REPORT No. 347

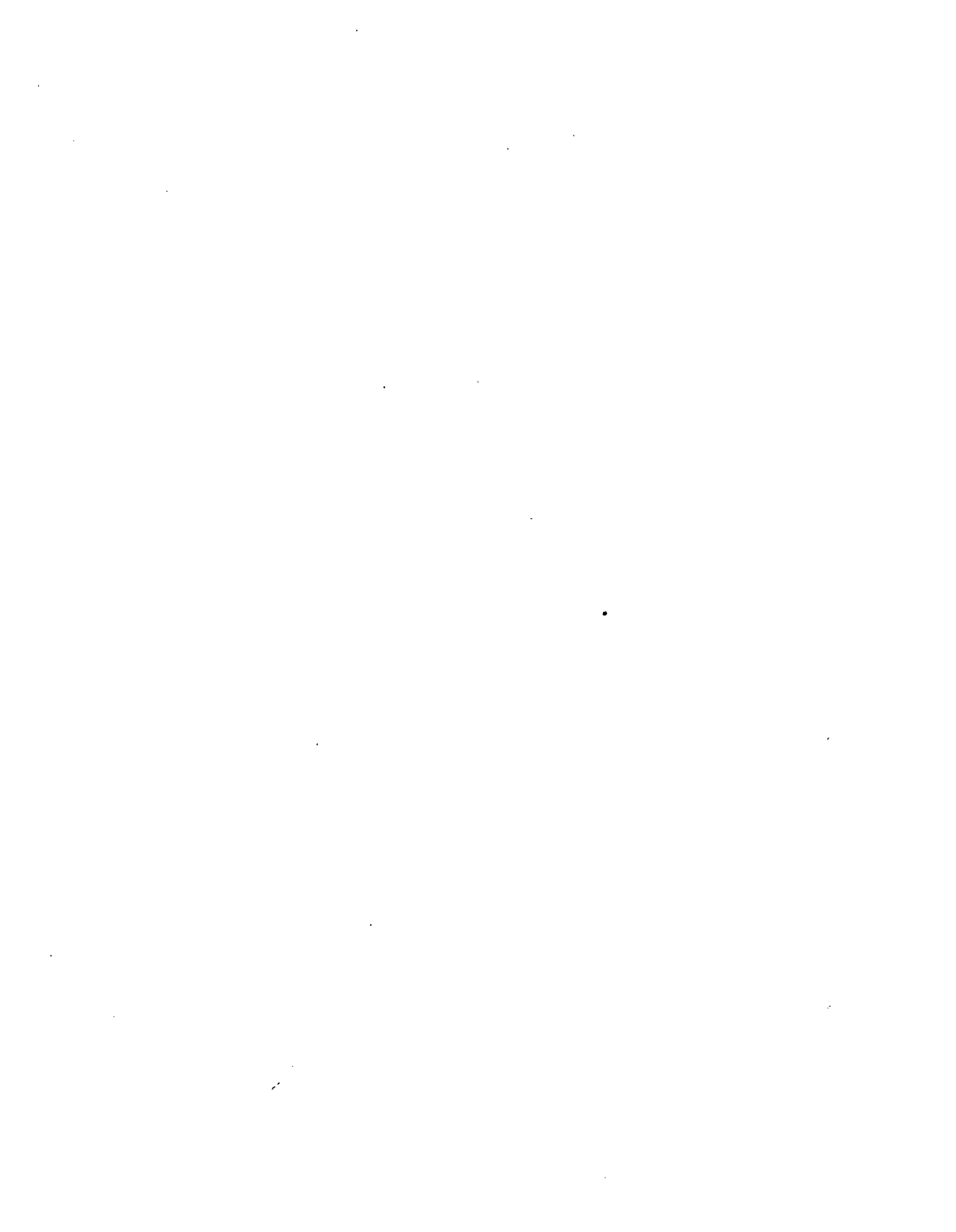## A METHOD OF CALCULATING
THE ULTIMATE STRENGTH OF CONTINUOUS BEAMS

By J. A. NEWLIN and GEO. W. TRAYER

Forest Products Laboratory

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A METHOD OF CALCULATING THE ULTIMATE STRENGTH OF CONTINUOUS BEAMS

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By J. A. Newlin[1] and Geo. W. Trayer[1]

SUMMARY

In the design of continuous beams subjected to transverse load only, it has been common practice to estimate maximum loads by substituting the numerical value of the modulus of rupture, as obtained in bending tests, in the usual equation of three moments. Further, for combined axial and transverse loading, two methods of calculation have been used. The more common one is the application of the generalized equation of three moments, while the other is an extension of the ordinary equation of three moments to allow for the moments introduced by the direct tension or compression load. In the second method, the deflection in the span at the point of maximum moment is calculated, neglecting the effect of axial load, and the product of the axial load and this deflection is added to the moment determined by the ordinary equation of three moments.

Both of these methods are used to calculate maximum loads, although neither is properly applicable beyond the elastic limit. The purpose of the study reported here was to investigate conditions after the elastic limit has been passed. As a result of the study, a method of calculation, which is applicable to maximum load conditions, has been developed. The method is simpler than the methods now in use and it applies properly to conditions where the present methods fail to apply.

The experimental work was conducted at the Forest Products Laboratory in cooperation with the Bureau of Aeronautics, Navy Department, and submitted to the National Advisory Committee for Aeronautics for publication. Over 300 continuous beams were tested under transverse load and under combined axial and transverse load. Loads obtained in test for beams of rectangular section were as much as 50 per cent in excess of loads calculated by the usual methods, with the average about 25 per cent. For I beams the average increase was about 40 per cent. Fortunately, the error in the usual calculation is on the side of safety, but it is too great to be neglected in good design.

INTRODUCTION

In employing the usual theory of three moments to calculate maximum load conditions, it is assumed that the relation of moments does not change when the elastic limit of the material is passed. When the elastic limit of the material has been exceeded at some point in the beam, however, the stiffness of the beam at that point is necessarily reduced. In other words, exceeding the elastic limit at some point is equivalent to a loss in modulus of elasticity at that point. This change in stiffness is accompanied by a shift in the points of contraflexure and a concomitant redistribution of moments. It is a fundamental principle of mechanics that if a continuous beam is stiffened between two successive points of contraflexure, these points of contraflexure will move away from the stiffened portion with a resulting increase in moment at that point. Conversely, if the same portion of the beam is made less stiff, the points of contraflexure will then move toward the portion of reduced stiffness.

This principle is the basis for the development of the proposed method of calculating the ultimate strength of continuous beams. Briefly stated, the investigation has made it possible to determine the true relation of the moments at maximum load. The beam can then be treated span by span and, when axial load accompanies transverse load, portions between two successive points of contraflexure, or between a point of contraflexure and a hinged support, may be considered separately.

Just where the points of contraflexure will move to after the elastic limit has been passed depends to a considerable extent on the toughness of the material, which is its capacity to continue to sustain load after the elastic limit has been passed. Furthermore, in arriving at a design basis, the extent of the damage caused through the beam having passed the elastic limit and the number of times the load will be repeated before failure occurs must be given careful consideration. The method of calculation herein proposed is based not on average material but upon material that will just pass an aircraft acceptance test, so that any increase in load obtained by this method of calculation may be safely counted on.

DESCRIPTION OF MATERIAL AND OF TEST SPECIMENS

Material.

In the study of continuous beams subjected to transverse loading only, a few beams were made of select Douglas fir (coast type) and a few of eastern-grown

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spruce, and the remainder of Sitka spruce cut in Oregon. The material for the combined axial and transverse load tests was also Sitka spruce from Oregon, which was air dried at the Forest Products Laboratory. This material varied in specific gravity from 0.32 to 0.46, with the determination based on its weight and volume when oven dry. Although the minimum specific gravity for acceptable aircraft material is 0.36, some material of low specific gravity was used in order to ascertain its behavior; this material was clear, straight grained, and of uniform texture. A few beams were also made of material including compression wood, to demonstrate the erratic behavior of such material; these beams contained compression wood to an extent readily discernible by an inspector or woodworker experienced in the use of spruce.

Major test specimens.

For transverse loading two styles of beams were used, namely, I and rectangular. All beams of both types were tested over two spans. Some of the I beams were routed throughout their length and others were left unrouted at the supports for varying distances.

For the combined axial and transverse loading the beams were either of the I or the box type. The panel arrangement was the same for both types, with 81 inches from the hinge to the first strut, 117 inches between struts, and a 40.66-inch overhang. The I beams were unrouted at the hinge fitting and at the two strut points and the box beams had filler blocks at these points.

Both styles were 2 inches wide and 4½ inches deep. The I beams had flanges 1 inch deep and webs ¾ inch thick. The box beams also had 1-inch flanges, but the web thickness was different for different beams.

MATCHING

The properties of the material in each beam were determined by making standard tests on small pieces cut from the plank from which the beam was made or from uninjured portions of the beam after test. Very often there was some slight difference in moisture content between these minor beams and the corresponding major beam, in which case adjustments were made. Values for the minor beams that are given in the tables have been adjusted to the moisture content of the major beam.

METHOD OF TEST
Major beams.

All beams subjected to transverse load only were tested over two spans with from one to eight concentrated loads in each span. The rate of loading was such that the rate of fiber strain was that specified for structural timber tests in the standards of the American Society for Testing Materials. Care was taken to select load and reaction blocks of the proper curvature,

so that the effect of crushing under the block might be compensated for by partial distribution of the load. Necessary provision was made to prevent bending in more than one plane.

For the combined axial and transverse loading I beams and box beams were tested as part of a truss. A diagrammatic sketch of this testing apparatus is shown in Figure 1. Load was applied to the beam through the evener system by lowering the movable head of the testing machine. A series of counter-weights supported the evener system during assembly and acted to prevent it from falling when a beam broke. Two truss heights were used. One height (47.5 inches) made the direct stress about 50 per cent of the total stress and the other height (30 inches) made the direct stress about 60 per cent of the total. The close spacing of the concentrated loads makes the moment curve for transverse load only agree very well with a moment curve for a uniform load (fig. 2). Failures usually occured in the inboard bay (fig. 2) and deflections were therefore taken in this critical portion.

Prior to the test to failure in the truss arrangement, the beams were subjected to a small transverse load applied in increments for the purpose of determining their stiffness. In this test they were supported at the two strut points and symmetrically loaded with two loads 44 inches apart.

Minor test specimens.

The minor specimens were tested in accordance with the standards of the American Society for Testing Materials.

ANALYSIS OF THE PROBLEM

The usual equation of three moments for transverse loads only and the generalized method of calculating stresses in continuous members subjected to both axial and transverse load are based on the assumption that the limit of elasticity is not exceeded. The design of wooden airplane spars, however, is based on maximum load stresses, and maximum loads are ordinarily estimated by substituting such stresses in formulas applicable only within the elastic limit. Such a procedure is satisfactory when designing a simply supported member. Further, the modulus of rupture, which is really not a stress at all, can be used with accuracy to estimate maximum loads for such a member, provided, of course, that when the member being designed is not rectangular in shape the modulus is corrected to take care of the form of the cross section.[3]

This procedure leads to difficulties, however, when designing a member the stiffness of which must be used in the equations of equilibrium in order to determine the reactions and from these the amount of stress

[3] National Advisory Committee for Aeronautics Report No. 181, Form Factors of Beams Subjected to Transverse Loading Only, by J. A. Newlin and G. W. Trayer.

at different points. The distribution of stresses in a member that can not be determined by a consideration of external forces only is fixed by the relative stiffness of the several parts of the structure. In general, that part which lacks stiffness will carry but little of the load while the stiff parts will take the brunt of the work.

