours of motor consumption at full throttle. In a weight-carrying machine more useful load can, of course, be carried as the fuel load required to attain the objective is diminished.

It becomes, therefore, of practical interest to examine the relations between these loads in greater detail than usual.

A machine can fly high or low, at maximum speed or at most economical speed, or at most economical power consumption. It is not at all evident a priori which of these or what combination of them is best under given conditions. The following study was primarily made to determine the conditions necessary to attain a given objective with the maximum bombing load and return.

It is, of course, evident that the calculations and theory as applied to bombing purposes will apply equally well for commercial load carrying purposes. The results are also directly applicable to the interesting questions of long distance flights, across the oceans or for purposes of exploration.

The investigation was made by two independent methods, one involving the usual performance estimate methods, the other based upon theoretical considerations.

As a specific example the data on an 800-horsepower, 15,000-pound bombing machine was used, but the method is applicable with slight variations to any machine.

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PART I.

NUMERICAL ANALYSIS.

By J. G. Coffin.

The essential data for the specific machine used in the calculations are given in Table I. The total wing and parasite resistances were computed for assumed total weights of 15,000, 13,000, 11,000, 9,000, and 7,000 pounds respectively The total resistance and required horsepower curves were then plotted against speed in the usual manner. See Table I and Fig. 1.

Table I.—*Summary of total resistances for a machine with a variable load. Area of wing=1,875 square feet.*

Weight and wing loading.	Speed in M. P. H.	Monoplane $K_y \times 10^4$.	Biplane K_y= .85 mono $K_y \times 10^4$.	Monoplane K_y/K_x.	Biplane K_y/K_x= .85 mono K_y/K_x.	Wing resistance =W/Bip. (L/D).	Parasite resistance ×1.10.	Total resistance wing+ parasite.	Required horse-power.
						Pounds.	*Pounds.*	*Pounds.*	
15,000 pounds or 8 pounds per square foot	70.0	19.2	16.3	16.2	13.8	1,090	609	1,699	318
	80.0	14.9	12.5	21.2	18.0	834	735	1,569	334
	90.0	11.6	9.9	21.7	18.4	814	937	1,751	421
	100.0	9.40	8.0	19.6	16.7	899	1,151	2,050	547
	110.0	7.88	6.6	16.8	14.3	1,050	1,400	2,450	719
13,000 pounds or 6.92 pounds per square foot	60.0	22.6	19.2	9.3	7.9	1,643	475	2,118	339
	70.0	16.6	14.1	19.8	16.8	773	609	1,382	258
	80.0	12.7	10.8	21.8	18.5	702	735	1,437	306
	90.0	10.0	8.55	20.5	17.4	718	937	1,656	404
	100.0	8.15	6.91	17.3	14.7	884	1,151	2,035	543
	110.0	6.72	5.71	15.0	12.7	1,022	1,400	2,422	710
11,000 pounds or 5.86 pounds per square foot	60.0	19.2	16.3	16.2	13.8	801	475	1,276	204
	70.0	14.1	12.0	21.5	18.3	602	609	1,211	226
	80.0	10.75	9.15	21.3	18.1	608	735	1,343	286
	90.0	8.48	7.23	18.2	15.5	712	937	1,649	396
	100.0	6.90	5.86	15.2	12.9	854	1,151	2,006	535
	110.0	5.70	4.84	13.0	11.1	996	1,400	2,396	702
9,000 pounds or 4.80 pounds per square foot	50.0	22.6	19.2	9.2	7.8	1,152	365	1,517	202
	60.0	15.7	13.3	20.6	17.5	515	475	990	158
	70.0	11.5	9.79	21.6	18.4	491	609	1,100	206
	80.0	8.82	7.50	18.6	15.8	570	735	1,305	278
	90.0	6.97	5.92	15.4	13.1	686	937	1,623	390
	100.0	5.66	4.80	13.0	11.1	815	1,151	1,966	524
	110.0	4.68	3.97	11.2	9.5	946	1,400	2,346	689
7,000 pounds or 3.74 pounds per square foot	42.7	24.0	20.4	6.6	5.6	1,250	270	1,520	173
	50.0	17.6	14.9	18.8	16.0	439	365	804	107
	60.0	12.3	10.4	21.8	18.5	378	475	853	136
	70.0	9.02	7.65	19.2	16.3	429	609	1,038	194
	80.0	6.85	5.84	15.1	12.8	547	735	1,082	274
	90.0	5.44	4.62	12.5	10.6	660	937	1,597	383
	100.0	4.41	3.74	10.6	9.0	777	1,151	1,928	514
	110.0	3.64	3.09	8.4	7.1	981	1,400	2,381	698

CONDITIONS FOR MAXIMUM RANGE.

It is evident that the *work* required to fly a given distance, being the product of the total resistance, or required propeller thrust, by the distance, is least when the thrust is kept at its minimum values. The points of minimum drag were, therefore, located on the resistance curves. These points determined the most economical speeds and the corresponding required powers. The powers thus determined are seen not to be minimum powers. The minimum powers are but slightly less than those corresponding to minimum resistance and occur at speeds slightly less than minimum resistance speeds. The minimum power is that for which the fuel consumption is least for a given *time* and as it turns out, is not the most economical power for flying the greatest distance. The speed corresponding to minimum power is the speed at which a machine should fly in order to remain aloft the greatest possible time.

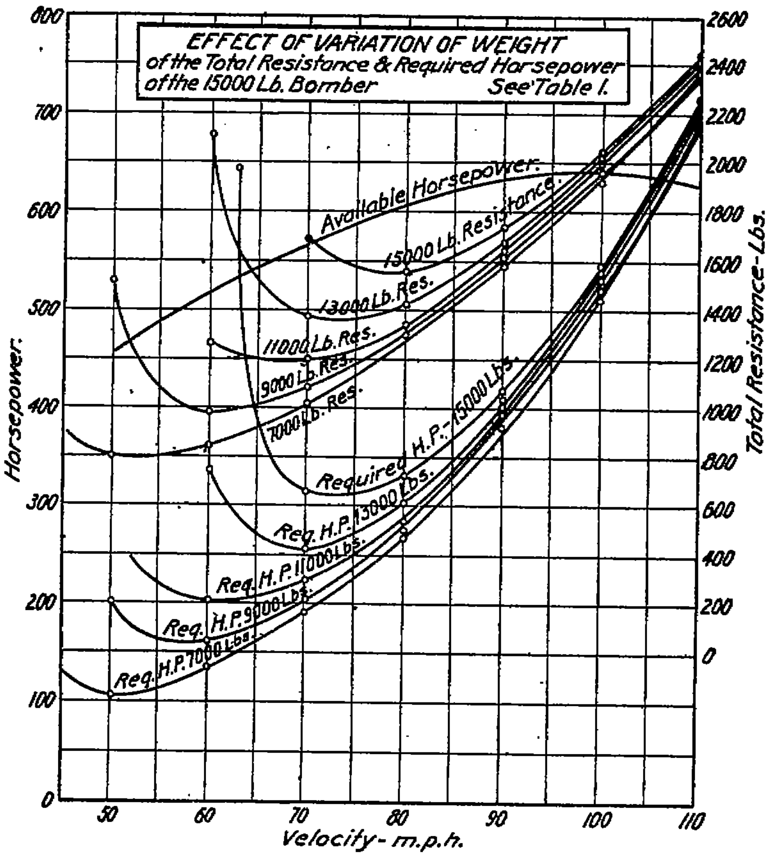

Fig. 1.

In figure 2 the weight of the machine is plotted against minimum resistance. The curves in figures 1 and 2 show that—

1. The maximum range speeds decrease as the load decreases. The plane must fly slower and slower as the load diminishes.

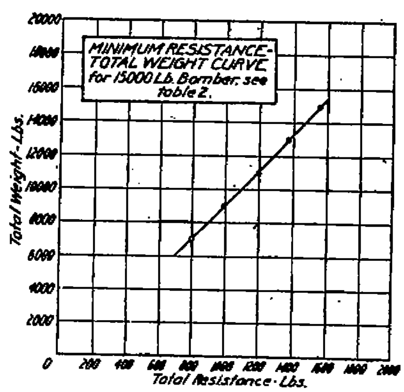

Fig. 2.

2. The maximum range powers decrease as the load decreases.

3. These powers are not minimum powers but are slightly greater and correspond to greater speeds than least power speeds.

