NACA-TR-631

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 631

AIRFOIL SECTION CHARACTERISTICS AS APPLIED TO THE PREDICTION OF AIR FORCES AND THEIR DISTRIBUTION ON WINGS

By EASTMAN N. JACOBS and R. V. RHODF

1938

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight $=mg$

g, Standard acceleration of gravity $=9.80665$ m/s² or 32.1740 ft./sec.²

m, Mass $=\frac{W}{g}$

I, Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu. ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure $=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0, Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s., the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_0, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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AIRFOIL SECTION CHARACTERISTICS AS APPLIED TO THE PREDICTION OF AIR FORCES AND THEIR DISTRIBUTION ON WINGS

By Eastman N. Jacobs and R. V. Rhode

SUMMARY

The results of previous reports dealing with airfoil section characteristics and span load distribution data are coordinated into a method for determining the air forces and their distribution on airplane wings. Formulas are given from which the resultant force distribution may be combined to find the wing aerodynamic center and pitching moment. The force distribution may also be resolved to determine the distribution of chord and beam components. The forces are resolved in such a manner that it is unnecessary to take the induced drag into account.

An illustration of the method is given for a monoplane and a biplane for the conditions of steady flight and a sharp-edge gust. The force determination is completed by outlining a procedure for finding the distribution of load along the chord of airfoil sections.

INTRODUCTION

This report originated in a request of the Bureau of Air Commerce, Department of Commerce, for a coordinated system of applying airfoil section data to the determination of wing forces and their distribution.

The system presented herein yields, within the limitations of our present knowledge of aerodynamics, a general solution of the resultant wing forces and moments and their distribution. For the sake of completeness and facility in use, the report contains a table of the important section parameters for many commonly used sections and all other necessary data required to solve the most practical design problems coming within the scope of the system.

Although the usefulness of the system extends into several phases of airplane design, its application to structural design is illustrated by following through a wing loading condition corresponding to that specified in reference 1.

Two basic principles underlie the system employed. First, a force coefficient is treated as the independent variable, thus eliminating, as far as possible, the angle of attack; and second, the forces are derived throughout in terms of certain basic parameters of the airfoil section, which are tabulated for each airfoil section. The method followed then builds up the forces progressively from simple combinations of certain basic forces and simple formulas involving the basic airfoil section parameters. As the forces are thus built up, they are resolved into any convenient components. This method also has another important advantage in that the induced drag, which is really only a component of the local lift at each section, may be entirely eliminated from the analysis.

In some problems it is desirable to know the location of the aerodynamic center of the wing and the pitching-moment coefficient about this center in order to construct the balance diagram of the complete airplane. Methods are therefore given for determining these two properties. For problems in which the aerodynamic center and the pitching moment are not required, a direct solution of the forces and force distribution can be made.

BASIC CONSIDERATIONS

The forces on a wing may be considered to be functions of the characteristics of the airfoil sections and of the spanwise distribution of lift. At a given section lift coefficient, the resultant air force and moment on the section are, according to wing theory, assumed to be independent of all geometric properties of the wing except the section shape; moreover, the forces and moments acting on any individual section may be considered to be independent of adjacent sections or of other characteristics of the wing, except as they affect the lift distribution and thus the local lift coefficient at that section.

The problem is thus divided into two parts: First, the determination of the spanwise lift distribution; and, second, the determination of the corresponding forces and moments at each section and the summation of these quantities to obtain the corresponding forces and moments for the entire wing. The spanwise lift distribution is obtained in terms of values of the local section lift coefficient c_{l_0} for a number of sections distributed along the span. The subscript zero is used to distinguish this section lift coefficient, perpendicular to the local relative wind at the section, from the lift coefficient c_l perpendicular to the relative wind at a great distance from the wing. The lower-case letters used for these coefficients have been chosen to distinguish the lift coefficient for a section ($c_l=dL/qcdy$) from the usual lift coefficient for the wing, C_L.

In order to permit easy reference, the symbols used in the text, the figures, and the tables are grouped in appendix C.

For many purposes, it is convenient to express the air forces in terms of components along two axes fixed with respect to the airplane rather than as the usual components, lift and drag. This resolution is conveniently accomplished from the c_{l_0} values, when the profile drag and other fundamental characteristics of the airfoil section are taken into account, by means of simple formulas involving parameters given for each airfoil section in a table of airfoil characteristics. This method has an important advantage in that the induced drag, which is really only a component of the c_{l_0} at each section, is entirely eliminated from the analysis.

For the purpose of determining the lift distribution corresponding to the c_{l_0} values along the span, the lift load along the span is considered as being made up of two independent parts that will be referred to as the "basic lift distribution" and the "additional lift distribution." The basic lift distribution is represented by the c_{l_0} distribution along the span when the total wing lift is zero. This basic lift distribution, which is the distribution arising by virtue of aerodynamic twist, may be considered to exist unaltered as the lift and angle of attack are changed. The additional lift distribution, as the name implies, represents the distribution of additional lift associated with changing the angle of attack. Wing theory indicates that, as long as the airfoil sections of the wing are working within a range of normal lift-curve slope, the form of the additional lift distribution is the same at all lift coefficients and is independent of wing twist, of aileron or flap displacements, and of other characteristics that affect only the basic lift distribution. Experiment shows that this deduction is approximately correct for wings with well-rounded tips. For such wings, the additional lift distribution is given as a function of the plan form and aspect ratio in terms of the additional lift coefficients $c_{l_{a1}}$, that is, the section additional lift coefficients for a wing lift coefficient of 1. The lift distribution for any wing is then found in terms of the wing lift coefficient C_L, the basic lift coefficient c_{l_b}, and the additional lift coefficient $c_{l_{a1}}$

$$c_{l_0}=c_{l_b}+C_L\, c_{l_{a1}} \tag{1}$$

GENERAL PROCEDURE

MONOPLANE

It is advisable first to choose a backward fore-and-aft reference axis x usually parallel to the reference axis, or thrust line, of the airplane and an upward z axis perpendicular to it. (See fig. 1.) Upward and backward air forces and distances are thus considered positive. Air-force components along these axes are then expressed at each section of the wing by

$$dX=c_x\, q\, c\, dy \tag{2}$$

and

$$dZ=c_z\, q\, c\, dy \tag{3}$$

where X and Z are the components of air load along the axes, and c_x and c_z are determined from c_{l_0} and the known characteristics and attitude of each airfoil section. The pitching moment about the origin contributed by each section is

$$dM=c_{m_{a.c.}}\, q\, c^2\, dy+c_x\, q\, c\, z\, dy-c_z\, q\, c\, x\, dy \tag{4}$$

where x and z are distances measured from the origin to the aerodynamic center of the airfoil section (see table I and appendix B) and the signs of the terms are so taken that stalling moments are positive.

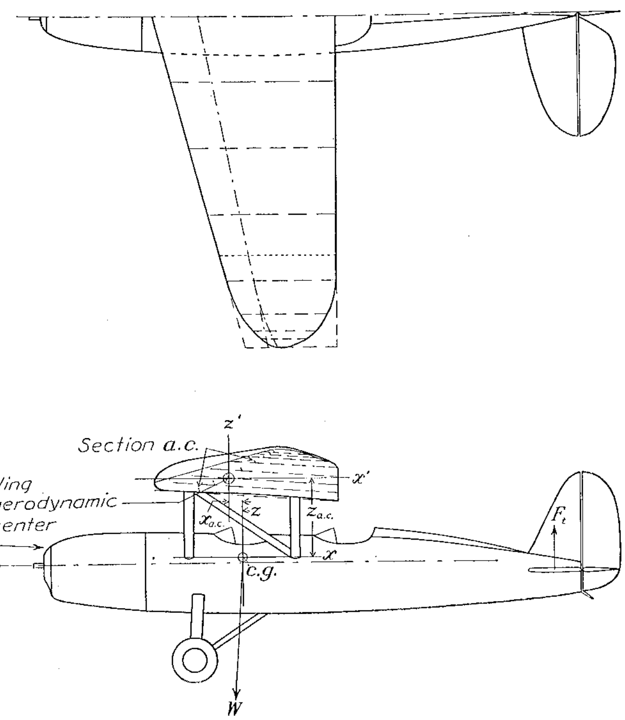


FIGURE 1.—Airplane drawing and balance diagram.

Thus far the origin has been arbitrarily chosen. If, with this arbitrarily chosen origin, the coordinates $x_{a.c.}$ and $z_{a.c.}$ of the aerodynamic center of the entire wing (fig. 1) are found, the origin of coordinates may then be moved to this point and from equation (4) there may be determined a value of $M_{a.c.}/q$ that has sensibly the same value for all flight conditions.

Aerodynamic center and additional lift distribution.—For the purpose of finding the aerodynamic center of the wing, it is necessary to consider only the additional distribution. In fact, the aerodynamic center of the wing may be considered as the centroid of all the additional loads. For wings with linear taper and rounded tips, values of L_a, giving the load distribution for $C_L=1$, may be found from table II for various sections along the span. The values of L_a were derived as outlined in reference 2. The corresponding values of $c_{l_{a1}}$ for various sections along the span are found from the relation $c_{l_{a1}}=\dfrac{L_aS}{cb}$. The corresponding values of c_{d_0} at each section are calculated using the method indicated in figure 2 or, if the profile-drag polar curve for the section is available, they may be read from it. Then

$$c_{x_{a1}}=c_{d_0}\cos\theta_{z_a}-c_{l_{a1}}\sin\theta_{z_a} \tag{5}$$

and

$$c_{z_{a1}}=c_{l_{a1}}\cos\theta_{z_a}+c_{d_0}\sin\theta_{z_a} \tag{6}$$

in which $\theta_{z_a}=\frac{c_{l_{a1}}}{a_0}+\alpha_{l_0}-i$; a_0, the section lift-curve slope, and α_{l_0}, the angle of attack of zero lift, are given in table I; and i is the incidence of the chord at each section with respect to the x axis.

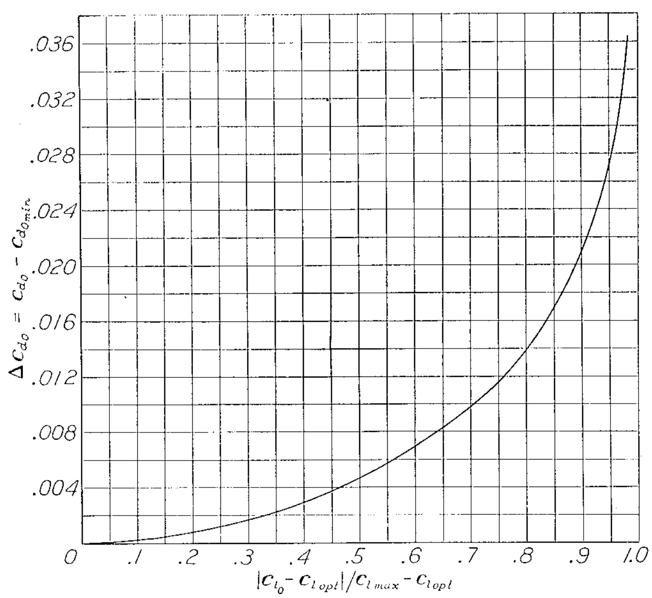


FIGURE 2.—Chart for the determination of the profile-drag coefficient c_{d_0} at any lift coefficient c_{l_0}.

The next step is to plot $c_{x_{a1}}c$, $c_{z_{a1}}c$, $c_{x_{a1}}cz$, and $-c_{z_{a1}}cx$ against y and to fair curves through the plotted points. Twice the area under each curve from $y=0$ to $y=b/2$ is then, respectively: X_{a1}/q, Z_{a1}/q, $M_{X_{a1}}/q$, $M_{Z_{a1}}/q$. The coordinates of the aerodynamic center of the wing are then found

$$x_{a.c.}=\frac{-M_{Z_{a1}}}{Z_{a1}} \tag{7}$$

$$z_{a.c.}=\frac{M_{X_{a1}}}{X_{a1}} \tag{8}$$

Pitching moment about the wing aerodynamic center.—The additional load distribution for $C_L=1$ and the position of the aerodynamic center are now known. The next step is the determination of the basic load distribution (that corresponding to $C_L=0$) and from it the basic pitching moment or the aerodynamic pitching moment of the wing about the aerodynamic center. The basic distribution for wings with linear twist may be obtained from table III in terms of the load parameter L_b for a number of sections along the span. The method of deriving the L_b values is given in reference 2. When the wing has partial-span flaps, the basic distribution may be obtained from reference 3. Following the system that was previously used, c_{l_b} values corresponding to the basic lift distribution are found for each section from $c_{l_b}=L_b\frac{\epsilon a_0 S}{cb}$, corresponding c_{d_0} values determined, and c_{x_b} and c_{z_b} calculated from the formulas

$$c_{x_b}=c_{d_0}\cos\theta_{z_b}-c_{l_b}\sin\theta_{z_b} \tag{9}$$

$$c_{z_b}=c_{l_b}\cos\theta_{z_b}+c_{d_0}\sin\theta_{z_b} \tag{10}$$

where $\theta_{z_b}=\frac{c_{l_b}}{a_0}+\alpha_{l_0}-i$.

Likewise are plotted curves of $c_{x_b}cz'$ and $-c_{z_b}cx'$, where z' and x' are the new coordinates of the section aerodynamic center from the aerodynamic center of the wing. The areas are then determined. In addition, another curve formed by plotting $c_{m_{a.c.}}c^2$ is drawn and the area determined. Twice these areas then give, respectively, $(M_{X_b}/q)_{a.c.}$, $(M_{Z_b}/q)_{a.c.}$, and M_s/q. The desired wing pitching moment about the aerodynamic center is found from

$$\frac{M_{a.c.}}{q}=\left(\frac{M_{X_b}}{q}\right)_{a.c.}+\left(\frac{M_{Z_b}}{q}\right)_{a.c.}+\frac{M_s}{q} \tag{11}$$

Lift distribution and total lift.—When the total wing lift or normal-force coefficients are known or specified by design conditions, the force distribution may be found immediately in terms of c_{l_0} values along the span from

$$c_{l_0}=c_{l_b}+C_Lc_{l_{a1}}$$

For wings having well-rounded tips, the lift distribution may thus be found in terms of the c_{l_b} and $c_{l_{a1}}$ values previously determined. This method will give a good approximation of the actual lift distribution in such cases. When, for any reason, the tip loads are of critical importance, that is, if the wing is tapered less than 2:1 and has a tip blunter than semicircular, the lift distribution should be determined according to the method given in appendix A or reference 4. If the wing plan form departs from a straight taper, the lift distribution should be determined from suitable theoretical methods (references 2 and 3). In any event, the loads are represented by the c_{l_0} distribution and may then be resolved to give chord and beam components and moments.

In general, the wing lift coefficient C_L' for the steady-flight condition preceding an accelerated-flight condition will be first determined. After the tail load and, finally, the wing lift L are determined from the balance diagram for the steady-flight condition, the corresponding wing lift coefficient is found

$$C_L'=\frac{L}{qS} \tag{12}$$

The wing lift coefficient C_L for an accelerated-flight condition may then be determined. For example, it may be that the acceleration is produced by a sharp-edge gust, and the wing lift coefficient is determined by the simplified formula

$$C_L = C_L' + m\frac{U}{V} \tag{13}$$

where C_L' has just been found, U/V is the ratio of the gust velocity to the flight velocity, and m is the slope of the wing lift curve, which may be determined from the values of a_0 or m_6, tabulated for the airfoil sections, by employing the method indicated later in figure 12.

The required lift distribution is then found in terms of the value of c_{l_0} at each section from

$$c_{l_0} = c_{l_b} + C_L c_{l_{a1}} \tag{14}$$

From these values of the lift coefficient at each section, the required coefficients representing the components of the air load may be computed and the total load components then determined as before by measuring the areas under curves representing $c_x\, q\, c$ and $c_z\, q\, c$. Some question exists, however, in regard to the values of c_{d_0} that should be used in the computation of c_x and c_z for the accelerated-flight condition.

Conditions and forces encountered instantaneously in accelerated-flight conditions after a suddenly changed angle of attack.—In an accelerated-flight condition the C_L value calculated from (13) and the c_{l_0} values from (14) may exceed the maximum lift coefficients. Such conditions are possible on entering a sharp-edge gust or in abrupt maneuvers owing to the considerable time required to accumulate the increased volume of reduced-energy air associated with the increased boundary-layer thickness or separated flow that will finally prevail at the increased angle of attack. Lift values should be based on the lift-curve slope extended without regard to the usual burbling. Such lift values are obtained simply by following the outlined procedure. The c_{d_0} values, however, deserve special consideration. The increasing profile-drag coefficients at the higher lift coefficients are likewise associated with a thickening boundary layer or a separating flow that will not occur at once when the angle of attack is suddenly increased.

The profile-drag coefficient for these transient conditions for a given lift, whether or not the lift exceeds the value given by wind-tunnel tests as the maximum, is undoubtedly less than the profile drag determined in the wind tunnel under steady conditions. The true value, however, is unknown and, in fact, a series of values increasing with time will exist. It may therefore be expedient in some cases to determine the force components on the wing by assuming that c_{d_0} retain its initial steady-flight value throughout the subsequent relative pitching motion of the wing. On the other hand, if it is desired to investigate the higher values that the profile drag will later assume, c_{d_0} may be found in the usual way from the wind-tunnel data unless c_{l_0} is greater than $c_{l_{max}}$, in which case some value of c_{d_0} may be assumed. The value $c_{d_0}=0.1$ is suggested.

The distribution of the resolved components and moments and the total wing components.—Values of c_{d_0} and c_{l_0} for the sections along the span having now been established, the distribution of the air-force components, given by values of c_x and c_z, may be found from

$$c_x = c_{d_0} \cos\theta_z - c_{l_0} \sin\theta_z \tag{15}$$

$$c_z = c_{l_0} \cos\theta_z + c_{d_0} \sin\theta_z \tag{16}$$

where

$$\theta_z = \frac{c_{l_0}}{a_0} + \alpha_{l_0} - i$$

The torsional moment contributed by each section about its aerodynamic center is simply

$$dM_{s_{a.c.}} = c_{m_{a.c.}}\, q\, c^2\, dy \tag{17}$$

For some problems, components and moments with respect to axes in the wing may be desired rather than the components given by c_x and c_z with respect to the airplane. For example, "chord-truss" and "beam" components may be desired at each section. These components represented by c_c and c_b may be obtained from a slight modification of (15) and (16).

$$c_c = c_{d_0}(1+\tan\theta_c \tan\phi)\cos\theta_c - c_{l_0}(1-\cot\theta_c\tan\phi)\sin\theta_c \tag{18}$$

$$c_b = c_{l_0}(1-\tan\theta_b\tan\phi)\cos\theta_b + c_{d_0}(1+\cot\theta_b\tan\phi)\sin\theta_b \tag{19}$$

where

$$\theta_c = \frac{c_{l_0}}{a_0} + \alpha_{l_0} - i_c$$

$$\theta_b = \frac{c_{l_0}}{a_0} + \alpha_{l_0} - i_b$$

$$\phi = i_b - i_c$$

and i_c is the incidence of the section chord with respect to the chord-truss direction (plane of the drag truss) and i_b is the incidence of the section chord with respect to the perpendicular to the beam direction (the perpendicular to the spar web). The distribution of the chord and beam components C and B may then be calculated from

$$dC = c_c\, q\, c\, dy \tag{20}$$

$$dB = c_b\, q\, c\, dy \tag{21}$$

Torsional moments contributed by the sections about some axes in the wing other than the axes of the aerodynamic centers of the sections as, for example, the wing torsional axis, may be desired in some instances. The moment about the torsional axis M_T is found from

$$dM_T = c_{m_{a.c.}}\, q\, c^2\, dy + c_c\, q\, c\, z_T\, dy + c_b\, q\, c\, x_T\, dy \tag{22}$$

where z_T is the distance of the torsional axis below the chord plane through the aerodynamic center and x_T is the distance of the torsional axis behind the beam plane through the aerodynamic center of the airfoil section.

