NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 632

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THE CRINKLING STRENGTH AND THE BENDING STRENGTH OF ROUND AIRCRAFT TUBING

By WILLIAM R. OSGOOD

1938

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight$=mg$

g, Standard acceleration of gravity=9.80665 m/s² or 32.1740 ft./sec.²

m, Mass$=\frac{W}{g}$

I, Moment of inertia$=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu. ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure$=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0, Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s., the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_0, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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National Bureau of Standards

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By William R. Osgood

SUMMARY

The upper limit of the column strength of structural members composed of thin material is the maximum axial stress such members can carry when short enough to fail locally, by crinkling. This stress is a function of the mechanical properties of the material and of the geometrical shape of the cross section. The bending strength, as measured by the modulus of rupture, of structural members is also a function of these same variables. Tests were made of round tubes of chromium-molybdenum steel and of duralumin to determine the crinkling strengths and the bending strengths in terms of the specified yield strength and the ratio of diameter to thickness. Empirical formulas are given relating these quantities.

INTRODUCTION

The column strength of structural members of closed sections, such as tubes, composed of thin material is found to increase with decreasing ratio of slenderness up to a limiting stress at which crinkling occurs. This stress may be called the crinkling strength. The crinkling strength is the upper limit of the column strength and is practically independent of the length of the member below the length at which it is first reached. The crinkling strength must be known for each geometric shape of cross section used in a compression member of a given material if the column strength is to be completely known.

The bending strength, measured by the modulus of rupture, is also a function of the shape of the cross section, as is well known, and for a given material must be determined for those shapes of cross section used in designing.

The interest of the Bureau of Aeronautics, Navy Department, and of the National Advisory Committee for Aeronautics in the strength of aircraft tubing led to the allotment of funds to the National Bureau of Standards for an investigation of the subject.

Almost all of the carefully made tests in the past to determine the crinkling strength or the bending strength of round tubing have been made on specimens with far higher ratios of diameter to thickness than are commonly used in aircraft. Timoshenko (reference 1) refers to many of these tests, and an excellent piece of work has been done recently by Hansen (reference 2).

In the present investigation the crinkling strength and the bending strength of round tubular specimens of chromium-molybdenum steel and of duralumin were studied. The diameters of the tubes ranged from 1 inch to 2 inches, and the thicknesses from 0.025 inch to 0.109 inch. The ratios of diameter to thickness ranged approximately from 15 to 100. The experimental work included comprehensive tests to determine the tensile properties of the chromium-molybdenum-steel tubing, the tensile and compressive properties of the duralumin tubing, 60 crinkling tests, and 38 bending tests. In this paper these tests are described, and the results are interpreted for practical use in design.

The author is indebted to the Aluminum Company of America for donation of all of the duralumin tubing, and to the Summerill Tubing Company for donation of three thin sizes of the chromium-molybdenum-steel tubing.

MATERIAL AND MATERIAL TESTS

THE MATERIAL AND ITS PREPARATION FOR TEST

Most of the chromium-molybdenum-steel tubing was manufactured to comply with Navy Department Specification 44T18c, Feb. 1, 1937: Tubing, Steel (Chrome-molybdenum) Round, Seamless (Aircraft Use).

The duralumin tubing was of the type commonly known as 17ST aluminum-alloy tubing. It complied with Navy Department Specification 44T21b, May 1, 1937: Tubing, Aluminum-alloy (Aluminum-copper-magnesium-manganese), Round, Seamless, Condition "T", heat treated.

The nominal cross-sectional properties of the tubes are given in table I.

One tensile specimen and one compressive specimen were taken from each tube, the remainder of the tube being available for crinkling-test and transverse-test specimens. In order to determine the actual cross-sectional properties of the specimens, more than half of the specimens were weighed and their lengths, outside diameters at the middle, and maximum and minimum thicknesses at the ends were measured. The densities of representative samples were determined by the Division of Weights and Measures of the National Bureau of Standards. The cross-sectional areas of the specimens were computed from the weights, the lengths, and the densities; and the average thicknesses and the section moduli were computed from the cross-sectional areas and the outside diameters.

TENSILE AND COMPRESSIVE TESTS

A tensile test was made of each tube, and a compressive test was made of each duralumin tube. Compressive tests were made of as many of the chromium-molybdenum-steel tubes as possible. The results were not used, however, since it was not possible to determine the yield strengths of the thin specimens; these specimens failed by crinkling before the yield strengths were reached.

The tensile tests were made in a pendulum, hydraulic machine having a capacity of 100,000 pounds and scale ranges of 0 to 10,000, 0 to 20,000, 0 to 50,000 and 0 to 100,000 pounds. Most of the compressive tests were made in a fluid-support, Bourdon-tube, hydraulic machine having a capacity of 100,000 pounds and scale ranges of 0 to 10,000, 0 to 50,000, and 0 to 100,000 pounds. Auxiliary nuts on the screws of this machine were tightened against the lower surface of the adjustable head to bring it into contact with the lower surface of the threads on the screws, so that rotation of the head relative to the platen of the machine due to clearance between the nuts of the head and the screws was obviated. The unsymmetrical position of the motor, the hand-wheel, and the other mechanism for raising and lowering the adjustable head causes it to exert on the portion of the two screws below it a constant moment of roughly 1,000 pound-inches in a plane normal to that of the screws. As a consequence the screws are slightly bent elastically and, as they tend to straighten out under load, produce rotation of the head. This condition, causing a slight eccentricity of loading, is especially undesirable in compression testing, but with the short specimens and comparatively low loads of the present investigation, the effect was not considered to be serious. Another possible source of error in making compressive tests in this type of machine arises from the possibility of rotation of the platen about a horizontal axis. The platen is rigidly connected to the piston of the hydraulic jack, which is packed, and the clearance between the cylinder and the piston permits rotation of the platen under eccentric load. This effect can be minimized by keeping as much of the piston in the cylinder as possible.

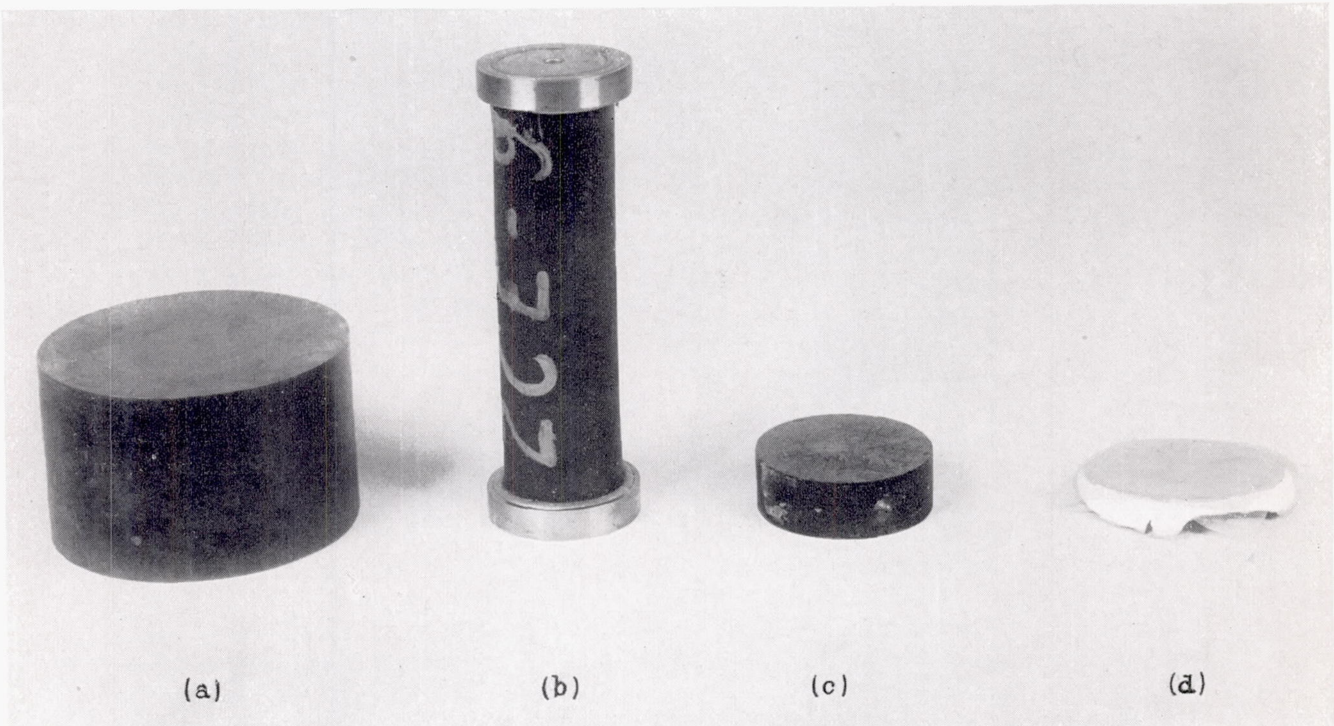

FIGURE 1.—Specimen (b), bearing blocks (a and c), and plaster cap (d).

Tensile tests were made of "full-tube" specimens as required in the specifications for the material. Compressive tests were made on specimens 4 or 5 (preferably 4 when possible) diameters long with ends machined plane and normal to the axis of the tubes. Each end of a specimen was embedded in Wood's metal to a depth of one-quarter of the diameter of the specimen, as shown in figure 1 (b). It is believed that somewhat more nearly uniform conditions are obtained at the ends by this procedure than by simply leaving the ends unsupported laterally. The specimen was then placed centrally on a ground, hardened-steel bearing block located centrally in the testing machine, figures 1 (a) and 2, and a similar, smaller block (fig. 1 (c)) was placed centrally on the upper end of the specimen. In order to secure as nearly uniform bearing as practically possible, the upper bearing block was capped with plaster of paris (fig. 1 (d)). A stiff "mix" was found most satisfactory. It was placed between two sheets of relatively nonabsorbent oiled tracing paper and transferred to the bearing block. Load was applied immediately, arbitrarily 500 pounds per inch of diameter of specimen. The plaster was allowed to set for at least 15 minutes before testing.

Strains were measured with a Ewing extensometer when possible, on a 2-inch gage length; otherwise Tuckerman strain gages were used for determinations of moduli of elasticity, and Huggenberger extensometers for determinations of yield strengths. The moduli of elasticity were obtained from stress-strain data by means of difference curves (reference 3) drawn for each of the tensile and compressive specimens. The stress-strain data used for determining the modulus of elasticity were taken after first loading the specimen to produce a strain of about 0.002 and then removing the load. This procedure made the determination of the modulus of elasticity more definite than a determination from readings taken during the first loading, particularly in the case of some of the chromium-molybdenum-steel specimens the initial stress-strain diagrams of which were apparently curved at all stresses, however low.

The tensile yield strengths were obtained from stress-strain diagrams according to the definitions in the Navy Department specifications applying; that is, the yield strength of the chromium-molybdenum steel was taken as the stress at a strain 0.002 in excess of the elastic strain corresponding to this stress, and that of the duralumin as the stress at a strain of 0.006. The compressive yield strength of the duralumin was obtained as the stress corresponding to the intersection with the stress-strain curve of a line drawn through the origin with a slope $\frac{2}{3}E$, where E is the modulus of elasticity (reference 4). Figures 3 and 4 show typical stress-strain diagrams.

