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AIRCRAFT RATE-OF-CLIMB INDICATORS

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SUMMARY

The theory of the rate-of-climb indicator is developed in a form adapted for application to the instrument in its present-day form. Compensations for altitude, temperature, and rate of change of temperature are discussed from the designer's standpoint on the basis of this theory. Certain dynamic effects, including instrument lag, and the use of the rate-of-climb indicator as a statoscope are also considered. Modern instruments are described. A laboratory test procedure is outlined and test results are given.

INTRODUCTION

Before rate-of-climb indicators were available, statoscopes were used by balloonists to detect departure from constant altitude. These instruments contained a closed chamber at atmospheric pressure. If the altitude changed, air would have to flow into or out of the chamber to equalize the pressure. This flow took place through a tube in which was trapped a small amount of liquid, so that the air passed in bubbles. The direction of motion of the bubbles showed whether the craft was rising or falling, and the frequency of occurrence showed the rate of change of altitude.

Later there was developed the balloon variometer, essentially a rate-of-climb indicator, in which the flow into the chamber took place through a capillary leak. The pressure difference across the leak, measured with a sensitive liquid manometer, provided a quantitative indication of the rate of climb. (See reference 1.) In some instruments, when it was desired to fly at constant altitude, the leak could be closed, whereupon the manometer would indicate the difference between the constant chamber pressure and the atmospheric pressure, thus providing a sensitive altimeter for use in holding the aircraft at a constant pressure altitude.

A U-tube manometer was usable as long as the rate-of-climb indicator was intended for use in the relatively steady balloon or dirigible, but the use of a liquid-type manometer presented serious difficulties in airplane maneuvers. On this account, a diaphragm cell was substituted for the manometer. For a fuller description of this type and earlier instruments, see reference 2. Considerable refinement was now possible in the instrument; the time lag has been decreased, and compensations for temperature effects and for change in sensitivity with altitude have recently been introduced, so that the indication corresponded reasonably well to the rate of climb in the standard atmosphere. Another recent improvement did away with the separate vacuum bottle that formed the closed chamber in service instruments and utilized the case of the indicator itself for this purpose. This change made possible a considerable saving in weight and increased the ease of installation.

Rate-of-climb indicators are now used both as indicators of level flight and as indicators of the rate of climb or descent. When the pilot is trying to maintain level flight, it is essential to know when the rate of climb is exactly zero and to have an immediate indication of small changes of altitude. Therefore, the instrument should have a zero point which does not shift under flight conditions and which may be conveniently adjusted to take care of secular changes. The time lag should be short enough to allow the instrument to respond to quick changes in altitude but not so short that the pointer becomes unsteady in gusty air.

A moderately accurate quantitative indication of the rate of climb or descent is desired in holding the airplane at an efficient climbing speed, in coming in for a landing, or in passenger service where there are restrictions on the rate of descent. The accuracy of present-day rate-of-climb indicators is quite adequate for these uses. The rate-of-climb indicator is the most important instrument on a glider, where it is essential to detect rising air currents promptly. It is a necessity on balloons and airships where altitude is controlled by valving lift gas and dropping ballast. In flight-testing aircraft there is desired a record and greater accuracy than is obtainable (even now) from rate-of-climb indicators and, accordingly, the rate of climb is usually computed from the indications of a barograph.

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THEORETICAL DISCUSSION

The elements of a leak-type rate-of-climb indicator are shown in figure 1. The interior C of the instrument forms an insulated chamber, which is connected to the static pressure line through a leak B. The outlet A is also connected to the interior of a diaphragm cell D, which therefore serves to measure the differential pressure across the leak. The displacement of the diaphragm is transmitted to the pointer by means of a suitable mechanical train, such as a link, bell crank, sector, and pinion.

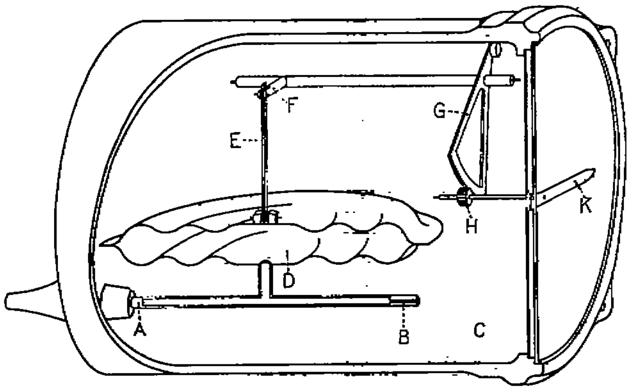

FIGURE 1.—Schematic diagram of a rate-of-climb indicator. A is the inlet from the static line; B, calibrated leak; C, chamber; D, diaphragm cell; E, link; F, crank; G, sector; H, pinion; and K, pointer.

DEFINITION OF SYMBOLS

The symbols used in the discussion are defined below. Quantities used only once will be defined in the text.

Z, pressure altitude in United States standard atmosphere.

$v=\frac{dZ}{dt}$, rate of climb in the standard atmosphere.

x, indication of instrument.

V, volume of chamber C (fig. 1).

t, time.

λ, time constant.

$\lambda_{10}=2.30\lambda=$time lag.

K, calibration coefficient.