The total forces and moments may then be found from the components or, more conveniently for Z and $M_{a.c.}$, from the summations previously made:

$$\frac{Z}{q}=\frac{Z_b}{q}+C_L\frac{Z_a}{q}$$

and $M_{a.c.}/q$ is a value obtained from (11). In order to find X/q, however, the c_x components should be summed.

Permissible approximations.—For the practical application of this method, certain approximations will often be justifiable. The approximations that will be found convenient and usually justifiable are made by assuming that

$$\cos\theta_z=1$$

and

$$c_{d_0}\sin\theta_z=0$$

The magnitude, but not the direction, of C_L and C_N may then be taken as the same; the following quantities are also equal in magnitude but not in direction:

$$c_{l_0},\ c_n,\ c_z$$

BIPLANE

The present unsatisfactory status of biplane theory and the large number of variables in the biplane shape or arrangement combine to prevent a completely rational solution of biplane problems by either theoretical or empirical methods. It is possible, however, to compute the forces and moments on "conventional" biplane wings by semiempirical methods that give fairly satisfactory results.

In general, the biplane calculations follow the principles and procedure previously outlined for the monoplane, the main extensions therefrom lying in the determination of the lift distribution between the wings and the determination of the biplane effect on the moments of the individual wings. The lift distribution between the wings is found according to the method developed by Diehl in references 5 and 6; the biplane effect on the moments of the individual wings is found according to a procedure outlined later in this report. Although a biplane has no aerodynamic center, a locus of points about which the pitching-moment coefficient of the cellule remains constant can be found. This locus is analogous to the aerodynamic center of the monoplane but lacks its practical utility. Nevertheless, since it leads to a better understanding of biplane phenomena, the locus of points of constant pitching moment will first be discussed.

Locus of points of constant pitching moment.—According to Diehl's solution of the lift distribution between the wings, the lift coefficients of the individual wings plotted as functions of the biplane coefficient are straight lines that intersect at some value of the biplane lift which is, in general, not equal to zero. A typical case is shown in figure 3. These individual wing lifts may be considered to have their points of application at the aerodynamic centers of the individual wings, because, as will be indicated later, the monoplane value of the aerodynamic center of either wing is not affected by the opposite wing; only the basic moment is affected.

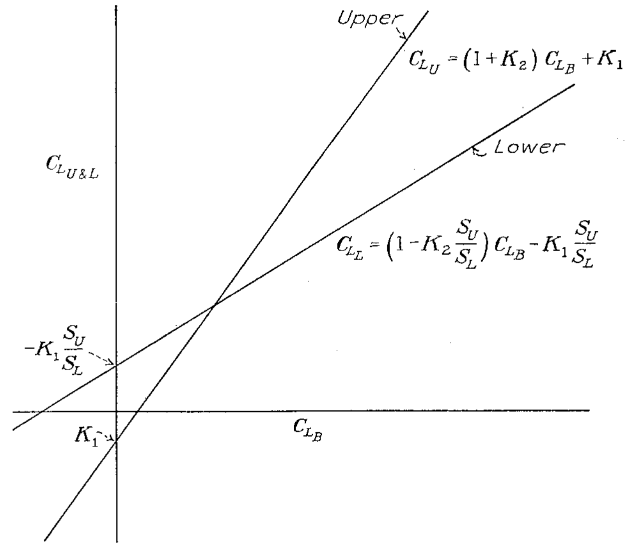


FIGURE 3.—Typical biplane wing lift curves.

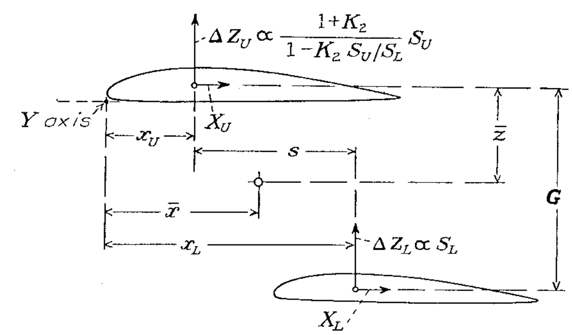


FIGURE 4.—Force diagram for determination of $\bar{x}$ and $\bar{z}$.

Now, if it be assumed that the biplane lift relations are equally applicable to the Z components,[1] it is clear that the location of the center of the Z components may be considered fixed in the direction of x, the ratio of the change in Z force on the upper wing to the change on the lower wing being constant. Reference to figure 4 shows that the x location of the locus of constant moment can be found from the relation

$$\bar{x}=\frac{\left(\dfrac{1+K_2}{1-K_2\dfrac{S_U}{S_L}}\right)S_Ux_U+S_Lx_L}{S_L+\left(\dfrac{1+K_2}{1-K_2\dfrac{S_U}{S_L}}\right)S_U}$$

[1] This assumption is perfectly valid in this case, since the slight error involved is within the error of the semiempirical method of determining the lift distribution.

in which K_2 is Diehl's biplane lift function, as indicated in figure 3.

Unlike the ratio of the Z forces, the ratio of the X components is not independent of the biplane lift because of the nonlinear relation between profile drag and lift in combination with the inequality in lift on the upper and lower wings and because of the trigonometric relation between the lift and its X component. The point about which the pitching moment remains constant therefore moves in the z direction with changes in lift or in X force ratio. Thus, according to figure 4, at any value of the biplane lift for which the X components may be determined

$$\bar{z}=\frac{X_L G}{X_U+X_L}$$

A graphic illustration of the behavior of $\bar{z}$ is given in figure 5, which shows values calculated for the bi-

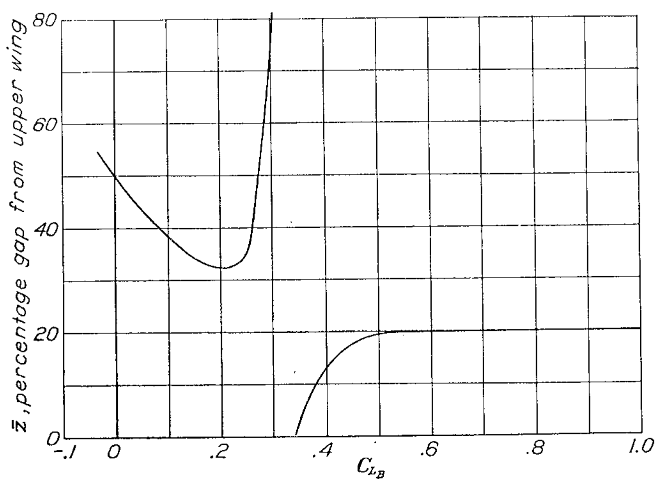


FIGURE 5.—Variation of $\bar{z}$ with biplane lift coefficient.

plane selected for the illustrative example given later in the report. At the higher values of the lift coefficient, the points of constant moment are close to the upper wing. In this condition both upper and lower X components act forward, the upper component being the larger. At a lift coefficient of about 0.33, $\bar{z}$ is indeterminate because the upper and lower X components are equal in magnitude but opposite in direction. In this condition the resultant force is in the z direction and the X components form a pure couple. At the lower lift coefficients both X components act rearward and are of nearly equal magnitude so that $\bar{z}$ is approximately half the gap.

It can be seen from the foregoing discussion that the biplane has no useful counterpart of the monoplane aerodynamic center. For this reason, biplane problems are best solved by proceeding directly to a solution of the forces and moments.

Lift coefficients of individual wings.—The first step in the biplane solution is to determine the lift coefficients of the individual wings as functions of the lift coefficient of the cellule. As previously indicated, this step may be performed according to the method developed by Diehl in references 5 and 6. When this method is used, however, it is recommended that, in cases involving large negative stagger, values of K_{20} be determined from a curve faired through the experimental points of figure 13 of reference 5, rather than from the linear relation between K_{20} and s/c (equation (15a), reference 5).

Distribution of force components.—The wing lifts corresponding to any biplane lift having been found, the force distribution on the individual wings is determined in the same manner as for monoplanes. This procedure neglects the effect of interaction of the individual wings and leads to some error, which is probably small in practical cases.

Pitching moment of biplane cellule.—The pitching moment of the whole cellule about any arbitrarily selected Y axis is found in the same manner as for the monoplane from a summation of the moments due to the Z and X components of force and to the section characteristics. To this total moment a correction, constant throughout the lift range, may be applied to staggered arrangements to obtain a more accurate result.

The correction is based on the fact, indicated by available test data, that the couple created by the lift forces on the individual wings of a staggered biplane with no decalage at zero cellule lift is exactly balanced by predominating increments of moment on the individual wings plus a secondary couple due to the biplane effect on the drags of the individual wings. The moment correction, therefore, constitutes simply a subtraction of the couple created by the K_1 forces due to thickness-gap ratio, stagger, and overhang from the total moment M previously found. Thus

$$M_{YY}=\Sigma M-(K_{10}+K_{11}+K_{13})\ S_U\ sq$$

where K_{10}, K_{11}, and K_{13} are Diehl's lift functions for thickness-gap ratio, stagger, and overhang and s is the stagger measured between the aerodynamic centers of the individual wings.

The function K_{12}, which is due to decalage, is not included in the correction.

Pitching moments of individual wings.—As previously mentioned, the couple due to the K_1 forces, if decalage is neglected, is exactly balanced by predominating opposite moments on the individual wings and a less important couple due to biplane effect on the drags. This drag moment is small compared with the K_1 couple and therefore negligible, since the K_1 couple itself is small. The K_1 couple may therefore be considered to be entirely balanced by increments of moment on the individual wings. No information exists, however, as to the distribution of these moment increments between the upper and lower wings; a consideration of this problem led to the conclusion that a reasonable assumption would be to divide the balancing couple equally between the wings. This assumption leads to very low increments of pitching-moment coefficient on the individual wings; in several cases that have been examined the values were well below 0.01. In view of such low values and the uncertainties in regard to the distribution, it is believed advisable to neglect these increments in computing the pitching moments of the individual wings.

Another biplane effect on the individual wing moments, however, should be taken into account. Its physical cause is not known at present, but it is probably due to the profile drag of the wings, which results in a pressure gradient from the leading to the trailing edge between the wings and to the curvature in the streamlines at each wing induced by the opposite wing. An examination of test data obtained both in flight and in wind tunnels showed that this biplane effect on the wing moments is, for all practical purposes, a linear function of the thickness-gap ratio given by the relation

$$\Delta c_{m_0\left(\frac{t}{G}\right)}=0.1\frac{t}{G}$$

These increments, for the data available, do not noticeably contribute to the resultant biplane moment; the total increment of moment on the upper wing must therefore be approximately equal and opposite to that on the lower wing.

In order to effect the practical application of these increments to the wings, it is assumed: (1) That the increment is distributed along the entire span of the shorter wing but only along that portion of the span of the longer wing that lies within the projected span of the shorter wing; and (2) that the increment of pitching-moment coefficient is distributed uniformly along the span of each wing between the limits of the pitching-moment distribution. On the basis of assumption (1), the value of $\Delta c_{m_0\left(\frac{t}{G}\right)}$ is found for the upper wing from the foregoing relation using the average value of t/G based on the lower wing for the portion of the span affected. Then

$$\Delta c_{m_0\left(\frac{t}{G}\right)_L}=-\Delta c_{m_0\left(\frac{t}{G}\right)_U}\times\frac{S_U'}{S_L'}\times\frac{c_U'}{c_L'}$$

where S_U' is the area of the portion of the upper wing involved.

S_L', the area of the portion of the lower wing involved.

c_U', average chord of the portion of the upper wing involved.

c_L', average chord of the portion of the lower wing involved.

LOAD DISTRIBUTION OVER AIRFOIL SECTION

The solution of the general problem has been completed except that the distribution of the air forces along the chord at each section has not been determined, the net section lift, drag, and pitching-moment coefficients having been employed heretofore rather than the distributed air loads at each section. Although the distribution of the air load around the airfoil section may not always be required, this distribution will be considered in order to make the analysis complete.

General procedure.—The previous analysis gives the section lift coefficient c_{l_0}, the method of finding the normal- and chord-force coefficients c_n and c_c, and the pitching-moment coefficient $c_{m_{a.c.}}$ at each section corresponding to any given loading condition of the complete airplane with which the designer is concerned. The corresponding distribution of the air load over the section will be given in terms of the normal-force coefficient by giving the distribution of the normal-pressure coefficient P along the chord of the section. Of course, this distribution gives no chord force but the chord force is known and may be considered as applied at the aerodynamic center. Its distribution will not be considered, the chord force being small and distributed over only a small distance equal to the wing thickness. Although the moment contributed by this distribution cannot be entirely neglected, the normal-force distribution will be slightly modified, more or less arbitrarily, so that it will give exactly the correct pitching moment about the aerodynamic center.

Determination of normal-pressure coefficients.—As previously stated, the distribution of the air load along the chord is found by determining the normal-pressure coefficient P, that is, the ratio of the pressure difference that may be considered as acting at any point along the chord to the dynamic pressure q. The distribution is defined by the values of P at a number of points along the chord. As with the span load distributions, it is convenient to consider the distribution as made up of two independent parts, one the distribution for zero normal force P_0 and the other an additional distribution giving all the normal force. The total normal-pressure coefficient at each point is then

$$P=P_0+c_nP_a \tag{23}$$

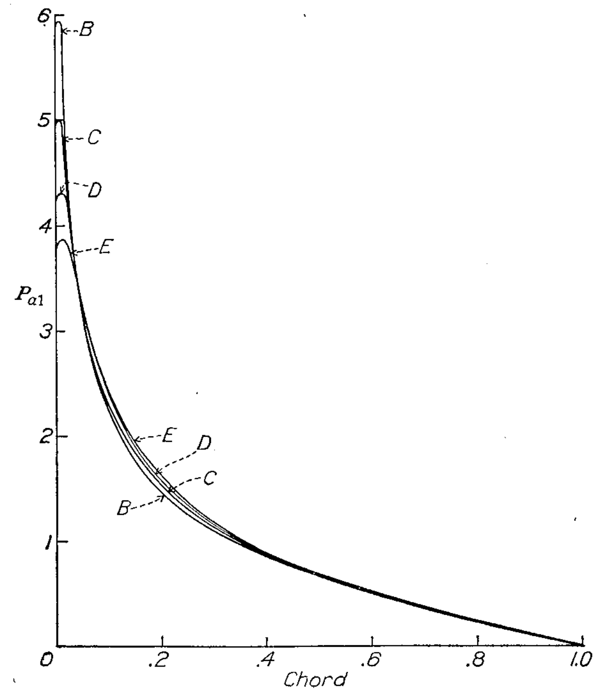
The value of P_a is found from

$$P_a=P_{a1}+\frac{x_{a.c.}}{c}P_{a.c.} \tag{24}$$

where values of P_{a1} and $P_{a.c.}$ are given by curves and tables for typical airfoils in figure 6. The designation of the airfoil class in this respect corresponds to a letter given for each section in the PD column of table I. Values of $x_{a.c.}/c$ are also found from table I by dividing by 100 the x coordinate of the aerodynamic center. A single table of the $P_{a.c.}$ distribution, which is taken as the same for all airfoils, is given in figure 6.

The value of P_0 is found from the so-called "basic distribution," thus

$$P_0=P_b-c_{n_b}P_a \tag{25}$$



Station	$P_{a.c.}$	P_{a1}			
		Class B	Class C	Class D	Class E
0	0	0	0	0	0
1.25	3.2	5.93	4.98	4.32	3.87
2.5	4.5	4.37	4.23	4.02	3.81
5	5.5	3.20	3.22	3.25	3.27
7	5.9	2.63	2.68	2.76	2.81
10	5.7	2.26	2.32	2.39	2.44
15	5.0	1.77	1.85	1.90	1.95
20	4.3	1.47	1.54	1.58	1.62
30	2.9	1.10	1.14	1.16	1.18
40	1.4	.86	.87	.88	.89
50	0	.67	.68	.68	.69
60	−1.4	.51	.51	.51	.51
70	−2.9	.38	.37	.37	.36
80	−4.3	.25	.24	.24	.23
90	−5.7	.13	.12	.12	.11
95	−5.5	.06	.06	.06	.06
100	0	0	0	0	0

NOTE.—Type A distributions have not yet been determined.

FIGURE 6.—Pressure distribution—additional.

The basic distribution P_b and the basic normal-force coefficient c_{n_b} are, in turn, found as the sum of two parts due respectively to moment and camber, thus

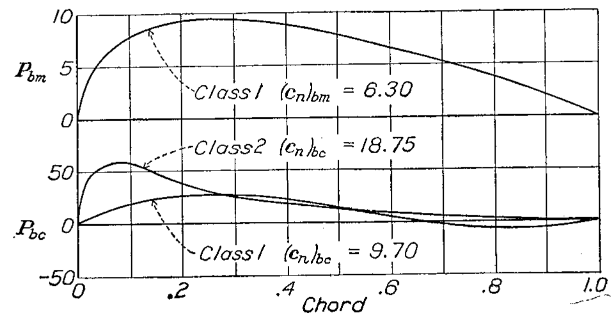
$$P_b = -c_{m_{a.c.}} P_{bm} + \frac{z_c}{c} P_{bc} \tag{26}$$

and

$$c_{n_b} = -c_{m_{a.c.}} (c_n)_{bm} + \frac{z_c}{c} (c_n)_{bc} \tag{27}$$

Values of P_{bm} and the corresponding values of $(c_n)_{bm}$ are given in figure 7, as well as values of P_{bc} and $(c_n)_{bc}$ for airfoils of classes as indicated in the airfoil table by the number following the letter in the PD column. For example, the number 10 indicates that P_{bm} is class 1 and P_{bc} is class 0. The zero signifies that P_{bc} and $(c_n)_{bc}$ are both zero. The values of $c_{m_{a.c.}}$ and the section camber z_c/c are both found from table I, z_c/c being found by dividing the mean camber as given, in percent of the chord, by 100.

When the actual calculation for any given airfoil section is made, values of P_0 and P_a should be calculated and tabulated for the standard stations along the chord. For any section normal-force coefficient c_n, the corresponding values of P are then found simply from (23) by multiplying the values of P_a by c_n and adding to P_0. The actual pressure difference acting at each point in pounds per square foot is, of course, obtained by multiplying by the dynamic pressure in consistent units.



Station	P_{bm}	P_{bc}		
	Class 1	Class 0	Class 1	Class 2
0	0	0	0	0
1.25	2.85		2.5	32.5
2.5	4.25		5.5	47.0
5	6.05		10.0	56.5
7.5	7.10		14.5	59.0
10	7.80		18.0	57.5
15	8.80		23.0	47.5
20	9.30		25.0	37.0
30	9.50		25.0	24.5
40	8.80		20.5	18.0
50	7.75		14.0	13.0
60	6.60		6.0	9.0
70	5.30		−2.5	5.5
80	3.75		−5.5	3.5
90	2.05		−4.5	1.5
95	1.10		−2.5	1.0
100	0		0	0
$(c_n)_{bm}$	=6.30	0	9.70	18.75 $=(c_n)_{bc}$

$$P_b = -c_{m_{a.c.}} P_{bm} + \frac{z_c}{c} P_{bc}$$

$$c_{n_b} = c_{m_{a.c.}} (c_n)_{bm} + \frac{z_c}{c} (c_n)_{bc}$$

FIGURE 7.—Pressure distribution—basic.

