REPORT No. 719

THE PROBLEM OF COOLING AN AIR-COOLED CYLINDER ON AN AIRCRAFT ENGINE

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SUMMARY

An analysis of the cooling problem has been made to show by what means the cooling of an air-cooled aircraft engine may be improved. Each means of improving cooling is analyzed on the basis of effectiveness in cooling with respect to power for cooling.

The altitude problem is analyzed for both supercharged and unsupercharged engines. The case of ground cooling is also discussed.

The heat-transfer process from the hot gases to the cylinder wall is discussed on the basis of the fundamentals of heat transfer and thermodynamics.

Adiabatic air-temperature rise at a stagnation point in compressible flow is shown to depend only on the velocity of flow.

INTRODUCTION

Whenever the cooling of an air-cooled engine cylinder is the factor limiting the power that may be developed in the cylinder, any possible increase in cooling becomes extremely important. It must not be assumed, however, that doubling or tripling the cooling of an air-cooled engine will in itself make possible doubling or tripling the power developed by the engine. If such an increase in power developed were possible, it could easily be obtained from liquid-cooled engines. Actually, there is no great difference in power developed per cubic inch displacement between good air-cooled and good liquid-cooled engines.

When the cooling of a cylinder is insufficient, any increase in cooling obtained will make possible (1) a reduction in the fuel consumption of the engine, (2) an increase in the power that may be taken from the engine, (3) an increase in the time between overhauls, or (4) a reduction of the likelihood of engine failure as a result of piston failure or of breakdown of the oil film between the piston and the cylinder.

Various means of improving the cooling of air-cooled aircraft engines are known. Each method of improving the cooling requires an increase in the power expended for cooling, greater mechanical skill in manufacture, or both. The various means of improving cooling are discussed in this report.

SYMBOLS

- A cross-sectional area of air passage, normal to flow
- c velocity of sound
- c_p specific heat at constant pressure
- c_v specific heat at constant volume
- $C_1, C_2, \ldots C_n$ proportionality constants
- C_D over-all drag coefficient of airplane
- d cylinder-wall thickness
- D hydraulic diameter of air passage $\left(\frac{4 \text{ cross-sectional area}}{\text{wetted perimeter}}\right)$
- f friction factor $\left(\frac{\Delta p}{q}\frac{D}{4L}\right)$
- f' fin effectiveness
- h surface heat-transfer coefficient, heat units per unit surface per degree temperature difference per unit time
- H total heat transferred per unit time
- H_T energy loss of air occurring up to a given point in a baffle
- $H_T/\Delta p$ energy loss of air occurring up to a given point in a baffle expressed as a fraction of total energy loss during passage of air through baffle
- I indicated horsepower
- k thermal conductivity of coolant
- k_m thermal conductivity of metal used in cylinder walls or cooling fins
- L length of air passage, characteristic length
- m, n exponents in the equation connecting Nusselt number, Prandtl number, and Reynolds number
- p pressure
- P power
- q dynamic pressure ($1/2\ \rho V^2$)
- Q volume rate of flow
- r radius of curvature
- r_b radius from center of cylinder to fin root
- R Reynolds number ($D\rho V/\mu$); compression ratio
- s fin spacing
- S surface area
- t fin thickness
- T absolute temperature

T_{ta} temperature of cooling air at inlet
T_o temperature of cooling air at outlet
T_w temperature of passage wall
U over-all heat-transfer coefficient from cylinder to air, based on outside area of cylinder wall, without fins
v volume
V velocity
w width of channel or fin
Δp pressure difference
Δt average temperature difference between cylinder wall and cooling air
ΔT temperature rise of cooling air due to adiabatic compression at cowling entrance
γ ratio of specific heats (c_p/c_v)
δ pressure ratio (p/p_0)
σ density ratio (ρ/ρ_0)
φ brake-horsepower ratio $[(b\ hp)/(b\ hp)_0]$
ψ indicated-horsepower ratio (I/I_0)
η_t heat-transfer efficiency
λ defined by $H \propto (\rho V)^\lambda$
μ viscosity of coolant
ρ mass density of coolant
ρV mass flow of coolant per unit time and open area

Subscripts:
$_0$ values at datum level
a values in main air stream
t values in tubes

ANALYSIS

Pye (reference 1) states that the power which must be dissipated from a gasoline engine through the cooling system is equivalent to about one-half the brake horsepower developed by the engine. It is shown in reference 2 that the power dissipated in cooling varied from about 70 percent of the indicated horsepower at low power output to 40 percent at high power output for an air-cooled radial engine enclosed in an NACA cowling. This excessive variation in heat dissipation was probably due to the nature of the tests, in which the heat carried away by spillage of heated air from the front of the cowling was not measured. The spillage would have varied considerably over the operating range used in the tests reported in reference 2.

The process of heat transference from the hot gases in the cylinder to the cooling air is naturally divided into three different phases, each of which may be separately treated. These phases are: the transfer of heat by convection and radiation from the cylinder gases to the piston and the cylinder wall; the conduction of heat through the piston and the cylinder wall to the outside surfaces, which are in contact with the cooling air; and the transfer of heat by convection from the cylinder wall and the fins to the cooling air. The problem of transferring this heat from the engine wall to the cooling air in the most efficient manner is the main topic of this paper.

Consider first the transfer of heat from the hot cylinder gases to the piston and the cylinder wall. The engine cylinder contains a varying quantity of gas moving at relatively high velocity and ranging in temperature from about atmospheric temperature up to about 4500° F. for engines now in use. Nusselt (reference 3) concluded from his work and from a review of the work of several other investigators that the heat transferred by radiation from the hot cylinder gases to the cylinder wall amounted to no more than 10 percent of the total heat transfer. Moss (reference 4), however, concluded that 46 percent of the total heat transfer is accomplished by radiation. The work on which the conclusions of references 3 and 4 were based was done with engines operating at fairly low speed and with maximum gas temperatures of about 3000° F. The heat transferred by radiation is roughly proportional to the difference in the fourth powers of the temperatures of the engine gases and the cylinder wall, but the heat transferred by conduction is proportional to the first power of this temperature difference, to the cube root of the absolute temperature of the engine gases (reference 3), and to the engine speed. Consequently, the effect of radiation in heat transfer cannot be dismissed as being unimportant in modern engines. The heat transferred by radiation may be a considerable portion of the total heat transferred.

The second phase of the over-all heat-transfer process is the conduction of the heat through the metal wall to the outside of the cylinder.

The permissible operating temperature of the inside wall of the cylinder is limited by considerations of lubrication and detonation. In order to obtain the maximum temperature difference between the outside wall of the cylinder and the cooling air, hence maximum cooling for any particular condition of air flow, the temperature drop through the wall should be made as small as possible by keeping d/k_m small.

The third phase of the heat transfer from the engine gas to the cooling air is the transfer of heat by convection from the outside of the cylinder wall to the cooling air. The heat transferred from the cylinder per second is

$$H = S\,U(\Delta t) \qquad (1)$$

For any particular cylinder, the area available for attaching cooling fins is fixed, so that the only means of increasing the cooling is by increasing U or Δt or both. These quantities can be simultaneously increased by increasing the mass flow of cooling air through the cooling system. The effect on Δt of increasing the mass flow is usually small; whereas, U increases as the 0.6 to 0.7 power of the mass flow. Very little can ordinarily be done about increasing the value of Δt; increasing U is therefore the only means of bringing about any large increase in the heat transfer from the cylinder to the cooling air.

MEANS OF IMPROVING COOLING

The amount of cooling obtained on a given cylinder may be increased in any of the following ways:

1. By increasing the mass flow of cooling air, leaving the cylinder and the fins unchanged
2. By using metal of higher thermal conductivity for the construction of the cylinder and the fins
3. By changing the fin design so that the value of the over-all heat-transfer coefficient is increased

These three means of improving cooling are discussed in order.

INCREASING COOLING BY INCREASING MASS FLOW OF COOLING AIR

The two extreme cases to be considered here are the cases most likely to occur in practical application: (1) The existing operating cylinder temperatures are satisfactory and it is desired to increase the power developed, thereby requiring more heat to be transferred at the same temperatures; and (2) the existing operating cylinder temperatures are too high and it is desired to reduce cylinder temperatures by increasing the mass flow of cooling air, the total heat transferred remaining unchanged.

McAdams (reference 5, p. 172) gives the relation between Nusselt number and Reynolds number for air in turbulent flow as

$$\frac{hD}{k}=0.020\left(\frac{\rho VD}{\mu}\right)^{0.8} \tag{2}$$

As k is proportional to μ, from the kinetic theory of gases, and as only small variations in k and μ will be experienced when the mass flow is changed, $k/\mu^{0.8}$ may be considered constant. Then, for any given air passage,

$$h=C_1(\rho V)^{0.8} \tag{3}$$

The over-all heat-transfer coefficient U of equation (1) may be expressed approximately in terms of S_r, the ratio of total cooling surface area to base area; h, the surface heat-transfer coefficient; and f', the fin effectiveness, as

$$U=S_rhf' \tag{4}$$

For any given fin arrangement, S_r is a constant and h is given by equation (3).