L, coefficient of rate of change of temperature.

n, ratio of time constant to damping constant of instrument with leak removed.

p, ρ, T, pressure, density, and absolute temperature of the free air.

p_c, ρ_c, T_c, pressure, density, and absolute temperature of air in chamber.

p_K, ρ_K, T_K, pressure, density, and absolute temperature of air at the leak.

μ, absolute viscosity of air (at leak).

α, leak coefficient.

g, acceleration due to gravity.

$\Delta p=p_c-p$, pressure drop across leak.

$m=\rho_c V=$mass of air in chamber.

$R=\frac{p}{T\rho}$, the gas constant for unit mass of air (3078 ft.²/sec.² ° C.).

F, volume rate of flow of air outward through leak.

a, rate of change in volume with respect to differential pressure.

b, rate of change in volume with respect to atmospheric pressure.

c, rate of change in volume with respect to instrument temperature.

$M=x/\Delta p$, sensitivity of pressure gage.

INITIAL ASSUMPTIONS

(a) The standard rate of climb will be based on the United States standard atmosphere (references 3 and 4), which was calculated from the La Place barometric equation:

$$dp=-\frac{gp}{RT}dZ \tag{1}$$

and, for altitudes below 35,000 feet, the following relation between altitude and temperature:

$$T=288-0.0019812Z \tag{2}$$

By elimination of Z between these two equations and integration and substitution of the numerical values of the constants, there results for the standard atmosphere a relation between the pressure and temperature:

$$T/288=(p/760)^{0.190} \tag{3}$$

(b) The relation between pressure drop across the leak and the flow F through it, may be put in the form

$$\Delta p=+\mu\alpha F \tag{4}$$

For the special case in which the leak is formed by a long capillary, equation (4) reduces to Poiseuille's law,

$$\Delta p=\frac{8l}{\pi r^4}\mu F \tag{5}$$

in which $\alpha=\frac{8l}{\pi r^4}$, where l is the length and r the radius of the capillary. The more general form is used since the leak need not be a capillary tube but may be much more complicated, perhaps involving separately or in combination, orifices or valves that are opened or closed by the mechanism of the device. The leak coefficient α can be said to be dependent on the physical dimensions of the leak but subject to control by the designer to effect altitude and temperature compensation. The leak coefficient is a constant for the special case of laminar flow in a long capillary but may be a slowly varying function of the air density and rate of flow for other types of restrictions.

(c) Air is assumed to be a perfect gas so that for a unit mass

$$p=\rho RT \tag{6}$$

(d) In order to simplify the mathematical discussion, use will be made, wherever possible, of the fact that the difference between the pressures in the chamber, the leak, and in the free air, and the difference between temperature in the chamber and in the leak, are small with respect to the absolute values of those quantities, so that the quantities $\frac{\rho_c}{\rho_K}, \frac{p_c}{p_K}$ and $\frac{T_c}{T_K}$ may be set equal to unity without serious error.

(e) For convenience, the relation between the indication and the pressure drop across the leak is put in the form

$$M\Delta p = x \tag{7}$$

where M is the sensitivity of the pressure gage. The sensitivity is subject to control by the designer in order to obtain the desired calibration and to effect temperature and altitude compensation. This relation omits consideration of the effect of inertia and friction in the indicating mechanism. As far as these effects are subject to analysis, they may be introduced by substitution of a suitable differential equation for equation (7).

DEVELOPMENT OF THE GENERAL EQUATION

If the air pressure p is decreasing (ascent), there will be a flow of gas outward through the leak of the instrument at a rate given by the equation:

$$F = -\frac{1}{\rho_K}\frac{dm}{dt} = -\frac{}{\rho_K}\left[\rho_c\frac{dV}{dt} + V\frac{d\rho_c}{dt}\right]$$

$$= -\frac{\rho_c}{\rho_K}\left[\frac{dV}{dt} + V\frac{1}{p_c}\frac{dp_c}{dt} - \frac{V}{T_c}\frac{dT_c}{dt}\right] \tag{8}$$

Setting $\frac{\rho_c}{\rho_K} = 1$ and substituting in equation (4), there is obtained

$$-\frac{\Delta p}{\mu\alpha V} = \frac{1}{p_c}\frac{dp_c}{dt} + \frac{1}{V}\frac{dV}{dt} - \frac{1}{T_c}\frac{dT_c}{dt} \tag{9}$$

The rate of change in volume is usually small and is incidental to the operation of the indicating means. It may be separated into three parts: due to changes in differential pressure Δp, to changes in atmospheric pressure p, and to changes in the instrument temperature T_c.

Therefore, one may set

$$\frac{dV}{dt} = a\frac{d\Delta p}{dt} + b\frac{dp}{dt} + c\frac{dT_c}{dt} \tag{10}$$

where the coefficients a, b, and c are the rates of change of volume with respect to differential pressure, absolute pressure, and instrument temperature, respectively.

Now, since $p_c = p + \Delta p$

$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{1}{p_c}\frac{dp}{dt} + \frac{1}{p_c}\frac{d\Delta p}{at}$$

or, recalling that p_c is very nearly equal to p, there is obtained

$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{1}{p}\frac{dp}{dt} + \frac{1}{p}\frac{d\Delta p}{dt} \tag{11}$$

The rate of climb, v, may be introduced by the use of the barometric formula. From equation (1) it follows that

$$\frac{1}{p}\frac{dp}{dt} = -\frac{g}{RT}\frac{dZ}{dt} = -\frac{gv}{RT} \tag{12}$$

in which T is related to the free-air pressure, p, by equation (3) for an instrument calibrated to the United States standard atmosphere.

