NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 377

A METHOD OF FLIGHT MEASUREMENT OF SPINS

By HARTLEY A. SOULÉ and NATHAN F. SCUDDER

1931

AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Symbol	Unit	Symbol
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of one kilogram	kg	weight of one pound	lb.
Power	P	kg/m/s		horsepower	hp
Speed		km/h	k. p. h.	mi./hr.	m. p. h.
		m/s	m. p. s.	ft./sec.	f. p. s.

2. GENERAL SYMBOLS, ETC.

W, Weight $= mg$

g, Standard acceleration of gravity $= 9.80665$ m/s$^2 = 32.1740$ ft./sec.^{2}

m, Mass $= \dfrac{W}{g}$

ρ, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg-m^{-4} s^2) at 15° C. and 750 mm $= 0.002378$ (lb.-ft.$^{-4}$ sec.^{2}).

Specific weight of "standard" air, 1.2255 kg/m$^3 = 0.07651$ lb./ft.^{3}.

mk^2, Moment of inertia (indicate axis of the radius of gyration k, by proper subscript).

S, Area.

S_w, Wing area, etc.

G, Gap.

b, Span.

c, Chord.

$\dfrac{b^2}{S}$, Aspect ratio.

μ, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

V, True air speed.

q, Dynamic (or impact) pressure $= \dfrac{1}{2}\rho V^2$.

L, Lift, absolute coefficient $C_L = \dfrac{L}{qS}$

D, Drag, absolute coefficient $C_D = \dfrac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_0} = \dfrac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i} = \dfrac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p} = \dfrac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C = \dfrac{C}{qS}$

R, Resultant force.

i_w, Angle of setting of wings (relative to thrust line).

i_t, Angle of stabilizer setting (relative to thrust line).

Q, Resultant moment.

Ω, Resultant angular velocity.

$\rho\dfrac{Vl}{u}$, Reynolds Number, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord 40 m/s, the corresponding number is 274,000.

C_p, Center of pressure coefficient (ratio of distance of c. p. from leading edge to chord length).

α, Angle of attack.

ϵ, Angle of downwash.

α_o, Angle of attack, infinite aspect ratio.

α_i, Angle of attack, induced.

α_a, Angle of attack, absolute.

(Measured from zero lift position.)

γ, Flight path angle.

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Langley Memorial Aeronautical Laboratory

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

NAVY BUILDING, WASHINGTON, D. C.

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A METHOD OF FLIGHT MEASUREMENT OF SPINS

By Hartley A. Soulé and Nathan F. Scudder

SUMMARY

A method is described involving the use of recording turn meters and accelerometers and a sensitive altimeter, by means of which all of the physical quantities necessary for the complete determination of the flight path, motion, attitude, forces, and couples of a fully developed spin can be obtained in flight. Data are given for several spins of two training type airplanes which indicate that the accuracy of the results obtained with the method is satisfactory.

The method was developed by the National Advisory Committee for Aeronautics at Langley Field as a part of a general study of the phenomenon of spinning. It is now being used in an investigation to determine how the spinning characteristics of several airplanes are affected by various changes in their inertia and geometric characteristics. A study is being made to extend the method to include measurements during the entry and recovery from a spin as well as during the fully developed spin.

INTRODUCTION

The discovery of dangerous spinning characteristics in a number of airplanes in recent years has focused attention on the problem of spinning. That the knowledge concerning the causes and prevention of spins and recovery from them is still inadequate is indicated by the prevalence of poor and even dangerous spinning characteristics in airplanes designed in accordance with the best current design practice. This incompleteness of knowledge can not be attributed entirely to a lack of study of the problem, for much has been learned from wind-tunnel investigations. The work of wind-tunnel investigators, however, has usually been handicapped by a lack of quantitative data of the motion and attitude of a spinning airplane. Such data, obtained in flight, therefore would not only be useful in a direct study of the problem but would also be an aid to wind-tunnel investigations.

The National Advisory Committee for Aeronautics, in connection with a general research on the spinning problem, has developed a method of determining the motion of spinning airplanes. Considerable work has been done on this subject by others, but none of the previous investigations has been sufficiently complete to be satisfactory. The more important work of this nature is given in References 1, 2, 3, 4, and 5. Reference 1 gives a thorough analytical treatment of spinning, as well as some results of flight measurements.

References 2, 3, 4, and 5 give experimental results obtained in flight by different methods. The data in Reference 2 were obtained by visual observation, those in Reference 3 by recording instruments, and those in References 4 and 5 by cinematographic means. Although the observations of Reference 2 were visual, the work is important because the effect of progressive changes of certain properties of the airplanes was investigated, whereas, in the experiments of References 3, 4, and 5 no changes in characteristics of the airplane were made.

At the time the flight investigation of spinning was started by the National Advisory Committee for Aeronautics some special information was desired with as little delay as possible. The use of flying models built to scale seemed the most promising line of attack at the time, and since a report on this phase of the investigation has not been published, it will be briefly outlined. Two models, one a one-sixteenth scale model of a training seaplane (NB–1) and the other a one-twelfth scale model of an observation airplane (O2–E) were used. The momental ellipsoids, as well as the dimensions, were made to scale, as discussed in Reference 6. The models were launched in a spin, and photographic and visual observations were made from which the attitude, rate of rotation, and the vertical velocity were obtained.

The NB–1 model was arranged so that the mass distribution could be changed without varying the weight or the center of gravity location. This was done by placing two equal weights on a rod passing through the center of gravity along the Z axis. The distance between the weights was varied by sliding them along the rod in opposite directions to positions equally spaced from the center of gravity. The spin of the model was observed for several positions of the weights. With the weights close to the center of gravity the X axis was inclined downwards about 30° from the horizontal. As the weights were progressively moved away from the center of gravity the inclination of the X axis increased until an angle of about 60° was attained. With further displacements of the weights the model developed a tendency to recover against the controls and finally would not spin at all. Some enlargements of photographic records obtained with this model are shown in Figure 1. With O2–E model an attempt was made to determine the effect of varied tail sizes, wing arrangements, and
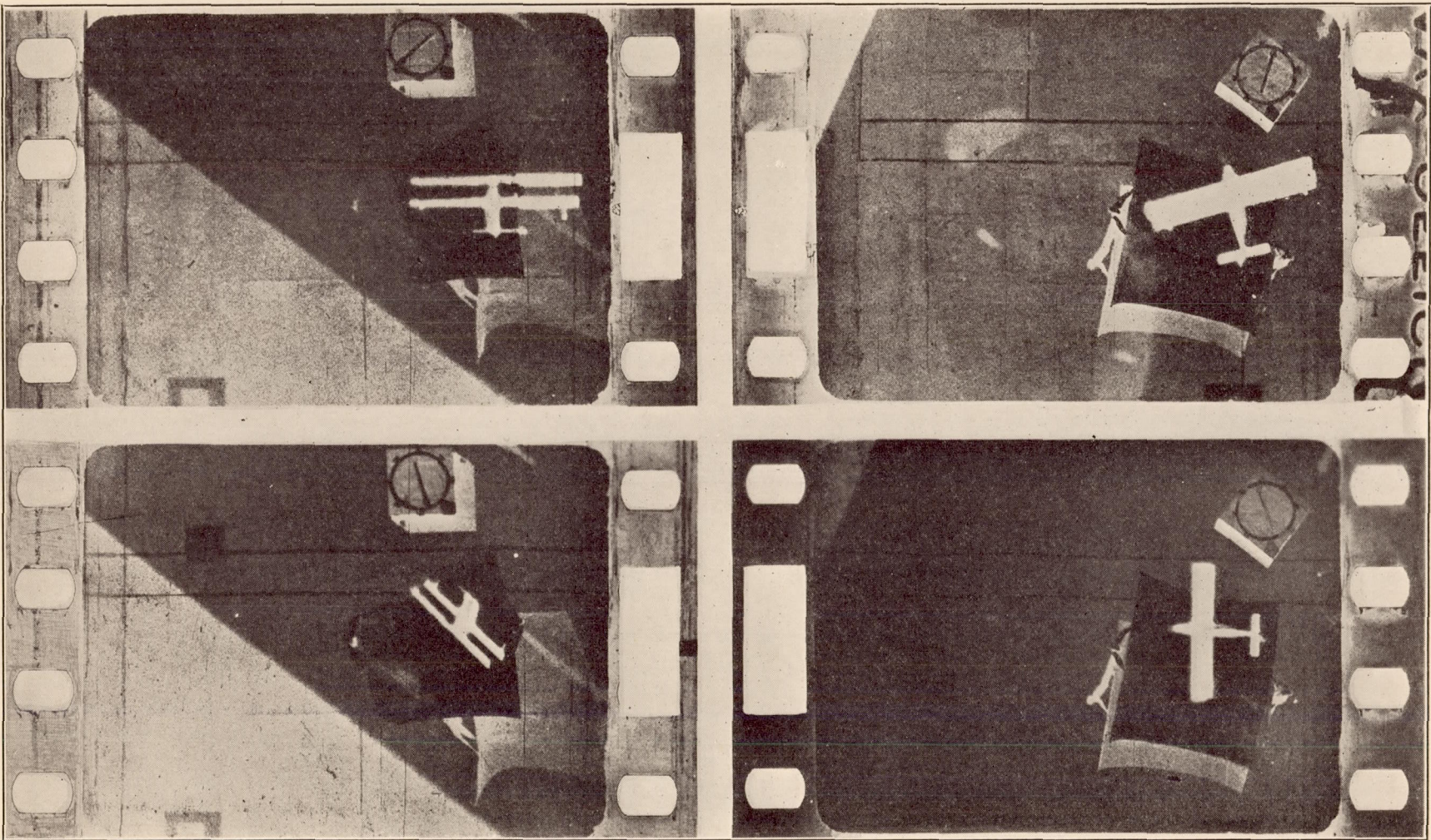
3