For values of f' between 0.50 and 0.95, which would include nearly all fins ordinarily used, the exact expression

$$f'=\frac{\tanh aw}{aw}\quad\text{(from reference 6)} \tag{5}$$

may be replaced by the simple approximation

$$f'=1.07-0.30\ aw \tag{6}$$

where

$$a=\sqrt{\frac{2h}{k_mt}}$$

Equation (6) gives values of f' within 1 percent of correct over the range stipulated.

Within small ranges of variation of f', the value of f' may be taken to vary as ρV to some power λ_1,

$$f'\propto(\rho V)^{\lambda_1} \tag{7}$$

This power λ_1 varies with f' and can be shown to be

$$\lambda_1=\frac{d\log f'}{d\log\rho V}=\frac{-0.12w\sqrt{\frac{2C_1}{k_mt}}(\rho V)^{0.4}}{1.07-0.3w\sqrt{\frac{2C_1}{k_mt}}(\rho V)^{0.4}} \tag{8}$$

Substitution of equations (3) and (7) in equation (4) gives

$$U\propto S_rC_1(\rho V)^{0.8}(\rho V)^{\lambda_1}\propto(\rho V)^{\lambda} \tag{9}$$

The power λ is shown as a function of f' in figure 1.

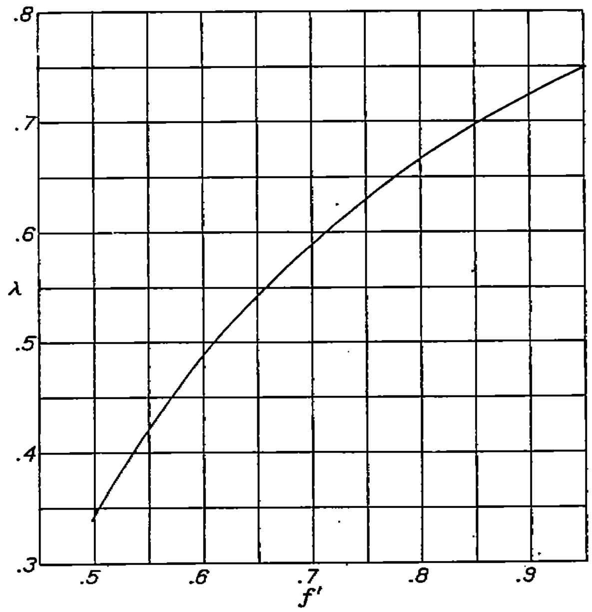

FIGURE 1.—Variation of λ with fin effectiveness f'. (λ is defined by $U\propto(\rho V)^{\lambda}$.)

It is shown in reference 7 that, for a given fin arrangement,

$$U=b(\rho V)^{\lambda}$$

and, for reasonable variations in the mass flow of cooling air, λ is constant. In the same reference, λ is shown experimentally to vary between 0.6 and 0.7 for conventional cylinder finning.

In the first case, it can be seen from equation (1) that the total heat transferred varies directly as U and, since the temperatures were assumed unchanged,

$$H\propto(\rho V)^{\lambda} \tag{10}$$

In the second case, where the total heat transferred is assumed constant, it is evident that

$$\Delta t\propto\frac{1}{U}\propto\frac{1}{(\rho V)^{\lambda}} \tag{11}$$

This equation indicates that the temperature difference between cylinder wall and cooling air will drop as the mass flow of cooling air is increased. Lower wall temperatures will result from this decrease in Δt because the cooling-air temperature remains unchanged.

Baffle design.—In reference 8, the importance of good baffle design in obtaining the largest possible mass flow between the fins for a given pressure drop was demonstrated. The air passage formed by the cylinder wall, the fins, and the baffle was shown to be simply a venturi, and the same principles that control venturi design were employed in obtaining maximum velocity between the fins for a given pressure drop.

It was shown in reference 8 how a conventional baffle then in use could be improved to obtain a 25-percent increase in mass flow without increasing the pressure drop. Since $P=Q\Delta p$, this increase in mass flow was obtained with a corresponding increase of 25 percent in the cooling power, for which an increase of 20 to 25 percent in total heat transfer was obtained.

For the best arrangement tested in reference 8, 36 percent of the total pressure drop across the cylinder was shown to take place in the fins, 50 percent at the baffle exit, and 14 percent at the exit from the exit duct. Only the pressure drop that occurred in the fins was useful in cooling the cylinder. On many modern engines, the useful pressure drop is only about 20 percent of the total drop.

Reference 8 further showed that:

1. A good baffle entrance is important and can easily be attained.
2. The baffle should be close fitting. (See table I.)
3. The radius of curvature of the baffle at the baffle exit should be greater than the fin width. Betz (reference 10) showed that the pressure loss when air travels around a 90° bend is

$$\Delta p_{90^\circ}=q\left[0.13+0.16\left(\frac{w}{r}\right)^{3.5}\right]$$

This relation shows that the pressure drop for a 90° bend rises sharply as the channel width w increases relative to the radius of curvature of the bend r. The results from reference 8 (fig. 22 of reference 8 has been reproduced herein as fig. 2) for a finned cylinder with ¾-inch fins show the importance of having a fairly large radius of curvature of the baffle exit.

4. The baffle exit duct should expand gradually, the included angle being about 6°. A baffle exit duct 2 or 3 inches long gives a large part of the possible kinetic-energy recovery. Figure 3, obtained from figures 17 and 18 of reference 8, shows cooling and baffle conductance as a function of exit-duct length.

5. The baffle-exit cross-sectional area should be approximately 1.5 times the free area between the fins. Decreasing the area of the baffle exit was found to improve the cooling at the rear of the cylinder and increasing the area was found to improve the cooling toward the front of the cylinder.

The results of cooling tests on a variety of baffle arrangements and on finned cylinders with streamline

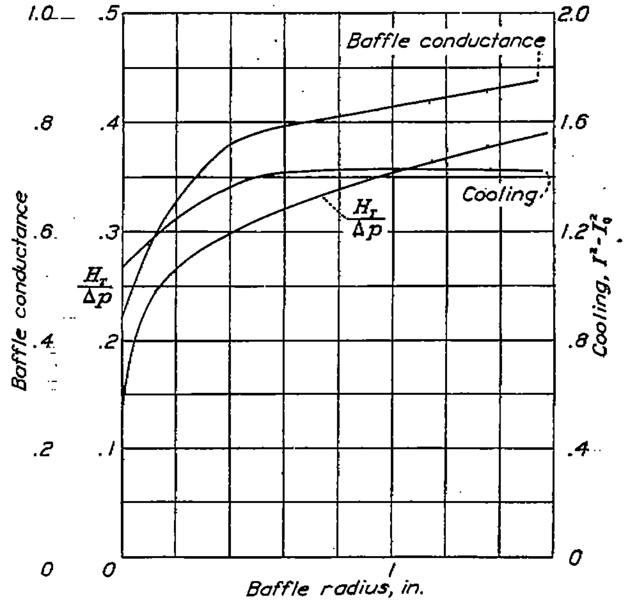

FIGURE 2.—Effect of baffle radius on $H_r/\Delta p$, baffle conductance, and cooling at 150° from front of cylinder at optimum exit-duct opening. Baffle exit opening, 1.0 inch; exit-duct length, 7.00 inches; cylinder diameter, 4.66 inches; fin space, 1/16 inch; fin width, ¾ inch. (Reproduced from reference 8.)

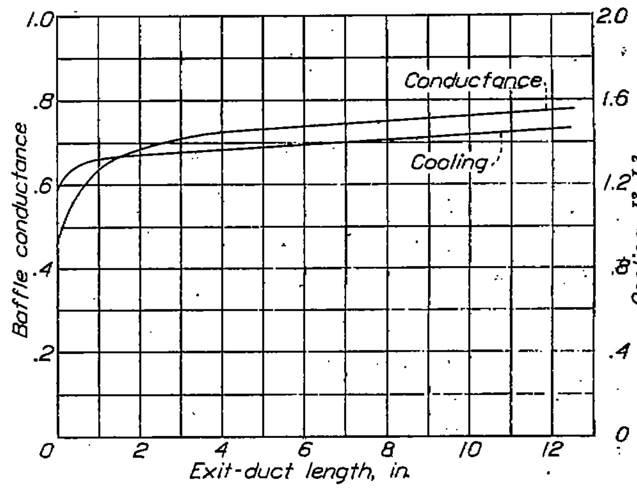

FIGURE 3.—Effect of exit-duct length on cooling and baffle conductance at 150° from front of cylinder at optimum exit-duct opening. Baffle exit opening, 1.50 inches; cylinder diameter, 4.66 inches; fin space, 1/16 inch; fin width, ¾ inch. (Data from reference 8.)

exits are given in table I. The cylinder and the baffle arrangements tested are shown in figure 4. The sketches in figures 4 (a) to 4 (j) are reproduced from figure 2 of reference 8. The test data for these arrangements are reproduced in table I from the table in reference 8. The arrangements shown as figures 4 (k) to 4 (m) are reproduced from figure 3 of reference 9.

