NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 599

FLIGHT TESTS OF THE DRAG AND TORQUE OF THE PROPELLER IN TERMINAL-VELOCITY DIVES

By RICHARD V. RHODE and HENRY A. PEARSON

1937

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length_____	l	meter_____	m	foot (or mile)_____	ft. (or mi.)
Time_____	t	second_____	s	second (or hour)_____	sec. (or hr.)
Force_____	F	weight of 1 kilogram_____	kg	weight of 1 pound_____	lb.
Power_____	P	horsepower (metric)_____	_____	horsepower_____	hp.
Speed_____	V	{ kilometers per hour_____	k.p.h.	miles per hour_____	m.p.h.
		{ meters per second_____	m.p.s.	feet per second_____	f.p.s.

2. GENERAL SYMBOLS

W, Weight $= mg$

g, Standard acceleration of gravity $= 9.80665$ m/s² or 32.1740 ft./sec.²

m, Mass $= \dfrac{W}{g}$

I, Moment of inertia $= mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m⁻⁴-s² at 15° C. and 760 mm; or 0.002378 lb.-ft.⁻⁴ sec.²

Specific weight of "standard" air, 1.2255 kg/m³ or 0.07651 lb./cu.ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\dfrac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure $= \dfrac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L = \dfrac{L}{qS}$

D, Drag, absolute coefficient $C_D = \dfrac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_o} = \dfrac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i} = \dfrac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p} = \dfrac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C = \dfrac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\dfrac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_0, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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Langley Memorial Aeronautical Laboratory

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By Richard V. Rhode and Henry A. Pearson

SUMMARY

The drag and torque of a controllable propeller at various blade-angle settings, and under various diving conditions, were measured by indirect methods on an F6C–4 airplane in flight. The object of these tests was (1) to provide data on which calculations of the terminal velocity with a throttled engine and the accompanying engine speed could be based and (2) to determine the possibility of utilizing the propeller as an air brake to reduce the terminal velocity.

The data obtained were used in the establishment of propeller charts, on the basis of which the terminal velocity and engine speed could be calculated for airplanes whose characteristics fall within the range of these tests. It was found that the propeller reduced the terminal velocity about 11 percent with the normal blade-angle setting of 19.0° and about 35 percent with a 5.5° setting. Indications were that the terminal velocity could be still further reduced by using even lower blade-angle settings. A method is given for the calculation of the terminal velocity with throttled engine and the engine speed.

INTRODUCTION

In cooperation with the Bureau of Aeronautics, Navy Department, and the Army Air Corps, the National Advisory Committee for Aeronautics has been making a study of rational methods for establishing the structural design conditions for airplanes. In the course of this study, a method was established in 1930 for calculating the terminal velocity of a diving airplane, taking propeller drag into account. The method was based on the results of small-scale propeller tests by Durand and Lesley (references 1 and 2), supplemented by the then unpublished results of a few tests of a 4-foot metal propeller in the N. A. C. A. propeller-research tunnel. Because of insufficient data on torque or power coefficients from these tests, no provision could be included for calculating the engine speed and the method was therefore based on the assumption of such an engine speed, which, for structural-design purposes, was limited to an arbitrary permissible value.

The interest aroused in this work because of the increasing use of the terminal-velocity dive in military tactics led to an extension of the study to determine the feasibility of using the propeller as an air brake to reduce the terminal velocity. As a result, the wind-tunnel tests of the 4-foot propellers were extended to include tests at the lower blade-angle settings and with different propeller-body combinations. At the same time, a program of dive tests to be made of a conventional airplane with a controllable propeller was formulated, the purpose of which was to evaluate the influence of the propeller under full-scale conditions at the high tip speeds associated with a terminal-velocity dive. The present report presents the results of the flight tests in a usable form for the quantitative determination of the influence of the propeller on the terminal velocity and the engine speed.

The flight tests were made in September 1932 by the N. A. C. A. at Langley Field, Va.

APPARATUS AND METHOD

A Navy F6C–4 airplane equipped with a Pratt & Whitney R–1340–CD engine was used in these tests. The pertinent data concerning this airplane are given in table I and a general view is given in figure 1. The propeller used was the Hamilton controllable model described in reference 3. This propeller was not completely adjustable in flight, as it could be set at only two positions, the locations of which depended upon the setting of stop nuts. As delivered, the range of blade-angle settings available was between 13° and 22°, which range was extended down to 5° for these tests by the use of special links. The pitch-changing mechanism consisted of a hydraulic piston and centrifugal weights, which actuated the blades through a system of push-pull rods. The action of the centrifugal weights tended to increase the blade angle; the engine-oil pressure, when acting on the piston, forced the blades to the lower setting.

The airplane was equipped with four synchronized standard N. A. C. A. photographically recording instruments—air-speed meter, tachometer, altimeter, and air-temperature thermometer—and a dive-angle indicator developed especially for these tests.

The diagram of figure 2 shows the simplicity of the dive-angle indicator. Its principal merit lies in the fact that it is not affected by accelerations, as its operation depends upon the reflection of a ray of sunlight onto a frosted-glass scale.

The air-speed head was mounted at the outer strut location on a boom one chord length forward of the leading edge of the wing, in order to reduce the interference on the air-speed measurements to a minimum. The air-speed installation was calibrated over a speed course, and a constant error of 2 percent for speeds between 130 and 150 miles per hour was found. It was assumed that the correction for the diving conditions was also 2 percent.

From data obtained in high-speed level flight the minimum drag coefficient of the airplane was calculated.

where W, weight of the airplane.

γ, flight-path angle.

$C_{D_{min}}$, minimum drag coefficient of the airplane.

q, dynamic pressure corresponding to the desired zero-thrust or basic terminal velocity.

S_w, wing area.