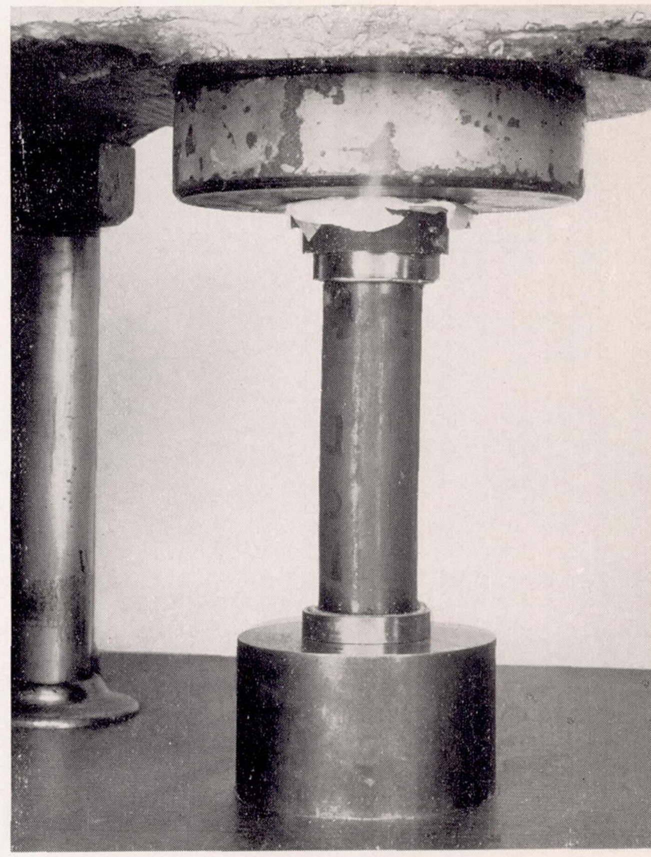

FIGURE 2.—Specimen in machine after test.

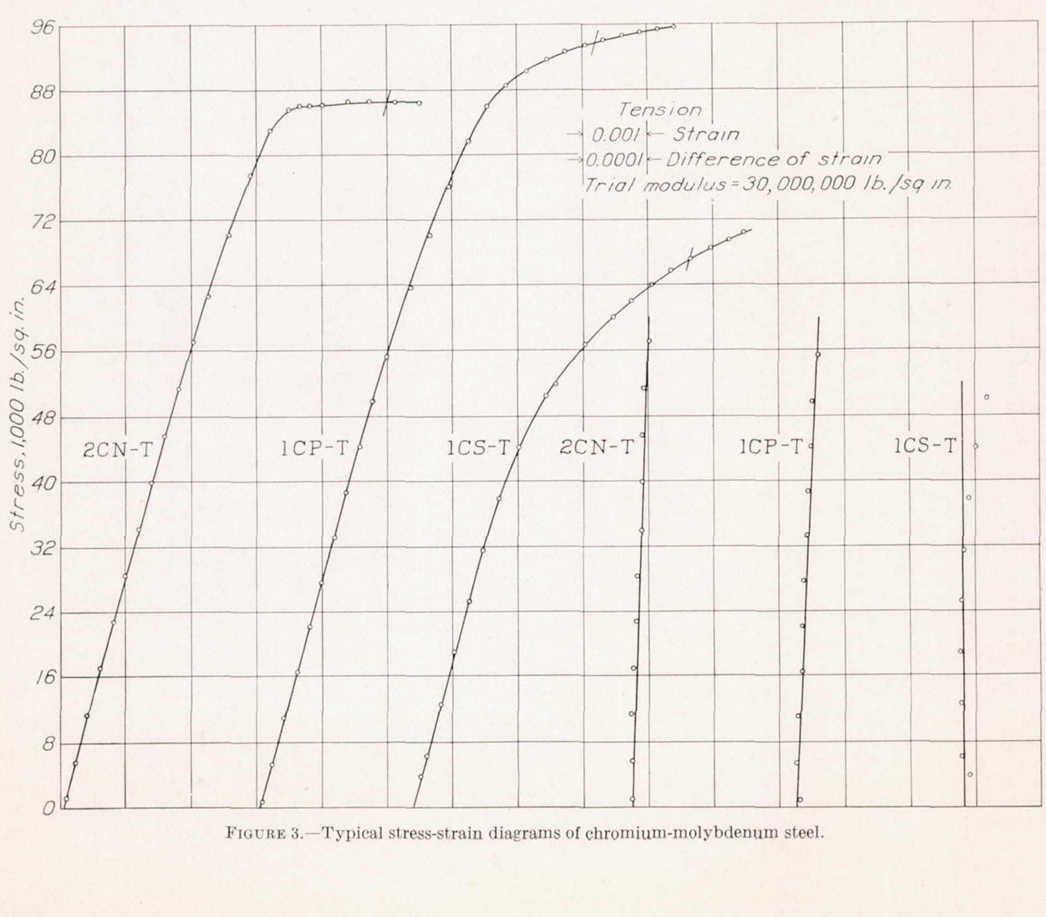

FIGURE 3.—Typical stress-strain diagrams of chromium-molybdenum steel.

CRINKLING TESTS

The procedure for making crinkling tests was exactly the same as that for making compressive tests, except that only the maximum loads were measured. Inasmuch as the compressive specimens failed by crinkling, these specimens also furnished values of the crinkling strength. Figure 5 shows some typical crinkling-test specimens after testing.

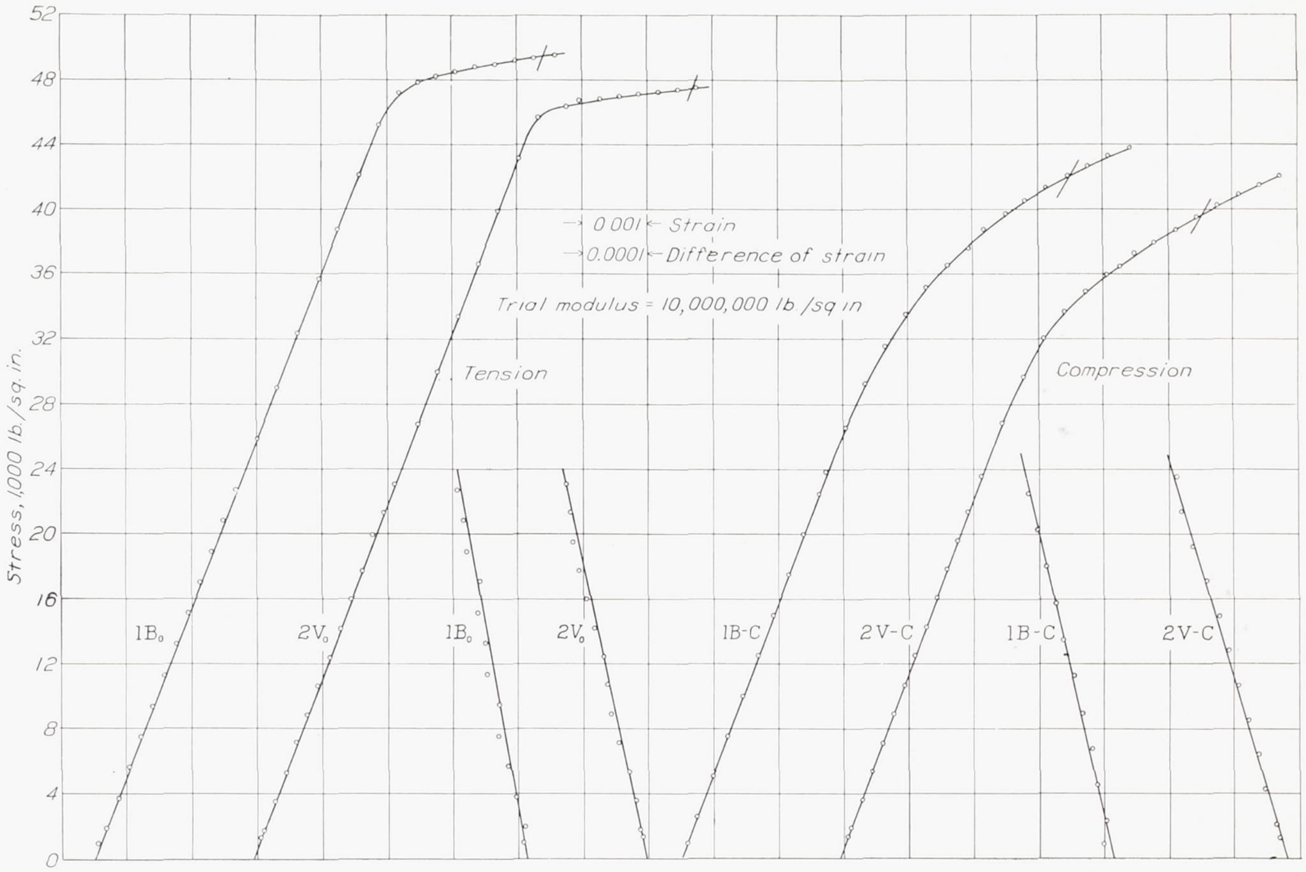

FIGURE 4.—Typical stress-strain diagrams of 17ST duralumin.

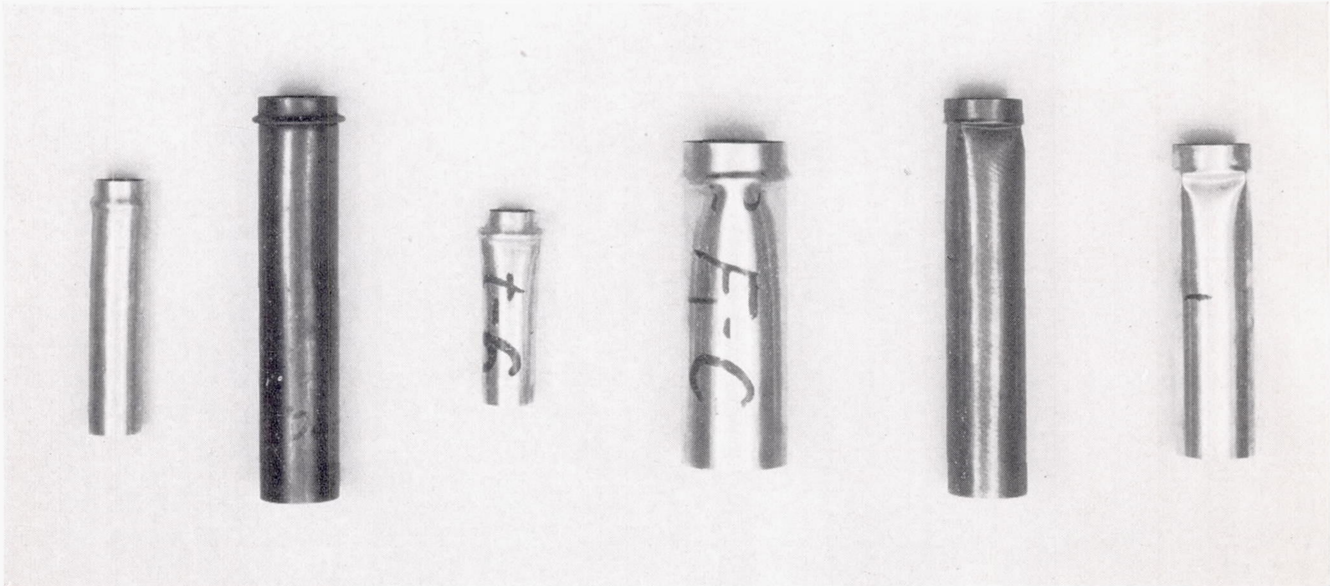

FIGURE 5.—Crinkling-test specimens.

