

## REPORT No. 726

### THE DESIGN OF FINS FOR AIR-COOLED CYLINDERS

By ARNOLD E. BIERMANN and HERMAN H. ELLERBROCK, Jr.

#### SUMMARY

*An analysis was made to determine the proportions of fins made of aluminum, copper, magnesium, and steel necessary to dissipate maximum quantities of heat for different fin widths, fin weights, and air-flow conditions. The analysis also concerns the determination of the optimum fin proportions when specified limits are placed on the fin dimensions. The calculation of the heat flow in the fins is based on an experimentally verified, theoretical equation. The surface heat-transfer coefficients used with this equation were taken from previously reported experiments.*

*In addition to the presentation of fin-design information, this investigation shows that optimum fin dimensions are inappreciably affected by the differences in air flow that are obtained with different air-flow arrangements or by small changes in the length of the air-flow path. For a given fin weight, the highest heat transfer can be obtained with fins of a magnesium alloy; pure copper and aluminum-alloy fins are only slightly inferior to magnesium-alloy fins and will dissipate several times more heat than steel.*

#### INTRODUCTION

Previous investigations on the subject of cooling surfaces by means of metal fins have shown that the heat dissipated can be expressed fairly accurately by an equation involving the fin dimensions, the thermal conductivity of the metal, and the surface heat-transfer coefficient  $q$  (references 1 to 5). Experimental values of  $q$  have been determined for a wide range of air-flow conditions (reference 5). From the information previously obtained, it is possible to calculate the over-all heat transfer of finned cylinders in the range of general interest.

The problem of fin design for any one air-flow condition involves the determination of the fin proportions that will give the greatest heat transfer for (1) a given weight of fin material or for (2) a given fin width. Previous investigations of optimum fin proportions (references 1 and 6) have generally been made to deter-

mine the greatest heat transfer for given weights of fin material. In these investigations it has been shown that, for every value of fin weight and air-flow condition, only one particular fin design will give a maximum heat flow.

In reference 6, fin-design information was presented for one cylinder diameter and one baffle arrangement. The more complete data of the value of  $q$  presented in reference 5 make it possible to widen the range of fin-design data. The object of the present report is to give fin-design information for several conditions of air flow, different cylinder diameters, and several metals. The criteria for excellence of fin design have been based upon the maximum heat transfer for a given fin weight and the maximum heat transfer for a given fin width.

#### SYMBOLS

$W_m$	specific weight of fin material, pounds per cubic inch
$D$	cylinder diameter at fin root, inches
$k_m$	thermal conductivity of metal, Btu per square inch through 1 inch per hour per °F
$k_a$	thermal conductivity of cooling air, Btu per square inch through 1 inch per second per °F
$l$	equivalent length for straight tube ( $\phi R_a$ ), feet
$M$	weight of fins, pounds per square inch of outside-wall area
$q$	surface heat-transfer coefficient, Btu per square inch total surface area per hour per °F temperature difference between surface and inlet cooling air
$q_{av}$	surface heat-transfer coefficient, Btu per square inch total surface area per hour per °F temperature difference between surface and average cooling air
$R_a$	average radius from center of cylinder to finned surface $\left(\frac{R_b}{12} + \frac{w}{2 \times 12}\right)$ , feet
$R_b$	radius from center of cylinder to fin root ( $D/2$ ), inches
$s$	average space between adjacent fin surfaces, inches

$t$	average thickness of fins, inches
$U$	over-all heat-transfer coefficient, Btu. per square inch outside-wall area per °F temperature difference between cylinder wall and inlet cooling air per hour
$V$	velocity of cooling air, feet per second
$w$	fin width, inches
$w'$	$w+t/2$ , inches
$g$	acceleration of gravity, feet per second per second
$\rho_1 g$	specific weight of cooling air in front of cylinder, pounds per cubic foot
$\rho_2 g$	specific weight of cooling air in rear of cylinder, pounds per cubic foot
$\rho_{av} g$	average specific weight of cooling air $(\rho_1 g + \rho_2 g)/2$ , pounds per cubic foot
$\rho_0 g$	specific weight of cooling air at 29.92 inches of mercury and 80° F (0.0734 lb per cu ft), pounds per cubic foot
$\mu$	absolute viscosity of cooling air, pounds per second per foot
$\varphi$	equivalent angle of curvature, 180° minus one-half the angle subtended by front opening of baffle
$\Delta p_1$	pressure difference across cylinder, inches of water
$\Delta p_2$	pressure difference caused by loss of velocity head from exit of skirt of baffle or jacket, inches of water
$\Delta p_{total}$	total pressure difference across set-up $(\Delta p_1 + \Delta p_2)$ , inches of water

#### ANALYSIS

In the cooling of engine cylinders, two outstanding requirements must be considered. One requirement is to protect the surfaces; the other is to reduce the knock or the tendencies to preignition caused by hot surfaces that come in contact with the fuel mixture. The lubricated working surfaces must be kept at a sufficiently low temperature to insure the maintenance of an adequate oil film. High piston and cylinder-wall temperatures usually cause sticking of the piston rings and rapid wear of the piston rings and the cylinder wall.

An additional problem of cooling is the prevention of undue distortion of the cylinder barrels, such as might be caused by uneven temperature distributions. Although difficulties arising from thermal distortion of cylinder barrels have been alleviated to some extent with specially ground pistons, it appears desirable to retain round cylinders by means of a uniform or otherwise satisfactory temperature distribution.

Of the several available methods of securing uniform temperatures around the cylinder, two methods are of particular interest. One method is so to distribute the effective fin area as to achieve the desired temperature

distribution. The other method is to control the air velocities around the cylinder by means of baffles surrounding the cylinder. In general, either of the foregoing methods will result in some loss in the maximum over-all heat transfer otherwise obtained for the same fin weight. In experimental work, it has been found that baffles designed for maximum over-all heat flow will not give a uniform temperature distribution and, conversely, baffles designed to give a uniform temperature distribution do so with a considerable sacrifice in over-all heat transfer. When the exhaust valves and the piston crowns are somewhat centrally located with respect to the finned surfaces that cool them, these parts generally are better cooled when a maximum over-all heat transfer of the finned surfaces is obtained at the expense of a uniform temperature distribution. In the present report, emphasis has therefore been placed on providing fins for obtaining high over-all heat transfer rather than on securing uniform cylinder temperatures.

The over-all heat-transfer coefficient  $U$  has been calculated from the following equation, which was derived in reference 1:

$$U = \frac{q}{s+t} \left[ \frac{2}{a} \left( 1 + \frac{w}{2R_b} \right) \tanh aw' + s \right] \quad (1)$$

where  $a = \sqrt{2q/k_{mt}}$  and  $k_m$  is the thermal conductivity of the metal (2.17 for steel; 7.66 for aluminum Y alloy; 18.04 for copper; and 7.54 for magnesium alloy). In this report aluminum and magnesium alloys are referred to as "aluminum" and "magnesium," respectively.

This equation has been experimentally verified (references 1 to 6) for fins of steel, copper, and aluminum alloy. Experiments have also shown that equation (1) holds equally well for rectangular or tapered fins, provided that the average values of the fin thickness and space are used in the calculations.

It has been found (reference 5) that the surface heat-transfer coefficient  $q$  can be correlated for each air-flow arrangement in terms of functions defining a single curve and involving the fin dimensions, the cylinder diameter, and the air-stream characteristics. Thus, for cylinders in a free air stream with and without baffles and for cylinders at a 45° fin-plane/air-stream angle,

$$\frac{qs}{k_a} = f_1 \left( \frac{V \rho_1 g s^2}{12 \mu D^{0.25} w^{0.45}} \right) \quad (2)$$

where  $V$  is the velocity of the free air stream and, for cylinders enclosed in a jacket and cooled by a blower,

$$\frac{qs}{k_a} = f_2 \left( \frac{V \rho_1 g s^2}{12 \mu D^{0.25}} \right) \quad (3)$$

where  $V$  is the velocity of the cooling air between the fins. Figure 1 shows the variation of  $q$  with fin and cylinder dimensions and air-stream characteristics for the four air-flow arrangements.

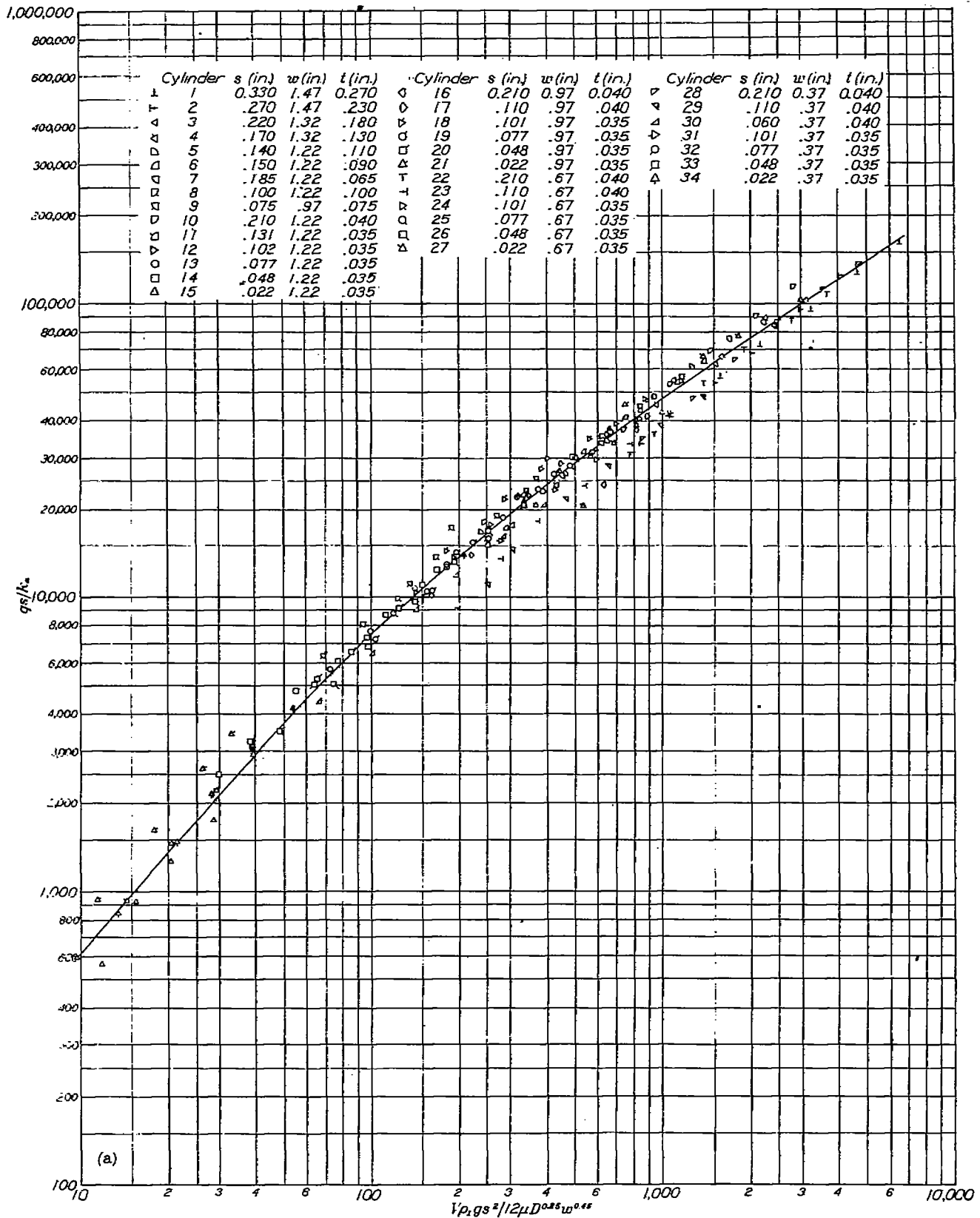
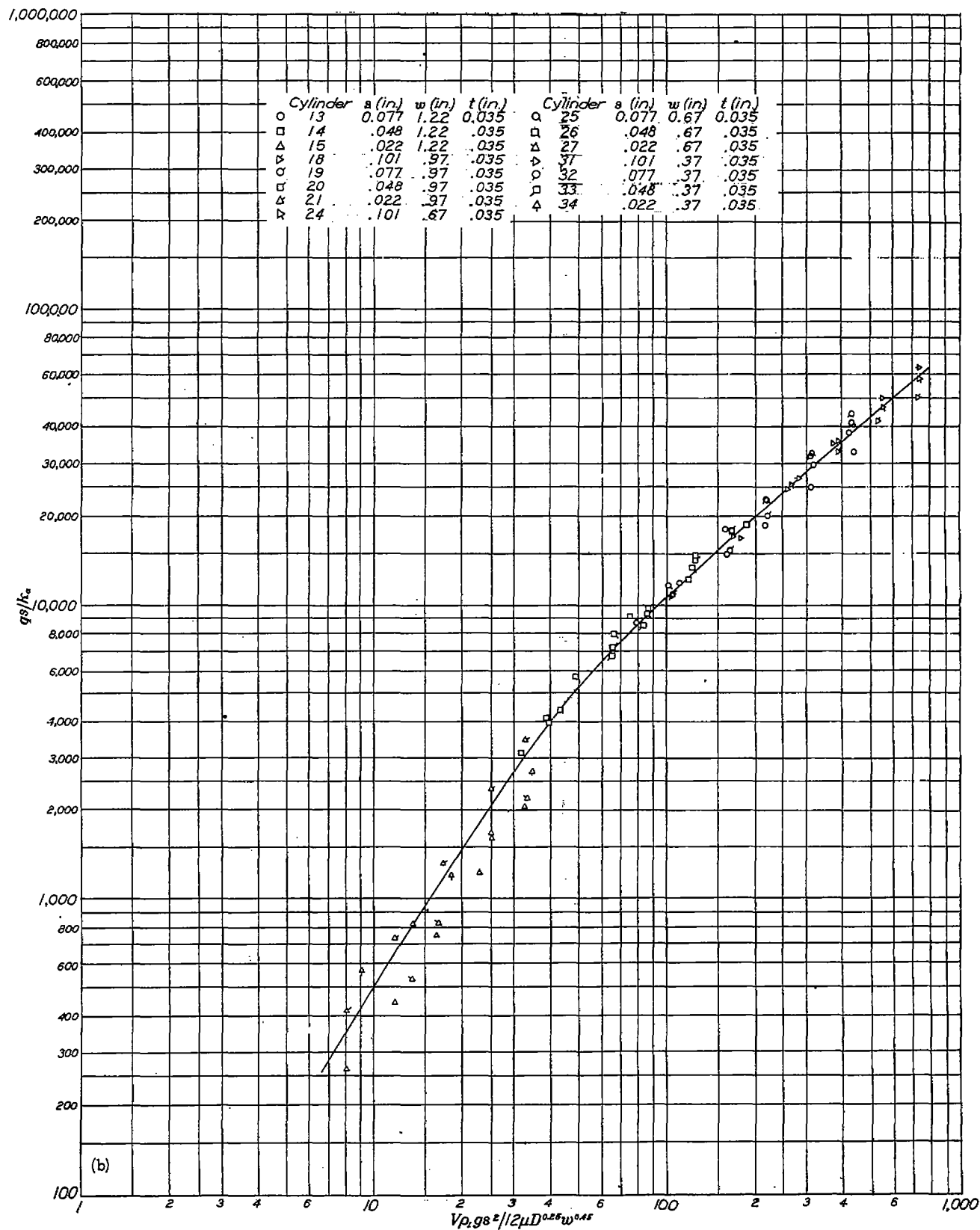
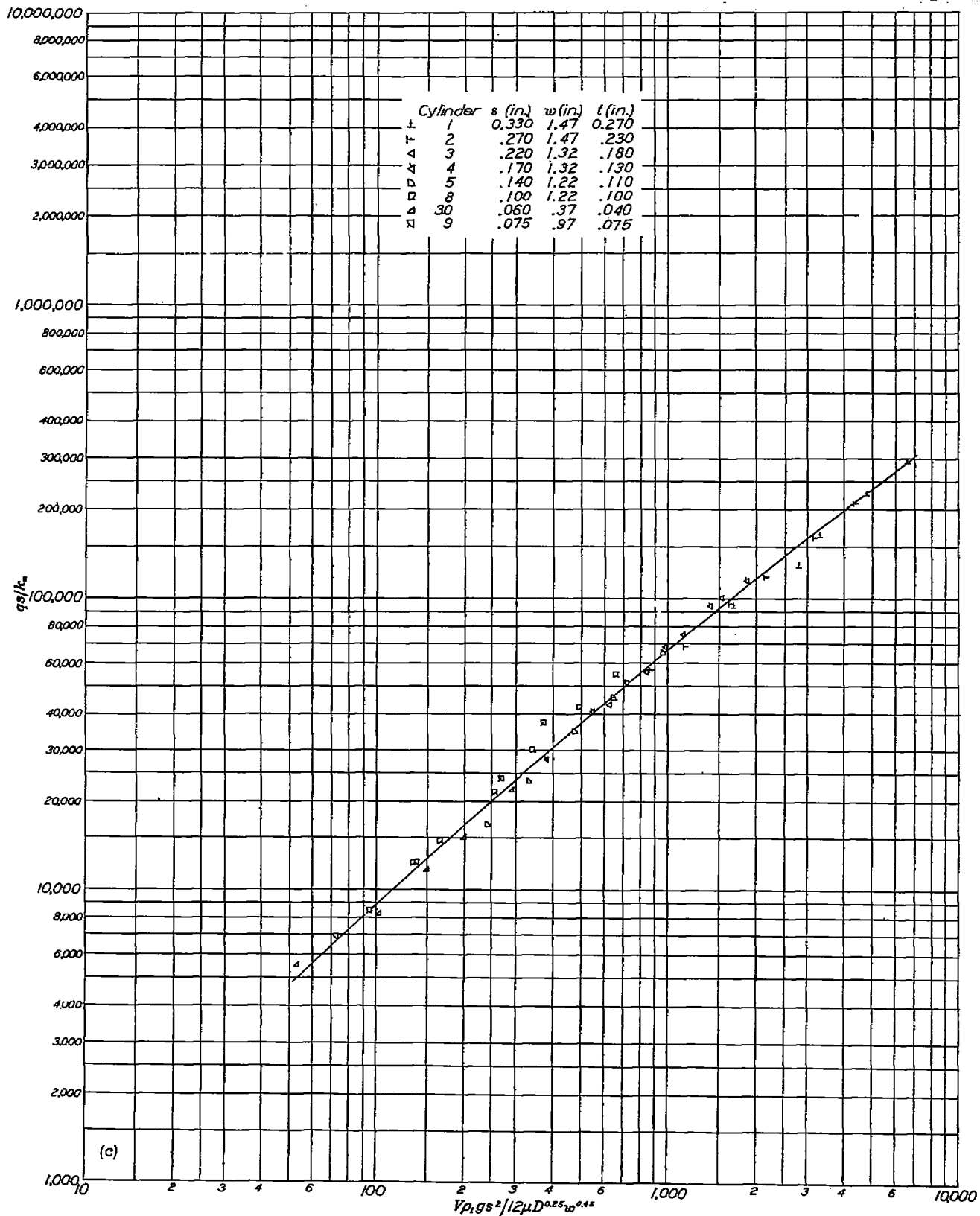