In order to obtain these dive angles in the flight tests, a curve of the elevation of the sun against time was plotted, and a pointer on the dive-angle indicator was set to indicate the proper dive angle corresponding to the elevation of the sun existing at the instant the dive
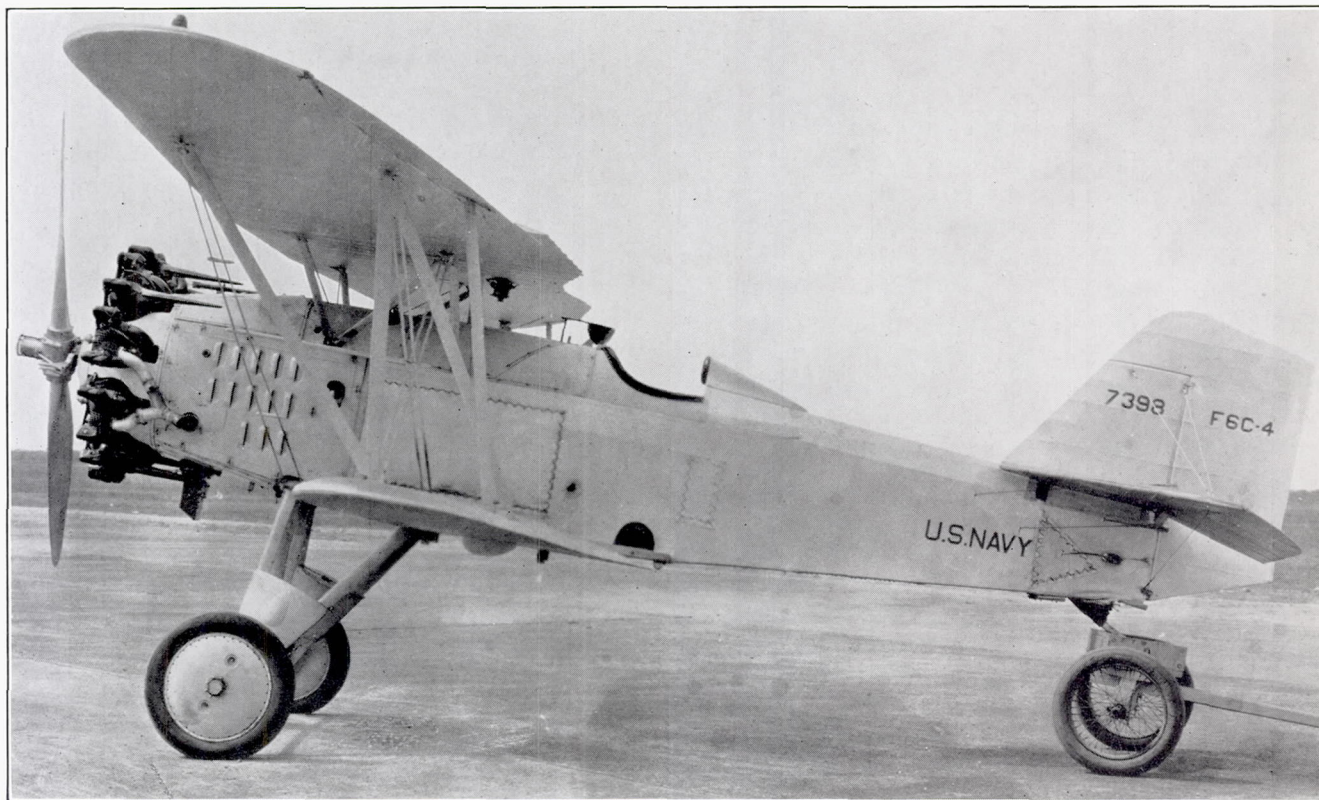

FIGURE 1.—The F6C–4 airplane.

The method employed consisted of deducting the calculated induced drag from the total drag, which had been evaluated from the known engine power and the estimated propeller efficiency. On the basis of a study of full-scale propeller-body tests, the propulsive efficiency was estimated in this case to be 75.5 percent.

The main tests consisted of terminal-velocity dives, with the engine fully throttled and with the ignition on, starting at 12,000 feet and continuing to approximately 5,000 feet altitude. The dives were made at various predetermined dive angles to simulate conditions for airplanes of various zero-thrust or "basic" terminal velocities. For each basic terminal velocity, tests were made with propeller blade-angle settings of 5.5°, 9.5°, 14.5°, 19°, and 22.5° at 0.75 radius.

The dive angles at which the tests were made were determined from the relation

$$\sin \gamma = \frac{C_{D_{min}}qS_w}{W}$$

was to be started. Continuous records of indicated air speed, engine speed, air temperature, and barometric pressure were taken throughout all the dives.

PRECISION

The corrected dynamic pressure measurements at terminal velocity are probably accurate to within 2 percent. During the entry into and accelerated portions of the dive, the precision may be slightly less because of lag in the air-speed system. The tachometer readings are correct to within 30 r. p. m. Barometric pressures were measured to a precision of about 2 percent, and the temperature to about 2° C. The maximum error in the dive angle was about 2° and was caused primarily by the inability of the pilot to maintain the airplane in a steady condition at all times.

RESULTS

The recorded measurements were first plotted as time histories of the quantities measured, to insure proper

evaluation of these quantities at the terminal velocity. A representative time history is shown in figure 3. From curves such as these, the indicated terminal velocities
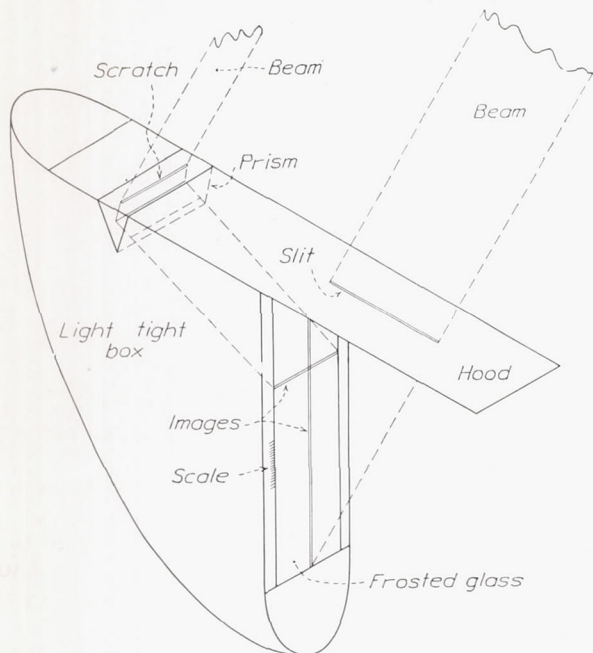

FIGURE 2.—Dive-angle indicator. Prism has blackened surface with horizontal scratch. In operation, pilot heads into the sun so that light through the slit in the hood makes a vertical image on the frosted glass. He then pushes into a dive until the horizontal image reaches a predetermined mark on the scale.