It is this principle of distribution of load that is involved in the development of the proposed method of calculation. Consider for the moment a continuous

Now in any continuous member what has happened by the time maximum load is reached? Certainly some part or parts have passed the elastic limit, thereby losing a portion of their stiffness. Just as soon, however, as these more highly stressed parts pass the elastic limit and lose stiffness, relief comes, more load is thrown on the parts that have retained their original stiffness, and the result is a maximum load considerably higher than that which the usual formulas would indicate.
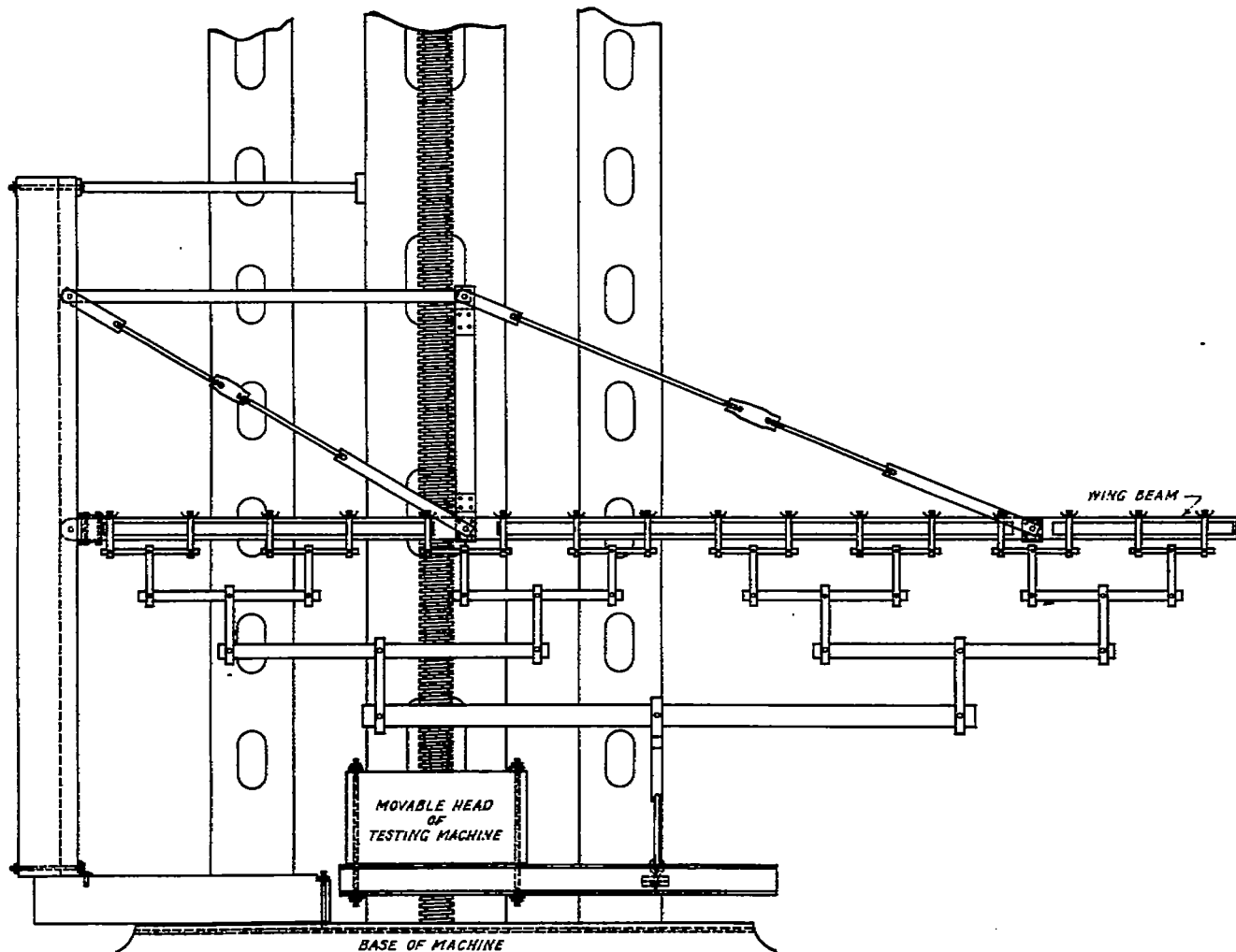
FIGURE 1.—Diagram of the loading apparatus for 2-bay and overhang test beams

beam. With a given loading within the elastic limit, certain bending moments occur out in the spans and at the supports. The slightest change in the stiffness of any portion, however, will cause a readjustment of these moments and an accompanying shift in the points of contraflexure. If the beam is stiffened between two successive points of contraflexure, these points move away from the stiffened portion with a resulting increase of moment in the stiffened part. Conversely, if the beam were made less stiff at any point, the adjacent points of contraflexure would move toward that point.

To illustrate, consider the simple case of a uniformly loaded continuous beam of uniform cross section, with an infinite number of equal spans perfectly aligned. Within the elastic limit the maximum moment in any span is one-half the moment at the supports, for all increments of load. If the load is increased so as to produce a stress at the supports in excess of the value at the elastic limit, a loss in stiffness results and this loss increases as the load increases, until failure occurs. Attending this loss in stiffness is a redistribution of moments, the points of contraflexure move toward the less stiff parts, and the ratio

HINGE   STRUT   STRUT

INBOARD BAY	OUTBOARD BAY	OVERHANG
81"	117"	40.66"

238.66"

DIMENSIONS OF 2-BAY TEST BEAM WITH OVERHANG

NOTE:-
THE FIGURES ON THE VERTICAL LINES ARE THE
PROPORTIONAL PART OF THE TOTAL LOAD.
THE FIGURES ON THE LEVEL ARMS ARE THE
ACTUAL LENGTHS IN INCHES.

DIAGRAM OF THE LOADING SYSTEM USED IN THE TESTS

MOMENT DIAGRAM FOR TRANSVERSE LOAD ONLY

SHEAR DIAGRAM FOR TRANSVERSE LOAD ONLY

FIGURE 2.—The loading of a typical continuous beam and the resulting moment and shear distribution

of maximum moment in the span to that at the support is no longer one-half. This action goes on until at failure the ratio of maximum moment out in the span to that at the supports approaches unity for tough material.

Now the principle involved in all this is one of fundamental mechanics, easy to understand and yet often difficult of application. The first step in the attack on the problem of application was an experi-

SOLUTION OF THE PROBLEM

In studying the behavior of continuous beams subjected to transverse load only, this experimental determination of the actual ratio of moments was made. The loading of beams continuous over two spans was so varied that the calculated ratio of the maximum moment in the span to that at the center support ranged from less than 10 to more than 80 per cent. Nearly 300 Sitka spruce beams, having specific gravi-
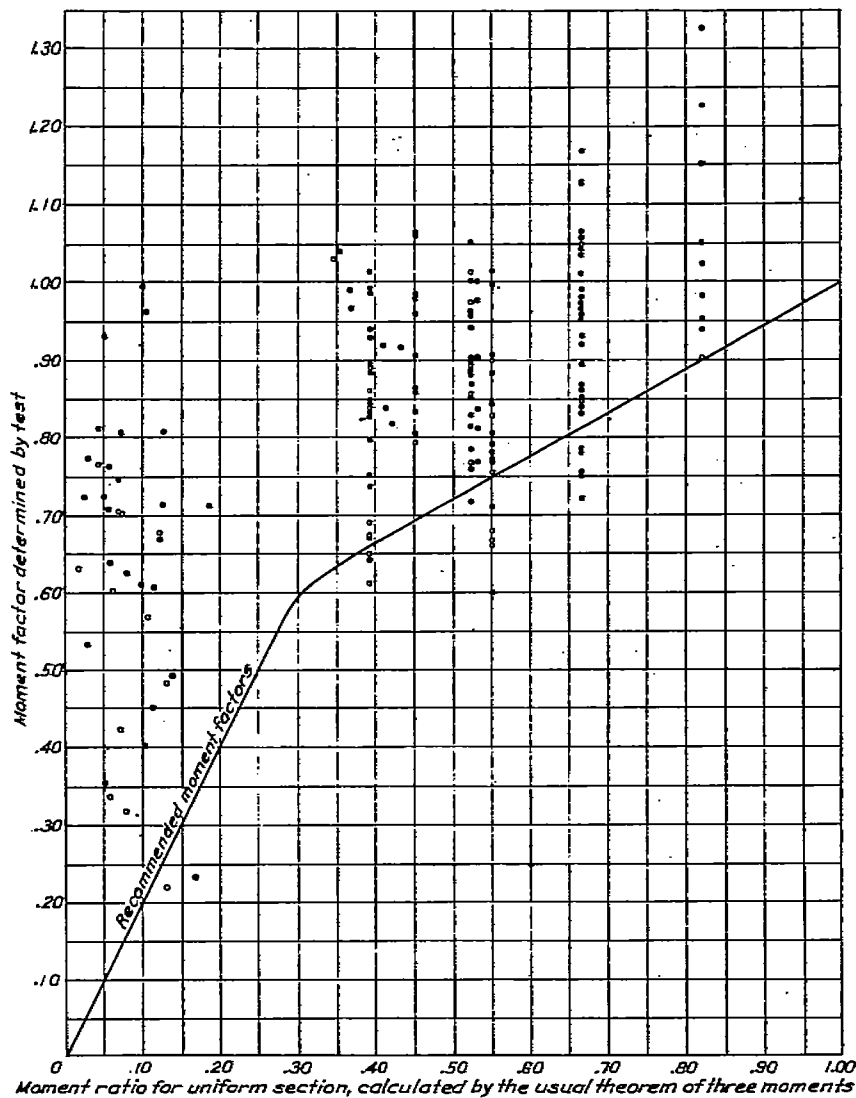
FIGURE 3.—Chart of the moment factor for calculating the strength of continuous beams

mental determination of the actual or true ratio of the maximum moments in the span and at the supports at the time of failure, as compared with the corresponding ratio while all the stresses were still within the elastic limit. With this information the location of the points of contraflexure at maximum load can be determined. The beam can then be treated span by span, or a portion of it between two successive points of contraflexure can be considered separately.

ties ranging from 0.32 to 0.46, were tested; the specific gravity values were based on the weight and the volume of the wood when it was oven dry. In addition, 15 beams of Douglas fir and 8 of eastern-grown spruce were also tested. In Figure 3 are plotted the true ratios of the moments at failure, as determined by these tests, against the calculated theoretical ratios for beams of acceptable material. The horizontal scale is the ratio of the maximum moment in the span

to that at the center support for a beam of uniform cross section, as obtained by means of the usual equation of three moments, and the vertical scale is the true relation of moments that existed at the time of failure. The curve is the locus of ratios recommended for the design of Sitka spruce beams; essentially they are the safe minimum actual ratios. Therefore, actual loads to cause failure will normally be considerably greater than those calculated from the curve of recommended ratios.

When a loading was chosen to give a selected calculated ratio of maximum moment in the span to moment at the center support for a beam of uniform cross section, it was found that almost any true ratio at failure, up to unity, could be obtained by selecting material varying in toughness. In fact, since it was impossible to always select minor test specimens that would give exactly the true properties of the material in the beam, and since information from which the proper radiuses of load blocks were determined was far from complete, the true ratio often apparently became greater than unity. As might be expected, material

low in toughness showed little redistribution of stresses.

The test values in Figure 3 are taken from Tables I, II, III, and IV. In Table I, for example, the values in one of the columns headed "Moment factor" are the ordinates for the abscissa standing above the column, which is there called "Calculated uniform section moment ratio." Beams that failed to meet the specific gravity and toughness requirements of the Bureau of Aeronautics were omitted from the figure. Such beams had been included in the tests only to determine the behavior of the abnormal material in them; this behavior is reported in the tables.

APPLICATION OF THE PROPOSED METHOD OF CALCULATION

Transversely loaded continuous beams of uniform cross section.

The application of the true ratio of moments to the simple case of a beam of uniform cross section can perhaps be best illustrated by taking a particular example.

TABLE I.—RECTANGULAR SITKA SPRUCE BEAMS, APPROXIMATELY 2 BY 2 INCHES IN CROSS SECTION, CONTINUOUS OVER TWO SPANS. THE CALCULATED MAXIMUM MOMENT IN THE SPAN VARIES FROM 0.452 TO 0.821 OF THAT AT THE CENTER SUPPORT, FOR THE DIFFERENT LOADINGS

Diagram 1-A

Diagram 1-B

Diagram 1-C

Diagram 1-A — Calculated uniform section moment ratio............0.524

	Major beam			Minor beam			
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
C-1	5,720	1.346	17.0	O-2	0.445	10,760	0.895
O-8	6,320	1.333	16.7	O-2-4	.455	10,630	1.054
C-5	6,090	1.340	16.4	C-4	.480	10,490	1.015
C-6	6,150	1.360	16.5	O-7	.458	11,800	.859
C-8	6,045	1.374	....	O-7-9	.458	11,220	.887
C-10	5,635	1.354	15.6	O-9	.429	11,580	.786
O-11	5,900	1.346	15.0	C-12	.462	11,310	.872
O-13	5,850	1.340	14.8	O-12-14	.437	11,370	.760
C-15	5,185	1.354	14.8	O-14	.441	11,320	.718
O-16	5,755	1.354	15.5	O-17	.450	10,770	.898
O-18	5,475	1.354	16.4	O-17-19	.468	10,240	.897
C-20	5,430	1.340	16.1	O-19	.446	10,350	.884
C-78	6,150	1.347	12.5	O-74	.468	10,850	.976
C-76	5,935	1.347	12.9	C-74-76	.464	10,270	1.005
C-78	5,490	1.354	12.4	C-77-79	.440	10,010	.855
C-80	6,200	1.300	11.6	O-79	.439	10,980	.959
O-81	6,305	1.300	11.5	C-82	.458	11,120	.965
C-83	5,660	1.347	11.7	C-82-84	.452	11,380	.815
C-86	5,780	1.347	12.3	C-85-87	.414	10,460	.944
C-88	5,700	1.307	11.6	O-87	.410	10,470	.905
C-94	3,340	1.347	12.9	O-93-95	.354	7,485	.697
C-96	4,090	1.367	12.5	O-95	.360	7,820	.854
C-102	4,660	1.333	13.0	C-101-103	.412	9,980	.708
C-104	4,990	1.354	12.8	C-103	.410	9,980	.817
C-105	4,770	1.374	11.9	O-106	.386	9,280	.831
C-107	4,790	1.333	13.0	C-106-108	.401	9,180	.892
Av.	5,497	1.351	13.9	....	.433	10,444	.877