The general equation for the leak-type of rate-of-climb indicator is now obtained by successive substitution into equation (9) from equations (10), (11), (12), and (7) and dividing by the coefficient of x:

$$\frac{\mu\alpha(V+ap)}{p}\frac{dx}{dt} + x = \frac{M\mu\alpha g(V+bp)}{RT}v + \mu\alpha M\left(\frac{V}{T_c} - c\right)\frac{dT_c}{dt} \tag{13}$$

INTERPRETATION OF THE GENERAL EQUATION

The general equation (13) may be rewritten in the form:

$$\lambda\frac{dx}{dt} + x = Kv + L\frac{dT_c}{dt} \tag{14}$$

in which

$$\lambda = \frac{\mu\alpha(V+ap)}{p} \tag{15a}$$

will be called the time constant,

$$K = \frac{M\mu\alpha g(V+bp)}{RT} \tag{15b}$$

the calibration coefficient, and

$$L = M\mu\alpha\left(\frac{V}{T_c} - c\right) \tag{15c}$$

the coefficient of the rate of change of temperature. The significance of these coefficients will now be discussed.

Calibration coefficient—altitude and temperature compensation.—If an instrument is subjected to a change of pressure at such a rate and under such conditions that the indication and the instrument temperature are constant, equation (14) will reduce to

$$x = Kv \tag{16}$$

It is desirable that an instrument be so constructed that the calibration coefficient K is, as nearly as possible, independent of altitude and of instrument temperature.

Of the various factors in the calibration coefficient K, the gas constant R and the acceleration of gravity g are constant; the viscosity μ at the leak, and the temperature T of the free air, are variable but not subject to control by the designer and therefore must be compensated for; and the sensitivity M of the pressure gage, the leak coefficient α, and the quantity $(V+bp)$ are subject to control by the designer.

Of the uncontrolled variables, for which compensation must be made, μ is nearly independent of the pressure and proportional to the absolute temperature T_c of the instrument. Compensation for its variation, known as temperature compensation, is obtained if the product of the three factors under the designer's control is maintained inversely proportional to the viscosity.

If, as is usually the case, it is desired that the instrument indicate the rate of change of altitude in the United States standard atmosphere, the temperature

T is that given by equation (2) as a function of the standard altitude or by equation (3) as a function of the pressure. In order to obtain altitude compensation, that is, compensation for the variation of T, in the standard atmosphere, it is the problem of the designer to adjust the dependence on pressure of the three design factors under his control so that their product increases linearly with the altitude.

In the present-day instrument the volume V is generally constant, its variation being only that incidental to the operation of the mechanism; also b will be small, and therefore the quantity $(V+bp)$ will change but slightly. Altitude and temperature compensation will therefore be obtained either by selection of the leak characteristic, or by the introduction of pressure-sensitive and temperature-sensitive auxiliary mechanisms that will produce the desired changes in the leak coefficient or in the pressure-gage sensitivity.

If an instrument is calibrated to indicate the rate of climb in the standard atmosphere, the true rate of climb may be obtained by multiplying the indication by the ratio of the existing free-air temperature to the temperature in the standard atmosphere at that pressure.

An instrument with constant α, M, and V could be graduated to read the true rate of climb in the existing atmosphere provided that the temperature at the leak be the same as the temperature of the free air. If the temperature at the leak were not the same as that of the free air, the ratio of the indication to the true rate of climb would be equal to T_K/T. (See reference 5.)

Effect of rate of change of temperature.—Since the instrument chamber does not have perfect thermal insulation, its temperature may change at a rate that will have a significant effect on the indication if it is exposed to surroundings at a greatly different temperature. Equation (14) may be written in the form:

$$\lambda\frac{dx}{dt}+x=K\left(v+\frac{L}{K}\frac{dT_c}{dt}\right) \tag{17}$$

where

$$\frac{L}{K}=\frac{R}{g}\frac{T}{T_c}\left(\frac{V-cT_c}{V+bp}\right)$$

In the foregoing ratio, the quantities $\frac{T}{T_c}$ and $\left(\frac{V-cT_c}{V+bp}\right)$ may be expected to have values not far from unity, and $\frac{R}{g}$ has the constant value of 96.0 feet per degree centigrade. Therefore $\frac{L}{K}$ will be of the order of 100 feet per degree centigrade.

The effect on the instrument of a change in temperature will be quite similar to that of a change in altitude. In particular, if the instrument temperature were rising at a rate of 1° C. per minute, the indication will be the same as though the altitude were increasing at a rate of about 100 feet per minute. Under extreme conditions during an altitude flight, or after an airplane has been taken outdoors from a hangar, a temperature change at this rate might be obtained.

This effect may be decreased by improving the insulation of the chamber or be compensated by the introduction of an auxiliary mechanism that will shift the pointer of the indicator by an amount proportional to the rate of change of temperature.

Time constant.—If the instrument is subjected to a varying rate of climb, the indication will not follow the rate of climb exactly. If the rate of climb is constant and the initial indication is not equal to it, the indication will not immediately assume its final value but will approach it slowly. If it is assumed that $\frac{dT_c}{dt}=0$ and the calibration is such that $K=1$, the indication x is given by the solution of the general equation (14) in the form:

$$\lambda\frac{dx}{dt}+x=v \tag{18}$$

in which v is the imposed rate of climb.