4. The total air resistances decrease almost in exact proportion to the weight of the plane.

FUEL CONSUMPTION.

The fuel consumption at maximum brake horsepower output is taken as 0.6 pound per horsepower hour, and for any reasonable throttled condition this number is increased to 0.7.

The available horsepower curve was obtained in the usual manner by assuming proper propeller efficiencies at slow and high speeds and multiplying these into the available brake horsepower. The fuel consumption per horsepower delivered by the propeller can therefore be computed by dividing the fuel consumption per brake horsepower of the motor by the propeller efficiency at that speed, or what is the same thing, multiplying 0.6 by 800 and dividing by the available horsepower at that speed. As mentioned above, these results are multiplied by $\frac{0.7}{0.6}$ in order to compensate for a slight loss of efficiency under throttled conditions. This procedure corresponds to experimental tests and if anything probably overestimates the fuel consumption.

With these data Table 2 and figure 3 were made. The fuel consumption is seen to be proportional to the weight.

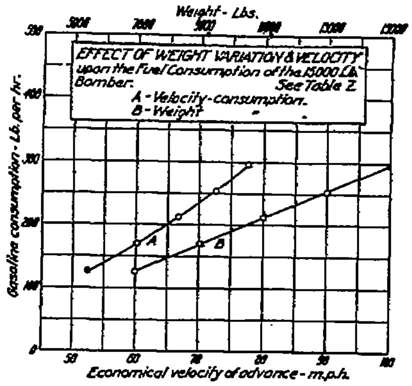

Fig. 3.

TABLE 2.—*Gas consumption in pounds per hour at the economical speed.*

Weight of machine.	Minimum total resistance.	L/D. or weight/tot. res.	Corresponding speed in miles per hour.	Corresponding horsepower.		Gasoline consumption, $\frac{0.7 \times 800}{\text{avail. H.P.}}$ in pounds/per horsepower.	Total gasoline consumption per hour.
				Required.	Available.		
Pounds.	*Pounds.*						*Pounds.*
15,000	1,560	9.62	77.5	320	603	0.927	296
13,000	1,372	9.47	72.5	265	583	.960	254
11,000	1,195	9.22	66.7	212	555	1.009	214
9,000	992	9.08	59.6	159	517	1.082	172
7,000	795	8.82	52.5	107	472	1.187	127

TABLE 2A.—*Gas consumption in pounds per hour at the maximum speed.*

Weight of machine.	Minimum total resistance.	L/D. or weight/tot. res.	Corresponding speed in miles per hour.	Corresponding horsepower.		Gasoline consumption, $\frac{0.6 \times 800}{\text{avail. H.P.}}$ in pounds/per horsepower.	Total gasoline consumption per hour.
				Required.	Available.		
Pounds.	*Pounds.*						*Pounds.*
15,000	2,265	6.63	105.4	636.0	636.0	0.755	480
13,000	2,260	5.75	105.9	635.5	635.5	.755	480
11,000	2,245	4.91	106.3	635.0	635.0	.756	480
9,000	2,225	4.05	106.8	634.9	634.9	.756	480
7,000	2,225	3.15	106.8	634.8	634.9	.756	480

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Figure 4 shows a curve giving the relation between the weight at any time and the corresponding distance flown. Starting with a full fuel load of 7,870 pounds, giving a total weight of 15,000 pounds, the machine was assumed to travel for a given time interval (two hours) at the weight, speed, gas consumption, and thrust corresponding to that weight. During the next two hours it was assumed to fly at new values corresponding to the new weight which is equal to the old weight less the fuel consumed in the preceding time interval, and so on.

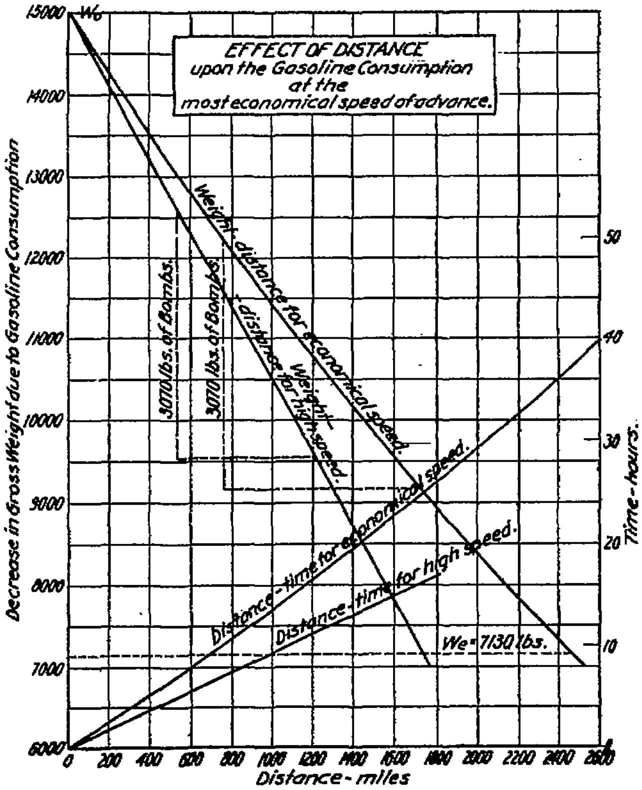

Fig. 4.

TABLE 2B.—*Ratios of L/D and angles of incidence for maximum range.*

MAXIMUM RANGE SPEEDS.

Loading pds./sq. ft.	Biplane L/D.	Velocity, miles per hour.	Biplane, Ky×10⁴.	Monoplane, Ky×10⁴.	Angle of incidence degrees.
8.00	15.55	77.5	13.8	15.65	5.3
6.92	15.63	72.5	13.2	15.55	5.2
5.86	15.63	66.7	13.15	15.50	5.2
4.80	15.38	59.6	13.5	15.90	5.45
3.74	15.30	52.5	13.55	15.95	5.5

HIGH SPEED.

Loading pds./sq. ft.	Biplane L/D.	Velocity, miles per hour.	Biplane, Ky×10⁴.	Monoplane, Ky×10⁴.	Angle of incidence degrees.
8.00	14.45	105.4	7.15	8.41	1.75
6.92	13.17	105.9	6.18	7.27	1.25
5.86	11.65	106.3	5.17	6.07	0.7
4.80	10.03	106.8	4.21	4.95	.2
3.74	3.06	106.8	3.28	3.86	—.35

TABLE 3.—*Time-weight variation computation for speeds.*

MAXIMUM RANGE.

Initial weight of machine in pounds.	Thrust in pounds.	Velocity, in miles per hour.	Gasoline consumed, pounds per hour.	Distance, miles.		Gasoline consumed in interval. Pounds.	Time in hours.
				S=Vt.	Total.		
15,000	1,560	77.5	297	155.0	155	594	2
14,406	1,500	76.3	286	152.6	307.6	572	4
13,834	1,450	74.9	273	149.8	457.4	546	6
13,288	1,403	73.3	261	146.6	604.0	522	8
12,766	1,355	71.3	250	142.6	746.6	500	10
12,266	1,305	70.5	240	141.0	887.6	480	12
11,786	1,265	69.0	230	138.0	1,025.6	460	14
11,326	1,225	67.5	220	135.0	1,160.6	440	16
10,886	1,185	66.2	210	132.4	1,293.0	420	18
10,466	1,145	64.7	201	129.4	1,422.4	402	20
10,064	1,110	63.5	193	127.0	1,549.4	386	22
9,678	1,065	62.2	184	124.4	1,673.8	368	24
9,310	1,030	61.0	177	122.0	1,795.8	354	26
8,956	995	59.7	169	119.4	1,915.2	338	28
8,618	965	58.6	162	117.2	2,032.4	324	30
8,294	930	57.4	155	114.8	2,147.2	310	32
7,984	900	56.3	148	112.6	2,259.8	296	34
7,688	872	55.1	142	110.2	2,370.0	284	36
7,404	840	54.1	136	108.2	2,478.2	272	38
7,132	815	53.3	132	106.6	2,584.8	264	40

The maximum possible range is seen to be 2,480 miles under these conditions.

DETERMINATION OF THE MAXIMUM LOAD FOR A GIVEN OBJECTIVE.