and the accompanying engine speeds were obtained. These quantities were then plotted against the appropriate blade-angle settings for each of the basic terminal velocities, as shown in figure 4. No flight-test points
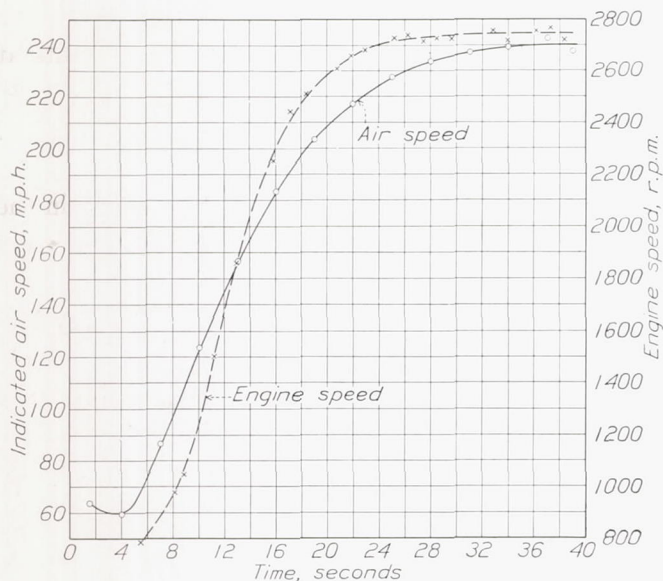

FIGURE 3.—Time history of a vertical dive. Blade-angle setting, 14.5° at 0.75 R.

are shown in this figure, as these curves are the results of cross-fairing an intermediate set of curves of the measured values. This cross-fairing was necessitated by the fact that the pilot found it impossible in some cases to dive at exactly the specified time, with the con-

sequence that the angle of dive did not correspond to an integral value of basic terminal velocity. The engine speeds given in figure 4 are those for a standard sea-level density. The engine speed at any other altitude can be obtained by multiplying these values by the square root of the ratio of the sea-level density to the density at altitude. It is assumed that the indicated terminal velocity does not change materially with altitude.
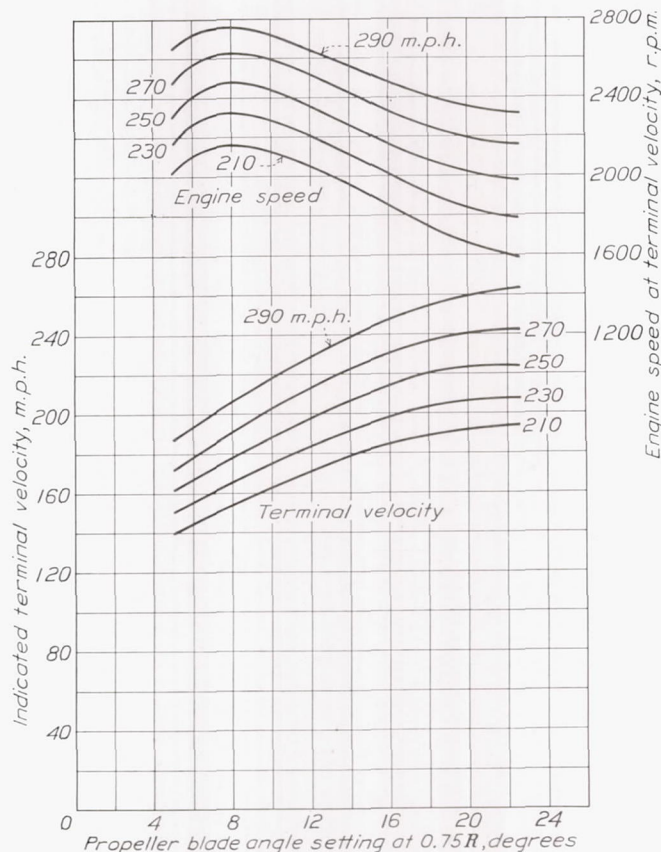

FIGURE 4.—Variation of engine speed and terminal velocity with propeller blade-angle setting for different zero-thrust velocities. The engine speeds are corrected to standard sea-level density. The parameter is zero-thrust terminal velocity.

The variation of air speed with engine speed during a number of dives is shown in figure 5. Two runs, representing the extreme values of the dive angles at which the tests were made, are shown for each blade-angle setting.

DISCUSSION

From figure 4 it can be seen that the terminal velocity decreases with blade-angle setting for the range investigated. Indications are that a further decrease in pitch would lower the limiting velocity still more. However, there is a critical value where a decrease in terminal velocity no longer accompanies a decrease in blade-angle setting, unless power is used to increase the engine speed. This fact is not apparent from the curves of figure 4, as the range of blade-angle settings could not be extended sufficiently low with the propeller used in these tests. The engine speed at terminal velocity increases as the blade angle decreases, down

to about 8°; thereafter, the engine speed decreases with decreasing blade angle.

There is some doubt whether the 22.5° points are correct, since there is a reversal in curvature between the 19.0° and 22.5° settings. Further, on the ground with the stop nuts set for 22.5° the engine speed was not sufficient for the centrifugal force to bring the blades quite against the stops. As the airplane was available for only a limited time, there was no opportunity to
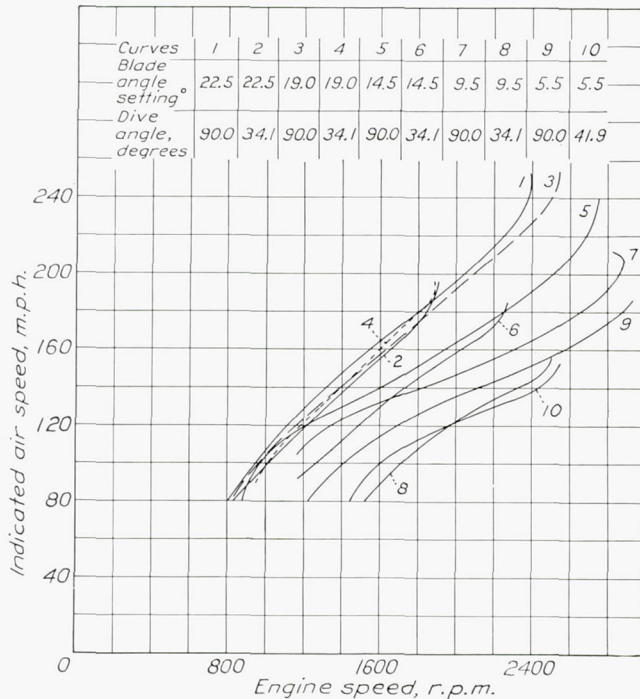

FIGURE 5.—Relation between engine speed and air speed for several dives made with the F6C–4 airplane.

construct the apparatus necessary to determine whether the blades were actually against the stops during the dives.

The significance of the curves in figure 5 is that the value of n/V at which the propeller operates during the major portion of any throttled dive is approximately constant. If the influence of tip speed is neglected, it may be said that the thrust coefficient is also nearly a constant, since the propeller, for a given blade-angle setting, operates at roughly the same value of nD/V. The over-all drag coefficient, which is the sum of the airplane and propeller drag coefficients, is thus approximately constant throughout any dive. This relation suggests that methods for the determination of time-altitude and velocity-altitude relations may be considered sufficiently precise for practical purposes if based on the assumption of a constant drag coefficient, which, of course, should include a proper allowance for the propeller.