For a constant value of v, the solution of this equation is the well-known expression for exponential decay

$$\frac{t}{\lambda}=\log_e\frac{x_0-v}{x-v} \tag{19}$$

where $x=x_0$ when the time $t=0$, and approaches the value of v as a limit, and where λ is the time constant. It may be seen that λ is the time required for the difference between the indication and its final constant value to decrease to $1/e$ times the initial difference.

By reference to equation (15a) it will be seen that λ increases with increase of the viscosity of air (that is, with increasing instrument temperature) and increases with decreasing pressure (or increasing altitude). The time constant may be decreased by decreasing the volume V of the chamber, or by decreasing the leak coefficient α. In order to keep the same value of the calibration coefficient K, a decrease in α or V makes necessary an increase in M, the sensitivity of the pressure gage.

The rate of change of volume with respect to differential pressure (coefficient a in equations (10) and (15a)) may affect the time lag considerably. For example, in an instrument for which V=500 cubic centimeters, p=20 millimeters of water at an indication of 2,000 feet per minute; a change in volume of 1 cubic centimeter for this indication would double the time lag. An increase in the sensitivity of the pressure gage without changing the calibration or too great an increase in the value of the coefficient a will decrease the time lag.

Thus far there has been no consideration of the effect of inertia and friction in the indicating mechanism.

DYNAMIC EFFECTS

These factors may be introduced by replacing equation (7) by a differential equation, such as

$$M\Delta p=l_2\frac{d^2x}{dt^2}+l_1\frac{dx}{dt}+x \tag{20}$$

where l_2 and l_1 are constants. In an actual installation, there may also be some external damping, as by a long line of small-bore tubing from the instrument to the static tube. Also the diaphragm cell of the pressure-measuring element may be vented to the outlet through a capillary leak. Consideration of the effect of any of these factors will lead to a general equation similar to equation (14) except that there will appear, in addition, terms containing second and higher derivatives of x with respect to time.

Draper and Schliestett (reference 6) have considered the problem in which the general equation involves the second derivative of x with respect to t.

Their equation,

$$\frac{\lambda}{n}\frac{d^2x}{dt^2}+\lambda\left(\frac{n+1}{n}\right)\frac{dx}{dt}+x=Kv \tag{21}$$

follows from a development similar to that used to obtain equation (14), neglecting the term involving the rate of change of temperature and substituting for equation (7) the differential equation

$$\frac{\lambda}{n}\frac{dx}{dt}+x=M\Delta p \tag{22}$$

It can be shown that the principal effect of the inclusion of second or higher derivatives in the general equation will be the addition to the solution of transient terms of rapid decay, the effect of which is important for a short time immediately after a change from one constant value of v to another, but may be neglected thereafter.

For example, consider an instrument for which $K=1$, $\lambda=4$ seconds, and $n=5$, which is subjected to an instantaneous change of rate of climb from an initial value of zero to a value v at time $t=0$. The solution of equation (18) for these conditions is:

$$v-x=ve^{-t/\lambda}$$

For the same initial conditions the solution of equation (21) is

$$v-x=\frac{5}{4}ve^{-t/\lambda}-\frac{1}{4}ve^{-5t/\lambda}$$

By the time $t=\frac{\lambda}{2}=2$ seconds the second term will be less than 3 percent of the first, and the two solutions will be, for practical purposes, equivalent except for a constant factor $n/(n-1)$

Of course, the elasticity and the damping of the mechanism play an important part in the reaction of the instrument to such external disturbances as vibration. For instance, it is important that the design be such that the resonance frequency of the indicator mechanism fall outside the frequency range of vibrations encountered in service.

SENSITIVITY IN INDICATING LEVEL FLIGHT

For the purpose of determining the sensitivity of the instrument in indicating level flight, assume that the rate of climb is a sine function of the time, that is,

$$v=v_0 \sin \omega t \tag{23}$$

where $2\pi/\omega$ is the period of the oscillation in rate of climb. Then equation (18) becomes

$$\lambda\frac{dx}{dt}+x=v_0 \sin \omega t$$

The steady state solution is of the form

$$x=\frac{v_0 \sin \omega t}{1+\lambda^2\omega^2}-\frac{\lambda\omega v_0 \cos \omega t}{1+\lambda^2\omega^2}$$

Two extreme cases may be noted.

(a) If $\lambda\omega$ is very small, the solution reduces to

$$x=v_0 \sin \omega t$$

and the instrument is seen to indicate the rate of climb for moderately slow changes of altitude.

(b) If $\lambda\omega$ is very large the solution becomes

$$x=\frac{-v_0 \cos \omega t}{\lambda\omega}$$

But the variation in altitude, as obtained by integrating equation (23), is

$$\Delta Z=-\frac{v_0}{\omega}\cos \omega t$$

and consequently

$$\lambda x=\Delta Z \tag{24}$$

Equation (24) shows that the response of the rate-of-climb indicator is proportional to the change of altitude for short-period variations in altitude. This fact makes the instrument a sensitive aid in maintaining level flight. Equation (24) is not only valid for short-period cyclic variations in altitude but is also approximately correct for any change in altitude, provided that the time required is less than the time constant λ.