Finally, consider briefly how the air pressures are divided between the upper and the lower surfaces. Pressure-distribution diagrams, given elsewhere, indicate the pressure on the upper and on the lower surface as measured from the static pressure as a reference. The designer, however, is not primarily concerned with these pressures but with the pressure differences across the wing covering which, of course, produce the air load on it. These pressure differences are a function of the internal pressure, that is, the pressure within the wing. If the wing is well vented, the internal pressure and the upper and lower covering loads may be estimated. For this purpose the lower-surface pressure distribution is estimated, remembering that the positive pressure cannot exceed by $1q$ the static pressure, and the upper-surface distribution determined from the known values of the differential-pressure coefficient P. If greater accuracy is required, the method of reference 7 or the results of reference 8 may be employed to calculate the pressure distribution on the lower surface.

SAMPLE CALCULATION

MONOPLANE

In order to make this example as general as possible, a case is chosen for which the design condition representing a 30-foot-per-second gust encountered at high speed causes the lift coefficient to exceed the usual maximum lift coefficient for the airfoil. The example does not, however, deal specifically with the procedure to be followed in cases for which portions of the wing are replaced by the fuselage or nacelles. The treatment, nevertheless, is exactly the same in such cases if the standard N. A. C. A. wing area, including those portions of the wing imagined as inside the fuselage or nacelles, is used for S. The solution is thus found by considering those portions of the wing to be actually present and undisturbed, the wing being imagined as extending continuously over those portions of the span. The calculated loads for these imaginary portions of the wings may later be applied to the fuselage and nacelles. In extreme cases a special treatment may be required. A wing of the U. S. A. 35 type is chosen so that some aerodynamic twist will be present in spite of the fact that the wing is not twisted with reference to the airfoil chords. The drag truss, for generality, is taken at an angle to the plane of the airfoil chords. The analysis is begun from the airplane drawing in figure 1 and from the following data on the airplane and wing:

Weight	1,000 lb.
Power	35 hp.
Propeller efficiency	75 percent.
High speed	95.3 f. p. s.
Wing incidence	4°.

Wing: U. S. A. 35 type, aspect ratio 5, rounded tips, area 180 sq. ft., root chord 8.268 ft., taper ratio 0.5, no geometric twist, beam direction perpendicular to chord, drag truss (chord direction) inclined upward at the leading edge with respect to the chord 4° at root to 2° at tip, airfoil section at root U. S. A. 35-A, at tip U. S. A. 35-B.

Calculation of wing aerodynamic center.—The first step in the procedure is to choose the reference axes. The axes are chosen, for generality, originating at the center of gravity with one axis parallel to the thrust line although, in this instance, some simplification might have resulted from choosing an axis in the direction of an airfoil chord because this direction is the same along the wing (no geometric twist) and perpendicular to the beam direction. Table IV is then filled in to give the necessary data for computing the aerodynamic center of the wing. The various columns leading first to the calculation of the additional-load curves for $C_L=1$ and finally to the position of the wing aerodynamic center are filled in as follows:

Column 1.—Stations along the span chosen arbitrarily or to agree with those in table II. These stations are indicated on the airplane drawing (fig. 1).

Column 2.—Values of L_a from additional-load table (table II) for aspect ratio 5, taper ratio 0.5.

Column 3.—Values of c from the airplane drawing.

Column 4.—Values of $c_{l_{a1}}$ from the multiplication of (2) by S/cb.

Column 5.—Values of a_0 from airfoil characteristics (table I) interpolating between U. S. A. 35-A and U. S. A. 35-B sections for intermediate sections of wing.

Column 6.—Values of $c_{l_{a1}}/a_0$ from (4) and (5).

Column 7.—Values of α_{l_0} by the same method as (5).

Column 8.—Values of $-i$, the incidence of the wing chords with respect to the x axis with reversed sign, from airplane drawing.

Column 9.—Values of c_{d_0}. The profile-drag coefficients are calculated for each section as indicated in table IV-A. The thickness ratio of each section t/c is obtained from the airplane drawing. Minimum profile-drag coefficients $c_{d_{0_{min}}}$ are obtained from a curve of profile-drag coefficient against section thickness, paralleling the typical curve given in reference 9 (fig. 91) but passing through the values indicated in table I for the U. S. A. 35-A and U. S. A. 35-B sections. Values of $c_{l_{opt}}$ and $c_{l_{max}}$ are obtained from table I. From the values in the preceding columns, the ratio $\dfrac{|c_{l_{a1}}-c_{l_{opt}}|}{c_{l_{max}}-c_{l_{opt}}}$ is computed. From this ratio and the curve of figure 2, the Δc_{d_0} values are obtained, which are added to the values of $c_{d_{0_{min}}}$ to give the desired c_{d_0}.

Column 10.—Values of θ_{z_a}. From the addition of (6), (7), and (8), where θ_{z_a} is $\left(\dfrac{c_{l_{a1}}}{a_0}+\alpha_{l_0}-i\right)$. (See equations (5) and (6).)

Columns 11 to 16.—From preceding columns.

Column 17.—Values of $c_{x_{a1}}$ from (13)+(14) following equation (5).

Column 18.—Values of $c_{z_{a1}}$ from (15)+(16) following equation (6).

Column 19.—Values of z from the airplane drawing, upward coordinate of aerodynamic center of section. May be obtained from airplane drawing after locating the aerodynamic center of the tip and the center sections from table I. The corresponding aerodynamic-center positions for the intermediate sections may be taken along the straight line joining these points except for the rounded-tip sections.

Column 20.—Values of x, backward coordinate of aerodynamic center of section, obtained from the airplane drawing as with (19).

Columns 21 to 24.—Products from previous columns.

These pitching-moment and loading results are plotted as in figures 8 and 9, and the areas measured to find $M_{Z_{a1}}/q$, $M_{X_{a1}}/q$, Z_{a1}/q, and X_{a1}/q. From these values the coordinates of the wing aerodynamic center are found from equations (7) and (8).

$$x_{a.c.}=\frac{-110.08}{180.0}=-0.612 \text{ ft.}$$

$$z_{a.c.}=\frac{15.08}{4.18}=3.6 \text{ ft.}$$

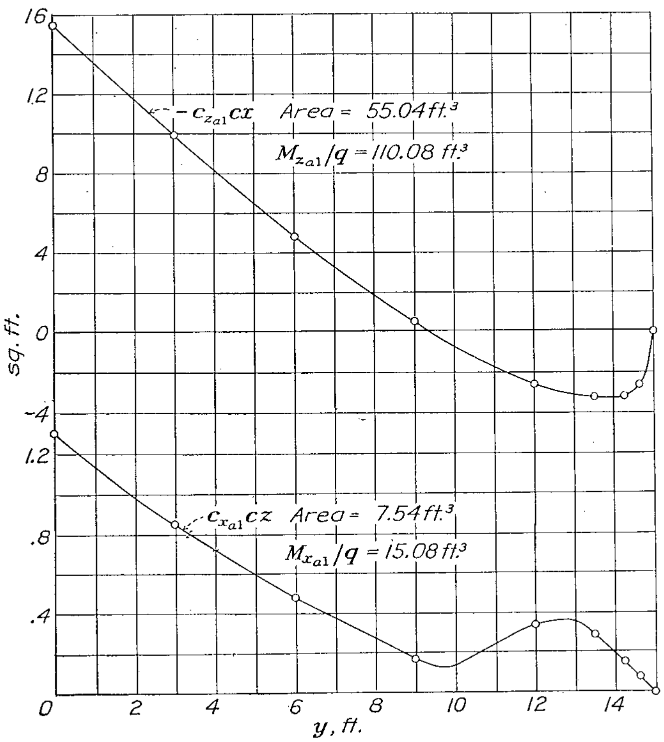


FIGURE 8.—Plots for the determination of the components of the additional wing pitching moments and the wing aerodynamic center.

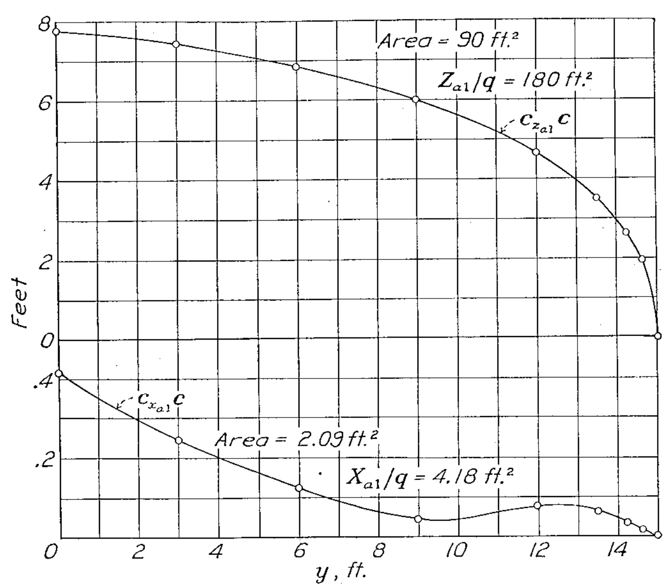


FIGURE 9.—Distribution of additional z and x components and determination of additional wing components.

Calculation of wing pitching moment about aerodynamic center.—The next step is to carry out practically the same procedure for the basic load distribution in order to find the wing pitching moment about the aerodynamic center. The origin of coordinates is moved to the aerodynamic center and another set of calculations is made for the basic load distribution as indicated in table V. The only differences worth noting are the different values taken from the tables, values for the basic load distribution L_b from table III, and the method of obtaining from these the c_{l_b} values in table V. The c_{l_b} values follow from those in the second column taken from table III, by multiplying by $\epsilon a_0 S/cb$.

The term ϵa_0 takes into account the aerodynamic twist of the wing, which is assumed to vary linearly along the span. The twist ϵ is measured with respect to the zero lift directions for the center and tip sections, being positive when the effective incidence is washed in from the center toward the tip. It is evident that the term ϵa_0 is a c_l difference between airfoil sections corresponding to the center and tip sections when the section angles of attack have the same relation as in the wing. In other words, ϵa_0 may be calculated as follows:

$$\epsilon a_0=[a_0(\alpha-\alpha_{l_0})]_{tip}-[a_0(\alpha-\alpha_{l_0})]_{center}$$

This procedure is strictly correct theoretically only when a_0 does not vary along the span. When a_0 varies, the best practical result is probably obtained by calculating ϵa_0 as a Δc_l for an α near the value at which the load distribution is desired.

The value of α may be taken as zero for the center section and, because no geometrical twist is present, the value of α at the tip is then also zero in this instance.

$$\begin{aligned}\epsilon a_0&=(-a_0\alpha_{l_0})_{tip}-(-a_0\alpha_{l_0})_{center}\\&=[-(0.099)(-5.2)]-[-(0.095)(-8.0)]\\&=0.515-0.760=-0.245\end{aligned}$$

Values of the factor $\epsilon a_0 S/cb$ are then obtained at each station along the span by which the values taken from table III are multiplied to obtain the c_{l_b} values. From the c_{l_b} values, the calculations proceed to the final results, which are given in the last columns of table V. These results are plotted and the areas determined to find $(M_{x_b}/q)_{a.c.}$, $(M_{z_b}/q)_{a.c.}$, and $(M_s/q)_{a.c.}$ as in figure 10. These values are added to obtain $M_{a.c.}/q$.

$$\frac{M_{a.c.}}{q}=0+4.84-113.36=-108.5$$

which, multiplied by q, gives $M_{a.c.}$, the required pitching moment of the wing about its aerodynamic center.

Calculation of forces and moments in accelerated-flight condition.—The exact procedure to be followed from this point on is dependent on the result desired. If a result meeting arbitrary design requirements is desired, the particular specified procedure will be followed. If, on the other hand, the most reliable actual air loads for a given design condition are desired, another procedure may be advisable.

From the method of references 1 and 10, for example, the applied load factor n_1 is determined and the wing normal-force coefficient C_{N_1} is taken as $n_1 s/q_L$ where s is the effective wing loading and q_L is the dynamic pressure for the design speed. Corresponding values of the chord-force coefficient c_c are obtained as more or less arbitrarily specified, and the pitching characteristics of the wing are rather arbitrarily given by specifying that the center-of-pressure position be taken as the most forward position for the wing between $C_L=C_{N_1}$ and $C_L=C_{L_{max}}$, unless C_{N_1} exceeds $C_{L_{max}}$, in which case a value taken from the extended center-of-pressure curve is specified. After C_L is calculated from the specified C_{N_1}, the lift-coefficient distribution may be found by adding the basic and additional lift coefficients in accordance with the relation

$$c_{l_0}=c_{l_b}+C_Lc_{l_{a1}}$$

and including, when necessary, the tip corrections given in appendix A. The corresponding specified values of the center of pressure and of c_c may then be applied at each section and the forces and moments resolved as desired for structural analysis.

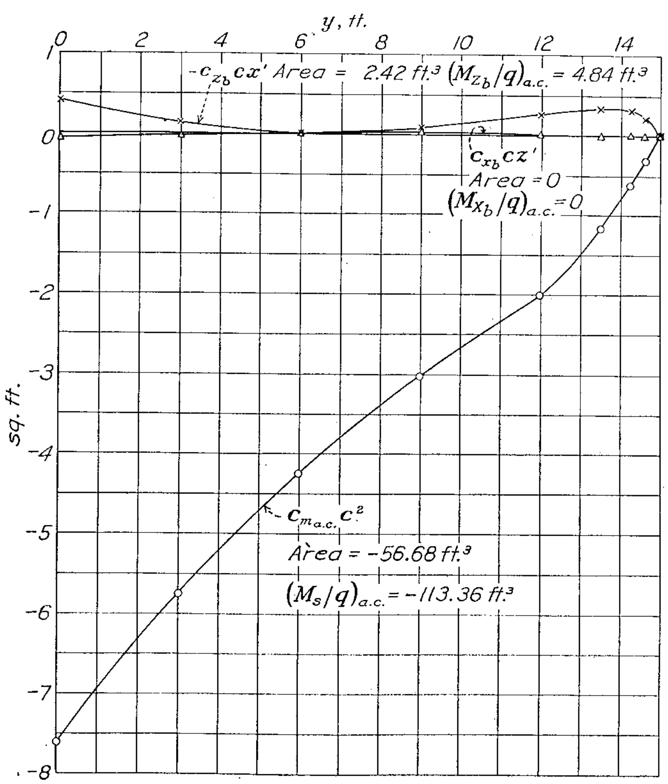


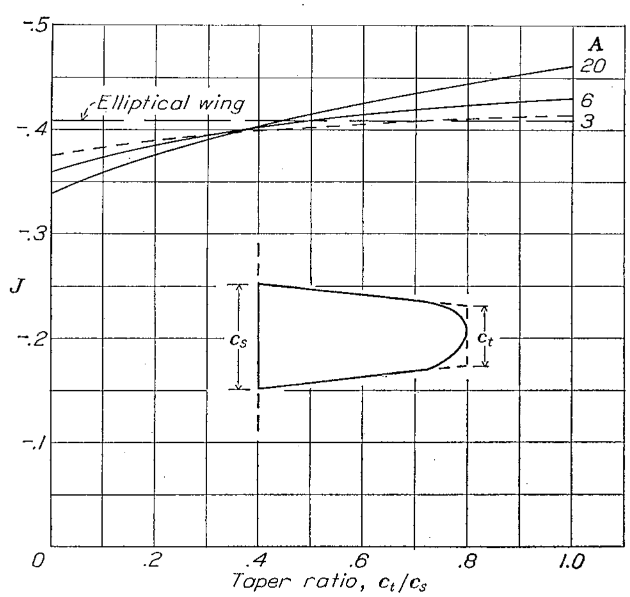
FIGURE 10.—Plots for the determination of the components of the basic wing pitching moment.

The foregoing procedure, however, will not be followed in this example. Specified design conditions and methods vary and, in many instances, it is believed that designers will wish to investigate loadings under conditions other than those specified. The example will therefore be carried through using the procedure that may be expected to give the best approximation to the actual air forces.

The first step is to obtain the lift coefficient C_L' corresponding to the steady-flight condition before entry into the gust. For the present example, this condition is represented by high-speed level flight. The corresponding C_L' value is obtained from the balance diagram for the airplane for this condition.

For the construction of the balance diagram, it is necessary to know the angle of the flight path so that the direction of the weight vector may be determined. A trial value of C_L' is first taken, assuming a down tail load of 30 pounds, $\frac{W+F_t}{qS}=0.530$. The wing angle of attack as measured by α_s, the angle of attack referred to the chord of the center section, may then be determined by the method indicated in figure 11:

$$\alpha_s=\frac{C_L}{a}+(\alpha_{l_0})_s+J\epsilon$$



Determine the angle of attack from:

$$\alpha_s=C_L/a+(\alpha_{l_0})_s+J\epsilon$$

where α_s, angle of attack referred to the chord of the central section of the wing.
C_L, wing lift coefficient.
a, wing lift-curve slope per degree.
$(\alpha_{l_0})_s$, angle of zero lift of the central section.
ϵ, angle of aerodynamic twist.
A, aspect ratio.
$C_L=a[\alpha_s-(\alpha_{l_0})_s-J\epsilon]$ or, angle of zero lift for the wing referred to the chord of the center section$=(\alpha_{l_0})_s+J\epsilon$.

FIGURE 11.—Lift in terms of angle of attack for tapered wings with twist.

From figure 12

$$a=f\frac{a_0}{1+\frac{57.3a_0}{\pi A}}$$

Taking $a_0=0.096$ as a mean value for the sections of the wing

$$a=0.999\frac{0.096}{1+\frac{57.3\times0.096}{\pi 5}}$$

$$=0.0711$$

The angle of zero lift for the root section $(\alpha_{l_0})_s$ is taken from table I. The twist ϵ is $\epsilon a_0/a_0$ or $-0.245/0.096=-2.55°$. The factor J from figure 11 is -0.408. Then

$$\alpha_s=\frac{C_L}{a}+(\alpha_{l_0})_s+J\epsilon$$

$$=\frac{0.530}{0.0711}-8+(-0.408)(-2.55)$$

$$=0.5°$$

As the incidence of the center section is 4°, the angle of the thrust line with the horizontal is $0.5°-4.0°=-3.5°$. The weight vector may therefore be drawn as indicated in figure 1 and the pitching moments may be taken about the wing aerodynamic center to determine the tail load F_t. The dynamic pressure is

$$q=\frac{1}{2}(0.002378)(95.3)^2$$

$$=10.79 \text{ lb./sq. ft.}$$

$$M_{a.c.}=\left(\frac{M_{a.c.}}{q}\right)q$$

$$=-108.52\times10.79$$

$$=-1{,}171 \text{ lb.-ft.}$$

Although for other purposes, such as balance calculations, a better moment analysis may be necessary, the thrust moment in this case may be determined with sufficient accuracy on the assumption that three-quarters of the thrust is used in overcoming parasite drag, which may be assumed to act approximately along the thrust axis and therefore to contribute no moment about the wing aerodynamic center. The thrust is

$$\frac{35\times550\times0.75}{95.3}=151.5 \text{ lb.}$$

Then writing the moment equation

$$(0.25\times151.5\times3.82)+(1{,}000\times0.82)-1{,}171-F_t14.62=0$$

$$F_t=-14.3 \text{ lb.}$$

The final value of the lift coefficient for steady flight C_L' may then be computed

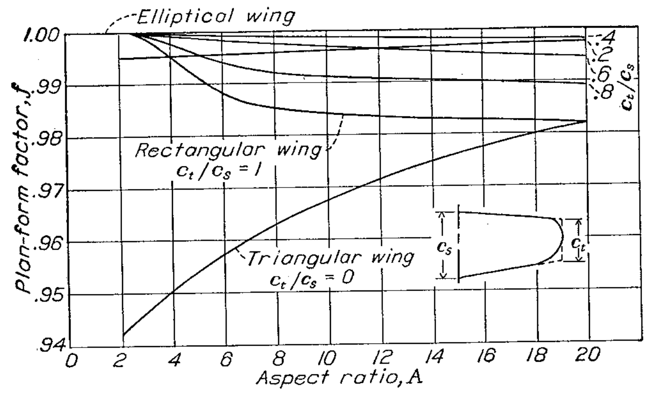
$$C_L'=\frac{1{,}000+14.3}{10.79\times180}$$

$$=0.522$$

The new wing lift coefficient C_L after entry into the gust is now determined from the slope m of the wing lift curve and the gust velocity U

$$C_L=C_L'+m\frac{U}{V}$$

$$C_L=0.522+4.07\frac{30}{95.3}=1.803$$



Compute the lift-curve slope from the equation:

$$m=f\frac{57.3a_0}{1+57.3a_0/\pi A} \text{ or } a=f\frac{a_0}{1+57.3a_0/\pi A} \text{ (per degree)}$$

where m, lift-curve slope (per radian).
a_0, section lift-curve slope (per degree).
a, wing lift-curve slope (per degree).
A, aspect ratio (b^2/S).
f, plan-form factor.