DERIVATION OF PROPELLER CHARTS

The coefficients that were found to be most adaptable for reducing propeller data in the negative range are defined as follows:

$$T_c = \frac{T}{\rho V^2 D^2}$$

and

$$Q_c = \frac{Q}{\rho V^2 D^3}$$

where T is the propeller thrust, lb.

Q, propeller torque, lb.-ft.

D, propeller diameter, ft.

V, air speed, ft. per sec.

ρ, mass density of air, slugs per cu. ft.

These coefficients were computed from the corresponding values of thrust and torque evaluated from the following relations:

$$T = W \sin \gamma - C_{D_{min}} \frac{\rho V^2}{2} S_w$$

$$Q = \frac{550 \ \text{f.hp.}}{2\pi n}$$

in which f.hp. is the friction horsepower of the engine and the other symbols have their usual significance.[1]

The experimental thrust and torque coefficients so computed for the 14.5° blade-angle setting are shown plotted against nD/V in figure 6. It will be noted that the points for the various dives made with this setting fall at nearly the same value of nD/V; further, it will be seen that the vertical displacement of the points tends to vary with tip speed. Results for the other blade-angle settings are similar in character to those for the 14.5° setting, but occur at different values of nD/V as indicated by the dashed lines of figure 7, which give the median lines through the test points for different blade-angle settings.

Because of the close grouping of the test points at each blade-angle setting, the establishment of a propeller chart (fig. 7) was necessarily based in part on information from other sources. The method and material used in establishing this chart are explained in the following paragraphs.

The form of the propeller-characteristic curves was determined from the tests by Durand and Lesley and from the unpublished results of the tests made in the

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[1] The friction horsepower used in these computations was obtained from a 50-hour endurance test of the Pratt & Whitney "Wasp" aircraft engine. The results are shown in fig. 9. The friction-power characteristics existing under the flight-test conditions may, for a number of reasons, have been at variance with the characteristics determined under the conditions of the engine test. Any such disagreement, of course, results in erroneously derived torque coefficients but, as will be shown later, these errors have a negligible influence on the terminal velocity calculated from the charts and only a small influence on the engine speed.

propeller-research tunnel. The quantitative establishment of the curves involved: (1) determination of the end points on the basis of data from outside sources;
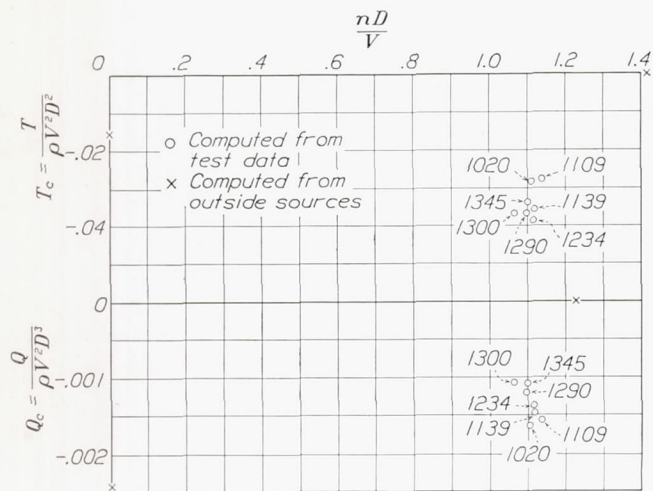

FIGURE 6.—Measured thrust and torque coefficients. Blade-angle setting, 14.5° at 0.75 R. All points labeled for tip speed.

(2) fairing of curves through the F6C–4 dive-test points; (3) establishment of tip-speed corrections, which were based largely on the dive-tests results but partly on tests in the propeller-research tunnel (reference 4).

The end points of the T_c curves at zero nD/V were established on the basis of a consideration of Diehl's formula (reference 5), Lock's formula (reference 6), and

the data given in reference 7. The quantitative values chosen represent a weighted mean of the data obtained from the three sources. The end points of the Q_c curves at zero nD/V were based entirely on the data of reference 7, which were the only data available.

Values of nD/V at zero T_c and Q_c were partly established by calculations based on the assumption that the aerodynamic characteristics of the blade element at 0.75 radius, considered as an airfoil, represent the action of the propeller as a whole in a condition near zero thrust. For these calculations the angle of zero lift was determined by Munk's method, given in reference 8. Since these points are affected appreciably by interference from the fuselage, consideration was also given to the slopes of the curves of reference 7, with an estimated allowance for fuselage interference, in combination with the requirement that the curves pass through the experimental points from the dive tests.

The propeller-characteristic curves were passed through these end points and through the experimental points (tip speed less than 1,050 feet per second) obtained in the dive tests. As thus drawn, the curves are applicable to cases involving propellers having the proportions of the one used in the dive tests.

In order to make the curves more convenient to apply, they have been corrected to a mean blade-width ratio of 0.1, as presented in figure 7. (Mean blade-width ratio is defined as the ratio of the mean blade