FIGURE 1.—Relation between factors involving  $g$ , fin dimensions, cylinder diameter, and air-stream characteristics.



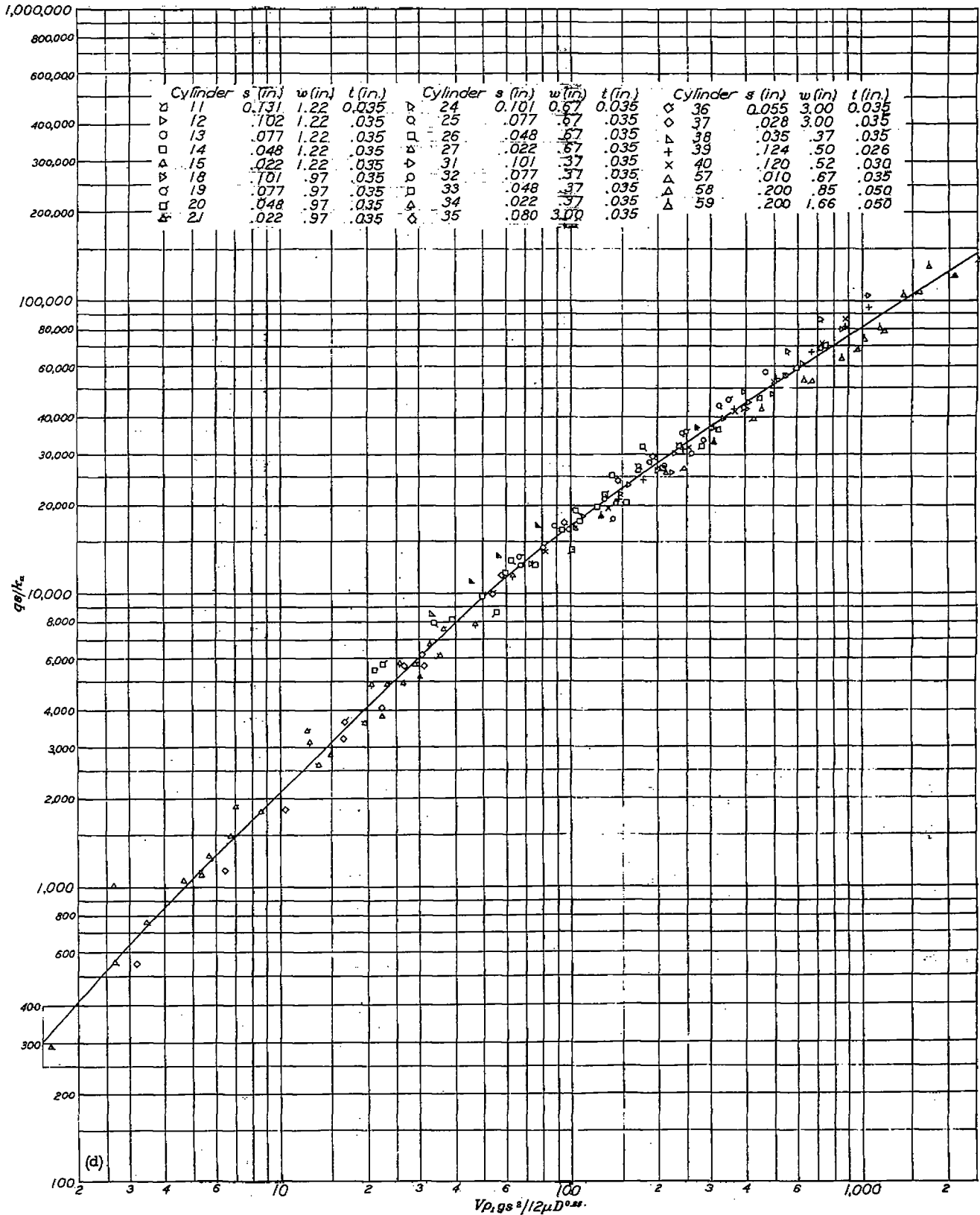
(b) Cylinder in free air stream, 140° baffles.

FIGURE 1.—Continued.



(c) Cylinder in free air stream, cylinder axis 45° to air stream.

FIGURE 1.—Continued.



(d) Cylinder enclosed in jacket, blower cooling.

FIGURE 1.—Concluded.

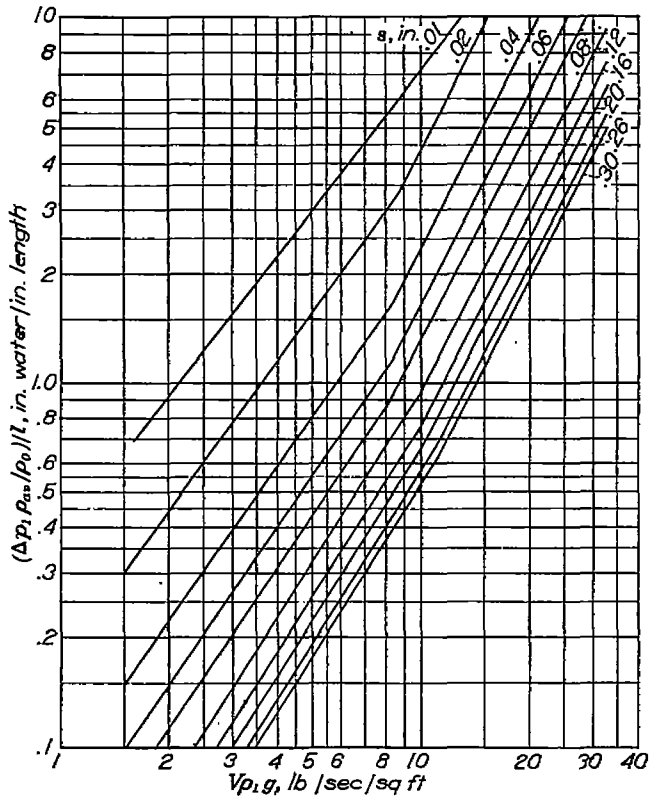


FIGURE 2.—Effect of weight velocity of cooling air on pressure difference per unit length of path for several fin spaces.

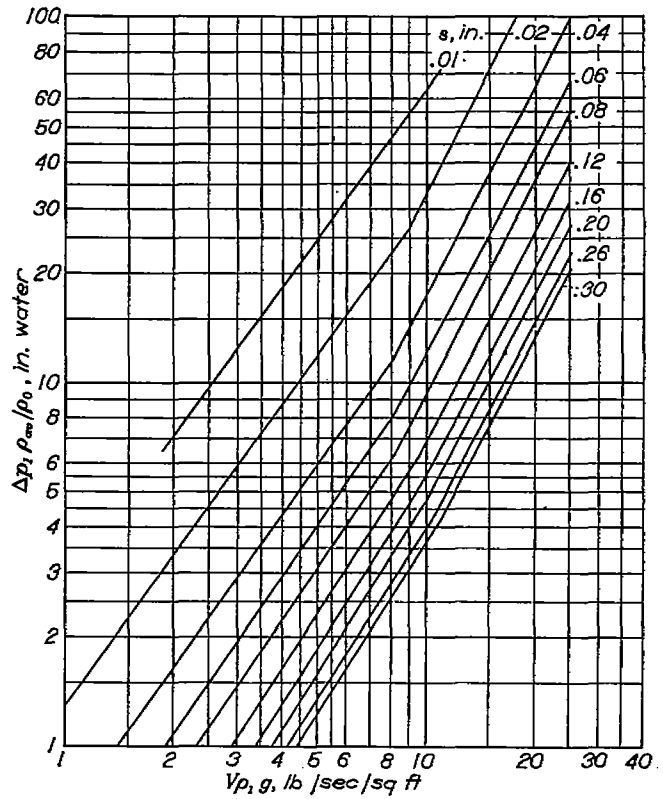


FIGURE 3.—Effect of weight velocity of cooling air on pressure difference across finned cylinders for several fin spaces. Average fin width, 0.035 inch; cylinder diameter, 4.66 inches; fin thickness, 0.035 inch; blower-cooling set-up.

For any one air-flow arrangement for which the pressure differences across the cylinders are available,  $q$  can be determined as a function of the pressure difference instead of the weight velocity of the cooling air. Previous tests of cylinders enclosed in jackets and cooled by a blower (references 5 and 7) have shown that the pressure difference across the cylinder  $\Delta p_1$  for a given fin space and weight velocity is proportional to the length of flow path  $l$  which, in turn, is determined by the cylinder diameter, the fin dimensions, and the jacket design. The pressure differences across the cylinders for various lengths of path, fin spaces, and weight velocities are presented in figure 2. From this figure, it can be shown that

$$\Delta p_1 \rho_{as}/\rho_0 = f_3(s, t, w, D, V \rho_1 g) \quad (4)$$

and, as

$$\Delta p_2 \rho_{at}/\rho_0 = f_4(V \rho_1 g) \quad (5)$$

it follows that

$$V \rho_1 g = f_5(s, t, w, D, \Delta p_{total} \rho_{as}/\rho_0) \quad (6)$$

The weight of fins per square inch of outside cylinder-wall area is given by the equation

$$M = W_m \frac{wt}{s+t} \left( 1 + \frac{w}{2R_s} \right) = W_m f_6(s, t, w, D) \quad (7)$$

where  $W_m$  is the specific weight of the fin material (0.282 for steel; 0.101 for aluminum Y alloy; 0.322 for copper; and 0.0648 for magnesium alloy).

From equations (1) and (3), it is evident that, for a given metal,

$$U = f_7(s, t, w, D, V \rho_1 g) \quad (8)$$

When fin weight is more important than fin width,  $w$  can be eliminated from equation (8) by means of equation (7). Then

$$U = f_8(s, t, M, D, V \rho_1 g) \quad (9)$$

The over-all heat-transfer coefficient  $U$  can be expressed as a function of  $\Delta p_{total} \rho_{as}/\rho_0$  by combining equations (6), (7), and (9), or

$$U = f_9(s, t, M, D, \Delta p_{total} \rho_{as}/\rho_0) \quad (10)$$

The method of obtaining optimum fins followed in this report is generally to hold constant the values of the variables that are specified by the design conditions and, from a plot of  $U$  against values of the remaining variables, to obtain the rest of the dimensions that give maximum heat transfer.

