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REPORT No. 92

ANALYSIS OF WING TRUSS STRESSES

INCLUDING

THE EFFECT OF REDUNDANCIES

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ANALYSIS OF WING TRUSS STRESSES.

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This report was prepared at the Langley Memorial Aeronautical Laboratory of the National Advisory Committee for Aeronautics under the direction of the Committee on Aerodynamics by Edward P. Warner and Roy G. Miller.

It has been the usual practice of airplane designers in making structural analyses to treat the airplane, not as a collected whole, but as an assemblage of separate units, and to carry through an analysis for each of these units in turn, ignoring members wherever necessary in order that the structure of each separate unit may be statically determinate. In wing truss analysis, for example, it is the invariable practice in making routine analyses to entirely ignore the effect of the stagger wires and the external drag wires, the forces acting on the truss being resolved into the planes of the lift bracing and the internal drag bracing and these bracing systems being designed strongly enough to carry the entire loads. When the stagger wires are taken into consideration at all, it is only on the assumption that the flying wire has been shot away and that the load must be carried from one lift truss to the other through the stagger wires. Obviously the members thus ignored will come into play under some conditions, and, in so doing, they will affect the stresses in the other members which are ordinarily taken into account. It is customary to fall back on the assertion that the ordinary method of analysis is on the safe side, but reliance on such a claim is always unscientific and unsatisfactory, and nowhere more so than in airplane design, where the loads acting are all dependent on the weight of the structure, and where it is therefore almost as undesirable to have one unit or group of members too strong and heavy relatively to the other members as to have one member too weak, since the excessive strength and weight of one increases the loads and stresses in all others. It is therefore eminently desirable that the analysis of the airplane structure should be carried through with the greatest possible refinement of detail, and that nothing should be left to guesswork or chance where it can be avoided.

To take one of the simplest cases as an illustration, it is evident that when an airplane is diving and the center of pressure is far to the rear of the rear spars the load on the rear truss will act upward and that on the front truss downward. If there were no restraint on the relative motion of the two systems of bracing the rear truss would therefore rise while the front one descended below its normal level, and the form would be distorted at each panel point, the truss being so warped as to decrease the angle of attack along the wing and to decrease the stagger near the tips of the wing. The physical reasoning on this point has been given at some length by Mr. John Case.[1] Other points at which there is uncertainty are the external drag wires, already alluded to, and the interaction between the fuselage and wings. The latter point was taken up in a recent report of the National Advisory Committee for Aeronautics,[2] but the analysis was not carried through in full and certain rough assumptions were made as to the tensions in the external drag wires.

The standard method of treating redundant members and statically indeterminate structures in general is furnished by the method of least work, originated by Castigliano. This method is commonly used in bridge design, and has found some application in other departments of engineering, but very little attempt has as yet been made to apply it to the needs of aeronautics.

The general means of application of the method of least work will be found discussed in any textbook on structures.[3] The application to airplanes has been briefly and simply discussed in

[1] The Importance of Incidence Wires in Strength Calculations, "Aeronautics," December 4, 1918.
[2] Fuselage Stress Analysis, by E. P. Warner and R. G. Miller, Report No. 76, National Advisory Committee for Aeronautics, Washington, 1920.
[3] The Theory of Structures, by C. M. Spofford, Chapter XVI, New York, 1915. Mechanics of Internal Work, by Church, New York, 1910.

Pippard and Pritchard's recent work on airplane structures (London, 1919), and Mr. Case,[4] in an extension of the article mentioned above, has treated mathematically the theory of the effect of incidence wires by this method, but a great deal of work on the subject remains to be done. The method pursued in this report is somewhat similar to that followed in the previous report on fuselage stresses. A representative airplane is chosen and the analysis carried through both with and without consideration of the redundancies for a number of different systems of loading, in order to give a concrete idea of the importance of the stagger wires and external drag wires and of the magnitude of the error involved by failing to take them into consideration when analyzing the stresses in an airplane of conventional type. The stresses in each member for the various conditions of loading have then been tabulated. The airplane chosen as an illustrative example closely resembles the JN4H, it being probable that more Americans are familiar with the general characteristics of this type than with any other. Assembly drawings of this airplane are given in figure 1.

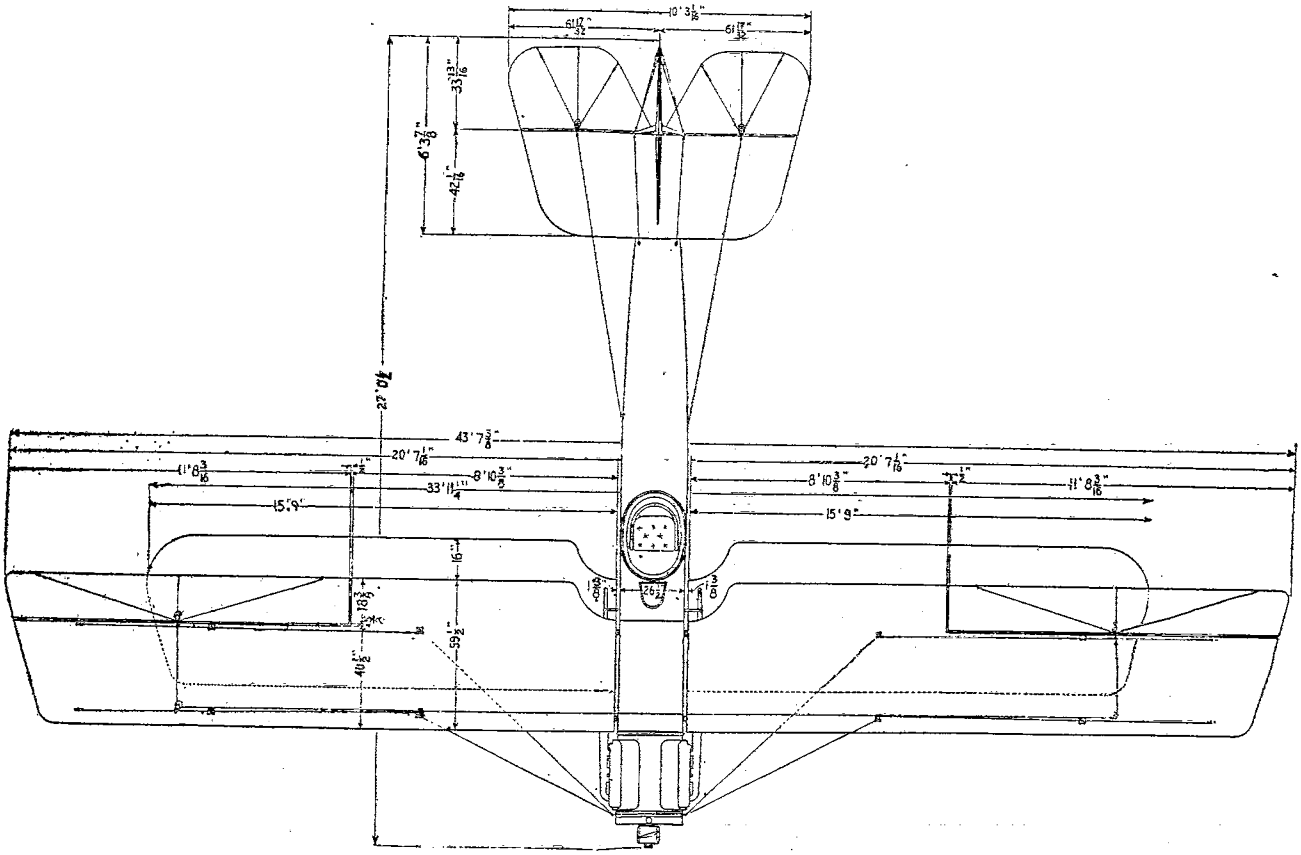

FIG. 1.