FIGURE 1.—Cinema films of NB–1 seaplane during spins

Nose down; movable weights far from *C. G.* along *Z* axis

Nose up; movable weights close to *C. G.* along *Z* axis

center of gravity locations. These latter tests, however, gave no important results.

The size of models used for tests such as those just described is limited by the height of free drop available. The tests were made in an airship hangar which was 105 feet high, and the largest practicable model span was about 3 feet. The small size was a serious limitation in that it prevented the mounting of instruments in the models. Furthermore, the results of such tests are subject to an unknown scale effect error. On account of these disadvantages in the flying model tests it was decided to continue the investigation with full-size airplanes.

A preliminary study of the problem indicated that many quantities would have to be measured to get a complete description of the motion of a spinning airplane, but that there was some choice in the combination of the quantities which were to be measured. Characteristics of the instruments at hand led to the selection of the following items for measurement: Angular velocities about the three airplane axes, accelerations along these axes, and the vertical velocity of the airplane. In addition to these quantities, record of the positions of the control surfaces and observations of engine speed were needed. Measurements of the moments of inertia of the airplane and of the propeller were likewise necessary in order that a complete determination of the forces and moments in a spin could be made.

The method of making these measurements and computing the results from the instrument records is the subject of this report. The results obtained with two airplanes in a trial of this method are included to illustrate it and show the accuracy obtained.

APPARATUS

Single component turn meters were used to record each of the three angular velocities about the airplane axes. The actuating mechanism of these instruments is an electrically driven gyroscope, and is fully described in Reference 7. A 3-component accelerometer, mounted at the c. g. of the airplane, recorded the components of acceleration along the airplane axes. This instrument depends for its operation on the deflection of spring-steel cantilever beams, and is completely described in Reference 8. The time was obtained by the use of an electric timer (Reference 9), which imposed time marks at 1-second intervals on the records.

Three separate methods for measuring altitude loss were tried, one using a camera obscura, one a recording statoscope, and one a special altimeter. The camera obscura method of measuring altitude loss is not entirely satisfactory, even though the results are quite accurate, because the maneuver must be made close to the ground; furthermore, the labor of working up the record is considerable. The recording statoscope

method was found to be inaccurate. A balancing type (Paulin) altimeter (Reference 10) was found to give results which checked the camera obscura values very closely, and since it was also easy to operate, it was selected for the tests. In this instrument the travel of the diaphragm is limited to only a few thousandths of an inch by stops. The pressure of the atmosphere on the diaphragm is balanced by a spring, the tension of which is adjustable by a hand-operated screw having a special pitch, and connected to a pointer indicating altitude in feet. When the tension of the spring balances the atmospheric pressure on the diaphragm the fact is shown by an indicator, and the position of the pointer then shows the altitude. The altimeter was mounted in the pilot's cockpit as shown in Figure 2.

A control position recorder (Reference 11) was used to record the positions of all three control surfaces. It was mounted as shown in Figure 2. Engine speed was obtained from the pilot's observations of the tachometer, which was part of the regular equipment of the airplane.

The recording instruments were controlled by two switches. One controlled the gyromotors of the turn meters and the other the film drum motors, source lights, and the timer. This arrangement permitted the gyromotors to be started in advance of the taking of records.

Vibration arising from the motor and other sources affected the recording accelerometer and turn meters. This trouble was lessened by mounting them, together with the batteries required for their operation, on a single panel which was in turn mounted on sponge rubber. Figure 3 shows the panel with the instruments in place.

In this investigation two training biplanes have thus far been used, the first a VE–7, and the second an NY–1. (See Figs. 4 and 5.) The important dimensions of these airplanes are as follows:

	VE-7	NY-1
Span, upper and lower	34 feet 1⅜ inches	34 feet 5⅝ inches.
Chord	4 feet 7½ inches	4 feet 6 inches.
Gap	4 feet 8 inches	4 feet 9 inches.
Stagger	1¼ inches	24 inches.
Dihedral	{1 degree 15 minutes	2 degrees 15 minutes.
	{1 degree 15 minutes	3 degrees.
Decalage	30 minutes	1 degree.
Airfoil	R. A. F. 15	U. S. A. 27.
Loading	7.7 pounds per square foot	8.5 pounds per square foot.
Weight during spin	2,190 pounds	2,390 pounds.
C. G. position	25 per cent M. A. C	25.8 per cent M. A. C.
Area, wings	284.5 square feet	282 square feet.
Length	24 feet 5⅜ inches	27 feet 9 inches.
Area, elevator	12.2 square feet	17.6 square feet.
Area, stabilizer	15 square feet	17.3 square feet.
Area, fin	1.77 square feet	6.6 square feet.
Area, rudder	5.77 square feet	13.0 square feet.
Area, ailerons	38.5 square feet	31.4 square feet.