When the lift-curve slope is normal, the following approximate equations may be used:

$$m=f\frac{57.3a_0}{1+1.843/A} \text{ or } m=f\frac{m_6}{0.761+1.403/A}$$

where m_6, lift-curve slope (per radian) for wing of aspect ratio 6 with rounded tips.

$$\text{Taper ratio}=\frac{c_t}{c_s}=\frac{\text{tip chord}}{\text{center chord}}.$$

FIGURE 12.—Values of f for computing the lift-curve slope.

where the value of m is found from figure 12 and table I.

The lift distribution is now determined by calculating the c_{l_0} values in table VI from

$$c_{l_0}=c_{l_b}+C_Lc_{l_{a1}}$$

where c_{l_b} is taken from table V and $c_{l_{a1}}$, from table VI. The calculations indicated in table VI proceed then to the determination of chord and beam components from equations (18) and (19).

It will be noted that the steady-flight value of the profile-drag coefficient c_{d_0}' has been used in the accelerated-flight condition. This procedure should be followed when a large forward-acting chord force is critical for the structure.

The last three columns of table VI give the required data on the air-force distribution as chord and beam forces and pitching moments per running foot of span. In order to complete the balance diagram, however, the total air forces and moments on the wing are required. The pitching moment of the entire wing in this case is the same as that previously found for the steady-flight condition, because $M_{a.c.}/q$ has not changed.

$$M_{a.c.}=\left(\frac{M_{a.c.}}{q}\right)q$$

$$=-108.5\times10.79=-1{,}171 \text{ lb.-ft.}$$

The total wing air-force component Z perpendicular to the thrust line may be found with sufficient accuracy from the Z_{a1}/q value previously determined without the necessity of resolving and summing the section forces

$$Z=\left(C_L\frac{Z_{a1}}{q}\right)q$$
$$=1.803\times180.0\times10.79$$
$$=3{,}510 \text{ lb.}$$

The total component X for the wing, however, should be found by resolving the section forces along the x direction and summing to find the total force component X. The values of c_{d_0} and c_{l_0} are taken from table VI and resolved by equation (15) to find the c_x values at the various sections along the span. In this example a large forward-acting chord component is assumed to be conservative so that the profile-drag coefficient c_{d_0} for the accelerated-flight condition is taken as equal to that in the preceding steady-flight condition for the determination of c_x. These values are then multiplied by c, plotted as in figure 13, and the area determined to give X. The result is

$$X=-42.2\times10.79=-455 \text{ lb.}$$

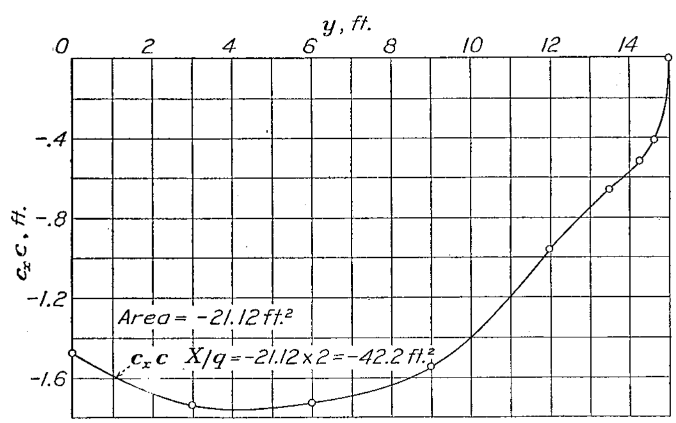


FIGURE 13.—Distribution of X component in the accelerated-flight condition.

It will be noted that the preceding calculations of the forces in the accelerated-flight condition have been made on the basis that the dynamic pressure q remains unaltered after encountering the gust. This condition is possible when the gust has a small angle with the vertical, and the procedure is further justified by the fact that the gust velocities specified have been largely determined on the basis of their effects on airplanes as indicated by the simple gust formula without taking into account such changes of velocity. In some instances, however, it may be desired to take into account the dynamic-pressure increase due to a gust, in which case the gust velocity should not be taken as nearly vertical but may be taken at an angle with the horizontal and the angle determined to give the maximum load.

The total air forces and moments are now known and may be applied at the aerodynamic center of the wing so that the balance diagram may be completed for the accelerated-flight condition, thus completing the solution with the exception of the determination of the air-force distributions over the ribs.

Calculation of the air-force distribution over a rib.— In order to complete the example, the rib distribution will be determined for the central section of the wing. Reference to table I will show the pressure-distribution classification of the U. S. A. 35–A section to be E10. The additional pressure distribution is therefore found from the class E P_{a1} distribution. Values of P_{a1} and $P_{a.c.}$ are taken from figure 6, and the additional pressure distribution is then calculated from equation (24)

$$P_a=P_{a1}+\frac{x_{a.c.}}{c}P_{a.c.}$$

where $x_{a.c.}$ is the distance of the section aerodynamic center forward of the quarter-chord point, from table I. The calculation may be carried out in tabular form as shown in table VII.

The basic pressure distribution as given by values of P_b is then found. From equation (26)

$$P_b=-c_{m_{a.c.}}P_{bm}+\frac{z_c}{c}P_{bc}$$

The designation in table I of the basic pressure distribution for this airfoil section is indicated by the number 10 following the E. The designation 10 indicates that P_{bm} is class 1 and P_{bc} is class zero, that is, $P_{bc}=0$. The basic pressures may then be computed as indicated in table VIII, taking values of P_{bm} from figure 7 and the value of the pitching-moment coefficient $c_{m_{a.c.}}$ from table I.

The zero lift distribution given by values of P_0 is then obtained by deducting a part of the P_a distribution corresponding to c_{n_b} according to equation (25)

$$P_0=P_b-c_{n_b}P_a$$

The value of c_{n_b} is obtained from equation (27)

$$c_{n_b}=-c_{m_{a.c.}}(c_n)_{bm}+\frac{z_c}{c}(c_n)_{bc}$$

where the values of $(c_n)_{bm}$ and $(c_n)_{bc}$ are given in figure 7, for the various distributions. In this case $(c_n)_{bm}=6.30$ and $(c_n)_{bc}=0$, hence

$$c_{n_b}=0.111\times6.30+0$$
$$=0.699$$

and

$$P_0=P_b-0.699P_a$$

For the accelerated-flight condition, the pressure distribution as given by values of P is then found from

$$P=P_0+c_nP_a$$

where c_n in this case is the same as the beam-component coefficient c_b and is taken from table VI for the center section,

$$P=P_0+1.695P_a$$

Finally, the actual pressure differences p are obtained by multiplying by the dynamic pressure, $q=10.79$ pounds per square foot. These pressures, calculated as indicated in table VIII and giving the final result, are plotted in figure 14.

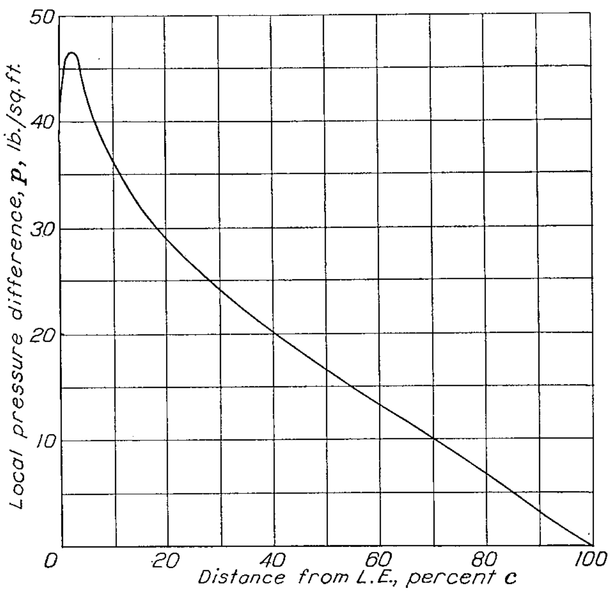


FIGURE 14.—Pressure distribution over center section in the accelerated-flight condition.

BIPLANE

The following example is given to illustrate the application of the method to biplane problems and also to illustrate the alternative method of finding the force distribution in a case where the empirical tip corrections may be important. A simple biplane (fig. 15)

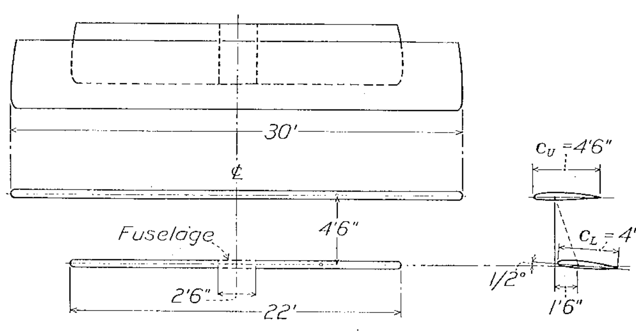


FIGURE 15.—Biplane cellule for illustrative example.

has been chosen in order to avoid, as far as possible, steps that have already been illustrated in the example for the general monoplane. The calculations are made for an airplane having the following characteristics:

Weight .... 1,636 lb.
High speed .... 100 m. p. h. ($q=25.6$ lb./sq. ft.).
Wing cellule:

Upper: N. A. C. A. 2412 section, span 30 ft., chord 4½ ft., area 135 sq. ft., no taper, no twist, no dihedral, incidence 0°.
Lower: N. A. C. A. 2412 section, span 22 ft., chord 4 ft., area (including projection through fuselage) 88 sq. ft.

Distance from leading edge of upper wing to *c. p.* of tail plane (l), 15 ft.
Distance from leading edge of upper wing to *c. g.* ($x_{c.g.}$), 1.75 ft.

The objective of the calculations in this example is the solution of the force and moment distribution along the upper wing in the 30-foot-per-second gust, corresponding to design Condition I of reference 1. In order to make the example more illustrative, the wing lift coefficient for the initial condition of steady flight is found from a balance computation, as in the monoplane example, but only the tail load is considered as an extraneous force. Because the biplane has no aerodynamic center, an exact balance can be obtained only through a process of trial and error; in the example the calculations are not repeated to obtain the exact solution.

Lift coefficients of individual wings.—The following data are pertinent to the solution of the lift distribution between the wings:

Effective stagger s_0 between one-third-chord points at zero lift, 1.67 ft.

Overhang, $\frac{30-22}{30}=0.267$.

$\frac{s_0}{c_L}=\frac{1.67}{4}=0.42$. $\qquad \frac{c_L}{c_U}=\frac{4}{4.5}=0.889$.

$\frac{G}{c_L}=\frac{4.5}{4}=1.125$. $\qquad \frac{t}{G}=\frac{0.12}{1.125}=0.1065$.

$\frac{t}{c}=0.12$.

With the foregoing data, the method of reference 6 yields—

$$K_{10}=-0.0178$$
$$K_{11}=0.0123$$
$$K_{12}=-0.0274$$
$$K_{13}=-0.0178$$
$$K_1=K_{10}+K_{11}+K_{12}+K_{13}=-0.0507$$
$$F_2K_{20}=0.0951$$
$$K_{21}=0.0083$$
$$K_{22}=0.0650$$
$$K_2=F_2K_{20}+K_{21}+K_{22}=0.1684$$

$$C_{L_U}=C_{L_B}+(K_1+K_2C_{L_B}) \tag{28}$$
$$=1.168C_{L_B}-0.0507$$

$$C_{L_L}=C_{L_B}-(K_1+K_2C_{L_B})\frac{S_U}{S_L} \tag{29}$$
$$=0.742C_{L_B}+0.0778$$

Wing lift coefficient in steady flight (first trial).—Neglecting tail load,

$$C_{L_B}'=\frac{W}{(S_U+S_L)q}=\frac{1,636}{(135+88)25.6}=0.286$$

With this value of C_{L_B}' the general method can be applied to each wing for the lift coefficients derived from equations (28) and (29) to find the moment about some arbitrary Y axis and, from this result, the tail load. The simplicity of the biplane cellule chosen permits, however, a relatively simple solution of the moment. Since the wings are rectangular and of constant section, the aerodynamic centers of each wing lie on the locus of the aerodynamic centers of the sections; the resultant wing forces may therefore be considered to apply at

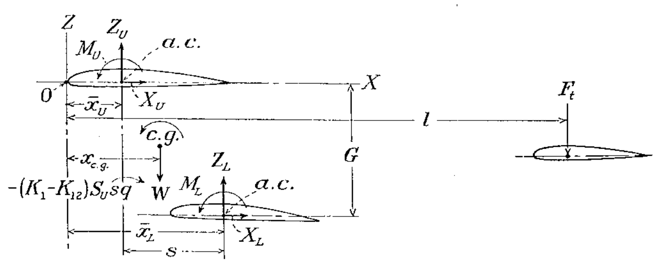


FIGURE 16.—Skeleton diagram of airplane for biplane example.

these centers. Reference to figure 16 indicates that the moment about the axis 0 may be expressed as

$$Z_U\,\bar{x}_U+Z_L\,\bar{x}_L+M_U+M_L+X_L\,G$$
$$=F_t l+W\,x_{c.g.}+(K_1-K_{12})S_U\,sq$$

in which s is now the stagger between the aerodynamic centers of the individual wings, and $(K_1-K_{12})S_U sq$ is the moment correction to allow for the increments of moment, which are not taken into account on the individual wings.

For the steady-flight condition the resultant forces at the aerodynamic centers of the individual wings are

$$Z_U'=C_{L_U}'\,q\,S_U$$
$$=(1.168C_{L_B}-0.0507)\times 25.6\times 135$$
$$=1{,}035 \text{ lb.}$$
$$Z_L'=C_{L_L}'\,q\,S_L$$
$$=(0.742C_{L_B}+0.0778)\times 25.6\times 88$$
$$=654 \text{ lb.}$$

The forces X_U' and X_L' are found by summation of the force components along the span in accordance with the general procedure described in the report. For this purpose the span distribution of c_{l_0}' (or c_z') has been found according to the alternative method given in appendix A, neglecting the tip corrections, which at low lift coefficients are very small.

$$X_U'=1.78 \text{ lb.}$$
$$X_L'=6.14 \text{ lb.}$$
$$M_U=C_{m_0}S_U c_U q=-684 \text{ lb.-ft.}$$
$$M_L=C_{m_0}S_L c_L q=-397 \text{ lb.-ft.}$$
$$K_1-K_{12}=-0.0234$$

With the foregoing data, and from the airplane characteristics, $F_t=69$ lb., acting downward. The corrected value of C_{L_B}' is

$$C_{L_B}'=\frac{1{,}636+69}{25.6\,(135+88)}=0.30$$

which is the value taken for the initial steady-flight condition prior to entry into the gust.

Wing lift coefficient in accelerated flight.—The increment in lift coefficient due to the gust is determined from the slope m of the cellule lift curve and the gust velocity U. The slope of the lift curve may be deter-

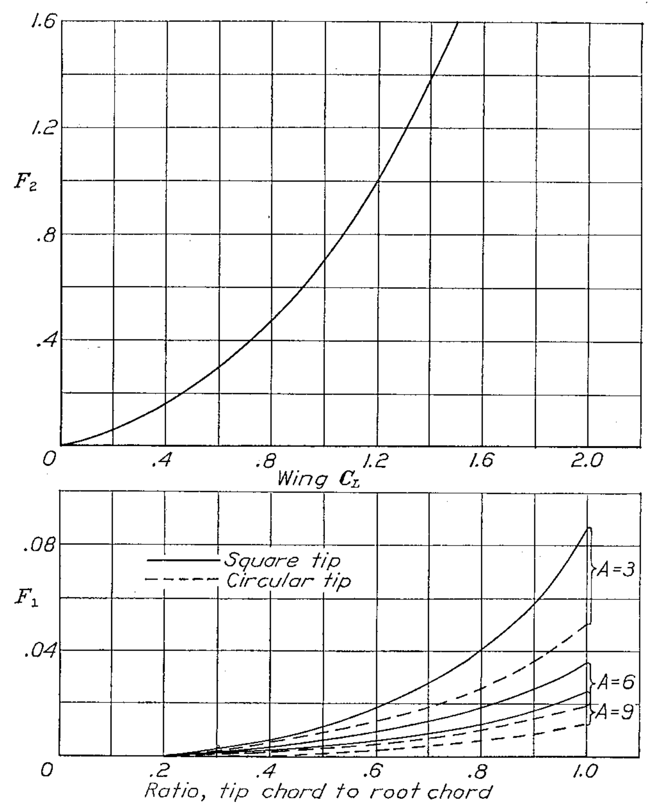


FIGURE 17.—Correction factors for wing C_L.

mined, as in the case of the monoplane, from the expression

$$m=f\frac{57.3a_0}{1+\dfrac{57.3a_0}{\pi A}}$$

For the biplane, f may be taken as unity and $A=\dfrac{(kb)^2}{S_U+S_L}$ in which Munk's span factor k may be determined from reference 11. In the present example,

$$m=\frac{57.3\times 0.097}{1+\dfrac{57.3\times 0.097}{\pi\dfrac{(1.02\times 30)^2}{135+88}}}=3.91$$

$$C_{L_B}=C_{L_B}'+m\frac{U}{V}$$
$$=0.30+3.91\frac{30}{146.6}=1.10$$

From this point, the distribution of forces and moments on the upper wing are to be determined by using the method for finding the span load distribution given in appendix A. The first step is to find the lift coefficient of the upper wing.