The design of fins for given values of  $M$  and  $\Delta p_{total} \rho_{as}/\rho_0$  is more difficult than for constant values of  $M$  and  $V \rho_1 g$  because the length of the flow path and the losses from the baffle exit enter into the calculations. Both the length of the flow path and the exit losses depend upon the fin dimensions and the baffle design. A method of designing fins for a constant pressure difference, using an average length of flow path and assuming that all the pressure difference is available for cooling, would considerably simplify the calculations. Computations have shown that the difference

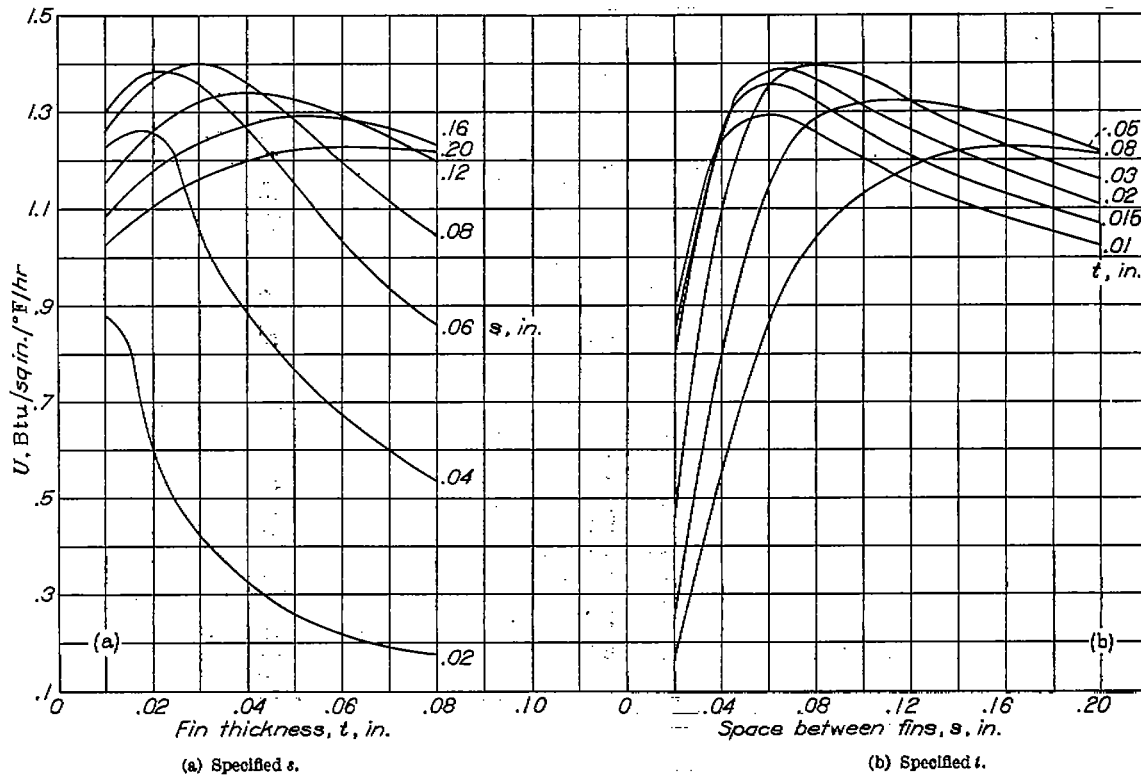


FIGURE 4.—Variation of  $U$  with  $s$  and  $t$  for constant  $M$ ,  $\Delta p_1 \rho_{as} / \rho_0$ , and  $D$ .

between the weight velocity based on the exact flow path and including the exit losses and the weight velocity based on an average flow-path length and neglecting the exit losses has very little effect on the correct fin proportions. Consequently, figure 3 has been used to determine pressure differences for this report.

It is evident from equation (10) that, for given values of cylinder diameter, weight of fins, and pressure drop,  $U$  is a function only of  $s$  and  $t$ . As an illustration, figure 4 (a) shows a plot of  $U$  against  $s$  and  $t$  for constant values of  $M$ ,  $\Delta p_1 \rho_{as} / \rho_0$ , and  $D$ . Figure 4 (b) is a cross plot of figure 4 (a). Either part of figure 4 clearly shows that, for one pair of values of  $s$  and  $t$ , the heat transfer is a maximum. The peak values of the curves of constant  $s$  shown in figure 4 (a) and of similar curves plotted for other values of  $M$  and  $\Delta p_1 \rho_{as} / \rho_0$  are shown in subsequent figures and are labeled "specified  $s$ " curves. Similarly, the peak values of the curves of constant values of  $t$  shown in figure 4 (b) and of similar curves plotted for other values of  $M$  and  $\Delta p_1 \rho_{as} / \rho_0$  are shown in subsequent figures and are labeled "specified  $t$ " curves. The specified  $s$  curves are used when a lower limit is set on the value of  $s$  and the specified  $t$  curves are used when a lower limit is set on the value of  $t$ .

For given values of  $M$ ,  $D$ ,  $\Delta p_1 \rho_{as} / \rho_0$  and a specified value of  $t$ , the value of  $s$  for which  $U$  is a maximum can also be found by setting the derivative of  $U$  with respect to  $s$  in equation (10) equal to zero and solving the resulting equation. In order to obtain an expression for the function in equation (10), an equation would

have to be fitted to the curve in figure 1. Making the substitution previously indicated in obtaining equation (10) would result in a complicated relationship. The work involved in solving the resulting equation for the optimum value of  $s$  would be considerably more than the work of obtaining figures 4 (a) and 4 (b) and picking the values of optimum  $s$  and  $t$  from these curves.

Plots of the type shown in figure 4 were obtained for other values of  $M$  and  $\Delta p_1 \rho_{as} / \rho_0$  by means of figures 1 and 3 and equations (1) and (7). For each value of  $s$  and  $t$  in figure 4, the associated value of  $w$  can be calculated from equation (7). The heat-transfer coefficient  $U$  can also be plotted against  $t$  for various values of  $w$  and the optimum value of  $t$  can be obtained for the maximum value of  $U$  for each value of specified  $w$ . In this case, the value of  $s$  is unrestricted and may be obtained from equation (7).

Figure 4 shows that, for given values of  $\Delta p_1 \rho_{as} / \rho_0$  and  $M$ , definite values of  $s$  and  $t$  exist for which  $U$  is a maximum. Although these values of  $s$  and  $t$  may be outside the practicable manufacturing range, a wide range of fins becomes available for values of  $U$  5 percent below the maximum. Figure 5 is a cross plot of figure 4,  $s$  having been plotted against  $t$  for several percentages of maximum  $U$ . It is evident from figure 5 that a single pair of values of  $s$  and  $t$  represents the optimum fin design as indicated by 100 percent  $U$ . In case the manufacture of these fins is impracticable because  $s$  and  $t$  are too small, some sacrifice in  $U$  must be made if the fin weight is to remain constant. For example, when



$U$  is decreased to 95 percent of the maximum value, an infinite number of pairs of values of  $s$  and  $t$  will give this heat transfer as shown by the points on the 95-percent line. When the minimum value of  $s$  is limited by manufacturing reasons, the points on line A give the maximum values of  $s$  and, if the value of  $t$  is limited, the points on line B give the maximum values of  $t$ . The shaded area between A and B is the only region of practical interest on these curves because the fins in this region give the indicated heat transfer with the highest values of  $s$  and  $t$ . The specified  $s$  curves in this report correspond to the values of  $s$  and  $t$  along the line A of graphs of the type shown in figure 5; the specified  $t$  curves correspond to the values of  $s$  and  $t$  along the line B of graphs of the type shown in figure 5. As is evident from figure 5, reasonably close approximations of values of  $s$  and  $t$  lying between the specified  $s$  line (A) and the specified  $t$  line (B) can be obtained by assuming a straight line between corresponding heat-transfer points on lines A and B. Similarly, in the charts presented later in this report, values of fin dimensions lying between the specified  $s$  and the specified  $t$  charts may be approximated by assuming a linear relationship between similar dimensions on each chart.

Very often the fin width is of more importance than the fin weight. In certain types of engine, such as in-line engines, the small distance between cylinders places a restriction on maximum fin width. From equations (6) and (8),  $U$  can evidently be written as a function involving width instead of weight.

$$U = f_{10}(s, t, w, D, \Delta p_{total}, \rho_a, \rho_o) \quad (11)$$

For a given cylinder diameter, fin width, and pressure drop,  $U$  is again evidently a function only of  $s$  and  $t$  and curves similar to those in figure 4 can be plotted. The curves of optimum fin proportions for specified  $s$  and  $t$  are obtained in a manner similar to that previously indicated for the case in which weight was the criterion and are shown later in the report. The specified  $s$  curves represent the best fins that fulfill the restrictions placed on  $s$  and  $w$ ; the specified  $t$  curves represent the best fins that fulfill the restrictions placed on  $t$  and  $w$ .

When the fin-design information is applied to engine cylinders, the fin proportions may be determined from an average value of the surface heat-transfer coefficient  $q$  for an entire cylinder circumference or may possibly be determined for each portion of the cylinder circumference from the local heat-transfer coefficients. As most aircraft-engine cylinders are composed of several cylindrical areas, it is believed to be most practicable in applying the fin-dimension information to consider each of these areas separately. The outside-wall surface of a conventional cylinder can thus be considered as five separate areas: The barrel, the lower head, the intake-valve stack, the exhaust-valve stack, and the curved surface between the intake-valve and the

exhaust-valve stacks. Further refinement that might be obtained by the consideration of smaller areas is believed unwarranted in view of the impracticability of changing fin sections and spacing from one point to another around a cylindrical surface.

In heat-transfer investigations, the heat-transfer coefficient is customarily based on the difference between the surface and the average fluid temperatures. The problem of determining fin proportions is, however, very much simplified when the coefficients are based on the intake cooling-air temperature. When the

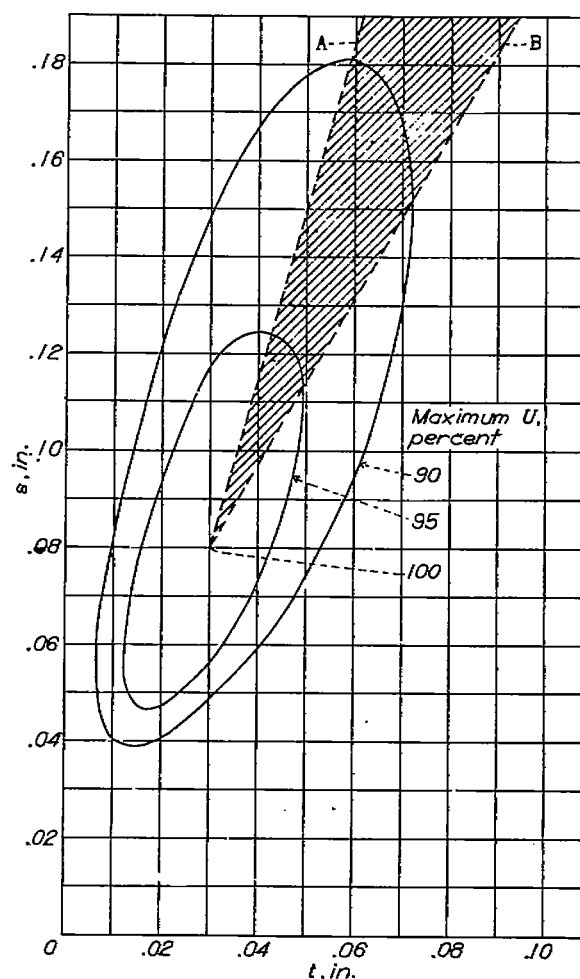


FIGURE 5.—Variation of  $s$  and  $t$  for several percentages of maximum heat-transfer coefficient. Constant  $M$  and  $\Delta p_{total}$ .

over-all heat-transfer coefficient  $U$  is calculated from the surface heat-transfer coefficient based on the average air temperature, it is necessary to determine the temperature rise of the air, which in turn depends upon the value of  $U$  being determined. In the present report, the over-all heat-transfer coefficients have therefore been based on the intake-air temperature.

Equations (10) and (11) show that  $U$  is a function of the cylinder diameter. In most of the calculations of this report, the length of the flow path was that for a 4.66-inch-diameter cylinder; this value is a representative average of the various diameters of the cylindrical

portions of a range of conventional aircraft-engine cylinders. The diameter of the valve stacks is usually much less than 4.66 inches whereas, except for small cylinders, the barrel and the head diameters are larger. In cases where the length of flow path is greatly different from that for a 4.66-inch-diameter cylinder, corrections can be made for differences in the temperature rise of the cooling air. Most attempts to obtain greater accuracy than can be obtained by using the data for the 4.66-inch diameter are unwarranted because calculations have shown that an appreciable change in flow-path length is required to effect much change in fin proportions although such a change in flow-path length will change the absolute values of  $U$ . In all calculations for determining  $g$ , the viscosity of the cooling air  $\mu$  was assumed to be  $130 \times 10^{-7}$  pounds per second per foot and the thermal conductivity  $k_a$  to be  $3.4 \times 10^{-7}$  Btu per inch per second per °F. The effect of variation of both  $\mu$  and  $k_a$  on the optimum fin proportions with the

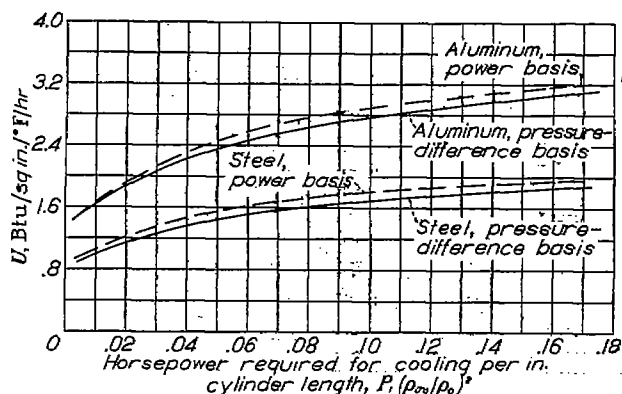


FIGURE 6.—Variation of maximum heat transfer with power required for cooling on pressure-difference and power bases. Fin volume, 0.45 cubic inch per square inch wall area; criterion, fin weight.

temperature range encountered is inappreciable, and the assumed values give results that are accurate enough for all practical purposes.

An exact determination of optimum fin proportions would require a different solution for every condition of air flow that might be caused by differences in baffle and cowling design. Furthermore, in order to cover the problem completely, it would also be necessary to determine fin proportions for variations of fin weight, fin dimensions, pressure drop, weight velocity, and power to cool. It has been necessary, in order to limit the scope of the present report, to choose for the final calculations a limited range of conditions believed to be of the greatest practical interest. The problem has been simplified, where possible, by eliminating several of the variables having little or no effect on the fin dimensions.

The determination of the most important of the foregoing variables will depend upon their application. For example, when the cooling-air flow through an engine is induced by the movement of the airplane through the air and the slipstream from the propeller,

the pressure difference available for forcing the air over the cylinders may be insufficient for cooling at the power output desired. In this case, it may be desirable to add fin weight to obtain sufficient cooling with the limited value of the pressure drop. When the cooling air is supplied by a blower, a wide range of pressures may be available and the power required for cooling may be equally as important as the pressure drop or the weight of the fins.

The determination of optimum fin dimensions for constant weight-velocity and power conditions is of interest only in special cases. If the cooling air is furnished by a blower and the power required for cooling is used as a criterion of fin design, the total blower power and not the power required to force the air across the cylinder should be used. The efficiency of a blower is particularly dependent on the pressure difference used and, if fins are designed for a constant cooling-air power, the pressure difference required may be such as to lie in a very inefficient part of the power curve of the blower.

The results of calculations to determine optimum fin dimensions for constant pressure drop, constant weight velocity, and constant power conditions show that, in general, the optimum fin space or thickness changes with the different bases. The desired values of  $s$ , when  $t$  is specified, are somewhat smaller for constant weight velocity and power than for constant pressure difference. When  $s$  is specified, the values of  $t$  are generally lowest for the constant weight-velocity condition and highest for the constant power condition.