Evidently in case a return trip is to be made without refueling the greatest distance for an objective is equal to or less than half this greatest range. It is easy to determine the greatest possible useful load by means of the weight-distance curve, figure 5, in the following way:

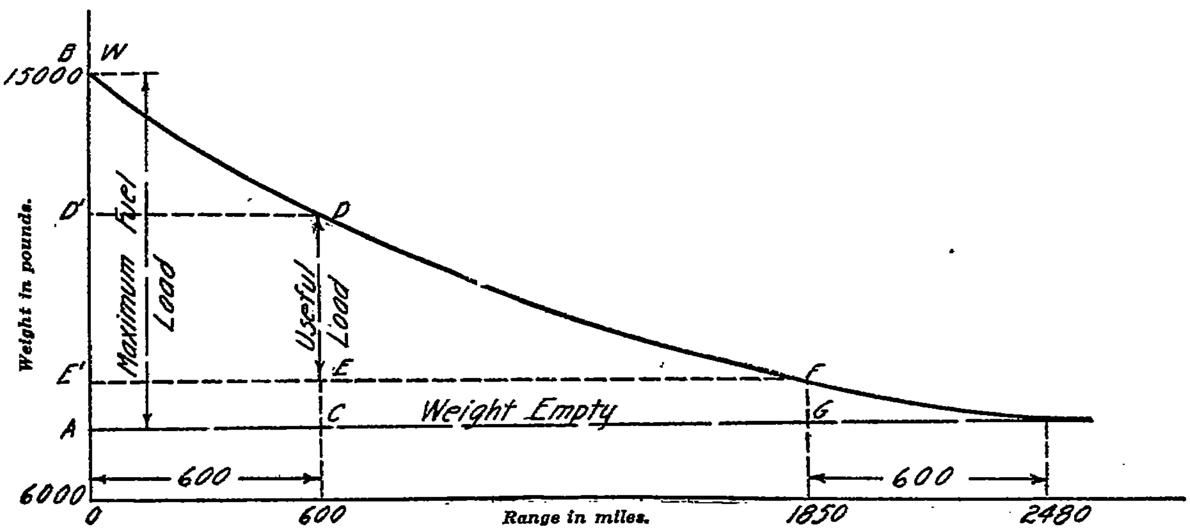

Fig. 5.

Suppose the objective is 600 miles distant. It requires AB–CD pounds of fuel to get there and GF pounds to get back after the load is deposited. Since the maximum load is AB pounds there will be left DE or D′E′ pounds for useful load. Calling the maximum range S, project the points on the curve for $s=600$, point D, and $s=S-600$, point F, on the weight axis, the weight included between these two points is the maximum load for that objective. This procedure is quite general. The load decreases to zero as the objective distance increases to half the maximum range and increases to the maximum load as the objective distance decreases to zero.

A curve (fig. 6) was determined by this method for this machine which gives directly the maximum useful load for any objective. This curve turns out to be practically a straight line. A proof that it should be very approximately straight is given in Part II.

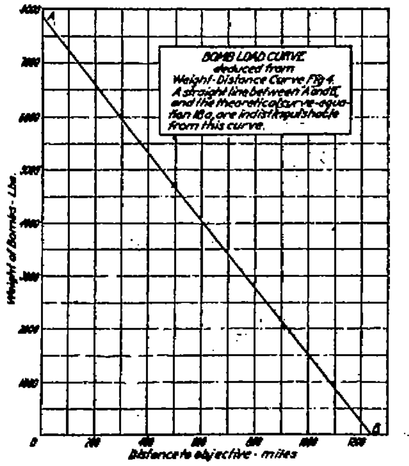

Fig. 6.

CONSEQUENCES OF FLYING AT MAXIMUM SPEED.

In Table 2a the results for flight at maximum speed are tabulated. The gas consumption is constant and the speed of advance was found to be constant to within about 1 per cent. The average value, 106.2 miles per hour was, therefore, used in the computations.

The weight-distance curve (fig. 4) is a straight line, as the fuel consumption and speed are very approximately constant.

The maximum range is considerably less than under greatest range conditions. The difference is 740 miles. A considerable gain in range is thus attained by flying at the proper angle and hence at proper speed.

The useful loads for maximum speed are considerably less than under best range conditions. For an objective 600 miles away the best conditions give a possible load of 4,050 pounds while at maximum speed this is reduced to 2,430 pounds, a reduction of 1,620 pounds.

For convenience, a comparison of the loads and ranges corresponding to them is made in the following tables:

TABLE 4.

Hours of fuel full open.	Bombing load in pounds.	Range at maximum speed.		Range at best speed.		Difference in miles.	
		Total.	Objective.	Total.	Objective.	Total.	Objective.
10	3,070	1,060	530	1,510	755	450	225
7½	4,270	790	395	1,130	565	340	170
4	5,950	420	210	610	305	190	95

TABLE 5.

Hours of fuel full open.	Range.		Bombing load maximum speed.	Bombing load best speed.	Difference in pounds.
	Total.	Objective.			
10	1,510	755	1,030	3,070	2,040
7½	1,130	565	2,738	4,270	1,532
4	610	305	5,090	5,950	860

For the shorter flights the differences decrease but they are considerable in all cases. The bombing load is increased by almost 190 per cent for maximum range speeds over maximum speed conditions for 10 hours fuel.

FLYING AT MINIMUM POWER.

The gas consumption at minimum power is practically identical with that at best range power. While the minimum power is slightly less than the power for best range speed, the speed is also less and the propeller efficiency is also slightly less. The net result is that the time of flight is about the same and the maximum range is diminished.

A calculation of the range at minimum power gives 2,400 miles instead of 2,480 miles.

For flight at minimum power the angle of attack is practically constant and slightly greater than that for best range speed.

TIME REQUIRED FOR ANY RANGE.

For convenience, the curves of elapsed time for any distance flown are given in figure 4 for both best range speeds and maximum speed conditions. By means of them the time of going and returning from any given objective may be read off. In particular it is seen that the maximum time of flight under high speed is 16.4 hours as against 38.0 hours for best range speed.

For a bombing raid on an objective at 600 miles, the total elapsed time to go and return is for maximum speed 11.25 hours and for best range speed 18.65 hours. It will be seen in the following how this time difference may be decreased by flying at high altitudes without changing the efficiency for best range conditions.

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PART II.

THEORETICAL ANALYSIS.

By J. G. Coffin.

THEORY OF MAXIMUM RANGE CONDITIONS.

Notation:

- W = weight of machine at any time.
- W_f = weight of machine fully loaded with fuel.
- W_e = weight of machine empty of fuel.
- T = thrust of propeller.
- η = efficiency of propeller.
- A = supporting wing area.
- V = speed of flight.
- L = lift per unit wing area per unit speed at ground level.
- D = drift per unit wing area under same conditions.
- R = parasite resistance per unit wing area under same conditions and is assumed constant.

L and D depend only on the flying angle of attack.

- s = distance traversed in time t.
- S = range.
- $\gamma = \rho / \rho_0$ = ratio of the density of the air at height h to its density at ground level.
- c = pounds of fuel (gas and oil) per brake horsepower hour consumed by motors.
- $a = \frac{c}{\eta}$ = pounds of fuel per useful horsepower hour delivered by propeller.

The lift coefficient K_y, as usually given, is proportional to the density, and we may, therefore write

$$K_y = k_y \rho = (k_y \rho_0) \rho / \rho_0 = L\gamma$$

where L is the value of K_y at ground level.

The fundamental equations for horizontal flight of the airplane are

$$W = L\gamma AV^2 \quad (1)$$

$$T\ \not{L} = D\gamma AV^2 + R\gamma AV^2 = (D+R)\gamma AV^2 \quad (2)$$

CONDITION FOR MINIMUM WORK.

The work done in flying a distance dl against a total drag T is Tdl. The total work done in flying a given distance s is, therefore,

$$\text{Work} = \int_0^s T dl$$

This work integral is evidently a minimum if T is always at its least possible value.

From equations (1) and (2) we get by division

$$T = W\left(\frac{D+R}{L}\right) \tag{3}$$

This equation shows, since γ has disappeared, that for a constant angle of attack and given weight the thrust is independent of the height at which the flight takes place, and also that for a constant angle of attack the thrust is proportional to the total weight.

It is for the first reason that no mention of altitude was made in Part I. The second statement is verified in figure 2, which plainly shows very approximate proportionality at all actual flying speeds.

It is convenient to employ the polar diagram in the following: This kind of diagram deserves greater popularity than it has yet received in aeronautical calculations in the United States. It consists in plotting L as ordinates against D as abscissae. Any point M on the curve corresponds to a given L and D and hence to a given angle of attack. This angle of attack is marked on the curve.