It should be noted that, although these tests were made on a model of a cylinder barrel, the same general principles apply to the cooling of the head.

Baffles designed on the basis of the results given in reference 8 have improved the cooling on a complete engine 28 percent over that obtained on an engine with conventional baffles, which had about ¼-inch clearance between fins and baffle and a very short exit duct.

The application of the results from reference 8 to full-scale engines is given in reference 11.

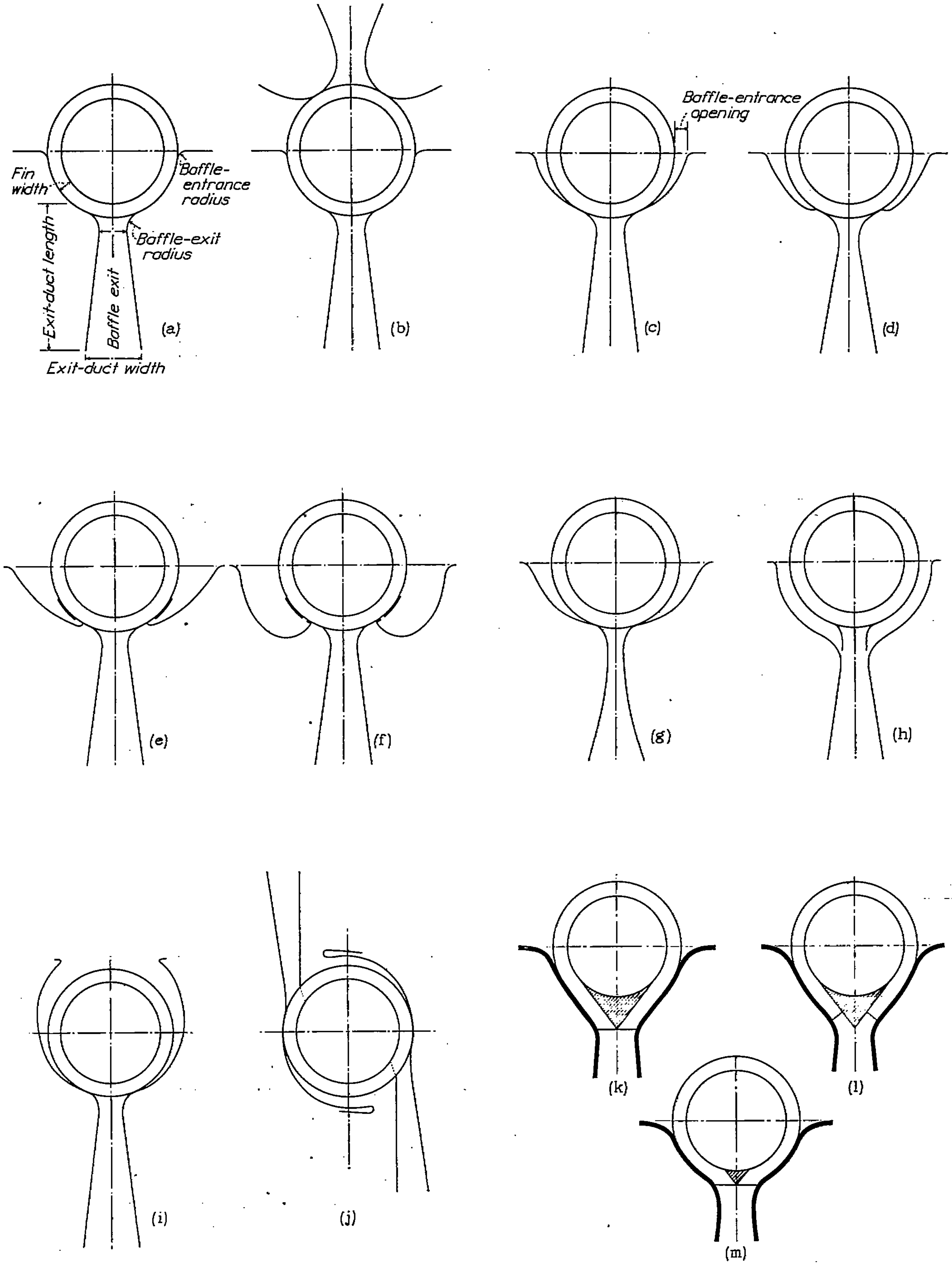

FIGURE 4.—Baffle arrangements tested in references 8 and 9.

TABLE I

TEMPERATURE DIFFERENCE BETWEEN CYLINDER WALL AND INLET AIR AT SEVERAL POSITIONS AROUND THE CYLINDER FOR THE DIFFERENT BAFFLE ARRANGEMENTS TESTED

[Data for baffles 1 to 15 taken from reference 8, and data for baffles 16 to 18 taken from figure 4 of reference 9]

Baffle	Figure	Baffle-exit radius (in.)	Baffle-exit width (in.)	Baffle-entrance width (in.)	Exit-duct length (in.)	Exit-duct width (in.)	Flared to (in.)	Temperature difference (° F)									
								Angular position around cylinder (deg)									
								0	30	60	90	105	120	135	150	165	180
1	--------	0.25	1.875	0.25	0	--------	--------	126	114	110	116	124	132	139	150	160	168
2	4 (a)	.25	1.875	(1)	0	--------	--------	126	114	110	113	119	125	132	140	148	155
3	4 (a)	.75	1.875	(1)	7	2.62	--------	120	106	100	99	103	108	114	124	136	144
4	4 (a)	.75	1.875	(1)	7	2.25	--------	120	106	100	101	106	110	115	122	128	136
5	4 (a)	.25	1.875	(1)	7	2.62	--------	126	114	109	112	115	120	125	133	141	151
6	4 (a)	.25	1.250	(1)	7	2.62	--------	127	114	110	112	118	122	128	135	140	147
7	4 (c)	.75	1.250	.50	7	2.25	--------	121	110	103	108	110	114	116	120	126	134
8	4 (c)	.75	1.250	1.00	7	2.25	--------	122	108	103	108	112	114	117	120	125	132
9	4 (d)	.75	1.250	1.00	7	2.62	--------	132	120	117	124	127	129	130	133	135	142
10	4 (e)	.75	1.250	2.00	7	2.25	--------	130	117	116	124	126	126	126	126	130	134
11	4 (f)	.75	1.250	2.62	7	2.25	--------	141	128	125	127	128	126	125	126	127	132
12	4 (g)	.75	1.250	1.25	7	2.25	1.00	132	116	114	120	125	126	126	128	130	134
13	4 (h)	.75	1.875	.75	7	2.62	--------	119	105	100	106	111	114	116	120	124	130
14	4 (i)	.75	1.250	1.25	7	2.25	--------	158	150	150	150	150	147	146	149	154	160
15	4 (j)	--------	.750	.62	7	1.25	--------	132	132	132	136	140	136	127	128	128	132
16	4 (k)	--------	--------	--------	--------	--------	--------	106	98	90	92	93	95	97	--------	102	98
17	4 (l)	--------	--------	--------	--------	--------	--------	108	98	90	90	91	98	108	108	124	124
18	4 (m)	--------	--------	--------	--------	--------	--------	98	100	93	91	--------	91	102	106	106	104

[1] Close.

Power increase associated with increasing the mass flow through a given cooling unit.—As previously pointed out, the increase in heat transferred obtained from increase in mass flow as a result of improved baffle design is attended by an increase in the power cost of cooling proportional to the increase in mass flow. Any further increase in heat transferred obtained by increasing the mass flow as a result of increased pressure drop increases the power required for cooling about as the cube of the mass flow or, as the friction factor drops slightly with increasing mass flow,

$$\text{Power}=Q\Delta p\propto(\rho V)\Delta p\propto(\rho V)^{2.9} \tag{12}$$

From equation (10) and the value of λ determined in reference 7, it may be seen that

$$H\propto(\rho V)^{0.65}$$

so that the power is related to the total heat transferred by

$$\text{Power }\propto H^{4.45}$$

From this relation, doubling the total heat transferred by increasing the mass flow would obviously require that the power used in cooling be increased about 20 times.