BENDING TESTS

Figure 6 shows the method of making the bending tests. The test specimen A was supported at the ends and was loaded symmetrically at the third-points until the bending moment in the middle portion became a maximum. It was desired to obtain failure in a part of the specimen that was free to assume its "natural" shape at failure, unaffected by local restraints or concentrated loads. In order to effect this condition, the loads were applied through stiff clamps B, which fit the tube snugly at the third-points. One of the clamps was always tightened to a sliding fit only, so that no excessive torsional stresses might be introduced in the specimen by any possible rotation of the clamps relative to each other about the axis of the specimen. The clamps held the tube circular at the loaded sections at all times and thus prevented flattening at these sections. The middle of the tube, on the other hand, subjected to the same bending moment, was free to deform at will and of course took a characteristic flattened shape at failure. All failures occurred at or near the middle. Preliminary tests showed that a distance of five diameters between loading points was sufficient to permit the middle section to assume its natural shape. All but three specimens, however, were tested with a distance of at least six diameters between loading points. The loads were applied through knife edges on the clamps by means of hangers C, which extended down to an equalizer. The equalizer bore on knife edges on the lower ends of the hangers and was itself loaded through a knife edge at the center by the movable head of a beam-and-poise testing machine having a capacity of 20,000 pounds. The scale ranges of the machine were 0 to 2,000 and 0 to 20,000 pounds. The specimen was usually cut 19 diameters long, the ends were plugged, and it was supported on knife edges D with the thinnest part of the specimen up (in compression). The supporting knife edges were spaced accurately by means of spacer bars (not shown in the figure) and rested on hard steel plates E which bore on hard steel balls. The balls were free to roll on other hard steel plates F thus practically eliminating axial stresses in the specimen. The whole assembly was carried on structural steel channels at the top of the testing machine. The hangers were long enough so that the error due to their not being parallel when the specimen failed was negligible. Stops G were provided at the ends of the specimen to prevent "the whole thing from rolling on the floor," which it did once or twice anyway. The stops did not make contact during a test.

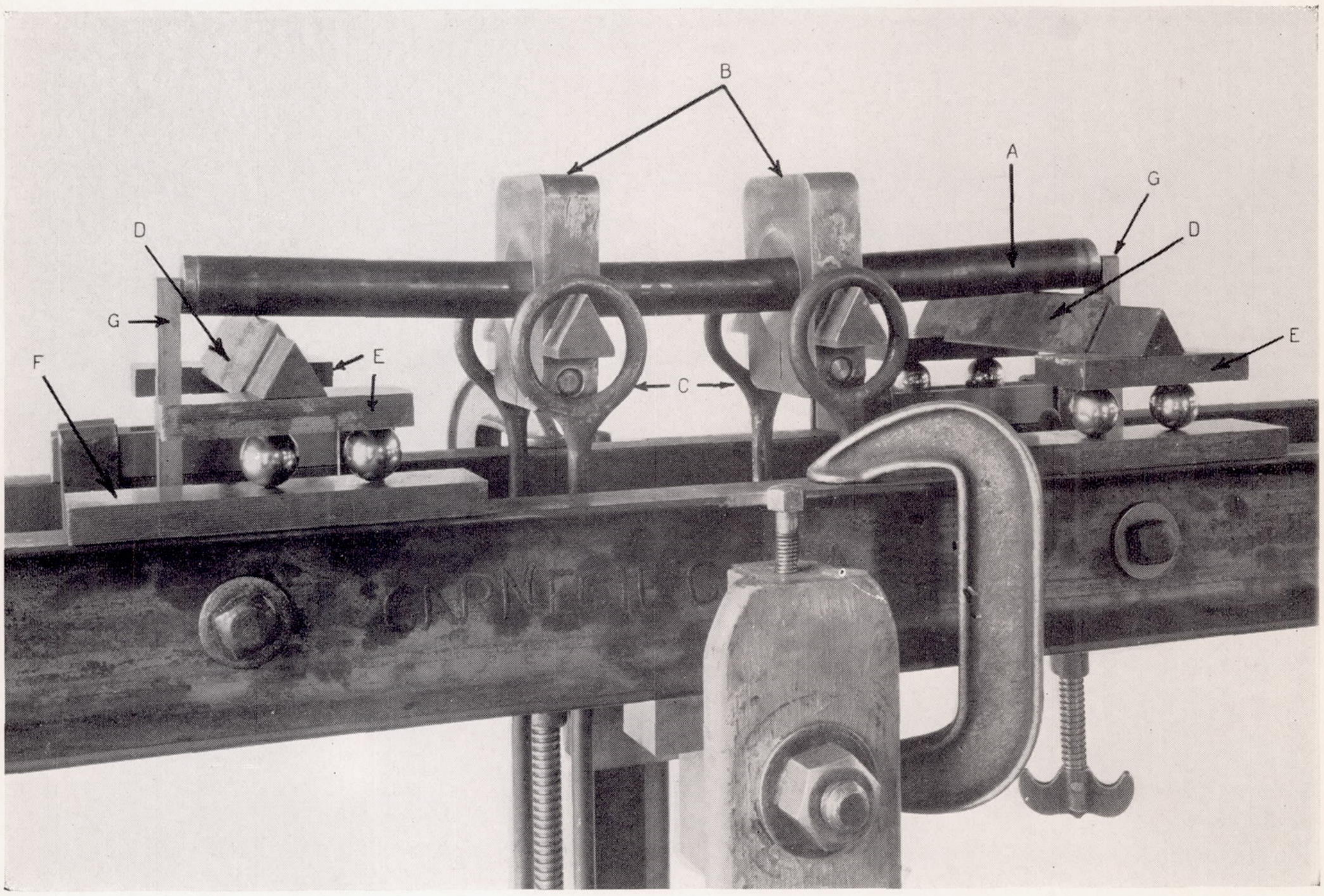

FIGURE 6.—Test to determine bending strength.

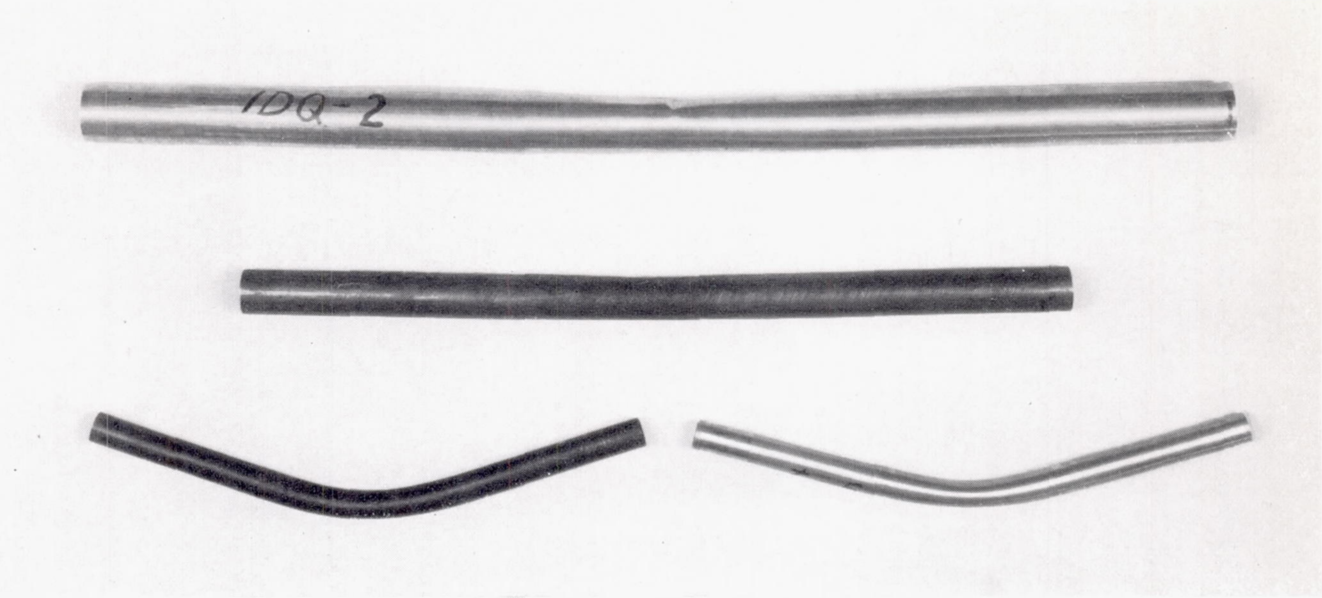

FIGURE 7.—Bending-test specimens.

Some of the specimens (the thick ones) deflected so much that the horizontal distance from a supporting knife edge to the knife edge on the nearer clamp was appreciably reduced. This distance and the corresponding load were consequently read simultaneously as the test proceeded. It was found in some cases that a maximum moment was reached for a load less than the maximum. Figure 7 shows some typical bending-test specimens after testing.

THEORY

The theory of elastic failure by buckling of thin circular cylinders has been presented by numerous authors (see reference 1) but no complete theory of plastic failure exists. Geckeler (reference 5) has presented a theory for the case of axially symmetrical buckling (single lobes reaching around the circumference of the tube). It is probable, by analogy with the elastic case, that the critical stress he obtains would not differ greatly from the theoretical critical stress for the most general (multilobed) type of buckling. Probably largely on account of the impossibility of satisfying the theoretical end conditions and ideal requirements of shape, homogeneity of material, etc., it has not been possible to obtain even an approximate check of any theory in the laboratory. For practical purposes the most valuable contribution of theory has been to give an indication of how the buckling or crinkling strength probably depends on the geometry and the mechanical properties of the material of the specimen or structural element.

Geckeler finds for the crinkling strength for axially symmetrical failure

$$f_{cr}=\frac{2tE}{d_m}\sqrt{\frac{\tau}{3(1-\mu^2)}} \tag{1}$$

where f_{cr} is the crinkling strength,

t, the thickness of the tube,

$d_m=d-t$, the mean diameter of the tube,

E, the modulus of elasticity of the material,

$\tau=\frac{\overline{E}}{E}=\frac{4E'/E}{(1+\sqrt{E'/E})^2}$, $\overline{E}$ being the double-modulus and E' the tangent modulus at the stress f_{cr},

μ, Poisson's ratio.