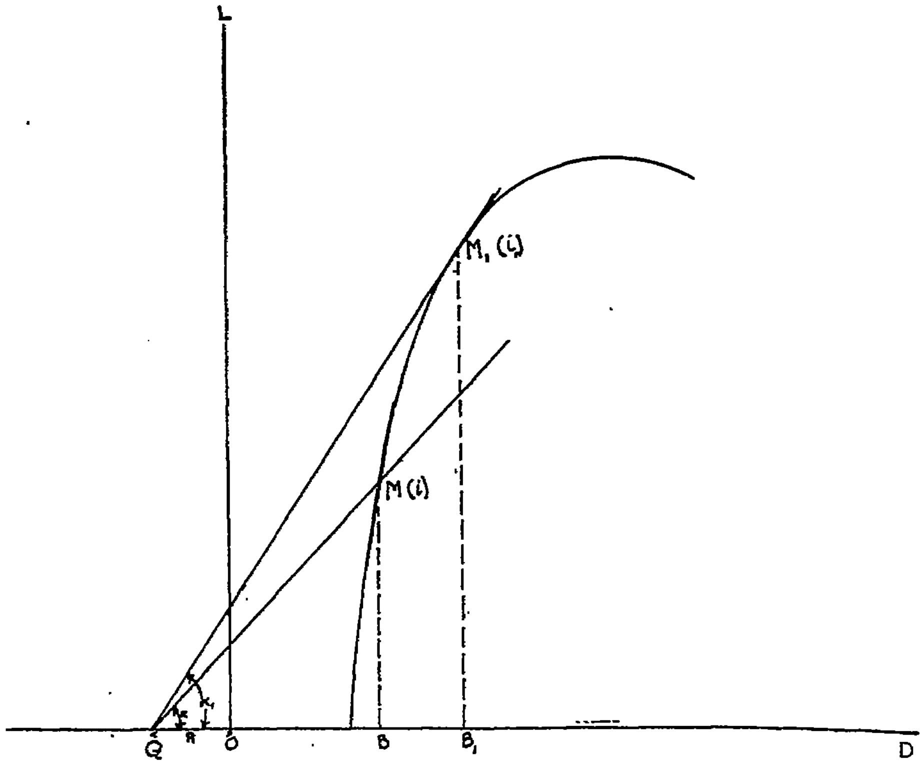

Fig. 7.

Lay off from O the distance OQ equal to R.

Then for any point M, OB = D, MB = L, and OQ = R.

The slope of the line $QM = \frac{L}{D+R} = \tan \alpha$

Consider equation (3). For a given weight and variable values of L and D, that is, for variable angles of attack, let us find the condition for minimum thrust.

$$dT = 0 = \frac{L\,(dD) - (D+R)\,dL}{L^2}$$

As L can not be infinite, the condition is

$$\frac{dL}{dD} = \frac{L}{D+R} \tag{4}$$

That is to say, for minimum thrust the tangent to the polar curve must have a value $\frac{L}{D+R}$. Looking at the polar diagram it is seen that the line QM_1 tangent at M_1 fulfills this condition and corresponds to a given angle of attack i_1. Since the polar is a given curve for the machine, it follows that

(*a*) for minimum thrust the machine must fly at a constant angle of incidence throughout the whole flight whatever the loading may be, and

(*b*) that these minimum thrusts are independent of altitude of flight.

In Part I, Table 2*b*, the angles of incidence corresponding to the least drag have been tabulated. They are remarkably constant, their average deviation from the mean 5.33° being only 0.12° and the greatest difference 0.3°.

For best range speeds, then, it follows that

$$W = C_1 \gamma V^2 \text{ where } C_1 = LA \tag{5}$$

and

$$T = C_2 \gamma V^2 \text{ where } C_2 = (D+R)\, A \tag{6}$$

where C_1 and C_2 are constants for flight at *any* constant angle of incidence. In particular for maximum range conditions they have the values corresponding to the maximum value of $\frac{L}{D+R}$ for the machine.

Let us suppose that the machine loses weight gradually due to fuel consumption alone. If the weight-distance curve can be deduced, the problem of bombing loads and ranges is solved, as explained in Part I.

DEDUCTION OF THE WEIGHT-TIME EQUATION.

Assume that the range of speeds is such that the engine runs at constant efficiency, burning "*a*" pounds of fuel per horsepower hour delivered by the propeller. If c = pounds of fuel consumed per brake horsepower hour and the propeller efficiency is denoted by η, then the fuel consumed per horsepower hour delivered by the propeller is given by $a = \frac{c}{\eta}$

The power at any speed is

$$TV = C_2 \gamma V^2 . V = C_2 \gamma \left(\frac{W}{C_1 \gamma}\right)^{3/2} = \frac{1}{\sqrt{\gamma}} \frac{C_2}{C_1^{3/2}} W^{3/2} \tag{7}$$

The rate of fuel consumption is aTV and in time dt a weight $aTVdt$ of fuel will be consumed, hence

$$aTVdt = \frac{1}{\sqrt{\gamma}} \frac{aC_2}{C_1^{3/2}} W^{3/2} dt = -dW \tag{8}$$

where $-dW$ is the loss of weight of the plane in time dt.

The weight of the plane at any time t is therefore, if W_f is the weight full,

$$W = W_f - \frac{a}{\sqrt{\gamma}} \frac{C_2}{C_1^{3/2}} \int_0^t W^{3/2} dt \tag{9}$$

Equation (8) is a differential equation for W at any time, and may be written

$$\frac{dW}{W^{3/2}} = -\frac{a}{\sqrt{\gamma}} \frac{C_2}{C_1^{3/2}} dt = -K dt \tag{10}$$

$$\text{where } K = \frac{a}{\sqrt{\gamma}} \frac{C_2}{C_1^{3/2}} \tag{11}$$

Solving and determining the constant of integration by the condition that when $t=0$, $W=W_f$, we get

$$\frac{1}{\sqrt{W}}=\frac{K}{2}t+\frac{1}{\sqrt{W_f}} \tag{12}$$

This equation is the desired relation between W and t and shows how the load diminishes as t increases. Since K contains γ, the rate of diminution of load depends upon the altitude.

The time of maximum flight can be obtained by letting W_f decrease to W_e, the weight of the empty machine.

$$t_{max}=\frac{2}{K}\left(\frac{1}{\sqrt{W_e}}-\frac{1}{\sqrt{W_f}}\right) \tag{13}$$

This maximum time of flight computed with equation (13), using 1.03 pounds per horsepower hour as the average fuel consumption, gives a value of $t_{max}=38.3$ hours, as compared with $t_{max}=38.0$ hours as taken from the time-weight curve in figure 4. The two methods again check very well. The formula arranged to give the time in hours is, for low levels,

$$t\ (\text{hours})=10550\left(\frac{1}{\sqrt{W_e^{\#}}}-\frac{1}{\sqrt{W_f^{\#}}}\right)$$

This time of flight diminishes proportionally to $\sqrt{\gamma}$.

DEDUCTION OF THE DISTANCE-TIME EQUATION.

The distance traversed in time dt is, using (5).

$$ds=V dt=\sqrt{\frac{W}{C_1\gamma}}\,dt$$

so that in time t

$$s=\frac{1}{\sqrt{C_1\gamma}}\int_0^t \sqrt{W}\,dt$$

which becomes, using (12),

$$s=\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\int_0^t \frac{\frac{K}{2}\,dt}{\frac{K}{2}t+\frac{1}{\sqrt{W_f}}}$$

Integrating and determining the constant by the condition that when $t=0$, $s=0$, we get

$$s=\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\log\frac{K/_2\,t+\frac{1}{\sqrt{W_f}}}{\frac{1}{\sqrt{W_f}}} \tag{14}$$

This equation is the desired relation between the distance flown and the time. The distance increases with the time and depends upon the altitude of flight. For a given time interval the distance flown increases proportionally to $\frac{1}{\sqrt{\gamma}}$.

DETERMINATION OF THE WEIGHT-DISTANCE EQUATION.