Diagram 1-B — Calculated uniform section moment ratio............0.667

	Major beam			Minor beam			
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
O-21	4,050	1.333	14.3	C-22	0.404	9,500	0.786
O-28	4,550	1.333	14.5	O-22-24	.398	9,240	.960
O-25	4,525	1.333	14.5	O-24	.392	9,055	.980
O-26	4,400	1.333	12.0	O-27	.410	9,660	.863
O-28	4,750	1.333	12.6	O-27-29	.407	8,960	1.059
O-30	4,750	1.333	....	O-20	.404	8,915	1.067
O-31	4,700	1.333	12.0	C-32	.413	9,760	.930
O-33	4,775	1.333	12.4	O-32-34	.409	9,040	.967
O-35	4,825	1.333	12.5	O-34	.405	9,660	.981
O-36	5,050	1.333	15.4	C-37	.411	10,070	.983
O-38	4,860	1.333	15.4	C-37-39	.408	9,760	.975
O-65	5,545	1.346	11.2	O-66	.430	11,400	.932
O-67	5,495	1.354	12.7	O-66-68	.418	10,270	1.050
O-70	5,545	1.346	13.5	O-69-71	.468	10,900	.992
O-72	5,725	1.374	11.9	C-71	.473	11,350	.958
5DF	6,150	1.320	10.5	5DF	.458	13,750	.853
Av.	4,981	1.338	13.0	....	.419	10,118	.968

Diagram 1-C — Calculated uniform section moment ratio............0.452

	Major beam			Minor beam			
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
C-121	5,275	1.347	12.8	C-122	0.437	10,010	0.992
C-123	5,290	1.333	13.0	C-122-124	.441	10,020	.982
C-126	5,720	1.347	14.2	C-125-127	.472	11,940	.808
C-128	5,785	1.354	13.7	C-127	.475	11,800	.909
C-129	6,800	1.333	13.3	C-125	.409	12,850	.987
C-131	5,695	1.347	14.4	O-130-132	.470	11,450	.807
C-134	4,850	1.354	13.4	C-133-135	.413	9,710	.867
C-136	4,825	1.360	13.1	C-135	.406	9,820	.836
C-137	5,475	1.354	13.5	C-133	.413	8,750	1.005
C-139	4,510	1.347	14.2	O-138-140	.405	9,100	.861
C-142	4,870	1.333	15.7	C-141-143	.437	8,800	1.066
O-144	5,300	1.340	12.8	O-143	.411	11,240	.795
Av.	5,356	1.346	13.6	....	.437	10,499	.917

TABLE I.—RECTANGULAR SITKA SPRUCE BEAMS, APPROXIMATELY 2 BY 2 INCHES IN CROSS SECTION, CONTINUOUS OVER TWO SPANS. THE CALCULATED MAXIMUM MOMENT IN THE SPAN VARIES FROM 0.452 TO 0.821 OF THAT AT THE CENTER SUPPORT, FOR THE DIFFERENT LOADINGS—Continued
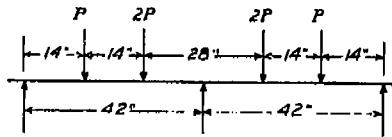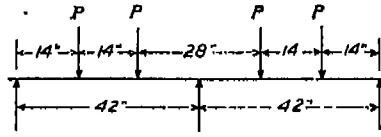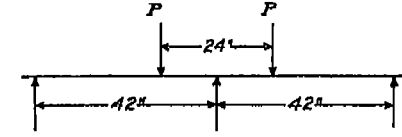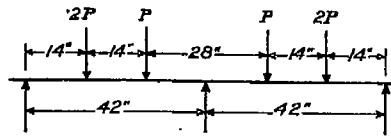
Diagram 1-D
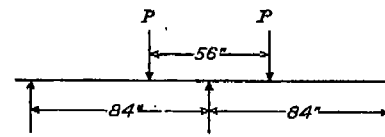
Diagram 1-E

Calculated uniform section moment ratio _____ 0.821

	Major beam				Minor beam		Moment factor
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	
	Pounds	I/C	Per cent			Lbs. per sq. in.	
C-41	5,975	1.354	12.6	C-42	0.482	11,570	1.152
C-43	5,250	1.333	13.3	C-42-44	.470	11,060	1.051
C-45	5,775	1.333	12.7	C-44	.459	11,350	1.152
C-46	5,715	1.333	13.4	C-47	.471	10,050	1.325
C-48	5,025	1.333	14.8	C-47-49	.464	9,400	1.227
C-50	5,300	1.333	12.4	C-49	.456	11,410	1.023
C-56	4,375	1.333	12.3	C-57	.426	9,690	.984
C-58	4,680	1.327	10.6	C-57-59	.418	10,760	.939
C-60	4,620	1.360	11.3	C-59	.411	10,690	.903
C-62	4,795	1.347	12.1	C-61-63	.429	10,760	.955
C-64	4,890	1.340	11.9	C-63	.434	11,450	.904
Av.	5,127	1.339	12.5	------------	.447	10,745	1.056

Calculated uniform section moment ratio _____ 0.533

	Major beam				Minor beam		Moment factor
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	
	Pounds	I/C	Per cent			Lbs. per sq. in.	
C-237	1,720	1.333	15.6	C-238	0.371	8,390	0.770
C-239	1,725	1.333	15.8	C-238-240	.375	8,165	.814
C-241	1,715	1.333	15.8	C-240	.379	7,980	.839
C-242	2,250	1.354	15.4	C-243	.460	9,420	.979
C-244	2,320	1.347	15.4	C-243-245	.460	9,630	1.004
C-246	2,270	1.360	15.3	C-245	.459	9,910	.907
Av.	2,000	1.343	15.6	------------	.417	8,916	.886

TABLE II.—RECTANGULAR DOUGLAS FIR AND EASTERN-GROWN SPRUCE BEAMS, APPROXIMATELY 2 BY 2 INCHES IN CROSS SECTION, CONTINUOUS OVER TWO SPANS. THE CALCULATED MAXIMUM MOMENT IN THE SPAN IS 0.667 OF THAT AT THE CENTER SUPPORT FOR THE LOADING SHOWN

Diagram 2

Calculated uniform section moment ratio............ 0.667
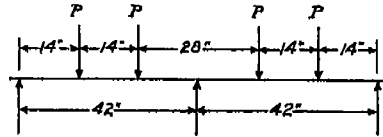
	Major beam				Minor beam		
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	I/C	Per cent			Lbs. per sq. in.	
1-DF	6,700	1,333	10.4	1-DF	0.549	12,780	1,044
8-DF	6,310	1,333	12.5	8-DF	.451	11,020	1,170
6-DF	5,800	1,340	10.6	6-DF	.468	11,070	1,086
8-DF	6,325	•1,347	12.3	8-DF	.526	13,660	.870
9-DF	6,450	1,333	10.4	9-DF	.516	12,570	1,012
12-DF	4,760	1,333	10.4	12-DF	.478	12,140	.696
13-DF	4,900	1,333	12.0	13-DF	.493	11,460	.789
14-DF	4,025	1,333	12.5	14-DF	.488	12,240	.729
15-DF	4,675	1,620	12.8	15-DF	.434	10,720	.822
16-DF	5,350	1,327	12.2	16-DF	.445	10,250	1,043
17-DF	7,200	1,333	12.0	17-DF	.588	16,110	.841
18-DF	6,750	1,327	11.7	18-DF	.619	16,340	.756
19-DF	5,725	1,333	11.5	19-DF	.509	12,890	.833
20-DF	6,175	1,333	13.1	20-DF	.750	11,750	1,046
21-DF	7,350	1,333	12.4	21-DF	.598	16,320	.849
Av.	5,960	1,333	11.8	----------	.527	12,755	.902

Calculated uniform section moment ratio............ 0.667

	Major beam				Minor beam		
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	I/C	Per cent			Lbs. per sq. in.	
2-ES	5,675	1,340	11.4	2-ES	0.421	10,650	1,058
4-ES	6,025	1,333	7.8	4-ES	.450	14,200	.781
5-ES	6,500	1,347	9.9	5-ES	.499	12,540	1,012
6-ES	5,850	1,333	9.0	6-ES	.437	12,250	.921
8-ES	7,355	1,333	9.6	8-ES	.454	13,200	1,180
9-ES	6,975	1,333	9.6	9-ES	.430	12,540	1,127
10-ES	5,650	1,347	8.4	10-ES	.409	11,940	.896
12-ES	4,775	1,333	8.8	12-ES	.444	11,560	.752
Av.	6,101	1,337	9.8	----------	.443	12,360	.960

TABLE III.—RECTANGULAR SITKA SPRUCE BEAMS, APPROXIMATELY 2 BY 2 INCHES OR 1.4 BY 2.5 INCHES IN CROSS SECTION, CONTINUOUS OVER TWO SPANS. THE CALCULATED MAXIMUM MOMENT IN THE SPAN IS EITHER 0.393 OR 0.556 OF THAT AT THE CENTER SUPPORT FOR THE LOADINGS SHOWN
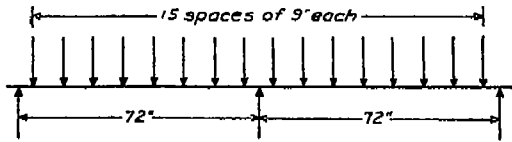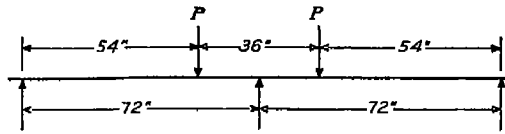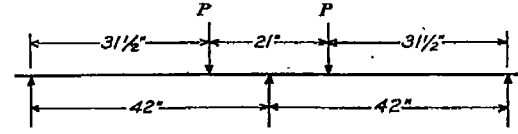

Diagram 3-A

Diagram 3-B

Diagram 3-C

Calculated uniform section moment ratio _____ 0.556

	Major beam				Minor beam		
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
D-1	2,000	1.490	9.7	D-1	0.427	7,680	0.376
D-2	2,575	1.340	8.8	D-2	.442	8,720	.560
D-3	3,000	1.490	8.6	D-3	.459	8,740	.594
D-4	2,300	1.333	9.3	D-4	.496	8,610	.475
D-5	2,675	1.490	9.5	D-5	.472	8,490	.521
D-6	2,825	1.333	8.4	D-6	.472	9,140	.600
D-7	2,450	1.490	12.6	D-7	.309	6,470	.706
D-8	2,040	1.380	13.5	D-8	.310	6,580	.603
D-15	4,950	1.333	9.8	D-15	.450	12,050	.902
D-18	3,075	1.327	18.6	D-18	.419	9,310	.668
D-19	3,500	1.327	14.6	D-19	.430	9,040	.836
A-1	2,400	1.314	10.9	A-1	.435	7,040	.710
A-2	3,685	1.300	9.5	A-2	.438	10,420	.771
A-3	3,425	1.293	9.8	A-3	.420	11,450	.600
A-4	4,225	1.307	10.5	A-4	.430	9,700	.998
D-29	3,580	1.155	11.0	D-29	.392	10,510	.846
D-30	3,620	1.167	12.2	D-30	.455	10,925	.808
D-31	3,425	1.167	11.7	D-31	.450	10,800	.757
D-9	3,165	1.447	10.4	D-9	.314	7,880	.781
D-11	3,130	1.459	11.8	D-11	.324	7,590	.800
D-13	2,500	1.427	9.8	D-13	.308	7,380	.622
D-40	2,675	1.479	9.2	D-40	.324	7,340	.596
D-41	2,560	1.333	10.3	D-41	.326	7,570	.686
D-42	2,725	1.480	9.9	D-42	.324	7,480	.655
D-16	5,200	1.425	------	D-16	.454	12,000	.886
D-46	4,825	1.333	10.4	D-46	.452	13,020	.783
D-47	3,450	1.333	9.9	D-47	.460	13,550	.440
D-48	3,000	1.333	10.0	D-48	.469	13,780	.331
D-51	3,420	1.314	14.4	D-51	.364	7,710	1.016
D-52	3,070	1.314	14.1	D-52	.359	7,540	.909
D-55	4,200	1.320	13.7	D-55	.450	10,770	.849
D-56	4,230	1.320	14.0	D-56	.450	11,400	.793
D-59	3,950	1.320	11.8	D-59	.442	12,090	.662
D-60	4,000	1.327	12.3	D-60	.446	11,585	.712
D-63	3,035	1.327	11.6	D-63	.436	13,500	.354
D-64	4,350	1.327	12.0	D-64	.482	11,890	.773
D-65	3,840	1.333	11.2	D-65	.476	11,420	.681
Av.	3,326	1.349	11.1	------	.417	9,775	.694