By introducing the nondimensional variables

$$\delta_s=\frac{S}{E}\frac{d_m}{t} \text{ and } \sigma_{crs}=\frac{f_{cr}}{S} \tag{2}$$

where S is the compressive yield strength, and by dividing equation (1) by S, the crinkling strength for axially symmetrical failure may be expressed in nondimensional form as

$$\sigma_{crs}=\frac{2}{\delta_s}\sqrt{\frac{\tau}{3(1-\mu^2)}} \tag{3}$$

If S is determined as the intersection with the stress-strain curve of a line through the origin having a slope βE $(0<\beta<1)$, equation (3) will represent one and the same curve for all materials having affinely related stress-strain curves [1] (reference 4). It is probable that

[1] Strictly, curves for which the quantity $\tau/(1-\mu^2)$ is the same for corresponding equal values of σ_{crs}, but the effect of variations in μ would, in any case, almost certainly be small.

the case of multilobed crinkling failure, just like that of single-lobed failure, is governed by some relation analogous to (3) between δ_s and σ_{crs}. It is to be expected then that any empirical relation found between δ_s and σ_{crs} as the result of tests in the laboratory will show less scatter than a relation for example between $\frac{d}{t}$ and f_{cr}.

The theoretical situation as regards the bending strength of thin circular cylinders is in a less satisfactory state than the theory that applies to the crinkling strength. Even the elastic case (reference 6) becomes so involved as to be quite intractable in any practical way. So far as is known, no one has attempted a solution of the plastic case. Nevertheless, because of the similarity in type between bending failures of thin tubes and crinkling failures, it seems reasonable to expect a closer relation to exist between

$$\delta_s = \frac{S}{E}\frac{d_m}{t} \text{ and } \sigma_{rs} = \frac{f_r}{S} \tag{4}$$

where f_r is the modulus of rupture, for materials having affinely related stress-strain curves, than between $\frac{d}{t}$ and f_r.

RESULTS

The results of the crinkling tests are given in table II and are shown for the chromium-molybdenum-steel specimens in figures 8 and 9 and for the duralumin specimens in figures 10 and 11. The results of the

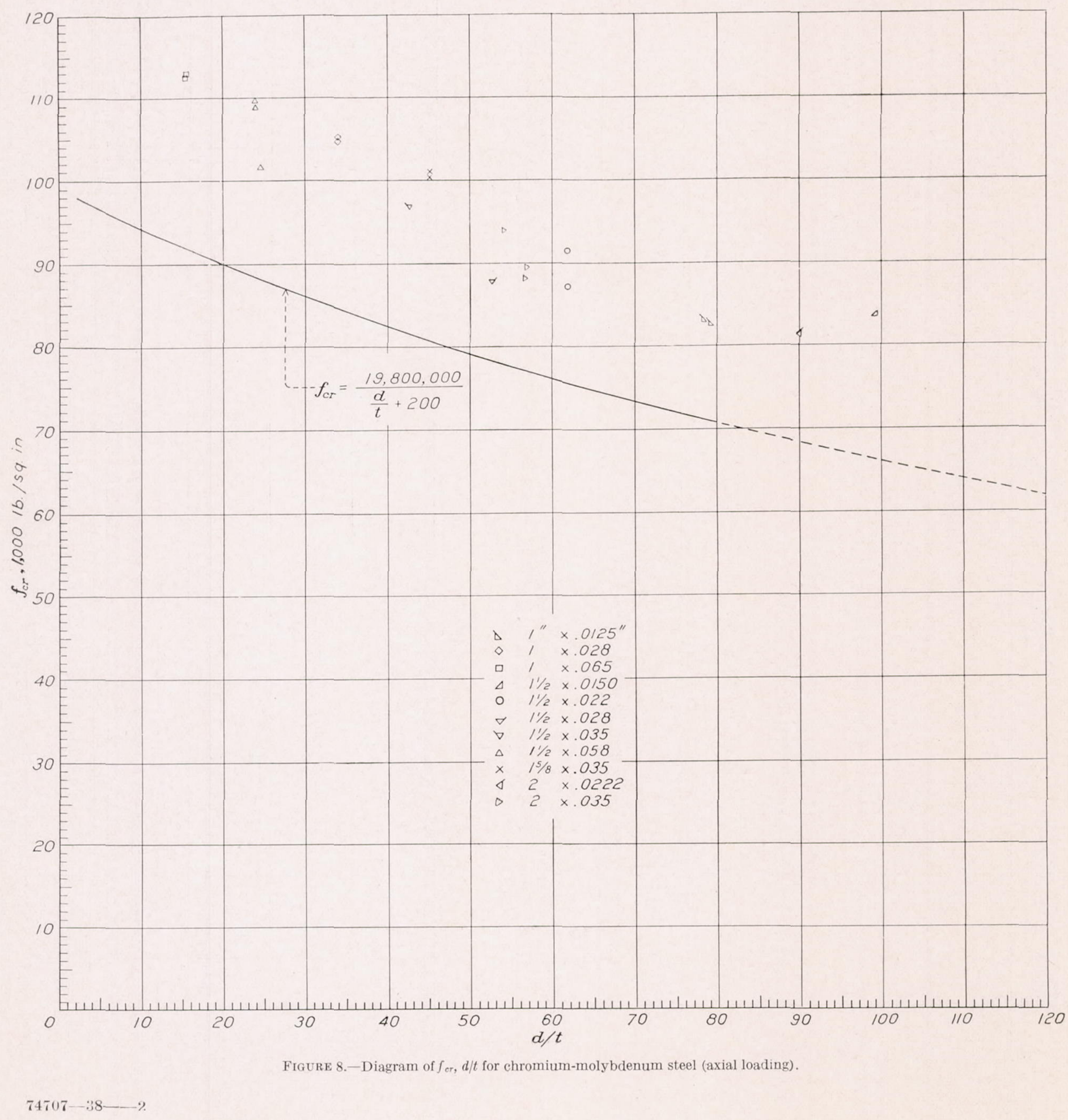

FIGURE 8.—Diagram of f_{cr}, d/t for chromium-molybdenum steel (axial loading).

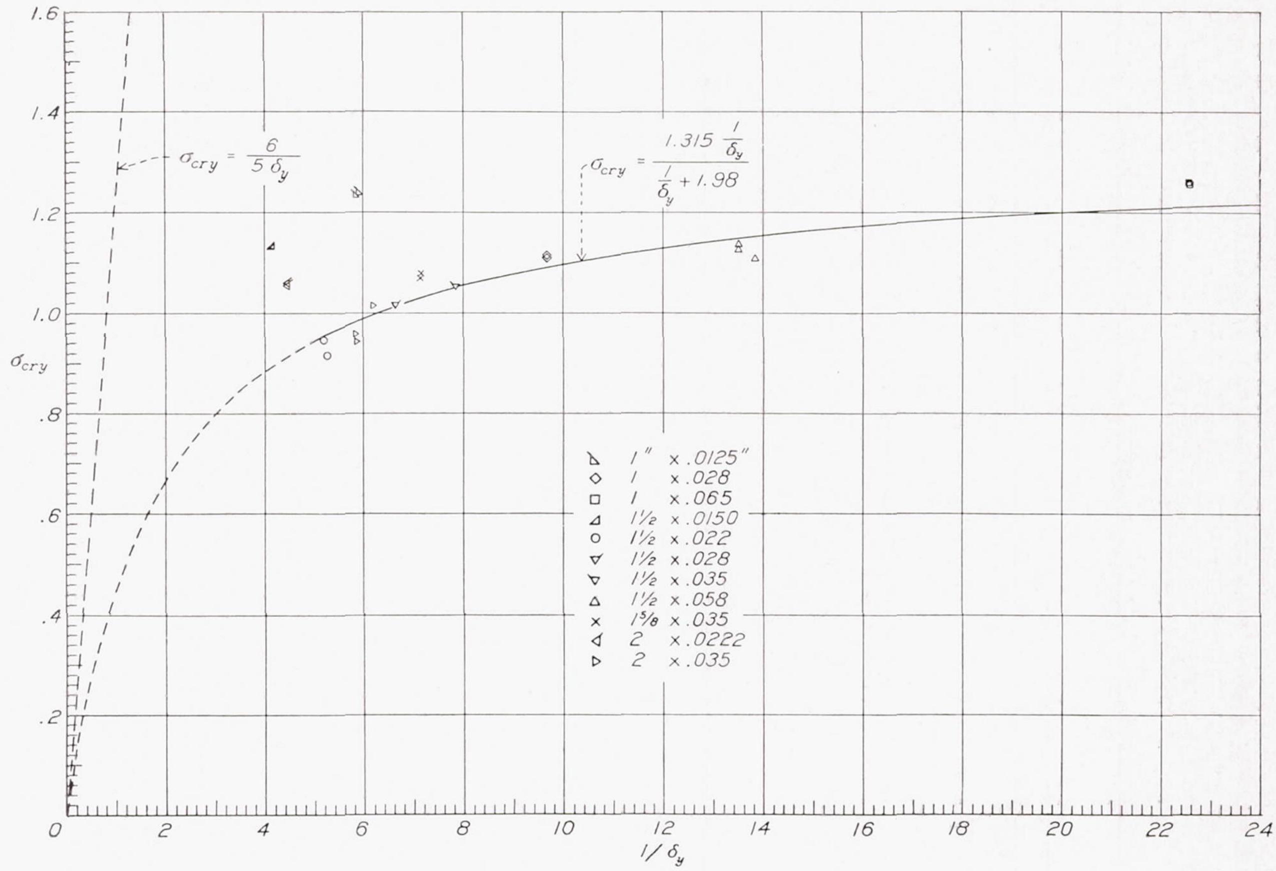

FIGURE 9.—Diagram of σ_{cry}, $1/\delta_y$ for chromium-molybdenum steel (axial loading).