Eliminate t between equations (12) and (14):

$$s=\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\log\frac{\frac{1}{\sqrt{W}}}{\frac{1}{\sqrt{W_f}}}=\frac{1}{K\sqrt{C_1\gamma}}\log\frac{W_f}{W} \tag{15}$$

where

$$\frac{1}{K\sqrt{C_1\gamma}}=\frac{\sqrt{\gamma}C_1^{3/2}}{aC_2\sqrt{C_1\gamma}}=\frac{C_1}{aC_2}=\frac{1}{a}\,\frac{L}{D+R} \qquad (16)$$

So that finally

$$s=\frac{1}{a}\cdot\frac{L}{D+R}\cdot\log\frac{W_f}{W}$$

Or, more specifically in terms of c and n,

$$s=\frac{375\,\eta}{c}\,\frac{L}{D+R}\cdot\log\frac{W_f}{W} \qquad (17)$$

This equation is true for any condition of flight at constant angle of incidence, where the ratio $\frac{L}{D+R}$ remains constant. For maximum range the maximum value of $\frac{L}{D+R}$ is of course used. This equation shows that as W diminishes s increases, and that the *distance* flown is independent of the flying altitude, since γ has disappeared.

The maximum range S can be obtained by finding the value of s for $W=W_e$.

Assume $W_f=15{,}000$ pounds.
$W_e=7{,}130$ pounds.

$$\text{Max.}\left(\frac{L}{D+R}\right)=9.2$$

$S=2{,}480$ miles.

We obtain for "a" a value of 1.03 pounds per horsepower hour.
This value checks very well with $a=1.033$ pounds per horsepower hour, the average value used in the preceding greatest range calculations, in Part I. This check is very satisfactory and shows that the two methods are in good agreement.

EQUATION CONNECTING USEFUL LOAD AND OBJECTIVE DISTANCE.

If we let $b=\frac{1}{a}\,\frac{L}{D+R}$ equation 17 becomes

$$s=b\log\frac{W_f}{W}$$

and hence

$$W=W_f e^{-s/b} \qquad (18)$$

Considering Fig. 5 it is easy to see that the load B for any given objective at distance s is evidently, if S = max. range

$$B=W_s-W_{S-s}=W_f(e^{-s/b}-e^{-(S-s)/b}) \qquad (19)$$

and since $W_e=W_f e^{-S/b}$ this becomes

$$B=W_f e^{-s/b}-W_e e^{s/b} \qquad (20)$$

We can thus compute the useful load for any objective. If the weight distance curve is plotted, however, it seems easier to plot the curve represented by (20) by the method explained above.

PROOF THAT LOAD-OBJECTIVE CURVE IS STRAIGHT.

In Part I the curve between useful load and distance to objective (Fig 6) was very approximately a straight line. The equation for this curve is

$$B = W_t(e^{-s/b} - e^{-(S-s)/b}) \tag{19}$$

where S is the maximum range, and s/b is a quantity which is smaller than unity and generally less than $\frac{1}{2}$.

This is not the equation of a straight line but turns out to be very closely straight as seen by the following:

Let $x = \frac{s}{b}$, $x_1 = \frac{S}{b}$ for the moment.

Expanding the exponentials and neglecting *cubes* and higher powers of x, we have

$$e^{-x} - e^{-(x_1 - x)} = 1 - x + \frac{x^2}{2} - \left[1 - (x_1 - x) + \frac{(x_1 - x)^2}{2}\right]$$

$$= (2 - x_1)\left(-x + \frac{x_1}{2}\right) \quad \text{a linear relation.}$$

The second powers have completely canceled.

Therefore

$$B = \frac{W_t}{b}\left(2 - \frac{S}{b}\right)\left(-s + \frac{S}{2}\right) \tag{21}$$

Wher

$$s = \frac{S}{2} \qquad B = o$$

When

$$s = o \qquad B = \frac{W_t}{b}\left(2 - \frac{S}{b}\right)\frac{S}{2}$$

But

$$W_t - W_e = W_t(1 - e^{-S/B}) = W_t\left(1 - 1 + \frac{S}{b} - \frac{S^2}{2b^2}\right) = \frac{W_t}{b}\left(2 - \frac{S}{b}\right)\frac{S}{2}$$

so that the straight line (20) passes through A and B as it should.

The curve then is a straight line to a high approximation and can always be taken as such.

EFFECT OF CLIMB AT START AND GLIDE AT END OF FLIGHT.

If the flight is so made that the plane is allowed to climb steadily as the load decreases, it must, of course, come down at the end of the flight when all the fuel is exhausted. The potential energy put into the plane by the consumption of a certain amount of fuel is then partially reemployed in the descent. A slight calculation shows that it is puerile to consider this effect, as shown below.

Assume that the plane rises under power to a height h with full fuel load and at the end of the flight descends with power shut off and without fuel. The potential energy which can not be regained is hence approximately the work done in raising the total weight of fuel to the maximum height reached. As shown previously, the expenditure of fuel is about 1 pound per horsepower-hour delivered by the propeller, which is equivalent to

$$550 \times 60 \times 60 = 1{,}980{,}000 \text{ foot-pounds of energy per pound of fuel.}$$

Assume 50 per cent useful load (all fuel), the work required to raise the plane to a height h is

$$\frac{W}{2} \times h \text{ foot-pounds,}$$

which is a considerably overestimated value, as the climb is gradual, and the *total* fuel load is not raised to the maximum height.

Considering the specific machine used in Part I, the work done in raising 7,500 pounds of fuel to a height of 10,000 feet, say, is 75,000,000 foot-pounds. This amount of work would require

$$\frac{75{,}000{,}000}{1{,}980{,}000}=38 \text{ pounds}=6.3 \text{ gallons of fuel}$$

(gas and oil), assuming 6 pounds to the gallon. As the total load is 1,250 gallons, this amount is about one-half of 1 per cent of the total fuel load, which is quite negligible in calculations of this nature.

As the percentage amount for any other type of machine would be about the same, we have made no effort to take this theoretically interesting part of the subject into consideration.

PRACTICAL SIMPLIFIED METHOD OF APPLYING THIS THEORY TO A SPECIFIC CASE.

The application of this theory to any specific case now reduces to the greatest simplicity as follows:

Data required. W_f = weight fully loaded with fuel
W_e = weight empty of fuel.
i_1 = angle of incidence for minimum total resistance.

1. Calculate the maximum range for an all fuel load by equation 17

$$S=\frac{\eta}{c}\left(\frac{L}{D+R}\right)_{max.}\log_e\frac{W_f}{W_e}$$

and plot the value of S/2 as abscissa for an ordinate B = o.

2. Plot W_f-W_e as an ordinate for S = o.

3. Connect these two points by a straight line and we have a diagram similar to the following figure:

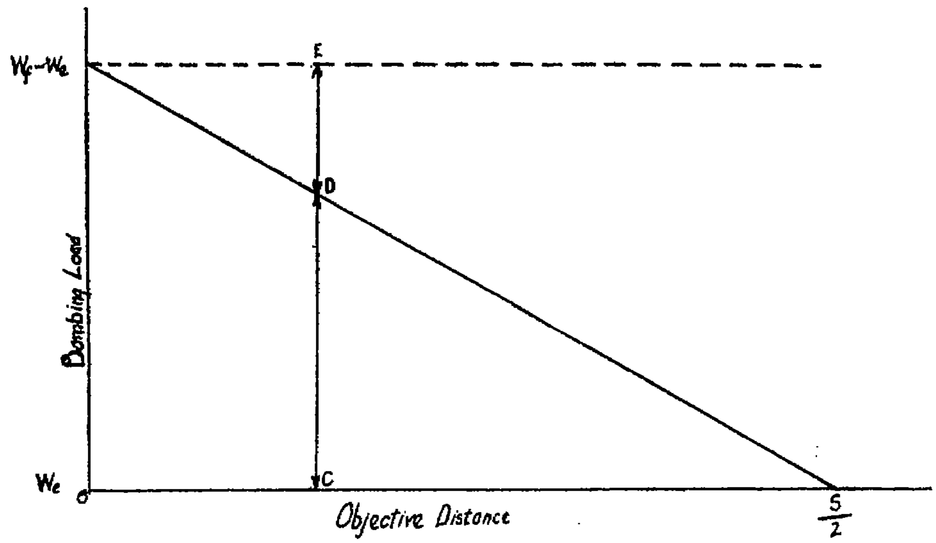

Fig. 8.

Any ordinate such as CD will give the maximum pounds of load that can be carried for an objective at distance OC. The distance DE will give the corresponding weight of gasoline to be carried. (See Fig. 6.)

For usual machines

$$S\text{ (miles)}=375\,\frac{2.30}{1.03}\left(\frac{L}{D+R}\right)_{max.}\log_{10}\left(\frac{W_f}{W_e}\right) \qquad (22)$$

EFFECT OF ALTITUDE ON SPEED, POWER, AND TIME.