INCREASING COOLING BY USING METAL OF HIGHER THERMAL CONDUCTIVITY IN CYLINDER HEAD

Current practice in engine-cylinder design is to make the fins and the wall of a cylinder of either steel or aluminum, a steel barrel and an aluminum head being the usual combination. The advantage of using a metal with high thermal conductivity for wall and fins can be well brought out by considering the effect on cooling of substituting copper for aluminum in only the cylinder head. Copper has about twice the tensile strength, twice the thermal conductivity, and three times the density of aluminum. The thickness of the wall being determined by the maximum pressure and the tensile strength of the wall metal, it follows that the copper wall would be about one-half as thick as the aluminum wall. The weight of the copper head including fins would then be between one and one-half and two times the weight of the aluminum head that it replaced.

The temperature drop through the wall of the copper cylinder head is only about one-fourth of the drop through the aluminum head, the same heat flow being assumed in each case. This reduction is the result of using a metal one-half the thickness of aluminum and having twice the thermal conductivity of aluminum.

The coefficient of thermal conductivity of the metal in the wall, together with the heat being conducted through the wall, determines the temperature gradient in the wall. The aluminum wall of the cylinder head is made relatively thick and the temperature drop through the wall is surprisingly high. If it is assumed that 35 horsepower must be conducted through the head wall and that the thermal conductivity coefficient for aluminum is 10 Btu per hour per square inch per degree Fahrenheit per inch, then, for a cylinder of 6-inch bore and having a hemispherical head, the gradient in the aluminum wall is about 157° F per inch. For a wall of ⅝-inch thickness the temperature drop through the wall is 98° F. For a copper wall, which has been shown to have only about one-fourth the drop of aluminum, the temperature drop through the wall is only 25° F.

Suppose that the inner surface of the cylinder-head wall has a temperature of 500° F and that the cooling air has a temperature of 100° F. On the basis of the temperature drop through the wall just mentioned, the temperature difference between the cooling air and the outside surface of the cylinder-head wall is 302° F for the aluminum head and 375° F for the copper head. These temperature differences indicate that the copper head would obviously dissipate about 25 percent more heat than the aluminum head, the same fin effectiveness being assumed. The substitution of copper for aluminum in the fins, however, considerably increases the value of the fin effectiveness (reference 6). On fins such as are now in use, for example, about 0.050 to 0.090 inch thick and 1.5 inches wide, the substitution of copper for aluminum would increase the effectiveness by about 10 percent. The total effect of substituting copper for aluminum is, according to the preceding rough calculation, to increase the cooling on the head by 27 percent.

The aluminum head, exclusive of valves and other connections, weighs approximately 5 pounds. This weight would be increased to 8 or 10 pounds by using a copper head. The power required to transport this increase in weight is small if the increase in cooling obtained is considered.

Application of the foregoing considerations would require the development of suitable alloys or protective coatings.

INCREASING COOLING BY IMPROVING THE FIN DESIGN

It has been shown in a previous section that the cooling of a cylinder could be increased as much as desired by sufficiently increasing the mass flow of cooling air. The cooling can also be increased by improved finning. Increased cooling by increased mass flow was also shown to be obtainable at a relatively large increase in power for cooling. Improved finning will also require an increase in power for cooling, but the extra power is used for cooling with relatively high efficiency.

The general principles of flow for cooling a finned cylinder are the same as for cooling in pipes. (See references 9 and 12.) Figures 5 and 6 demonstrate that this similarity extends even to the point that the curved passages on a cylinder have a higher heat-transfer coefficient than a straight passage. This same effect for pipes is shown in reference 5 (pp. 117 and 159). The air passages formed by the fins, the cylinder wall, and the baffle are, in reality, only small pipes of length approximately equal to the baffle length and diameter equal to the hydraulic diameter of the passage. Figure 5 gives, as a solid curve, the friction factor f as a function of Reynolds number for air flow within circular tubes (reference 5, p. 110). The broken curve in the laminar region was derived on the assumption of viscous laminar flow and is the theoretical curve for laminar flow between parallel planes. McAdams (reference 5, p. 117) states that, in the turbulent region, the friction factor for pipes of circular cross section may be applied to

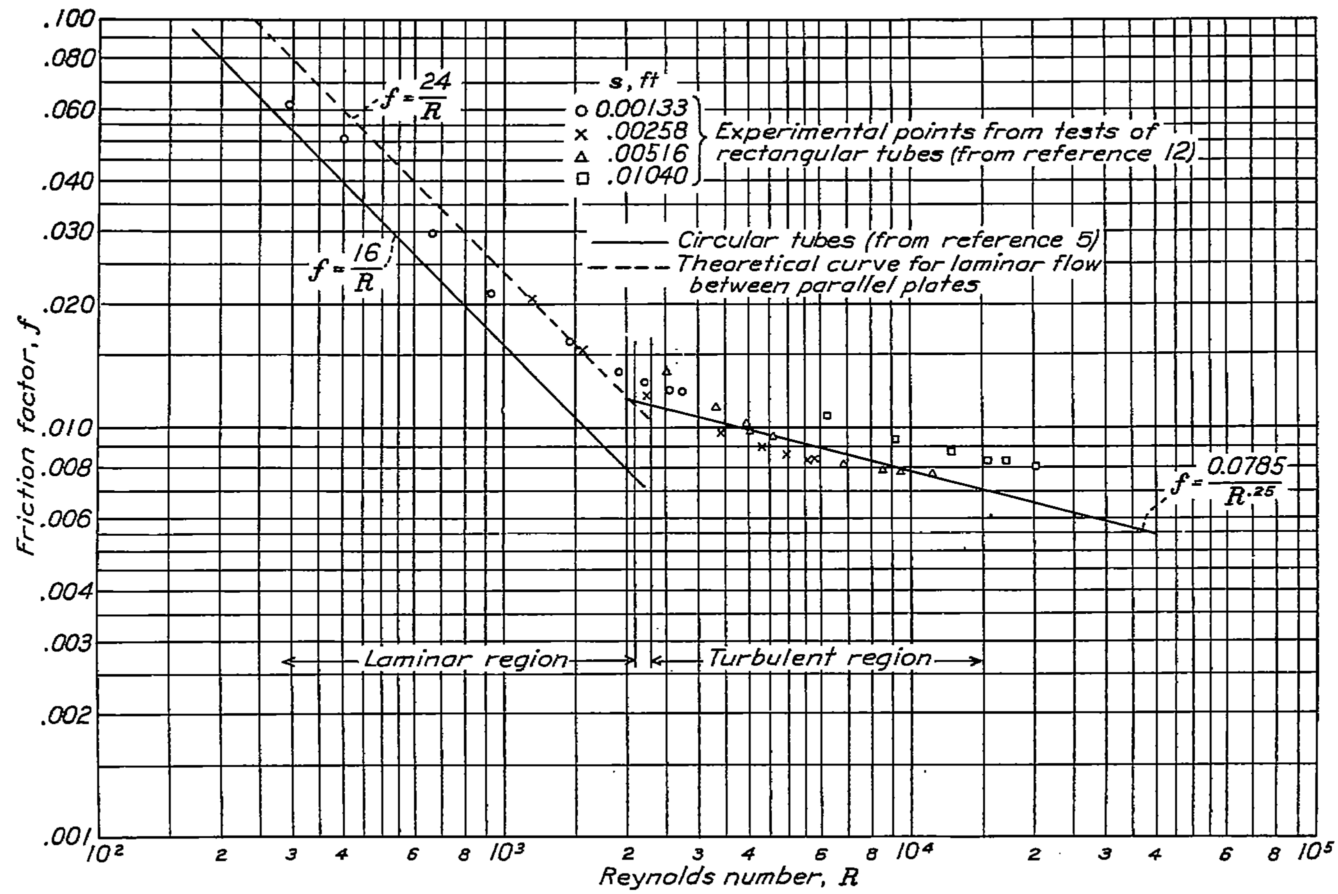

FIGURE 5.—Variation of friction factor $\frac{\Delta p}{q}\frac{D}{4L}$ with Reynolds number $D\rho V/\mu$ for fully developed air flow through circular and rectangular tubes.

tubes of any cross section, provided that the hydraulic diameter of the tube is used in place of the diameter of the circular pipe. Points obtained from tests (reference 12) on segments of finned cylinders with various fin spacings are also shown in figure 5. The agreement between these points and the curve for pipes of circular cross section is good, especially in the turbulent region, and cooling is almost invariably accomplished in this region. It must be recognized that the pressure drop calculated from the curve in figure 5 is the drop occurring while the air is passing between the fins. If these results are to be applied to an engine cylinder, the exit loss must be added. This loss is controlled largely by the baffle design, which has been previously discussed.