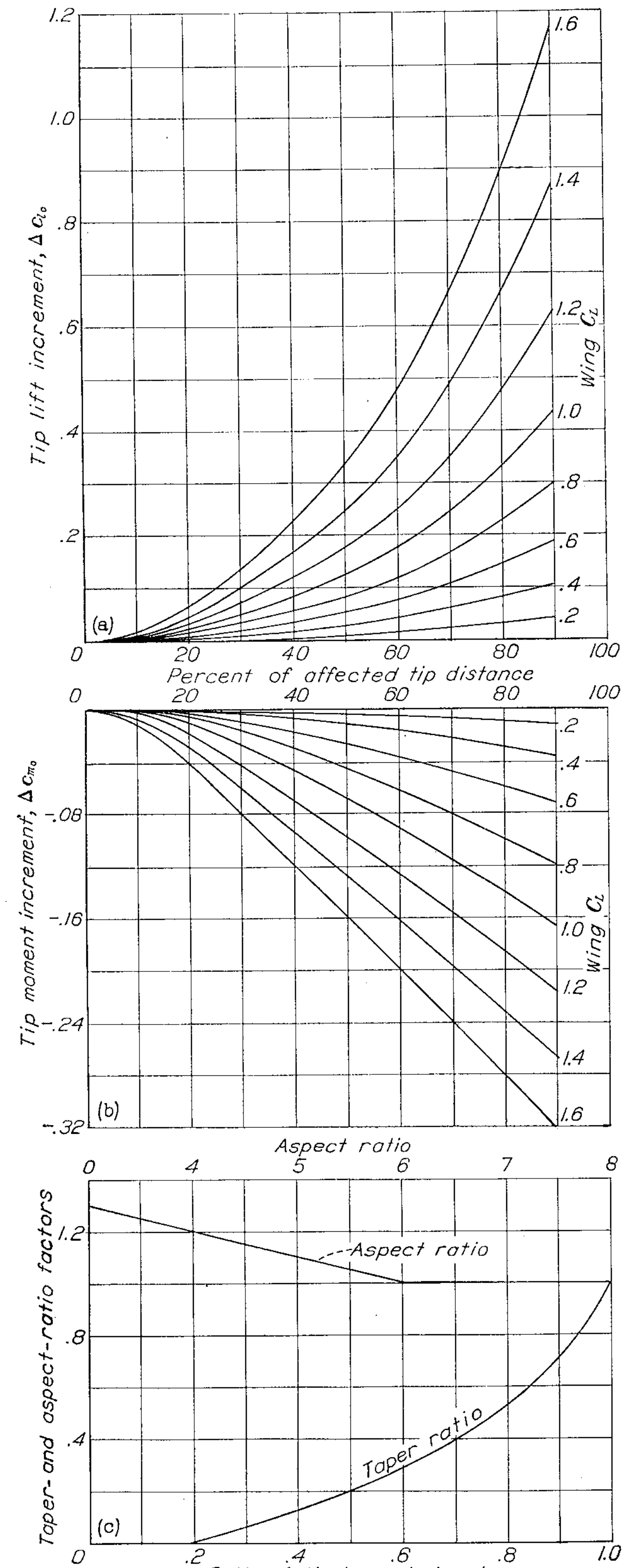


FIGURE 18.—Tip corrections.

1. From C_{L_B} and equation (28)

$$C_{L_U}=1.168\times1.10-0.0507=1.233$$

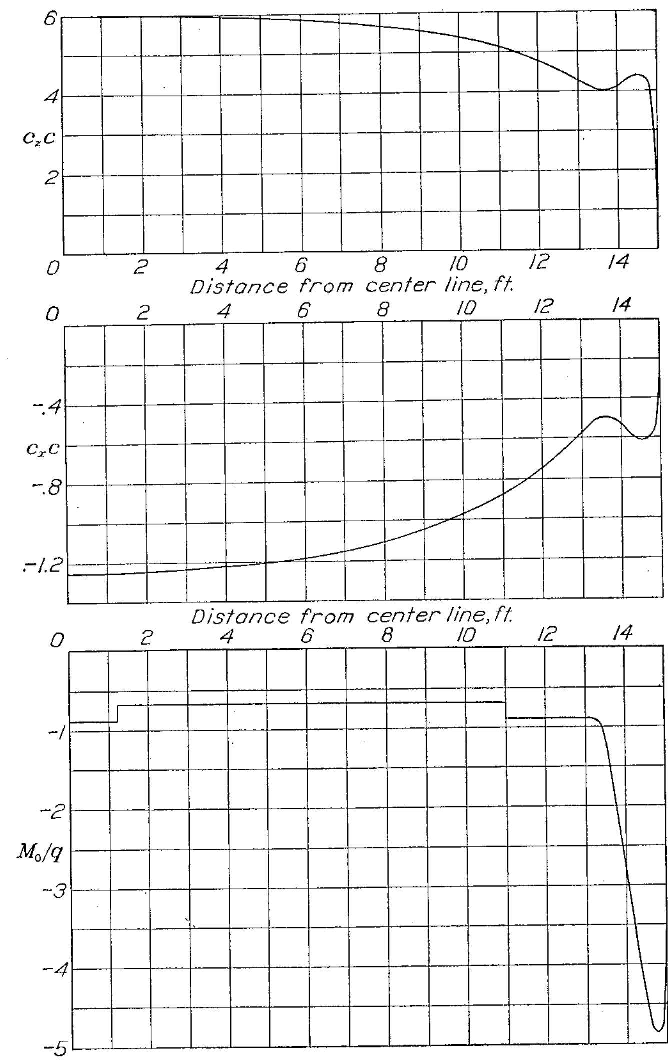


FIGURE 19.—Distribution of forces and moments on upper wing of biplane.

2. In order to reduce C_{L_U} to allow for the tip correction, F_1 and F_2 are found from figure 17

$$F_1=0.038$$

$$F_2=1.06$$

$$F_1\times F_2=0.040$$

The value of C_{L_U} used to enter the charts is (tables IX and X)

$$C_{L_U}''=1.233-0.040=1.193$$

3. The aspect ratio of the upper wing is

$$\frac{(30)^2}{135}=6.67$$

4. The distribution of $c_{l_{a1}}$ is found from table IX, which gives the distribution for aspect ratio 6. The aspect ratio of the upper wing will be considered herein as sufficiently close to 6 to require no interpolation.

Values of $c_{l_{a1}}$ are tabulated in column 2 and values of $C_{L_U}'' \times c_{l_{a1}}$ are tabulated in column 3 of table XI.

5. Since there is no twist, the values of c_{l_b} are zero.

6. The tip increments are determined from figure 18 and are tabulated in column 4 of table XI.

7. The tip corrections are added to the values of c_{l_0}'' to obtain the final c_{l_0} values tabulated in column 5 of table XI.

From this point the procedure follows the general method of this report to find the values of $c_z c$ of column 14 and $c_x c$ of column 17. The values of Δc_{d_0} given in table XI have been computed for the initial steady-flight condition in accordance with the principle of the delay in the growth of the boundary layer in accelerated flight. These values give the distribution required; they are plotted in figure 19.

In order to find the moment distribution, the basic section moment coefficients are tabulated in column 18, and to these are added the increments due to t/G and the tip effect. The value of $\Delta c_{m_0\left(\frac{t}{G}\right)}$ is found from the expression

$$\Delta c_{m_0\left(\frac{t}{G}\right)} = 0.1\left(\frac{t}{G}\right)_{av}$$

$$= 0.1 \times 0.1065 = 0.011$$

These values are applied only along that portion of the span of the wing which lies between the projected tips of the lower wing, as indicated in column 19.

The tip-moment increments are found from figure 18 and are tabulated in column 20. The resultant distribution of c_{m_0} is given in column 21, and the final values of moment are tabulated in column 22 and plotted in figure 19.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *March 25, 1938.*

APPENDIX A

DETERMINATION OF SPAN LOAD DISTRIBUTION FOR SPECIAL CASES

The tables of span load ordinates (tables II and III) referred to in the development of the method and used in the monoplane example are, in general, suitable for the determination of resultant wing forces and for the determination of force distribution for structural applications except in cases, such as some externally braced wings, in which the tip loading has an important influence. For such excepted cases empirical tip corrections should be applied, in accordance with the following procedure. Also, in cases in which the plan form departs widely from the straight tapered shape or in which there are discontinuities in twist such as occur with partial-span flaps, the span load distribution should be determined from the basic wing theory. For such cases, the method discussed in reference 3 is recommended.

The results from reference 4 are to be used. The following procedure should be utilized for obtaining the span load distribution with special tip corrections:

1. From the conditions of the problem determine C_L, based on the actual wing area, for which the distribution is to be determined.
2. Find $\Delta C_L(F_1 \times F_2)$ from figure 17, interpolating when necessary. Subtract ΔC_L from the value of C_L found from step (1).
3. Determine the geometric aspect ratio of the actual wing.
4. From table IX find the $c_{l_{a1}}$ distribution and multiply by the value from step (2).
5. Add to the distribution found in step (4) the c_{l_b} distribution from table X, reduced or increased in proportion to the actual twist.
6. Find the tip corrections (Δc_{l_0}) from figure 18(a). The affected distance is 40 percent of S/b. The tip increments of figure 18(a) are multiplied by both the aspect-ratio and taper-ratio factors given in figure 18(c).
7. Add the Δc_{l_0} increments, corrected for aspect ratio and taper, to the distribution of step (5).
8. Add to the section c_{m_0} values the tip Δc_{m_0} increments from figure 18(b) corrected for aspect ratio and taper ratio by the factors given in 18(c).

APPENDIX B

TABLE OF AIRFOIL CHARACTERISTICS (TABLE I)

A form of presentation of the airfoil characteristics has been adopted that permits all the characteristics necessary for the solution of problems such as those considered in this paper to be compactly presented. All such characteristics for a given airfoil section are presented by entries across a single line of a table. Characteristics are given in this form for certain well-known and commonly used airfoil sections in table I. The information presented for each section is discussed in the following paragraphs under subheadings corresponding to the column headings in the table.

Airfoil: The first column of the table gives the commonly used designations of the airfoil sections.

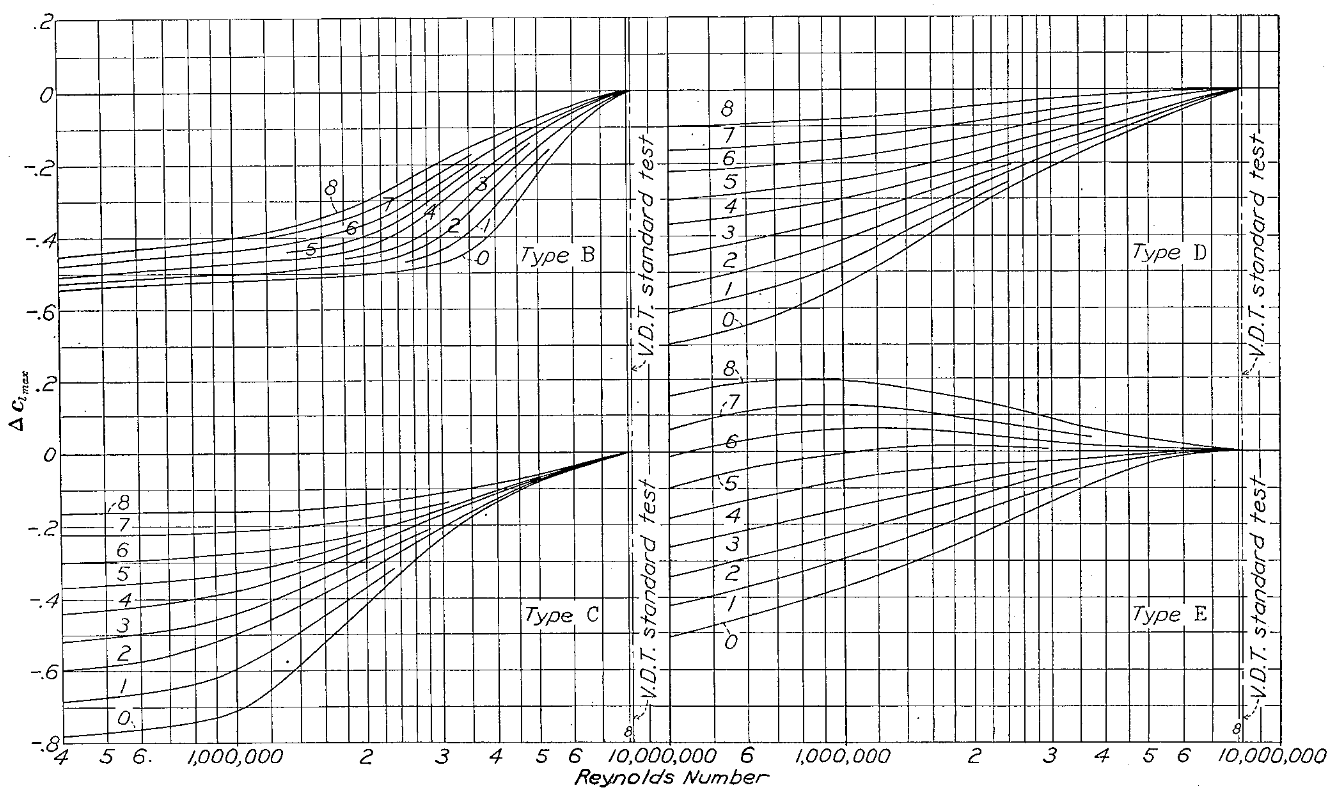
Reference: The second column gives the reference to an N. A. C. A. report or technical note (R or N), in which additional data for the section, including the official table of ordinates, may be found.

CLASSIFICATION

Chord: The letters in this column classify the airfoil sections with respect to the type of their chord. The letter A designates a chord joining the extremities of the mean line, the N. A. C. A. 2412, for example; B designates the chord as being tangent to the lower surface; and C designates an arbitrary chord from which the section ordinates are specified.

PD: The letters and numbers in column PD classify the airfoil section with respect to the character of the pressure distribution about the section. The letter refers to the character of the additional and the accompanying numbers to the character of the basic pressure distribution. The section of the present paper that discusses load distribution over an airfoil section, figures 6 and 7 that give the various distributions for the airfoil classes indicated, and the sample calculation of the pressure distribution about the U. S. A. 35–A section should be referred to for further details. The typical pressure distributions employed are based on Theodorsen's method (reference 7) modified to improve the agreement with experiment. The modified method and some experimental results may be found in reference 12. No data are available for class A airfoils.

SE: The character of the scale effect as affecting the maximum lift coefficient is indicated by the classification in column SE. The numbers and letters correspond to the designations of the typical scale-effect curves presented in figure 20 except that no data are available for class A airfoils. This information is necessary for determinations of stalling speeds. The Reynolds Number corresponding approximately to the stalling speed is first determined. Then from the curve of figure 20 corresponding to the designation in the SE column, the increment $\Delta C_{L_{max}}$ corresponding to this Reynolds Num-



To obtain the section maximum lift coefficient at the desired Reynolds Number, apply to the standard-test value the increment indicated by the curve that corresponds to the scale-effect designation of the airfoil. For type A, $\Delta c_{l_{max}}=0$.

FIGURE 20.—Scale-effect corrections for $c_{l_{max}}$.

ber is obtained. This increment is added to the standard test value of the maximum lift coefficient given in table I to obtain the maximum lift coefficient to be expected for the particular airfoil section in flight at the Reynolds Number corresponding to the stalling speed. Application of the section data to the prediction of the $C_{L_{max}}$ of tapered wings may be found in reference 2.

This method for the prediction of maximum lift coefficients in flight is based on scale-effect tests of a number of related airfoils. The experimental data and a more complete discussion of the subject may be found in reference 13.

$C_{L_{max}}$: Under the heading $C_{L_{max}}$ the airfoil sections are classified according to the character of the lift-curve peak. The airfoils are classified A, B, C, and D in accordance with behavior in the neighborhood of maximum lift.

In type A the lift is more or less steady until it breaks suddenly to a lower value without an appreciable change of angle of attack.

In type B the lift becomes so unsteady and erratic as to preclude the taking of measurements for a range of angles of attack beyond an angle referred to as that of maximum lift.

In type C, before reaching the lift referred to as the "maximum," the lift breaks intermittently from a rather definite value to another rather definite but lower value, and then returns to the higher value. As the angle of attack is increased, the breaks become more frequent and of longer duration. The maximum lift is taken as the higher value occurring at an angle of attack at which it is a maximum or beyond which the higher value can no longer be determined with confidence.

In type D the lift is reasonably steady in the neighborhood of the maximum or any breaks occurring are small so that average values of the lift are measured throughout the range and the lift coefficients are represented by a continuous curve in the neighborhood of the maximum.

FUNDAMENTAL SECTION CHARACTERISTICS

Effective Reynolds Number: The values in this column represent the values of the Reynolds Number at which the section characteristics should be considered as applying to flight. The effective Reynolds Number is obtained from the actual test Reynolds Number by the application of a factor to allow for the effects of turbulence present in the tunnel. The turbulence factor 2.64 has been used for the variable-density tunnel. Comparative tests (reference 14) indicate that, at the effective Reynolds Number, maximum lift results from the tunnel tend to agree with those in flight.

$c_{l_{max}}$: This column gives the maximum lift coefficients corrected to represent values for the airfoil sections.

α_{l_0}: In this column are tabulated the angles of zero lift in degrees.

a_0: This column gives the slope (per degree) of the curve of lift coefficient against section angle of attack, that is, the lift-curve slope for a section of a wing of infinite span. The corresponding slopes for wings of finite span are found from the a_0 values by the method indicated in figure 12.

$c_{l_{opt}}$: The optimum lift coefficient, that is, the lift coefficient corresponding to the minimum profile-drag coefficient for the section, appears in this column. The profile-drag coefficient for the section at any lift coefficient may be inferred approximately from $c_{l_{opt}}$, $c_{d_{0_{min}}}$, and $c_{l_{max}}$ by the method indicated in figure 2.

$c_{d_{0_{min}}}$: The values in this column give the minimum profile-drag coefficients. The values given, however, are not the ones read from the usual plots of profile-drag coefficient made directly from the test data. They are corrected for the different skin-friction coefficients to be expected at the effective rather than at the test Reynolds Number (see footnote on p. 21 of reference 13) and for support interference. The support-interference correction, which gives an important reduction of drag for the thicker airfoils, was evaluated only recently and results published heretofore do not include the correction. Furthermore, another small correction is applied to these data to allow for a tip effect present in the tests of rectangular-tip airfoils. A corresponding correction has been applied to certain other characteristics including a_0 and the maximum lift coefficient; other characteristics are indirectly affected. A discussion of this subject may be found in reference 13.

$c_{m_{a.c.}}$: The values in this column give the pitching-moment coefficients referred to the aerodynamic center of the section rather than to the usual quarter-chord point. The aerodynamic center, by definition, is the point about which the pitching-moment coefficient is constant. Experimental results indicate that, by the use of an empirically derived aerodynamic-center position as suggested by Diehl, a constant pitching-moment coefficient $c_{m_{a.c.}}$ may be specified for each section that does not depart from the measured pitching-moment coefficients by more than the experimental error, over the range of lift coefficients between zero lift and slightly below maximum lift.

a. c.: In these two columns the coordinates of the aerodynamic center ahead of and above the quarter-chord point are given in percentage of the chord.

DERIVED AND ADDITIONAL CHARACTERISTICS

$c_{l_{max}}/c_{d_{0_{min}}}$: The values of this ratio are given because the ratio has been employed as a speed-range index. Strictly speaking, for this purpose, values of $c_{l_{max}}$ and $c_{d_{0_{min}}}$ should not be taken at the same value of the Reynolds Number; but the method has the advantage of simplicity and is of some value in comparing airfoil sections.

c. p. at $c_{l_{max}}$: Values are given in this column representing the center-of-pressure position in percentage of the chord behind the leading edge, or the forward end of the chord. The values are the measured values.

Wing characteristics $A=6$: Wing characteristics are given for a wing of aspect ratio 6 having the given airfoil section and for a modified rectangular plan form with rounded tips. (Tip length approximately one chord.) The values of m_6 represent the slope of the curve of lift coefficient against angle of attack expressed as changes in lift coefficient per radian. The values of $C_{D_{min}}$ represent the minimum drag coefficients for the wings.

Thickness.—Data are given in three columns that refer to the airfoil section thickness at the indicated representative stations. The thicknesses are measured along perpendiculars to the chord and are expressed in percentages of the chord.

Camber.—The camber expressed in percentage of the chord is represented by giving the maximum displacement of the mean line from the straight line joining its extremities.