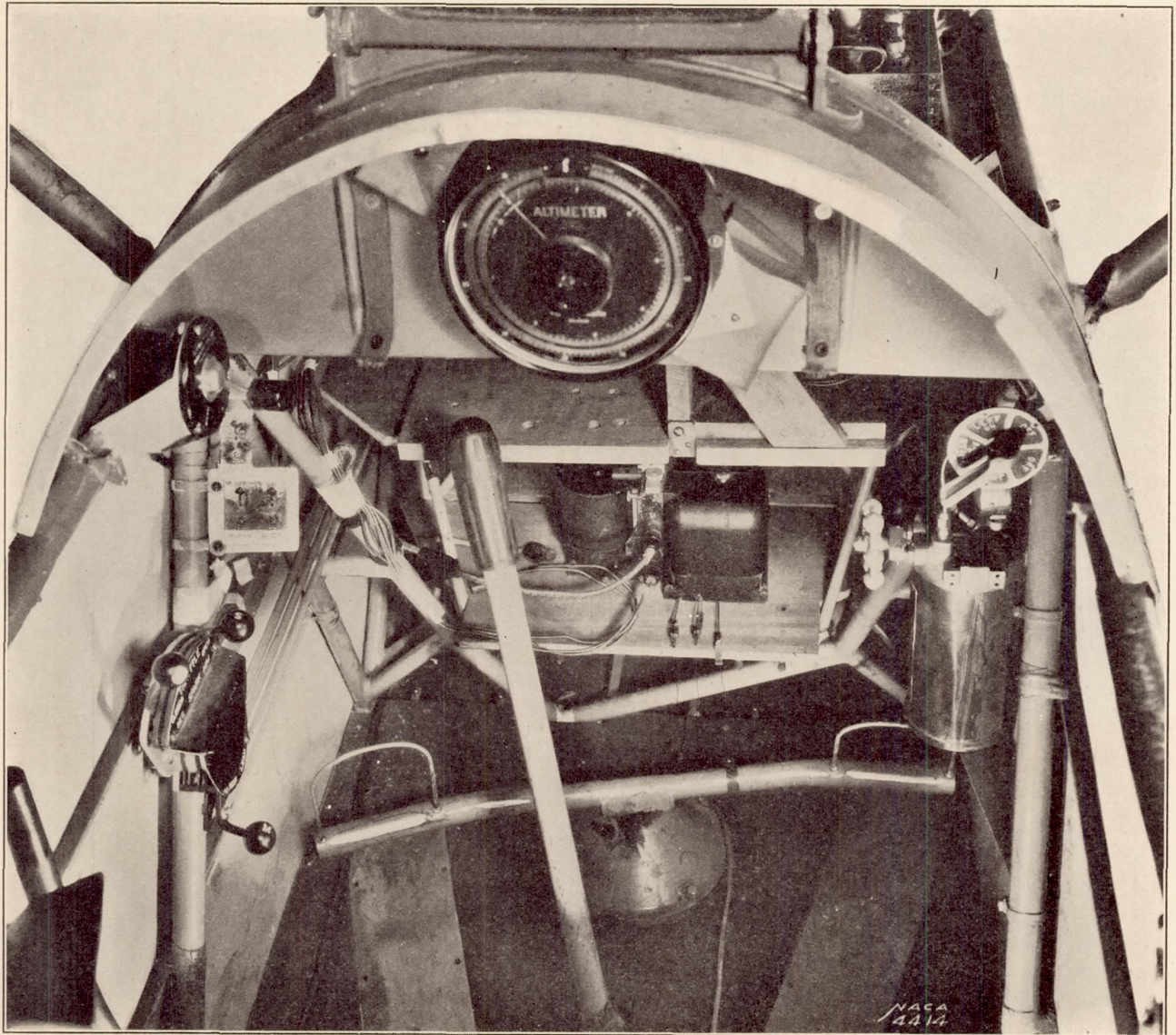|	VE-7	NY-1
Momental ellipsoid:		
A principal	1,676 slug feet ^{2}	2,380 slug feet.^{2}
B principal	1,586 slug feet ^{2}	2,567 slug feet.^{2}
C principal	2,342 slug feet ^{2}	3,887 slug feet.^{2}
Angle between the principal X and body X axes	−12 degrees 32 minutes	−1 degree 20 minutes.
Moment of inertia of propeller.	4.0 slug feet ^{2}	4.7 slug feet.^{2}