Calculated uniform section moment ratio _____ 0.393

	Major beam				Minor beam		
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
D-20	2,525	1.327	14.3	D-20	0.421	9,030	0.673
D-21	2,675	1.327	12.6	D-21	.416	9,560	.673
D-22	2,640	1.268	12.2	D-22	.409	8,890	.831
D-23	2,575	1.268	12.5	D-23	.382	7,860	.994
D-24	2,675	1.280	12.8	D-24	.389	8,400	.931
D-25	2,460	1.268	12.3	D-25	.368	7,970	.894
D-26	2,410	1.268	11.5	D-26	.369	8,120	.830
D-10	2,065	1.327	12.5	D-10	.318	7,500	.650
D-12	1,860	1.333	12.5	D-12	.317	6,900	.616
D-27	2,650	1.167	11.7	D-27	.386	10,630	.692
D-28	2,625	1.180	11.7	D-28	.392	10,750	.648
D-32	2,590	1.333	13.1	D-32	.462	11,210	.421
D-33	2,375	1.333	14.4	D-33	.424	10,130	.437
D-36	2,395	1.480	10.2	D-36	.330	7,780	.652
D-37	1,915	1.333	11.8	D-37	.331	7,770	.498
D-38	1,920	1.468	10.6	D-38	.333	7,380	.447
D-39	1,920	1.333	11.9	D-39	.320	7,850	.488
D-43	3,900	1.347	10.9	D-43	.470	12,610	.798
D-44	4,040	1.347	11.0	D-44	.455	12,640	.850
D-34	3,060	1.293	13.8	D-34	.412	9,450	.942
D-35	2,920	1.294	12.2	D-35	.420	9,310	.886
D-53	2,880	1.300	13.8	D-53	.365	7,800	.834
D-54	2,360	1.300	14.3	D-54	.369	7,680	.846
D-57	3,050	1.320	14.1	D-57	.466	10,940	.674
D-58	3,185	1.320	14.1	D-58	.477	10,840	.753
D-61	3,050	1.333	12.3	D-61	.466	11,350	.611
D-62	2,965	1.333	12.1	D-62	.453	10,770	.643
Av.	2,636	1.314	12.5	------	.397	9,301	.712

Calculated uniform section moment ratio _____ 0.393

	Major beam				Minor beam		
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Moment factor
	Pounds	*I/C*	*Per cent*			*Lbs. per sq. in.*	
C-110	5,140	1.333	14.0	C-109-111	0.440	9,540	0.841
C-112	5,890	1.333	13.0	C-111	.452	10,790	.864
C-113	5,550	1.354	13.0	C-109	.423	9,310	.988
C-115	5,115	1.333	13.9	C-116	.435	10,150	.738
C-118	5,745	1.333	14.5	C-117-119	.434	9,610	1.015
C-120	6,055	1.327	13.0	C-119	.436	10,910	.896
Av.	5,582	1.336	13.6	------	.437	10,052	.890

TABLE IV.—RECTANGULAR SITKA SPRUCE BEAMS, APPROXIMATELY 2 BY 2 INCHES IN CROSS SECTION, CONTINUOUS OVER TWO SPANS, WITH THE END SUPPORTS AT A LOWER LEVEL THAN THE CENTER SUPPORT. THE CALCULATED MAXIMUM MOMENT IN THE SPAN AVERAGES FROM 0.077 TO 0.360 OF THAT AT THE CENTER SUPPORT, FOR THE THREE LOADING CONDITIONS SHOWN
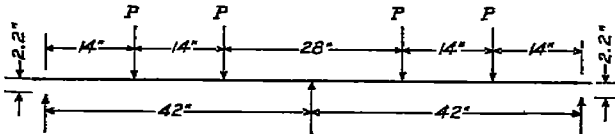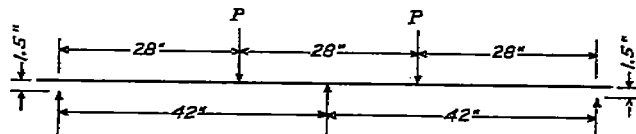

Diagram 4-A

Diagram 4-B

	Major beam				Minor beam				
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Modulus of elasticity times I	Calculated uniform section moment ratio [1]	Moment factor
	Pounds	I/C	Per cent			Lbs. per sq. in.	EI		
C-145	4,210	1.333	12.4	C-145	0.443	10,410	2,286,000	0.050	0.730
C-147	4,290	1.333	12.3	C-146-148	.422	10,180	2,306,000	.031	.774
C-157	4,065	1.340	10.5	C-158	.426	10,920	2,392,000	.059	.639
C-159	3,575	1.360	12.8	C-158-160	.462	9,500	2,516,500	—.062	.602
C-162	4,035	1.313	12.2	C-161-163	.444	8,290	2,241,000	—.105	.964
C-164	2,880	1.333	13.0	C-163	.440	7,280	2,195,000	—.186	.713
C-189	4,325	1.347	13.3	C-190	.408	10,400	2,242,000	.063	.746
C-191	4,630	1.347	13.1	C-190-192	.407	10,540	2,221,000	.072	.808
C-217	4,175	1.328	14.3	C-218	.406	10,020	2,177,000	.057	.764
C-219	3,960	1.320	14.6	C-218-220	.398	10,140	2,123,000	.074	.702
C-221	3,330	1.320	13.7	C-220	.390	10,890	2,069,000	.131	.477
C-269	5,200	1.333	13.0	C-269	.480	13,140	2,798,000	.069	.705
C-270	3,350	1.333	11.7	C-270	.412	10,150	2,515,000	—.029	.533
C-271	4,275	1.333	13.5	C-271	.393	10,210	2,582,000	—.044	.766
C-272	3,525	1.333	13.9	C-272	.369	9,240	2,166,000	.122	.669
C-273	2,410	1.333	11.2	C-273	.360	9,810	1,808,000	.124	.312
C-274	1,900	1.313	11.2	C-274	.359	9,590	1,958,000	.085	.195
C-275	2,175	1.326	11.3	C-275	.367	8,420	2,005,000	.060	.281
C-276	2,475	1.320	11.0	C-276	.380	9,810	1,735,000	.038	.336
C-277	4,375	1.306	12.1	C-277	.461	12,220	2,495,000	.080	.625
C-278	4,575	1.313	12.1	C-278	.491	13,520	2,676,000	.107	.568
C-279	5,350	1.306	12.5	C-279	.491	11,330	2,490,000	.052	.931
C-290	5,175	1.320	11.9	C-280	.493	11,970	2,630,000	.044	.813
Av.	3,837	1.328	12.5		.423	10,388	2,289,300	.077	.637

	Major beam				Minor beam				
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Modulus of elasticity times I	Calculated uniform section moment ratio [1]	Moment factor
	Pounds	I/C	Per cent			Lbs. per sq. in.	EI		
C-197	2,870	1.333	13.3	C-198	0.406	9,420	1,760,000	0.103	0.401
C-199	3,160	1.333	14.5	C-198-200	.405	9,540	1,634,000	.139	.492
C-201	2,440	1.333	13.6	C-200	.404	9,520	1,508,000	.189	.231
C-207	3,880	1.333	13.3	C-208	.417	9,830	1,725,000	.127	.714
C-209	3,395	1.333	14.8	C-206-210	.416	9,340	1,700,000	.115	.606
C-211	3,075	1.333	14.0	C-210	.415	9,900	1,675,000	.071	.422
C-212	4,400	1.347	13.8	C-213	.417	9,170	1,745,000	.100	.996
C-214	2,525	1.333	14.8	C-213-215	.414	8,670	1,814,000	.062	.353
C-216	2,675	1.333	13.7	C-215	.411	9,510	1,882,000	.078	.317
C-222	4,265	1.333	12.9	C-223	.411	10,600	2,274,000	.127	.809
C-224	4,010	1.320	14.5	C-223-225	.416	10,190	2,231,000	.027	.724
C-226	4,105	1.314	13.0	C-225	.422	11,410	2,178,000	.038	.610
C-227	3,290	1.354	14.7	C-228	.412	10,150	1,990,000	.113	.450
C-229	3,455	1.360	15.4	C-228-230	.421	9,520	1,710,000	.130	.219
C-231	3,385	1.347	14.9	C-230	.430	9,520	1,530,000	.167	.492
C-232	3,015	1.354	14.1	C-233	.418	9,260	1,658,000	.122	.678
C-234	3,545	1.354	15.0	C-233-235	.410	8,890	1,834,000	.057	.708
C-236	3,400	1.333	14.5	C-235	.402	9,170	2,080,000	.018	.631
Av.	3,327	1.333	14.1		.414	9,645	1,828,200	.101	.547
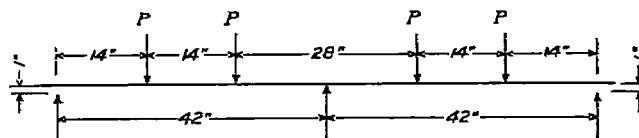
Diagram 4-C

	Major beam				Minor beam				
Number	Maximum load	Section modulus	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Modulus of elasticity times I	Calculated uniform section moment ratio [1]	Moment factor
	Pounds	I/C	Per cent			Lbs. per sq. in.	EI		
C-166	3,110	1.333	11.5	C-165-167	0.467	8,650	2,619,000	0.277	0.620
C-168	3,025	1.333	11.7	C-167	.448	7,390	2,343,000	.263	.740
C-169	3,325	1.333	11.9	C-170	.442	7,800	2,230,000	.302	.785
C-171	3,705	1.333	12.5	C-170-172	.440	7,660	2,116,000	.314	.937
C-182	4,950	1.333	14.7	C-181-183	.418	9,820	2,300,000	.367	.992
C-184	4,740	1.333	14.5	C-183	.418	9,550	2,242,000	.368	.969
C-185	4,800	1.333	14.4	C-186	.392	9,160	2,232,000	.355	1.042
C-187	4,655	1.347	15.0	C-186-188	.405	8,860	2,248,000	.347	1.032
C-194	5,150	1.333	13.4	C-193-195	.433	10,760	2,174,000	.410	.922
C-196	4,925	1.333	13.8	C-195	.445	11,020	2,188,000	.414	.840
C-202	3,415	1.333	15.6	C-203	.387	7,720	1,561,000	.408	.830
C-204	3,640	1.333	17.2	C-203-205	.376	8,250	1,591,000	.421	.820
C-206	4,210	1.333	18.9	C-205	.366	8,840	1,621,000	.433	.918
Av.	4,127	1.334	14.2		.419	8,878	2,112,700	.360	.881

[1] When the supports are not on a level, the calculated ratio of maximum moment in the span to moment at the support changes as the amount of load changes. The ratio given in this column was obtained by first solving for P by substituting in the equation

$$-PX_1-\frac{3EI\Delta}{L^3}=MRXI/C$$

where X_1 is some function of the span, depending on the loading, and Δ is the subsidence of the end supports, and then substituting the value of P in the equation

$$RL-PX_1=MRXI/C$$

where X_2 is some function of the span, depending on the loading, and R is an end reaction. Knowing R, the maximum moment in the span can be determined; the maximum moment divided by $(MRXI/C)$ gives the moment factor.

Assuming, for instance, the loading shown in Diagram 1–A of Table I, and using the following symbols:

C = distance from the neutral axis to the extreme fiber.

I = moment of inertia of the cross section of the beam.

L = distance between supports.

M = maximum moment.

P = concentrated load.

R = reaction at an end support.

S = normal unit stress on a fiber.

the usual equation of three moments gives the following results:

$$\text{Moment at center support} = -\frac{14}{27}PL$$

Maximum moment in the span is at the single P loads and equals

$$+\frac{22}{81}PL$$

The maximum load, therefore, would be predicted by this method from the equation

$$\frac{14}{27}PL = M = \frac{SI}{C}$$

from which

$$P = \frac{27}{14}\frac{SI}{CL}$$

and

$$\text{Total load} = 6P = 11.57\frac{SI}{CL}$$

In the proposed method, however, the calculation proceeds as follows:

The calculated ratio of the maximum moment in the span to that at the center support is

$$\frac{22}{81}PL \div \frac{14}{27}PL = 0.524$$

The moment at the center support is

$$RL - \frac{4}{3}PL$$

and the maximum moment in the span, which is under the single P loads, is

$$\frac{RL}{3}$$

From Figure 3, for a calculated ratio of 0.524 within the elastic limit, the moment factor at failure may be taken as 0.735; that is, the moment at the center support multiplied by 0.735 equals the maximum moment in the span, at maximum load. Since these two moments are opposite in sign the equation expressing the relation is

$$0.735 \left(RL - \frac{4}{3}PL\right) = -\frac{RL}{3}$$

or

$$R = 0.917\ P$$

The moment at the center support is then equal to

$$0.917\ PL - \frac{4}{3}PL = -0.416\ PL$$

and the maximum load would be estimated by

$$0.416\ PL = M = \frac{SI}{C}$$

from which

$$P = \frac{SI}{0.416\ CL}$$

$$\text{Total load} = 6P = 14.42\frac{SI}{CL}$$

This load is $\frac{14.42}{11.57}$ or 124.7 per cent of that obtained by the method in common use.