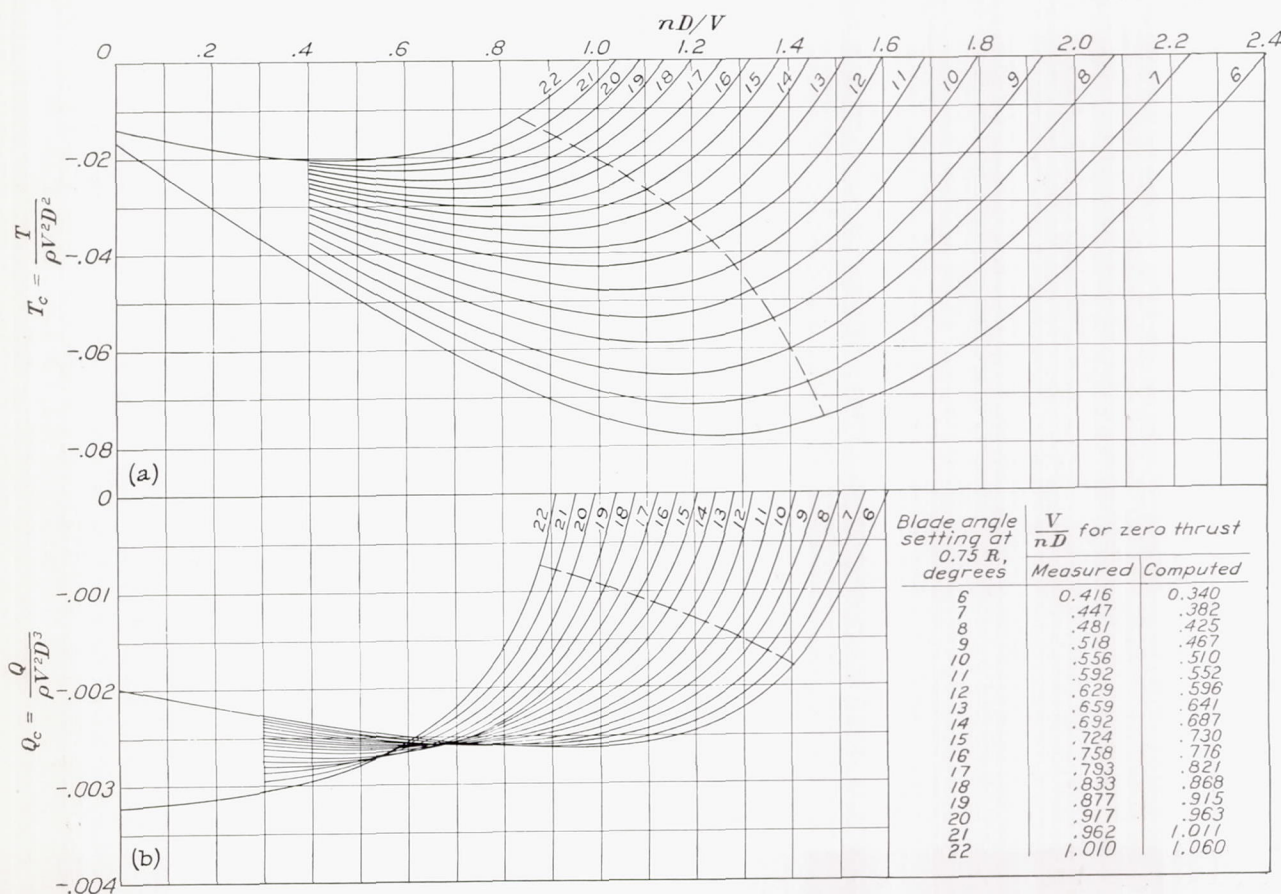| Blade angle setting at 0.75 R, degrees	$\frac{V}{nD}$ for zero thrust	
	Measured	Computed
6	0.416	0.340
7	.447	.382
8	.481	.425
9	.518	.467
10	.556	.510
11	.592	.552
12	.629	.596
13	.659	.641
14	.692	.687
15	.724	.730
16	.758	.776
17	.793	.821
18	.833	.868
19	.877	.915
20	.917	.963
21	.962	1.011
22	1.010	1.060

FIGURE 7.—Propeller characteristics at negative slip for a mean blade-width ratio of 0.1 based on the radius.

width between $0.2R$ and R to R, where R is the radius.) The mean blade-width ratio is a measure of the blade area when the diameter is known. This area must be taken into account in applying a single general set of propeller characteristics to any particular case, in the same manner that the wing area must be taken into account in dealing with wing forces. The coefficients therefore vary directly with the blade area or with the mean blade-width ratio. Since the curves of figure 7 apply to propellers having a mean blade-width ratio of 0.1, the coefficients must be multiplied by the ratio of the actual mean blade-width ratio to 0.1 when using the curves for any other case.

The curves of figure 7 are labeled for blade-angle setting in degrees at $0.75R$ for metal propellers based on either the Clark Y or RAF–6 sections. In order to make the charts more general, values of V/nD for zero thrust are given in two forms, either one of which may be used in lieu of blade-angle setting for selecting the curves in cases involving sections other than the Clark Y or RAF–6. Measured values of V/nD for zero thrust should be used only if the measurements have been made with the proper body interference. Computed values are determined on the basis of a setting determined at the $0.75R$ section and with the zero-lift angle of that section found by Munk's method as given in reference 8.

TIP-SPEED CORRECTION FACTORS

As given in figure 7, the propeller characteristics apply only to cases in which the tip speeds are below the critical value, and they agree well with the flight-test data only for such cases. When the tip speed is above the critical value (approximately 1,050 feet per second), which is the usual case in a dive, the characteristics are different from those given in figure 7. This effect is apparent from figure 6, where the points shift with increasing tip speed. In general, it may be said that there is, for a given propeller and propeller load, a separate set of characteristics for each tip speed above the critical value. The characteristics will, in general, also vary with load at a given tip speed because of variations in the blade deflection with changing load. The characteristics at the higher tip speeds may be determined approximately by introducing conversion factors, which can be used to transform the basic characteristics into those applicable at various tip speeds above the critical value. A method used in determining such conversion factors on the basis of the F6C–4 data follows.

It can be shown qualitatively that as the tip speed increases above the critical value, the value of nD/V for a given value of T_c also increases. Further, it can be shown that at a given value of nD/V the value of Q_c decreases numerically with increasing tip speed above the critical value. These considerations imply that as the tip speed increases above the critical value,

the curves of T_c are shifted to the right and the curves of Q_c are shifted upward. The conversion factors evolved are based on these considerations with their numerical values determined by comparing results calculated from the characteristics of figure 7 with the experimental results.

Specifically, the terminal velocities and the engine speeds were calculated for the various dive angles, using as given data the measured weight and the drag coefficient of the airplane, the friction-horsepower curve of the engine, and the propeller characteristics of figure 7. The factors necessary to convert the calculated engine speeds to the experimental values were plotted against tip speed. The mean curve drawn through these points is the conversion curve for nD/V. In a similar manner, conversion factors for
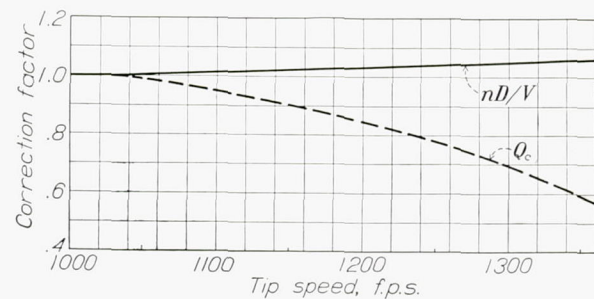

FIGURE 8.—Correction factors for tip speed.

Q_c at the corrected values of nD/V were plotted to give a conversion curve for Q_c. These conversion factors include both the influence of blade deformations with changing load and the influence of tip speed. They are shown in figure 8.