APPENDIX C

SYMBOLS

BASIC CONSIDERATIONS

S, wing area.
L, lift.
q, dynamic pressure ($1/2\rho V^2$).
c, chord.
b, span.
$C_L=\frac{L}{qS}$, wing lift coefficient.
$c_l=\frac{dL}{qcdy}$, section lift coefficient.
c_{l_0}, section lift coefficient acting perpendicular to local relative wind.

Subscripts:

$a1$, refers to additional part of load distribution for $C_L=1$.
a, refers to additional part of load distribution for any C_L.
b, refers to basic part of load distribution for $C_L=0$.
y, distance along lateral airplane axis.

GENERAL PROCEDURE

Monoplanes

x, distance along longitudinal airplane axis.
z, distance along normal airplane axis.
$x_{a.c.}$, $z_{a.c.}$, x and z coordinates of wing aerodynamic center.
L_a, additional load parameter, $c_{l_{a1}}c\frac{b}{S}$.
x', z', x and z coordinates with respect to a system of axes originating at the wing aerodynamic center.
L_b, basic load parameter, $c_{l_b}\frac{c}{\epsilon a_0}\frac{b}{S}$.
X, Z, components of air force in the x and z directions.
c_x, c_z, section force coefficients.
M, wing pitching moment.
$M_{a.c.}$, wing pitching moment about wing aerodynamic center.
M_s, part of wing moment due to section pitching moments about their aerodynamic centers.
M_X, M_Z, parts of wing moment due to X force and Z force.
M_T, wing pitching moment about torsional axis.
z_T, x_T, distances of the torsional axis below the chord plane through the section aerodynamic center and behind the beam plane through the section aerodynamic center.
$c_{m_{a.c.}}$, section pitching-moment coefficient about section aerodynamic center.
c_{d_0}, section profile-drag coefficient acting parallel to local relative wind.
a_0, section lift-curve slope (per degree).
α_{l_0}, section angle of attack for zero lift.
i, incidence of section chord with respect to x axis.
c_t, tip chord (for rounded tips, c_t is the fictitious chord obtained by extending leading and trailing edges to the extreme tip).
c_s, chord at center of wing or plane of symmetry.
A, aspect ratio, b^2/S.
ϵ, aerodynamic twist, assumed linear, and measured as the angle between the zero lift directions of the center and tip sections, positive for washin.
C_L', wing lift coefficient for steady-flight condition preceding accelerated-flight condition.
m, slope of wing lift curve (per radian).
m_6, slope of lift curve for nontapered wing with rounded tips and aspect ratio 6 (per radian).
$c_{d_{0_{min}}}$, minimum section profile-drag coefficient.
$c_{l_{opt}}$, optimum section lift coefficient, lift coefficient corresponding to $c_{d_{0_{min}}}$.
V, flight velocity or air speed.
U, velocity of gust.
a, slope of wing lift curve (per degree).
f, plan-form factor.
B, beam component of force.
C, chord component of force.
c_b, c_c, section coefficients of beam and chord components.

$$\theta_z=\frac{c_{l_0}}{a_0}+\alpha_{l_0}-i$$

$$\theta_b=\frac{c_{l_0}}{a_0}+\alpha_{l_0}-i_b$$

$$\theta_c=\frac{c_{l_0}}{a_0}+\alpha_{l_0}-i_c$$

$$\phi=i_b-i_c$$

i_b, incidence of section chord with respect to the perpendicular to the beam direction.
i_c, incidence of section chord with respect to the chord-truss direction.
C_N, wing normal-force coefficient.
c_n, section normal-force coefficient.

BIPLANES

$\overline{x}$, $\overline{z}$, distances defining the locus of the aerodynamic centers of the biplane cellule.

K_1, K_2, etc., Diehl's biplane lift functions (references 5 and 6).

G, gap.

t, thickness of wing.

s, stagger, distance between aerodynamic centers of upper and lower wings measured parallel to x axis.

s_0, stagger, distance between ⅓-chord points of upper and lower wings measured parallel to the zero-lift direction.

M_{YY}, net biplane pitching moment about arbitrary y axis.

$\Delta c_{m_0}\left(\frac{t}{G}\right)$, increment in section moment coefficient due to biplane parameter t/G.

S_U', portion of upper wing area to which the t/G moment correction applies.

S_L', portion of lower wing area to which the t/G moment correction applies.

c_U', average chord of the portion of the upper wing corresponding to S_U'.

c_L', average chord of the portion of the lower wing corresponding to S_L'.

Subscripts:

U, refers to upper wing.

L, refers to lower wing.

B, refers to biplane.

LOAD DISTRIBUTION OVER AIRFOIL SECTION

P, normal-pressure coefficient, p/q.

p, the pressure difference across the wing section at any station along the chord.

P_0, value of P for the pressure distribution at zero lift.

P_a, value of P for the additional part of the pressure distribution when the additional section lift coefficient is 1.

P_{a1}, $\frac{x_{a.c.}}{c}P_{a.c.}$, components of P_a. (See fig. 6 and equation (24).)

$\frac{x_{a.c.}}{c}$, distance in terms of chord of section aerodynamic center forward of quarter-chord point. (See table I.)

P_b, value of P for the basic part of the pressure distribution.

$-c_{m_{a.c.}}P_{bm}$, $\frac{z_c}{c}$ P_{bc}, components of P_b. (See fig. 7.)

$\frac{z_c}{c}$, camber ratio; distance, in terms of chord, of the minimum height of the section mean line above the straight line joining its extremities. (See table I.)

c_{n_b}, section normal-force coefficient corresponding to basic pressure distribution.

$-c_{m_{a.c.}}(c_n)_{bm}$, $\frac{z_c}{c}(c_n)_{bc}$, components of c_{n_b}. (See fig. 7.)

SAMPLE CALCULATION

W, weight of airplane.

F_t, force on horizontal tail surfaces.

n_1, applied wing load factor, basic design Condition I (a), reference 1.

s, effective wing loading, reference 1.

q_L, dynamic pressure corresponding to design high speed.

C_{N_1}, wing normal-force coefficient corresponding to n_1.

α_s, wing angle of attack based on chord of central section.

$(\alpha_{l0})_s$, angle of zero lift of the central section.

J, parameter for determining angle of zero lift of twisted wing. (See fig. 11.)

c_{d_0}', section profile-drag coefficient for steady-flight condition.

q', dynamic pressure for steady-flight condition.

BIPLANE

l, distance from y axis to $c.p.$ of horizontal surfaces.

F_1, F_2, factors for reducing wing lift coefficient to allow for tip increment. (See fig. 17.)

k, Munk's span factor.

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TABLE I.—AIRFOIL SECTION CHARACTERISTICS [1]

Airfoil	N. A. C. A. reference (R=report; N=technical note)	Classification				Fundamental section characteristics									Derived and additional characteristics that may be used for structural design							
		Chord	PD	SE	$C_{L_{max}}$	Effective Reynolds Number (millions)	$c_{l_{max}}$	α_{l0} (deg.)	a_0 (per deg.)	$c_{l_{opt}}$	$c_{d_{0_{min}}}$ (1)	$c_{m_{a.c.}}$	a. c. (percent c from c/4) Ahead	a. c. (percent c from c/4) Above	$\frac{c_{l_{max}}}{c_{d_{0_{min}}}}$	c. p. at $c_{l_{max}}$ (percent c)	Wing characteristics A=6; round tips m_6 (per radian)	Wing characteristics A=6; round tips $C_{D_{min}}$	Thickness (percent c) at— 0.15c	Thickness (percent c) at— 0.65c	Thickness (percent c) at— Maximum	Camber $100\frac{z_c}{c}$ (percent c)
Clark Y	R416	B	C10	C4	D	8.37	1.68	−5.0	0.092	0.12	0.0071	−0.069	1.1	4	237	29	4.07	0.0072	10.53	8.30	11.70	3.9
Clark YM-15	N412	C	D10	D4	C	8.38	1.70	−5.2	.094	.10	.0076	−.068	1.1	7	224	30	4.14	.0079	13.51	10.63	15.00	4.0
Clark YM-18	N412	C	E10	E4	C	8.24	1.60	−5.1	.091	.07	.0085	−.064	1.4	5	188	30	4.04	.0086	16.21	12.72	18.00	4.0
Curtiss C-72	N412	B	C10	C4	D	7.98	1.74	−5.6	.095	23	.0071	−.084	1.0	3	245	30	4.18	.0075	10.53	7.39	11.73	4.0
Göttingen 387	N428	B	D10	D6	D	8.40	1.70	−6.6	.097	.30	.0076	−.093	.7	4	224	32	4.24	.0081	13.40	9.69	14.85	5.9
Göttingen 398	N412	B	D10	D5	D	8.11	1.68	−6.0	.094	.15	.0076	−.081	.4	1	221	31	4.14	.0079	12.50	9.27	13.75	4.9
N-22	N412	B	C10	C4	D	8.11	1.72	−5.4	.096	.17	.0075	−.075	.6	4	229	30	4.20	.0076	11.25	8.36	12.37	4.3
N. A. C. A. CYH	N412	B	C11	C3	D	8.06	1.58	−2.9	.095	.08	.0065	−.027	.7	6	243	28	4.18	.0066	10.53	8.30	11.70	3.1
N. A. C. A.-M6	N412	A	C11	C3	D	8.00	1.51	−.8	.095	.03	.0066	.002	−.4	0	229	26	4.18	.0066	10.29	9.00	12.01	2.4
R. A. F. 15	R352	C	A10	A2	D	8.51	1.30	−2.2	.096	.25	.0060	−.053	1.7	10	217	29	4.20	.0063	6.38	5.04	6.38	2.6
U. S. A. 27	N412	B	C10	C6	B	8.06	1.71	−4.7	.094	.30	.0075	−.078	1.8	5	228	30	4.14	.0084	10.40	8.70	11.12	5.6
U. S. A. 35-A	R628	B	E10	E6	D	8.42	1.52	−8.0	.095	.38	.0094	−.111	.8	5	162	34	4.18	.0099	16.60	11.90	18.18	7.3
U. S. A. 35-B	N412	B	C10	C5	B	8.30	1.81	−5.2	.099	.35	.0071	−.076	.5	5	255	30	4.31	.0076	10.56	7.54	11.61	4.6
N. A. C. A. 0006	R628	A	A10	A	D	8.47	.91	0	.098	0	.0051	0	.7	2	178	35	4.28	.0051	5.35	4.13	6.00	0
N. A. C. A. 0012	R628	A	C10	C0	A	8.37	1.66	0	.099	0	.0060	0	.6	3	277	26	4.32	.0060	10.69	8.27	12.00	0
N. A. C. A. 2212	R460	A	C12	C3	B	8.42	1.72	−1.8	.099	.12	.0062	−.029	.9	5	277	27	4.31	.0062	10.69	8.25	12.00	2.0
N. A. C. A. 2409	R460	A	B10	B2	B	8.14	1.62	−1.7	.099	.08	.0060	−.044	.7	4	270	28	4.31	.0062	8.02	6.20	9.00	2.0
N. A. C. A. 2412	R628	A	C10	C2	B	8.24	1.72	−2.0	.098	.14	.0061	−.043	.5	3	282	28	4.28	.0062	10.71	8.27	12.00	2.0
N. A. C. A. 2415	R460	A	D10	D2	C	8.00	1.66	−1.7	.097	.10	.0068	−.040	1.4	5	244	28	4.24	.0068	13.39	10.34	15.00	2.0
N. A. C. A. 2418	R460	A	E10	E2	C	7.98	1.53	−1.9	.094	.06	.0076	−.038	1.1	2	201	27	4.14	.0076	16.08	12.39	18.00	2.0
N. A. C. A. 4412	R628	A	C10	C4	D	7.92	1.74	−4.0	.098	.32	.0071	−.088	.8	2	245	31	4.28	.0073	10.77	8.28	12.00	4.0
N. A. C. A. 23006	R610	A	A12	A	D	8.29	1.17	−1.2	.100	.15	.0057	−.012	1.0	8	205	26	4.34	.0058	5.34	4.13	6.00	1.8
N. A. C. A. 23009	R610	A	B12	C2	A	8.26	1.66	−1.1	.099	.08	.0059	−.009	.9	7	281	25	4.32	.0060	8.02	6.21	9.00	1.8
N. A. C. A. 23012	R610	A	C12	D2	A	8.37	1.74	−1.2	.100	.08	.0060	−.008	1.2	7	290	25	4.34	.0061	10.69	8.25	12.00	1.8
N. A. C. A. 23015	R610	A	D12	D2	A	8.37	1.73	−1.1	.098	.10	.0067	−.008	1.1	6	258	24	4.28	.0068	13.36	10.35	15.00	1.8
N. A. C. A. 23018	R610	A	E12	E2	B	8.16	1.58	−1.2	.097	.08	.0074	−.006	1.7	6	214	24	4.24	.0074	16.04	12.39	18.00	1.8
N. A. C. A 23021	R610	A	F12	E2	B	8.21	1.50	−1.2	.092	.07	.0080	−.005	2.3	7	188	24	4.07	.0080	18.70	14.44	21.00	1.8

[1] Recently airfoil data from the variable-density tunnel have been found to be subject to a correction for support interference. These corrections have been applied to the data in table I, hence the discrepancy between the drag coefficients in the table and those appearing in previous publications and in the calculations of the example of this report.

TABLE II.—ADDITIONAL SPAN LIFT DISTRIBUTION DATA
VALUES OF L_a FOR ROUNDED-TIP WINGS

A \ $\frac{c_t}{c_s}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Spanwise station $\frac{y}{b/2}=0$											
2	1.439	1.400	1.367	1.339	1.316	1.301	1.298	1.292	1.290	1.287	1.282
3	1.489	1.430	1.385	1.350	1.322	1.302	1.288	1.275	1.263	1.253	1.246
4	1.527	1.452	1.400	1.360	1.329	1.302	1.279	1.260	1.242	1.226	1.211
5	1.559	1.473	1.414	1.369	1.333	1.301	1.272	1.248	1.225	1.204	1.186
6	1.585	1.492	1.428	1.378	1.338	1.300	1.267	1.237	1.211	1.187	1.163
7	1.609	1.510	1.440	1.386	1.340	1.300	1.264	1.232	1.203	1.176	1.149
8	1.629	1.534	1.456	1.392	1.344	1.300	1.264	1.229	1.198	1.165	1.135
10	1.661	1.558	1.473	1.409	1.355	1.306	1.264	1.222	1.187	1.152	1.120
12	1.686	1.578	1.490	1.420	1.361	1.308	1.261	1.219	1.180	1.143	1.109
14	1.708	1.592	1.502	1.429	1.366	1.309	1.260	1.214	1.172	1.136	1.100
16	1.726	1.610	1.513	1.433	1.368	1.309	1.255	1.208	1.165	1.127	1.090
18	1.741	1.623	1.525	1.441	1.370	1.308	1.252	1.203	1.160	1.118	1.080
20	1.755	1.632	1.531	1.446	1.372	1.307	1.250	1.199	1.152	1.109	1.070
Spanwise station $\frac{y}{b/2}=0.2$											
2	1.369	1.329	1.300	1.279	1.267	1.260	1.258	1.256	1.253	1.250	1.248
3	1.405	1.346	1.308	1.279	1.260	1.248	1.241	1.234	1.228	1.221	1.214
4	1.434	1.363	1.318	1.284	1.260	1.243	1.232	1.220	1.209	1.193	1.186
5	1.459	1.377	1.324	1.288	1.260	1.240	1.223	1.208	1.194	1.181	1.168
6	1.477	1.388	1.329	1.290	1.259	1.236	1.218	1.200	1.184	1.169	1.151
7	1.491	1.393	1.332	1.291	1.259	1.236	1.214	1.193	1.174	1.157	1.138
8	1.502	1.401	1.338	1.294	1.261	1.236	1.212	1.189	1.168	1.148	1.129
10	1.513	1.411	1.347	1.299	1.265	1.236	1.209	1.182	1.158	1.137	1.114
12	1.520	1.417	1.349	1.302	1.265	1.233	1.202	1.172	1.148	1.126	1.102
14	1.527	1.423	1.354	1.307	1.268	1.232	1.201	1.170	1.144	1.119	1.094
16	1.532	1.428	1.358	1.308	1.269	1.232	1.199	1.164	1.135	1.110	1.087
18	1.539	1.429	1.359	1.309	1.270	1.231	1.195	1.160	1.130	1.103	1.078
20	1.547	1.431	1.360	1.311	1.271	1.230	1.190	1.155	1.123	1.098	1.069
Spanwise station $\frac{y}{b/2}=0.4$											
2	1.217	1.190	1.178	1.172	1.172	1.171	1.170	1.169	1.169	1.168	1.168
3	1.220	1.191	1.176	1.166	1.161	1.160	1.159	1.158	1.157	1.156	1.155
4	1.223	1.192	1.173	1.162	1.156	1.151	1.149	1.148	1.147	1.146	1.145
5	1.226	1.193	1.172	1.159	1.149	1.142	1.140	1.138	1.136	1.134	1.133
6	1.229	1.193	1.171	1.155	1.145	1.138	1.132	1.128	1.127	1.126	1.125
7	1.229	1.193	1.170	1.152	1.140	1.131	1.124	1.121	1.120	1.119	1.118
8	1.229	1.192	1.168	1.150	1.138	1.128	1.120	1.116	1.113	1.111	1.110
10	1.228	1.192	1.167	1.148	1.132	1.121	1.113	1.108	1.104	1.102	1.100
12	1.228	1.192	1.166	1.145	1.125	1.111	1.107	1.102	1.099	1.094	1.090
14	1.228	1.191	1.161	1.136	1.116	1.104	1.100	1.096	1.090	1.087	1.082
16	1.228	1.189	1.158	1.131	1.112	1.101	1.097	1.091	1.086	1.081	1.075
18	1.228	1.186	1.152	1.129	1.111	1.100	1.092	1.087	1.080	1.076	1.070
20	1.228	1.182	1.149	1.127	1.110	1.098	1.089	1.083	1.078	1.071	1.065
Spanwise station $\frac{y}{b/2}=0.6$											
2	0.970	0.976	0.984	0.992	1.003	1.010	1.012	1.014	1.016	1.018	1.019
3	.950	.962	.975	.985	.996	1.004	1.011	1.018	1.023	1.030	1.038
4	.932	.948	.962	.978	.992	1.002	1.008	1.014	1.023	1.035	1.050
5	.920	.938	.953	.971	.988	1.000	1.008	1.015	1.024	1.038	1.053
6	.909	.930	.949	.966	.981	.993	1.002	1.013	1.024	1.039	1.055
7	.900	.920	.940	.959	.975	.989	1.000	1.012	1.024	1.039	1.054
8	.891	.916	.938	.956	.972	.988	.999	1.011	1.024	1.039	1.053
10	.881	.907	.929	.947	.961	.976	.992	1.008	1.023	1.039	1.052
12	.872	.901	.923	.941	.958	.972	.989	1.006	1.022	1.038	1.051
14	.868	.895	.918	.937	.953	.969	.986	1.003	1.019	1.035	1.049
16	.861	.888	.912	.931	.948	.966	.983	1.000	1.017	1.033	1.048
18	.858	.883	.906	.925	.944	.963	.981	.998	1.015	1.032	1.047
20	.851	.876	.898	.920	.940	.959	.978	.995	1.012	1.028	1.046