From equation (5) it follows that for a given weight the speed must increase as γ diminishes and the exact relation is

$$V_h = \frac{V_o}{\sqrt{\gamma}} \tag{23}$$

where h is the height characterized by γ. From equation (7) it follows that the power required for flight increases in the same proporation as the speed so that

$$P_h = \frac{P_o}{\sqrt{\gamma}} \tag{24}$$

This is otherwise evident as the thrust does not change.

From the latest experimental tests of motor horsepower and motor efficiency at altitudes, it is found that the power falls off almost in proportion to the density so that

$$P_h' = P_o'\gamma \tag{25}$$

Hence, to fly at an altitude characterized by γ, a power P_o' at the ground, which becomes $P_o'\gamma$ at the height, corresponding to γ, must be provided, such that

$$P_o'\gamma = \frac{P_o}{\sqrt{\gamma}} \tag{26}$$

If the maximum available horsepower P_o max. at the ground is provided, the plane will rise until

$$P_o \text{ max. } \gamma = \frac{P_o}{\sqrt{\gamma}} \tag{27}$$

or to such an altitude as is characterized by

$$\gamma = \left(\frac{P_o}{P_o \text{max.}}\right)^{2/3} \tag{28}$$

In order to provide a constant thrust the propeller must increase in angular speed according to the same law as the plane since propeller blades are aerodynamically similar to wings, so that

$$n = (\text{RPM})_h = \frac{(\text{RPM})_o}{\sqrt{\gamma}} \tag{29}$$

It follows immediately from this that since V and n increase in the same ratio then $\frac{V}{nD}$ remains constant with altitude, and hence the propeller efficiency.

There are thus several reasons why flight at a high level will be better than at low.

(*a*) The motor running full open will probably use less fuel per horsepower than has been assumed for throttle, say, in the ratio of 0.6 to 0.7.

(*b*) The motor running at a higher speed can develop slightly more power with proper adjustment, which will increase the height, and therefore the speed.

(*c*) A very good third reason is that the duration of the flight will be considerably lessened and this, together with

(*d*) the increased safety due to high altitude and greater flying speed lead to the conclusion that: For bombing purposes the aviator should fly at a certain predetermined constant angle of attack; he should allow the plane to rise as the load diminishes.

Since the work consumed in rising to the higher level is at least partially returned when the machine glides down at the end of the trip without power, these works have not been considered.

The aviator will thus attain the greatest range, carry the greatest load, secure the greatest safety and speed consistent with these conditions and ecomomize in time as well.

NOTE ON CEILINGS.

By employing the equation (28):

$$\gamma=\left(\frac{P_o}{P_o \text{max.}}\right)^{2/3}$$

based upon the assumption $P_h = P_o\gamma$, the ceiling for any corresponding weight of machine may be found if the available and required horsepower is known.

The following table of computations refers to the machine considered in Part I.

Table of ceiling computations.

Weight in pounds.	Available horsepower P_m.	Required horsepower P_o.	$\frac{P_o}{P_m}^{2/3}$	Ceiling.	
				(a)	(b)
15,000	600	320	0.658	12,500	10,350
13,000	582	265	.592	16,000	13,900
11,000	555	212	.527	19,600	17,900
9,000	518	159	.455	23,800	21,800
7,000	477	107	.368	29,600	25,600

The column (*a*) under the heading of ceiling in the above table has been obtained from a γ-altitude curve based upon data obtained from report No. 14 of the National Advisory Committee for Aeronautics.

The column (*b*) under "Ceiling" has been obtained from data on the average performance values of a number of machines.

REPORT NO. 69.

PART III.

EFFECT OF WIND ON RANGE CALCULATIONS.

By J. G. Coffin. *September 1, 1919.*

BEST FLIGHT SPEED.

If there is a retarding or a helping wind it will be shown below that the conditions for maximum range must be changed.

The following is a proof of an important method for finding the proper attitude of flight with or without winds.

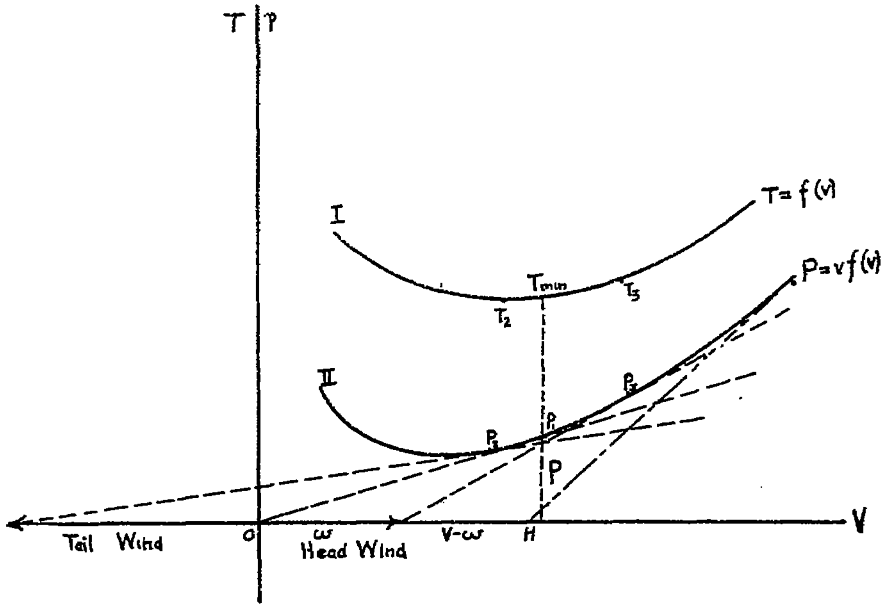

Fig. 9.

Let curve I be the required thrust-speed curve and II the required power-speed curve for the machine.

Consider a machine of constant weight W which is flying with air speed V against a wind speed w. The ground speed is then $V-w$.

In order to fly a *ground* distance ds it will take a time $dt=\frac{ds}{V-w}$.

If the thrust is T and "a" is the rate of gas consumption per *delivered* power the gas consumed in flying this distance is

$$aPdt=aP\frac{ds}{V-w} \tag{30}$$

As "a" is assumed constant and ds is fixed, for this expression to be a minimum we must have

$$\frac{d}{dV}\frac{P}{V-w}=0=\frac{(V-w)\,P'-P}{(V-w)^2}$$

and since $V-w$ can not be infinite the condition is:

$$P'=\frac{P}{V-w}$$

This means that the subtangent to the power-speed curve is $(V-w)$ and the equation is fulfilled by the following construction: Lay off w, figure 9, on the V-axis, to the right of the origin, if a contrary wind, or to the left if a following wind, and draw a tangent from this point to the P–V curve; it is seen that the slope or tangent P′ is equal to $\frac{P}{V-w}$. For calm air the tangent is drawn from the origin. As the slope of a line drawn from the *origin* to any point on the P–V curve is always $\frac{P}{V}=\frac{VT}{V}=T$ it follows that the thrust varies as the slope of such a line, and as the tangent from the origin to the P–V curve has evidently the minimum slope, this shows that in calm air the machine must fly at minimum thrust, as is otherwise evident.

Thus the minimum points of the T–V curves lie directly over the points of tangency of lines from the origin to the P–V curves.

If there is a head wind this condition of minimum thrust no longer holds and more power is required for most economic flight which corresponds, of course, to a greater thrust.

As the power curve is limited to the right by the maximum output of the power plant it is seen that for *economical* flight there is a limiting head wind corresponding to the distance OH, where H is the intersection of the tangent to the P–V curve at its limit with the V-axis. It is, of course, possible to make headway against stronger winds, but the condition for economical flight in such a case is no longer fulfilled. When w is a helping wind the tangent is drawn from a point on the left of O and it is evident that as the following wind increases in speed it pays to use less and less power, the limit for an infinite wind being minimum power. In other words, it pays to let the wind carry the machine along with the least use of the power plant. Curiously enough, this corresponds to a thrust greater than the minimum which is proper in calm air.

While for economic flight in calm air the machine must fly at minimum thrust and hence at maximum L/D [1] for the machine for all loads, this simplicity does not obtain for economic flight in the wind. Not only does the L/D change for a given load with varying winds, but also for a constant wind it varies with the load. Fortunately these variations are small for any reasonable head winds and for a change of load equal to the weight of the machine empty. Referring to figure 9, the proper L/D's for the machine and hence the proper angles of incidence may be determined by the method demonstrated above.