The method of least work is really nothing more than a simple method of analyzing the geometry of a structure. It is obvious that if a structure would deflect under load, and any particular redundant tension member were absent, in such a manner as to increase the distance between the points at which the ends of that redundant member are actually attached, the redundant member will resist and reduce the deflection and will modify the strains in the other members and the distribution of load among them. Castigliano's theorem offers an easy and straightforward route to the determination of this new distribution, which could otherwise be found only by a tedious process of trial and error. There are certain points which make it very difficult to apply the method of least work to airplane structures in the normal manner. The end fixation of the members is uncertain, there being an initial yield in the terminals and fittings which it is usually impossible to take into theoretical consideration. Furthermore, the stresses acting on some of the members are a combination of bending and direct end loading, and it would

[4] Incidence Wires in the Strength Calculation of Wind Structures, "Aeronautics," December 18 and 25, 1918, and January 1 and 8, 1919.

be extremely difficult to take full account of the effects of both types of stress. It appears probable that it will be safe in least work analyses of the wing structure to ignore the wooden members entirely. The tensile strength of airplane wire is about 200,000 pounds per square inch, and its modulus of elasticity is about 30,000,000 pounds per square inch. The strength of spruce in direct compression is, on the other hand, about 4,500 pounds per square inch and its modulus of elasticity is about 1,600,000 pounds per square inch. If all the members were perfectly elastic up to their ultimate strength, the strain per unit length at the instant before rupture would be a little less than one-half as great for spruce as for wire. Furthermore, the unit stress in the spruce members is always a much smaller proportion of the ultimate strength than is that in wires, because most of the wooden members are long columns of a small sectional radius of gyration, and the unit stress must therefore be kept low in order that failure may not occur by buckling. Since the work done in stressing a member depends largely on the strain imposed, being directly proportional to strain for a given stress, it is clear that the work done in stressing the wooden members will be much less than that done on the wires, and that the effect on total work and its derivatives of any change in the stress in the wooden members will therefore be relatively slight. Reliance has not, however, been placed solely on this approximate physical reasoning. An analysis has been carried through for one type of loading, taking the wooden members fully into account so far as their end loads are concerned, and the tabulation of results shows, as has just been predicted, that the effect of the wooden members is small enough to be neglected under ordi-

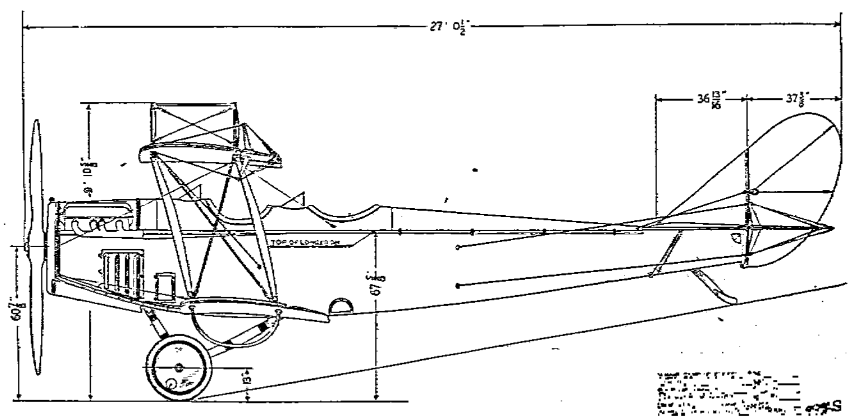

FIG. 2.

nary circumstances. The comparative analyses with and without consideration of the spars and struts will be fully discussed in their proper place.

Another question which has a considerable effect on the stress when there are redundant members is that of initial tension in the wires, and the uncertainty prevailing as to the initial tension is often used as an argument against the undertaking of further refinement of the methods of stress analysis. There is some justice in this argument for, as will be shown later, the initial tension does vary widely between different members in the same airplane and between corresponding members in different machines. To show what the maximum effect of initial tension is likely to be, an analysis has been carried through with the maximum probable initial tension in each wire.

In the application of the method of least work to aeronautical structures there arises a problem not so often encountered in the design of indeterminate bridge structures, in that some of the members are capable only of taking tension. It is necessary, then, to make some assumption in starting the analysis as to which one of an opposed pair of tension members will be in tension when all the loads are acting, and then to carry the analysis through, disregarding entirely the members opposed to those which are believed to carry tension in the final result and treating the working members for the moment as though they could take either tension or compression. If, however, the final result shows a compression in a wire, it is necessary to repeat the whole analysis with the opposing wire taken into consideration throughout in place of the one which had a stress of opposite sign to that expected. It is usually possible, after a little practice, to guess which wire of any pair will carry tension, and the trial and error method just outlined therefore does not often have to be invoked.

The method of least work is essentially a check method. It can not be used for initial calculations, as it is necessary to know the sizes of all the members before the work equations can be written. In this respect it is like the "Berry method" of wing spar analysis by the generalized equation of three moments.

The cases treated in this report are five in number, two of them relating to loadings experienced in the air and the other three to comparison with other types of analysis and to the effects of modifying factors. The loadings considered are those experienced at a high angle of attack and a high speed, as in pulling out of a dive abruptly and in a vertical dive at limiting speed. The other three cases deal with the effect of wooden members, the effect of initial tension, and with the determination of the stresses encountered when the structure is loaded in accordance

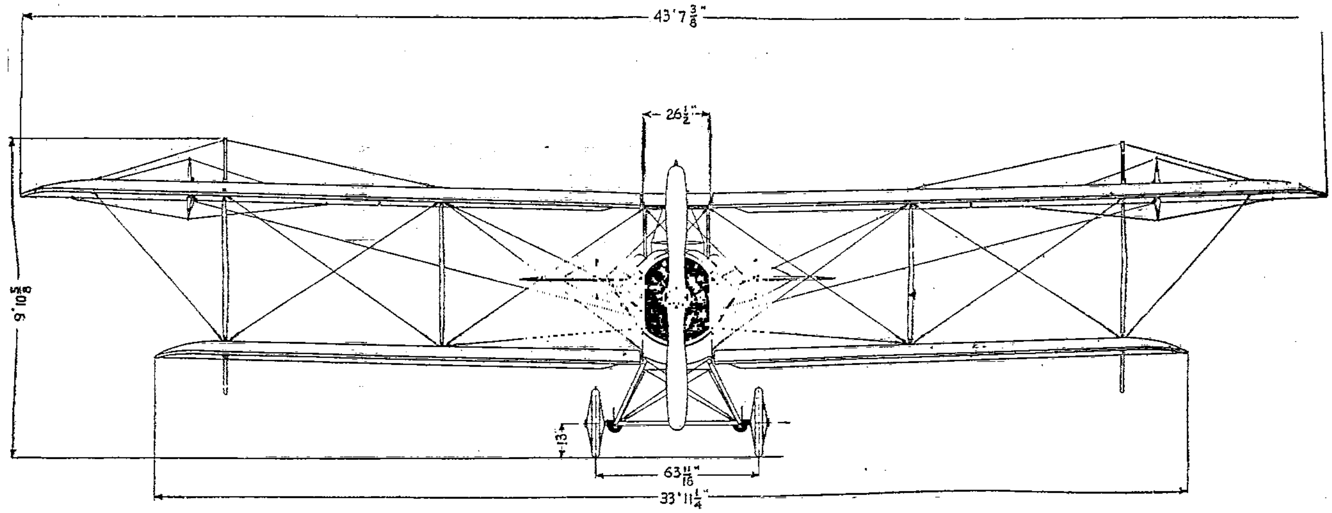

Fig. 3.

with a suggested set of specifications for static testing recently drawn up by the staff of the National Advisory Committee for Aeronautics.

CASE I.

As a first application of the analysis, the airplane was assumed to flatten out of a dive very abruptly, so that the angle of attack reached 12° in combination with a speed of 100 miles per hour. The total air load under these conditions is 5.43 times the weight of the airplane. Accelerometer tests on pursuit airplanes, conducted at the Royal Aircraft Establishment, have never shown a dynamic load factor in excess of 4.2 under the most violent handling, and ordinary stunting does not impose loads in excess of three times the weight. The conditions assumed are therefore at least as severe as any that would ever be encountered in flattening out of a dive.

A perspective view of the left wing truss, with every wire numbered, is shown in figure 2. The first step in the analysis is, as already pointed out in the introduction, to determine which wires are placed in tension by the loads being considered, as all wires which do not carry tension must be disregarded entirely. The possible redundancies are the stagger wires (not more than one at each panel acting in any given case), the two external drag wires 20 and 21, and the landing wires in the inner bay, 16′ and 17′. It is possible for the landing and flying wires to be stressed at the same time, even though there is no initial tension anywhere, as the center section struts can carry no tension and the lift reaction on the upper wing at the center section may be carried in whole or in part by the landing wire, being transmitted thence to the fuselage through the inner interplane strut and the inner flying wire. Since the point of attachment of the lower end of the landing wire is itself deflected upward by the normal lift load, that wire will not take the center section reaction if there is any other member capable of carrying it in a reasonably direct fashion. This is the case in the front truss of the airplane here analyzed, as the wire 18 carries the reaction. The rear truss, however, is supported at the center section only by the wire 19, which runs so obliquely that a relatively large vertical deflection of the upper rear spar at the center section would ensue if there were no other restraining member. If this deflection proceeds far enough, the rear landing wire comes into play, and it is therefore necessary to take this wire into account as one of the redundancies.