FIGURE 2.—Altimeter, control switches, and control-position-recorder in pilot's cockpit
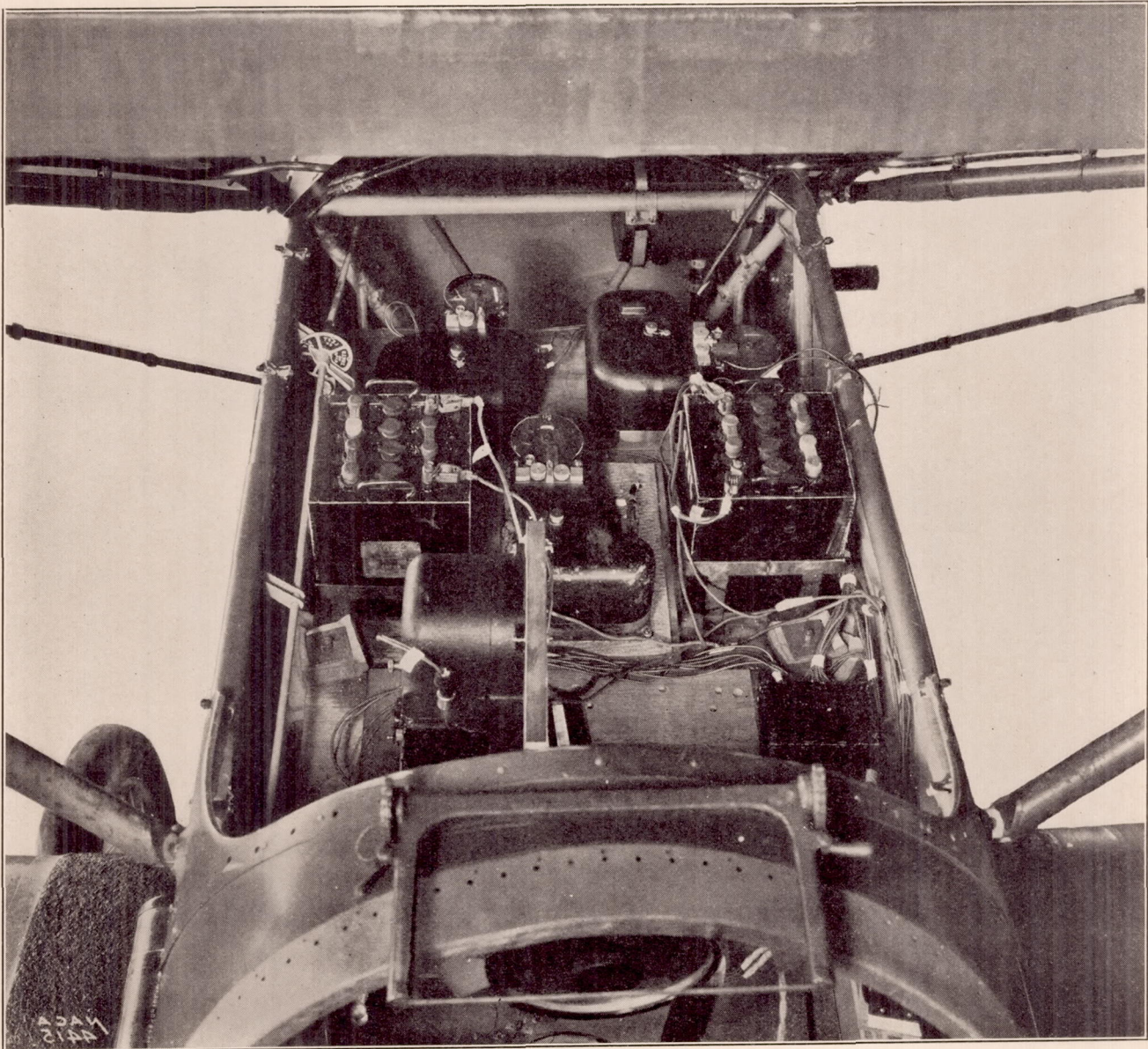
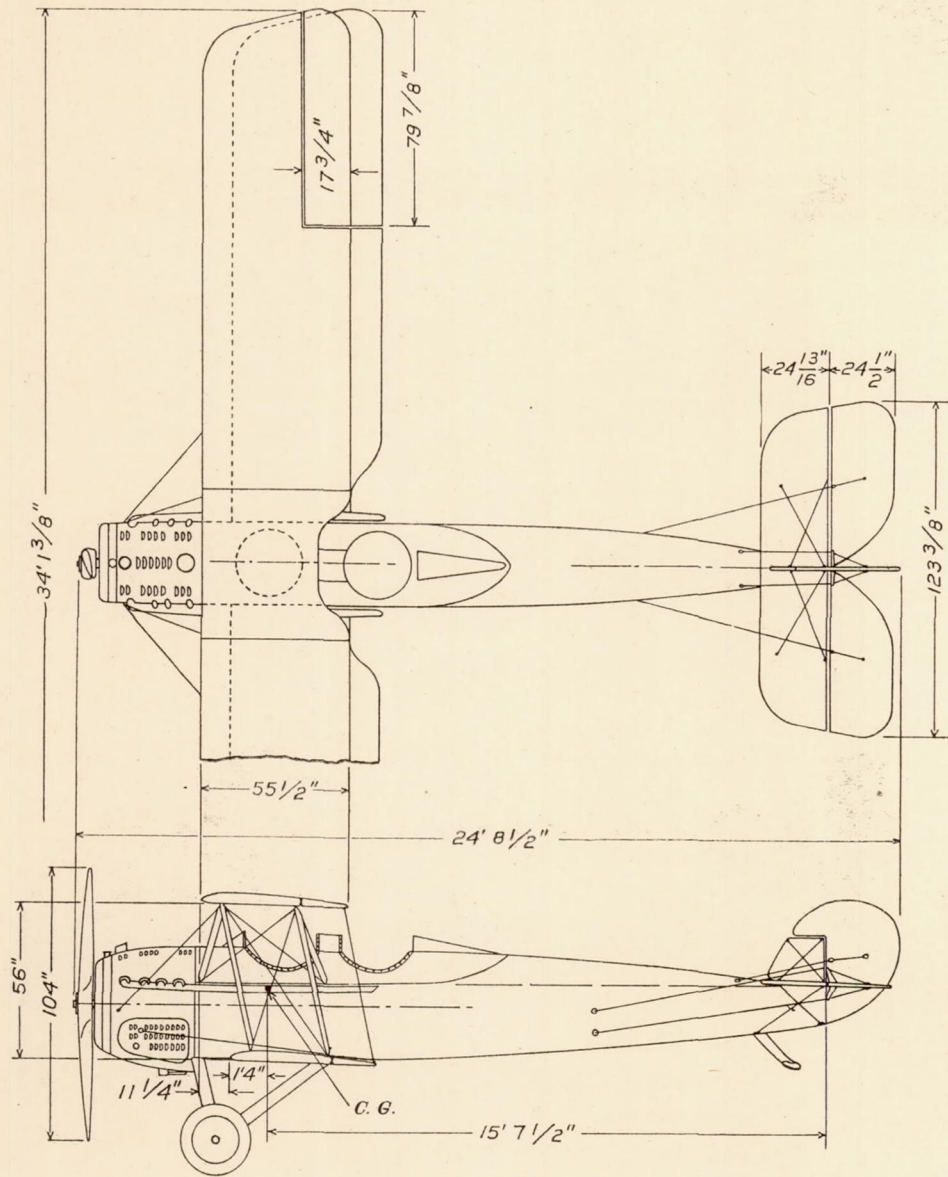FIGURE 3.—Instrument panel mounted in front cockpit of NY–1 airplane
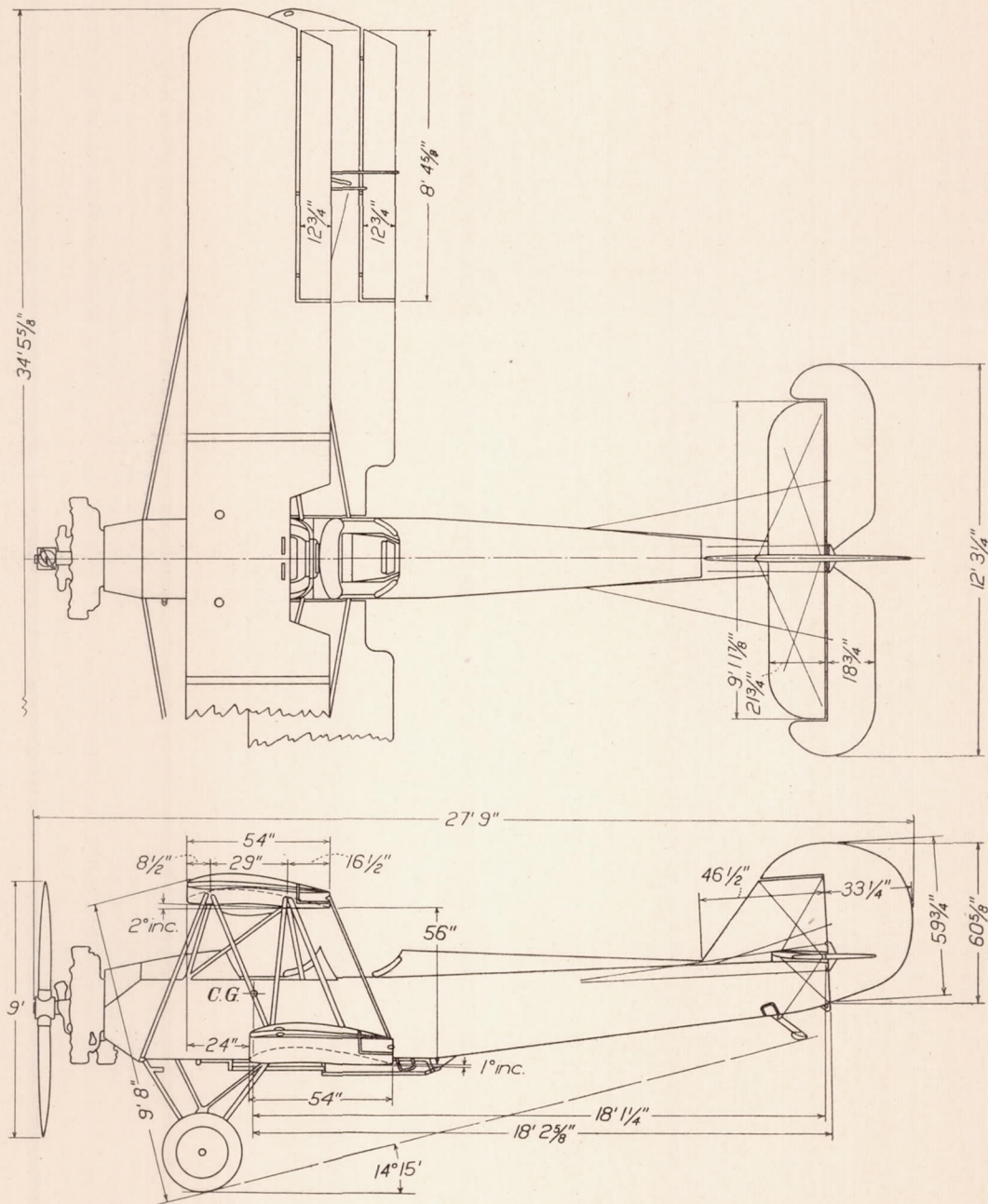
FIGURE 4.—VE–7 airplane

FIGURE 5.—NY-1 airplane

TEST PROCEDURE

The airplane is flown to an altitude considered safe under the circumstances, and the altimeter is set a thousand feet lower. This causes a large deflection of the balance needle. The gyroscope motors are kept running for some time before the spin is started, in order to warm up the parts affected by their heat. The airplane is put into a spin from a stall and allowed to spin a thousand feet before records are taken. At the instant the altimeter balance needle indicates that the proper altitude has been reached the records are started. The altimeter is then immediately set for a thousand feet lower, and the record is stopped just as the balance needle again indicates the altitude set on the scale. By this procedure records are obtained during spins for a thousand feet of altitude loss after time has been allowed for the spin to become steady and the instruments to reach full deflection. Any effect of lag in the instruments is thus eliminated. After the spin is completed the airplane is returned to the hangar to be weighed. The result of this weighing is taken as the weight of the airplane during the spin, since very little fuel is used in landing and taxying to the hangar.

COMPUTATION OF RESULTS

The average value for each of the deflections recorded on the film is obtained by finding the area under the curve and dividing this area by the length. This process eliminates the effect of oscillations, since the film records include several complete oscillations in the cases in which periodic fluctuations occurred. The average value of the deflection with the calibration curves for the instruments gives the angular velocity and the acceleration recorded. The time for the record is derived from the timing dots or lines, and since the record was made during a known altitude change, the vertical velocity may be derived. A set of film records is reproduced in Figure 6.

With actual values of rotation, forces and vertical velocity at hand, together with knowledge of the weight and momental ellipsoid of the airplane, computation of the final results becomes possible. The plan of computation is outlined here. The detailed equations and notation, including the definition of the systems of axes, will be found in the Appendix.