A \ $\frac{c_t}{c_s}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Spanwise station $\frac{y}{b/2}=0.8$											
2	0.615	0.678	0.712	0.731	0.740	0.745	0.746	0.746	0.747	0.747	0.748
3	.589	.659	.700	.726	.743	.754	.764	.772	.782	.790	.799
4	.568	.644	.691	.723	.746	.764	.781	.795	.806	.816	.824
5	.548	.632	.685	.720	.748	.769	.790	.808	.822	.834	.845
6	.531	.619	.675	.717	.748	.775	.800	.820	.838	.851	.862
7	.517	.609	.670	.713	.748	.778	.802	.827	.845	.861	.875
8	.504	.600	.663	.710	.748	.779	.808	.834	.854	.872	.886
10	.486	.585	.653	.704	.748	.783	.815	.842	.868	.887	.905
12	.472	.576	.648	.702	.748	.788	.850	.850	.877	.899	.919
14	.462	.569	.641	.699	.748	.789	.825	.858	.887	.911	.933
16	.456	.564	.638	.698	.748	.791	.830	.862	.894	.921	.944
18	.450	.559	.636	.698	.750	.796	.835	.870	.901	.930	.953
20	.444	.545	.629	.698	.753	.801	.842	.878	.909	.937	.962
Spanwise station $\frac{y}{b/2}=0.9$											
2	0.378	0.465	0.508	0.525	0.531	0.534	0.535	0.536	0.537	0.538	0.539
3	.352	.447	.500	.528	.543	.552	.559	.564	.568	.571	.575
4	.331	.435	.495	.532	.554	.569	.581	.590	.598	.603	.609
5	.314	.424	.490	.531	.550	.583	.600	.613	.622	.630	.636
6	.300	.416	.487	.531	.565	.595	.615	.631	.643	.652	.659
7	.290	.410	.484	.535	.572	.603	.628	.646	.660	.671	.678
8	.282	.403	.481	.536	.579	.612	.638	.658	.673	.686	.696
10	.266	.383	.472	.541	.590	.628	.656	.679	.698	.712	.723
12	.253	.376	.469	.542	.597	.639	.669	.698	.718	.736	.751
14	.245	.370	.468	.545	.602	.648	.684	.715	.739	.759	.776
16	.239	.366	.468	.547	.609	.659	.698	.729	.756	.780	.801
18	.234	.367	.470	.552	.618	.669	.710	.743	.773	.800	.822
20	.231	.368	.473	.550	.625	.679	.722	.759	.791	.819	.846
Spanwise station $\frac{y}{b/2}=0.95$											
2	0.231	0.296	0.334	0.358	0.370	0.379	0.381	0.383	0.386	0.388	0.390
3	.209	.290	.339	.369	.389	.401	.407	.412	.416	.418	.420
4	.191	.286	.342	.378	.402	.420	.428	.434	.440	.444	.446
5	.176	.281	.344	.384	.415	.436	.449	.458	.463	.469	.471
6	.166	.278	.346	.392	.428	.451	.466	.475	.482	.490	.496
7	.155	.272	.346	.398	.438	.464	.481	.494	.502	.510	.515
8	.148	.261	.346	.403	.446	.475	.495	.510	.521	.529	.534
10	.138	.255	.346	.410	.460	.495	.520	.538	.553	.566	.575
12	.132	.254	.348	.419	.473	.511	.542	.566	.583	.598	.608
14	.129	.252	.349	.423	.482	.529	.562	.588	.609	.628	.640
16	.126	.252	.351	.432	.495	.546	.581	.610	.635	.655	.671
18	.122	.254	.357	.439	.503	.558	.598	.629	.658	.682	.702
20	.121	.258	.364	.449	.516	.569	.613	.648	.680	.707	.730
Spanwise station $\frac{y}{b/2}=0.975$											
2	0.132	0.172	0.207	0.239	0.263	0.272	0.274	0.277	0.279	0.281	0.282
3	.119	.166	.210	.250	.278	.289	.291	.294	.298	.300	.301
4	.107	.163	.214	.258	.288	.304	.308	.311	.315	.319	.322
5	.098	.158	.217	.269	.304	.320	.322	.328	.333	.338	.342
6	.089	.158	.219	.272	.314	.332	.340	.344	.350	.357	.361
7	.081	.158	.222	.278	.320	.342	.351	.359	.366	.373	.381
8	.077	.158	.228	.283	.328	.352	.363	.374	.383	.391	.400
10	.069	.158	.233	.295	.343	.373	.390	.403	.415	.428	.438
12	.068	.161	.242	.308	.360	.395	.413	.430	.448	.461	.473
14	.066	.163	.248	.320	.376	.413	.438	.458	.478	.495	.510
16	.064	.166	.255	.331	.394	.435	.463	.488	.510	.529	.546
18	.063	.169	.263	.346	.412	.461	.492	.518	.539	.560	.580
20	.062	.171	.271	.363	.435	.483	.515	.544	.570	.593	.615

TABLE III.—BASIC SPAN LIFT DISTRIBUTION DATA

VALUES OF L_b FOR ROUNDED-TIP WINGS

A \ $\frac{c_t}{c_s}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Spanwise station $\frac{y}{b/2}=0$											
2	-0.118	-0.121	-0.122	-0.122	-0.122	-0.121	-0.121	-0.121	-0.120	-0.120	-0.120
3	-.153	-.160	-.162	-.163	-.165	-.164	-.164	-.163	-.162	-.161	-.160
4	-.183	-.192	-.197	-.199	-.199	-.199	-.198	-.197	-.196	-.194	-.192
5	-.211	-.221	-.224	-.226	-.225	-.225	-.224	-.224	-.221	-.219	-.218
6	-.235	-.248	-.253	-.253	-.252	-.252	-.250	-.247	-.244	-.243	-.242
7	-.256	-.269	-.275	-.276	-.274	-.272	-.270	-.268	-.264	-.261	-.258
8	-.274	-.288	-.293	-.293	-.291	-.290	-.288	-.285	-.282	-.279	-.276
10	-.304	-.318	-.322	-.323	-.321	-.320	-.318	-.315	-.311	-.305	-.299
12	-.329	-.342	-.350	-.349	-.348	-.346	-.341	-.337	-.331	-.323	-.317
14	-.350	-.364	-.370	-.370	-.368	-.365	-.360	-.355	-.350	-.342	-.334
16	-.367	-.380	-.386	-.385	-.382	-.379	-.375	-.370	-.362	-.358	-.348
18	-.384	-.399	-.405	-.403	-.400	-.393	-.387	-.380	-.376	-.368	-.360
20	-.398	-.411	-.417	-.415	-.410	-.404	-.399	-.392	-.386	-.378	-.369
Spanwise station $\frac{y}{b/2}=0.2$											
2	-0.076	-0.080	-0.082	-0.085	-0.086	-0.086	-0.086	-0.085	-0.085	-0.084	-0.083
3	-.098	-.108	-.111	-.112	-.113	-.113	-.113	-.113	-.112	-.110	-.108
4	-.117	-.130	-.135	-.138	-.137	-.137	-.137	-.137	-.137	-.135	-.132
5	-.131	-.148	-.156	-.159	-.159	-.158	-.158	-.158	-.157	-.156	-.152
6	-.145	-.162	-.173	-.176	-.176	-.176	-.176	-.176	-.175	-.172	-.170
7	-.156	-.178	-.189	-.192	-.192	-.192	-.191	-.191	-.190	-.190	-.189
8	-.168	-.189	-.200	-.204	-.204	-.205	-.205	-.206	-.205	-.204	-.204
10	-.182	-.207	-.220	-.224	-.225	-.225	-.226	-.226	-.225	-.225	-.225
12	-.197	-.226	-.239	-.240	-.239	-.238	-.238	-.238	-.237	-.237	-.237
14	-.206	-.234	-.248	-.249	-.248	-.248	-.248	-.248	-.248	-.248	-.248
16	-.212	-.242	-.256	-.258	-.257	-.256	-.256	-.256	-.256	-.256	-.255
18	-.219	-.247	-.260	-.264	-.265	-.265	-.265	-.265	-.265	-.264	-.262
20	-.222	-.255	-.269	-.271	-.271	-.271	-.272	-.272	-.272	-.272	-.270
Spanwise station $\frac{y}{b/2}=0.4$											
2	-0.006	-0.011	-0.013	-0.015	-0.016	-0.016	-0.016	-0.016	-0.016	-0.016	-0.015
3	-.002	-.010	-.012	-.015	-.016	-.016	-.016	-.016	-.017	-.018	-.018
4	0	-.006	-.011	-.012	-.016	-.016	-.018	-.019	-.020	-.020	-.021
5	.004	-.004	-.010	-.012	-.016	-.018	-.020	-.021	-.021	-.022	-.023
6	.009	-.002	-.008	-.012	-.016	-.018	-.020	-.021	-.022	-.024	-.026
7	.012	-.001	-.010	-.013	-.017	-.018	-.020	-.022	-.025	-.027	-.029
8	.014	0	-.008	-.012	-.017	-.019	-.021	-.025	-.029	-.030	-.030
10	.021	.007	-.002	-.010	-.017	-.020	-.022	-.027	-.030	-.032	-.032
12	.028	.009	-.001	-.010	-.017	-.021	-.025	-.029	-.032	-.036	-.038
14	.036	.013	0	-.010	-.017	-.021	-.028	-.031	-.035	-.040	-.042
16	.043	.019	.002	-.008	-.016	-.022	-.029	-.034	-.038	-.041	-.045
18	.049	.022	.004	-.008	-.015	-.022	-.031	-.038	-.041	-.043	-.046
20	.050	.023	.006	-.006	-.014	-.022	-.031	-.038	-.041	-.046	-.049
Spanwise station $\frac{y}{b/2}=0.6$											
2	0.052	0.052	0.051	0.050	0.050	0.050	0.050	0.050	0.049	0.049	0.048
3	.070	.069	.068	.068	.068	.068	.068	.068	.068	.068	.068
4	.085	.082	.081	.080	.080	.080	.080	.080	.080	.080	.080
5	.099	.095	.092	.091	.091	.091	091	.091	.090	.090	.090
6	.109	.107	.104	.102	.101	.101	.100	.100	.100	.100	.100
7	.119	.117	.114	.112	.111	.110	.110	.110	.110	.109	.108
8	.128	.122	.121	.120	.120	.119	.119	.118	.118	.117	.116
10	.139	.138	.135	.132	.131	.130	.130	.129	.128	.126	.124
12	.148	.145	.141	.140	.140	.139	.137	.135	.134	.132	.130
14	.155	.152	.150	.148	.145	.142	.141	.140	.139	.138	.135
16	.160	.158	.154	.151	.149	.146	.143	.141	.140	.139	.136
18	.165	.162	.160	.158	.152	.148	.145	.142	.140	.139	.138
20	.170	.169	.165	.159	.152	.148	.147	.143	.141	.140	.140

A \ $\frac{c_t}{c_s}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Spanwise station $\frac{y}{b/2}=0.8$											
2	0.072	0.079	0.080	0.082	0.083	0.085	0.085	0.086	0.086	0.084	0.081
3	.088	.098	.101	.102	.104	.108	.109	.110	.110	.108	.106
4	.100	.113	.120	.123	.126	.128	.128	.130	.130	.130	.129
5	.109	.125	.135	.138	.140	.143	.147	.148	.148	.148	.149
6	.115	.135	.148	.152	.156	.160	.160	.162	.163	.164	.165
7	.121	.142	.158	.163	.169	.172	.173	.173	.174	.174	.175
8	.126	.149	.164	.174	.180	.182	.182	.183	.183	.184	.184
10	.136	.160	.178	.188	.195	.200	.201	.202	.203	.201	.198
12	.145	.170	.188	.200	.208	.212	.214	.216	.216	.214	.210
14	.152	.182	.200	.210	.216	.221	.223	.227	.228	.225	.220
16	.159	.186	.205	.216	.222	.229	.232	.233	.236	.232	.229
18	.161	.197	.215	.224	.230	.235	.239	.242	.243	.242	.238
20	.166	.201	.220	.232	.237	.241	.243	.248	.248	.248	.247
Spanwise station $\frac{y}{b/2}=0.9$											
2	0.059	0.068	0.072	0.073	0.075	0.076	0.075	0.075	0.075	0.075	0.075
3	.068	.083	.092	.098	.099	.100	.100	.100	.100	.100	.100
4	.074	.098	.111	.118	.121	.122	.123	.123	.123	.123	.123
5	.081	.107	.122	.131	.138	.140	.141	.141	.142	.142	.142
6	.087	.117	.136	.148	.154	.159	.160	.160	.160	.160	.160
7	.090	.123	.146	.160	.167	.171	.171	.172	.172	.172	.172
8	.092	.131	.153	.170	.179	.182	.183	.184	.185	.186	.187
10	.098	.139	.166	.184	.197	.201	.203	.205	.207	.209	.210
12	.100	.147	.178	.198	.210	.218	.221	.225	.228	.229	.230
14	.102	.156	.188	.208	.220	.231	.238	.241	.243	.245	.246
16	.103	.161	.197	.219	.231	.241	.249	.253	.258	.259	.260
18	.105	.166	.202	.228	.243	.252	.260	.263	.269	.271	.275
20	.107	.172	.211	.233	.248	.260	.268	.273	.279	.282	.285
Spanwise station $\frac{y}{b/2}=0.95$											
2	0.038	0.051	0.058	0.059	0.060	0.060	0.060	0.060	0.059	0.059	0.058
3	.044	.063	.073	.078	.079	.080	.080	.080	.080	.079	.078
4	.050	.072	.076	.092	.095	.097	.099	.100	.100	.100	.099
5	.052	.083	.100	.107	.110	.112	.113	.114	.116	.117	.116
6	.054	.088	.109	.119	.122	.128	.130	.132	.132	.131	.130
7	.056	.093	.116	.130	.135	.140	.144	.148	.150	.149	.145
8	.057	.100	.125	.140	.146	.152	.158	.160	.161	.160	.159
10	.058	.107	.138	.152	.162	.171	.178	.182	.186	.187	.183
12	.059	.112	.143	.165	.179	.189	.198	.200	.202	.205	.204
14	.060	.116	.151	.174	.190	.202	.211	.215	.218	.221	.222
16	.061	.121	.159	.184	.203	.218	.222	.229	.233	.236	.238
18	.061	.126	.166	.194	.213	.229	.236	.241	.248	.251	.255
20	.061	.128	.173	.203	.225	.239	.245	.251	.259	.265	.271
Spanwise station $\frac{y}{b/2}=0.975$											
2	0.019	0.030	0.035	0.037	0.037	0.037	0.037	0.036	0.036	0.035	0.034
3	.022	.039	.045	.049	.050	.051	.052	.054	.053	.052	.051
4	.026	.043	.054	.060	.062	.064	.068	.069	.069	.068	.067
5	.029	.051	.065	.070	.071	.075	.078	.081	.082	.083	.083
6	.030	.055	.071	.079	.082	.088	.091	.094	.097	.097	.097
7	.030	.060	.078	.087	.091	.098	.101	.107	.110	.110	.110
8	.030	.062	.081	.091	.100	.107	.112	.120	.121	.121	.121
10	.031	.067	.090	.105	.115	.124	.132	.138	.141	.142	.143
12	.031	.069	.095	.115	.131	.141	.149	.153	.160	.161	.162
14	.031	.071	.102	.127	.143	.155	.163	.171	.175	.177	.178
16	.031	.077	.111	.138	.156	.169	.178	.182	.188	.190	.191
18	.032	.083	.121	.150	.169	.182	.191	.197	.200	.201	.202
20	.032	.086	.128	.158	.178	.193	.202	.208	.210	.212	.213

TABLE IV.—CALCULATION OF THE ADDITIONAL FORCES AND MOMENTS LEADING TO THE DETERMINATION OF THE WING AERODYNAMIC CENTER

1	2	3	4	5	6	7	8	9	10	11	12
Station from center line $\frac{y}{b/2}$	L_a	c (ft.)	$c_{l_{a1}}$	a_0 (per deg.)	$\frac{c_{l_{a1}}}{a_0}$ (deg.)	α_{l_0} (deg.)	$-i$ (deg.)	c_{d_0}	θ_{z_a} (deg.)	$\cos\theta_{z_a}$	$\sin\theta_{z_a}$
0	1.301	8.27	0.944	0.095	9.9	−8.0	−4	0.0157	−2.1	0.999	−0.0366
.2	1.240	7.44	1.000	.096	10.4	−7.4	−4	.0159	−1.0	1.000	−.0175
.4	1.142	6.62	1.035	.097	10.7	−6.9	−4	.0157	−.2	1.000	−.0035
.6	1.000	5.79	1.036	.097	10.7	−6.3	−4	.0147	.4	1.000	.0070
.8	.775	4.92	.945	.098	9.6	−5.8	−4	.0124	−.2	1.000	−.0035
.9	.583	3.84	.911	.099	9.2	−5.5	−4	.0117	−.3	1.000	−.0052
.95	.436	2.83	.924	.099	9.3	−5.3	−4	.0115	0	1.000	0
.975	.320	2.04	.942	.099	9.5	−5.3	−4	.0115	.2	1.000	.0035
1.0	0	0		.099		−5.2					

1	13	14	15	16	17	18	19	20	21	22	23	24
Station from center line $\frac{y}{b/2}$	$c_{d_0}\cos\theta_{z_a}$	$-c_{l_{a1}}\sin\theta_{z_a}$	$c_{l_{a1}}\cos\theta_{z_a}$	$c_{d_0}\sin\theta_{z_a}$	$c_{x_{a1}}$	$c_{z_{a1}}$	z (ft.)	x (ft.)	$c_{x_{a1}}c$ (ft.)	$c_{z_{a1}}c$ (ft.)	$c_{x_{a1}}cz$ (ft.²)	$-c_{z_{a1}}cx$ (ft.²)
0	0.0157	0.0346	0.943	−0.0006	0.0503	0.942	3.14	−1.98	0.416	7.79	1.31	15.42
.2	.0159	.0175	1.000	−.0003	.0334	1.000	3.43	−1.34	.248	7.44	.85	9.97
.4	.0157	.0036	1.035	−.0001	.0193	1.035	3.76	−.70	.128	6.85	.48	4.80
.6	.0147	−.0073	1.036	.0001	.0074	1.036	4.04	−.08	.043	6.00	.17	.48
.8	.0124	.0033	.945	0	.0157	.945	4.34	.56	.077	4.65	.34	−2.60
.9	.0117	.0047	.911	−.0001	.0164	.911	4.54	.94	.063	3.50	.29	−3.29
.95	.0115	0	.924	0	.0115	.924	4.68	1.24	.033	2.61	.15	−3.24
.975	.0115	−.0033	.942	0	.0082	.942	4.78	1.40	.017	1.92	.08	−2.69

TABLE IV-A.—CALCULATION OF c_{d_0} VALUES FOR TABLE IV

Station	t/c	$c_{d_{0_{min}}}$	$c_{l_{opt}}$	$c_{l_{max}}$	$\frac{\lvert c_{l_{a1}}-c_{l_{opt}}\rvert}{c_{l_{max}}-c_{l_{opt}}}$	Δc_{d_0}	c_{d_0}
0	0.1818	0.0116	0.38	1.57	0.47	0.0041	0.0157
.2	.1745	.0112	.37	1.62	.50	.0047	.0159
.4	.1653	.0106	.37	1.66	.52	.0051	.0157
.6	.1536	.0100	.36	1.72	.50	.0047	.0147
.8	.1380	.0092	.36	1.76	.42	.0032	.0124
.9	.1280	.0089	.35	1.79	.39	.0028	.0117
.95	.1224	.0086	.35	1.80	.40	.0029	.0115
.975	.1194	.0084	.35	1.80	.41	.0031	.0115
1	.1161	.0083	.35	1.81			