As for the two external drag wires, No. 20, which runs downward and forward from the rear upper spar, is obviously in tension, as the upward deflection of the lift truss and the rearward deflection of the drag truss both act to extend that wire. No. 21, while it is extended by the deflection of the drag truss, is so much shortened by the much larger movement of the lift truss that it carries no tensile stress, and is therefore disregarded. There are, then, four redundancies in all, including the two stagger wires which are acting. One stagger wire at each panel point always comes into play, but the load may shift from one diagonal to the other as the type of loading changes. In the particular case under consideration it is the long diagonal, running downward from front to rear, which acts at both panel points, chiefly because the front lift truss carries more load than the rear, the center of pressure being far forward, and conse-

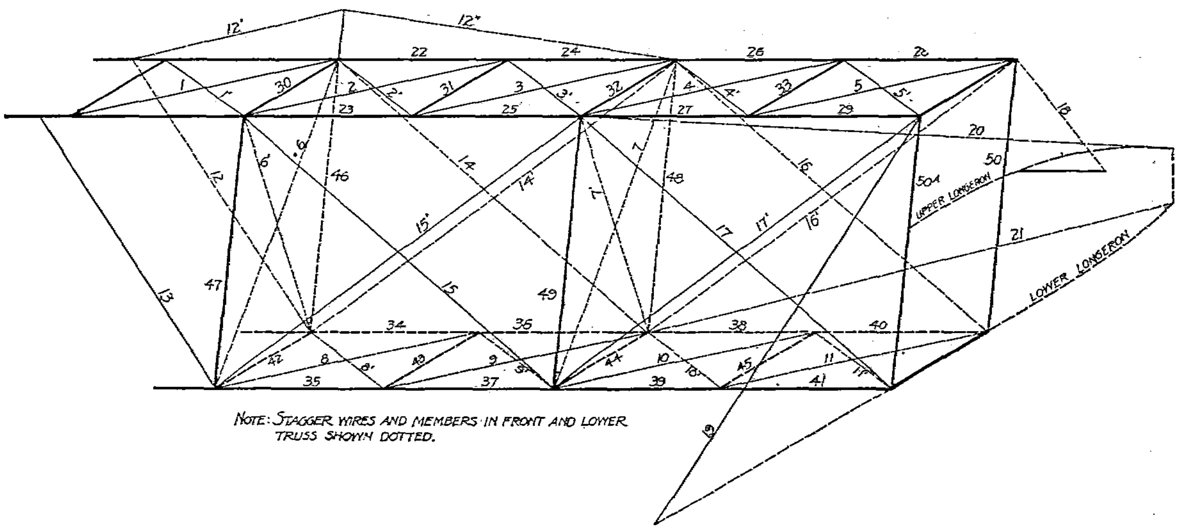

FIG. 4.

quently has a larger deflection. The long stagger wire accordingly comes into play to equalize the deflections. The distribution of the drag load also acts to stress the same wire, as the wire No. 20 acts as a partial support for the upper wing at the inner panel point, and the length of the portion of the lower wing which is cantilevered beyond its last support (not counting the stagger wires as supports) in respect of drag is therefore greater than the length of the corresponding portion of the upper wing. Part of the drag of the lower wing is therefore transferred to the upper and carried by it to the fuselage, instead of the reverse, which is generally assumed and which would hold true if it were not for the external drag wires.

The center section struts are incapable of sustaining any tension, and the reactions must therefore be taken, in the nonredundant analysis, by the wires 18 and 19. The horizontal components of the pulls in these wires combine with the center section drag truss reaction to produce an unbalanced force in the plane of the wing, and one of the center section struts must be thrown into compression to take the force. In the case under discussion at present, the unbalanced force being to the rear, the forward strut is in compression and the rear one is inoperative. The tension in 18 is very large because of the small angle which it makes with the forward strut.

The mean resultant air load on the wings was found to be 36.6 pounds per square foot. In this, as in all other cases, the variation of unit loading between the wings and the variation along the spars were neglected, the load per running foot being assumed to be constant. The load was distributed between the front and rear spars in the usual manner, the center of pressure being 33 per cent of the way back on the chord. The lift and drag reactions at the several panel points were then determined, and each lift reaction resolved into the lines of the drag struts and the interplane struts. The perfectly general method of carrying through the work would be to resolve every force into those two lines and a line parallel to the wing spars, and also to determine the direction cosines of every member of the truss with respect to a nonrectangular system of axes parallel, respectively, to the wing spars, to the drag struts, and to the interplane struts,[5] and then to write the equations of equilibrium at every point. Having done this, the solution becomes practically automatic. It is possible, however, to very much shorten the work by a judicious use of the method of sections, especially if the stresses in the wooden members need not be determined. The first part of the problem is to solve for the stresses in all members except the redundant ones, ignoring those entirely; and this is identical with the ordinary stress analysis.

The analysis with wires 6, 7, 20, and 17′ ignored being completed, each of these, in turn, is assumed to carry a tension of 1 pound, and the stresses which every other member of the truss would bear, due to this tension, were there no other loads acting, are computed and tabulated. The total stress in any member can then be expressed by the formula:

$$T_x = f_i + T_6 \times f_6 + T_7 \times f_7 + T_{20} \times f_{20} + T_{17}' \times f_{17}'$$

where T_x is the total stress in the member in question, f_i the stress due to air loads with redundancies omitted from consideration, f_6, f_7, f_{20}, and f_{17}', the stresses due to tensions of 1 pound in 6, 7, 20, and 17′, respectively, and T_6, T_7, T_{20}, and T_{17}', the stresses which actually exist in those redundant members when the structure is loaded.

The work done in elongating the member x is

$$W_x = \frac{T_x^2 \times l}{2AE}$$

where l is the length of the member, A its cross-section area, and E the modulus of elasticity of the material composing it. Writing T in this expression in terms of T_6, T_7, T_{20}, and T_{17}', and doing the same for the expressions for W_y, W_z, and so on, for every member of the structure, the total work of deformation for any set of values of the stresses in the redundancies can be obtained by summation. In order that the work may be a minimum, as required by Castigliano's theorem, its partial derivatives with respect to each of the independent variables (in this case the tensions in the redundant wires) must all be equal to zero. Differentiating the expression for total work with respect to T_6, T_7, and so on for each of the redundancies in turn, four simultaneous equations in four unknowns are obtained, and these can at once be solved. The stresses on all the members taken into account in the usual type of analysis and ordinarily considered as nonredundant can then be determined by substituting in equations of the form given for T_x the values just found for the stresses in the redundancies by solution of the simultaneous equations for the work derivatives.

The carrying through of this process shows the tensions in the redundancies to be 87 pounds in No. 6, 143 pounds in No. 7, 707 pounds in No. 20, and 1 pound in No. 17′. The important figures in connection with each member of the truss are tabulated below. Of special interest are the listings of factors of safety as found by the ordinary statical analysis with all stagger wires and external drag wires disregarded and as found by the complete analysis with these members fully taken into account. It should be borne in mind that these are true factors of safety or "material factors," based on the worst possible loading, and are less than one-fifth as large as the hypothetical "factors of safety" which are usually specified and which are based on the loading in normal rectilinear horizontal flight in smooth air.

The presence of the redundant members reduces the stress in 11 wires and increases it in only 3 (not including the redundancies themselves). The beneficial effect on the worst-stressed members is, however, slight.

[5] This system of axes would be rectangular if there were no stagger.

The stress in the rear inner landing wire is negligible and has been omitted from consideration in computing the factors of safety. Furthermore, the effect of 17′ is actually even a little less than would appear, as the rear portion of the fuselage is subjected to a downward dynamic load, and the point of attachment of the lower end of 19 is therefore deflected downward relative to the points of attachment of the lower wing spars, so that 19 carries a larger share of the upward reaction at the center section of the upper wing than it would if its lower end remained exactly fixed. The effect of the landing wires will therefore be disregarded in all subsequent cases. The possibility of their having an effect is only mentioned as a warning that they should sometimes be taken into account, as the share of the center section load carried by the landing wires increases rapidly as the obliquity of the center section wires is increased.

CASE I.