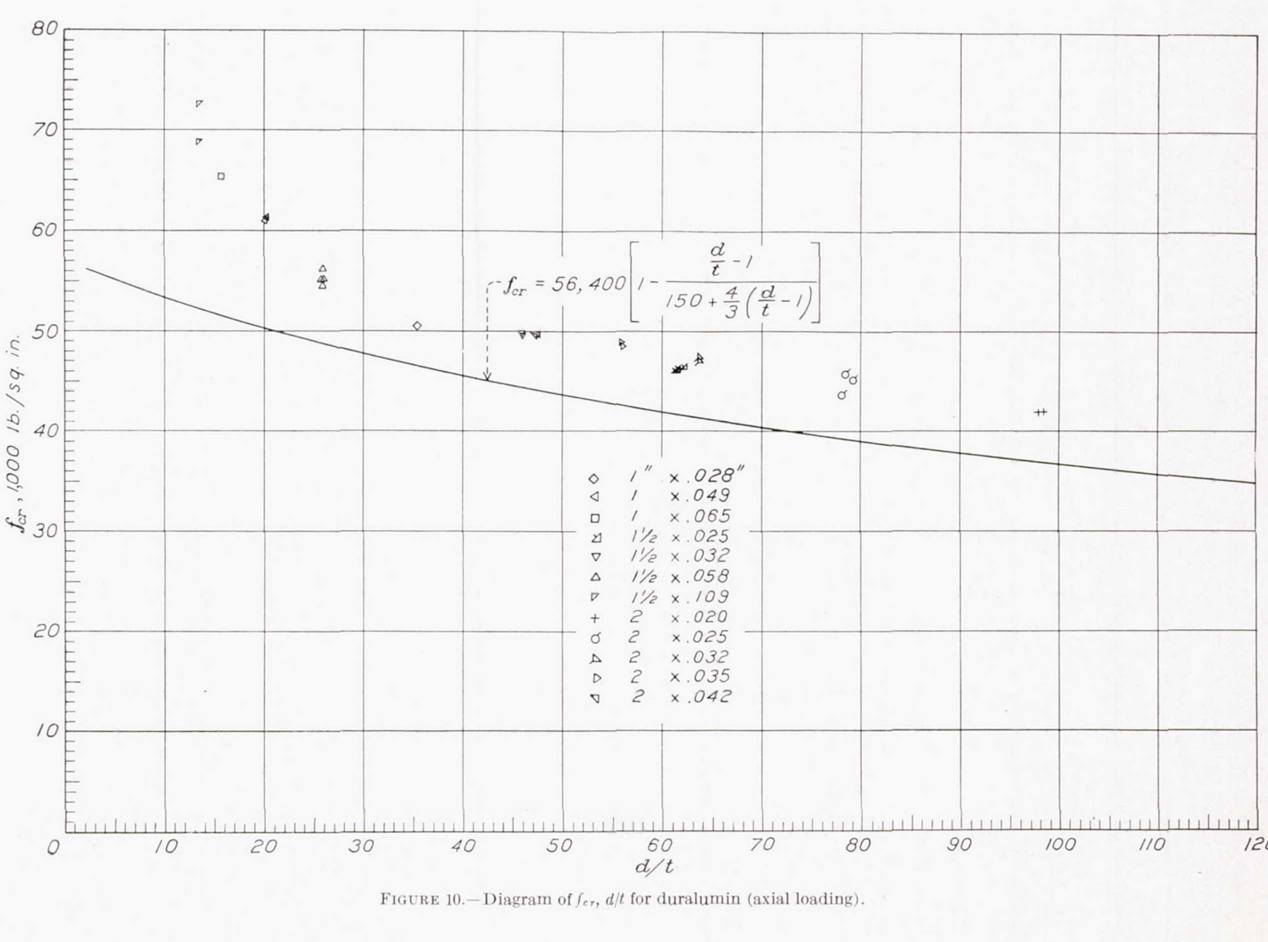

FIGURE 10.—Diagram of f_{cr}, d/t for duralumin (axial loading).

bending tests are given in table III and are shown for the chromium-molybdenum-steel specimens in figures 12 and 13 and for the duralumin specimens in figures 14 and 15. The measured diameters and the computed average thicknesses have been used in determining the values of d/t in tables II and III and in the figures. The crinkling strength, f_{cr}, is the maximum axial load carried by the specimen divided by its cross-sectional

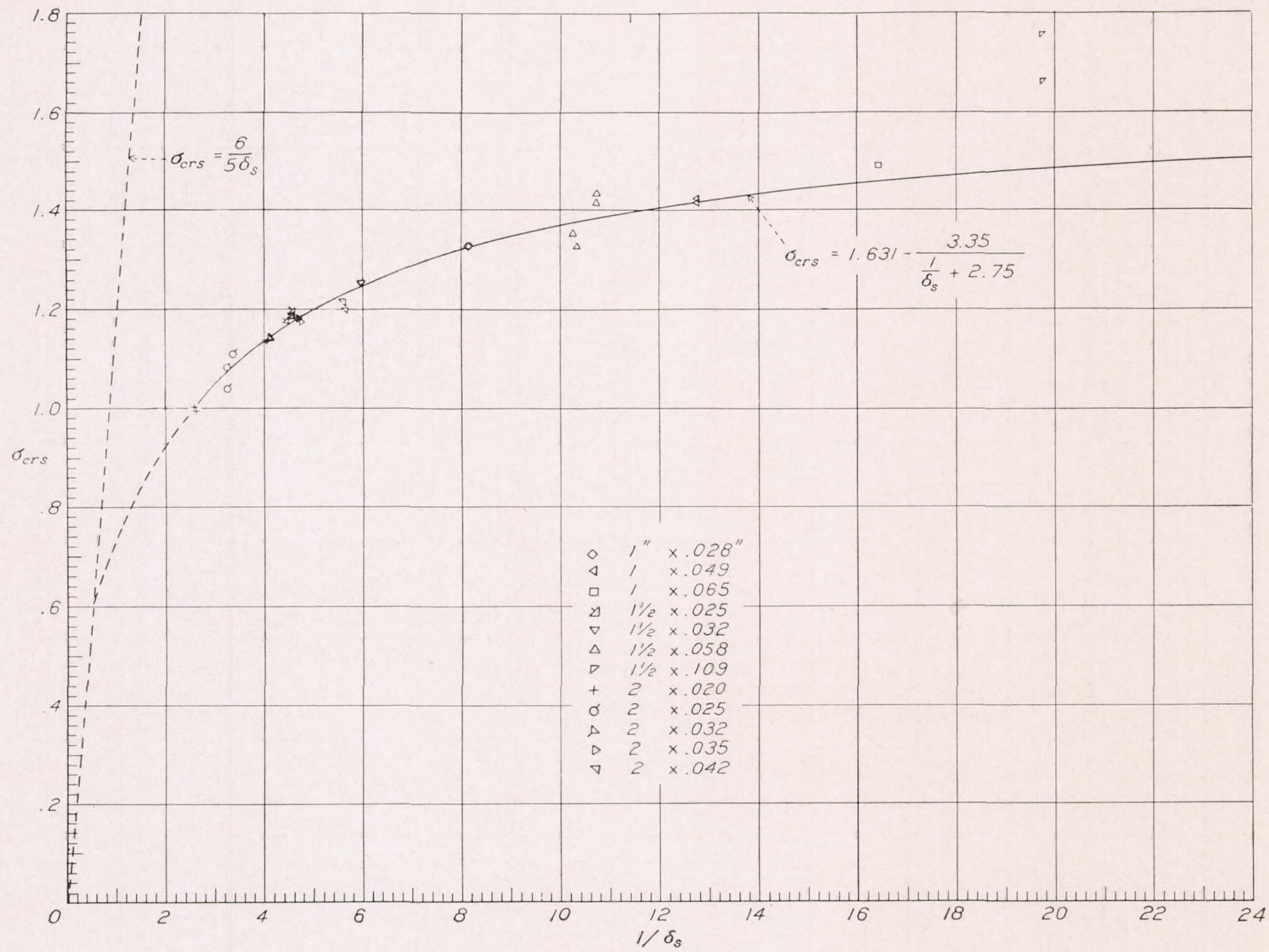

FIGURE 11.—Diagram of σ_{crs}, $1/\delta_s$ for duralumin (axial loading).

area. The modulus of rupture, f_r, has been computed from the common flexure formula

$$f_r = \frac{Mc}{I}$$

where M is the maximum moment resisted by the specimen, and I/c is its section modulus. The data in figures 8, 10, 12, and 14 have been plotted in the conventional way and in figures 9, 11, 13, and 15 in terms of the nondimensional variables [2] δ and σ. Because compressive yield strengths of the thin chromium-molybdenum-steel tubes could not be obtained, the tensile yield strengths, Y, were used in the expressions for δ and σ; that is,

$$\delta_y = \frac{Y}{E}\frac{d_m}{t} \text{ and } \sigma_{cry} = \frac{f_{cr}}{Y},\ \sigma_{ry} = \frac{f_r}{Y} \qquad (5)$$

were used in figures 9 and 13 instead of δ_s and σ_{crs}, σ_{rs} as defined in equations (2) and (4). It is seen that the scatter of the points for the chromium-molybdenum-steel specimens in the σ_y, $1/\delta_y$-diagrams of figures 9 and 13 is materially less than in the f, d/t-diagrams of figures 8 and 12 except for the three sets of points representing the three sizes of tubes, 1 by 0.0125 inch, 1½ by 0.0150 inch, and 2 by 0.0222 inch. For the duralumin specimens, the points in the σ_s, $1/\delta_s$-diagrams of figures 11 and 15 lie on somewhat smoother curves than the points in the f, d/t-diagrams of figures 10 and 14. Greater improvement in the nondimensional representation would be expected for the chromium-molybdenum-steel data than for the duralumin data because the former material was somewhat more variable in its mechanical properties than the latter.

It is not difficult to find an explanation for the apparently anomalous behavior of the three chromium-molybdenum-steel tubes mentioned. The tensile stress strain curves for these tubes were gradually curved from low stresses on (1CS–T, fig. 3), whereas all the other tubes had stress-strain curves with relatively sharp knees. Since the material is supposed to be

[2] Where δ, σ_{cr}, σ_y, σ_s, σ, and f are used, the context will indicate whether δ_s or δ_y; σ_{crs}, σ_{cry}, σ_{rs}, or σ_{ry}; and f_{cr} or f_r is meant.