Since some of the data are taken with regard to the airplane axes and some with regard to the ground axes, it is necessary to find the relation between the two sets of axes. Two simple relations are used for the purpose. The first relation is that the direction of the axis of resultant rotation is vertical in the ground system of axes. Its direction in the body system of axes may be found from the components of rotation measured with the turn meters. The second relation is that the component of force perpendicular to this axis of rotation lies along the X ground axis. Its direction in the body system of axes may be computed from the force components registered by the accelerometers and the direction of the axis of rotation just found. The direction of the Y ground axis in the body system of axes may then be found by computing the direction of a line perpendicular to this ground X–Z plane. After selecting the correct combination of direction cosine values, the directions of each of the body axes are known in the ground system of axes. In the course of these steps it is necessary to compute the resultant force and its direction, which in itself is one of the results.

The next step consists in finding the direction of the flight path in the ground system of axes and then finding its direction in the body system of axes in terms of the angle of side slip and angle of attack. The direction of the flight path in the ground system of axes may be found after computing the radius of spin and the horizontal velocity from the horizontal component of force and the resultant rate of rotation. This horizontal velocity (along the Y ground axis) added vectorially to the measured vertical velocity gives the direction of the flight path in the ground system of axes.

The third step involves computation of the aerodynamic couple acting on the airplane. This is balanced by the resultant of the inertia couples of the airplane and of the propeller. The Euler equations are used in deriving the components of couple for the airplane, and if the angular velocities about the principal axes are computed, the simple form of the equations may be employed. Computation of the components of the propeller inertia couples is made in the same way as such a computation would be made for a disk, since an analysis of the case leads to the conclusion that the average of the fluctuating couple acting on the propeller is equal to the steady couple a disk having the same moment of inertia would experience. Finally, for the purpose of determining whether the results check the requirement of the equations of motion, the direction of the resultant couple computed for the airplane alone is found in the ground system of axes. If the data are exact, the component of couple about the Z ground axis should be zero.

The three steps just described involving the derivation of the angular relation between the two systems of axes defining the attitude, the angles of attack and side slip, the radius of the spin, vertical velocity, resultant force acting, and resultant couple acting, give much important information, but for the purpose of the study of spinning there are some other desired quantities that may be computed. The spin
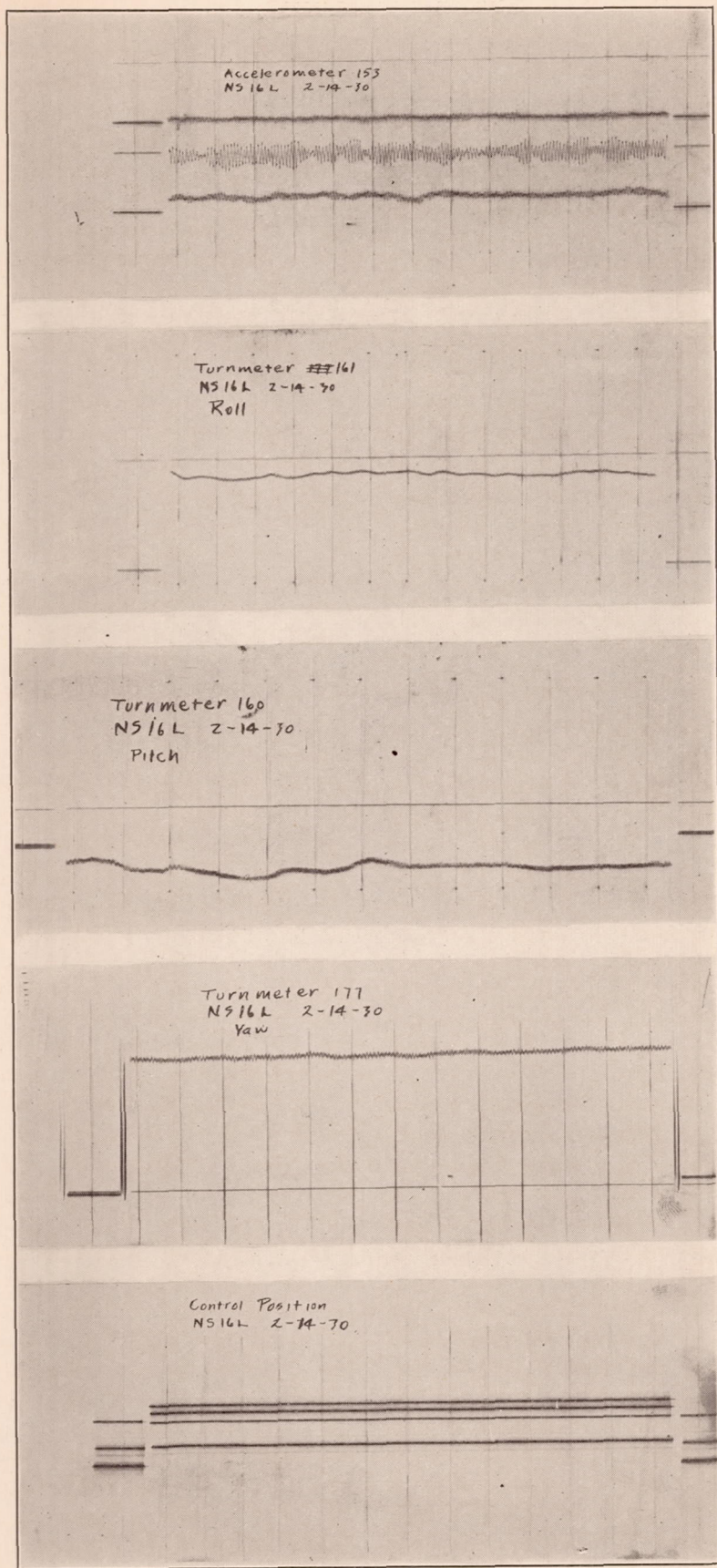
FIGURE 6.—Film record

coefficient, an important one of these, is included in the table of results. It is computed as follows:

$$\text{Spin coefficient } \Lambda = \frac{\Omega b}{2V}$$

where $\Omega =$ resultant angular velocity
$b =$ span
$V =$ velocity along the flight path.

Another quantity, which may be of value in this study, is the position of the center of pressure for the whole airplane with regard to the c. g. It can be easily derived, since the aerodynamic moment is the sole source of the couple that balances the dynamic couples arising from the airplane and propeller.

RESULTS

Some samples of the data taken in flight are shown in Table I and the numerical results mentioned above are shown in Table II. In the last column of Table I remarks are given relating to the position of the controls during the spin. By "normal" is meant a spin in which the ailerons are neutral, the elevator hard up, and rudder hard over. When ailerons are described as "with the spin" the control column was moved hard over so that the deflection of the ailerons would normally give a rolling moment in the direction of roll of the particular spin. The opposite sense is meant by "ailerons against spin."

Since it is difficult to transform the data in Tables I and II into a mental picture of the attitude of the airplane, several views of an airplane model set in the proper position for two of the spins are given in Figure 7. In the first view, for each spin, the axis of the camera is directed outward along the radius of spin through the c. g. In the second, the axis of the camera is directed along the Y ground axis toward the c. g., and in the third the axis of the camera is directed downward through the c. g. along the Z ground axis (parallel to the spin axis). In order to further assist in visualizing the positions, the projections of the ground axes have been drawn on the background.

PRECISION

The recording instruments were calibrated frequently to eliminate errors due to calibration changes. The effect of lag, which is small in any event, can be neglected, since several seconds were allowed for the instruments to come to full deflection before the records were taken. The effect of temperature changes on these instruments was investigated and found to be negligible for the turn meters, but appreciable for the accelerometer. However, the error in recorded accelerations due to this effect is not more than 2 per cent.

The method of calculating vertical velocity from the vertical displacement measured by the Paulin alti-

meter and the time for this displacement as given by the timing intervals on the record was checked against the camera obscura method. The vertical velocities found by the two methods in actual spins check within 2 per cent. As it is believed that the error in working up camera obscura records is possibly 1 per cent, the total error in the vertical velocity may be 3 per cent. Corrections for variation of atmospheric conditions from those of standard air were not made because the magnitude of such corrections is small compared with other errors of measurement of loss of altitude and because of the difficulty of obtaining a record of temperatures. Furthermore, the records show that the variation of vertical velocity in similar spins was within 3 per cent of the average value; hence, the actual vertical velocities of the spins must have been very nearly constant and the error of measurement small. The effect of changes of air density on the spin itself should be not great, since all the records were taken at about the same standard altitude (3,000 feet).