Twenty-six beams were tested with this loading; the results of the tests are given in Table I. The average actual load was 41 per cent greater than that obtained through calculation with the use of the ordinary equation of three moments. This exceptional increase is accounted for by the fact that practically all of the material was of the highest quality. Tables I, II, III, and IV show the results of tests with other combinations of loads and other species of wood under similar loading.

Transversely loaded continuous beams reinforced at the supports.

The conventional I beam in an airplane wing is left unrouted at the strut points to accommodate fittings and to resist the high shear stresses usually present there. For the same reason box beams have filler blocks at these points. Some designers treat such spars as if they were routed throughout their entire length; the results of some 60 tests of routed beams, on the other hand, show that this method gives estimated maximum loads far below what can actually be obtained. Further, if the principle of the shift in point of contraflexure after the elastic limit has been passed is applied to the problem, a method of calculation that presents itself checks actual test loads within very narrow limits.

For a beam of uniform cross section, Figure 3 shows the relation of maximum moment in any given span to the moment at the support, for the span selected, that has the greater moment. In other words, then, for such beams the moment factor taken from the chart tells to what fraction of its capacity to resist stress the section in any given span is stressed when the section at the major support for that span is offering its maximum resistance. If this last idea is extended to include beams that are reinforced at the supports, maximum load can be estimated rather accurately. The procedure is as follows:

Considering the beam to be of uniform cross section throughout its length, first compute the relation of maximum moment in any span to that at the adjoining support having the greater moment, by the usual equation of three moments. Then take from Figure 3 the moment factor for the relation thus computed. This moment factor tells to what fraction of its capacity the section in the span is stressed when the reinforced section at the support reaches its ultimate capacity.

The tests showed this procedure to be safe. In test, when the maximum capacity of the reinforced section at the support was reached, the moment in

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the span actually had already exceeded the product of the moment factor and resisting capacity of the section at that point. This was probably due to the fact that a solid section will bend to a sharper curvature at maximum load than a routed section of the same exterior dimensions.

The results of several series of tests on I beams tested over two spans and left unrouted for varying distances at the center support are given in Tables V, VI, and VII. These data show that the method of calculation is accurate, with a slight margin on the side of safety.

TABLE V.—SITKA SPRUCE I BEAMS, CONTINUOUS OVER TWO SPANS, WITH TWO CONCENTRATED LOADS IN EACH SPAN
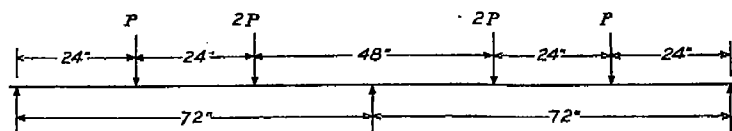

Diagram 5

2 BY 3.5 INCH I BEAMS WITH ⅝-INCH FLANGES

| Major beam | | | | Minor beam | | | Type of failure | Computed shear stress | Maximum load | | | Routing |
Number	Section modulus in span	Section modulus at support	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture			Actual	By old method	By new method	
	Z_r	Z_s	Per cent			Lbs. per sq. in.		Lbs. per sq. in.	Pounds	Pounds	Pounds	
C-291	3.10	3.89	6.6	C-292	0.411	12,840	Shear	1,405	4,760	4,470	6,950	Unrouted for 16 inches at center support; rest ½-inch web.
C-293	3.14	3.88	6.5	C-292-4	.411	12,210	...do	1,360	4,660	4,310	6,590	Do.
C-295	3.10	3.88	6.4	C-294	.411	11,660	...do	1,475	5,085	4,070	6,290	Do.
C-296	3.13	3.95	10.2	C-297	.462	14,030	...do	1,355	4,920	4,940	7,710	Do.
C-298	3.16	3.16	8.9	C-297-9	.461	14,650	Flange split	720	2,815	5,210	6,480	Routed entire length; ½-inch web.
C-300	3.12	3.95	9.6	C-299	.460	13,900	Shear	1,580	5,600	4,880	7,640	Unrouted for 16 inches at center support; rest ½-inch web.
C-301	3.16	3.97	8.3	C-302	.426	8,390	Brash [1]	980	3,600	2,980	4,640	Unrouted for 18 inches at center support; rest ½-inch web.
C-303	3.16	3.93	8.1	C-302-4	.410	8,790	...do	555	1,910	3,125	4,805	Do.
C-305	3.14	3.97	7.8	C-304	.394	9,230	...do	920	3,450	3,260	5,105	Do.

1.4 BY 2.5 INCH I BEAMS WITH ¼-INCH FLANGES

| Major beam | | | | Minor beam | | | Type of failure | Computed shear stress | Maximum load | | | Routing |
Number	Section modulus in span	Section modulus at support	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture			Actual	By old method	By new method	
C-306	1.250	1.466	10.1	C-307	.411	9,710	Shear	975	2,265	1,472	2,160	Unrouted for 6 inches at center support; rest ½-inch web.
C-308	1.240	1.459	11.0	C-307-9	.413	9,710	Buckling	950	2,160	1,460	2,150	Do.
C-310	1.230	1.456	10.1	C-309	.415	10,570	Shear	795	1,850	1,578	2,335	Do.
C-311	1.256	1.459	12.9	C-312	.433	9,600	Buckling	670	2,425	1,472	2,140	Unrouted for 6 inches at center support; then ¾-inch web for 24 inches; rest ½-inch web.
C-313	1.245	1.446	12.9	C-312-14	.437	9,800	Shear	815	2,100	1,480	2,150	Unrouted for 6 inches at center support; rest ½-inch web.
C-315	1.265	1.459	11.7	C-314	.441	11,700	...do	890	2,710	1,793	2,590	Unrouted for 6 inches at center support; then ⅝-inch web for 24 inches; rest ½-inch web.
C-316	1.257	1.459	12.0	C-317	.455	11,240	...do	785	2,825	1,712	2,490	Unrouted for 6 inches at center support; then ¾-inch web for 24 inches; rest ½-inch web.
C-318	1.243	1.446	10.1	C-317-19	.453	11,630	...do	890	2,255	1,752	2,560	Unrouted for 6 inches at center support; rest ½-inch web.
C-320	1.230	1.436	11.2	C-319	.451	10,450	...do	870	2,585	1,558	2,280	Unrouted for 6 inches at center support; then ⅝-inch web for 24 inches; rest ½-inch web.

The calculated uniform section moment ratio for this loading is 0.524.
The moment factor from Figure 8 is 0.735.
The form factor for the 2 by 3.5 inch beams is 0.70.
The form factor for the 1.4 by 2.5 inch beams is 0.754.

[1] The material in C-301, C-303, and C-305 was not acceptable. The toughness averaged about 32, whereas the recommended minimum is 75.

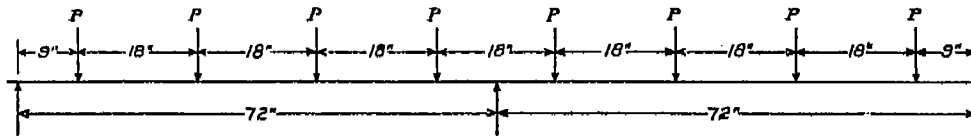# TABLE VI.—SITKA SPRUCE I BEAMS, CONTINUOUS OVER TWO SPANS, WITH FOUR CONCENTRATED LOADS IN EACH SPAN

Diagram 6

1.4 BY 2.5 INCH I BEAMS WITH ¼-INCH FLANGES

Major beam				Minor beam				Computed shear stress	Maximum load			Routing
Number	Section modulus in span	Section modulus at support	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture	Type of failure		Actual	By old method	By new method	
	Z_1	Z_2	Per cent			Lbs. per sq. in.		Lbs. per sq. in.	Pounds	Pounds	Pounds	
C-321	1.24	1,448	8.4	C-322	0.414	12,440	Shear	1,090	3,255	2,505	3,525	Unrouted for 5 inches at center support; rest ¼-inch web.
C-323	1.264	1,438	10.2	C-322-4	.418	11,600	do	830	3,000	2,550	3,395	Unrouted for 5 inches at center support; rest ⅝-inch web.
C-325	1.235	1,438	8.6	C-324	.423	12,850	do	1,105	3,190	2,580	3,615	Unrouted for 5 inches at center support; rest ¼-inch web.
C-326	1.215	1,414	9.0	C-327	.365	9,080	Compression[1]	680	2,465	1,785	2,498	Unrouted for 6 inches at center support; ⅝-inch web to 10½ inches each side of center; rest ¼-inch web.
C-328	1,255	1,459	11.5	C-327-9	.371	8,555	do	850	2,515	1,745	2,440	Unrouted for 6 inches at center support; rest ¼-inch web.
C-330	1,235	1,435	10.4	C-329	.377	9,220	do	665	2,730	1,850	2,585	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-331	1,230	1,425	10.0	C-332	.452	11,880	do	1,005	3,485	2,363	3,295	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-333	1,226	1,447	12.3	C-332-4	.455	10,390	do	1,055	3,040	2,071	2,940	Unrouted for 6 inches at center support; rest ¼-inch web.
C-335	1,240	1,448	7.2	C-334	.459	13,000	Shear	780	3,030	2,620	3,680	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-336	1,235	1,447	8.7	C-337	.484	14,940	do	1,105	3,810	3,000	4,225	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-338	1,243	1,436	9.1	C-337-9	.490	15,880	do	1,155	3,415	3,105	4,320	Unrouted for 6 inches at center support; rest ¼-inch web.
C-340	1,214	1,416	9.1	C-339	.496	16,100	do	990	4,170	3,180	4,460	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-341	1,230	1,435	10.0	C-342	.421	11,790	do	970	3,325	2,355	3,310	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-343	1,237	1,435	9.5	C-342-4	.428	11,700	do	1,005	2,940	2,350	3,282	Unrouted for 6 inches at center support; rest ¼-inch web.
C-345	1,246	1,446	9.7	C-344	.436	11,200	do	900	3,625	2,265	3,170	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-346	1,215	1,415	9.4	C-347	.405	10,550	do	765	3,100	2,085	2,920	Do.
C-348	1,227	1,435	9.5	C-347-0	.426	11,110	do	1,030	2,925	2,213	3,115	Unrouted for 6 inches at center support; rest ¼-inch web.
C-350	1,212	1,413	8.7	C-340	.448	12,140	Compression[1]	785	3,540	2,390	3,355	Unrouted for 6 inches at center support; 1-inch web to 10½ inches each side of center; rest ¼-inch web.
C-351	1,234	1,435	9.5	C-352	.432	11,610	Shear	845	3,480	2,328	3,260	Unrouted for 6 inches at center support; ¾-inch web to 10½ inches each side of center; rest ¼-inch web.
C-353	1,233	1,425	9.2	C-352-4	.426	11,940	do	1,005	2,875	2,393	3,325	Unrouted for 6 inches at center support; rest ¼-inch web.
C-355	1,233	1,425	9.8	C-354	.419	11,690	Compression[1]	850	3,850	2,342	3,260	Unrouted for 6 inches at center support; 1-inch web to 10½ inches each side of center; rest ¼-inch web.

The calculated uniform section moment ratio for this loading is 0.595.
The moment factor from Figure 8 is 0.775.
The form factor for the sections out in the span is 0.754 except for beam C-328 where, with a ¾-inch web, it becomes 0.81.

[1] By compression failure is meant a compression failure followed by a tension failure.