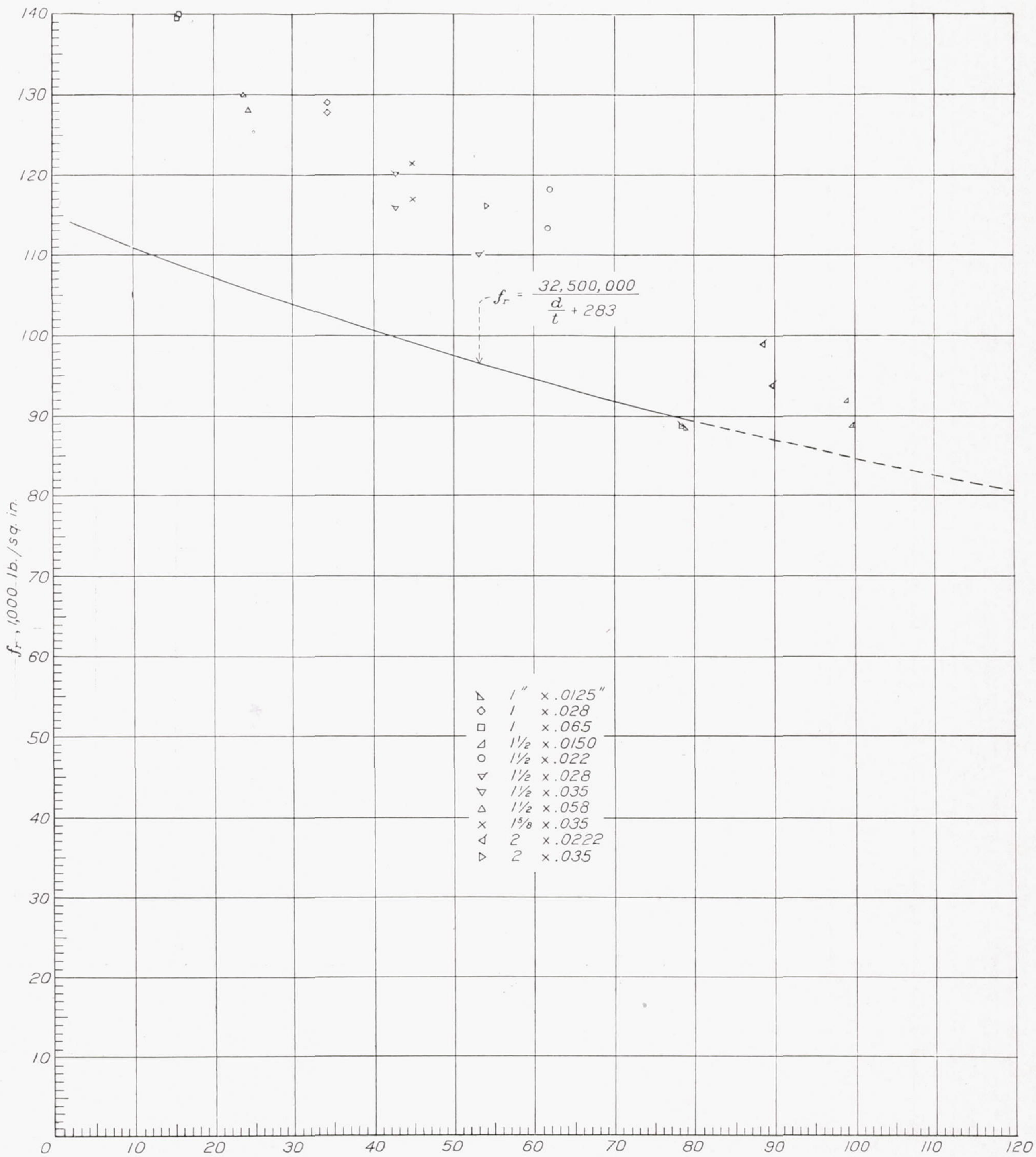

FIGURE 12.—Diagram of f_r, d/t for chromium-molybdenum steel (transverse loading).

normalized, it is not unreasonable to expect similar stress-strain curves in compression. If this is the case, then for a given yield strength S for the two types of material and at a given stress $f>S$ at failure,[3] the value of τ, which depends on the tangent modulus, will be greater for the material with the blunt-knee stress-strain curve than for the material with the sharp-knee stress-strain curve. Now, unquestionably, the strength increases with τ and decreases with d/t (whether equation (1) is right or not) and, consequently, for a given strength, represented by f, a high value of τ will be associated with a high value of d/t and a low value of τ with a low value of d/t. It is to be expected, then, that the three sets of points representing the tests on the thin tubes in figures 7 and 11 would be shifted to the left. They are "off the beaten track" of the other

[3] The statement that follows is true not only for $f>S$ but for f greater than the stress at which the slopes of the two stress-strain curves become equal. The values of E are assumed to be equal.

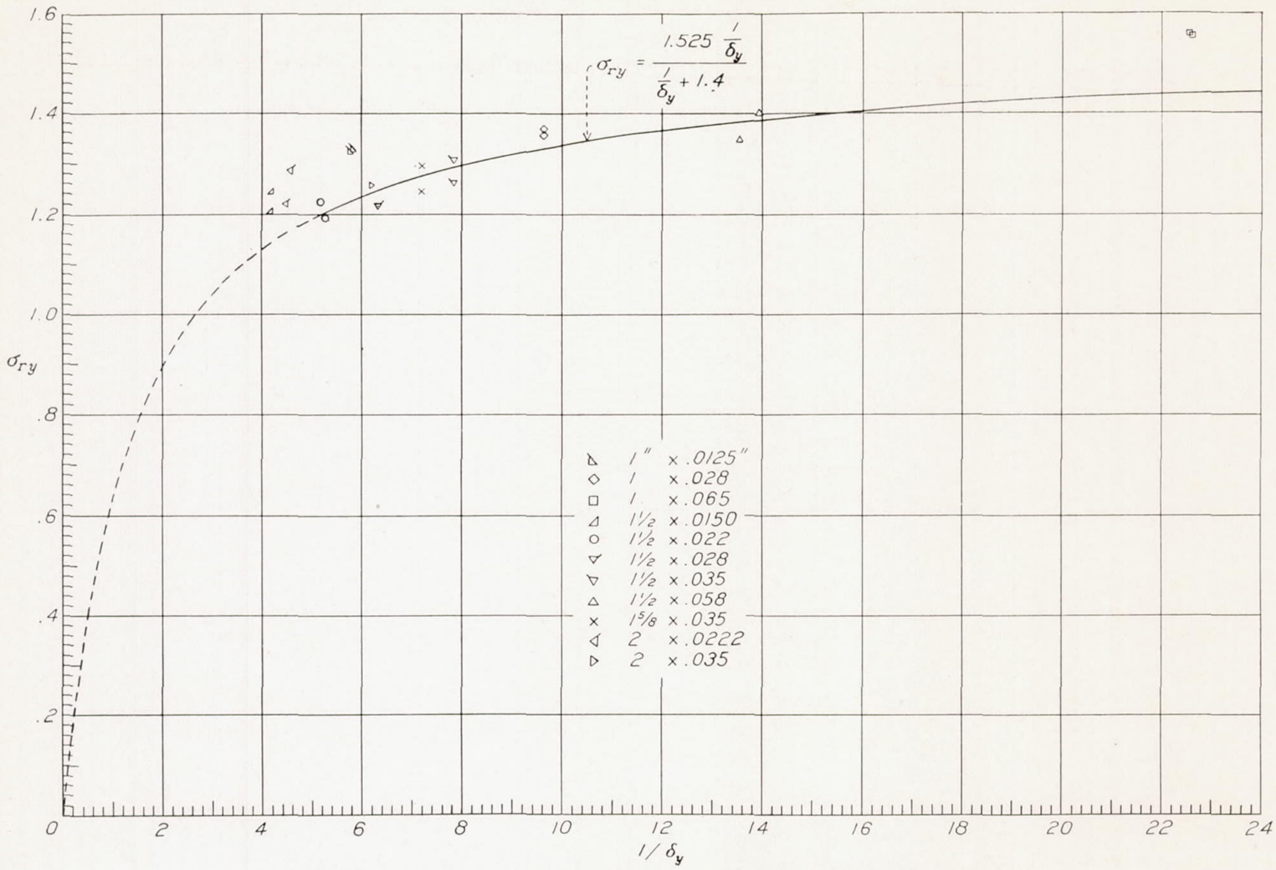

FIGURE 13.—Diagram of σ_{ry}, $1/\delta_y$ for chromium-molybdenum steel (transverse loading).

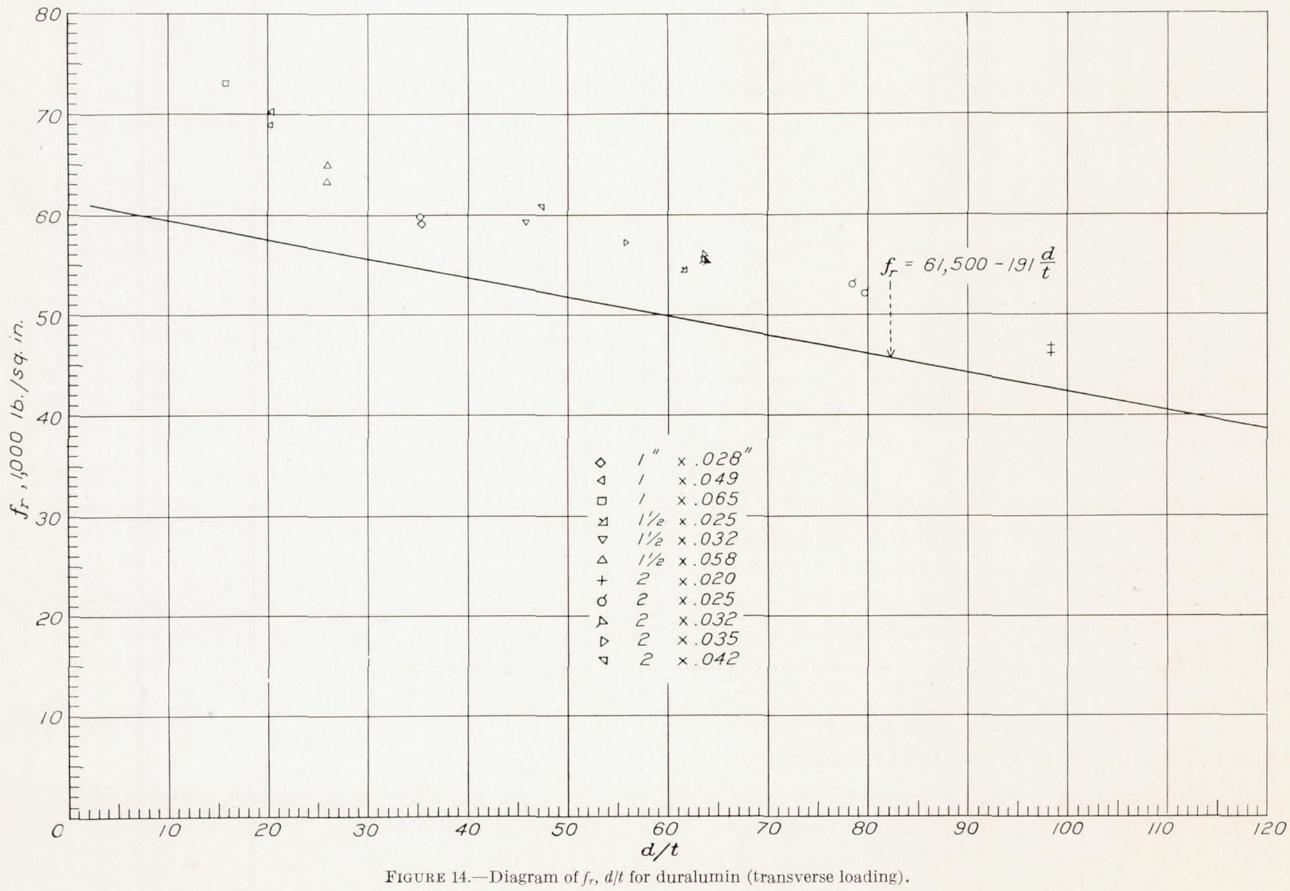

FIGURE 14.—Diagram of f_r, d/t for duralumin (transverse loading).