No.	Stress without redund.	Stress due to 1 pound in No. 6.	Stress due to 1 pound in No. 7.	Stress due to 1 pound in No. 20.	Stress with all redund.	Ultimate strength of member.	F. S. without redund.	F. S. with all redund.
1	162				162	2,600	16.0	16.0
2	518	.683			577	2,600	5.02	4.50
3	433	.683			492	2,600	6.00	5.29
4	905	.654	.654	−1.008	343	4,000	4.61	11.6
5	827	.654	.654	−1.008	264	4,000	4.84	15.1
6		1.000			87	2,000		23.0
7			1.000		143	2,000		14.0
8	257	− .683			198	2,600	10.1	13.1
9	180	− .683			120	2,600	14.5	21.6
10	686	− .654	− .654		536	4,000	5.82	7.45
11	608	− .654	− .654		458	4,000	6.59	8.74
12	508				508	4,000	7.87	7.87
13	367				367	4,000	10.9	10.9
14	2,456	−1.318			2,341	8,400	3.42	3.58
15	1,772	1.318			1,886	8,400	4.74	4.45
16	5,143	−1.272	−1.272		4,851	8,400	1.64	1.73
17	3,711	1.272	1.272	− .523	3,633	8,400	2.26	2.31
18	2,606	1.282	1.282	−1.975	1,505	4,200	1.61	2.79
19	342				342	2,000	5.85	5.85
20				1.000	707	4,200		5.94
22	−2,152	1.052			−2,062			
23	−2,174	−1.583			−2,310			
24	−1,750	1.583			−1,613			
25	−2,510	−2.114			−2,692			
26	−5,425	3.107	.9922		−5,019			
27	−6,087	−3.600	−1.485	.4189	−6,365			
28	−4,742	3.600	1.485	− .7599	−4,739			
29	−6,710	−4.093	−1.979	1.179	−6,586			
30	−2,327	− .4300			− 269			
31	− 295	− .4300			− 332			
32	− 460	− .4300	− .4300		− 557			
33	− 565	− .4300	− .4300	.6625	− 214			
34	− 320				− 320			
35	31	.5301			76			
36	520	− .5301			474			
37	− 109	1.060			− 18			
38	2,619	−2.112			2,437			
39	788	2.606	.4932		1,071			
40	3,136	−2.606	− .4932		2,843			
41	329	3.099	.9864		734			
42	− 162				− 162			
43	− 134	.4300			− 96			
44	− 309	.4300			− 272			
45	− 421	.4300	.4300		− 324			
46	− 878				− 878			
47	− 633	− .7950			− 702			
48	−2,366	.7950			−2,298			
49	−1,707	− .7950	− .7950		−1,887			
50	−1,790	−1.020	−1.020	1.585	− 948			

CASE Ia. (*Effect of wooden members.*)

The loading taken in this case was the same as in the last, but full allowance was made for the effect of the wooden members, in so far as their end loads were concerned, the stresses in these members and the work done in elongating or shortening being computed exactly as for the wires, and the equations of total work being enlarged to include the work which goes into storing strain energy in the spars and struts. The strain energy of flexure has not been taken into consideration, as its variation due to the redundancies is slight, and the accurate computation of flexural work would be an undertaking of great difficulty, requiring a series of successive approximations to allow for the departure of intermediate panel points from the straight line connecting the outermost and innermost supports of the wing truss. It is only because of such departures that the work of flexure is changed by the redundant members, and these redundancies therefore have no effect on the energy of flexure in airplanes which have no intermediate panel points, the wing truss on each side consisting of a single bay and an overhang.

The introduction of the wooden members into the work equations gives a larger stress in two of the redundancies than was found in Case I, while the stress in the third (No. 7) remains practically unchanged. The tension in No. 6 was increased to a rather surprising extent. In only one wire (No. 18) does the allowance for the struts and spars change the computed tension by as much as 5 per cent of its ultimate strength, and the effect in that one wire, as well as in most of the others, is to reduce the computed stress.

The effect of redundancies on the stresses in the wooden members themselves is small in most instances, but is not by any means small enough to be negligible. The loads in the worst-stressed portions of the wing spars are reduced by from 15 per cent to 25 per cent by the redundant wires, chiefly by the effect of No. 20. The stress in the intermediate compression rib in the inner bay of the upper wing is cut down about 85 per cent by the external drag wires. The interplane struts are but little affected, with the exception of the front center section strut, the stress in which is 64 per cent smaller than it would be if the redundant members were removed.

A tabulation, similar to that for Case I, of the stresses with and without allowance for the redundant members is given below. The differences between the stresses found in Case I and Case Ia, or the errors due to failing to include the wooden members in the work equations have been included in the tabulation.

It appears from the comparison of the results in this case and in Case I that the assumption originally made was a reasonably accurate one, and that it is safe to omit the wooden members from consideration for any except the most refined work.

CASE Ia.

[Stresses without redundancies, and effects of unit stresses in redundancies, are the same as in Case I.]

No.	Stress with all redund.	Difference between Ia and I (absolute magnitude).	No.	Stress with all redund.	Difference between Ia and I (absolute magnitude).	No.	Stress with all redund.	Difference between Ia and I (absolute magnitude).
1	162	0	18	1,186	−319	35	113	+ 37
2	624	+ 47	19	342	0	36	437	+ 37
3	539	+ 47	20	911	+204	37	56	[1] + 38
4	180	−163	22	−1,989	− 73	38	2,291	−146
5	102	−162	23	−2,420	+110	39	1,262	+191
6	155	+ 68	24	−1,504	−109	40	2,662	−181
7	140	− 3	25	−2,839	+147	41	949	+215
8	151	− 47	26	−4,803	−216	42	− 162	0
9	74	− 46	27	−6,473	+108	43	− 67	− 29
10	493	− 43	28	−4,667	− 72	44	− 242	− 30
11	414	− 44	29	−6,550	− 36	45	− 296	− 28
12	508	0	30	− 299	+ 30	46	− 878	0
13	367	0	31	− 362	+ 30	47	− 757	+ 55
14	2,251	− 90	32	− 587	+ 30	48	−2,243	− 55
15	1,977	+ 91	33	− 88	+126	49	−1,942	+ 55
16	4,767	− 84	34	− 320	0	50	− 648	−300
17	3,621	− 12						

[1] The stress is changed in sign in this case.

CASE II.

The loading in this case was that encountered in a vertical dive at 120 miles per hour. This is considerably below the limiting speed of the JN, but is as fast as it is likely to be dived. It was assumed that the upload on the rear truss was equal to the down load on the front truss, and the resultant force on the wings was therefore parallel to the chords. Since the angle of attack was negative, there was some lift on the wings under these conditions, but not enough to balance the down load on the tail. In the particular machine used as an illustration the angle of zero normal force is −5°, the zero lift angle for the Eiffel 36 section being unusually small. The true angle of attack in a vertical dive would probably be nearer −4° than −5°. The components of load acting perpendicular to the wing chord were 45 pounds per foot, giving a total force of about 1.6 times the weight of the airplane in each lift truss (including both the right and left sides of each truss). This is unusually large, the Eiffel 36 wing having an exceptionally large diving moment at the angle of zero lift. The loading in diving the JN to 120 miles per hour is about equal to that which would be found at the terminal velocity with most airplanes using the R. A. F. 15 or other similar section. The load parallel to the wing chord (front and rear trusses together) was 7.22 pounds per foot, so that the total distributed load on the drag trusses, including the parasite resistance of the interplane bracing, but not including the components in the planes of the wings, due to stagger, of the lift truss reactions, was about 29 per cent of the weight of the airplane. In a dive to the terminal velocity this force may rise to as much as 50 per cent of the weight of the airplane for a machine with fine lines and low parasite resistance.

The front king-post bracing above the upper wing is stressed by the down-load on the front truss, and the stresses in the two lift trusses therefore are not quite symmetrical with respect to each other. If the two systems of trussing were parallel throughout, the stagger would have no effect on the net reactions in the plane of the wing, as the effects of the inclination of the lift bracing would be equal and opposite at the front and rear panel points and would exactly cancel out; but this is not actually the case, since the king-post overhang bracing lies in a plane perpendicular to the wing chord instead of being parallel to the lift truss proper.

There are three redundancies in this case, Nos. 6′, 7′, and 21. The stagger wires acting are those which run upward and to the rear, as might be expected, since the rear truss tends to move up and the forward one down, and the stagger wires acting are those which are thrown into tension in resisting this relative displacement of the lift trusses. The work equations were 424 pounds in 6′, 427 pounds in 7′, and 485 pounds in 21. It might perhaps have been anticipated that No. 20 would be in tension, as the rear truss considered alone tends to move upward and to the rear and both of these components of motion would elongate No. 20, but analysis shows that No. 20 would carry a considerable compressive load if it were capable of sustaining such a load. The physical explanation of this is dual. In the first place, the pull in stagger wires Nos. 6′ and 7′ tend to draw the upper wing forward. Secondly, and more important, the load in the rear truss is carried by the flying wires, while that in the front truss falls on the landing wires. These, being single in each bay, elongate more under a given load than do the double flying wires, and, if the two trusses were not connected together in any way, the front one would deflect downward more than the rear one would yield upward. Since the two are connected by the stagger wires and must move substantially together, the effect of the dissymmetry of the lift and antilift bracing is to cause the wing cell to deflect downward as a whole. The upper rear spar therefore moves, not upward and backward as it would if there were no redundancies, but forward and downward. Incidentally, this serves as an excellent illustration of the intricacies of a redundant structure and of the manner in which the stress in any member depends on the form and strength of every other member. For example, if the antilift wires as well as the lift wires, were double there is but little doubt that the upper drag wire (No. 20), as well as the lower one, would carry a considerable tensile load during a dive instead of going slack.