Reference 13 gives the relation for the over-all heat-transfer coefficient based on cylinder-wall area as

$$U=\frac{h}{s+t}\left[\frac{2}{a}\left(1+\frac{w}{2r_b}\right)\tanh aw+s\right] \tag{13}$$

where

$$a=\sqrt{\frac{2h}{k_m t}}$$

The coefficient U may be computed as a function of fin spacing, fin width, thermal conductivity of the fins, and surface heat-transfer coefficient h, which may be calculated from the Nusselt number. Figure 6 gives

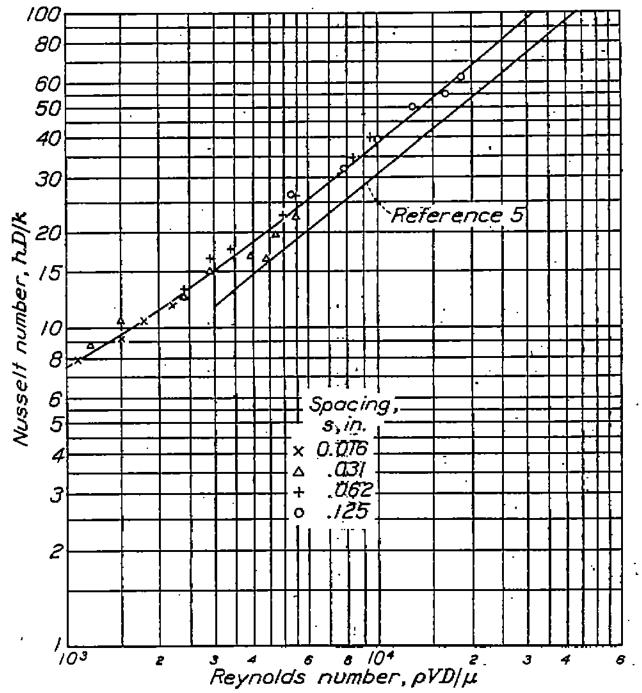

FIGURE 6.—Variation of Nusselt number with Reynolds number for several fin spacings.

the Nusselt number as a function of Reynolds number for curved segments of finned cylinders with several fin spacings. The curve for fully developed turbulent flow within tubes (from reference 5, p. 172) is included for comparison. Figure 7 shows, for a steel cylinder, values of U so computed plotted against s for a constant air speed of 100 miles per hour between the fins. Each curve is for a particular fin width. In all cases, the fin thickness is optimum.

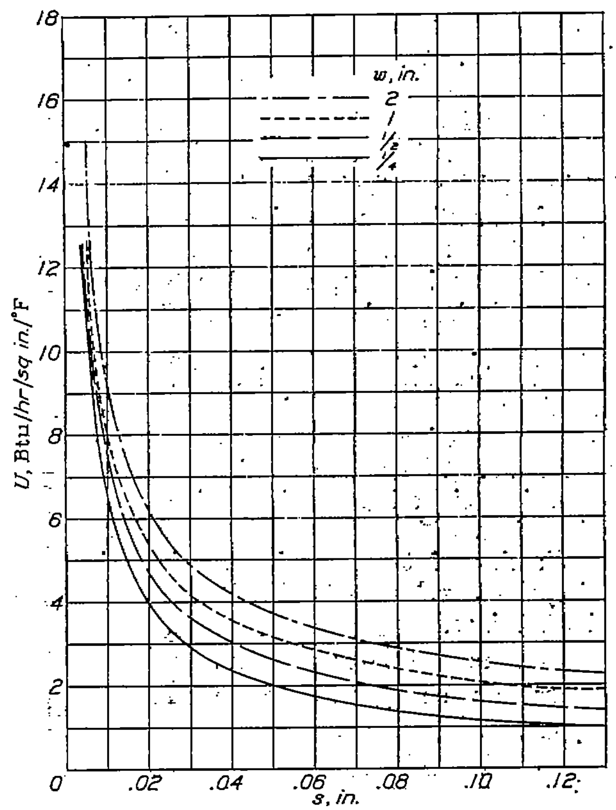

FIGURE 7.—Variation of over-all heat-transfer coefficient with spacing for four fin widths, at 100 miles per hour and optimum fin thickness. Steel cylinder; k_m=2.17 Btu/(hr) (sq in.) (°F/in.).

Figure 8 shows a similar set of curves for steel, aluminum, and copper fins. These curves show the ultimate heat transfer that can be obtained by the use of material of high thermal conductivity. This advantage of high thermal conductivity is in addition to that previously considered where only the effect of the temperature drop through the wall upon the temperature difference between outer cylinder surface and cooling air was considered.

The variation of cooling-air and cylinder-wall temperatures with the length of air passage is shown in figure 9 for the condition of constant heat transfer per unit passage length. This condition is nearly that prevailing on an air-cooled engine cylinder. Note that, for this condition, the temperature difference between the cylinder wall and the cooling air is constant along the passage length. Hence, it follows that any increase in the temperature rise of the cooling air in its passage around the cylinder will result in increased wall-temperature variation around the cylinder.

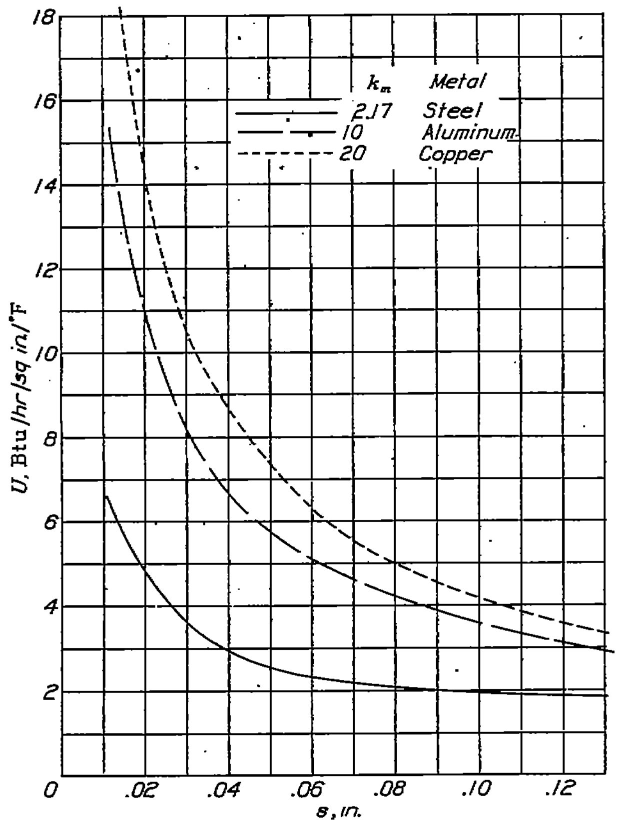

FIGURE 8.—Variation of over-all heat-transfer coefficient with spacing for three thermal conductivities, at 100 miles per hour and optimum fin thickness. w, 1 inch.

Greater temperature rise of the cooling air will result from a decrease in fin spacing, unless the mass flow of cooling air is maintained by either reducing the baffle length or increasing the pressure drop available for cooling. If the mass flow of cooling air is maintained by either of these methods, then a change to smaller fin spacing will reduce cylinder-wall temperatures at all points in the baffle with a small increase in wall-temperature variation around the cylinder.

If the heat-transfer coefficient of a cylinder is increased by increasing the mass flow of cooling air, then the temperature rise of the cooling air, and hence the wall-temperature variation, will become smaller. This procedure will also reduce cylinder-wall temperatures in the baffle but, as has been pointed out, will require a considerable increase in power consumed for cooling.

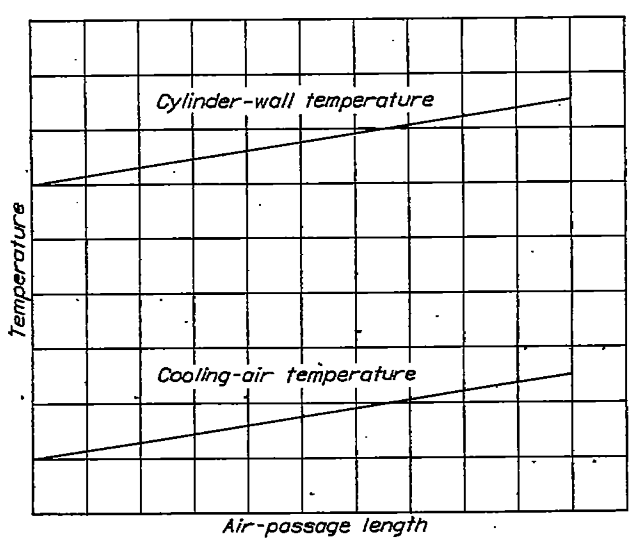

FIGURE 9.—Variation of cooling-air and cylinder-wall temperatures along an air passage for the condition of constant heat transfer per unit passage length.