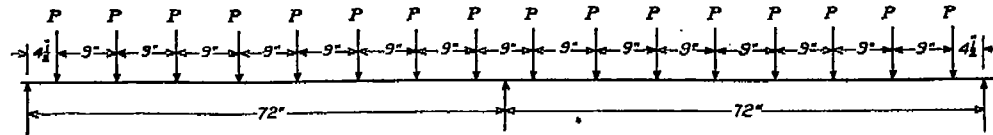# TABLE VII.—SITKA SPRUCE I BEAMS, CONTINUOUS OVER TWO SPANS, WITH EIGHT CONCENTRATED LOADS IN EACH SPAN

Diagram 7

1.4 BY 2.5 INCH I BEAMS WITH ½-INCH FLANGES

Major beam				Minor beam			Type of failure	Computed shear stress	Maximum load			Routing
Number	Section modulus in span	Section modulus at support	Moisture content	Number	Specific gravity based on weight and volume when oven dry	Modulus of rupture			Actual	By old method	By new method	
	Z_r	Z_s	Per cent			Lbs. per sq. in.		Lbs. per sq. in.	Pounds	Pounds	Pounds	
C-356	1.254	1.458	8.4	C-357	0.386	11,210	Compression[1]	1,165	3,840	2,330	3,320	Unrouted for 6 inches at center; then taper web to ¼ inch at 13½ inches from center; rest ½-inch web.
C-358	1.253	1.460	9.0	C-357-9	.394	11,020	---do---	885	3,280	2,295	3,270	Unrouted for 6 inches at center; then 1-inch web to 4½ inches from center; then ¾-inch web to 13½ inches from center; rest ½-inch web.
C-360	1.255	1.460	8.0	C-359-61	.398	11,050	Shear	1,025	3,650	2,300	3,272	Unrouted for 6 inches at center; then ¾-inch web to 4½ inches from center; then ⅝-inch web to 13½ inches from center; rest ½-inch web.
C-362	1.260	1.470	8.5	C-361	.393	10,120	Compression[1]	1,240	3,825	2,120	3,020	Unrouted for 6 inches at center; then taper to ½-inch web at 13½ inches from center; rest ½-inch web.
C-363	1.256	1.458	7.6	C-364	.394	11,350	---do---	1,265	3,700	2,370	3,360	Do.
C-365	1.262	1.470	7.4	C-364-6	.402	11,900	Shear	1,130	3,860	2,495	3,550	Unrouted for 6 inches at center; then 1-inch web to 4½ inches from center; then ¾-inch web to 13½ inches from center; rest ½-inch web.
C-367	1.265	1.470	7.3	C-366-8	.406	12,100	Compression[1]	1,130	3,860	2,540	3,610	Unrouted for 6 inches at center; then ¾-inch web to 4½ inches from center; then ⅝-inch web to 13½ inches from center; rest ½-inch web.
C-369	1.268	1.470	8.4	C-368	.401	11,150	---do---	1,200	3,650	2,350	3,325	Unrouted for 6 inches at center; then taper to ½-inch web at 13½ inches from center; rest ½-inch web.
C-370	1.246	1.447	8.7	C-371	.437	13,070	---do---	1,545	4,750	2,710	3,840	Unrouted for 9 inches at center; rest ¼-inch web.
C-372	1.239	1.447	9.1	C-371-3	.440	12,990	---do---	1,395	4,225	2,675	3,818	Unrouted for 6 inches at center; then taper to ½-inch web at 13½ inches from center; rest ½-inch web.
C-374	1.244	1.447	8.7	C-373-5	.444	13,120	---do---	1,305	4,110	2,710	3,855	Unrouted for 9 inches at center; rest ¼-inch web.
C-376	1.241	1.447	8.7	C-375	.447	12,880	---do---	1,435	4,450	2,060	3,788	Unrouted for 6 inches at center; then taper to ½-inch web at 13½ inches from center; rest ½-inch web.
C-377	1.228	1.425	8.4	C-378	.501	6,960	Brash	795	3,000	1,420	2,015	Unrouted for 30 inches at center; rest ¼-inch web.
C-379	1.255	1.458	8.2	C-378-80	.510	8,450	---do---	650	2,400	1,763	2,502	Unrouted for 6 inches at center; then taper to ½-inch web at 15 inches from center; rest ½-inch web.
C-384	1.246	1.447	7.3	C-385	.443	13,520	Compression[1]	1,125	4,280	2,800	3,975	Unrouted for 30 inches at center; rest ¼-inch web.
C-386	1.246	1.447	7.9	C-385-7	.444	13,340	---do---	1,145	4,090	2,760	3,918	Unrouted for 6 inches at center; then taper to ½-inch web at 15 inches from center; rest ½-inch web.
C-388	1.252	1.458	7.6	C-387-9	.446	13,710	---do---	1,210	4,650	2,850	4,060	Unrouted for 30 inches at center; rest ½-inch web.
C-390	1.246	1.447	7.8	C-389	.446	13,600	---do---	1,200	4,490	2,815	3,998	Unrouted for 6 inches at center; then taper to ¼-inch web at 15 inches from center; rest ½-inch web.
C-391	1.219	1.415	9.5	C-392	.463	10,160	Brash	685	2,310	2,055	2,920	Unrouted for 30 inches at center; rest ½-inch web.
C-393	1.232	1.426	11.1	C-392-4	.452	9,990	Compression[1]	720	3,675	2,040	2,895	Unrouted for 53 inches at center; rest ½-inch web.
C-395	1.226	1.426	11.0	C-394-6	.434	10,300	X grain	670	3,460	2,100	2,980	Unrouted for 76 inches at center; rest ½-inch web.
C-397	1.214	1.426	10.0	C-396	.427	10,470	---do---	770	3,950	2,115	3,035	Unrouted for 99 inches at center; rest ½-inch web.

The material in C-377, C-379, and C-391 was very poor and would not pass the toughness requirement.
The calculated uniform section moment ratio for this loading is 0.556.
The moment factor from Figure 3 is 0.753.
The form factor for the section in the span is 0.754.

[1] By compression failure is meant a compression failure followed by tension failure.

As already stated, the first steps in the calculation for maximum load are to compute the relation of the maximum moment in the span selected to that at the adjoining support having the greater moment, and then to determine from Figure 3 what fraction of the capacity in the span is developed when the section over the support is stressed to its maximum. The method should be applied, in turn, to every span in the beam under investigation. In order to treat span by span a beam that extends over several supports, it is necessary to learn to what fraction of its capacity the section at the other support for the same span is stressed. This fraction is determined in exactly the same way as the fraction for the section in the span. From the relation of the moment at this support to that at the support with the greater moment as it is ordinarily calculated, determine from Figure 3 what fraction of the capacity of the section at the secondary support will be developed when the section at the major support is stressed to a maximum. These relations can be conveniently expressed algebraically as follows:

Assuming a uniform cross section and using the ordinary equation of three moments:

Let M_a = moment at support a.

M_b = moment at support b.

M_{ab} = maximum moment in span ab.

M_b be greater than M_a.

Z_a = section modulus at a.

Z_b = section modulus at b.

Z_{ab} = section modulus in span ab.

F_a = form factor at support a.

F_b = form factor at support b.

F_{ab} = form factor in span ab.

S = modulus of rupture.

The ratio $\dfrac{M_{ab}}{M_b}$ determines the moment factor K_{ab}, which is taken from Figure 3, and the ratio $\dfrac{M_a}{M_b}$ determines the moment factor K_a.

The moment developed for the three points at maximum load may then be written

$$M_a = F_a S Z_a K_a$$

$$M_b = F_b S Z_b$$

$$M_{ab} = F_{ab} S Z_{ab} K_{ab}$$

With these moments known, any span can be treated separately, and the load that will give these relations can thus be determined.

To illustrate the application further, consider a simple case, running through the actual calculations. Take, for example, beam C-326 in Table VI, loaded as shown in the diagram for this table.

Form factor in span = 0.754.

Form factor at center support = 1.00.

Modulus of rupture = 9,030 pounds per square inch.

Section modulus in span = 1.215.

Section modulus at support = 1.414.

For a beam of uniform cross section and using the ordinary equation of three moments:

$$\text{Maximum moment in span} = +\frac{157}{512} PL$$

$$\text{Moment at center support} = -\frac{33}{64} PL$$

The calculated moment ratio therefore is

$$\frac{157}{512} PL \div \frac{33}{64} PL = 0.595.$$

For this ratio the moment factor from Figure 3 is 0.775.

Maximum resisting moment at the center support
= 9,030 × 1.414 = 12,760 inch-pounds.

Maximum moment developed in the span at failure
= 0.754 × 9,030 × 1.215 × 0.775
= 6,410 inch-pounds.

Taking moments about the center support,

$$72R - 144P = -12,760 \text{ inch-pounds}$$

The maximum moment in the span occurs at the second load from the end reaction and is

$$27R - 18P = +6410 \text{ inch-pounds}$$

Solving these two equations,

$$R = 445 \text{ pounds}$$

and substituting in the first equation,

$$P = 311 \text{ pounds}$$

and

$$\text{Total load } 8P = 2488 \text{ pounds}$$

The slight discrepancy between this value and that given in the table is accounted for by the fact that in obtaining the values in the table nominal dimensions were used for the calculation of the section moduli.

The actual test load on this beam was 2,465 pounds, which agrees with the calculated load within 1 per cent. The load estimated by the usual equation of three moments was 1,785 pounds, which is 28 per cent lower than the test load. The real accuracy of the proposed method, however, can be checked only by an

examination of those beams in Tables V, VI, and VII that failed in bending rather than in shear. Such an examination will show that, even though the calculated loads are much higher than those estimated by the ordinary equation of three moments, they are safe. In no case, except the one used in the preceding example, did the beam fail to support the expected load.

Continuous beams under combined axial and transverse loading.

In applying the proposed method of calculation to continuous members subjected to combined loading, the first step is to determine where the points of contraflexure will be when the maximum load is reached. If these points are known, consideration of the problem may be restricted to the portion of the beam between two successive points of contraflexure or between a point of contraflexure and a hinged end, with a resulting simplification of the solution for each span. This solution will then involve merely the solution of a column under combined loading. Correct procedure, however, requires bearing in mind the fact that when the section at the support is stressed to its maximum capacity, the section out in the portion of the span suspended between two points of contraflexure has to be limited to a certain fraction of its stress capacity. Since these relations exist, it is necessary only to investigate conditions out in the span. Such procedure will usually give a slight factor of safety, since the moment introduced by the axial load is usually less at the support than in the span.

As previously stated, the relation of moments for transverse load only is calculated by means of the usual equation of three moments, with the assumption that the cross section of the beam is uniform throughout. Moment factors are then taken from Figure 3, and considering one span at a time the positions of the points of contraflexure are located. The positions of the points of contraflexure so determined will not coincide exactly with their position at maximum load under combined loading, but calculations and

tests have shown that the differences are so small that they can be neglected with safety.

Sections between points of contraflexure can now be investigated separately but, as already suggested, checking the section out in the span suspended between two points of contraflexure is sufficient. Under combined loading that section is limited to a certain fraction of its stress capacity. For transverse load only this fraction is the moment factor but, with axial loads present, the fraction of stress capacity may be increased as follows:

Let K = moment factor.

$\quad \rho$ = ratio of direct stress to total stress.

U = maximum load modulus at the ratio ρ. Then the design modulus for the portion of the beam out in the span is

$$KU + \rho(U - KU) \qquad (1)$$

The application of the preceding principles can be best illustrated by following through an actual calculation. For convenience, let

W = maximum total actual transverse load on the entire beam.

W' = fictitious total transverse load.

R_h = reaction at the hinge.

X = distance from the hinge to the point of contraflexure in the inboard span.

Then, for example, consider the first beam in Table VIII, No. CC–2.

Maximum total load W = 5,565 pounds

The maximum moment in the span between the hinge and the inboard strut occurs at the second load from the hinge (fig. 2). For a transverse load only and a uniform section, the ordinary equation of three moments shows it equal to