A more exact analysis would take into account the manner in which the change of altitude occurred and the second-order lag effects discussed in the preceding section. For an instrument in which $\lambda=4$ seconds and $n=5$, the indication for an instantaneous change of altitude is about two-thirds that calculated from equation (24) and is about 56 percent for an altitude change taking place at a constant rate in a time equal to the time constant. In experimental tests on typical instruments, the maximum indication after a rapid change of altitude was found to be about 70 percent of the indication calculated from equation (24).

The sensitivity of the rate-of-climb indicator to sudden changes of altitude may be illustrated by the following example. A rate-of-climb indicator whose time constant is 4 seconds is subjected to a sudden altitude change of 10 feet. By equation (24) the maximum indication would be

$$x=10 \text{ ft./4 sec.}=150 \text{ ft./min.}$$

If the correction be made for second-order dynamic effects, where $n=5$, this value will be reduced to $x=100$ ft./min. for an instantaneous change or to $x=84$ ft./min. for a change taking place in 4 seconds. Now, for a rate-of-climb indicator having a scale of 4,000 ft./min. per revolution of the pointer, the pointer motion for this 10-foot change of altitude would be 9° to 7.5° of arc. In a sensitive altimeter with 1,000 feet per pointer revolution, the pointer motion for a 10-foot change of elevation is 3.6° of arc. Thus the rate-of-climb indicator is ordinarily a little more than twice as sensitive to small, sudden changes of altitude as the sensitive altimeter.

The desirability of a sensitive indication of rapid changes in altitude, combined with the desirability of a short time lag, created a demand for an instrument having the smallest time constant feasible. When the time constant was reduced below 4 seconds, however, the instrument became unsteady in gusty air. The present value of the time constant in the neighborhood of 4 seconds is a compromise between quick response and steadiness.

EFFECT OF ADDITIONAL RESTRICTIONS

It is possible to modify the performance of the instrument somewhat by introducing additional obstructions to the flow of air in the instrument.

A restriction in the line leading to the diaphragm that damps pressure changes within the cell will avoid the unsteadiness associated with a short time lag. In the use of the instrument to indicate level flight, the effect of this damping is to increase the time interval between a sudden change in static pressure and the maximum indication of the instrument, although there will be no change in the time constant. The instrument will not respond to extremely quick changes of pressure, such as occur in gusty air, but the sensitivity of the instrument to pressure changes associated with the variations of the altitude of the aircraft as a whole will not be greatly affected. There will be a reduction of sensitivity of the instrument, when used as a rate-of-climb indicator, in the ratio $\left(1-\frac{1}{n}\right)$.

A restriction in the static line, ahead of both the diaphragm connection and the leak, will not change the sensitivity of the instrument as an indicator of rate of climb but will increase the time lag and consequently decrease the sensitivity of the instrument as an indicator of level flight.

DESCRIPTION OF TYPICAL RATE-OF-CLIMB INDICATORS

Most rate-of-climb indicators in present-day use are similar in principle to that sketched in figure 1, for which the theory has been developed in previous sections. The mechanism is enclosed in a standard 2¾-inch dial case, of length from 4 to 6 inches. The weight of the instrument is about 1¼ pounds. In common airplane rate-of-climb indicators, the pointer moves one-half revolution for a rate of climb of 2,000 feet per minute. The calibration extends from 2,000 to −3,000 feet per minute, although most instruments will indicate rates of climb as great as ±4,000 feet per minute. Instruments are equipped with a knob for resetting the pointer to zero.

The designs of the mechanisms by the various manufacturers differ chiefly in the methods by which the various compensations are obtained. For this reason the description of the various instruments will be limited largely to the means of obtaining the various compensations, together with an indication of significant departures from the typical design.

The rate-of-climb indicators used in airships, balloons, and gliders are similar to those used in airplanes except that a greater sensitivity and a smaller range are ordinarily desired.

Kollsman.—In the Kollsman rate-of-climb indicator, the correction for altitude and instrument temperature

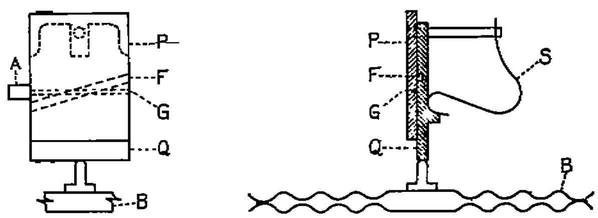

FIGURE 2.—Leak assembly of Kollsman rate-of-climb indicator. A is the inlet from the static line; B, diaphragm cell; F, groove in movable plate; G, groove in stationary plate; P, stationary plate; Q, movable plate; and S, spring.

is obtained by variation of the length of the capillary forming the leak. The leak (fig. 2) is formed by a groove G engraved on a fixed flat plate P upon which is laid another movable flat plate Q. A larger groove F is cut in the movable plate at an angle to the first and open to the chamber at both ends. Thus the groove G forms a small channel of capillary size that extends from the edge of the plate, where the connection to the static-pressure line A is made, to the point where the two grooves meet. The length of this channel may be varied by changing the position of the upper plate, which bears against a diaphragm cell B filled with air under pressure. By a proper choice of the pressure to which the cell is filled, it is possible to obtain simultaneous temperature and altitude compensation. If the instrument temperature decreases, the pressure in the auxiliary cell falls, the cell contracts, and the plate is moved so as to lengthen the capillary. This action increases the value of the leak coefficient α (see equation (15b)) to compensate for the decrease of viscosity with decrease in temperature. If the altitude increases, the pressure in the instrument falls, the cell expands, and the capillary shortens. The decrease of α compensates for the decrease, with increasing altitude, of the factor T in the calibration coefficient K.