Although the optimum  $s$  and  $t$  are somewhat different for the conditions of constant-pressure difference and constant power to cool  $P_1$ , the difference between the heat transfer obtained for a given power to cool and the heat transfer obtained for a given pressure drop is very slight, as is shown in figure 6. These curves were obtained by determining the optimum fin designs for several constant assumed powers and pressure differences. In these calculations, the fin weight was held constant for each metal. The slight difference in  $U$  shown by these curves makes the design of fins from a power-to-cool basis of little interest. An advantage of fins designed on a pressure-difference basis is that the optimum thickness and space are greater than for fins designed on a power-to-cool basis.

Optimum fin designs were also determined for three air-flow arrangements: Cylinders in a free air stream, with and without baffles, and cylinders at a 45° fin-plane/air-stream angle. The calculations were based on a constant fin weight and a constant air-stream velocity. These results show that differences in air flow caused by these air-flow arrangements do not materially affect the best fin dimensions.

From the foregoing results, it is believed that fin-design information for cylinders enclosed in a jacket will apply with reasonable accuracy to other conditions

of flow as caused by different baffle arrangements. As the power-to-cool and the weight-velocity bases are of interest only in special cases, the fin-design data submitted in this report have been calculated for constant values of pressure difference across a cylinder-jacket arrangement. Although the material of this report was derived from data for cylindrical barrels, the small effect of different air-flow characteristics on the optimum fin dimensions would appear to justify the application of the material of the report to complicated shapes such as cylinder heads.

The conditions covered in this report with fin weight as the basis of design are listed in table I and the conditions with fin width as the basis of design are listed in table II.

TABLE I.—CONDITIONS FOR WHICH CALCULATIONS WERE MADE USING FIN WEIGHT AS CRITERION

Fin material	Cylinder diameter at fin root (in.)	Fin volume (cu in. per sq in. wall area)	Fin weight (lb per sq in. wall area)	Pressure difference (in. of water)
Steel.....	4.66	0.04	0.0113	1, 4, 8, 12
		.20	.0564	
		.45	.1269	
Aluminum.....	4.66	1.20	.3384	1, 4, 8, 12
		0.04	0.0040	
		.20	.0202	
Copper.....	4.66	.45	.0455	4
		.12	.0387	
		.40	.1288	
Magnesium.....	4.66	0.04	0.0026	4
		.20	.0130	
		1.20	.0778	
		2.00	.1296	

TABLE II.—CONDITIONS FOR WHICH CALCULATIONS WERE MADE USING FIN WIDTH AS CRITERION

Fin material	Cylinder diameter at fin root (in.)	Fin width (in.)	Pressure difference (in. of water)
Steel.....	4.66	0.4	4
		.6	
		.8	
		1.0	
		1.5	
Aluminum.....	4.66	0.5	4
		1.0	
		1.5	
		2.0	
		2.5	

The values of the over-all heat-transfer coefficients  $U$  for the various conditions listed in these tables and from which curves such as shown in figure 4 were drawn have been tabulated in nine tables, which are available upon request from the National Advisory Committee for Aeronautics.

The values of the peaks of the curves of the type shown in figure 4 were used in plotting the final charts which are presented later in the report and in which both fin weight and fin width are used as criterions.

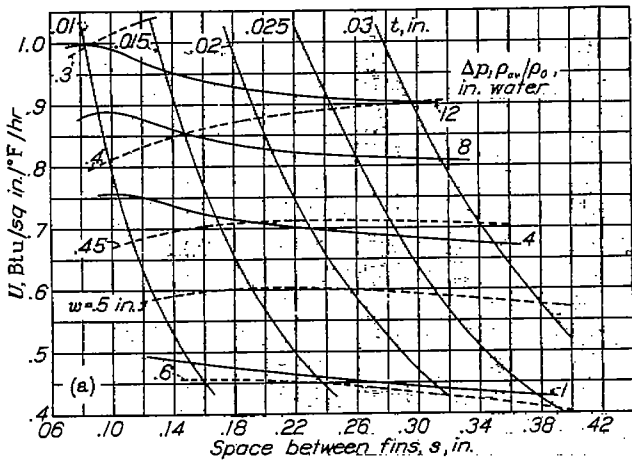
The peaks of some of the curves of the type shown in figure 4 are fairly flat; the values of  $s$  and  $t$  may consequently be varied somewhat without changing the heat transfer.

OPTIMUM FIN DESIGNS WITH LIMITED FIN WEIGHT SPECIFIED SPACE AND THICKNESS

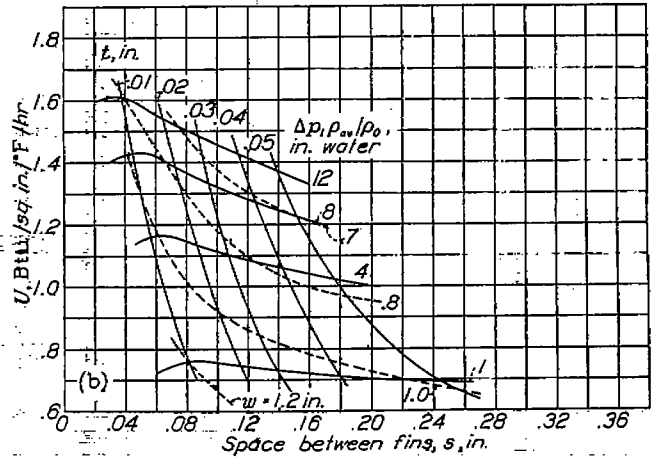
Figures 7, 8, 9, and 10 show the relation between the optimum fin dimensions and  $U$  when  $s$  or  $t$  is specified for both steel and aluminum with fin weight as the criterion. As each graph is for a constant weight of material, it is apparent that the peaks of the pressure-difference curves represent the fin designs that will give the maximum heat transfer for the given weight and pressure difference. The values of  $U$ ,  $s$ ,  $t$ , and  $w$  at the peak point are the same for both the specified  $s$  and the specified  $t$  charts.

Several characteristics of these graphs are of particular interest. The wide range over which both  $s$  and  $t$  may be varied without much change in  $U$  is very noticeable, especially for steel at low fin weights and low pressure differences. In general, the peak point of  $U$  occurs at smaller values of  $s$  and  $t$  as the pressure difference is increased. The fins indicated by the peak points, particularly for aluminum, are generally too thin for practical use. Although the value of  $s$  can be varied over quite a range without affecting maximum  $U$ , it may be desirable in some engine installations to limit  $s$  to small values in order to have a minimum volume of air passing through the engine cowling.

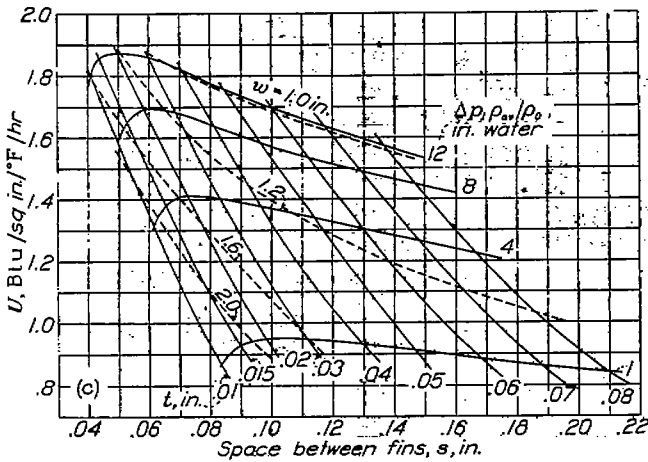
Information similar to that already presented for steel and aluminum fins is shown for copper and magnesium fins for a pressure difference of 4 inches of water in figures 11 to 14. Copper is of particular interest owing to its high thermal conductivity, and the use of magnesium is significant because of its low weight combined with fairly good thermal conductivity. A comparison of the proportions of fins of steel, magnesium, aluminum, and copper for maximum heat transfer shows that fins of metals having a high thermal conductivity are extremely thin. A comparison of the maximum heat transfer obtainable with steel, aluminum, copper, and magnesium is shown in figure 15 for different fin weights. These data were taken from the peak points of the curves of figures 7 to 14. Magnesium alloy of the thermal conductivity chosen has a slight advantage over the other metals; whereas copper and aluminum, although somewhat less effective than magnesium, are equally good, both being several times as effective as steel for a given fin weight. A plot similar to that of figure 15 could be made showing maximum  $U$  against width of fins as the criterion. Such a plot would show a definite advantage for copper with aluminum, magnesium, and steel following in the order of their relative effectiveness.



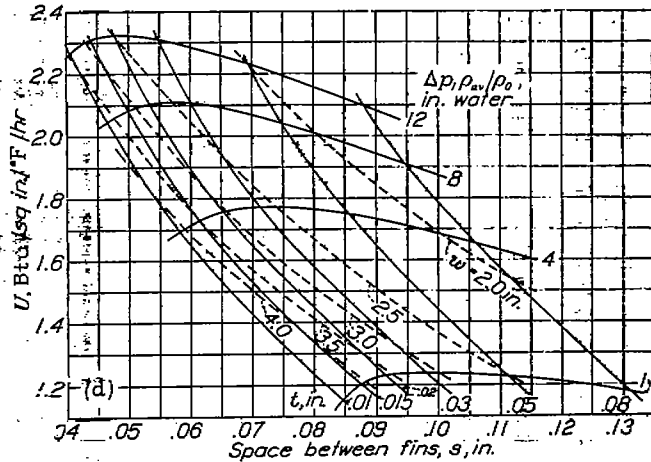
(a) Fin weight  $M$ , 0.0113 pound per square inch wall area.



(b) Fin weight  $M$ , 0.0564 pound per square inch wall area.

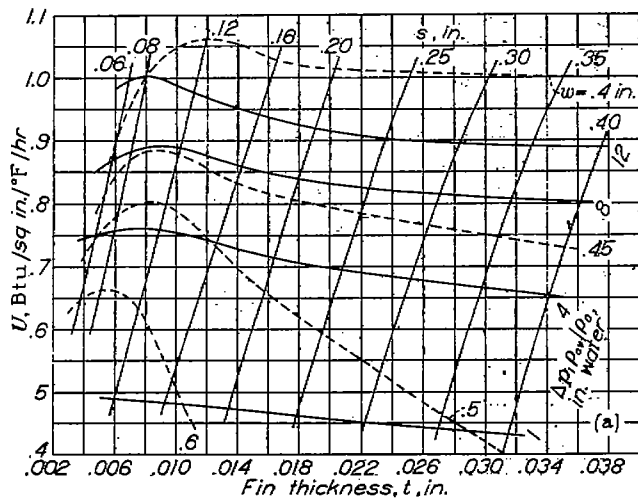


(c) Fin weight  $M$ , 0.1289 pound per square inch wall area.

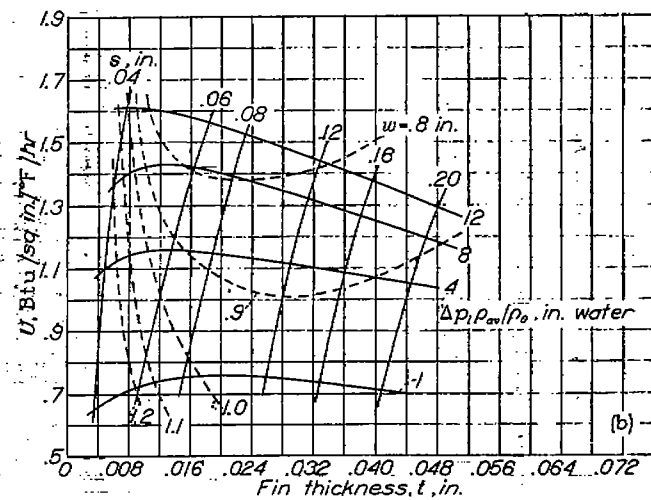


(d) Fin weight  $M$ , 0.3384 pound per square inch wall area.

FIGURE 7.—Optimum dimensions for steel fins with specified fin thickness. Criterion, fin weight.

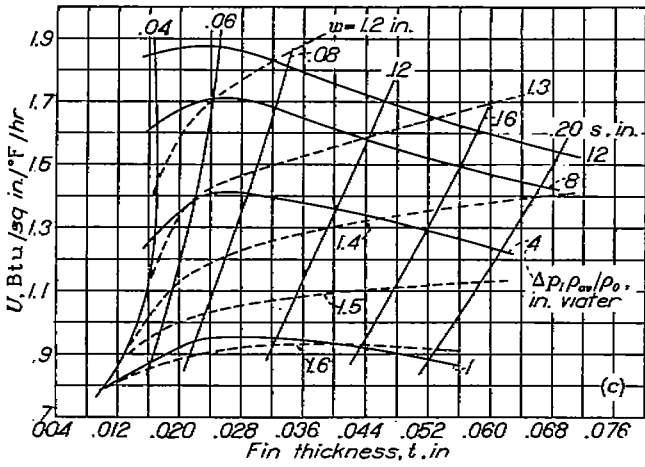


(a) Fin weight  $M$ , 0.0113 pound per square inch wall area.

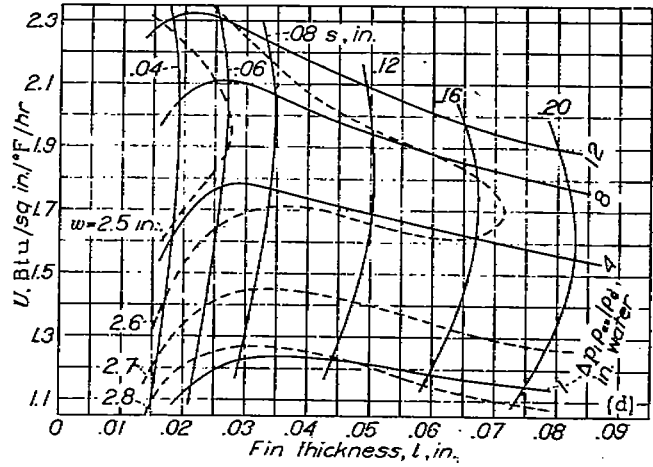


(b) Fin weight  $M$ , 0.0564 pound per square inch wall area.

FIGURE 8.—Optimum dimensions for steel fins with specified fin space. Criterion, fin weight.

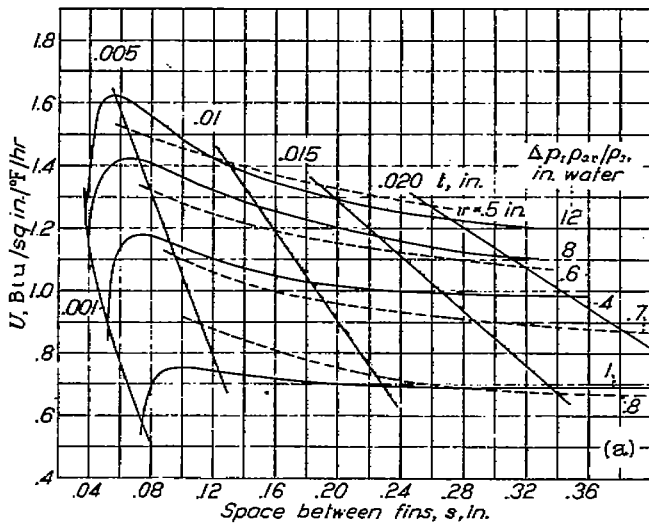


(c) Fin weight  $M$ , 0.1289 pound per square inch wall area.