Assuming a head wind of w miles per hour draw tangents to the required horsepower curves from abscissa $+w$. Read off on the thrust curves the thrusts corresponding to the points of tangency on the power curves, divide these thrusts by the corresponding weight of the machine and the values thus obtained are the D/L's corresponding to economical flight under the assumed conditions.

SINGLE CURVE METHOD.

A much simpler method will now be described to accomplish the same result requiring the drawing of but a single curve for the whole procedure.

The method is based upon the following considerations:

The equations for horizontal flight may be written

$$W=LAV^2$$
$$T=DAV^2$$
$$P=TV$$

From these we obtain

$$V=\frac{1}{\sqrt{LA}}\cdot W^{1/2} \tag{31}$$

$$T=\frac{D}{L}\cdot W \tag{32}$$

$$P=\frac{1}{\sqrt{LA}}\cdot\frac{D}{L}W^{3/2} \tag{33}$$

[1] In the following D represents the total drag on the machine and corresponds to (D+R) in the preceding pages.

These equations show that as the load changes the corresponding speeds for any given angle of incidence vary as $W^{1/2}$, the thrusts as W and the powers as $W^{3/2}$.

Consider now any required power-speed curve. Figure (9).

The required power curves for any other weight W_1 can be calculated from this given curve by multiplying the speeds by $\left(\frac{W_1}{W}\right)^{1/2}=\lambda^{1/2}$ and the corresponding powers by $\left(\frac{W_1}{W}\right)^{3/2}=\lambda^{3/2}$ and plotting these values on the same sheet. If required the thrust curves can be obtained by plotting $\frac{P\lambda^{3/2}}{V\lambda^{1/2}}=\frac{P}{V}\lambda$ against $V^{1/2}$.

Consider what effect a change in loading has upon the equation of condition for economical flight.

$$\frac{dP}{dV}=\frac{P}{V-w}$$

becomes for a new loading W_1

$$\frac{dP\cdot\lambda^{3/2}}{dV\cdot\lambda^{1/2}}=\frac{P\lambda^{3/2}}{V\lambda^{1/2}-w}$$

where

$$\lambda=\frac{W_1}{W}$$

This reduces to

$$\left(\frac{dP}{dV}\right)=\frac{P}{V-\frac{w}{\lambda^{1/2}}} \qquad (34)$$

which indicates that instead of plotting P−V curves for various loads and drawing tangents from the abscissa w it is sufficient to plot but one curve, and as the load increases draw tangents from abscissas $\frac{w}{\lambda^{1/2}}$ as the *load* changes. The point of tangency determines values of P and V which correspond to a required power P $\lambda^{3/2}$ and a flying speed V $\lambda^{1/2}$ for the new condition.

As the main interest here is to find the variations in L/D, or, what is the same thing, in D/L, we continue as follows:

Since $\frac{P\lambda^{3/2}}{V\lambda^{1/2}}=\frac{P\lambda}{V}$ is the new corresponding thrust, and since the new thrust divided by the new load W_1 gives the new D/L, it is seen that

$$\frac{\frac{P}{V}\lambda}{W_1}=\frac{T\lambda}{W_1}=\frac{T}{W}=\left(\frac{D}{L}\right)_1 \qquad (35)$$

and hence in order to determine the D/L for any new loading it is merely necessary to draw a tangent from $\frac{w}{\lambda^{1/2}}$, and the ratio $\frac{P}{V}$ read off on the *original* scale is the corresponding value of the $\frac{D}{L}$ required.

This single curve is preferably plotted for some simple load such as 1,000 pounds or 10,000 pounds. The D/L's will then come out directly by dividing the power by the speed, and by changing the position of the decimal point.

EXAMPLE OF USE OF THE ONE-CURVE METHOD.

In order to check the proposed method against the usual multicurve method, three P-V curves were plotted for the same machine with loads of 7,000 pounds, 10,000 pounds, and 13,000 pounds, respectively. The machine weighs 7,000 pounds empty.

These curves are shown in Fig. (10). The L/D's for various wind speeds were derived from them and compared with the L/D's taken from the 10,000-pound curve using modified wind speeds as described above.

The agreement is very satisfactory, as shown in the table (6) below.

When using the curve for a 7,000-pound load the modified wind speeds corresponding to w are

$$w\sqrt{\frac{10}{7}}=1.195\ w,$$

and for a load of 13,000 pounds they are

$$w\sqrt{\frac{10}{13}}=.877\ w$$

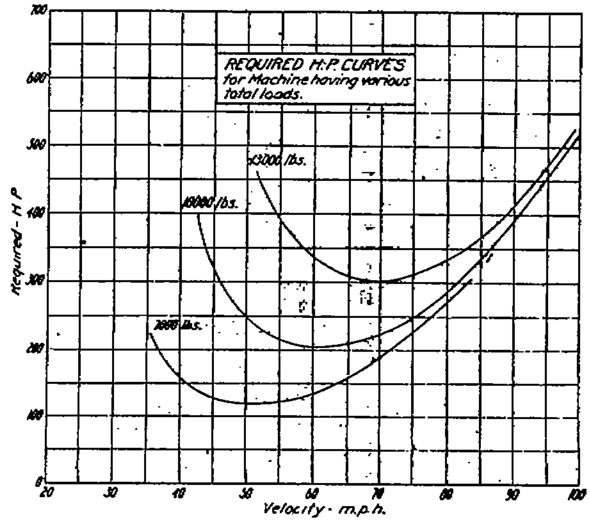

Fig. 10.

TABLE (6).

Actual wind speeds, *w*, in miles per hour.	L/D from 7,000-pound P-V curve.	L/D from 10,000-pound P-V curve.	Modified wind speeds used, 1.195 *w*.	L/D direct from 10,000-pound curve.	L/D from 13,000-pound P-V curve.	L/D from 10,000-pound P-V curve.	Modified wind speeds used, 0.877 *w*.
		(7,000 pounds.)		*(10,000 pounds.)*		*(15,000 pounds.)*	
−40	8.26	8.31	−47.80	8.31	8.35	8.33	−35.06
−20	8.35	8.37	−23.90	8.35	8.38	8.40	−17.53
0	8.41	8.43	0	8.44	8.43	8.43	0
10	8.41	8.43	11.95	8.41	8.41	8.42	8.77
20	8.29	8.31	23.90	8.32	8.41	8.42	17.53
30	8.04	8.03	35.85	8.19	8.25	8.25	26.60
40	7.47	7.45	47.80	7.96	8.04	8.07	35.06
50	5.95	5.90	59.75	7.21	7.62	7.60	43.82

Several interesting results appear from the values obtained.

1. The influence of wind on the L/D is greater for light loads than for heavy. A change in L/D from 8.43 to 5.90 is found for the 7,000-pound machine as against a change from 8.43 to 7.60 for the 13,000-pound machine, these values corresponding to a change in head-wind speed of 50 miles per hour.

2. This influence still holds although much less noticeably for helping winds, the L/D changing from 8.43 to 8.30 for the light machine as against 8.43 to 8.34 for the heavy machine.

3. For any reasonable head wind that could be flown against in long-distance flight, the change in the L/D is small, running from 8.43 to 8.00 for the 7,000-pound case to 8.43 to 8.25 for the 13,000-pound case, these values corresponding to a head wind of 0 and 30 miles per hour, respectively.

As practically it is difficult to fly at a given angle with mathematical accuracy the main result of these figures is to show that as the head winds increase in speed it is necessary to slightly diminish the flying angle; exactly how much depending on a preliminary calculation as outlined above.

The instructions to the pilot can be given in either of two ways:

(a) Proper flying angles for any given wind.

(b) Proper air speed for any given wind.