In one model, used by the air lines in transport service, the first quadrant of the scale represents a rate of climb of 500 feet per minute. At this point additional springs become effective so that the second quadrant of the scale is used for the range between 500 and 2,000 feet per minute. Thus, the pointer motion for small rates of climb is doubled and the instrument made more sensitive as an indication of level flight, without a decrease in the range of the instrument.

The mechanism is placed in a Dewar flask, closed in front by a metal face plate, backed with a one-half inch layer of cork. The thermal insulation is such that the temperature of the instrument changes at a very small rate, so that the effect of a changing temperature (see equation (14), last term, and equation (17)) is small enough to be neglected.

Pioneer.—The Pioneer rate-of-climb indicator has a leak unit of novel design. The leak is designed to have characteristics intermediate between that of an orifice and a capillary so that no separate mechanism to compensate for altitude is needed.

For a leak in which laminar flow obtains, as in a long capillary tube, the pressure drop at a given volume flow is nearly independent of the density. For an instrument using such a leak, the leak coefficient α (equation (4)) would be constant and, if the other design factors of the calibration coefficients K (equation (15b)) are constant, the value of K would increase with decreasing values of T; that is, with increasing altitude. On the other hand, if the leak were equivalent to an orifice the pressure drop at constant volume flow would be proportional to the density. For an instrument in which the leak is an orifice, the value of α would be proportional to the density and K would decrease with decreasing density or increasing altitude. An instrument in which the leak is equivalent to a combination of orifice and capillary would have intermediate altitude characteristics. For this instrument, a combination is used for which the value of K is nearly constant between sea level and 30,000 feet.

A bimetallic element is used to compensate for changes in instrument temperature. In some models a means for compensating for rate of change of instrument temperature is also provided. Details of this feature have not been released.

LABORATORY TEST PROCEDURE AND TEST DATA

Calibration at room temperature.—The calibration at room temperature serves to show whether the instrument is properly calibrated, has a linear scale, and is properly compensated for altitude. A diagram of the apparatus used at the National Bureau of Standards is shown in figure 3. The instrument R is placed in a low-pressure chamber or bell jar C together with a calibrated sensitive altimeter A and means such as a buzzer B, for providing sufficient vibration to eliminate the effect of friction in either instrument. The outlet of the test chamber is connected to a mercurial barometer M and to a needle valve V for controlling the flow of air. The needle valve is connected by means of a two-way stopcock T to a suction line S and to a line P supplying clean air under a pressure of an inch or two of mercury. The needle valve is set so as to pass air into or out of the chamber at such a rate that the indication of the rate-of-climb indicator is constant. The indication of the rate-of-climb indicator is compared with the standard rate of climb determined by timing with a stop watch the change in indication of the altimeter between two previously determined altitudes, say, 2,000 and 4,000 feet. The calibration of the altimeter may be checked independently with the aid of the mercury barometer.

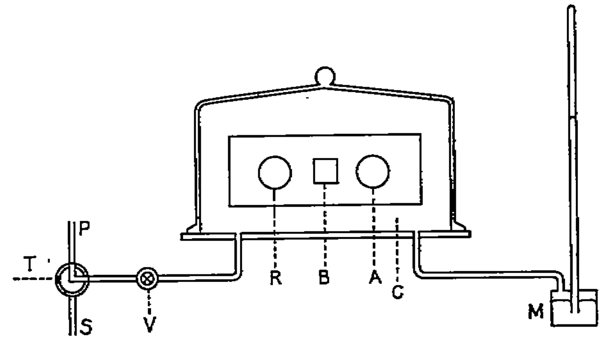

FIGURE 3.—Sketch of apparatus for making scale-error tests. A is the altimeter; B, buzzer; C, bell jar; M, mercury barometer; P, pressure line; R, rate-of-climb indicator; S, suction line; T, two-way stopcock; and V, needle valve.

Comparison of scale errors determined at altitudes near sea level with those determined at higher altitudes will provide a measure of the accuracy of the altitude compensation. The performance of five typical 1938 airplane instruments is shown in table I.

TABLE I.—SCALE ERRORS IN FEET PER MINUTE OF FIVE RATE-OF-CLIMB INDICATORS

[Scale errors are positive if indicated rate of climb or descent is greater than the standard rate. Changes in scale errors are positive if the indication is greater at the low temperature]

Interval (thous. of ft.)	Standard rate of climb (ft./min.)	Instruments				
		A	B	C	D	E
		Scale errors at 23° C.				
2 to 4	1,000	+5	−30	+35	+5	+30
2 to 4	2,000	+5	−15	+45	+35	+50
4 to 2	1,000	−15	−15	+20	−10	+20
4 to 2	2,000	−5	−35	+20	−15	+15
4 to 2	3,000	+25	−50	+40	−55	+40
15 to 17	2,000	+110	+70	+135	+125	+140
17 to 15	2,000	+100	+65	+105	+80	+105
28 to 30	2,000	+160	+90	+130	+140	+165
30 to 28	2,000	+145	+95	+120	+110	+155
		Change in scale errors between 23° C. and −35° C. corrected for zero shift				
2 to 4	2,000	−10	−30	0	−10	+20
15 to 17	2,000	−70	−70	−30	−60	+15
28 to 30	2,000	−15	−15	+25	−20	+15
		Zero shift between 23° C. and −35° C.				
		+30	−40	0	−20	−20