The pull of the stagger wires, drawing the upper wing forward, also has the effect, not very generally foreseen or allowed for, of throwing a load on the antidrag wires in the upper wing. A load on these wires is expected at large angles of attack, particularly in airplanes with little or no stagger, but its appearance in a vertical dive seems rather curious until a thorough analysis is made.

The nature of the load distribution in the center section is quite different from that at a large angle of attack, although three of the four members involved are active in each case. In a dive, the front wire (No. 18) takes no load. Both struts are in compression, and the forward tendency of the upper wing, due to the pull of the stagger wires, is resisted by a tension in No. 19.

A tabulation of stresses, similar to that given for Case I, appears below. There has been no recomputation of redundancies with the work done in the spars and struts taken into account in this case, but the final stresses in the wooden members have been computed with allowance for the redundancies found by writing the work equations for the wires alone.

The factors of safety in the wires are high and fairly uniform. The stresses in the wooden members, with a few exceptions (chiefly the internal drag struts), are reduced by the introduction of the redundant wires. This is particularly true of the worst-stressed portions of the spars, the maximum direct compressive loads being reduced by about 72 per cent. It is a curious fact that every bay of every spar is in compression in a dive, the effect of the stagger wires and of the king-post bracing being sufficient to overcome the tension which might normally be expected to appear in the upper front and lower rear spars.

The stagger wires are of enormous benefit as regards the lift trusses. In the lift and antilift wires, as in the spars, the stresses are from 55 per cent to 70 per cent lower than they would be if the stagger wires were removed.

CASE II.

No.	Stress without redund.	Stress with all redund.	F. S. without redund.	F. S. with redund.	No.	Stress without redund.	Stress with all redund.	F. S. without redund.	F. S. with redund.
1	+ 133	+133	19.6	19.6	25	−1,222	−300	...	...
2	+ 72	...	36.3	...	26	+1,225	−253	...	...
2′	...	+289	...	4.50	27	−3,094	−783	...	...
3	+ 115	...	22.6	...	28	+1,375	−610	...	...
3′	...	+245	...	5.30	29	−3,255	−416	...	...
4	+−200	...	20.0	...	30	− 45	−225	...	...
4′	...	+488	...	4.10	31	− 73	−180	...	...
5	+ 240	...	16.7	...	32	− 131	−379	...	...
5′	...	+447	...	4.47	33	− 158	−320	...	...
6′	...	+424	...	4.72	34	− 976	−421	...	...
7′	...	+427	...	4.68	35	+ 90	−188	...	...
8	+ 55	+415	47.6	6.27	36	− 933	−101	...	...
9	+ 94	+455	28.6	5.72	37	+ 17	−539	...	...
10	+ 142	+457	28.2	8.75	38	−2,581	−825	...	...
11	+ 182	+497	22.0	8.05	39	+ 751	−597	...	...
12′	+ 513	+513	3.90	3.90	40	−2,474	−480	...	...
12″	+ 518	+518	3.86	3.86	41	+−614	−972	...	...
13	+−211	+211	19.9	19.9	42	− 34	−259	...	...
14′	+1,223	+528	3.44	7.96	43	− 59	−284	...	...
15	+1,053	+359	7.98	23.1	44	− 93	−300	...	...
16′	+2,206	+770	1.90	5.46	45	− 119	−326	...	...
17	+2,206	+860	3.81	9.77	46	− 530	−530	...	...
18	+ 529	...	7.94	...	47	− 377	−377	...	...
19	+ 202	+502	9.90	3.98	48	− 999	−580	...	...
21	...	+485	...	8.67	49	−1,015	−596	...	...
22	− 410	−632	...	...	50	− 625	−615	...	...
23	−1,133	−523	...	...	50A	...	−210	...	...
24	− 354	−820	...	...					

CASE III.

The loading in this case was one devised by the authors and recommended for use as a standard in sand-load tests. It was based on an attempt to distribute the load over the wings in such a manner that both lift trusses and both drag trusses would simultaneously reach the worst load which they ever encounter in flight. The total load on the wings was taken as 5.3W. The center of gravity of the load was placed at 37 per cent of the chord from the leading edge, and the chord was assumed to be inclined at 6.5° to the horizontal, the trailing edge being lower than the leading edge (the wing truss, of course, being inverted for sand-load test). The load per running foot was 84 pounds in the front truss and 78 pounds in the rear.

The solution was exactly similar to those for Cases I and II and calls for no special comment. Since the load was nearly equally distributed between the front and rear trusses the stresses in the stagger wires were extremely small, different diagonals being stressed at the two panels and the stress in the short diagonal at the outer panel point being less than 1 pound. The larger pull in the long stagger wire at the inner panel point is due to the forward reaction of the upper external drag wire on the upper wing at that point. Both external drag wires carry some load, the upper one taking more than the lower, as the upper wing deflects more freely in the direction of the drag truss than does the lower and as the upper drag wire also assists in carrying the lift.

The nature of the stress distribution in the redundancies causes a very peculiar reversal of direction of stress in the internal drag bracing of the upper wing. The direction of the load-carrying diagonal reverses twice, so that the load-carrying members are arranged as in a Warren truss, but with all the members in tension. The compression ribs at the points where these reversals occur carry no load at all, and a sand load in accordance with these specifications would therefore be unduly easy on the upper drag truss in the inner bay. The stress in the upper front and lower rear spars, also, are considerably less in Case III than in Case I, particularly in the inner bays. The drag wires in the inner bay of the lower wing and some of the compression ribs in both upper and lower wings, on the other hand, are stressed more severely in the sand load than they ever would be in flight. The comparison of the results of the various analyses serves to emphasize the impossibility of devising any single sand load which will truly simulate all of the "worst conditions" that may be encountered in the air.

CASE III.

No.	Stress without redund.	Stress with all redund.	F. S. with all redund.	No.	Stress without redund.	Stress with all redund.	F. S. with all redund.
1	392	392	6.63	26	−2,182	−1,923	....
2	835	835	3.11	27	−6,353	−5,870	....
3	947	946	2.75	28	−2,182	−2,164	....
4′	−1,495	8	2.47	29	−8,684	−7,169	....
5	1,598	95	42.2	30	− 388	− 388	....
6′	....	1	33.30	31	− 561	− 561	....
7	....	201	9.98	32	+ 186	− 887	....
8	396	396	6.56	33	− 35	− 35	....
9	497	497	5.22	34	+ 254	+ 254	....
10	1,090	800	5.00	35	− 70	− 70	....
11	1,192	902	4.43	36	+ 562	+ 562	....
12	404	404	9.89	37	− 456	− 457	....
13	376	376	10.6	38	+2,507	+2,338	....
14	1,954	1,955	4.30	39	+ 175	+ 390	....
15	1,819	1,818	4.62	40	+3,327	+2,941	....
16	4,093	3,879	2.17	41	− 724	− 290	....
17	3,810	3,217	2.61	42	− 128	− 128	....
18	4,302	1,358	3.09	43	− 282	− 282	....
19	351	351	5.70	44	− 521	− 394	....
20	....	1,620	2.59	45	− 751	− 555	....
21	....	207	20.3	46	− 699	− 699	....
22	−1,507	−1,508	....	47	− 650	− 650	....
23	−2,644	−2,643	....	48	−1,883	−1,860	....
24	− 857	− 858	....	49	−1,753	−1,912	....
25	−3,380	−3,379	....	50	−3,251	− 887	....

EFFECT OF INITIAL TENSIONS.

The analyses of the first two cases have been based on the assumption that all of the wires are just taut but with no initial tension. Actually, even if it were possible to secure such an adjustment it would not be desirable to do so, as some initial tension is necessary in order to keep the structure from vibrating badly and to hold it in proper alignment. It is therefore necessary to investigate the effect of initial stress on the distribution of load.