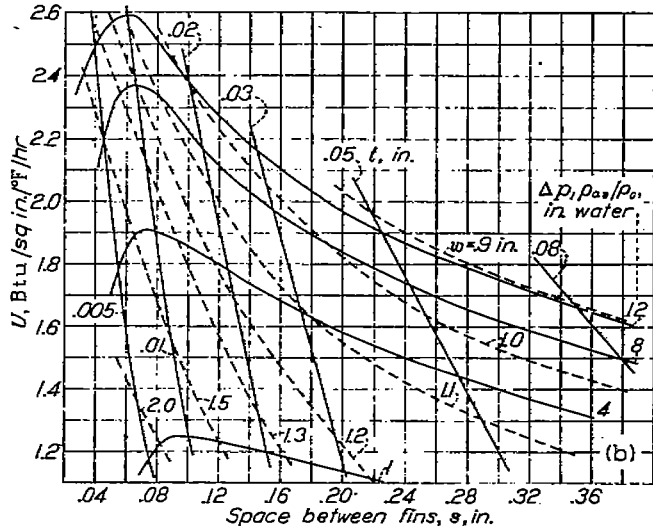


(d) Fin weight  $M$ , 0.3384 pound per square inch wall area.

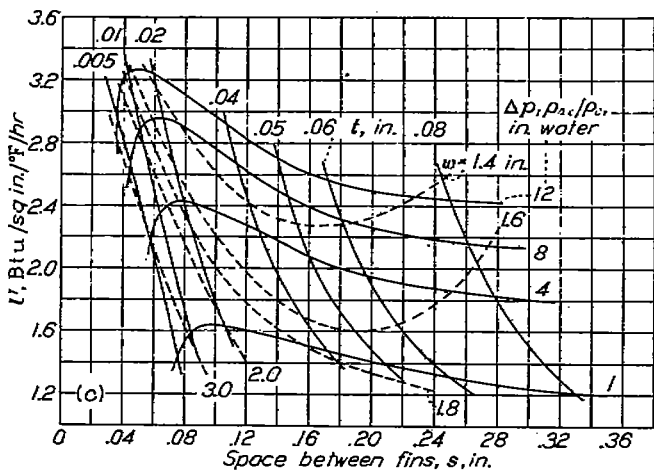
FIGURE 8.—Optimum dimensions for steel fins with specified fin space. Criterion, fin weight—Continued.



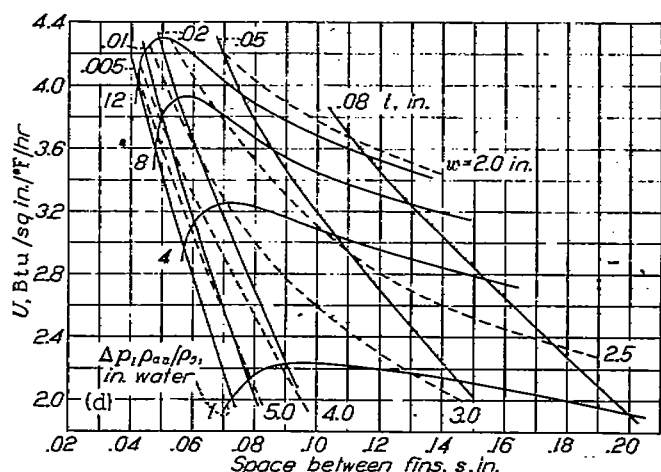
(a) Fin weight  $M$ , 0.0040 pound per square inch wall area.



(b) Fin weight  $M$ , 0.0202 pound per square inch wall area.

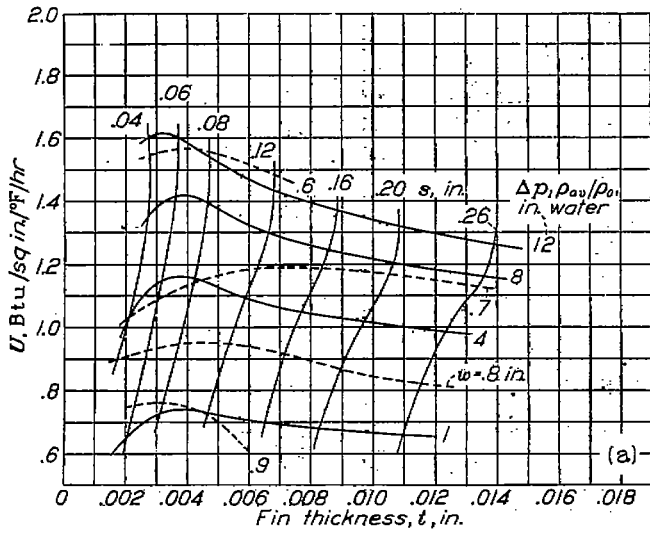


(c) Fin weight  $M$ , 0.0455 pound per square inch wall area.

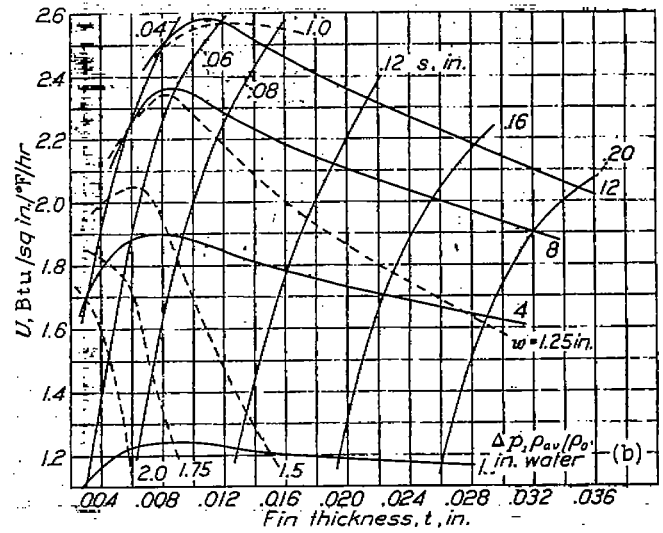


(d) Fin weight  $M$ , 0.1212 pound per square inch wall area.

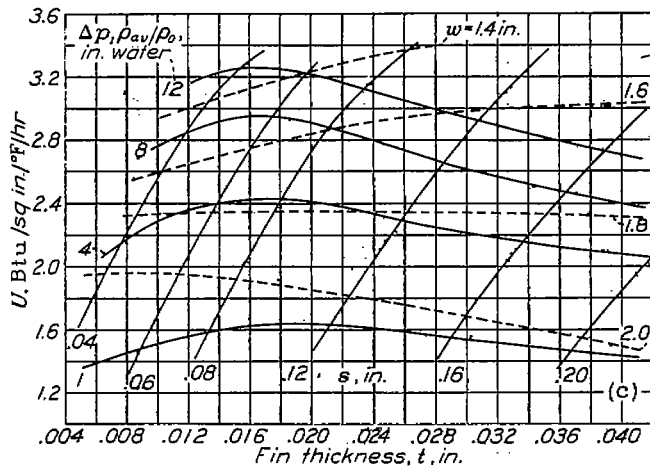
FIGURE 9.—Optimum dimensions for aluminum fins with specified fin thickness. Criterion, fin weight.



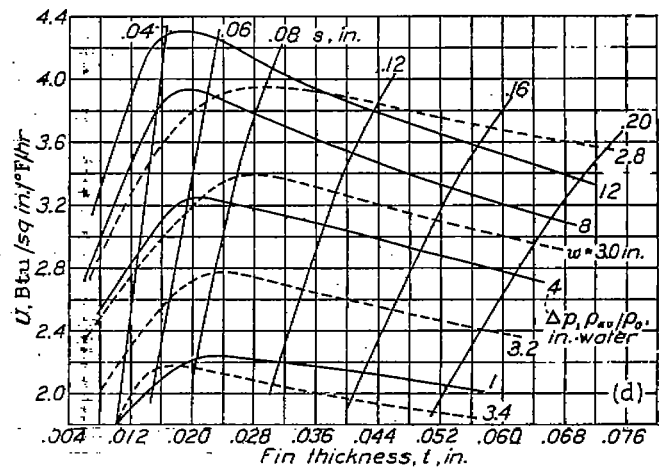
(a) Fin weight  $M$ , 0.0010 pound per square inch wall area.



(b) Fin weight  $M$ , 0.0302 pound per square inch wall area.



(c) Fin weight  $M$ , 0.0455 pound per square inch wall area.



(d) Fin weight  $M$ , 0.1212 pound per square inch wall area.

FIGURE 10.—Optimum dimensions for aluminum fins with specified fin space. Criterion, fin weight.

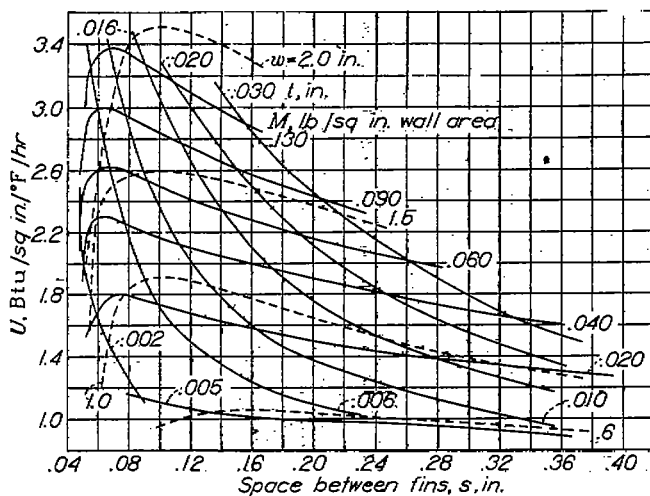


FIGURE 11.—Optimum dimensions for copper fins with specified fin thickness. Criterion, fin weight;  $\Delta p_1 \rho_{av} / \rho_o$ , 4 inches of water.

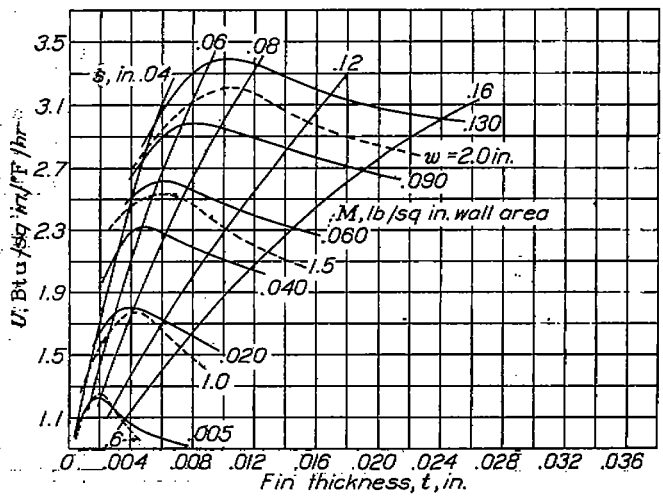


FIGURE 12.—Optimum dimensions for copper fins with specified fin space. Criterion, fin weight;  $\Delta p_1 \rho_{av} / \rho_o$ , 4 inches of water.

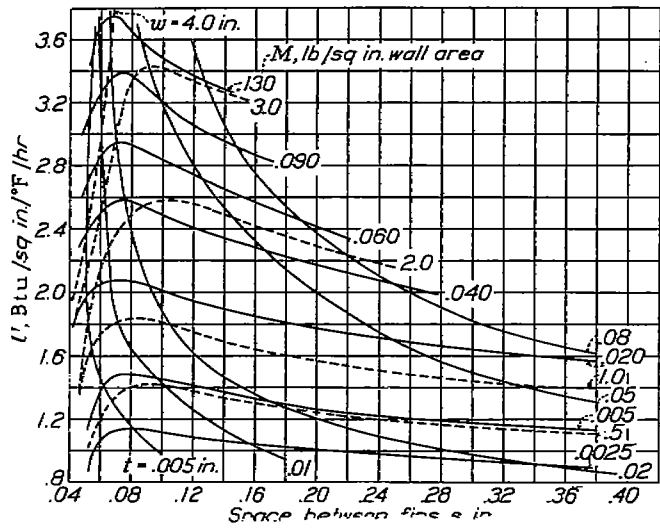


FIGURE 13.—Optimum dimensions for magnesium fins with specified fin thickness. Criterion, fin weight;  $\Delta p_{\rho_a/\rho_f}$  4 inches of water.

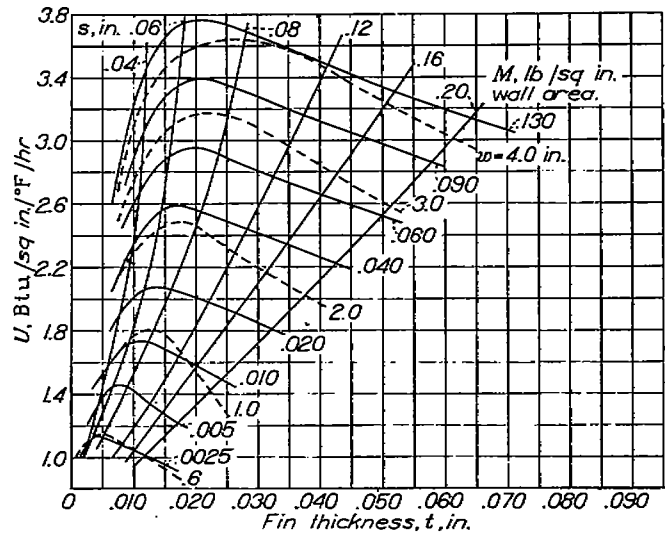


FIGURE 14.—Optimum dimensions for magnesium fins with specified fin space. Criterion, fin weight;  $\Delta p_{\rho_a/\rho_f}$  4 inches of water.