Temperature test.—The accuracy of the temperature compensation may be determined by comparison of the scale errors at room temperature with those obtained at a low temperature, say, −35° C. Because of the effect of changing chamber temperature on the indication, considerable care must be taken to insure that the temperature of the instrument is constant during the test. For this reason, and because of the thermal insulation of the instrument chamber, the apparatus should be maintained at the test temperature for at least 2 hours. Air admitted to the bell jar should be carefully dried to prevent all possibility of condensation in the leak. It is better to dry the air by precooling to condense out moisture than by chemical means.

After temperature equilibrium has been established, the zero shift is observed, and the scale errors are determined by the same procedure as was used in the calibration test. The performance of typical 1938 airplane instruments with respect to temperature compensation is also shown in table I.

Time lag.—The time constant of a rate-of-climb indicator is usually calculated by means of equation (19) from observations made on the decay of the indication of the instrument. The instrument is first subjected to a certain rate of change of pressue which is then suddenly reduced to zero. The time required for the indication to fall from one definite value to another is observed. Although any pair of indications may be used, it has been found convenient to determine the time required for the indication to fall from 2,000 to 200 feet per minute, when the initial maximum indication is about 3,000 feet per minute and the final indication is zero. From equation (19) it is seen that this time of decay is given by

$$t=\lambda \log_e \left(\frac{2000-0}{200-0}\right)=2.3\lambda$$

The time of decay between two indications whose ratio is 10 :1 will be called the time lag λ_{10} and is seen to be 2.3 times the time constant λ.

A determination of the time lag will be based on timing the decay of the indication. Of more significance in the instrument is the decay of the differential pressure Δp across the leak. If the time constant of this decay is desired, there may be used an equation of the form of equation (19), except that the actual differential pressures are substituted for the indications.

In the development of equation (19), it was assumed that the calibration constant K was constant and of unit value. If the scale errors are such that the relation between indication and rate of climb is not substantially linear, the decay of indication will not be exponential and the time lag will have only an arbitrary significance. An error in zero setting may be allowed for by assigning to v, in equation (19), the value of the indication eventually reached when the rate of climb is zero.

In a determination of time lag a small error in zero setting will cause an error of the second order, provided the average of the time of decay in ascent and descent is taken. Thus, if the decay from positive rates of climb were being observed, and the zero setting were such that the instrument indicated +20 feet per minute at zero rate of climb, the time of decay would be

$$t=\lambda \log_e \frac{2000-20}{200-20}=2.41\lambda$$

For decay from negative rates of climb and the same zero setting,

$$t=\lambda \log_e \frac{-2000-20}{-200-20}=2.22\lambda$$

The time of decay is about 5 percent high in one case and 4 percent low in the other, but the average is within ½ percent of the time obtained with the correct zero setting.

Three procedures for determining the time lag will be discussed. The first two give equivalent results and the third yields results not in agreement, for reasons which will be given.

(a) The instrument is set to zero and placed in the test apparatus, as described in the calibration test. Starting with a pressure in the bell jar lower than that of the atmosphere by an amount corresponding to a difference of, say, 2,000 feet, one admits air to the test chamber so that the pressure approaches that of the atmosphere at a rate corresponding to a constant indication between 2,500 and 3,000 feet per minute. When the pressure in the chamber is exactly equal to that of the atmosphere, the flow of air is stopped and the chamber opened to the atmosphere, as by disconnecting the tubing between the needle valve and the bell jar or by a valve with a large opening. The time required for the indication to fall from 2,000 to 200 feet per minute is measured with a stop watch. The observation is repeated except that the pressure is initially greater than the atmospheric pressure, and air is withdrawn from the chamber. The average of the two measured times is taken as λ_{10}.

(b) A small pressure is applied to the static pressure connection of the instrument. When the pressure is quickly released by opening the instrument to the atmosphere, the indication of rate of climb will rise to a maximum, then return to zero. The initial pressure is chosen of such magnitude (about 4 inches of water) that the maximum indication is between 2,500 and 3,000 feet per minute. The time of decay from 2,000 to 200 feet per minute is observed. The procedure is repeated, but with an initial suction, thus obtaining a rate of descent. The average of these two values for the time of decay is taken as the time lag λ_{10}. This method gives a result in good agreement with that obtained with the first method, provided that the maximum indication in this method is approximately equal to the initial steady indication in the first method.

(c) With the instrument set to zero and placed in the test chamber, just as in method (a), the pressure in the chamber is made to change at such a rate that the indication of the instrument is constant at 2,500 feet per minute. At the desired pressure altitude the flow of air is cut off by closing a valve, the indication decays to zero, and the time of decay from 2,000 to 200 feet per minute is measured.