APPLICATION OF CHARTS TO THE CALCULATION OF TERMINAL VELOCITY

PRINCIPLES INVOLVED

The fundamental principles involved in any calculation of terminal velocity where propeller drag is to be taken into account are: (1) At terminal velocity the component of weight along the flight path must equal the total drag; (2) the shaft power of the propeller must equal that absorbed in friction by the engine. Obviously, the point of intersection of the curves of shaft power of the propeller and of power absorbed in friction by the engine, plotted against velocity, meets the conditions required.

Specifically, the following procedure is employed in the calculation of terminal velocity and engine speed:

1. Assume a series of terminal velocities in the interval given by the following formula whose solutions roughly approximate the F6C–4 data:

$$V_{t_{ind}} = K(0.0178\,\theta + 0.89 \pm 0.05)$$

where $V_{t_{ind}}$ is the indicated terminal velocity, in miles per hour.

K, the indicated terminal velocity with zero thrust, in miles per hour.

θ, the difference, in degrees, between the normal high-speed blade-angle setting and that on which the calculations are based. The angle θ is positive when the blade-angle setting under consideration is ~~larger~~ smaller than the normal setting.

2. Compute T_c for the series of assumed velocities from the formula

$$T_c = \frac{W \sin \gamma - C_{D_{min}} \frac{1}{2} \rho V^2 S_w}{\rho V^2 D^2}$$

3. At the appropriate blade-angle setting obtain from figure 7 the values of nD/V and Q_c corresponding to the computed thrust coefficients.

4. Compute the values of n from the known values of nD/V, D, and V.

5. Compute the propeller torques from the formula

$$Q = Q_c \rho V^2 D^3$$

6. Using the computed values of Q and n, compute the shaft horsepower of the propeller from the formula

$$P = \frac{2\pi Q n}{550}$$

7. Plot the results from step 6 against those from step 1.

8. Plot the friction horsepower of the engine against the velocities of step 1.

The curve of power absorbed in friction by the engine against velocity is obtained from a curve of friction horsepower against engine speed using the values of n from step 4. The intersection of the two curves gives the point satisfying the conditions and is the calculated terminal velocity. The speed of the engine can be found by plotting the computed values of n against the assumed velocities and finding n existing at the calculated terminal velocity. The foregoing procedure involves no corrections for tip speed or mean blade-width ratio. The manner in which these corrections are applied is best shown by an illustrative example. A complete series of calculations will not be given but a sample computation using the final calculated terminal velocity for an F6C-4 airplane will be used.

ILLUSTRATIVE EXAMPLE

Given:

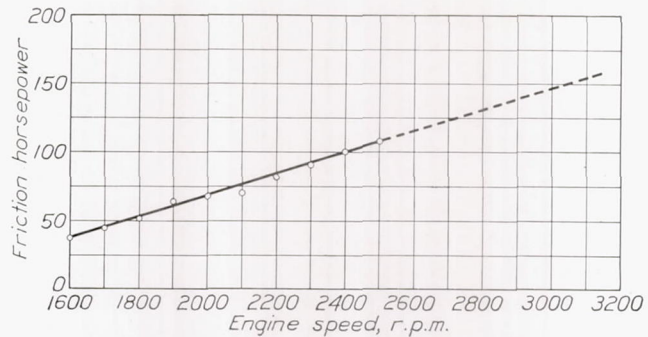|	
Airplane	F6C-4.
Weight (W)	2,830 lb.
Wing area (S_w)	252 sq. ft.
Minimum drag coefficient ($C_{D_{min}}$)	0. 0513.
Engine	Pratt & Whitney R-1340-CD.

Friction-horsepower curve (fig. 9)

FIGURE 9.—Friction horsepower for P. & W. R-1340-CD engine.

Propeller:

Diameter (D)	9 ft.
Mean blade-width ratio	0. 123.
Blade-angle setting at 0.75 R	19. 0°.

It is required to find:

1. The indicated terminal velocity in a vertical dive ($\gamma = 90°$) at 3,000 ft.
2. The propeller revolution speed at terminal velocity at this altitude.

Assume $V_{t_{ind}} = 258.2$ m. p. h.

$$= 378.9 \text{ f. p. s.}$$
$$q = 170.6 \text{ lb./sq. ft.}$$

Negative propeller thrust, $T = W \sin \gamma - C_{D_{min}} q S_w$

$$T = 2830 \times 1 - 0.0513 \times 170.6 \times 252 = 623 \text{ lb.}$$

$$T_c = \frac{T}{\rho V^2 D^2} = \frac{T}{2qD^2} = \frac{623}{2 \times 170.6 \times 81} = 0.02252$$

T_c corrected to mean blade-width ratio of 0.1 to allow entry into charts $= \dfrac{0.1 \times 0.02252}{0.123} = 0.01831$

$\dfrac{nD}{V}$ at $T_c = 0.01831$ for $19.0° = 0.940$ (fig. 7).

$$n = \frac{0.940 \times 378.9 \times \sqrt{\rho_0/\rho}}{9} = 41.35 \text{ r. p. s.}$$

$\sqrt{\rho_0/\rho}$ at 3,000-foot altitude $= 1.045$ (reference 9).

Tip speed $= \sqrt{(\pi D n)^2 + \rho_0/\rho V_{t_{ind}}^2} = 1,235$ f. p. s.

Correction factor for $\dfrac{nD}{V} = 1.038$ (fig. 8).

Correction factor for $Q_c = 0.80$ (fig. 8).

Corrected $\dfrac{(nD)}{V} = \dfrac{(nD)'}{V} = 1.038 \times 0.940 = 0.975$.

Corrected $n = n' = 1.038 \times 41.35 = 42.9$ r. p. s. (r. p. m.)$' = 60 \times 42.9 = 2,575$.

Q_c at $\dfrac{(nD)'}{V} = 0.00094$ (fig. 7).

Q_c corrected for tip speed $= 0.80 \times 0.00094 = 0.000752$.

Q_c corrected to mean blade-width ratio, $0.123 = \dfrac{0.123}{0.1} \times$

$0.000752 = 0.000925.$

$Q = Q_c \, 2q \, D^3 = 230$ lb.-ft.

Shaft horsepower of the propeller, $P = \dfrac{2\pi Q n'}{550} = 113.0.$

At a value of n equal to 42.9 r. p. s. and an engine speed of 2,575 r. p. m., the horsepower absorbed in friction by the engine, using the engine friction-horsepower curve, is 113.5. Since the conditions of equilibrium are satisfied, i. e., the total drag equals the weight and the shaft horsepower of the propeller equals that absorbed by the engine, the indicated terminal velocity is 258.2 miles per hour and the engine speed is 2,575 r. p. m.