It is not correct to apply the method of least work in a straightforward manner, taking the derivatives of the work done by the external loads along, or of the change in total strain energy due to the imposition of the external loads, as might at first be assumed to be the case. The partial derivative of the total strain energy with respect to the stress in any member is equal to the deflection, parallel to the line of that member, of the point at which the force representing the stress is considered to be applied, this deflection being measured from the point at which there would be no stress in the member in question. If the frame of the structure is lined up with initial tensions in some or all of the members, the deflections which are desired in order to establish the conditions of geometrical equilibrium of the truss are those measured from the strained lengths of the members before the external loads are applied, and it is therefore necessary to make a deduction for the initial deflections due to straining of the redundant members against each other. The equations based on the work derivatives, and defining the relations between the final stresses in the redundant members, must then be written:

$$\frac{dW}{dT_x}-\frac{dw}{dt_x}=0$$

where W is the work done by external loads, w the work of deformation when the initial stresses alone are acting, T_x the final tension in any redundant member and t_x the initial tension. It is not necessary, however, to re-write all the equations, as it is sufficient to carry through the analysis and compute the stresses without regard to the initial tensions, and then to add to the stress in each member that due to initial stress in the redundancies. It is evident that this is the case, as the equations for W and w in each member are homologous, except that the terms involving only one unknown stress do not appear in the latter, since those terms are due to the external loads. The derivatives $\frac{dW}{dT_x}$ and $\frac{dw}{dt_x}$ are then identical, except that the second involved t where the first has T, the subscripts remaining the same, and that the first has a pure numerical term which is lacking in the other. The terms combine, when the second expression is subtracted from the first, in such a way that neither T nor t appears singly, but always in the combination $(T-t)$, and it would therefore have been sufficient to write in the first place

$$\frac{dW}{d(T_x-t_x)}=0$$

where W is given the fictitious value $\frac{(T-t)^2 1}{2AE}$ instead of $\frac{(T^2-t^2)1}{2AE}$, which is the true change in strain energy caused by the application of the external loads. The solution of the simultaneous equations then gives T_x-t_x for the redundant members, and the initial stresses must be added in to secure the total final load.

The effect of initial tension can best be illustrated by giving a couple of simple examples. As a first instance the pin-jointed structure shown in figure 3, and consisting of bars cross braced with wires, may be selected. It is assumed that the bars are so large in proportion to the wires that their strain may be neglected, and that the two diagonal wires are of equal size. If an initial tension F be placed in one wire there must be an equal and opposite initial tension resisting it in the other diagonal member in order that the structure may be in equilibrium. If an external load 0.707 P be applied as shown in the figure wire No. 1 will carry a tension of P pounds while No. 2 goes slack if there is no initial tension. If there is initial tension No. 2 will shorten by exactly the same amount that No. 1 lengthens, and the resultant tension in No. 1 will be $F+\frac{P}{2}$, while that in No. 2 is $F-\frac{P}{2}$. The tensions will vary in this manner as P is increased until $P-2F$, at which time the tensions are P and O. Thereafter the stresses are the same as if there had been no initial tension. If this very simple problem had been treated by least work with initial tension the stresses determined would have been $+\frac{P}{2}$ for 1 and $-\frac{P}{2}$ for 2. Adding these stresses algebraically to the initial tensions in the two members the same result is obtained as was just given as a result of elementary geometrical reasoning.

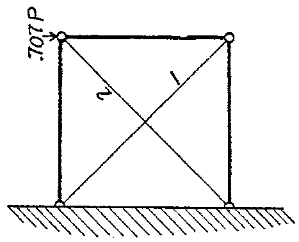

FIG. 5.

If, in this problem, No. 2 had only half the cross-section area of No. 1 the initial tensions in the two would, as before, be equal. An applied load superimposed on the original stresses would, however, produce twice as great an effect in 1 as in 2, since the increase in tensile strain of 1 as the structure deforms must be equal to the decrease of strain in 2. The unit stresses in the two are then equal if they are of the same material, and the total stresses are proportional to the cross-sectional areas. It follows from this that the total loads in the two wires are given, so long as they both remain in tension, by the formulae $F+\frac{2P}{3}$ and $F-\frac{P}{3}$ and that the lighter wire will not become slack until $P=3F$.

To afford some indication of the initial tensions existing in airplanes rigged in the field under average conditions and without using a tensiometer, tensiometer measurements of the stresses in all the exposed wires were made for 6 JN4H airplanes, four of them rigged by four different Army crews and the remaining two by a civilian crew. The averages are tabulated below, together with the mean deviations showing how widely the tensions in corresponding wires varied in the several machines. In the case of the flying wires, the mean deviations given are the mean deviations of the total stress in the two parallel wires from the mean value of that total, and the figures in parentheses, immediately under those mean deviations, are the means of the differences between the tensions in two parallel wires on the same airplane. The tensimeter readings taken in this way do not directly represent the true initial tensions, as the weight of the cellule is an external load which was being carried by the landing wires at the time when these measurements were made. The tensions read in the landing wires were therefore a little higher than the true initial tensions, while the values for the flying wires were correspondingly too low. This effect, amounting to about 60 pounds in some wires, has been corrected for in compiling the table of means. The magnitudes of the mean deviations in initial tensions strongly indicate the advisability of using a tensiometer and straining all wires in accordance with a schedule specified by the builder of the airplane. This method has been tried in rigging one or two machines at Langley Field, the tensiometer being used by mechanics with no previous experience with such an instrument, and a great improvement in the rigging was manifested. Where it had been common for one or more wires to vibrate badly at all engine speeds when the initial tension was adjusted by feel in the usual manner, there was no vibration except at one critical speed on the machine rigged by tensiometer.

It has been assumed that the probable maximum of initial tension in any particular wire given reasonably competent and careful rigging, is equal to the mean of the tensions for the six machines examined plus twice the mean deviation. This is not by any means an absolute maximum, and it was exceeded in some wires on several of the airplanes examined, but it represents a figure which need not and should not ever be exceeded. These probable maxima have also been included in the tabulation above. In the case of the stagger wires, where both wires remain in tension and it is only the amount of unbalanced tension or the difference between the two, which must be taken into account, the assumption in the analysis has been that the wire stressed by external loads (the long one) has an initial tension equal to the average for the six airplanes plus the mean deviation and that the short wire carries a stress less than the average by an amount equal to the mean deviation for that member. The difference between the two is therefore twice the average of their mean deviation.

TABLE OF MEAN INITIAL TENSION ON SIX JN4HS.

Wire No.	Average tension.	Mean deviation.	Probable maximum.	Wire No.	Average tension.	Mean deviation.	Probable maximum.
6	665	129	923	15	691	107	905
6′	498	90	678	(Double)		(83)	
7	737	170	1,077	15′	730	215	1,160
7′	497	105	707	16	645	118	881
12	110	47	204	(Double)		(92)	
(Double)		(33)		16′	693	167	1,027
12′	205	64	333	17	684	192	1,068
13	85	38	161	(Double)		(54)	
(Double)		(27)		17′	728	122	972
13′	181	62	305	18	466	105	676
14	721	138	997	19	630	87	804
(Double)		(64)		20	225	31	287
14′	838	138	1,114	21	269	86	441

The differences between the initial tensions in any given pair of opposed wires can be computed, if the initial tensions in the redundancies are known, on the usual assumption of frictionless pin joints. Any discrepancy between the difference of stress thus computed and that found by actual measurement is then due to the partial rigidity of the joints and the continuity of the spars. If, when the structure is in perfect alignment, there is a difference between the computed and measured stresses in the nonredundant members, it shows that the wings are warped and that they have had to be initially stressed to draw them into alignment. In the average of the six machines measured this discrepancy was largest in the inner bay of the rear truss, where it amounted to a deficiency of about 200 pounds in the tension in the flying wires. This is largely due to the relative bowing of the left rear spars in order to give "droop" to that wing and balance the engine torque.

The effect of the maximum probable initial tension has been computed for all three of the loadings thus far treated, and the results are tabulated below. In general, the effect on the worst-stressed members is injurious, and the initial tensions should therefore be kept as small as possible without permitting excessive vibration. In tabulating the stresses due to initial tension it has been assumed in every case that the excess tension is in that stagger wire where it will increase the stress, as both stagger wires of an opposed pair are in tension at all times with the usual initial tension. It will be noted that the factors of safety in the stagger wires are low, as their initial tensions are a large proportion of their ultimate strengths. The change of tension in the stagger wires under load is therefore small in comparison with the initial tension, the stress in one wire increasing while that in the other decreases so that the change in each wire is equal to approximately half the tension computed by the least work analysis. This is in accordance with the results of sand load tests, where tensiometer measurements after the application of each load have shown that the stresses in the stagger wires vary only a little from their initial values. In addition to always taking the worst condition as regards the initial distribution of load between the stagger wires, the stresses in the external drag wires have been taken as the probable minimum, instead of the probable maximum, wherever that would be the worst condition as regards the resultant stress in any particular member.