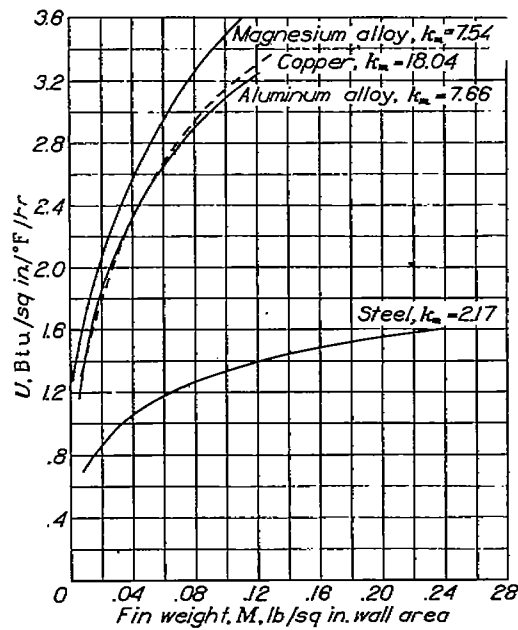
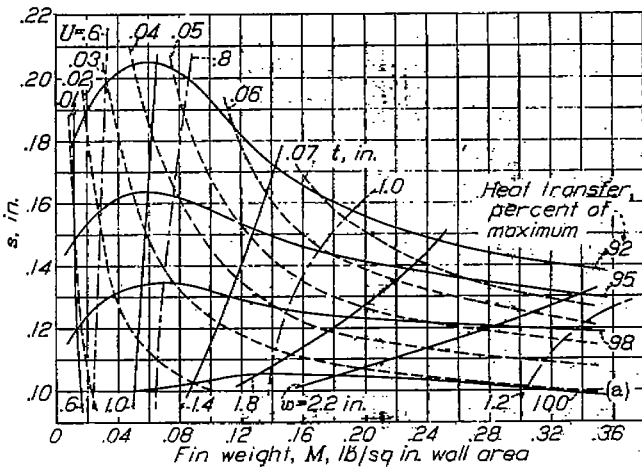
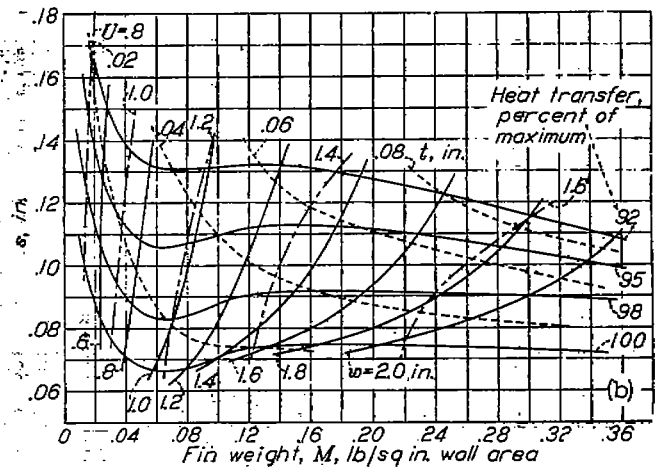


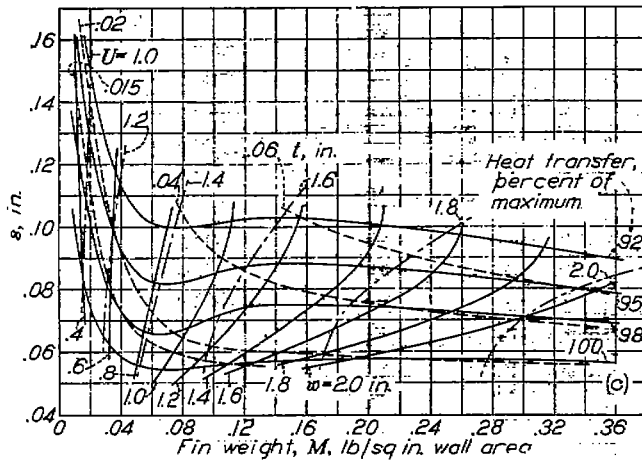
FIGURE 15.—Variation of maximum over-all heat-transfer coefficient with weight of fins for several fin materials. Criterion, fin weight;  $\Delta p_{\rho_a/\rho_f}$  4 inches of water.



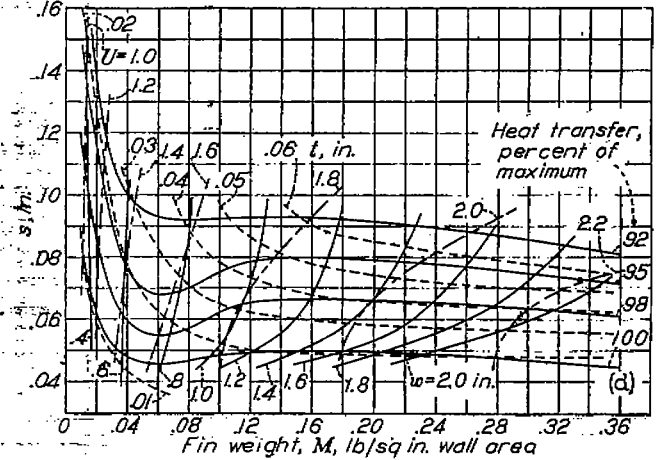
(a) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 1 inch of water.



(b) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 4 inches of water.

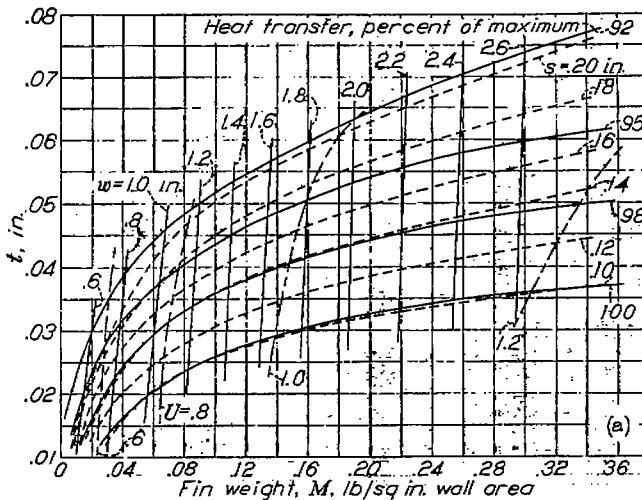


(c) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 8 inches of water.

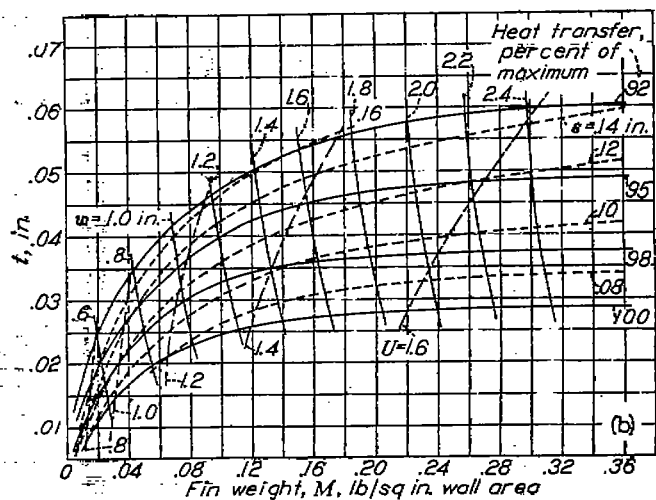


(d) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 12 inches of water.

FIGURE 16.—Optimum dimensions of steel fins for various percentages of maximum heat transfer. Specified fin thickness; criterion, fin weight.



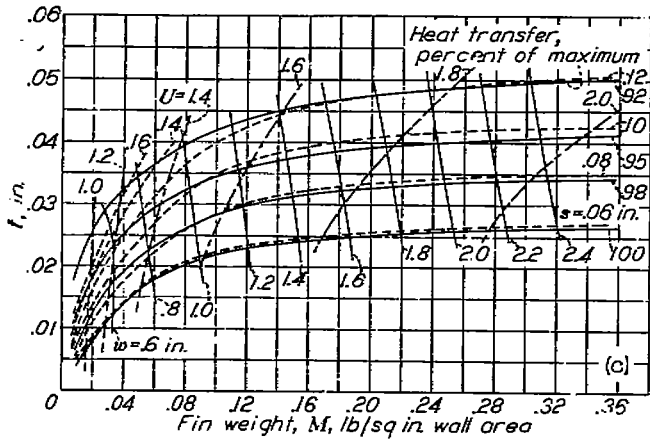
(a) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 1 inch of water.



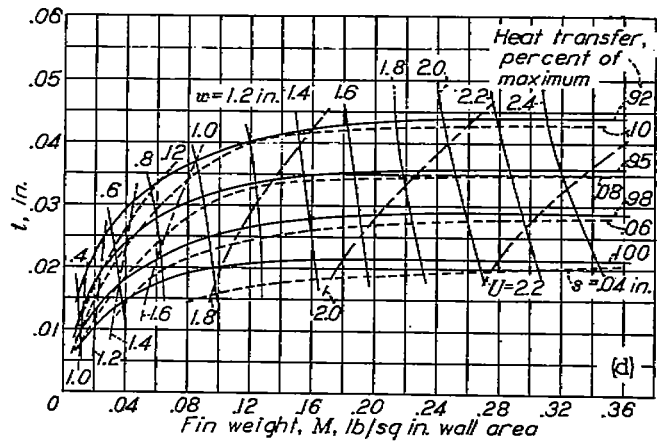
(b) Pressure difference  $\Delta p_{10} = \rho_0 v_0^2$ , 4 inches of water.

FIGURE 17.—Optimum dimensions of steel fins for various percentages of maximum heat transfer. Specified fin space; criterion, fin weight.



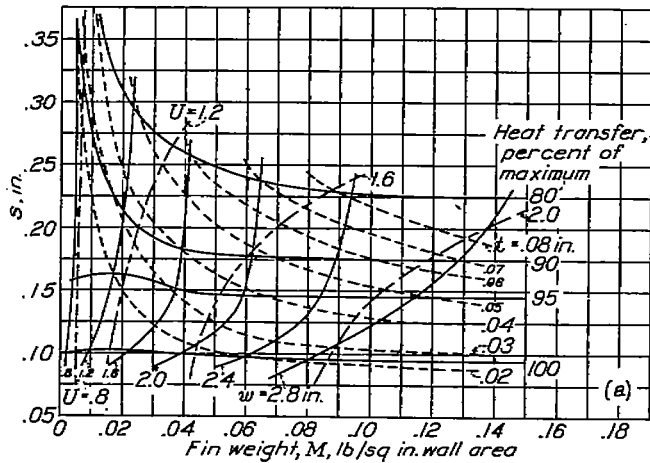


(c) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 8 inches of water.

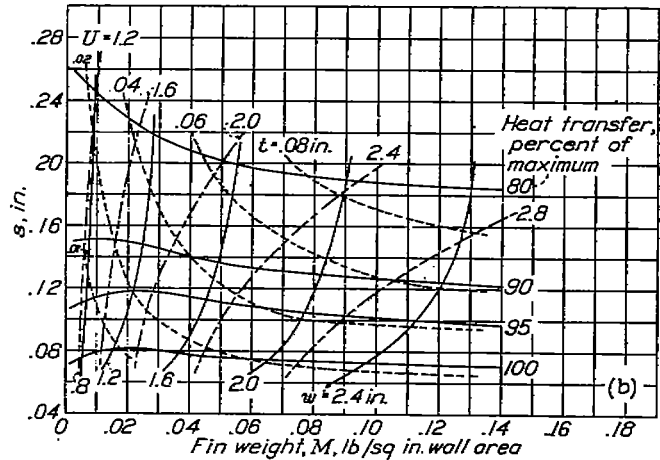


(d) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 12 inches of water.

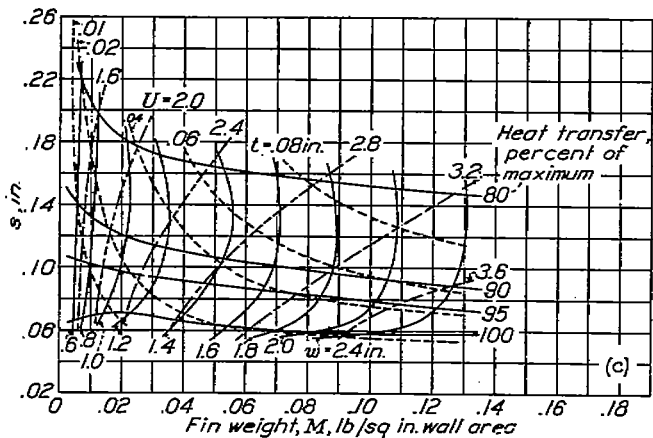
FIGURE 17.—Optimum dimensions of steel fins for various percentages of maximum heat transfer. Specified fin space; criterion, fin weight—Continued.



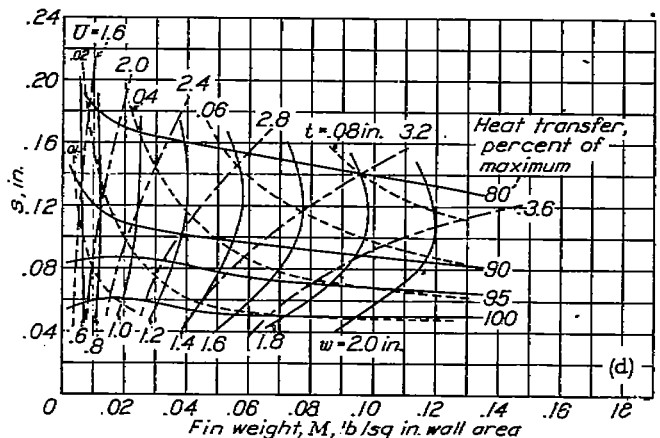
(a) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 1 inch of water.



(b) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 4 inches of water.

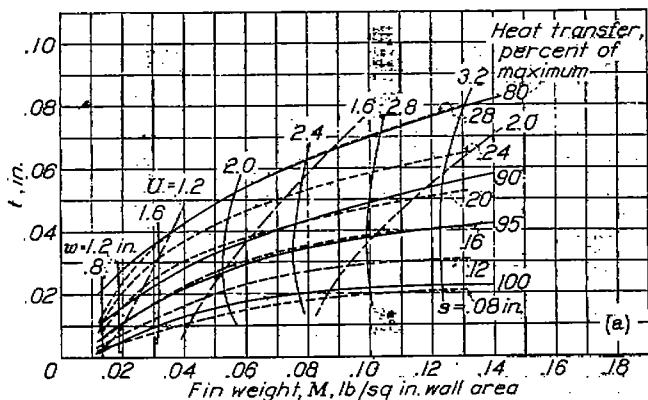


(c) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 8 inches of water.

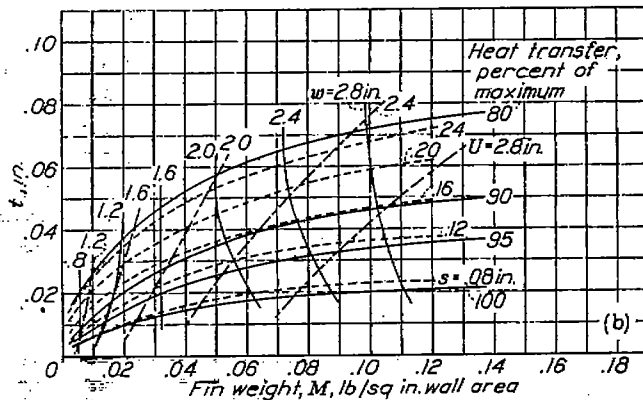


(d) Pressure difference  $\Delta p_{10} = p_1 - p_2$ , 12 inches of water.