If, in the preceding example, the problem had been solved for a minimum altitude of 6,000 feet, the values for the indicated terminal velocity would have been 258.0 miles per hour, and the accompanying engine speed, 2,676 r. p. m. The influence of air density on the indicated terminal velocity is seen to be slight, but its influence is appreciable on the engine speed, which varies approximately inversely with the square root of the density.

It has been previously stated that errors in the friction-horsepower curve have but a small influence on the final result. A critical analysis, based on figure 7, of the interrelations of the several variables involved indicates that this statement is true for all reasonable cases. It is perhaps sufficient here, however, to point out that in figure 7 the steepness of the Q_c curves in the neighborhood of the dotted line indicates that fairly large variations of Q_c may occur without greatly affecting the engine speed at given values of D and V. At the same time, small variations in nD/V do not result in as large a change in thrust. Hence, it would be expected that quite large variations in friction horsepower can be taken up by the propeller without greatly affecting either the engine or the airplane speed. As an extreme example, if the friction horsepower of the engine used in the illustrative example is doubled, the terminal velocity is found to be 256 miles per hour and the engine speed about 2,400 r. p. m. These values compare with the original values of 258.2 miles per hour and 2,575 r. p. m., differing by 0.85 percent and 6.8 percent, respectively.

It has been found, in most cases, that the propeller operation in a throttled dive will be defined by characteristics falling close to the dotted lines of figure 7. To operate at greatly lower values of nD/V for any blade-angle setting would require an abnormally small propeller, while to operate at much higher values would require the application of engine power.

COMPARISON OF EXPERIMENTAL AND CALCULATED RESULTS

A comparison between the experimental and calculated results using the tip-speed corrections is made in table II. This comparison merely indicates the degree to which factors other than those included in the method of calculation affect the result. Part of the discrepancies are, however, attributable to experimental error. It will be seen that the percentage error in the terminal velocity is small, the maximum being 4.3 percent, while the average is less than half that value. The average errors in the engine speed are slightly higher, with the maximum error 6.8 percent. As these comparisons cover a wide range of blade-angle settings and dive angles, the agreement is considered to be reasonably good.

Table III includes a comparison between calculated and experimental results for three airplanes on which data were available. The agreement for airplanes A and B is good in regard both to terminal velocity and engine speed. These airplanes were somewhat similar to the F6C–4 airplane in their general features; in particular, the power plants were of the same type and the performances were similar. Hence, a good agreement between the calculated and experimental results on these airplanes was perhaps to be expected.

In the case of airplane C the agreement in terminal velocity is poor although the agreement in engine speed is fair. The experimental results indicate a very slight reduction in terminal velocity due to the propeller, whereas the calculated results indicate a reduction of the same order as those noted for the other airplanes listed. As far as can be determined, there is no unusual feature in airplane C to account for this discrepancy. The airplane minimum drag coefficients as determined from three independent sources agreed within 2 percent. Although the drag coefficient used in the calculations holds for a Reynolds Number corresponding to high-speed level flight and there is evidence that a reduction in drag coefficient with increasing Reynolds Number is to be expected, the influence of such a scale effect should not be felt in this case alone. In other words, the influence of scale effect is implicitly allowed for roughly in the method of calculation because of the empirical nature of the method. There is a possibility that the degree of turbulence in the slipstream with the propeller operating at negative thrust may have a critical effect on the drag of some parts of the structure within the slipstream. At the present state of knowledge it would be practically impossible to take such a phenomenon into account.

It is somewhat difficult, because of the lack of experimental cases, to say whether the method of calculation as presented will generally hold good. It is felt that within the following limitations the method will yield satisfactory results except in cases where unusual or unpredictable influences occur.

LIMITATIONS

1. The propeller-body combination should be approximately similar to that of the F6C–4.

2. Blade-angle settings should not be extrapolated, particularly in the low range.

3. Mean blade-width ratios should not be less than 0.09 nor more than 0.17.

4. The propeller blade sections should be based on either the Clark Y or RAF–6 sections and should be of normal thicknesses.

5. Tip-speed correction factors should not be extrapolated.

RULES OF THUMB

In calculated results for a number of airplanes of widely different characteristics, such as those listed in table III, consistent trends which indicate the feasibility of quick rules have been noted. Thus, the percentage reduction in terminal velocity caused by the propeller in a vertical dive with engine fully throttled and with normal blade-angle setting is given by the equation

$$R \text{ (percent)} = 0.011 \ V_{t_b} + 9.7$$

in which V_{t_b} is the terminal velocity (m. p. h.) in a vertical dive with no thrust in standard sea-level conditions of atmosphere.

Engine speed (r. p. m.) is given by the equation

$$N = 15.6 \ V_t - \frac{V t^2}{45}$$

in which V_t is the terminal velocity with the foregoing correction for the propeller effect.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *August 22, 1933.*

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TABLE I

CHARACTERISTICS OF F6C–4 AIRPLANE

|	
Type	Tractor biplane, landplane.
Engine	Pratt & Whitney, R–1340–CD.
Horsepower	450 at 2,100 r. p. m.
Weight (as flown)	2,815 and 2,830 lb.
Principal dimensions:	
Span (upper wing)	31 ft. 6 in.
Span (lower wing)	26 ft.
Length	22 ft. 6 in.
Height	9 ft. 6 in.
Total wing area	252 sq. ft.
Gap	4 ft. 5$\frac{5}{16}$ in.
Stagger	3 ft. 2$\frac{1}{2}$ in.
$C_{D_{min}}$ (from flight tests)	0.0513