$$+1.62W$$

TABLE VIII.—RESULTS OF TESTS ON BEAMS HAVING A 2-BAY AND OVERHANG TRUSS ARRANGEMENT WITH A 47¼-INCH HEIGHT OF TRUSS

I BEAMS.—NOMINAL DIMENSIONS; 2 BY 4½ INCHES WITH 1-INCH FLANGES AND ¼-INCH WEBS

Beam number	Maximum load W	Moisture content	Specific gravity	Toughness	I_r	C	Z_s	Z_r	A_r	Web thickness	Form factor	Moment factor	Suspended span	Transverse load moment	Axial load P	Bending stress S'	Direct stress P/A	Actual total stress S=S'+P/A	Expected stress by new method	S'/S	Modulus of rupture	Compression parallel to grain	Modulus of elasticity	Maximum load modulus	Remarks
	Lbs.	P. ct.		Inch-pounds	Inches⁴	Inches	Ins.³	Ins.³	Square inches	Inch			Inches	Inch-pounds	Pounds	Lbs. per sq. in.	Lbs. per sq. in.	Lbs. per sq. in.	Lbs. per sq. in.		Lbs. per sq. in.	Lbs. per sq. in.	1,000 lbs. per sq. in.	Lbs. per sq. in.	
OO-2	5,565	12.1	0.331	87	13.58	2.250	6.75	6.04	5.90	0.755	0.77	.633	57.65	9,955	12,980	2,280	2,370	4,600	4,416	0.485	8,130	4,558	1,371	5,873	
OO-3	5,060	12.1	.308	55	13.57	2.250	6.75	6.03	5.89	.750	.77	.633	57.65	9,050	12,710	2,097	2,160	4,257	4,183	.492	7,920	4,170	1,140	5,106	
OO-4	5,315	12.3	.335	68	13.48	2.245	6.72	6.00	5.87	.753	.77	.633	57.65	9,510	13,350	2,175	2,275	4,450	4,427	.489	8,370	4,420	1,264	5,397	
OO-5	6,000	11.6	.322	101	13.40	2.240	6.69	5.98	5.88	.754	.77	.633	57.65	10,735	15,080	2,512	2,575	5,087	4,886	.494	7,900	4,680	1,364	5,359	
OO-6	5,310	12.4	.320	75	13.68	2.250	6.78	6.06	5.90	.752	.77	.633	57.65	9,500	13,350	2,112	2,260	4,372	4,802	.483	7,930	4,390	1,311	5,228	
OO-7	5,470	12.6	.324	86	13.40	2.240	6.69	5.98	5.86	.754	.77	.633	57.65	9,790	13,750	2,277	2,350	4,627	4,282	.492	7,890	4,428	1,260	5,225	
OO-8	7,290	12.5	.430	111	13.23	2.235	6.66	5.92	5.78	.740	.77	.633	57.65	13,050	18,400	3,012	3,182	6,194	5,730	.486	10,420	5,730	1,795	6,811	
OO-9	7,060	13.3	.402	155	13.16	2.235	6.63	5.89	5.76	.740	.77	.633	57.65	12,680	17,750	2,848	3,082	5,930	5,611	.480	10,880	5,415	1,895	6,811	
OO-10	5,240	12.9	.396	70	13.20	2.235	6.63	5.91	5.79	.740	.77	.633	57.65	9,370	13,170	2,165	2,274	4,439	5,185	.487	10,000	5,040	1,291	6,315	Compression wood.
OO-11	4,545	13.4	.416	66	13.27	2.235	6.66	5.94	5.80	.740	.77	.633	57.65	8,135	11,430	1,777	1,970	3,747	4,945	.474	9,495	4,840	1,301	5,998	Do.
OO-12	6,855	11.7	.428	118	13.24	2.240	6.64	5.91	5.81	.740	.77	.633	57.65	12,270	17,225	2,812	2,965	5,777	5,967	.487	10,960	6,190	1,720	7,265	Do.
OO-13	8,565	10.0	.456	198	13.24	2.240	6.64	5.91	5.81	.740	.77	.633	57.65	9,955	13,980	2,040	2,406	4,446	7,802	.459	13,270	7,620	2,097	8,788	Fitting failure.
OO-17	7,780	9.6	.413	118	13.34	2.240	6.68	5.95	5.82	.748	.77	.633	57.65	13,925	19,550	3,404	3,357	6,761	6,137	.504	11,880	5,925	1,620	7,532	
OO-18	7,400	9.8	.425	93	13.19	2.235	6.62	5.90	5.80	.750	.77	.633	57.65	13,250	18,590	3,200	3,203	6,403	5,485	.500	10,560	5,990	1,640	7,039	

BOX BEAMS.—NOMINAL DIMENSIONS; 2 BY 4½ INCHES WITH 1-INCH FLANGES; THE WEB THICKNESS GIVEN IS THE COMBINED THICKNESS OF TWO WEBS

Beam number	Maximum load W	Moisture content	Specific gravity	Toughness	I_r	C	Z_s	Z_r	A_r	Web thickness	Form factor	Moment factor	Suspended span	Transverse load moment	Axial load P	Bending stress S'	Direct stress P/A	Actual total stress S=S'+P/A	Expected stress by new method	S'/S	Modulus of rupture	Compression parallel to grain	Modulus of elasticity	Maximum load modulus	Remarks
OO-14	4,260	11.5	0.336	100	10.42	2.225	5.46	4.68	4.04	0.650	0.75	0.633	56.85	7,375	10,700	2,075	2,649	4,724	4,854	0.439	8,900	5,115	1,436	5,790	Filler block too short at strut point.
OO-15	3,640	10.2	.336	100	10.47	2.225	5.50	4.70	4.05	.648	.75	.633	56.85	6,318	9,145	1,682	2,259	3,941	5,115	.427	9,290	5,415	1,459	6,067	Failed laterally.
OO-16	3,880	10.4	.330	100	10.48	2.235	5.48	4.69	4.00	.624	.75	.633	56.85	6,736	9,745	1,827	2,435	4,262	5,186	.428	9,225	5,610	1,461	6,158	Filler block too short at strut point.
OO-19	5,100	8.1	.363	121	12.05	2.250	6.38	5.36	4.05	.217	.67	.633	54.40	8,365	12,820	1,973	3,164	5,137	5,294	.384	10,220	5,740	1,573	6,159	Two-ply spruce webs.
OO-20	5,135	8.0	.363	121	11.81	2.250	6.23	5.25	4.08	.310	.69	.633	55.10	8,550	12,900	2,113	3,161	5,274	5,497	.401	10,250	6,040	1,506	6,445	Do.
OO-21	5,190	8.3	.363	121	11.50	2.245	6.08	5.12	4.10	.406	.71	.633	56.30	8,875	13,050	2,231	3,182	5,413	5,436	.412	10,440	5,710	1,679	6,405	Do.
OO-22	4,840	7.8	.363	121	11.22	2.245	5.88	5.00	4.09	.503	.73	.633	56.40	8,290	12,150	2,150	2,070	5,130	5,172	.421	9,480	5,550	1,549	6,118	Do.

The specific gravity is based on the weight and the volume of the wood when it is oven dry.

I_r = moment of inertia of routed section.

C = ½ height.

Z_s = section modulus $\left(\dfrac{I}{C}\right)$ of unrouted or filled section.

Z_r = section modulus $\left(\dfrac{I}{C}\right)$ of routed section.

A_r = area of routed section.

The maximum load modulus was calculated from a separate chart made for each beam in accordance with the properties of the material in the beam.

The calculations are for the panel from the hinge to the first strut.

The calculations were made with a slide rule.

At the inboard strut the moment under the same conditions is

$$-4.68W$$

Moment ratio for a uniform section $=\dfrac{1.62W}{4.68W}=0.346$

Moment factor from Figure 3 = 0.633

Maximum resisting capacity at the strut

    = modulus of rupture × section modulus × form factor   (2)

    = 8130 × 6.75 × 1.00 = 54,900 inch-pounds.

Maximum resisting capacity in the span

    = modulus of rupture × section modulus × form factor × moment factor   (3)

    = 8130 × 6.04 × 0.77 × 0.633 = 23,940 inch-pounds.

To determine where the point of contraflexure will be at failure, considering transverse load only, it is necessary to determine what fictitious total transverse load W' on the entire beam will cause the preceding moments, and what the hinge reaction is.

Taking moments about the inboard strut and remembering that each load in the inboard span is 0.06786 of the maximum total load (fig. 2), the following equation results:

$$81 R_h - 13.74 W' = -54,900$$

Taking moments about the second load from the hinge,

$$24.3 R_h - 1.099 W' = +23,940$$

Solving these two equations,

$$R_h = 1,591 \text{ pounds}$$

and

$$W' = 13,380 \text{ pounds}$$

To determine the position of the point of contraflexure, try a point between the load 56.7 inches from the hinge and the load 72.9 inches from the hinge.

$$0 = 1,591X - 0.06786 \times 13,380\,[(X-8.1) + (X-24.3)$$
$$+ (X-40.5) + (X-56.7)]$$
$$X = 57.65 \text{ inches.}$$

Under combined loading, therefore, the member is 57.65 inches long from the hinge to the point of contraflexure, with four concentrated loads on it. The axial load on it is equal to 2.512W; the factor 2.512 is determined by the height of the truss.

$$2.512W = 2.512 \times 5,565 = 13,980 \text{ pounds}$$

Let M = total maximum bending moment in the span.

M' = maximum bending moment caused by transverse load.

P = axial load.

S = total unit stress of combined transverse and axial loads.

S' = total bending stress.

Then by Johnson's formula, in which P is the axial load,

$$S' = \frac{M'C}{I - \dfrac{PL^2}{9.6E}}\qquad(4)$$

The maximum moment resulting from transverse load only, for the four concentrated loads on the 57.65-inch span, is determined thus:

$$57.65\,R_h = 0.06786 \times 5,565 \times 101 \qquad (\text{fig. 2})$$

$$R_h = 661.5$$

and

$$661.5 \times 24.3 - 377.5 \times 16.2 = 9,955 \text{ inch-pounds}$$

Since the axial load P is 13,980 pounds and the modulus of elasticity E given in Table VIII is 1,371,000 pounds per square inch,

$$S' = \frac{9,955 \times 2.25}{13.58 - \dfrac{13,980 \times (57.65)^2}{9.6 \times 1,371,000}} = 2,230 \text{ pounds per sq. inch}$$

$$\frac{P}{A} = \frac{13,980}{5.90} = 2,370 \text{ pounds per sq. inch}$$

$$\text{Total stress } S = \overline{4,600} \text{ pounds per sq. inch}$$

The total stress of 4,600 pounds per square inch is the stress in the suspended span that was produced by the actual test load of 5,565 pounds, as determined in accordance with the method of calculation herein proposed. It is the stress that existed in the suspended span when failure occurred at the strut point. At failure,

$$\frac{S'}{S} = \frac{2,230}{4,600} = 0.485$$

Employing the proposed method of estimating the stress in the span when failure occurs at the strut point, and making the calculation without using the value of the actual test-load, the result is a stress somewhat smaller than 4,600 pounds per square inch; this fact substantiates again the assertion that the procedure is conservative for average material. The estimated stress in the span at which failure should occur at the strut joint is obtained as follows: For the modulus of rupture, form factor, and similar properties of this beam the maximum load modulus [4] at a ratio of S' to S of 0.485 is equal to 5373. Now the moment factor is 0.633 and, since 0.485 is the ratio of bending to total stress, $(1-0.485)$ is the ratio of direct to total stress. The design modulus is then calculated by means of formula (1), thus:

$$0.633 \times 5373 = 3400$$
$$(5373 - 3400)\,(1 - 0.485) = \underline{1016}$$
$$\text{Design modulus} = \overline{4416} \text{ pounds per square inch.}$$

Therefore, by means of the proposed method and using the properties as determined from this particular piece, the design modulus, which is the expected stress in the span when failure occurs at the strut point, is found to be 4,416 pounds per square inch. The actual load produced a stress of 4,600 pounds per square inch before the beam failed.

[4] National Advisory Committee for Aeronautics Report No. 188, Stresses in Wood Members Subjected to Combined Column and Beam Action, by J. A. Newlin and G. W. Trayer.

The stresses in the beam in the outboard bay can be calculated in the same manner when these stresses are near a doubtful value. Here they are sufficiently below the inboard bay stresses to make such calculation unnecessary.

In order to permit full appreciation of the effect of the quality of the material on the load that a continuous beam will sustain, a detailed discussion of all the combined loading tests seems worth while.

DISCUSSION OF THE INDIVIDUAL COMBINED LOADING TESTS

Agreement with the moment factor theory.

The first six beams in Table VIII, although low in specific gravity, were clear, straight-grained, and of uniform texture. Each beam carried more load than would be expected from substituting its own individual modulus of rupture, crushing strength, and modulus of elasticity in the proposed formula. As previously pointed out in this report, the recommended moment factors of Figure 3 represent what should be expected of border-line material, and of course material of average or higher toughness should give higher loads than the proposed formula would indicate.

Beams CC-3, CC-4, and CC-6 of Table VIII, because of their low toughness, sustained loads that produced stresses only slightly in excess of the allowable value recommended for design, while CC-2 and CC-7 with greater toughness show a greater difference between the allowable stress recommended for design and that produced by the actual test load. (For convenience, the ratio of the actual stress to the allowable design stress will hereafter be called the "improvement ratio.") CC-5, with a toughness that would place it in acceptable stock in spite of its low specific gravity, showed the greatest improvement ratio; it sustained a load equal to that which would be obtained by substituting recommended stresses for aircraft timber of Sitka spruce in the proposed formula.

Beams CC-8, CC-9, CC-17, and CC-18 range in specific gravity from the average of the species up. All carried loads producing stresses higher than those that would be expected from calculation by means of the proposed formula.

Beams CC-10, CC-11, and CC-12 contained some compression wood, and the results for them are therefore erratic. The material in these beams, although not acceptable for aircraft construction, was purposely used to show that it is unsuitable for continuous beams in spite of the fact that it is of high specific gravity. As a rule, compression wood is not uniformly distributed throughout the cross sections of a piece of timber, especially if the piece is of large size. Rather it is localized along certain annual growth rings, or it varies over the cross section in its degree of development. Hence, when a beam contains compression wood that occurs locally, the load that the beam will carry depends primarily upon whether the compression

wood happens to be at some point of high tensile stress. At failure, CC-12 was well up to the load indicated by the proposed formula, while CC-10 and CC-11 reached only about three-fourths of the formula load. In the test of CC-13 a fitting failed at the maximum load recorded in the table and the results, therefore, in no way represent the strength of the material.