In a few cases the influence of the initial stress is great enough to control the direction of the diagonal which carries load in the internal drag bracing, the load shifting from the drag to the antidrag wires, or vice versa, if the excess unbalanced tension is transferred from one stagger wire to the opposed member. In some cases this leads to difficulty where the worst loads in the spars and in the internal drag wires occur under different conditions of initial adjustment and where the worst load in the spars corresponds with a reversal of stress and a transfer to the opposite diagonal from that which normally carries the tension in the internal truss. When this occurs it would be necessary, in order to secure strictly accurate results, to carry the whole analysis through from the start with the antidrag wires included and the drag wires omitted, but a close approximation can be made without the necessity of repeating the work in this manner. This approximation is based on the assumption that a compression in one diagonal of a rectangular frame can be replaced by a tension in the opposite diagonal, an assumption which would be true if the frame were exactly symmetrical and if the drag and antidrag wires were of the same size. If any particular combination of initial tensions gives a negative result for the total force in a drag wire this wire is therefore replaced by the opposed member, and it is assumed that the resultant stress determined is unchanged in magnitude but reversed in sign. A correction has to be applied to the stresses in the spars in the panel where this reversal occurs, as the drag and antidrag wires do not affect the same portions of the spars. In the second panel from the tip of the upper wing, for example, the stress in 22 (see figure 1) is affected by a force in 2′ but not by one in 2, whereas exactly the opposite is the case with 23. It would therefore be necessary, in arbitrarily passing from 2 to 2′ as the load-carrying member, to subtract (algebraically) from the direct load on each spar panel an amount equal to the component parallel to the transverse axis of the stress in the wire. The correction is subtractive in each case, as there is taken away from 23 a tension due to the fictitious compression in 2, while there is added to 22 a compression arising from the real tension in 2′, this tension being equal in magnitude, as already noted, to the theoretical compression found in 2. In the tabulation, wherever an approximation of this sort has been made the stress for the member affected is placed in parentheses.

In the members (interplane struts and compression ribs) directly interposed between two points of attachment of stagger wires, the fact that both wires remain in tension under all conditions has been allowed for. The final stress in any stagger wire is approximately equal to the initial stress plus or minus half the computed stagger wire tension (the stress being increased in the diagonal which was originally assumed to be stressed, decreased in the other). This, again, is only an approximation, but approximations are essential if the work is not to be complicated beyond all endurance by the introduction and simultaneous treatment of about 20 redundancies.

INITIAL TENSIONS, CASE I.

No.	Stress without initial tension.	Stress due to initial tension in 6.	Stress due to initial tension in 7.	Stress due to initial tension in 20.	Total resultant stress.
1	163	0	0	0	163
2	577	197	0	0	774
3	482	197	0	0	679
4	343	189	276	−164	644
5	264	189	276	−164	565
6	86	923	0	0	[1] 966
7	140	0	1,077	0	[1] 1,147
8	198	131	0	0	329
9	120	131	0	0	251
10	536	126	116	0	778
11	458	126	116	0	700
12	508	0	0	0	508
13	367	0	0	0	367
14	2,341	253	0	0	2,594
15	1,886	381	0	0	2,267
16	4,851	244	226	0	5,321
17	3,633	368	537	− 85	4,453
18	1,505	371	541	−322	2,095
19	342	0	0	0	[2] 804
20	677	0	0	287	964
22	−2,062	−202	0	0	−2,264
23	−2,310	−458	0	0	−2,768
24	−1,613	−304	0	0	−1,917
25	−2,692	−611	0	0	−3,303
26	−5,019	−597	−177	0	(−5,824)

No.	Stress without initial tension.	Stress due to initial tension in 6.	Stress due to initial tension in 7.	Stress due to initial tension in 20.	Total resultant stress.
27	−5,307	−1,040	−627	312	−6,662
28	−4,739	− 691	−264	−2,181	(−6,005)
29	−5,528	−1,183	−833	436	−7,108
30	− 269	− 395	0	0	− 647
31	− 332	− 124	0	0	− 456
32	− 557	− 124	−463	0	[1] −1,114
33	− 214	− 124	−182	108	− 411
34	− 320	0	0	0	− 320
35	76	153	0	0	229
36	474	102	0	0	576
37	− 18	....	0	0	[3] (228) (222)
38	2,437	406	0	0	2,843
39	1,071	753	208	0	2,032
40	2,843	500	88	0	3,431
41	734	806	416	0	(2,042)
42	− 162	−360	0	0	[1] − 503
43	− 96	− 83	0	0	− 179
44	− 272	− 83	−375	0	− 701
45	− 324	− 83	− 76	0	− 483
46	− 878	−671	0	0	[1] 1,514
47	− 702	−734	0	0	[1] 1,401
48	−2,298	−153	−699	0	[1] −3,045
49	−1,887	−230	−856	0	[1] −2,917
50	− 948	−295	−430	258	−1,415

INITIAL TENSIONS, CASE II.

No.	Stress without initial tension.	Stress due to initial tension in 6′.	Stress due to initial tension in 7′.	Stress due to initial tension in 21.	Total resultant stress.
1	133	0	0	0	133
2′	289	132	0	0	420
3′	245	132	0	0	377
4′	487	126	116	0	729
5′	447	126	116	0	689
6′	424	498	0	0	[1] 710
7′	427	497	0	0	[1] 711
8	415	132	0	0	547
9	455	132	0	0	586
10	457	126	116	− 74	624
11	497	126	116	− 74	664
12′	513	0	0	0	513
13	518	0	0	0	518
14′	528	380	0	0	908
15	359	380	0	0	739
16′	770	367	540	− 18	1,657
17	860	367	540	0	1,766
19	502	109	100	− 3	708
21	485	0	0	441	926
22	−632	−102	0	0	− 734
23	−523	−304	0	0	− 836
24	−820	−205	0	0	−1,025
25	−300	−456	0	0	− 756
26	−253	−504	− 87	0	− 844
27	−783	−894	−421	0	−2,098

No.	Stress without initial tension.	Stress due to initial tension in 6′.	Stress due to initial tension in 7′.	Stress due to initial tension in 21.	Total resultant stress.
28	−610	− 599	−174	0	−1,383
29	−416	−1,036	−629	0	(−2.092)
30	−225	− 360	0	0	[1] − 473
31	−180	− 83	0	0	− 263
32	−379	− 83	−376	0	[1] − 725
33	−320	− 83	− 76	0	− 479
34	−421	− 304	0	0	− 725
35	−188	− 102	0	0	− 290
36	−101	− 456	0	0	− 557
37	−539	− 205	0	0	− 743
38	−825	− 894	421	−328	(−2,767)
39	−597	− 504	−504	56	−1,132
40	−480	−1,036	−629	−623	(−2,996)
41	−972	− 599	−599	112	−1,633
42	−259	− 360	0	0	− [1] 507
43	−284	− 83	0	0	− 367
44	−300	− 83	−376	49	− [1] 646
45	−326	− 83	− 76	49	− 436
46	−530	− 734	0	0	[1] − 1,055
47	−377	− 734	0	0	[1] − 902
48	−580	− 229	−856	0	[1] − 1,455
49	−596	− 229	−856	0	[1] − 1,471
50	−615	− 237	−348	12	− 1,188
50A	−210	− 65	− 60	− 22	− 22

[1] The maximum probable stress here is not equal to the sum of the figures in the first four columns, as the stress i
both diagonals remain in tension, varies only half as
in one stagger wire of each pair. The stress in inter
interposed, is therefore less, in most cases, when t

other component of stress from an entirely different source.
imum probable initial tension found by measurement. Com-
as any initial tension in 19 would have to be balanced by a
analysis.
gger wire carries an excess of initial tension.

CASE III.