TABLE V.—CALCULATION OF THE BASIC FORCES AND MOMENTS LEADING TO THE DETERMINATION OF THE WING PITCHING MOMENT

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Station from center line $\frac{y}{b/2}$	L_b	c (ft.)	c_{l_b}	a_0 (per deg.)	$\frac{c_{l_b}}{a_0}$ (deg.)	α_{l_0} (deg.)	$-i$ (deg.)	c_{d_0}	θ_{z_b} (deg.)	$\cos\theta_{z_b}$	$\sin\theta_{z_b}$	$c_{d_0}\times\cos\theta_{z_b}$	$-c_{l_b}\times\sin\theta_{z_b}$
0	−0.225	8.27	0.0400	0.095	0.42	−8.0	−4	0.0132	−11.6	0.980	−0.2011	0.0129	0.0080
.2	−.158	7.44	.0312	.096	.32	−7.4	−4	.0126	−11.1	.981	−.1925	.0124	.0060
.4	−.018	6.62	.0040	.097	.04	−6.9	−4	.0122	−10.9	.982	−.1891	.0120	.0008
.6	.091	5.79	−.0231	.097	−.24	−6.3	−4	.0115	−10.5	.983	−.1822	.0113	−.0042
.8	.143	4.92	−.0427	.098	−.44	−5.8	−4	.0108	−10.2	.984	−.1771	.0106	−.0076
.9	.140	3.84	−.0536	.099	−.54	−5.5	−4	.0104	−10.0	.985	−.1754	.0102	−.0094
.95	.112	2.83	−.0582	.099	−.59	−5.3	−4	.0101	−9.9	.985	−.1719	.0099	−.0100
.975	.075	2.04	−.0540	.099	−.55	−5.3	−4	.0099	−9.9	.985	−.1719	.0098	−.0093

1	15	16	17	18	19	20	21	22	23	24	25	26
Station from center line $\frac{y}{b/2}$	$c_{l_b}\times\cos\theta_{z_b}$	$c_{d_0}\times\sin\theta_{z_b}$	c_{x_b}	c_{z_b}	$c_{x_b}c$ (ft.)	$c_{z_b}c$ (ft.)	x' (ft.)	z' (ft.)	$c_{x_b}cz'$ (ft.²)	$-c_{z_b}cx'$ (ft.²)	$c_{m_{a.c.}}$	$c_{m_{a.c.}}c^2$ (ft.²)
0	0.0392	−0.0027	0.0209	0.0365	0.173	0.302	−1.37	−0.47	−0.08	0.41	−0.111	−7.59
.2	.0306	−.0024	.0184	.0282	.137	.210	−.73	−.18	−.02	.15	−.104	−5.76
.4	.0039	−.0023	.0128	.0016	.085	.011	−.09	.15	.02	.00	−.097	−4.25
.6	−.0227	−.0021	.0071	−.0248	.041	−.144	.53	.43	.02	.08	−.090	−3.02
.8	−.0420	−.0019	.0030	−.0439	.015	−.216	1.17	.73	.01	.25	−.083	−2.01
.9	−.0528	−.0018	.0008	−.0546	.003	−.210	1.55	.93	.00	.33	−.079	−1.16
.95	−.0573	−.0017	−.0001	−.0590	.000	−.167	1.85	1.07	.00	.31	−.078	−.62
.975	−.0532	−.0017	.0005	−.0549	.001	−.112	2.01	1.17	.00	.23	−.077	−.32

TABLE VI.—CALCULATION OF THE CHORD AND BEAM COMPONENTS

[Numbers in parentheses are columns]

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Station from center line $\frac{y}{b/2}$	c_{l_b}	$C_Lc_{l_{a1}}$	c_{l_0}	$c_{m_{a.c.}}$	c_{d_0}'	c_{d_0}	$\frac{c_{l_0}}{a_0}$ (deg.)	α_{l_0} (deg.)	$-i_b$ (deg.)	$-i_c$ (deg.)	θ_b (deg.)	θ_c (deg.)	ϕ (deg.)	$\cos\theta_c$	$\sin\theta_c$	$\cos\theta_b$	$\sin\theta_b$
0	0.040	1.702	1.742	−0.111	0.0120	0.0120	18.3	−8.0	0	4.0	10.3	14.3	4.0	0.969	0.2470	0.984	0.1788
.2	.031	1.803	1.834	−.104	.0116	.0116	19.1	−7.4	0	3.6	11.7	15.3	3.6	.965	.2639	.979	.2028
.4	.004	1.865	1.869	−.097	.0110	.0110	19.3	−6.9	0	3.2	12.4	15.6	3.2	.963	.2689	.977	.2147
.6	−.023	1.866	1.843	−.090	.0104	.0104	19.0	−6.3	0	2.8	12.7	15.5	2.8	.964	.2672	.976	.2199
.8	−.043	1.703	1.660	−.083	.0093	.0093	16.9	−5.8	0	2.4	11.1	13.5	2.4	.972	.2335	.981	.1925
.9	−.054	1.642	1.588	−.079	.0090	.0090	16.0	−5.5	0	2.2	10.5	12.7	2.2	.976	.2199	.983	.1822
.95	−.058	1.666	1.608	−.078	.0087	.0087	16.2	−5.3	0	2.1	10.9	13.0	2.1	.974	.2250	.982	.1891
.975	−.054	1.698	1.644	−.077	.0085	.0085	16.6	−5.3	0	2.0	11.3	13.3	2.0	.973	.2301	.981	.1960

1	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Station from center line $\frac{y}{b/2}$	$\tan\phi$	$\tan\theta_c$	$\cot\theta_c$	$\tan\theta_b$	$\cot\theta_b$	$\tan\theta_c\tan\phi$	$\cot\theta_c\tan\phi$	$\tan\theta_b\tan\phi$	$\cot\theta_b\tan\phi$	$c_{l_0}(1-\tan\theta_b\tan\phi)\cos\theta_b$	$c_{d_0}(1+\cot\theta_b\tan\phi)\sin\theta_b$	$c_{d_0}(1+\tan\theta_c\tan\phi)\cos\theta_c$	$-c_{l_0}(1-\cot\theta_c\tan\phi)\sin\theta_c$	$c_b=$ (28)+(29)	$c_c=$ (30)+(31)	dB/dy (lb./ft.)	dC/dy (lb./ft.)	$dM_{a.c.}/dy$ (ft.-lb./ft.)
0	0.0699	0.2549	3.923	0.1817	5.500	0.0178	0.2741	0.0127	0.3842	1.692	0.0029	0.0118	−0.3124	1.695	−0.3006	151.3	−26.8	−81.9
.2	.0629	.2717	3.681	.2071	4.829	.0171	.2315	.0130	.3037	1.772	.0031	.0114	−.3720	1.775	−.3606	142.5	−28.9	−62.1
.4	.0559	.2811	3.558	.2199	4.548	.0157	.1989	.0123	.2542	1.804	.0030	.0108	−.4026	1.807	−.3918	129.1	−28.0	−45.9
.6	.0489	.2811	3.558	.2254	4.437	.0137	.1740	.0110	.2170	1.779	.0028	.0101	−.4067	1.782	−.3966	111.3	−24.8	−32.6
.8	.0419	.2419	3.971	.1962	5.097	.0101	.1664	.0082	.2136	1.615	.0022	.0091	−.3231	1.617	−.3140	85.8	−16.7	−21.7
.9	.0384	.2272	4.401	.1853	5.396	.0087	.1690	.0071	.2072	1.550	.0019	.0089	−.2902	1.552	−.2813	64.3	−11.7	−12.6
.95	.0367	.2309	4.333	.1926	5.193	.0085	.1590	.0071	.1906	1.568	.0019	.0086	−.3043	1.570	−.2957	47.9	−9.0	−6.7
.975	.0349	.2364	4.230	.1998	5.005	.0083	.1476	.0070	.1747	1.602	.0020	.0084	−.3223	1.604	−.3139	35.3	−6.9	−3.5

TABLE VII.—CALCULATION OF P_a

Station $100\frac{x}{c}$	P_{a1}	$P_{a.c.}$	$P_{a.c.}\frac{x_{a.c.}}{c}=P_{a.c.}\times 0.008$	P_a
0	0	0	0	0
1.25	3.87	3.2	.03	3.90
2.50	3.81	4.5	.04	3.85
5	3.27	5.5	.04	3.31
7.5	2.81	5.9	.05	2.86
10	2.44	5.7	.05	2.49
15	1.95	5.0	.04	1.99
20	1.62	4.3	.03	1.65
30	1.18	2.9	.02	1.20
40	.89	1.4	.01	.90
50	.69	.0	.00	.69
60	.51	−1.4	−.01	.50
70	.36	−2.9	−.02	.34
80	.23	−4.3	−.03	.20
90	.11	−5.7	−.04	.07
95	.06	−5.5	−.05	.01
100	0	0	0	0

TABLE VIII.—CALCULATION OF PRESSURE DISTRIBUTION

Station $100\frac{x}{c}$	P_{bm} class 1	$-P_{bm}c_{m_{a.c.}}$	P_{bc}	$\frac{z_c}{c}P_{bc}$	P_b	$-c_{n_b}P_a$	P_0	c_nP_a	P	p (lb./sq.ft.)
0	0	0	0	0	0	0	0	0	0	0
1.25	2.85	.32	0	0	.32	−2.73	−2.41	6.61	4.20	45.3
2.5	4.25	.47	0	0	.47	−2.69	−2.22	6.52	4.30	46.4
5	6.05	.67	0	0	.67	−2.31	−1.64	5.61	3.97	42.8
7.5	7.10	.79	0	0	.79	−2.00	−1.21	4.85	3.64	39.3
10	7.80	.87	0	0	.87	−1.74	−.87	4.22	3.35	36.2
15	8.80	.98	0	0	.98	−1.39	−.41	3.37	2.96	31.9
20	9.30	1.03	0	0	1.03	−1.15	−.12	2.80	2.68	28.9
30	9.50	1.05	0	0	1.05	−.84	.21	2.03	2.24	24.2
40	8.80	.98	0	0	.98	−.63	.35	1.52	1.87	20.2
50	7.75	.86	0	0	.86	−.48	.38	1.17	1.55	16.7
60	6.60	.73	0	0	.73	−.35	.38	.85	1.23	13.3
70	5.30	.59	0	0	.59	−.24	.35	.58	.93	10.0
80	3.75	.42	0	0	.42	−.14	.28	.34	.62	6.7
90	2.05	.23	0	0	.23	−.05	.18	.12	.30	3.2
95	1.10	.12	0	0	.12	−.01	.11	.02	.13	1.4
100	0	0	0	0	0	0	0	0	0	

TABLE IX.—THEORETICAL DISTRIBUTION OF $c_{l_{a1}}$

[Straight tips, $C_L=1.0$]

Taper ratio	Rib location in percent semispan							
	0	20	40	60	80	90	95	97.5
Aspect ratio 4								
0.0	0.754	0.890	1.025	1.185	1.444	1.650		
.1	.790	.917	1.030	1.140	1.304	1.342	1.326	1.173
.2	.828	.945	1.036	1.109	1.175	1.125	.976	.783
.3	.870	.972	1.043	1.088	1.090	.995	.818	.645
.4	.911	1.000	1.053	1.073	1.030	.910	.734	.567
.5	.954	1.027	1.066	1.064	.986	.844	.674	.513
.6	.997	1.058	1.081	1.056	.953	.794	.632	.476
.7	1.039	1.086	1.093	1.050	.927	.756	.596	.446
.8	1.082	1.112	1.104	1.045	.906	.727	.566	.422
.9	1.124	1.136	1.112	1.041	.889	.703	.541	.402
1.0	1.166	1.158	1.117	1.039	.873	.684	.519	.385
Aspect ratio 6								
0.0	0.783	0.920	1.030	1.154	1.340	1.470	1.618	1.800
.1	.807	.937	1.030	1.113	1.225	1.267	1.180	1.028
.2	.838	.954	1.034	1.088	1.141	1.110	.965	.777
.3	.874	.975	1.044	1.073	1.079	1.002	.847	.660
.4	.914	1.000	1.056	1.065	1.028	.927	.763	.588
.5	.953	1.026	1.064	1.060	.994	.876	.705	.539
.6	.992	1.051	1.070	1.055	.970	.840	.663	.506
.7	1.030	1.075	1.078	1.050	.950	.810	.632	.480
.8	1.067	1.097	1.086	1.046	.932	.782	.609	.458
.9	1.103	1.114	1.094	1.042	.914	.757	.589	.437
1.0	1.137	1.129	1.104	1.038	.898	.734	.571	.417
Aspect ratio 8								
0.0	0.797	0.930	1.030	1.136	1.294	1.375	1.448	1.620
.1	.820	.945	1.037	1.099	1.194	1.228	1.160	.964
.2	.849	.964	1.042	1.072	1.118	1.106	.973	.768
.3	.884	.983	1.046	1.058	1.065	1.013	.852	.660
.4	.919	1.004	1.051	1.050	1.028	.942	.773	.600
.5	.952	1.027	1.060	1.048	1.000	.896	.725	.564
.6	.987	1.050	1.068	1.049	.979	.860	.694	.538
.7	1.021	1.071	1.077	1.049	.960	.832	.669	.515
.8	1.055	1.090	1.083	1.047	.944	.808	.646	.494
.9	1.086	1.103	1.090	1.043	.930	.788	.624	.474
1.0	1.114	1.113	1.094	1.038	.917	.770	.605	.454

TABLE X.—THEORETICAL DISTRIBUTION OF c_{l_b}

[Straight tips, $C_L=0$; $\epsilon=10°$; $A=6$]

Taper ratio	Rib location in percent semispan									
	0	20	40	50	60	70	80	90	95	97.5
0	0.218	0.165	0.043	−0.023	−0.082	−0.128	−0.166	−0.174	−0.154	−0.122
.1	.208	.158	.040	−.025	−.086	−.133	−.170	−.179	−.160	−.127
.2	.200	.154	.037	−.028	−.089	−.137	−.175	−.190	−.168	−.134
.3	.191	.149	.034	−.030	−.093	−.142	−.182	−.202	−.178	−.146
.4	.184	.145	.030	−.033	−.096	−.148	−.189	−.216	−.194	−.160
.5	.175	.140	.026	−.037	−.100	−.154	−.198	−.233	−.214	−.176
.6	.165	.134	.023	−.039	−.104	−.160	−.210	−.250	−.239	−.192
.7	.153	.126	.019	−.041	−.107	−.167	−.223	−.275	−.270	−.214
.8	.137	.117	.016	−.044	−.112	−.175	−.240	−.307	−.313	−.264
.9	.115	.103	.012	−.046	−.115	−.185	−.260	−.354	−.396	−.370
1.0	.078	.083	.008	−.048	−.116	−.197	−.287	−.421	−.581	−.800

TABLE XI.—COMPUTATION OF THE FORCE DISTRIBUTION ON THE UPPER WING OF BIPLANE

[Numbers in parentheses are columns]

1	2	3	4	5	6	7	8	9	10	11	12
Distance from center line (ft.)	$c_{l_{a1}}$	$c_{l_0}''=C_{L_U}''c_{l_{a1}}$	Δc_{l_0}	$c_{l_0}=(3)+(4)$	c_{l_0}/a_0 (deg.)	$\theta_z=\alpha_{l_0}-i+(6)$ (deg.)	$\cos\theta_z$	$\sin\theta_z$	$c_{l_0}c\cos\theta_z$	Δc_{d_0}	$c_{d_0}=c_{d_{0min}}+(11)$
0	1.137	1.357	0	1.357	14.0	12.2	0.978	0.2111	5.965	0.0004	0.0075
3	1.130	1.349	0	1.349	13.9	12.1	.978	.2097	5.935	.0004	.0075
6	1.105	1.319	0	1.319	13.6	11.8	.979	.2045	5.810	.0004	.0075
9	1.039	1.240	0	1.240	12.8	11.0	.982	.1906	5.475	.0003	.0074
12	.897	1.070	0	1.070	11.0	9.2	.987	.1605	4.750	.0002	.0073
13	.798	.952	0	.952	9.8	8.0	.990	.1395	4.240	.0002	.0073
13.5	.735	.877	.022	.899	9.3	7.5	.992	.1300	4.013	.0002	.0073
14	.638	.761	.160	.921	9.5	7.7	.991	.1340	4.107	.0001	.0072
14.5	.480	.573	.410	.983	10.1	8.3	.989	.1449	4.375	----------	.0071

1	13	14	15	16	17	18	19	20	21	22
Distance from center line (ft.)	$c_{d_0}c\sin\theta_z$	$c_zc=(10)+(13)$	$c_{l_0}c\sin\theta_z$	$c_{d_0}c\cos\theta_z$	$c_x=(15)+(16)$	c_{m_0}	$\Delta c_{m_0}\left(\frac{t}{a}\right)$	Δc_{m_0}	(18)+(19)+(20)	c^2 (21)
0	0.007	5.972	−1.291	0.033	−1.258	−0.044	0	0	−0.044	−0.891
3	.007	5.942	−1.273	.033	−1.240	−.044	.011	0	−.033	−.669
6	.007	5.817	−1.214	.033	−1.181	−.044	.011	0	−.033	−.669
9	.006	5.481	−1.064	.033	−1.031	−.044	.011	0	−.033	−.669
12	.005	4.760	−.773	.033	−.740	−.044	0	0	−.044	−.891
13	.005	4.245	−.598	.033	−.565	−.044	0	0	−.044	−.891
13.5	.004	4.017	−.526	.033	−.493	−.044	0	−.010	−.054	−1.093
14	.004	4.111	−.556	.032	−.523	−.044	0	−.090	−.134	−2.716
14.5	.005	4.380	−.641	.032	−.609	−.044	0	−.170	−.214	−4.340

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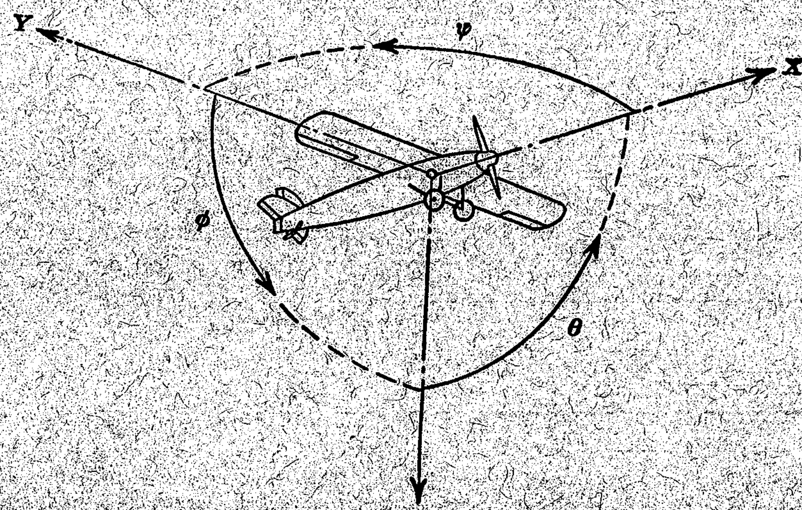

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\dfrac{L}{qbS}$ (rolling) $C_m=\dfrac{M}{qcS}$ (pitching) $C_n=\dfrac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter
p, Geometric pitch
p/D, Pitch ratio
V', Inflow velocity
V_s, Slipstream velocity
T, Thrust, absolute coefficient $C_T=\dfrac{T}{\rho n^2 D^4}$
Q, Torque, absolute coefficient $C_Q=\dfrac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P=\dfrac{P}{\rho n^3 D^5}$
C_s, Speed-power coefficient $=\sqrt[5]{\dfrac{\rho V^5}{P n^2}}$
η, Efficiency
n, Revolutions per second, r.p.s.
Φ, Effective helix angle $=\tan^{-1}\left(\dfrac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.
1 metric horsepower = 1.0132 hp.
1 m.p.h. = 0.4470 m.p.s.
1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.
1 kg = 2.2046 lb.
1 mi. = 1,609.35 m = 5,280 ft.
1 m = 3.2808 ft.