Certain difficulties were encountered in the tests of box beams CC-14, CC-15, and CC-16, as explained in the "Remarks" column of the table. The results of these tests, therefore, can not be used as a check on the efficiency of the proposed method of calculation.

The other box beams, CC-19, CC-20, CC-21, and CC-22, were carefully matched, and all had 2-ply 45° Sitka spruce webs of a thickness that was different for each beam. The results show clearly the effect of using relatively thin webs. For web thicknesses of 0.10 inch and 0.15 inch the shear stresses in the webs were greater than the recommended stress for this type of construction and, although no shear failure occurred, wrinkling of the plywood took place and caused a reduction in the ultimate load. The actual loads for beams CC-19 and CC-20, therefore, produced maximum stresses less than the values that would be expected from the proposed method for the properties of the particular material used. For beams CC-21 and CC-22, the expected and the actual stresses are practically the same.

Table IX, in which are given the results of tests made after the ratio of axial load to transverse load had been increased, shows beams, CC-23, CC-24, and CC-33, of material above the average in quality. CC-23 and CC-24 have practically the same specific gravity and, considering that factor alone, should give about the same results. CC-23 with a toughness of 155, however, shows a better improvement ratio than CC-24 with its toughness of 120. On the other hand, CC-33 with a specific gravity even higher than these two and a toughness of 179 had a stress at failure only slightly in excess of that allowable. The usual failure in these tests was compression at the strut followed by complete failure in the span between the hinge and inboard strut. CC-33, however, failed at the inboard edge of the unrouted portion at the strut, indicating that the length of the unrouted portion, although about right for practically all other I beams, was slightly short for this one beam. With such a failure the beam would not be expected to surpass materially the load indicated by the formula.

The other six beams in Table IX were of low specific gravity. All except one show loads higher than those indicated by the proposed formula and all except this same one carry loads that compare favorably with the design load calculated by means of the proposed method and with the stresses recommended for Sitka spruce for aircraft service. This one beam, CC-29, has a very low toughness value and a very low specific gravity, which accounts for its behavior.

TABLE IX.—RESULTS OF TESTS ON BEAMS HAVING A 2-BAY AND OVERHANG TRUSS ARRANGEMENT WITH A 30-INCH HEIGHT OF TRUSS

I-BEAMS.—NOMINAL DIMENSIONS: 2 BY 4½ INCHES WITH 1-INCH FLANGES AND ¾-INCH WEBS

Beam number	Maximum load W	Moisture content	Specific gravity	Toughness	I_r	C	Z_s	Z_r	A_r	Web thickness	Form factor	Moment factor	Suspended span	Transverse load moment	Axial load P	Bending stress S'	Direct stress $\frac{P}{A}$	Actual total stress $S=S'+\frac{P}{A}$	Expected stress by new method	$\frac{S'}{S}$	Modulus of rupture	Compression parallel to grain	Modulus of elasticity	Maximum load modulus
	Pounds	*Per cent*		*Inch-pounds*	*Inches⁴*	*Inches*	*Inches³*	*Inches³*	*Sq. in.*	*Inch*			*Inches*	*Inch-pounds*	*Pounds*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*		*Lbs. per sq. in.*	*Lbs. per sq. in.*	*1,000 lbs. per sq. in.*	*Lbs. per sq. in.*
CC-23	6,500	9.0	0.427	155	12.99	2.225	5.53	5.84	5.79	0.768	0.77	0.633	57.65	11,040	25,890	2,300	4,470	7,770	6,988	0.425	12,160	7,530	1,744	8,280
CC-24	5,025	8.0	.430	120	13.08	2.225	5.59	5.88	5.80	.747	.77	.633	57.65	8,990	20,000	2,448	4,448	6,896	6,384	.415	9,300	5,805	1,408	6,327
CC-25	4,980	8.2	.348	105	13.06	2.225	5.56	5.87	5.81	.754	.77	.633	57.65	8,700	19,350	2,374	3,380	5,704	5,699	.416	9,980	6,160	1,384	6,725
CC-26	4,740	7.7	.353	77	13.11	2.225	5.56	5.88	5.84	.757	.77	.633	57.65	8,490	18,870	2,367	3,231	5,598	5,192	.423	9,360	5,445	1,272	6,148
CC-29	4,275	6.7	.320	69	13.07	2.225	5.56	5.87	5.88	.760	.77	.633	57.65	7,650	17,020	2,057	2,919	4,976	5,192	.413	8,985	5,670	1,229	6,121
CC-30	4,670	7.1	.328	78	13.06	2.225	5.56	5.87	5.82	.766	.77	.633	57.65	8,360	18,590	2,249	3,194	5,443	5,381	.413	9,490	5,760	1,344	6,343
CC-31	5,500	6.6	.344	87	12.99	2.225	5.53	5.84	5.80	.758	.77	.633	57.65	9,840	21,990	2,821	3,772	6,593	5,936	.428	10,750	6,215	1,448	7,045
CC-32	5,420	6.9	.349	71	13.06	2.225	5.56	5.87	5.87	.759	.77	.633	57.65	9,700	21,680	2,810	3,708	6,518	5,618	.431	9,845	6,035	1,389	6,648
CC-33	6,040	6.8	.462	179	12.64	2.205	5.41	5.73	5.73	.748	.77	.633	57.65	10,810	24,030	2,950	4,195	7,145	7,116	.413	12,640	7,535	1,826	8,886

The specific gravity is based on the weight and the volume of the wood when it is oven dry.

I_r = moment of inertia of routed section.

C = ½ height.

Z_s = section modulus $\left(\frac{I}{C}\right)$ of unrouted or filled section.

Z_r = section modulus $\left(\frac{I}{C}\right)$ of routed section.

A_r = area of routed section.

The maximum load modulus was calculated from a separate chart made for each beam in accordance with the properties of the material in the beam.

The calculations are for the panel from the hinge to the first strut.

The calculations were made with a slide rule.

Actual loads sustained.

For the design stress values of 9,400 pounds per square inch modulus of rupture, 5,000 pounds per square inch compressive strength, and 1,300,000 pounds per square inch modulus of elasticity adopted for spruce by the Federal Aeronautical Bureaus, each I beam of Table VIII, with its 47½-inch height of truss, should carry a load of about 5,900 pounds. These standard stresses are based on a moisture content of 15 per cent and 3 seconds duration of stress. The beams shown in Table VIII are of lower moisture content but the difference in the rate of loading should offset the difference in moisture content. Of the first six beams, all of which are of low specific gravity, only one reached this load. It is beam CC-5, which has a toughness of 101 inch-pounds. The standard design stresses for spruce are also based on a minimum specific gravity of 0.36 except that material with a toughness of 90 or more is acceptable at any value of specific gravity. CC-2 and CC-7 are just under this requirement of 90 and their loads are only 6 or 8 per cent below the design load. The other three beams are decidedly low in toughness as well as in specific gravity, and are correspondingly low in load.

Beams CC-8, CC-9, CC-17, and CC-18 are well up in strength properties and therefore they exceed, by a considerable margin, the design load that is based on the standard stress values for spruce.

Of the box beams, CC-14, CC-15, and CC-16 had filler blocks only 10 inches long at the inboard strut. Although preliminary tests of I beams had shown that, with an unrouted portion less than 14 inches long at this point, the full strength of the beam could not be developed, a block 4 inches shorter was tried in the box beams. Two of the beams failed at the ends of these blocks at loads less than those calculated and the other failed laterally before the load was great enough to cause failure at the end of the filler block.

In this connection, it should be borne in mind that box beams with 45° webs are less stiff laterally than beams with the parallel-perpendicular type of webs.

Beams CC-19, CC-20, CC-21, and CC-22 were alike except for web thickness and both their specific gravity and their toughness are acceptable. With these beams, however, the slowness of the rate of loading did not quite offset the difference in moisture content. The design load based on standard stresses is about 4,600 pounds and all four exceed that by enough to offset the lower moisture content when duration of stress is taken account of in spite of the fact that the maximum loads of beams CC-19 and CC-20 were reduced by the wrinkling of the excessively thin webs, as already explained.

This design load of 4,600 pounds is considerably less than the design load for the I beams, but the web thickness also is considerably less and in addition to

that only one-half the thickness of the plywood can be used in calculating the moment of inertia; the result is a considerable reduction in moment of inertia [5] for beams of the dimension used.

The results in Table IX are for a combination of axial load and transverse load that makes the direct stress about 60 per cent of the total stress. For this loading and the standard design stresses for Sitka spruce at 15 per cent moisture content, a maximum load of about 4,360 pounds would be expected.

Beams CC-23, CC-24, and CC-33, all of which are of material above the average, sustained loads enough in excess of 4,360 pounds to show that the low moisture content had compensated for the duration of the loading. The other seven beams were below the minimum specific gravity permissible. CC-25, however, would have been accepted because of its high toughness, and CC-31 and CC-32 are border-line material. All three gave acceptable loads. CC-26, CC-29, and CC-30 are the lowest in specific gravity, and their toughness is too low to make them acceptable in spite of their specific gravity. Their loads reflect the fact that they are not of acceptable quality.

EFFECT OF REVERSING THE STRESSES

A question may be raised as to the strength of a continuous beam under a reversal of stresses after it has once been subjected to a load approaching its maximum. A few tests were made to determine what might be expected. Three continuous beams and several standard bending-test specimens were cut from the same plank for comparison. It so happened that the material in the selected plank was exceptionally tough. In fact its average toughness for standard specimens was 165 inch-pounds, whereas the minimum requirement for aircraft spruce is 75. The loading used on these continuous beams was that illustrated in Table V. With this high toughness the true ratio of maximum moment in the span to moment at the support at failure was taken as 95 per cent, whereas the moment factor from Figure 3 is only 73½ per cent. On this basis it is reasonable to assume that the beam would carry 50 per cent more load than the value estimated by means of the usual equation of three moments. The recommended figure of 73½ per cent would give an increase of only 25 per cent. To illustrate the safety in reversing stresses, however, the higher figure is used in the following discussion.

The first beam was run to 112½ per cent of what should be expected from the old method of calculation,

which is three-fourths of what is expected from the new method. After this the load was reversed and was run up just as high. This double operation was repeated eight times. The beam was then run to failure and reached a load approximately 85 per cent of what was expected on the 95 per cent moment-factor basis.

For the next beam, the first run was 125 per cent of what would be expected from the equation of three moments, or five-sixths of what would be expected from the proposed method on the 95 per cent basis. The loads were then reversed and run up to two-thirds of what would be expected from the proposed method of calculation. This double operations was repeated three times. On the next trial, the five-sixths load was not reached.

The third beam was run up to five-sixths of what would be expected from the proposed method of calculation, on the 95 per cent true ratio basis, after which it was released and again run up to this same load. After 50 repetitions, the load was reversed and run up to two-thirds of the estimated load. A slight failure developed but, when the loads were again reversed to their original direction, the full estimated maximum load was obtained.

Now, how does this compare with the action of a beam simply supported? As compared with the first beam on the three-fourths load basis, a beam simply supported took 17 repetitions where the continuous beam took 8. With the second condition, of five-sixths normal load and two-thirds reversed load, a simple beam sustained 11 repetitions, whereas the continuous beam withstood but 3. For the third condition of repeating a five-sixths load fifty times, and then reversing, the simple beam stood the two-thirds reversed load as did the continuous beam and, like the continuous beam, reached its maximum load within 5 per cent when the loads were again applied in the original direction. Hence the recommended true ratio for design is conservative, and will give beams that will stand limited repeating and reversal of stress. Minimum toughness and specific gravity requirements will insure material with a fairly good improvement ratio as indicated by the moment factor, and the chances of poor tensile and poor compressive strength occurring at the same time with a low moment factor is remote.

CONCLUSIONS

The maximum transverse load that a continuous beam will sustain can not be determined with any reasonable degree of accuracy by using the modulus of rupture in the usual equation of three moments. Similarly, the maximum load stresses that are used to calculate the strength of a strut subjected to combined axial and transverse load can not be used in

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[5] Although the modulus of elasticity of 45° plywood is only about one-sixth of that for the species, the Forest Products Laboratory has recommended that one-half the thickness of 45° webs be used in calculating the moment of inertia, since such webs offer a great resistance to shear, thereby reducing distortion and providing a better distribution of stress across the flanges. Furthermore, in calculating the area by which the axial load is divided to give the direct stress, only one-half the plywood is included.

the generalized equation of three moments, which applies to continuous members under combined loading. The equation of three moments represents well the relation of moments for elastic conditions, but does not represent the true relation of moments after the elastic limit has been passed.

The usual incorrect procedure just described will yield estimated maximum loads that are considerably smaller than actual loads. The errors, therefore, are on the side of safety but they usually are too large to be neglected.

The method of calculation proposed in this report is not only simpler than those in common use but it also yields results that several hundred tests have shown to be accurate.

The principle upon which the proposed methods of calculation are built is a fundamental principle of mechanics and consequently is applicable to other materials as well as to wood.

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