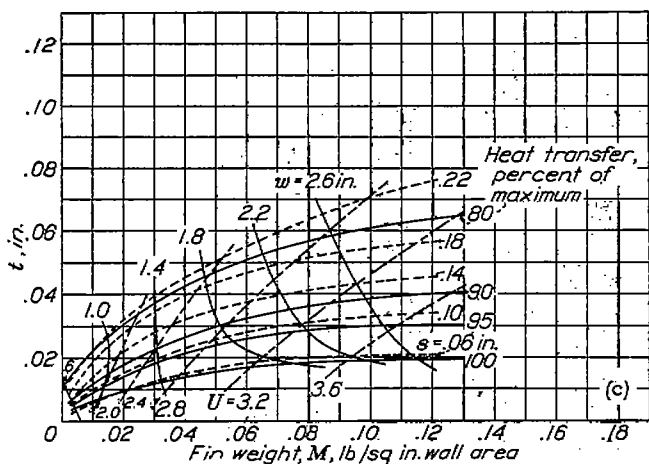
FIGURE 18.—Optimum dimensions of aluminum fins for various percentages of maximum heat transfer. Specified fin thickness; criterion, fin weight.



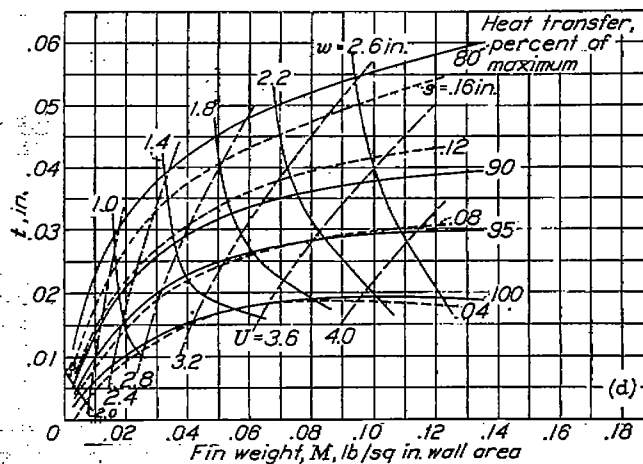
(a) Pressure difference  $\Delta p_1 p_2 / \rho_0$ , 1 inch of water.



(b) Pressure difference  $\Delta p_1 p_2 / \rho_0$ , 4 inches of water.



(c) Pressure difference  $\Delta p_1 p_2 / \rho_0$ , 8 inches of water.



(d) Pressure difference  $\Delta p_1 p_2 / \rho_0$ , 12 inches of water.

FIGURE 19.—Optimum dimensions of aluminum fins for various percentages of maximum heat transfer. Specified fin space; criterion, fin weight.

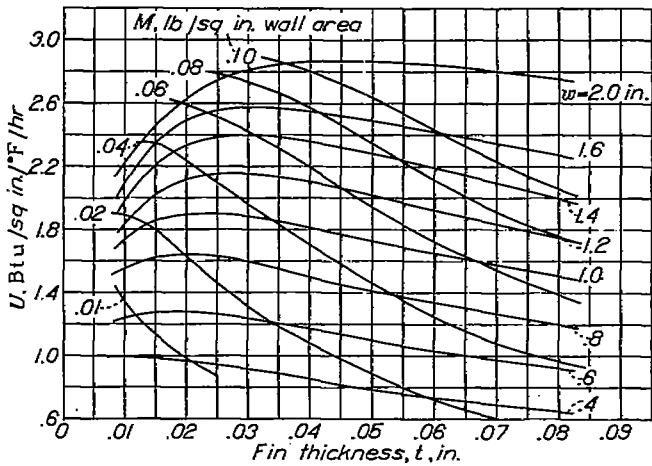
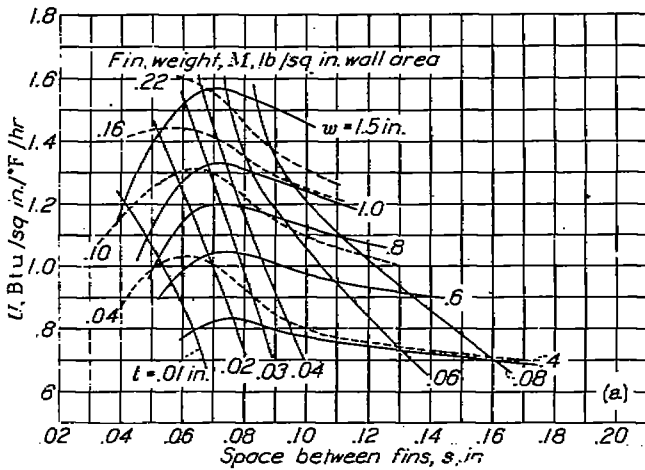
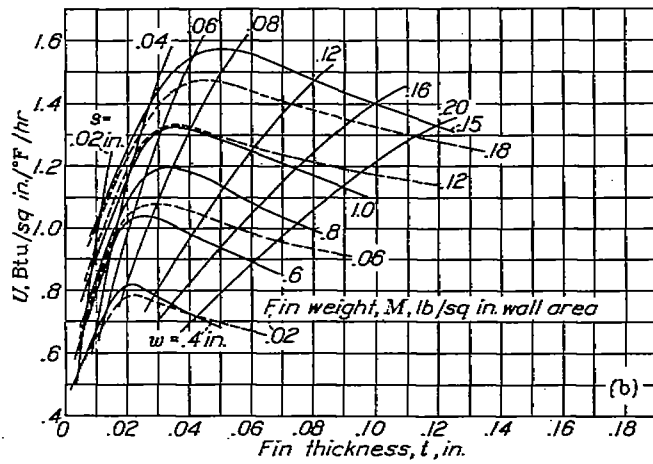


FIGURE 20.—Optimum dimensions for aluminum fins with specified fin width. Criterion, fin weight;  $\Delta p_{10} = \Delta p_0$ , 4 inches of water; optimum fin space constant at 0.072 inch.

The data in figures 7 to 10 have been cross plotted in figures 16 to 19 with the fin weight as abscissa. These plots show the fin dimensions for maximum  $U$  and also the fin dimensions when certain percentage reductions in maximum  $U$  are allowed in order to obtain easily constructed fins. The usefulness for design purposes of the data plotted in this manner will be shown later. Figures 16 and 18 show that, for a given pressure difference, the optimum spacing remains practically constant for maximum heat transfer over a large range of fin weights. The same is true for the optimum thickness as shown in figures 17 and 19 at the higher pressure differences. The optimum spacing for the maximum heat transfer at a given pressure difference is approximately the same for steel and aluminum over a large range of fin weights.

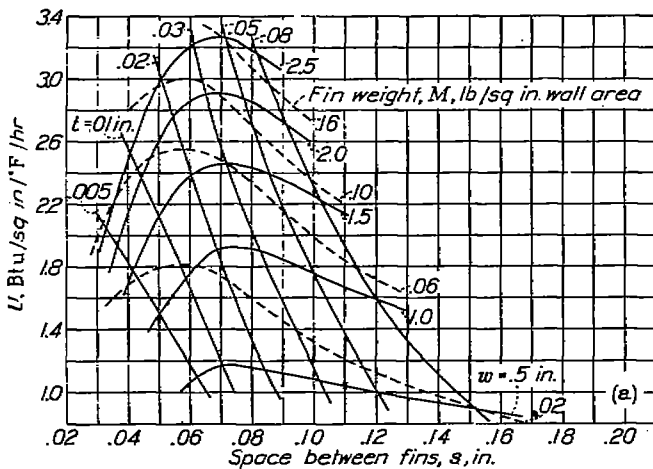


(a) Specified  $t$  chart.

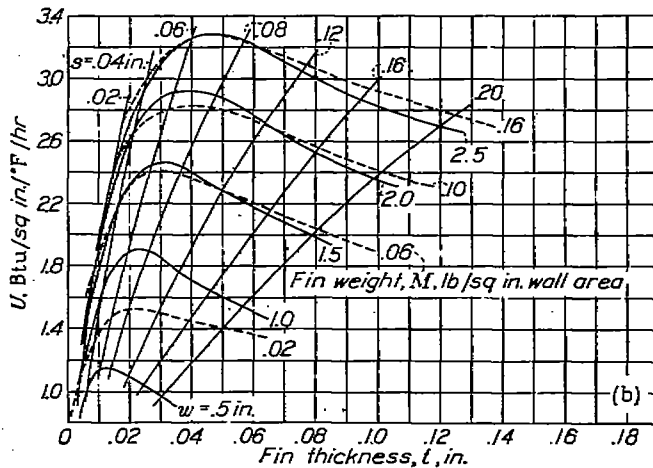


(b) Specified  $s$  chart.

FIGURE 21.—Optimum dimensions for steel fins. Criterion, fin width;  $\Delta p_{10} = \Delta p_0$ , 4 inches of water.



(a) Specified  $t$  chart.



(b) Specified  $s$  chart.

FIGURE 22.—Optimum dimensions for aluminum fins. Criterion, fin width;  $\Delta p_{10} = \Delta p_0$ , 4 inches of water.

## SPECIFIED WIDTH

In certain cases of fin design, limiting values of the fin width  $w$  will be more important than limiting values of  $s$  or  $t$ . One chart for aluminum fins in which  $w$  is specified and in which fin weight is the criterion is shown in figure 20. In the calculations for figure 20,  $U$  was found to be a maximum when  $s$  was 0.072 inch regardless of fin width or fin weight. This figure also shows that, for every fin width, only one fin thickness and fin weight give a maximum heat transfer.

## OPTIMUM FIN DESIGNS WITH LIMITED FIN WIDTH

For certain cases in which fin width is limited, it will be desirable to obtain a maximum heat transfer irrespective of fin weight. The fins for the adjacent surfaces of the cylinder heads of in-line engines are an example of this type. In this case, an addition of fin width may cause a corresponding increase in the length of the engine.

Curves of maximum  $U$  when fin width is limited are shown in figures 21 and 22 for steel and aluminum, respectively. In these figures, the fin spacing corresponding to the peak point of each width of curve slightly decreases as the fin width increases. The optimum spacing for both steel and aluminum is approximately 0.07 inch for the range of fin widths shown. The fin dimensions at the peak points of each constant-width curve are the same whether fin space or fin thickness is specified.

When fins of different metals are compared on the basis of width as the criterion, metals of high thermal conductivity are obviously superior. For this reason, copper should prove of definite advantage in applications where  $w$  is limited.

## APPLICATION OF RESULTS

The following examples are intended to illustrate not only the use of the material of the report but also possible improvements in fin design. For simplicity in the solution of these examples, it will be assumed that the total heat from the cylinder changes inappreciably with change in cylinder temperature.

Two methods of designing fins for a new engine cylinder are possible. One method consists in obtaining the ratio of the heat-transfer coefficient required to cool the new cylinder to the heat-transfer coefficient of an existing cylinder from a consideration of relative power and size of the cylinders and then in obtaining a fin design that gives this ratio of the heat-transfer coefficients for fins located at similar positions on the two cylinders. The heat-transfer coefficients of the fins on both cylinders can thus be determined from the data given in this report.

The second method consists in estimating the quantity of heat to be dissipated and in using the heat-transfer coefficients given in this report for obtaining

the fin dimensions. As the fins were tested under somewhat different conditions of air flow than may exist in flight and, furthermore, as the estimation of the heat to be dissipated is rather indeterminate, the accuracy of the second method is questionable. In the first method, however, differences in flow conditions should not appreciably change the relative heat-transfer coefficient of different fins when both coefficients are used under the same conditions. The first method is therefore believed to be more reliable.

**Example 1.**—Let it be required to lower the wall temperature of an aluminum-alloy, cylindrical surface having  $s = 0.142$  inch,  $t = 0.08$  inch,  $w = 0.6$  inch, and  $D = 7.0$  inches from  $480^\circ$  F to  $380^\circ$  F, assuming a specific weight of the air  $\rho_{at}g$  of 0.0734 pound per cubic foot, an air temperature of  $80^\circ$  F, and a pressure difference,  $\Delta p_1$ , of 4 inches of water. Let it also be assumed that both minimum fin weight and narrow fin width are desirable and that, for manufacturing reasons,  $s$  and  $t$  shall not be less than 0.08 inch and 0.03 inch, respectively. The final choice of fin dimensions will be made upon inspection of the several resulting fin designs.

As previously stated, the graphs of this report are for a  $D$  of 4.66 inches. Other diameters will affect  $U$  but will not materially affect the fin dimensions. Any change in  $U$  effected by changing fin dimensions for either of two different diameters will cause a proportional change in  $U$  for either diameter; this fact will be demonstrated in the present example.

The over-all heat-transfer coefficient for the original cylinder is obtained from equation (1) as follows: From figure 1,  $qs/k_a$  can be determined from

$$qs/k_a = f_2 \left( \frac{V \rho_1 g s^2}{12 \mu D^{1/4}} \right)$$

From figure 3 at  $s = 0.142$  inch,  $V \rho_1 g = 7.6$  pounds per second per square foot.

$$\frac{V \rho_1 g s^2}{12 \mu D^{1/4}} = \frac{7.6 \times 0.142^2}{12 \times 130 \times 10^{-7} \times 4.66^{1/4}} = 670$$

From figure 1,  $qs/k_a = 63,200$

$$q = \frac{63200 \times 0.00000034}{0.142} = 0.1512 \text{ Btu per square}$$

inch per  $^\circ$ F per hour

$$w' = w + \frac{t}{2} = 0.6 + 0.04 = 0.64 \text{ inch}$$

$$a = \sqrt{2q/k_m t} = \sqrt{(2 \times 0.1512) / (7.66 \times 0.08)} = 0.703$$

$\tanh aw' = 0.422$  (See reference 1, fig. 15.)

$$\begin{aligned} U &= \frac{q}{s+t} \left[ \frac{2}{a} \left( 1 + \frac{w}{2R_b} \right) \tanh aw' + s \right] \quad (1) \\ &= \frac{0.1512}{0.142 + 0.08} \left[ \frac{2 \times 0.422}{0.703} \left( 1 + \frac{0.6}{4.66} \right) + 0.142 \right] \\ &= 1.03 \text{ Btu per square inch per } ^\circ\text{F per hour} \end{aligned}$$

A reduction of temperature from 480° F to 380° F requires

$$U_2 = U_1 \frac{\Delta t_1}{\Delta t_2} = 1.03 \left( \frac{480 - 80}{380 - 80} \right) = 1.37$$

The fin dimensions as determined for a  $U$  of 1.37 Btu per square inch per °F per hour, minimum  $s$  of 0.08 inch, and minimum  $t$  of 0.03 inch are as follows:

Solution	Chart	Criterion	$w$ (in.)	$s$ (in.)	$t$ (in.)	$M$ (lb/sq in.)	$U$ (Btu/sq in./°F/hr)
1	Specified $t$ (fig. 18 (b))	Weight	0.90	0.22	0.03	0.014	1.37
2	Specified $s$ (fig. 19 (b))	do.	1.10	.24	.03	.015	1.37
3	Specified $w$ (fig. 20)	do.	.68	.072	.03	.022	1.37
4	Specified $t$ (fig. 22 (a))	Width	.70	.095	.03	.020	1.37
5	Specified $s$ (fig. 22 (b))	do.	.70	.137	.03	.015	1.37
	Original cylinder		.60	.142	.08	.025	1.03

From the foregoing table, the designer can pick the combination of fin dimensions that will best suit his particular requirements. When the values for this table were calculated, solutions with minimum  $s=0.08$  inch and minimum  $t=0.03$  inch were not obtainable from all charts. It was felt, however, that the inclusion of the next best solution possible in these cases would be of interest.