Tests show that method (c) yields a time of decay for both ascent and descent less than by either of the other two methods. The difference depends on the particular apparatus used and may be 20 percent or more. In method (c) the requirement of constant pressure in the test chamber is not satisfied during the timing interval but, because of temperature changes, the pressure is changing at a rate that introduces appreciable error. Suppose the time lag in ascent is being measured. While air is being withdrawn from the test chamber, the remaining air will be cooled slightly below the temperature of the surroundings. After the flow is stopped, the air in the test chamber will warm slowly, and consequently the pressure will increase. This condition is equivalent to a negative, instead of zero, rate of climb during the timing interval. When the time lag in descent is measured, the test chamber is warmed when air is being introduced; it cools slightly when the flow is stopped. Consequently, the pressure will decrease during the timing interval and the rate of climb will be positive. The equivalent rate of climb or descent due to these temperature adjustments may easily be of the order of 100 feet per minute or more.

Assume that an instrument tested by method (a) or (b) has an observed time of decay of 9.2 seconds. From equation (19) the time constant in seconds is

$$\lambda=9.2/\log_e\frac{2000-0}{200-0}=4.0$$

Tested in descent by method (c) the same instrument would have a time of decay, in seconds, assuming a rate of climb during the timing interval of +100 feet per minute,

$$t=4.0\log_e\frac{-2000-100}{-200-100}=7.8$$

For the test in ascent, the rate of climb would be −100 feet per minute during the timing interval, and the time of decay in seconds is

$$t=4.0\log_e\frac{2000-(-100)}{200-(-100)}=7.8$$

The 15-percent discrepancy between 9.2 and 7.8 seconds is of the order observed in practice. It follows that method (c) may be used for comparative tests but, in order to obtain the time constant, it will be necessary to determine a correction factor for the particular testing apparatus and the type of instrument.

Typical present-day instruments have a time constant at sea level of the order of 4 seconds and a time lag λ_{10} as determined by method (a) or (b) of about 9 seconds. Equation (15a) indicates that the time constant varies inversely as the atmospheric pressure and therefore both it and the time lag increase with increasing altitude. If the other factors affecting the lag are constant, the values of the time constant and the time lag at 30,000 feet will be about 3.4 times that at sea level. In most instruments the ratio is somewhat less, because the lag is affected if the altitude compensation is made by control of the leak. The ratio of the time lag at 30,000 feet to that at sea level ranges from about 1.5 to 3.0 for a wide variety of instruments tested in the past 15 years, with the more recent types having a ratio between 1.5 and 2.0.

Vibration test.—The vibration test is designed to indicate the effect on the instrument of the vibration encountered in a service installation. It being out of the question to subject the instrument to all the possible modes of vibration that may be encountered, the practice has been to specify a standard vibration for the laboratory test. Accordingly, the instruments are subjected to a translational vibratory motion in a circular path one-thirty-second inch in diameter in a plane inclined 45° from the horizontal. The frequency range is 1,000 to 2,500 cycles per minute. The zero reading and the pointer oscillation are observed over the range of vibration frequencies. In addition, the instruments are vibrated for a 3-hour period at a constant frequency, usually at about 1,800 cycles per minute. The zero readings before and after the test are then compared.

The performance of a typical instrument under vibration is shown in the following table.

PERFORMANCE IN VIBRATION TEST

Frequency (c. p. m.)	Pointer oscillation (ft./min.)	Change of reading (ft./min.)
1,200	5	10
1,800	10	20
2,400	50	50
Change in zero after 3 hours' vibration, 10		

Overpressure test.—Certain airplane maneuvers, such as the power dive, will involve rates of descent, and occasionally rates of climb, far greater than the maximum indication of the instrument. The overpressure test will indicate whether the diaphragm or mechanism will be injured by the high differential pressures built up during such maneuvers. The instrument is subjected to a pressure change corresponding to a climb from sea level to 30,000 feet at a rate of 20,000 feet per minute, followed by a descent to sea level at a rate of 30,000 feet per minute. For a good modern instrument, the zero reading 1 minute after the completion of this test will not differ from that before the test by more than 50 feet per minute; and, in 10 minutes, the zero reading will have returned almost to its original value.

Leak test.—It is necessary that the case of the rate-of-climb indicator be free from leaks since the case, vented to a static tube in the free air, is normally at a pressure differing from that in the cockpit of the airplane. In the test for case leaks, the static-pressure connection of the instrument is sealed off, and the pressure of the surroundings is reduced by 4 inches of water. A change in indication will be observed if the instrument leaks appreciably. Leaks are most likely to occur at the rim of the glass dial cover or at the packing of the shaft of the knob with which the zero setting is made.

INSTALLATION IN AIRCRAFT

In the usual airplane installation the case of the rate-of-climb indicator is connected to a static tube. The resistance to the flow of air should be kept low by providing a wide-bore line to the static tube. (See reference 7.) Although the lag due to the resistance of the tubing will not affect the response of the rate-of-climb indicator as seriously as that of the altimeter or air-speed indicator, it will somewhat reduce the response of the instrument when used as a level-flight indicator.

The static line should be leakproof. If an altimeter or an air-speed indicator is connected to the static line, a suction sufficient to cause an appreciable indication of the most sensitive instrument may be applied to the static tube and the holes sealed off. A change of indication after sealing will indicate a leak. If the rate-of-climb indicator is the only instrument in the static line, a suction of, say, 4 inches of water may be applied and the static holes sealed. If the indication returns to zero after sealing, and the instrument indicates a descent when the holes are opened after an interval of 1 minute, the line may be regarded as leakproof.

Besides being leakproof the lines must be free of oil, dust, and water to avoid stoppage or entry to the instrument. A drain may be provided for the removal of water.

National Bureau of Standards,
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