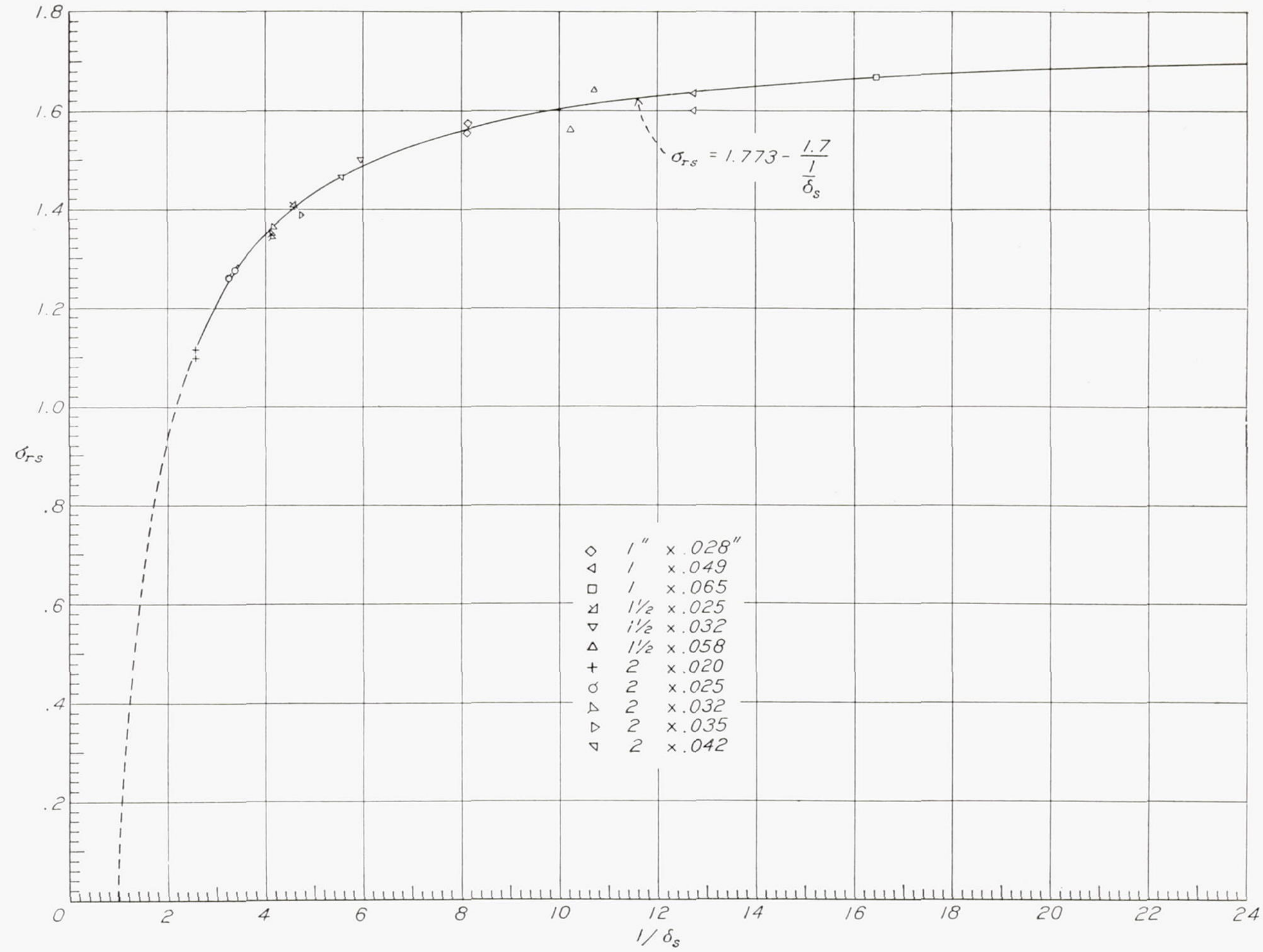

FIGURE 15.—Diagram of σ_{rs}, $1/\delta_s$ for duralumin (transverse loading).

points because the stress-strain curves of the material they represent are not even approximately affinely related to the stress-strain curves of the other material.

It will be noted that smooth curves drawn to represent the σ, $1/\delta$-data would rise concave upward to meet the points for which $1/\delta$ is greater than 19. These points represent the strengths of relatively thick specimens, and it is to be expected that the strengths of such specimens would increase rapidly with decrease in ratio of diameter to thickness. If $d/t=2$, the tube becomes a solid bar, the crinkling strength becomes infinite, and the bending strength becomes very high, depending now on the tensile strength of the material. These high values of crinkling strength and bending strength, however, have no practical significance since the deformations required to obtain them are so great as to be quite intolerable in a structure. The more or less abrupt rise in strength at low values of the ratio of diameter to thickness is analogous to that which occurs in columns at low values of the ratio of slenderness.

Empirical curves have been fitted to the σ, $1/\delta$-data of figures 9, 11, 13, and 15. In doing so, the points for which $1/\delta$ was greater than 19 have not been used, in accordance with the preceding discussion, nor have the three sets of points for the chromium-molybdenum-steel specimens previously discussed been taken into account, since they represent essentially a different material. The omission from consideration of all these points is on the safe side. All four curves are hyperbolas. They are shown solid in the figures for the range covered by the tests, and they are extended as dotted curves. The crinkling strength, in nondimensional form, of the chromium-molybdenum-steel tubes was found to be given by

$$\sigma_{cry}=\frac{1.315\frac{1}{\delta_y}}{\frac{1}{\delta_y}+1.98}, \qquad \frac{1}{\delta_y}>5 \tag{6}$$

The crinkling strength, in nondimensional form, of the duralumin tubes was found to be given by

$$\sigma_{crs}=1.631-\frac{3.35}{\frac{1}{\delta_s}+2.75}, \qquad \frac{1}{\delta_s}>2.5 \tag{7}$$

The modulus of rupture, in nondimensional form, of the chromium-molybdenum-steel tubes was found to be given by

$$\sigma_{ry}=\frac{1.525\frac{1}{\delta_y}}{\frac{1}{\delta_y}+1.4}, \qquad \frac{1}{\delta_y}>5 \tag{8}$$

The modulus of rupture, in nondimensional form, of the duralumin tubes was found to be given by

$$\sigma_{rs}=1.773-\frac{1.7}{\frac{1}{\delta_s}}, \qquad \frac{1}{\delta_s}>2.5 \tag{9}$$

Figures 9 and 11 show also the theoretical curve (straight line) for axially symmetrical elastic failure. If one assumes, for convenience, $\mu=\frac{1}{3}\sqrt{\frac{2}{3}}=0.2722$, one obtains from equation (3)

$$\sigma_{cr}=\frac{6\sqrt{\tau}}{5\delta} \tag{10}$$

and, since for elastic failure, $\tau=1$

$$\sigma_{cr}=\frac{6}{5\delta} \tag{11}$$

When σ_{cr} is plotted against $1/\delta$, equation (11) may be represented by the straight line shown in the figures.

It immediately appears that, by substituting for τ in equation (10) the expression found from column tests, or otherwise, a theoretical relation between δ and σ_{cr} might be obtained in the plastic range. This substitution might be done for the range of values of σ for which an expression for τ was reliable; but no agreement with the results of crinkling tests would be expected because crinkling failures obtained in the laboratory in the plastic range are not stability failures but bending failures. This condition is necessarily true because of the impossibility of satisfying the end and other conditions required by theory for a stability failure.

It remains to obtain expressions for the crinkling strength and the bending strength of tubing that just complies with specifications. Such expressions may be obtained immediately from equations (2) and equations (4) to (9), inclusive. The specified minimum yield strength of chromium-molybdenum-steel tubing such as used in this investigation is, according to Navy Department Specification 44T18c for tubing not over 0.188 inch thick, 75,000 pounds per square inch, and the modulus of elasticity may be taken as 29,800,000 pounds per square inch (reference 4). Substituting these values in equations (5) and then replacing δ_y and σ_{cry} in equation (6) by the expressions obtained from equations (5), and solving for the crinkling strength, f_{cr}, gives after rounding off,

$$f_{cr}=\frac{19\ 800\ 000}{\frac{d}{t}+200}, \qquad \frac{d}{t}<80 \tag{12}$$

in lb per sq in.

The specified minimum tensile yield strength of duralumin tubing such as used in this investigation is, according to Navy Department Specification 44T21b for Condition "T" heat-treated tubing, 40,000 pounds per square inch. The average ratio of compressive yield strength to tensile yield strength of the tubes used in this investigation was found to be 0.864. The average value of the modulus of elasticity was 10,610,000 pounds per square inch. Substituting $S=0.864\times 40{,}000=34{,}560$ pounds per square inch and $E=10{,}610{,}000$ pounds per square inch in equations (2) and then replacing δ_s and σ_{crs} in equation (7) by the expressions obtained from equations (2), and solving for the crinkling strength, f_{cr}, gives, after rounding off,

$$f_{cr}=56\ 400\left[1-\frac{\frac{d}{t}-1}{150+\frac{4}{3}\left(\frac{d}{t}-1\right)}\right], \qquad \frac{d}{t}<125 \tag{13}$$

in lb per sq in.

An expression for the modulus of rupture of chromium-molybdenum-steel tubing that just complies with Navy Department Specification 44T18c for tubing not over 0.188 inch thick may be found, as just outlined, from equations (5) and (8):

$$f_r=\frac{32\ 500\ 000}{\frac{d}{t}+283}, \qquad \frac{d}{t}>80 \tag{14}$$

in lb per sq in.

An expression for the modulus of rupture of duralumin tubing that just complies with Navy Department Specification 44T21b for Condition "T" heat-treated tubing may be found, as outlined, from equations (4) and (9):

$$f_r=61\ 500-191\frac{d}{t}, \qquad \frac{d}{t}<125 \tag{15}$$

in lb per sq in.

The curves representing equations (12), (13), (14), and (15) are shown [4] in figures 8, 10, 12, and 14, respectively. They indicate, for the range of values of ratio of diameter to thickness covered, the crinkling strengths and the moduli of rupture that may be expected from tubing which just complies with the applicable specifications noted.

DISCUSSION

As explained in the introduction, the crinkling strength is the upper limit of column strength. With the determination of the crinkling strength, it now becomes possible to indicate where the column curves for the two materials of this investigation must be "cut off" at their upper ends for tubing of a given ratio of diameter to thickness, namely, at the stresses given by equations (12) and (13).

[4] The two points in figure 12 that are below the curve represent the results of tests on specimens the material of which did not comply with the requirement of the specification for chemical composition nor yield strength (1CS–T, fig. 3).

If the stresses from equations (12) and (13) are equated to the highest average column stresses (ratio of slenderness equal to zero), the highest values of d/t may be found for which the column curves for the two materials apply over their entire range without cutting off at the top.[5] For chromium-molybdenum-steel tubes, the limiting column stress was found in reference 4 to be 79,400 lb. per sq. in., and, if one equates this quantity to the right-hand side of equation (12),

$$79\ 400=\frac{19\ 800\ 000}{\frac{d}{t}+200}$$

and solves for d/t, one obtains $d/t=50$. Similarly, for duralumin tubes the limiting column stress was found in reference 4 to be 42,700 lb. per sq. in., and if one equates this quantity to the right-hand side of equation (13),

$$42\ 700=56\ 400\left[1-\frac{\frac{d}{t}-1}{150+\frac{4}{3}\left(\frac{d}{t}-1\right)}\right]$$

and solves for d/t, one obtains $d/t=55$. The column formulas given in reference 4 may therefore be used over their entire respective ranges for tubing for which d/t does not exceed 50 in the case of chromium-molybdenum steel and d/t does not exceed 55 in the case of duralumin.

The question may arise as to whether clamps used in practice for transferring transverse loads to tubing are sufficiently effective in holding the tube round to prevent failure at the clamp. Preliminary bending tests made with several types of clamps, both with clamps furnished by manufacturers and more flexible clamps made for the purpose, indicated that, so long as the transverse load was applied through a tension member, the clamp would not weaken the tube. The type of connection most likely to weaken a tube locally is a weld. A relatively small compression member transferring its load to the tube in question through a weld might easily promote a dent and cause local failure at lower stresses than those given by equation (14).