It is possible to check the accuracy of the rotation and acceleration records, since the vertical component of force on unit mass, when acceleration is in units of gravity, should be 1.0. Inspection of the values given in column 14, Table II, shows that for most of the flights this component of force checks within 3 per cent. The calculated values involving only the rotation and acceleration records are subject to approximately the same error. Values involving vertical velocity in addition to the above records are subject to a possible error of about 6 per cent, except for angles, in which case the error is probably about 0.5°.

The accuracy of the experimentally determined momental ellipsoids, which were used in the computation of the results, will be discussed in a report soon to be published on this subject. It will suffice to say that the errors are less than 2 per cent for the VE–7 and, due to improved apparatus, less than 1 per cent for the NY–1.

DISCUSSION

The data given in the tables were obtained in flights made to test the feasibility of the method. They are measurements of spins of only two airplanes and there were no changes made in any of the aerodynamic or mass characteristics in either case, so that a general study of the factors which influence spinning is not possible; but they are satisfactory for the purpose of establishing the suitability of this system of measuring the elements of a spin. Attention is called to the close agreement of all the values in the similar spins, flights Nos. 16, 17, and 18. The air was quiet and conditions steady on the day they were made; consequently, the spins were probably almost identical.

The computations made in this study are extensive and somewhat involved; hence, the accuracy of the
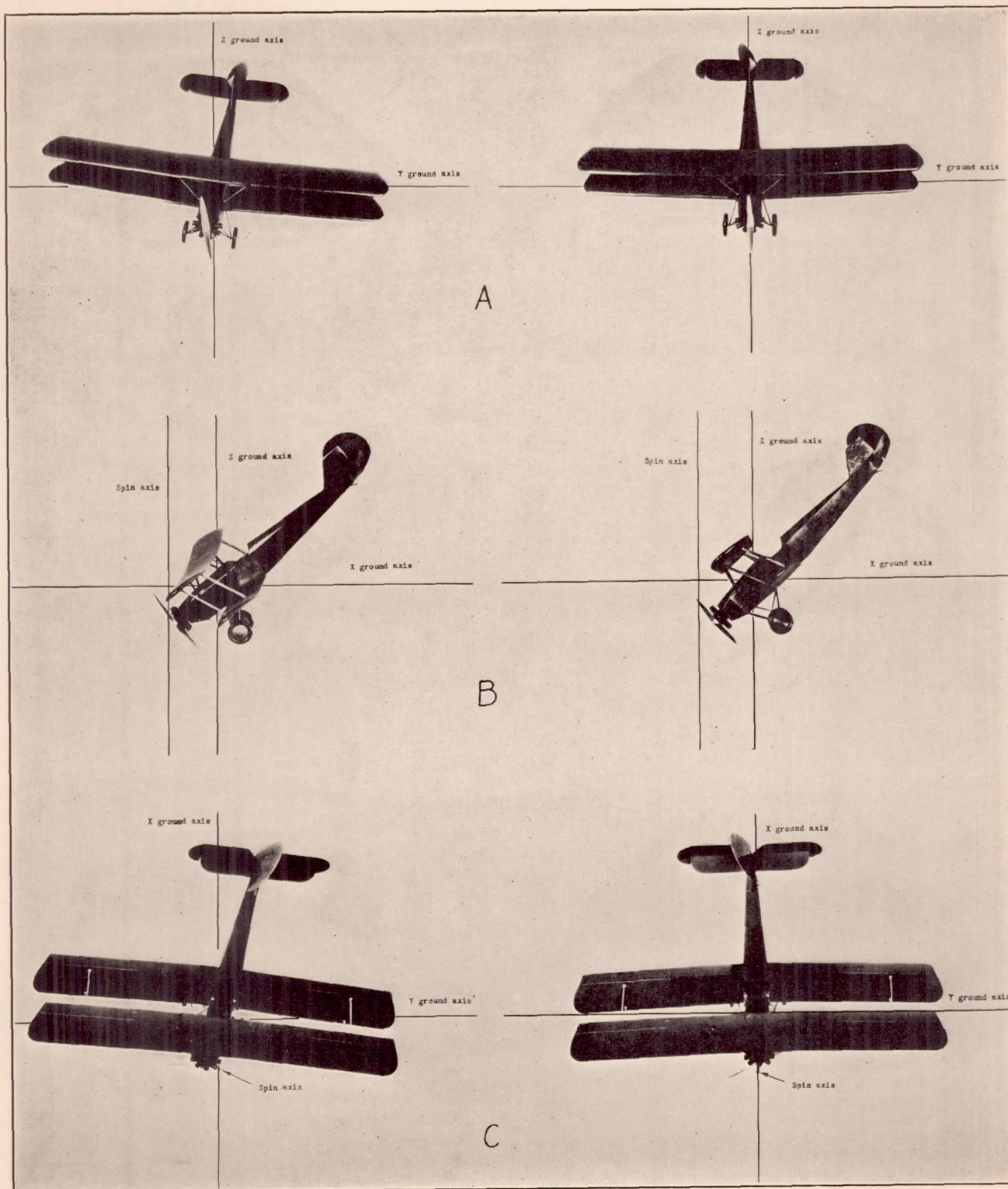
FIGURE 7.—Model of NY–1 airplane in attitude of spins

A, looking along radius of spin; B, looking along Y ground axis; C, looking along Z ground axis

measured data must be very great for the results to be of value. The accuracy obtained thus far appears to be satisfactory. However, a few improvements in the instrument installation, notably, provision of a constant temperature compartment for the accelerometer and some improvements in the details of the flight procedure, are being effected, which will give greater accuracy than that reported herein.

An investigation of the spinning characteristics of the NY–1 airplane by this method is now in progress. In this work the effect of changes of several quantities is being determined. Among the variables under consideration at present are mass distribution, balance, wing loading, and size of tail surfaces. A study is being made to extend the method to include entry and recovery from a spin, as well as the fully developed spin, so that these phases may also be investigated.

CONCLUSIONS

The method described herein for measuring completely the angular and linear velocities, dimensions of the flight path, and the forces and couples acting on an airplane in a spin is satisfactory. The precision of the results is sufficient to make possible studies of the effect of altering the properties of the airplane.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *October 7, 1930.*

APPENDIX

Rotation:

p angular velocity in roll referred to body axes.

q angular velocity in pitch referred to body axes.

r angular velocity in yaw referred to body axes.

Ω resultant angular velocity.

Force:

X [1]   inertia and gravity force component along X body axis.

Y inertia and gravity force component along Y body axis.

Z inertia and gravity force component along Z body axis.

R resultant force.

Z'' vertical component of resultant force in ground axes.

X'' horizontal component of resultant force in ground axes.

In body axes: (Unprimed letters refer to body axes; subscripts refer to vector.)

l_R, m_R, n_R direction cosines of resultant force.

$l_{Z''}, m_{Z''}, n_{Z''}$ direction cosines of resultant rotation (Z ground axis).

$l_{X''}, m_{X''}, n_{X''}$ direction cosines of radius of spin (X ground axis).

$l_{Y''}, m_{Y''}, n_{Y''}$ direction cosines of Y ground axis.

l_Q, m_Q, n_Q direction cosines of resultant gyroscopic couple.

In ground axes: (Double primed letters refer to ground axes; subscripts refer to vector.)

l_V'', m_V'', n_V'' direction cosines of flight path.

l_Y'', m_Y'', n_Y'' direction cosines of Y body axis.

l_X'', m_X'', n_X'' direction cosines of X body axis.

l_Q'', m_Q'', n_Q'' direction cosines of resultant gyroscopic couple.

In principal axes: (Superscript IV refers to principal axes, subscripts to vector.)

$l_Q^{IV}, m_Q^{IV}, n_Q^{IV}$ direction cosines of resultant gyroscopic couple.