The power for any particular arrangement can be calculated from

$$\text{Power}=AV\Delta p$$

since Δp and A will be known and V can be calculated from the equation relating pressure gradient to Reynolds number for turbulent flow (reference 5, p. 111):

$$\frac{dp}{dL}=0.098\frac{1}{R^{0.20}}\frac{\rho V^2}{D} \tag{14}$$

Although the power increases faster than the heat transfer, decreasing the spacing is still an economical means of improving the cooling if manufacturing difficulties are neglected.

FRONT COOLING

Cooling on the front of the cylinder of an air-cooled radial engine is accomplished by large-scale turbulence, as first mentioned in reference 14; the subject has been covered in reference 15 and will not be reviewed here. Suffice it to say that front cooling has proved entirely adequate in the past and, with some improvement in fin design, will probably be adequate for future needs.

GROUND COOLING

An airplane engine must be supplied with sufficient cooling for any required condition of operation. The definition of these conditions in flight is relatively simple because there both the cooling requirements and the cooling available can be definitely stated. It is sometimes difficult to meet certain arbitrary cooling requirements on the ground, as, for instance, that the engine cool with a certain fraction of full power.

Such requirements can invariably be met on any engine type by penalizing the airplane with extra weight or poor performance in flight. The analysis of two extreme cases is enlightening: (1) an engine required to cool with reduced power, and (2) an engine required to cool with full power.

Based on the experimental work of reference 2, for constant cylinder temperatures,

$$\Delta p=\Delta p_1\left(\frac{I}{I_1}\right)^{1.75} \tag{15}$$

where Δp is the required pressure drop for satisfactory cooling for a given indicated horsepower I, and Δp_1 is the known pressure drop for satisfactory cooling at some other indicated horsepower I_1. When the engine was operating at full power, Δp_1 was assumed to be 40 pounds per square foot and I_1 to be 1100 horsepower.

If the engine is operated at one-half top speed, I is calculated to be about 150 horsepower and equation (15) gives the required Δp as 1.2 pounds per square foot. This pressure drop can be obtained by proper design.

If the engine requires a pressure drop of 40 pounds per square foot for cooling at full power in flight, approximately this same pressure drop will be required on the ground. The problem is simply one of providing a pressure drop of 40 pounds per square foot across the engine. Two methods for the solution of this problem are obvious: (1) To provide the required pressure by means of the propeller and its slipstream; and (2) to use auxiliary means, such as a blower.

The use of the propeller and the propeller slipstream for cooling has been investigated in references 16 and 17. In reference 17, the nature of the flow in front of the cowling was studied. A round-edge disk was set in the front of the cowling and it was shown that an annular opening between the disk and the cowling for the entry of the air would materially increase the pressure drop for cooling. The amount of increase depended upon the blade-angle setting and the solidity of the portion of the propeller operating in front of this annular opening. Obviously, both the nose of the cowling and the disk must be close to the propeller for maximum pressure. The thrust-distribution curves for various propellers give a clear picture of the function of the propeller in providing pressure drop for cooling. The maximum pressure occurs near the 0.6 radius point; the difficulty of providing adequate pressure drop for cooling at full power on the ground when the cowling diameter is small relative to the propeller diameter is obvious. Propellers with blade sections extending to the hub markedly increase the pressure drop available for cooling provided that the blade angle of the propeller is suitable for the local velocity. Propellers having round blade shanks near the hub may be equipped with sleeves having airfoil sections to give the same result. Suction at the exit slot is provided by flaps, which operate on the propeller slipstream. The effectiveness of flaps depends almost entirely upon their being located in a relatively high velocity.

The previously discussed conventional means of augmenting the cooling are usually sufficient to provide adequate cooling for most ordinary cooling conditions. In the future, however, cases may very possibly occur where these conventional means may fail to provide adequate cooling. It will then be necessary to resort to blower cooling.

The efficiency of the blower will be about 70 percent for a good design. For the 1100-horsepower engine assumed, approximately 11 horsepower of useful work will be required to cool the engine. At 70-percent efficiency, this value amounts to a power expenditure of about 15 horsepower for ground cooling. An excess pressure will be supplied when there is any air speed. This excess pressure can be used to force air out the exit slot at an increased speed and thus, as the exit slot is almost 100 percent efficient, the excess work will be recovered. The only loss, then, is that due to the inefficiency of the blower.

The main consideration in the use of blower cooling concerns mechanical difficulties, weight, and effect of blower cooling on the front cooling as opposed to the difficulty, for ordinary cowlings, of producing sufficient pressure drop for full-power cooling on the ground with large-diameter propellers on normal-size cowlings.

Two conditions of ground cooling have been discussed, namely, with the engine idling and at full power. The discussion has shown that certain means are available for giving cooling on the ground. Each of these means must be examined in the light of what a particular method of improving cooling does to the airplane performance. When this examination is concluded, it may very possibly be more practicable to alter the cooling requirement by restricting operation on the ground and during take-off than to take the penalty in performance. From considerations of economy and simplicity, every detail of cowling, flaps, and propeller should obviously be arranged to give maximum cooling; only then, if the cooling is insufficient, should blower cooling be resorted to.

PRESSURE AVAILABLE FOR COOLING

The pressure available for cooling with a conventional cowling has been the subject of several studies (references 14, 16, 17, and 18). These studies, which cover the whole range of operation from ground to cruising condition, show the fundamental principles involved in the problem and give information suitable for design purposes.

The pressure on the ground being entirely dependent on the slipstream from the propeller, the distribution of the velocity in this slipstream determines the pressure available for cooling. An understanding of this fact immediately explains the variation in pressure available from propellers of various designs and diameters. Some of these variations are illustrated by figure 10 where Δp available for cooling is plotted against air speed. Dynamic pressure q is also shown as a function of air speed. Curves for two engine conductances are given for two typical three-blade propellers having diameters of 10 feet. Propeller F is Bureau of Aeronautics plan form 4893 and propeller C is Bureau of Aeronautics plan form 5868–9. The nacelle was 52 inches in diameter. In the computation of these curves, the engine was assumed to be developing 750 horsepower.

The curves for propeller F, which has airfoil sections close to the hub, may be used almost regardless of relative diameter of nacelle and propeller as long as the propeller disk loading remains the same. Propeller C, however, which has round blade shanks near the hub, will give a lower Δp if the propeller diameter is increased relative to the nacelle diameter or vice versa.

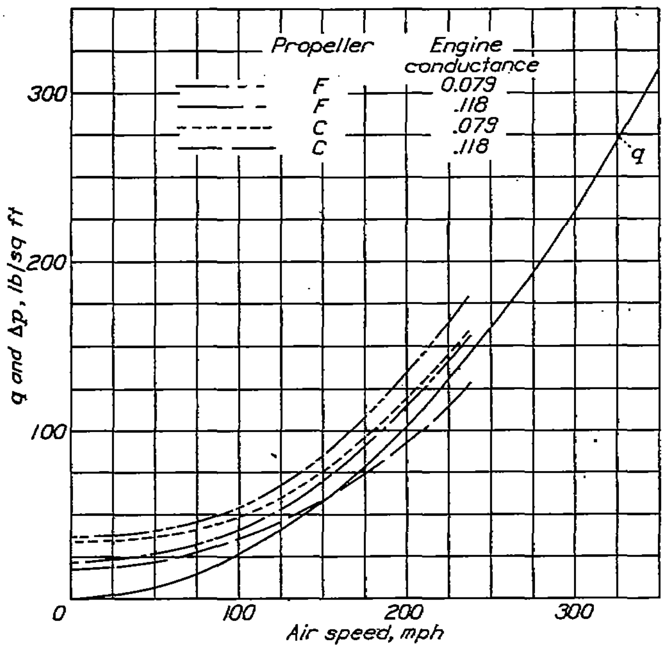

FIGURE 10.—Variation with air speed of Δp available for cooling using an NACA cowling with flaps in combination with two propellers and engine conductances.

HOT SPOTS

Some of the reasons for requiring increased cooling of an engine cylinder are: (1) to reduce detonation, (2) to maintain an oil film on the cylinder wall, and (3) to maintain the volumetric efficiency of the engine. Hot spots tend to amplify any tendency to detonate. The hottest points on the inside of the cylinder are the exhaust valve, the spark plug, and the piston. Any attempt to cool these points by improving the over-all cooling is very indirect and consequently very expensive as regards power for cooling.

The cylinder wall and the piston must have a relatively low temperature to insure lubrication between the two surfaces. Cooling the cylinder wall was formerly sufficient to insure proper lubrication. As the power per cylinder has been increased, however, the problem of cooling the cylinder wall has become more acute. The piston must now be cooled by more direct means, such as by the oil, to insure proper lubrication and to provide a workable temperature for the piston crown.

Cooling tests of model cylinders have shown that the rear of the cylinder is most difficult to cool. The location of the spark plug at this point multiplies the difficulties. The rear of the head could be cooled quite satisfactorily if there were no spark plug and if it were possible to supply the same finning that is employed elsewhere on the head. If the exhaust valve were in front, it also could be better cooled and a much simpler baffle arrangement could be used.