It may not be out of place here to call attention to possible failure by transverse shear. The transverse shearing strength of tubing has not been studied in the present investigation. All that can be said is that no evidence of failure due to transverse shear was observed in any specimen. Hansen (reference 2) found, for much thinner tubes, that the modulus of rupture was unaffected by shear when the cantilevered end of the tube was as short as three diameters.

[5] The use of equations (12) and (13) assumes that the column curves are based on the same compressive yield strength as the crinkling curves. If the compressive yield strength of the column material is higher, as it was for the duralumin tubes of reference 4, the values of d/t obtained will be on the conservative side.

NATIONAL BUREAU OF STANDARDS,
WASHINGTON, D. C., *March 28, 1938.*

TABLE I

NOMINAL CROSS-SECTIONAL PROPERTIES OF TUBES

Diameter d (in.)	Thickness t (in.)	Ratio $\frac{d}{t}$	Area A (sq. in.)	Section modulus I/c (in.^{3})
CHROMIUM-MOLYBDENUM STEEL				
1	0.0125	80.0	0.0388	0.00887
1	.028	35.7	.0855	.02021
1	.065	15.4	.1909	.04193
1½	.0150	100.0	.0700	.02572
1½	.022	68.2	.1022	.03720
1½	.028	53.6	.1295	.04678
1½	.035	42.9	.1611	.05765
1½	.058	25.9	.2628	.09121
1⅝	.035	46.4	.1748	.06803
2	.0222	90.0	.1379	.06746
2	.035	57.1	.2161	.10432
DURALUMIN				
1	0.028	35.7	0.0855	0.02021
1	.049	20.4	.1464	.03319
1	.065	15.4	.1909	.04193
1½	.025	60.0	.1153	.04202
1½	.032	46.9	.1476	.05303
1½	.058	25.9	.2628	.09121
1½	.109	13.8	.4763	([a])
2	.020	100.0	.1244	.06097
2	.025	80.0	.1551	.07564
2	.032	62.5	.1978	.09581
2	.035	57.1	.2161	.10432
2	.042	47.6	.2534	.12386

[a] No bending tests were made of this size.

TABLE II

RESULTS OF CRINKLING TESTS

CHROMIUM-MOLYBDENUM STEEL

Specimen	$\frac{d}{t}$	Crinkling strength f_{cr} (lb./sq.in)	$\frac{1}{\delta_y}=\frac{E}{Y}\frac{t}{d_m}$	$\sigma_{cry}=\frac{f_{cr}}{Y}$	Specimen	$\frac{d}{t}$	Crinkling strength f_{cr} (lb./sq.in)	$\frac{1}{\delta_y}=\frac{E}{Y}\frac{t}{d_m}$	$\sigma_{cry}=\frac{f_{cr}}{Y}$
1CL-Cr	15.5	112500	22.58	1.253	1CO-C	54.1	94000	6.19	1.018
1CL-9	15.6	112900	22.55	1.257	2CO-C	56.7	88200	5.85	.945
2CE-9	24.0	109600	13.51	1.135	2CO-Cr	56.9	89500	5.84	.959
2CE-C	24.0	108900	13.51	1.127	1CM-C	61.8	91500	5.16	.946
1CE-1a	24.6	101500	13.87	1.108	2CM-C	61.8	87200	5.22	.917
1CK-9	34.0	105300	9.68	1.115	1CS-3	78.4	82900	5.80	1.239
1CK-C	34.0	104800	9.68	1.110	1CS-C	79.1	82600	5.74	1.234
1CT-C	42.7	96900	7.86	1.052	1CU-C	89.8	81200	4.47	1.056
1CP-3C	45.2	100900	7.12	1.076	1CU-3	89.8	81100	4.47	1.054
1CP-C	45.2	100300	7.12	1.069	1CC-3	99.3	83600	4.16	1.132
2CN-C	52.7	87900	6.64	1.019	1CC-C	99.4	83700	4.16	1.132

DURALUMIN

Specimen	$\frac{d}{t}$	Crinkling strength f_{cr} (lb./sq.in)	$\frac{1}{\delta_s}=\frac{E}{S}\frac{t}{d_m}$	$\sigma_{crs}=\frac{f_{cr}}{S}$	Specimen	$\frac{d}{t}$	Crinkling strength f_{cr} (lb./sq.in)	$\frac{1}{\delta_s}=\frac{E}{S}\frac{t}{d_m}$	$\sigma_{crs}=\frac{f_{cr}}{S}$
1DR-1	13.5	72500	19.74	1.756	F-C	47.4	49900	5.59	1.211
1DR-C	13.5	68700	19.74	1.663	E-4	55.8	48800	4.73	1.178
v-6C	15.7	65400	16.43	1.494	E-C	55.9	48400	4.73	1.175
t-C	20.1	61000	12.77	1.416	1V-C	61.4	46000	4.56	1.185
t-6	20.2	61300	12.74	1.423	V-C	61.5	46200	4.57	1.195
b-C	25.6	55100	10.34	1.324	V-7	61.5	46000	4.57	1.185
1b-4C	25.8	55100	10.73	1.431	2V-C	62.2	46300	4.44	1.172
b-7	25.9	56100	10.25	1.350	DH-4	63.7	47400	4.14	1.145
1b-C	25.9	54400	10.71	1.413	DH-C	63.7	47000	4.14	1.141
1p-C	35.3	50500	8.14	1.328	1B-C	78.1	43600	3.26	1.038
1p-9	35.3	50500	8.13	1.328	1B-5	78.4	45600	3.24	1.081
X-4	45.9	49700	5.98	1.254	B-4C	79.2	45200	3.37	1.109
X-C	45.9	49600	5.98	1.253	1DQ-1	97.9	41900	2.62	.994
1F-3C	47.3	49700	5.61	1.198	1DQ-C	98.4	41800	2.60	.995

TABLE III

RESULTS OF BENDING TESTS

CHROMIUM-MOLYBDENUM STEEL

Specimen	$\frac{d}{t}$	Modulus of Rupture f_r (lb./sq. in.)	$\frac{1}{\delta_y}=\frac{E}{Y}\frac{t}{d_m}$	$\sigma_{ry}=\frac{f_r}{Y}$	Specimen	$\frac{d}{t}$	Modulus of Rupture f_r (lb./sq. in.)	$\frac{1}{\delta_y}=\frac{E}{Y}\frac{t}{d_m}$	$\sigma_{ry}=\frac{f_r}{Y}$
1CL-1a	15.5	139500	22.60	1.553	1CN-3	53.1	110000	6.31	1.215
1CL-3	15.6	140000	22.52	1.559	1CO-3	54.4	116100	6.16	1.258
2CE-3	24.0	129900	13.51	1.345	2CM-2	61.9	113300	5.22	1.191
1CE-3a	24.5	128200	13.92	1.400	1CM-3	62.1	118200	5.13	1.222
1CK-7	34.3	128900	9.61	1.365	1CS-2	78.4	88800	5.80	1.327
1CK-3	34.3	127800	9.61	1.354	1CS-1	79.0	88500	5.75	1.322
1CT-2	42.8	120100	7.84	1.304	1CU-1	88.6	99000	4.54	1.288
1CT-1	42.8	116000	7.84	1.260	1CU-2	89.8	93800	4.47	1.220
1CP-2	44.9	121400	7.16	1.294	1CC-2	99.3	91800	4.16	1.243
1CP-1	44.9	116900	7.16	1.246	1CC-1	100.0	89000	4.13	1.204

DURALUMIN

Specimen	$\frac{d}{t}$	Modulus of Rupture f_r (lb./sq. in.)	$\frac{1}{\delta_s}=\frac{E}{S}\frac{t}{d_m}$	$\sigma_{rs}=\frac{f_r}{S}$	Specimen	$\frac{d}{t}$	Modulus of Rupture f_r (lb./sq. in.)	$\frac{1}{\delta_s}=\frac{E}{S}\frac{t}{d_m}$	$\sigma_{rs}=\frac{f_r}{S}$
v-3	15.7	73100	16.49	1.669	E-3	55.8	57200	4.73	1.389
t-4	20.2	70300	12.73	1.632	V-4	61.6	54400	4.56	1.408
t-5	20.2	69000	12.73	1.600	DH-2	63.7	56100	4.15	1.361
b-5	25.9	65000	10.24	1.561	DH-3	63.7	55600	4.14	1.350
1b-5	25.9	63200	10.71	1.642	DH-1	63.9	55300	4.13	1.342
1p-6	35.3	59100	8.14	1.556	1B-4	78.4	52900	3.24	1.259
1p-8	35.3	59900	8.13	1.576	B-5	79.6	52100	3.36	1.277
X-3	45.9	59400	5.98	1.501	1DQ-2	98.4	46200	2.60	1.099
1F-4	47.4	60900	5.59	1.467	1DQ-4	98.4	46800	2.60	1.115

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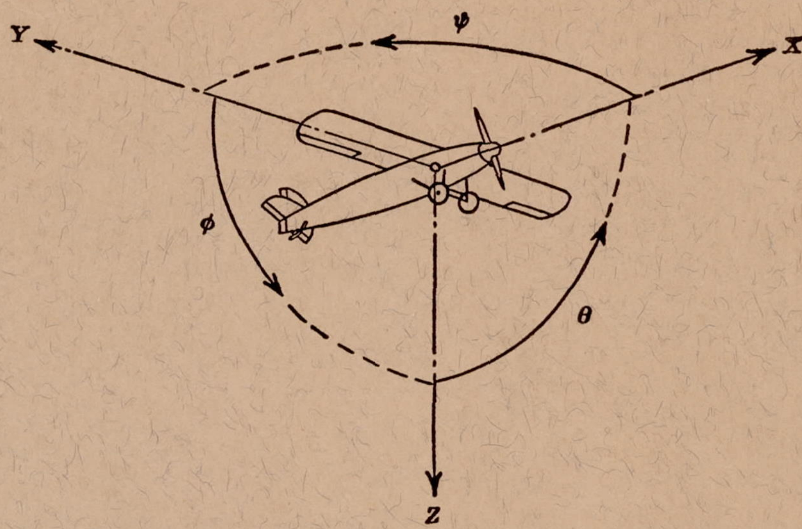

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $C_m=\frac{M}{qcS}$ (pitching) $C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter
p, Geometric pitch
p/D, Pitch ratio
V', Inflow velocity
V_s, Slipstream velocity
T, Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
Q, Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$
P, Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$
C_s, Speed-power coefficient $=\sqrt[5]{\frac{\rho V^5}{P n^2}}$
η, Efficiency
n, Revolutions per second, r.p.s.
Φ, Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.
1 metric horsepower = 1.0132 hp.
1 m.p.h. = 0.4470 m.p.s.
1 m.p.s. = 2.2369 m.p.h.
1 lb. = 0.4536 kg.
1 kg = 2.2046 lb.
1 mi. = 1,609.35 m = 5,280 ft.
1 m = 3.2808 ft.

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