TABLE II

COMPARISON BETWEEN CALCULATED AND CROSS-FAIRED EXPERIMENTAL VALUES

Dive angle (deg.)	Basic terminal velocity (m. p. h.)	Blade-angle setting at 0.75 R (deg.)	Indicated terminal velocity				Engine speed			
			Calculated (m. p. h.)	Experimental (m. p. h.)	Difference (m. p. h.)	Difference (percent)	Calculated (r. p. m.)	Experimental (r. p. m.)	Difference (r. p. m.)	Difference (percent)
90	290	22.5	271.2	263.0	8.2	3.1	2,272	2,320	−48	−2.1
90	290	19.0	258.7	257.0	1.7	.7	2,483	2,385	98	4.1
90	290	14.5	241.4	241.0	.4	.2	2,670	2,540	130	5.1
90	290	9.5	215.4	214.5	.9	.4	2,745	2,730	15	.5
90	290	5.5	195.0	190.0	5.0	2.6	2,713	2,680	33	1.2
59	270	22.5	252.3	242.0	10.3	4.3	2,114	2,160	−46	−2.1
59	270	19.0	240.8	239.5	1.3	.5	2,310	2,225	85	3.8
59	270	14.5	224.2	224.5	−.3	−.1	2,483	2,400	83	3.5
59	270	9.5	200.1	199.5	.6	.3	2,562	2,605	−43	−1.7
59	270	5.5	180.8	175.0	5.8	3.3	2,530	2,520	10	.4
47	250	22.5	233.6	225.0	8.6	3.8	1,953	1,980	−27	−1.4
47	250	19.0	223.1	224.0	−.9	−.4	2,140	2,055	85	4.1
47	250	14.5	207.9	209.0	−1.1	−.5	2,309	2,225	84	3.8
47	250	9.5	185.3	185.0	.3	.2	2,377	2,460	−83	−3.4
47	250	5.5	167.1	164.0	3.1	1.9	2,340	2,360	−20	−.8
38	230	22.5	214.1	208.0	6.1	2.9	1,786	1,780	6	.3
38	230	19.0	204.9	205.5	−.6	−.3	1,954	1,875	79	4.2
38	230	14.5	191.4	193.0	−1.6	−.8	2,117	2,085	32	1.5
38	230	9.5	170.5	172.0	1.5	−.9	2,177	2,295	−118	5.1
38	230	5.5	153.4	153.0	.4	.3	2,124	2,215	−91	−4.1
31.75	210	22.5	196.7	194.0	2.7	1.4	1,640	1,585	55	3.5
31.75	210	19.0	187.8	190.0	−2.2	−1.2	1,785	1,700	85	5.0
31.75	210	14.5	175.6	181.0	−5.4	−3.0	1,930	1,925	5	.3
31.75	210	9.5	156.6	160.0	−3.4	−2.1	1,995	2,140	−145	−6.8
31.75	210	5.5	141.0	142.0	−1.0	.7	1,951	2,060	−109	−5.3

TABLE III

COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

Airplane	Engine type and power	Sea-level high speed (m.p.h.)	Propeller diameter (ft.)	Mean blade width ratio	Blade-angle setting at 0.75 R (deg.)	Dive angle (deg.)	$V_{t_{ind}}$ (sea level) zero thrust (m.p.h.)	$V_{t_{ind}}$ (sea level) closed throttle (m.p.h.)	R.p.m. (sea level) closed throttle	Percentage reduction due to propeller	Experimental $V_{t_{ind}}$ (sea level) closed throttle (m.p.h.)	Experimental r.p.m. (sea level) closed throttle	Percentage error in calculated $V_{t_{ind}}$	Percentage error in calculated r.p.m.
A	P & Ww 450-2,100	140	9	0.1285	17.0	90	288.0	253.3	2,600	12.0	258.0	2,600	−1.8	0
B	P & Ww 450-2,100	160	9	.125	18.0	90	290.5	254.7	2,528	12.3	255.0	[1] 2,500	−1.2	1.1
B	P & Ww 450-2,100	160	9	.138	18.0	90	290.5	252.0	2,520	13.2	255.0	[1] 2,500	−1.2	.8
C[2]	P & Wh 575-2,100	131	10	.134	16.0	41	246.3	219.4	2,260	10.9	238.0	2,200	−7.8	2.7
D	Wright R-1510	194	8.5	.166	26.0	90	416.0	361.5	2,720	13.1	--------	--------	--------	-------
E	Wright R-1820F1	201	9.5	.129	23.3	90	430.0	371.5	2,775	13.6	--------	--------	--------	-------

[1] Indicated r. p. m.   [2] Calculations made for 4,000 feet.
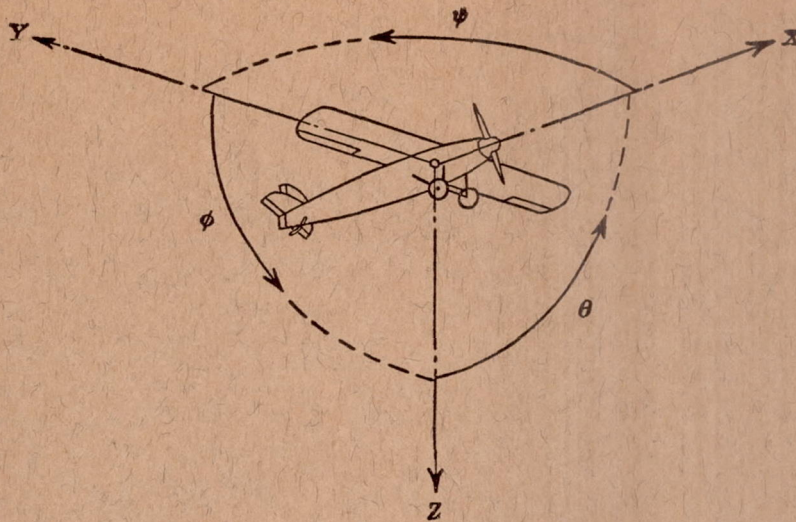
Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal___	X	X	Rolling_____	L	$Y \longrightarrow Z$	Roll_____	ϕ	u	p
Lateral_____	Y	Y	Pitching____	M	$Z \longrightarrow X$	Pitch____	θ	v	q
Normal_____	Z	Z	Yawing_____	N	$X \longrightarrow Y$	Yaw_____	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS} \qquad C_m = \frac{M}{qcS} \qquad C_n = \frac{N}{qbS}$$

(rolling)　　　　(pitching)　　　　(yawing)

Angle of set of control surface (relative to neutral position), δ.　(Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D,　Diameter

p,　Geometric pitch

p/D,　Pitch ratio

V',　Inflow velocity

V_s,　Slipstream velocity

T,　Thrust, absolute coefficient $C_T = \dfrac{T}{\rho n^2 D^4}$

Q,　Torque, absolute coefficient $C_Q = \dfrac{Q}{\rho n^2 D^5}$

P,　Power, absolute coefficient $C_P = \dfrac{P}{\rho n^3 D^5}$

C_s,　Speed-power coefficient $= \sqrt[5]{\dfrac{\rho V^5}{P n^2}}$

η,　Efficiency

n,　Revolutions per second, r.p.s.

Φ,　Effective helix angle $= \tan^{-1}\left(\dfrac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp. $=76.04$ kg-m/s $=550$ ft-lb./sec.

1 metric horsepower $=1.0132$ hp.

1 m.p.h. $=0.4470$ m.p.s.

1 m.p.s. $=2.2369$ m.p.h

1 lb. $=0.4536$ kg.

1 kg $=2.2046$ lb.

1 mi. $=1,609.35$ m $=5,280$ ft.

1 m $=3.2808$ ft.