No.	Stress without initial tension.	Stress with worst initial tension.	No.	Stress without initial tension.	Stress with worst initial tension.	No.	Stress without initial tension	Stress with worst initial tension.
1	392	392	18	1,358	1,886	35	−70	−172
2	835	1,033	19	351	351	36	562	−664
3	946	1,144	20	1,620	1,907	37	−457	−660
4′	8	538		207	648	38	2,338	2,664
5	95	396		−1,508	−1,711	39	390	1,609
6′	1	678		−2,643	−3,103	40	2,941	3,393
7	201	1,187	24	−858	−1,163	41	−290	−951
8	396	526	25	−3,379	−3,992	42	−128	−488
9	497	627	26	−1,923	−2,779	43	−282	−365
10	800	968	27	−5,870	−7,320	44	−394	−756
11	902	1,018	28	−2,164	−3,732	45	−555	−663
12	404	404	29	−7,169	−8,756	46	−699	−1,370
13	376	376	30	−388	−785	47	−650	−1,384
14	1,955	2,210	31	−561	−685	48	−1,860	−2,687
15	1,818	2,204	32	−887	−1,417	49	−1,912	−3,088
16	3,879	4,432	33	−35	−35	50	−887	−1,217
17	3,217	4,039	34	254	−254			

The results of the investigations, as recorded in these tables, emphasize the great importance of initial tension, the deleterious effects of which have too seldom been appreciated. In almost every instance the stresses under the worst probable distribution of initial tensions are greater than those which arise from the air load alone, either with or without redundancies. In short, the stagger wires, as they are usually set up, are actually harmful and weaken the structure under most conditions of flight, whereas they should be an important element of strength. The initial tensions in the external drag wires are much more innocuous, although the values selected there should always be as small as are consistent with the rigidity of the structure and with freedom from vibration when in flight. The stagger wires, being disposed in directly opposed pairs, can and should be so adjusted that there will be little or no unbalanced tension to affect the remainder of the truss. Even if this is done, however, the initial tensions should be kept small to ease the strain on the stagger wires themselves and on the interplane struts and drag struts or compression ribs which make up the parallelogram frames at panel points. If the alignment of the air plane is carried out with a tensiometer the element of guesswork is definitely removed, the factors of safety in some important and badly stressed members are increased by from 25 per cent to 50 per cent, and the time required for rigging is increased very little, if at all. In fact, it is probable that a crew which has had a little experience with a tensiometer can work quite as rapidly with as without it, as the amount of trial and error required to bring the machine into true alignment is less than by the ordinary method.

In order that mechanics may have some reliable guide for use in rigging the designers of airplanes should draw up schedules of initial tensions to be used. The primary principle to be followed in drawing up such a schedule is that there should be no unbalanced tension in either of two directly opposed members. In a rectangular frame this means that the initial tension must be equal. (This of course applies to the total tensions where there are two or more members in parallel. Where, for example, two flying wires oppose a single landing wire the initial tension in each flying wire should be just half that in the landing wire.) Where the frame is not rectangular, but has two parallel sides, as in the stagger panels of an airplane with stagger or in the lift truss of a machine with interplane struts sloping outwardly and with the same amount of dihedral in the upper and lower wings, the condition is that the diagonal wires should have equal components perpendicular to the parallel sides. In the case of a stagger panel, this means that the tensions in the two stagger wires should be inversely proportional to the sines of the angles which they make with the wing chords, so that the long diagonal has the larger tension.

In drawing up a tension schedule the periods of vibration of all the wires should be high enough not to synchronize with the natural period of the engine, and should be approximately the same throughout the structure. The fundamental frequency of a stretched wire can be shown[6] to be equal to $\frac{1}{2l}\sqrt{\frac{T}{m}}$, where l is the length of the wire, T the tension, and m the

[6] Textbook on Sound, J. H. Poynting & J. J. Thomson: p. 88.

mass per unit length, which is of course directly proportional to the sectional area and so to the strength of the wire. The tension to give a constant frequency must therefore be proportional to the ultimate strength and to the square of the length, and it is necessary that very long wires be supported at some intermediate point, as the initial tension required to prevent vibration if this were not done would be dangerously high. It has been found by actual experiment that an initial tension of 220 pounds in the upper drag wire of a JN is enough to prevent vibration. Since this wire carries an additional load of about 140 pounds when flying normally with a load factor of 1, the total resultant tension for satisfactory results is 360 pounds, and this may be taken as a basis for the determination of the other tensions. The flying and landing wires are substantially equal in length to the upper drag wire, but they have an intermediate point of support where they cross each other. The area of all these members are the same, and the resultant tension in the flying and landing wires must therefore be at least 90 pounds (the effective length being halved). With a load factor of 2, which is as high a value as is likely to be maintained steadily, the air load reduces the stress in the inner landing wires by about 630 pounds (the total air load on the wires in the inner bay being 1,880 pounds, of which two-thirds is taken by an increase in the stress in the double flying wires, while the remaining third shows as a reduction in the landing wire tension), and the initial tensions therefore should be at least 720 pounds. The initial tension in each flying wire, as already noted, should be half this amount. In the outer bay a tension of 390 pounds in the landing wires is sufficient, as the air load effect there is less. The length of the long stagger wire is approximately two-thirds that of the upper drag wire, and there is a center support where the two stagger wires cross. The area of the stagger wire is about half that of the external drag wire, so that the resultant tension for Nos. 6 and 7 in the conspectus only needs to be one-eighteenth of that for No. 20, or 20 pounds. Under normal conditions of flight (load factor of 2 or less) the tension in the stagger wires is not changed more than 30 pounds by the air load, and the initial tension thus does not need to exceed 50 pounds. Making some extra allowance to secure rigidity, 150 pounds for the long wire and 120 pounds for the short one appears ample, and tests in flight have shown it to be so.

The complete tension schedule for the JN is given below, and will serve as a guide in drawing up such schedule for other machines of similar type.

Member.	Initial tension (including weight of wings).	Member.	Initial tension (including weight of wings).
Inner front flying wires (each)	360	Stagger wires, long	150
Inner rear flying wires (each)	270	Stagger wires, short	120
Outer flying wires (each)	180	Front center section wires	150
Inner landing wires	780	Rear center section wires	230
[illegible]	420	Upper drag wire	220
[illegible]	70	Lower drag wire	340
[illegible]	400		

The pulls in the flying and landing wires in the inner bay are not exactly balanced because the vertical components of the tensions in the external drag wires are balanced by modification of the flying wire stresses.

PRACTICAL CONCLUSIONS AND SUMMARY.

The conclusions to be drawn from this work will first be tabulated and will then be examined more in detail where they call for such examination.

(*i*) The making of a least work analysis of a new design for at least one case is thoroughly justified. The labor of making such an analysis is not excessive and it gives an idea of the nature and magnitude of the true stresses which can not be obtained in any other way.

(*ii*) The wooden members may be omitted from consideration in the work equations without causing any serious error.

(*iii*) The effect of the stagger wires is unimportant when the load is approximately equally distributed between the front and rear trusses. In diving the effect of the stagger wires is very important, and greatly reduces the load on the lift trusses. The effect of the stagger wires depends in part on the arrangement of the external drag wires. If there is no external drag wire attached to the upper wing, and if the center section wires have as little forward inclination as they have on the JN, the stagger wires running upward from front to rear will be in tension at all times, transferring drag from the upper to the lower wing, and must be taken into account.

(*iv*) The tension in the external drag wires varies widely with the conditions of loading. Only very rarely are both wires stressed at the same time, and most of the work now done by the two wires could be accomplished equally well by a single one.

(*v*) The initial tensions are almost always excessive, particularly in the stagger wires, and are sometimes so large as to be dangerous, especially as regards the compression ribs at the lift truss panel points. The initial tension is sometimes so high that the total effect of the redundancies becomes harmful, whereas it should be distinctly beneficial to the total strength of the truss.

RECOMMENDATIONS.

I. Only one external drag wire should be used on each side of the plane of symmetry That one can be kept in tension nearly all the time, whereas, as already noted, it is only rarely that the upper and lower drag wires are in tension simultaneously. The structure should of course be designed to fly normally (not to be stunted) without any external drag wires at all. A single drag wire should be attached at the lower front spar, so that it will resist the downward and backward deflection of the truss during a dive. If two external wires are used the second one should be attached either to the upper front or the upper rear spar. The first position is probably the more effective in most instances, as the drag wire then relieves the very heavy load on the front lift truss at large angles. The same result can be obtained without the use of a second drag wire by increasing the strength of the flying wires in the inner bay and attaching them to the fuselage a little forward of the wing spars, as has been done in several recent designs, in order that they may resist the drag on the upper wing. Attachment of the drag wire at the lower rear spar should not be employed.

II. The stagger wire which runs upward from front to rear carries a heavy load at times and may well be made stronger than the other diagonal. If a steel tube, with no opposing member, is used for stagger bracing it should run upward from front to rear. If there is no drag wire attached to the upper wing, such a tube need not be designed to carry a compressive load of more than one-eighth the weight of the airplane, but it should be capable of sustaining a tension equal in magnitude to the total weight of the machine. If picture-frame struts are used, and they are highly recommended, they should be designed to carry from five to eight times as large a compressive load in the direction of the long diagonal (for a machine with positive stagger) as in the direction of the other diagonal.

III. Airplanes should be rigged, whenever possible, by means of a tensiometer and in accordance with a schedule of initial tensions to be provided by the designer. Detailed instructions for drawing up such a schedule have already been given. In particular, the tensions in the stagger wires should be far less than has been the common practice, and opposing members should exactly balance each other. One great advantage of the picture-frame strut is that it eliminates all danger of excessive initial tension.