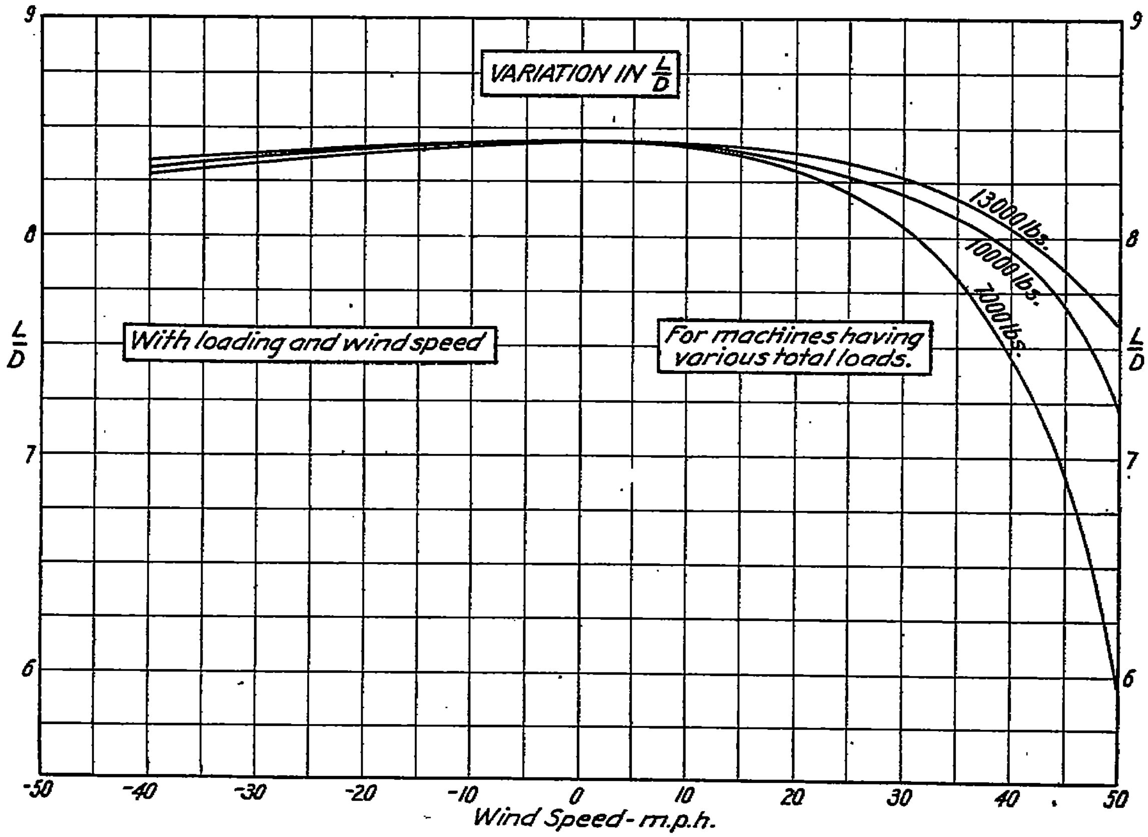

Fig. 11.

A plot of the values of L/D against wind speeds for the three loadings is shown in figure (11). These L/D values correspond to definite air speeds at a given altitude and definite angles of incidence which can also be placed upon the plot. Such a chart will give with sufficient exactness the proper flying angles for practical navigation under economical conditions.

RANGE FORMULÆ INCLUDING EFFECT OF WINDS.

The time-weight equation (13)

$$t=\frac{2}{K}\left(\frac{1}{\sqrt{W}}-\frac{1}{\sqrt{W_f}}\right)$$

Where

$$K=\frac{a}{\sqrt{\gamma}}\frac{C_2}{C_1^{3/2}}$$

is naturally unchanged, since for a given angle of incidence the *time* in which the fuel is consumed can not depend on whether there is a wind or not.

The time-distance equation must be modified. Since the ground distance is the important factor the wind modifies the range. If at any instant the air-speed is V and the wind speed w, the ground speed is $V-w$, and in time dt a ground distance

$$dS=(V-w)dt \tag{36}$$

will be covered. With an obvious substitution, this becomes

$$dS=\sqrt{\frac{W}{C_1\gamma}}\,dt-w\,dt$$

and

$$S=\frac{1}{\sqrt{C_1\gamma}}\int_0^t W^{1/2}\,dt-\int_0^t w\,dt$$

Using equation (13) this becomes

$$S=\frac{1}{\sqrt{C_1\gamma}}\,\frac{2}{K}\int_0^t\frac{\frac{K}{2}\,dt}{\frac{K}{2}\,t+\frac{1}{\sqrt{W_f}}}-w\int_0^t dt=\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\log_e\left(\frac{K}{2}\,t+\frac{1}{\sqrt{W_f}}\right)-wt+\log C$$

The constant of integration log C is determined by the condition that when $t=O$ $S=O$ and

$$\log C=-\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\log_e\frac{1}{\sqrt{W_f}}$$

so that finally

$$S=\frac{2}{K}\,\frac{1}{\sqrt{C_1\gamma}}\log_e\frac{\frac{K}{2}\,t+\frac{1}{\sqrt{W_f}}}{\frac{1}{\sqrt{W_f}}}-wt$$

Eliminating t and giving K its value we have

$$S=\frac{1}{a}\left(\frac{L}{D}\right)\log\frac{W_f}{W}-\frac{2}{a}\left(\frac{L}{D}\right)\sqrt{\gamma LA}\left(\frac{1}{\sqrt{W}}-\frac{1}{\sqrt{W_f}}\right)w \tag{37}$$

The L/D which appears in this equation is an average value given by the preliminary calculation as in Table (6) corresponding to loads and wind speeds for which the range is desired.

It would, of course, be possible to introduce an empirical expression for L/D in terms of W which could be integrated, but no practical advantage would accrue on account of the impossibility of obeying the mathematically exact conditions in actual flight.

The expression (31) for S can be put into either of the following forms by simple transformations:

$$\begin{aligned}S&=\frac{1}{a}\left(\frac{L}{D}\right)\log\frac{W_f}{W}-\frac{2}{a}\left(\frac{L}{D}\right)\left(\frac{1}{V}-\frac{1}{V_f}\right)w\\&=\frac{1}{a}\left(\frac{L}{D}\right)\log\frac{W_f}{W}-wt\\&=\frac{2}{a}\left(\frac{L}{D}\right)\left[\log\frac{V_f}{V}-w\left(\frac{1}{V}-\frac{1}{V_f}\right)\right]\\&=\frac{1}{a}\left(\frac{L}{D}\right)\log\frac{W_f}{W}-\frac{2}{a}\,w\left(\frac{W}{P}-\frac{W_f}{P_f}\right)\end{aligned}$$

In all of these expressions the altitude of flight is assumed to be substantially constant.

SUMMARY.

The relation between useful load and range has been worked out by two distinct methods.

Part I employs no new theory and is made by the usual performance estimate methods. It would involve considerable plotting of curves and is cumbersome.

Part II gives a theoretical solution. This solution checks remarkably well with the previous one in every particular. It leads to an elegant and simple solution for any specific case.

Directions are given for the application of the results of this paper to any machine. The total time required for this complete calculation should not take over 15 minutes.

The results of interest for calm air are:

1. The machine should fly at a constant angle of attack, the angle corresponding to the minimum value of $\frac{\text{Weight}}{\text{Total resistance}}$.
2. It is practically immaterial whether the machine flies high or low as far as range is concerned.
3. There is an advantage in flying high in that the time is much reduced.
4. The resistance is proportional to the weight at a given altitude.
5. The result of flying at maximum speed is a very much diminished range, or for a given range a very much diminished useful load.
6. The result of flying at minimum power is to slightly reduce the range.
7. The times of flight at the same level for flying at best range speed and at minimum power speed are practically the same.
8. The condition for best range is shown.
9. The weight-time curve is deduced.
10. The range-time curve is deduced.
11. The weight-range curve is deduced.
12. The effect of altitude has been taken into account.
13. The time is greatly diminished for flying at corresponding levels.
14. The theory checks closely with the ordinary methods of Part I.

Part III gives a theoretical solution of the effect of wind on range. First, a proof of a method for determining the L/D and air speed for the machine under any wind conditions is given. A new method is shown wherein but one P–V curve is required for any load and any wind speed.

Variations in L/D for changes in load and wind speed are derived and checked against the usual methods.

The weight-distance formula is derived as modified by winds.

The results of interest for flight in winds are:

1. The angle of attack changes but slightly when flying against winds of reasonable strength, and but very slightly when flying with winds of any strength.
2. The altitude of flight affects the range. The reason being that higher speeds are attained at higher altitudes and the ratio of air speed to wind speed changes. However, as wind speeds change with altitude it does not seem worth while to go into the matter more fully.
3. Other things being equal, it is slightly advantageous to fly high, especially as to time of flight.
4. The weight-time curve is unchanged.
5. The range-time curve is deduced.
6. The weight-range curve is deduced.

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