The cooling problem for air-cooled engines is to determine the cooling requirements of each part of the cylinder and of its associated parts, that is, of the piston and the valves, and then to provide adequate cooling for each part according to its needs. Only in this way can the air-cooled cylinder be made to function free from detonation and with low fuel consumption. The problem of cooling distribution, however, is much more critical for the air-cooled than for the liquid-cooled cylinder.

From cooling considerations, the sleeve-valve engine obviously has a decided advantage over the poppet-valve type. The use of sleeve valves would eliminate the valve-cooling problem; the piston then becomes the most critical point. The critical nature of piston cooling is demonstrated by the frequent failure of the oil film on the cylinder resulting in blow-by, loss of compression, and subsequent failure of the piston. The piston can be cooled (1) through the cylinder wall and (2) by means of the oil. The first method is the most common and, for low-power and low-compression engines, is adequate. The second method, if mechanical difficulties are neglected, is more direct, cheaper in power for cooling, and more adequate. In spite of any objection, method (2) must be employed as engine power and compression increase.

VARIATION OF TOTAL REQUIRED HEAT TRANSFER WITH INDICATED HORSEPOWER

The heat-transfer process on the inside of the cylinder, neglecting radiation, follows the same general laws of heat transfer as for forced convection over any body:

$$\frac{hL}{k}\propto\left(\frac{c_p\mu}{k}\right)^n\left(\frac{\rho VL}{\mu}\right)^m \tag{16}$$

For a given engine, c_p, μ, k, and L will be nearly constant so that, for practical purposes,

$$h\propto(V\rho)^m \tag{17}$$

The actual heat transferred is

$$H\propto Sh\Delta t_1\propto S(V\rho)^m\Delta t_1 \tag{18}$$

where Δt_1 is the mean effective temperature difference between the cylinder wall and the gases in the cylinder.

Assume that, during engine operation, the charge has the same air-fuel ratio throughout the operating range. Thus, when combustion takes place, the heat per unit weight of charge remains the same. The temperature rise due to compression is a function of only the compression ratio.

The temperature at any point is merely a function of air-fuel ratio and of compression ratio; that is, the temperatures are always the same at a given part of the cycle.

The equation for adiabatic compression from condition 1 to condition 2 is

$$\frac{p_1}{p_2}=\left(\frac{v_2}{v_1}\right)^\gamma=\left(\frac{1}{R}\right)^\gamma \tag{19}$$

$$\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}=\frac{T_1}{T_2}=\left(\frac{1}{R}\right)^{\gamma-1} \tag{20}$$

$$T_2=T_1(R)^{\gamma-1} \tag{21}$$

where R is compression ratio.

Equation (21) shows that the temperature T_2 remains the same regardless of quantity of charge for a given compression ratio and air-fuel ratio.

The velocity of movement of the charge in the cylinder is proportional to the speed of the engine. The density of the charge is directly proportional to the charge present which, in turn, is proportional to the indicated horsepower. (See reference 1.) The value of the heat-transfer exponent m for turbulent flow usually falls in the neighborhood of 0.8.

For convenience, consider two cases: (1) an engine operating at constant speed but with variable power due to variation in weight of charge, and (2) an engine having constant charge per cycle but with varying speed.

It is to be remembered that, in both cases, the temperature of the gases in the cylinder is fixed by the engine.

In case (1), the velocity and the temperature are fixed and the heat dissipated is proportional to $\rho^{0.8}$ or $I^{0.8}$. In case (2), the temperature and the density remain constant and the heat dissipation depends upon $V^{0.8}$ or $(\text{rpm})^{0.8}$. The indicated horsepower is proportional to the engine speed. Thus, the heat dissipated is proportional to $I^{0.8}$.

It follows that, regardless of whether the power is varied by changing engine speed or charge weight, the heat to be dissipated is proportional to $I^{0.8}$.

This conclusion is in agreement with experimental data obtained by Pye (reference 1), which showed that

$$H \propto I^{0.82} \tag{22}$$

Pinkel (reference 7) obtained values of 0.64 to 0.68 for the exponent of I in tests on cylinder heads and 0.64 to 0.85 in tests on cylinder barrels.

As a further check on the foregoing conclusions, the experimental work of reference 2 gave the empirical equation for pressure needed for cooling

$$\frac{\Delta p}{\Delta p_1}=\left(\frac{I}{I_1}\right)^{1.75} \tag{15}$$

Reference 12 gives

$$\frac{\Delta p}{\Delta p_1}=\left(\frac{V}{V_1}\right)^{1.75} \tag{23}$$

where V is the air velocity between the fins.

Then

$$\frac{I}{I_1}=\frac{V}{V_1}=\left(\frac{h}{h_1}\right)^{1.25} \tag{24}$$

since $h \propto V^{0.8}$.

As the cylinder temperatures were held constant in the work described in reference 2, it follows that

$$\frac{H}{H_1}=\left(\frac{I}{I_1}\right)^{0.8} \tag{25}$$

This subject has been treated in reference 19 in which similar conclusions were reached.

THE EFFECT OF ALTITUDE ON COOLING REQUIREMENT FOR AN UNSUPERCHARGED ENGINE

The cooling on a cylinder being dependent on the temperature and the density of the cooling air and on the pressure drop available for cooling, changes in altitude may have a great effect on cooling. The effect of altitude on cooling will now be considered.

Assume that C_D is constant with changes in altitude and further assume that the engine power P developed at any altitude is defined by

$$P=\varphi P_0 \tag{26}$$

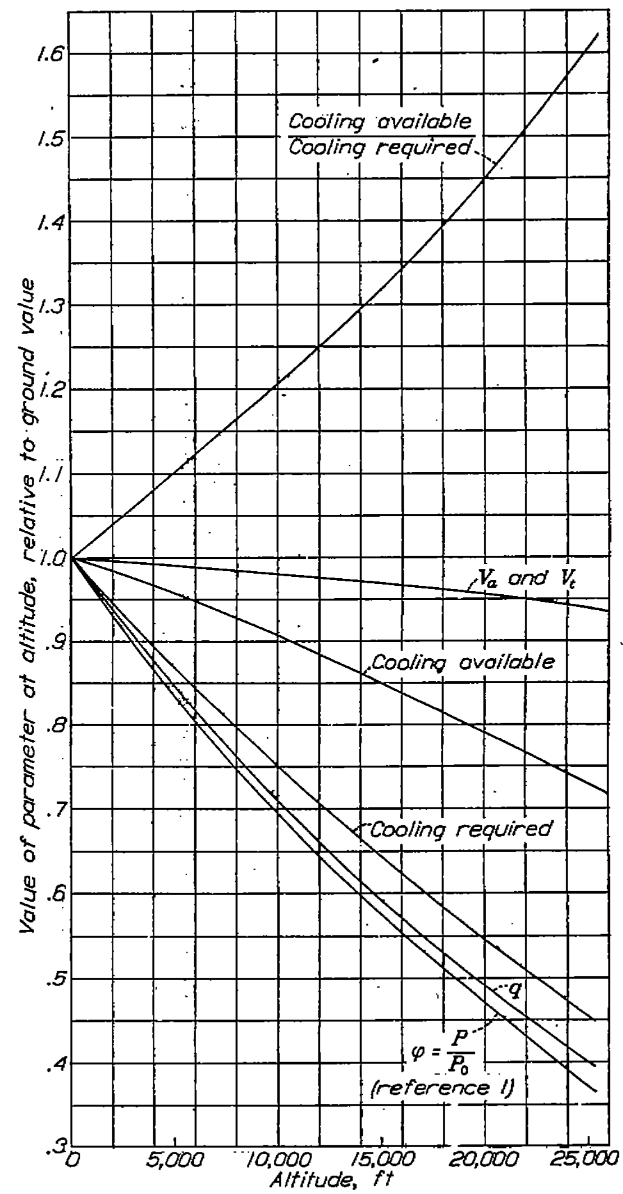

FIGURE 11.—Variation with altitude of several performance and cooling parameters for an airplane having an unsupercharged engine.

In figure 11, φ is shown as a function of altitude (data obtained from reference 1).

These assumptions are sufficiently accurate for use in calculating cooling parameters.

Equating engine thrust power to airplane drag power,

$$P=C_Dq_aSV \tag{27}$$

from which it may be shown that

$$q_a=\frac{\rho^{1/3}P^{2/3}}{2}\left(\frac{2}{C_DS}\right)^{2/3} \tag{28}$$

and

$$q_a=\sigma^{1/3}\varphi^{2/3}(q_a)_0 \tag{29}$$

Solving equation (29) for V yields

$$V=\varphi^{1/3}/\sigma^{1/3}V_0 \tag{30}$$

(See fig. 11.)