σ angle between resultant rotation and resultant force vectors.

i, j, k proportional to direction cosines of radius of spin in body axes.

δ helix angle of flight path measured from vertical.

[1] The capital letters, X, Y, and Z may refer to either a force along the axis or the axis itself.

V velocity along flight path.

v'' horizontal velocity in ground axes.

w'' vertical velocity in ground axes.

α angle of attack referred to X axis of airplane.

β angle of side slip (or yaw).

W weight of airplane.

μ angle between flight path and X axis (body)

p^{IV} component of rotation about principal X axis.

q^{IV} component of rotation about principal Y axis.

r^{IV} component of rotation about principal Z axis.

L^{IV} couple about principal X axis.

M^{IV} couple about principal Y axis.

N^{IV} couple about principal Z axis.

Q resultant gyroscopic couple.

A moment of inertia about principal X axis.

B moment of inertia about principal Y axis.

C moment of inertia about principal Z axis.

τ angle between body X axis and principal X axis.

b span.

I moment of inertia of propeller about its axis of rotation.

ω angular velocity of propeller with respect to the airplane.

ΔL couple due to propeller about X body axis of airplane.

ΔM couple due to propeller about Y body axis of airplane.

ΔN couple due to propeller about Z body axis of airplane.

The airplane body axes have their origin in the c. g. and are directed as follows:

Axis	Direction
X	Forward, parallel with thrust axis in plane of symmetry;
Y	To starboard (right), perpendicular to plane of symmetry;
Z	Downward, perpendicular to X axis in plane of symmetry.

Sense of positive angular velocities and moments is in the following order: X to Y, Y to Z, and Z to X. The axes called ground axes have their origin in the c. g. of the airplane and have directions simply related to the ground, although, as the airplane travels around its

15

path, these axes turn with the c. g. The sketch (fig. 8) gives an instantaneous arrangement of the ground axes.
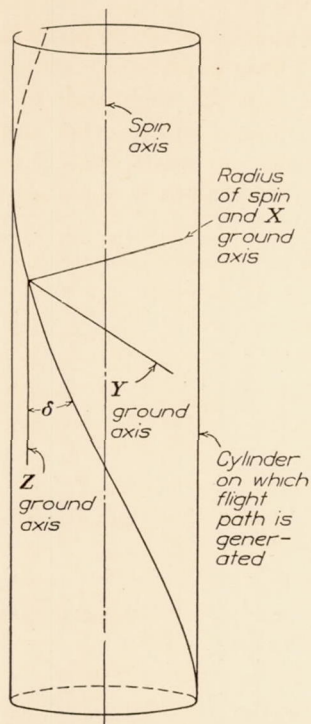

FIGURE 8.—Instantaneous arrangement of ground axes

DERIVED VALUES

The derivation of l_R, m_R, n_R and $l_{Z''}$, $m_{Z''}$, $n_{Z''}$, Ω, and R is indicated by their definitions.

$$\cos\sigma = (l_R\, l_{Z''} + m_R\, m_{Z''} + n_R\, n_{Z''}).$$
$$X'' = R \sin \sigma.$$
$$Z'' = R \cos \sigma \ (=1.0 \text{ if data are exact}).$$
$$\text{Radius} = \frac{X''g}{\Omega^2}.$$
$$v'' = \frac{Z''g}{\Omega}, \quad V = \sqrt{v''^2 + w''^2}$$

$$\left.\begin{array}{l} i = l_R - l_{Z''} \cos \sigma \\ j = m_R - m_{Z''} \cos \sigma \\ k = n_R - n_{Z''} \cos \sigma \end{array}\right\}$$ Direction components of line perpendicular to spin axis intersecting force line at unit distance form the origin.

$$l_{X''} = \frac{i}{\sqrt{i^2 + j^2 + k^2}}$$
$$m_{X''} = \frac{j}{\sqrt{i^2 + j^2 + k^2}}$$
$$n_{X''} = \frac{k}{\sqrt{i^2 + j^2 + k^2}}$$

Direction cosines of the Y'' ground axis in the body axes are found from a solution of:

$$l_{Z''}l_{Y''} + m_{Z''}m_{Y''} + n_{Z''}n_{Y''} = 0$$
$$l_{X''}l_{Y''} + m_{X''}m_{Y''} + n_{X''}n_{Y''} = 0$$
$$l_{Y''}l_{Y''} + m_{Y''}m_{Y''} + n_{Y''}n_{Y''} = 1$$

$$l_{Y''} = \begin{vmatrix} m_{Z''} & n_{Z''} \\ m_{X''} & n_{X''} \end{vmatrix} \qquad m_{Y''} = - \begin{vmatrix} l_{Z''} & n_{Z''} \\ l_{X''} & n_{X''} \end{vmatrix}$$

$$n_{Y''} = \begin{vmatrix} l_{Z''} & m_{Z''} \\ l_{X''} & m_{X''} \end{vmatrix}$$

$$\delta = \tan^{-1}\frac{v''}{w''}$$
$$l''_V = 0$$
$$m''_V = \pm \sin \delta \ [(-) \text{ for right hand,}$$
$$\qquad\qquad\qquad (+) \text{ for left-hand spins.]}$$
$$n''_V = \cos \delta$$
$$l''_Y = -m_{X''} \qquad l''_X = l_{X''}$$
$$m''_Y = m_{Y''} \qquad m''_X = l_{Y''}$$
$$n''_Y = \pm m_{Z''} \qquad n''_X = \pm l_{Z''} \ [(+) \text{ for right}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\text{hand,}$$
$$\qquad\qquad\qquad\qquad\qquad (-) \text{ for left}$$
$$\qquad\qquad\qquad\qquad\qquad\text{hand spins.]}$$

$$\beta = \sin^{-1}\left(m''_V\, m''_Y + n''_V\, n''_Y\right) \quad \begin{array}{l}\text{(angle between a line} \\ \text{and a plane.)}\end{array}$$
$$\mu = \cos^{-1}\left(m''_V\, m''_X + n''_V\, n''_X\right)$$
$$a = \cos^{-1}\left(\frac{\cos\mu}{\cos\beta}\right)$$

Since the motion in the spin is steady, the Euler equations reduce to the following expressions for the moments acting on the airplane about the principal axes. These moments are produced by the aerodynamic forces on the airplane.

$$L^{IV} = -(B-C)q^{IV}r^{IV}$$
$$M^{IV} = -(C-A)r^{IV}p^{IV}$$
$$N^{IV} = -(A-B)p^{IV}q^{IV}$$

The values of p^{IV}, q^{IV}, r^{IV} are derived from p, q, r as follows:

$$p^{IV} = p \cos \tau + r \sin \tau$$
$$q^{IV} = q$$
$$r^{IV} = r \cos \tau - p \sin \tau$$

Derivation of l'^{IV}_Q, m'^{IV}_Q, n'^{IV}_Q and Q is indicated by their definitions.

Then:

$$l_Q = l_Q^{IV} \cos \tau - n_Q^{IV} \sin \tau$$

$$m_Q = m_Q^{IV}$$

$$n_Q = n_Q^{IV} \cos \tau + l_Q^{IV} \sin \tau$$

and

$$l_Q'' = l_X'' l_Q + m_X'' m_Q + n_X'' n_Q$$

$$m_Q'' = l_Y'' l_Q + m_Y'' m_Q + n_Y'' n_Q$$

$$n_Q'' = l_Z'' l_Q + m_Z'' m_Q + n_Z'' n_Q \ (= 0 \text{ if data are exact.})$$

Distance of center of pressure of whole airplane aft of

$$\text{c. g.} = \frac{M^{IV} + \Delta M}{WZ}.$$

Gyroscopic couples due to the propeller:

$$\left.\begin{array}{l} \Delta M = I \omega r \\ \Delta N = I \omega q \end{array}\right\}$$ These expressions are approximations, but the magnitude of the effect compared to the other similar quantities is so small that a more complete treatment is not warranted.

Spin coefficient $= \dfrac{\Omega b}{2V}.$

TABLE I—MEASURED QUANTITIES

Flight No.	Date	Airplane	Components of rotation (rad./sec.)			Components of force (g)			Vertical velocity ft./sec.	Remarks
			p	q	r	X	Y	Z		
4 L[1]	Dec. 11, 1928	VE-7	−2.42	0.914	−1.62	+0.0894	−0.0950	+1.84	77.7	Normal.
6 R[1]	Jan. 9, 1929	VE-7	1.99	.812	1.77	+.0296	+.1143	+1.52	80.5	Do.
8 R	Jan. 14, 1929	VE-7	2.17	.422	1.64	+.0280	+.0313	+1.74	83.4	Do.

Revolutions per minute for VE-7 450 (practically constant for all spins)

2 R	Jan. 29, 1930	NY-1	1.70	.126	1.67	−.0329	−.0333	+1.42	92.1	Normal, motor stopped.
3 R	do	NY-1	1.64	.120	1.81	−.0260	−.0351	+1.40	93.4	Do.
5 R	Feb. 1, 1930	NY-1	1.68	.060	1.76	−.0472	−.0379	+1.39	83.3	Do.
1 R	Jan. 25, 1930	NY-1	1.45	.076	1.78	−.0236	−.0375	+1.37	86.1	Normal.
6 R	Feb. 5, 1930	NY-1	1.80	.099	1.67	−.0286	−.0333	+1.47	86.2	Do.
7 R	Feb. 6, 1930	NY-1	1.90	.041	1.57	−.0222	−.0433	+1.56	89.8	Do.
8 R	Feb. 8, 1930	NY-1	1.75	−.001	1.81	−.0076	−.0340	+1.41	84.8	Do.
9 R	Feb. 10, 1930	NY-1	1.82	.787	2.13	−.0010	+.0565	+1.38	80.1	Aileron with spin.
12 R	Feb. 11, 1930	NY-1	1.60	−.325	1.97	−.0083	−.0806	+1.34	80.6	Aileron against spin.
13 R	Feb. 12, 1930	NY-1	2.40	−.453	2.02	−.0277	−.1158	+1.64	89.9	Elevator down.
16 L	Feb. 14, 1930	NY-1	−1.58	.291	−1.86	−.0202	+.0268	+1.31	78.9	Normal.
17 L	do	NY-1	−1.58	.291	−1.86	−.0100	+.0251	+1.30	82.9	Do.
18 L	do	NY-1	−1.58	.286	−1.88	−.0100	+.0335	+1.29	76.9	Do.

Revolutions per minute for NY-1 500 (practically constant for all spins)

[1] R, right-hand spin; L, left-hand spin.

TABLE II.—COMPUTED QUANTITIES

Flight No.	Airplane	Resultant rotation rad./sec.	Resultant force (g)	Radius of spin, feet	Helix angle °	Angle of attack °	Angle of side-slip °	Dynamic couple components, lb.—ft.			Resultant couple lb.—ft.	C. P. position aft of c. g., feet	Spin coefficient	Vertical force (g)	Direction cosine of couple in ground axes		
								L^{IV}	M^{IV}	N^{IV}					l_Q''	m_Q''	n_Q''
4 L	VE-7	3.05	1.84	5.3	11.8	30.6	[1]6.1	−1,436	−2,812	−146.9	3,161	0.78	0.655	1.075	−0.09198	−0.9956	0.00009
6 R	VE-7	2.78	1.53	5.0	9.7	38.2	−7.8	+1,306	−2,236	−101.3	2,592	.78	.581	.955	.09385	−.9954	.00006
8 R	VE-7	2.75	1.74	5.9	11.0	35.6	2.1	+653	−2,427	−59.5	2,514	.73	.552	1.057	.07313	−.9972	0
2 R	NY-1	2.39	1.43	5.9	8.3	43.8	5.2	285	−4,292	39.2	4,302	1.27	.443	.974	−.0400	−.9992	−.00002
3 R	NY-1	2.45	1.40	5.1	7.4	47.2	4.5	294	−4,463	36.0	4,473	1.34	.448	1.015	−.0442	−.9990	−.00001
5 R	NY-1	2.43	1.39	5.4	8.4	45.6	7.0	141	−4,437	18.3	4,439	1.34	.497	.971	−.0411	−.9992	−.00002
1 R	NY-1	2.29	1.37	5.4	7.9	50.2	6.0	182	−3,831	20.0	3,836	1.07	.455	1.048	−.0473	−.9989	−.00002
6 R	NY-1	2.46	1.47	5.8	9.2	42.2	6.9	223	−4,550	32.5	4,555	1.17	.486	.979	−.0319	−.9993	.00000
7 R	NY-1	2.46	1.56	6.4	9.8	39.1	8.8	87	−4,520	14.2	4,520	1.13	.465	.974	−.0387	−.9992	−.00001
8 R	NY-1	2.52	1.41	5.0	8.4	45.6	8.4	−3	−4,778	−.4	4,777	1.28	.508	1.011	−.0343	−.9995	.00000
9 R	NY-1	2.91	1.38	3.5	7.2	47.7	−8.7	2,258	−5,796	260.2	6,226	1.61	.622	1.022	+.0077	−1.0000	−.00001
12 R	NY-1	2.56	1.34	4.2	7.4	51.5	14.7	−864	−4,696	−94.3	4,775	1.32	.542	1.039	−.0704	−.9976	−.00001
13 R	NY-1	3.17	1.65	4.1	8.0	40.4	16.2	−1,243	−7,355	−199.4	7,462	1.77	.602	1.043	−.0637	−.9980	+.00001
16 L	NY-1	2.46	1.31	4.6	8.0	48.3	1.1	−730	−4,407	−83.9	4,468	1.55	.533	.974	+.0488	−.9988	.00000
17 L	NY-1	2.46	1.30	4.6	7.6	48.4	0.7	−729	−4,398	−83.5	4,459	1.56	.507	.975	+.0493	−.9988	−.00001
18 L	NY-1	2.47	1.29	4.5	8.0	48.5	1.3	−724	−4,436	−82.3	4,496	1.58	.549	.973	+.0588	−.9983	−.00001

[1] Positive angle is side-slip outward; negative, inward.

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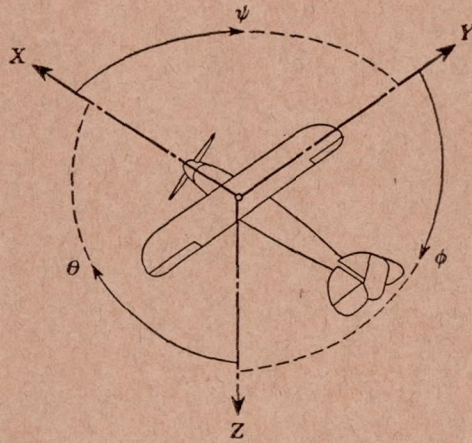
Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal___	X	X	rolling_____	L	$Y \longrightarrow Z$	roll_____	ϕ	u	p
Lateral_____	Y	Y	pitching____	M	$Z \longrightarrow X$	pitch_____	θ	v	q
Normal_____	Z	Z	yawing_____	N	$X \longrightarrow Y$	yaw_____	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS} \qquad C_m = \frac{M}{qcS} \qquad C_n = \frac{N}{qbS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.

p, Geometric pitch.

p/D, Pitch ratio.

V', Inflow velocity.

V_s, Slipstream velocity.

T, Thrust, absolute coefficient $C_T = \dfrac{T}{\rho n^2 D^4}$

Q, Torque, absolute coefficient $C_Q = \dfrac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P = \dfrac{P}{\rho n^3 D^5}$.

C_s, Speed power coefficient $= \sqrt[5]{\dfrac{\rho V^5}{Pn^2}}$.

η, Efficiency.

n, Revolutions per second, r. p. s.

Φ, Effective helix angle $= \tan^{-1}\left(\dfrac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.

1 kg/m/s = 0.01315 hp

1 mi./hr. = 0.44704 m/s

1 m/s = 2.23693 mi./hr.

1 lb. = 0.4535924277 kg

1 kg = 2.2046224 lb.

1 mi. = 1609.35 m = 5280 ft.

1 m = 3.2808333 ft.