Figure 12 gives relative density, relative pressure, and standard temperature for standard atmosphere for altitudes up to 35,000 feet. The data for figure 12 were taken from reference 20.

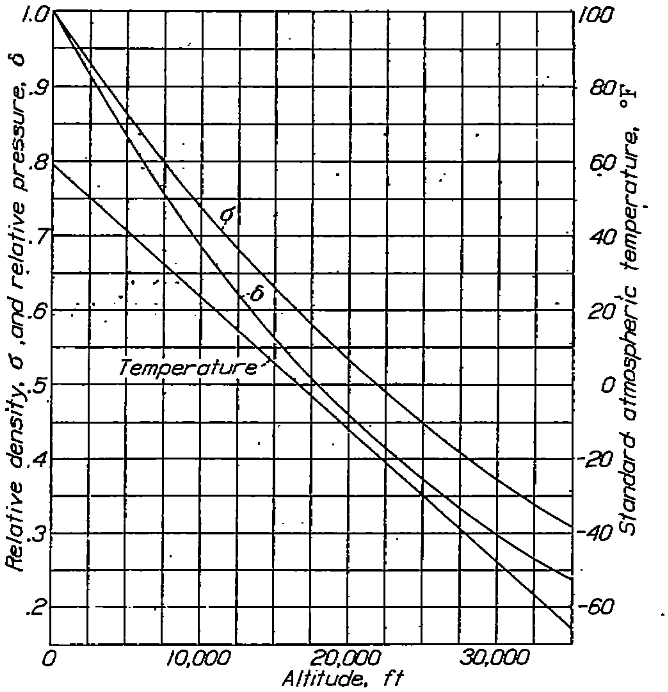

FIGURE 12.—Variation of relative density, relative pressure, and temperature of the standard atmosphere with altitude. (Data from reference 20.)

It can be shown that, within close limits, V_t varies directly with the flight speed V and hence with altitude as $(\varphi/\sigma)^{1/3}$. (See fig. 11.) Since the variations of V_t, ρ, T, and hence of μ, with altitude are all known, the variation of the Reynolds number with altitude can be calculated.

By the use of equations (3), (4), and (6) and on the assumption that $T_w=350°$ F and $f'=0.77$ at the ground, the variation of cooling available with altitude can be calculated. (See fig. 11.)

Earlier in this paper, the total required heat transfer was shown to vary as the eight-tenths power of the indicated horsepower, so that

$$H/H_0=\psi^{0.8}\approx\varphi^{0.8} \tag{31}$$

(The ratio of indicated horsepower ψ is taken as equal to the ratio of brake horsepower φ. These two values differ by less than 5 percent for altitudes up to 25,000 feet.) The cooling required then varies directly with $\varphi^{0.8}$. This curve of cooling required is given in figure 11, and the curve of $\frac{\text{cooling available}}{\text{cooling required}}$ is given in the same figure.

Figure 11 shows that, for an unsupercharged engine, cooling becomes relatively easier with increasing altitude.

EFFECT OF ALTITUDE ON COOLING REQUIREMENTS FOR A SUPERCHARGED ENGINE

The cooling requirements for a supercharged engine can be calculated from the foregoing analysis for an unsupercharged engine by taking $\varphi=1$ at all altitudes below the critical. The results of such a calculation for a supercharged engine are shown in figure 13. Cooling becomes more difficult with increasing altitude up to the critical altitude; and, beyond the critical altitude, cooling again becomes easier with increasing altitude.

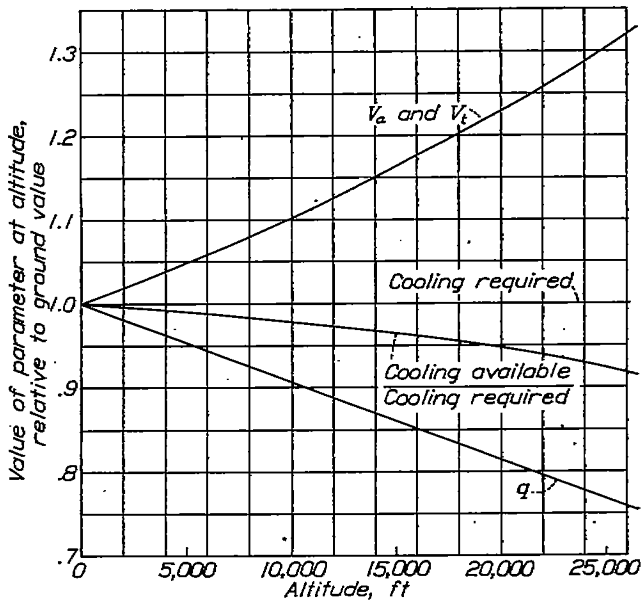

FIGURE 13.—Variation with altitude of several performance and cooling parameters for an airplane having a supercharged engine.

TEMPERATURE RISE OF COOLING AIR DUE TO ADIABATIC COMPRESSION AT ENTRANCE

The temperature rise of the cooling air due to adiabatic compression at the cowling entrance may be calculated from the equation for stagnation pressure in front of a body in compressible flow (reference 21):

$$p_2=p_1\left(1+\frac{\gamma-1}{2}\frac{V^2}{c^2}\right)^{\frac{\gamma}{\gamma-1}} \tag{32}$$

The formula for adiabatic change of temperature and pressure of a gas is

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{33}$$

where p absolute pressure

V airplane speed

c velocity of sound at altitude at which airplane is flying

T absolute temperature

Subscript 1 refers to the free air stream at the flying altitude. Subscript 2 refers to air conditions in the cowling entrance. Combining equations (32) and (33) gives

$$T_2=T_1\left(1+\frac{\gamma-1}{2}\frac{V^2}{c^2}\right) \tag{34}$$

from which

$$\Delta T=T_2-T_1=T_1\frac{\gamma-1}{2}\frac{V^2}{c^2} \tag{35}$$

Laplace's formula for the velocity of sound in a gas gives

$$c^2=\gamma\frac{p}{\rho} \tag{36}$$

When equation (36) is substituted in (35) and, since $\gamma=c_p/c_v$, the equation becomes

$$c_p\Delta T=\frac{T_1\rho}{p}(c_p-c_v)\frac{V^2}{2} \tag{37}$$

Since $pv=p/\rho=RT$ for one slug of substance and $c_p-c_v=R$ for one slug of substance,

$$c_p\Delta T=\frac{V^2}{2\times 778} \tag{38}$$

where ΔT degrees Fahrenheit

V feet per second

c_p Btu per slug per degree Fahrenheit (7.73 for air)

Equation (38) shows that, for adiabatic compressible flow under standard conditions, the temperature rise of the cooling air at the entrance is a function of velocity and specific heat and is independent of altitude. This temperature rise can be calculated for the anticipated top speed of the airplane and allowance for it can be made in the design calculations.

Values of ΔT have been calculated and are shown for several values of V in table II.

TABLE II

ADIABATIC TEMPERATURE RISE AT ANY STAGNATION POINT DUE TO THE FORWARD VELOCITY OF THE AIRPLANE

V (mph)	ΔT (°F)
100	1.8
200	7.2
300	16.1
400	28.6
500	44.6
600	64.2

CONCLUSIONS

The problem of cooling an air-cooled cylinder of an aircraft engine has been analyzed. The cooling may be improved by:

(1) Increasing the mass flow. Doubling the cooling requires 20 times the power for cooling.

(2) Improving the fin design. Wider fins give increased cooling with a power increase for cooling proportional to the width. Reducing the fin spacing by one-half its value gives about a 50-percent increase in cooling and a 100-percent increase in power for cooling if the length-diameter ratio of the air passage is kept constant and the fin thickness is kept optimum.

(3) Using a material of higher thermal conductivity. The temperature drop through the cylinder wall may be decreased, thus giving better cooling without change of other conditions.

The problem of ground cooling was reviewed from considerations of the cooling requirements and of the means by which such requirements can be met. It was concluded that, for normal operation of landplanes, satisfactory cooling could be obtained by the use of propeller and cowling flaps and that seaplanes with long take-offs would undoubtedly require auxiliary means of cooling.

The mechanism of heat transfer from hot gases to the cylinder wall was analyzed on the basis of thermodynamics, and the heat transferred was shown to be proportional to the eight-tenths power of the indicated horsepower, regardless of whether the horsepower was varied by changing the cylinder charge or by varying the engine speed.

The cooling problem at altitude for an unsupercharged and a supercharged engine has been analyzed. It was shown that the cooling is:

(1) Accomplished at altitude without change in power for cooling or fin design for an unsupercharged engine.

(2) More difficult, up to critical altitude, for a supercharged engine. An arrangement that just cools satisfactorily on the ground would have approximately a 10-percent deficiency in cooling ability at 25,000 feet.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *April 22, 1940.*

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