The percentage increase in  $U$  obtained can now be compared with the corresponding increase in  $U$  had a diameter of 7 inches been used:

$U$  for the original cylinder with a  $D$  of 7 inches = 0.928 Btu per square inch per °F per hour.

$U$  for solution 4 for a  $D$  of 7 inches = 1.28 Btu per square inch per °F per hour.

If all fins are now assumed to be on a 7-inch-diameter cylinder, the increase in  $U$  for the fin design of solution 4 over the original design is 38 percent. The corresponding increase in  $U$  for solution 5 is 32 percent. These values check with fair accuracy the increase of 33 percent obtained for a diameter of 4.66 inches.

**Example 2.**—Let it be required to determine whether the fin design of solution 4 of example 1 will be adequate for cooling at an altitude of 23,000 feet, if the same total heat is assumed to be dissipated with a cylinder-wall temperature of 380° F and a pressure difference  $\Delta p_1$  of 4 inches of water.

At 23,000 feet, the cooling-air temperature is -23° F, and  $\rho_1 g$  is 0.0368 pound per cubic foot. From the foregoing,

$$U_{altitude} = \frac{1.28(380 - 80)}{(380 + 23)} = 0.953 \text{ Btu per square inch per °F per hour}$$

The weight velocity between the fins is proportional to  $\Delta p_1 \rho_1 g$ . When  $\rho_1 g = 0.0734$  pound per cubic foot,

$$\Delta p_1 = 4 \times \frac{0.0368}{0.0734} = 2 \text{ inches of water}$$

From figure 3,  $V \rho_1 g = 4.1$  pounds per second per square foot when  $\Delta p_1 = 2$  inches of water,  $\rho_1 g = 0.0734$  pound per cubic foot, and  $s = 0.095$  inch. The value of  $U$  as determined by equation (1), as in example 1, is 0.973 Btu per square inch per °F per hour, which is greater than the  $U$  required and therefore the fin design is satisfactory. If the calculated  $U$  had been less than that required (0.953), a new fin design would have been necessary.

**Example 3.**—Let it be required to determine how much the power of a cylinder having the fin dimensions of the original cylinder of example 1 with a wall temperature of 480° F can be increased without exceeding a wall temperature of 380° F by substituting a new fin design having a value of  $s$  not less than 0.14 inch, of  $t$  not less than 0.08 inch, and of  $w$  not greater than 1.5 inches. Let the cooling-air temperature and the pressure difference available for cooling be the same as in example 1. The possible solutions from the data of this report, which are for a cylinder diameter of 4.66 inches, are:

Solution	Chart	Criterion	$w$ (in.)	$s$ (in.)	$t$ (in.)	$M$ (lb/sq in.)	$U$ (Btu/sq in./°F/hr)
1	Specified $t$ (fig. 18 (b))	Weight	1.5	0.26	0.08	0.055	1.80
2	Specified $s$ (fig. 19 (b))	do.	1.5	.14	.025	.029	1.90
3	Specified $w$ (fig. 20)	do.	1.5	.072	.08	.107	2.13
4	Specified $t$ (fig. 22 (a))	Width	1.5	.14	.12	.10	1.81
5	Specified $s$ (fig. 22 (b))	do.	1.5	.20	.08	.059	1.08

For solution 5,  $U$  is equal to 2.104 Btu per square inch per °F per hour for a  $D$  of 7 inches.

The following equation, which expresses the power in terms of the cylinder temperature and  $U$ , can be derived from reference 8.

$$\frac{I_2}{I_1} = \frac{U_2 \left( \frac{T_{h_2} - T_{a_1}}{T_g - T_{h_2}} \right)^{\frac{1}{n'}}}{U_1 \left( \frac{T_{h_1} - T_{a_1}}{T_g - T_{h_1}} \right)^{\frac{1}{n'}}} = \frac{2.104 \left( \frac{380 - 80}{1150 - 380} \right)^{1.64}}{0.928 \left( \frac{480 - 80}{1150 - 480} \right)^{1.64}} = 1.83$$

where

- $I$  indicated horsepower
- $T_h$  average temperature over cylinder-wall surface, °F
- $T_{a_1}$  inlet temperature of cooling air, °F
- $T_g$  effective gas temperature inside cylinder, °F
- $n'$  an exponent

These calculations indicate that the new fin design should permit an increase in indicated power output of

almost twice that of the original value before the specified temperature limit of 380° F is attained.

Example 4.—The effect on  $U$  of decreasing the value of  $s$  and  $t$  used in example 3 to minimum values of 0.07

inch and 0.035 inch, respectively, and of a maximum value of  $w=2$  inches will be illustrated. The six possible solutions available from the data of this report are:

Solution	Chart	Criterion	$w$ (in.)	$s$ (in.)	$t$ (in.)	$M$ (lb/sq in.)	$U$ (Btu/sq in. °F/hr)
1.....	Specified $t$ (fig. 18 (b)).....	Weight.....	2	0.095	0.035	0.074	2.68
2.....	Specified $s$ (fig. 19 (b)).....	do.....	2	.145	.035	.054	2.30
3.....	Specified $w$ (fig. 20).....	do.....	2	.072	.035	.100	2.85
4.....	Specified $w$ (fig. 20).....	do.....	2	.072	.045	.120	2.87
5.....	Specified $t$ (fig. 22 (a)).....	Width.....	2	.070	.038	.105	2.91
6.....	Specified $s$ (fig. 22 (b)).....	do.....	2	.070	.041	.110	2.92

If solution 4 is taken as the accepted design, the increase in  $U$  over the original design ( $U=1.98$ ) is 45 percent.

Example 5.—Let it be required to determine how much the heat transfer of the steel barrel of a cylinder having an  $s$  of 0.115 inch, a  $t$  of 0.026 inch, a  $w$  of 0.5 inch, and a diameter of 4.66 inches can be increased when the limiting dimensions are: Minimum  $s$ , 0.07

inch; minimum  $t$ , 0.03 inch; and maximum  $w$ , 1.2 inches.

With a  $\Delta p_1 \rho_{av} / \rho_0$  of 4 inches of water,  $U$  is 1.08 Btu per square inch per °F per hour for the original cylinder. Only four solutions are possible from the graphs of this report because the curve for steel with a specified  $w$  and with weight as the criterion has been omitted for the sake of brevity.

Solution	Chart	Criterion	$w$ (in.)	$s$ (in.)	$t$ (in.)	$M$ (lb/sq in.)	$U$ (Btu/sq in. °F/hr)
1.....	Specified $t$ (fig. 16 (b)).....	Weight.....	1.2	0.089	0.03	0.107	1.31
2.....	Specified $s$ (fig. 17 (b)).....	do.....	1.2	.095	.03	.102	1.33
3.....	Specified $t$ (fig. 21 (a)).....	Width.....	1.2	.070	.038	.155	1.40
4.....	Specified $s$ (fig. 21 (b)).....	do.....	1.2	.070	.041	.160	1.43

Solution 4 gives a 32-percent increase of  $U$  over the original cylinder. This increase of  $U$  is, however, obtained at the expense of an increase of fin weight of 460 percent.

The foregoing examples illustrate methods of improving the fin design of a given cylinder. Another problem, as has been noted, is the determination of fin dimensions for a new cylinder design. For practical purposes, the solution of such a problem may be determined as follows.

From reference 8 it can be shown that

$$\frac{U_a}{U_b} = \left( \frac{a_{1a}}{a_{1b}} \right) \left( \frac{a_{0b}}{a_{0a}} \right) \left[ \frac{(I/v)_a}{(I/v)_b} \right]^{n'} \frac{(T_g - T_h)_a (T_h - T_{a1})_b}{(T_g - T_h)_b (T_h - T_{a1})_a}$$

where subscript  $a$  denotes one cylinder; subscript  $b$  denotes another cylinder;  $a_1$ , inside cylinder-wall area;  $a_0$  outside cylinder-wall area;  $T_b$ ,  $T_{a1}$ ,  $T_g$ ,  $I$ , and  $n'$  have been previously defined in example 3; and  $v$  is displacement volume. For simplicity, it will be assumed as in the foregoing examples that the total heat from the cylinder changes inappreciably with change in cylinder temperature and, furthermore, that the ratio of  $a_1/a_0$  is 1, which is justified except for thick-wall cylinders. Then

$$\frac{U_a}{U_b} = \left[ \frac{(I/v)_a}{(I/v)_b} \right]^{n'} \frac{(T_h - T_{a1})_b}{(T_h - T_{a1})_a}$$

From the pressure difference available for cooling,  $U_a$  can be determined from the fin dimensions for an existing cylinder from the material of the present report. The foregoing equation can then be solved for  $U_b$  from

known values of  $(T_b - T_{a1})_a$  and  $(I/v)_a$  at the pressure difference available and from required values of  $(T_b - T_{a1})_b$  and  $(I/v)_b$ . The determination of fin proportions for obtaining the desired heat transfer for the new cylinder  $U_b$  is similar to that for the other examples presented.

### INCREASING THE COOLING BY USING HIGH AIR VELOCITIES

In the foregoing examples, improvements in heat transfer have been made by increasing the effective fin

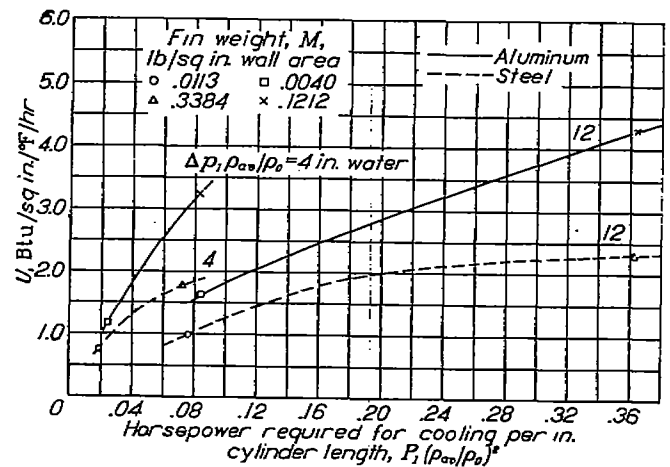


FIGURE 23.—Variation of maximum heat transfer with power required for cooling. Criterion, fin weight.

area. Corresponding increases can be made by using higher air velocities. In references 6 and 9, however, it has been shown that, from considerations of power

required for cooling  $P_1$ , the method of increasing the fin area is greatly superior to the method of using higher air velocities. Figure 23 has been prepared to illustrate this same point for optimum fin designs. The power required for cooling was calculated for the fin designs giving maximum heat transfer for several fin weights from figures 16 (b), 16 (d), 18 (b), and 18 (d) and is plotted in figure 23. For a given  $U$ , the power required is in some cases three times as great for a pressure difference of 12 inches of water as it is for a pressure difference of 4 inches of water for both steel and aluminum. The fin weight corresponding to any value of  $U$  can be obtained from the original figures. From considerations of the power required for cooling, it is thus apparent that, in order to increase the heat transfer, a greater effective fin area should be used in preference to increasing the air velocity. The problem of determining how much the fin weight should be increased in order to decrease the power required for cooling depends upon the particular engine-airplane combination involved.

#### INCREASING THE COOLING BY USING SHORT FLOW PATHS

In the application of closely spaced fins, a definite advantage has been noted (reference 4) in making flow paths as short as practicable. Short flow paths increase the heat transfer because of the lower air-temperature rise and the higher weight velocities of the cooling air for the same pressure difference as long flow paths. It has been noted in the present report that, for the range of cylinder diameters and fin widths used on conventional aircraft-engine cylinders, the flow path does not change enough to affect appreciably optimum fin proportions. For very short flow paths, however, the optimum fin spacing decreases as the flow path decreases, as has been noted in reference 10.

Calculations have been made to compare the optimum fin spacing obtained with aluminum cylinders having a flow-path length of approximately 8 inches with the optimum fin spacing obtained for cylinders having a flow-path length of 1 inch. In both cases, the pressure difference assumed was 1 inch of water. The corresponding weight velocities were obtained from figure 3 for the long-path cylinder ( $\Delta p_{1, \rho_{air}}/\rho_0=1$ ) and from figure 2 for the short-path cylinder [ $(\Delta p_{1, \rho_{air}}/\rho_0)/l=1$ ]. The over-all heat-transfer coefficients for the short-path cylinder were calculated from the values of the surface heat-transfer coefficients at the front of the cylinders tested in the work reported in reference 5. The fin weight was taken as the criterion in these calculations and a weight of 0.0455 pound per square inch of wall area was used for both cases.

The following table gives the optimum spacings and over-all heat-transfer coefficients for the long and the short paths for the several thicknesses assumed.

Specified $t$ (in.)	Optimum spacing (in.)		$U$ (Btu/sq in./°F/hr)	
	Short path	Long path	Short path	Long path
0.005	0.087	0.076	3.66	1.44
.01	.041	.083	3.88	1.69
.02	.056	.105	3.99	1.82
.05	.110	.210	3.25	1.39
.08	.140	.300	2.76	1.19

The foregoing table shows that the decrease in the length of path from 8 inches to 1 inch reduces the optimum spacing to approximately one-half its original value and increases the heat transfer a little more than twice its original value. It is thus apparent that short paths are advantageous and that the optimum fin dimensions are appreciably different for extreme differences in the length of the flow path. The difficulties in the breaking up of a long flow path into more than two paths in parallel presents some practical objections.

#### CONCLUSIONS

The charts presented in this report indicate that:

1. The fin spacing and the fin thickness for maximum heat transfer at a given pressure difference are practically constant for a large range of fin weights, with the spacing increasing and the thickness decreasing at very low fin weights.
2. The optimum fin spacing and thickness decrease slightly with increase of the pressure difference.
3. For a given fin weight, the highest heat transfer can be obtained with fins of a magnesium alloy. In this respect, pure copper and aluminum-alloy fins are only slightly inferior to magnesium-alloy fins and will transfer several times more heat than steel.
4. For a given fin width, the highest heat transfer can be obtained with metals having a high thermal conductivity. Of the metals considered, the highest heat transfer will be obtained when copper is used; aluminum, magnesium, and steel follow in the order of their respective effectiveness.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,  
LANGLEY FIELD, VA., June 28, 1939.

#### REFERENCES

1. Biermann, Arnold E., and Pinkel, Benjamin: Heat Transfer from Finned Metal Cylinders in an Air Stream. Rep. No. 488, NACA, 1934.

2. Schey, Oscar W., and Rollin, Vern G.: The Effect of Baffles on the Temperature Distribution and Heat-Transfer Coefficients of Finned Cylinders. Rep. No. 511, NACA, 1934.
3. Schey, Oscar W., and Ellerbrock, Herman H., Jr.: Blower Cooling of Finned Cylinders. Rep. No. 587, NACA, 1937.
4. Biermann, Arnold E.: Heat Transfer from Cylinders Having Closely Spaced Fins. T. N. No. 602, NACA, 1937.
5. Ellerbrock, Herman H., Jr., and Biermann, Arnold E.: Surface Heat-Transfer Coefficients of Finned Cylinders. Rep. No. 676, NACA, 1939.
6. Biermann, Arnold E.: The Design of Metal Fins for Air-Cooled Engines. SAE Jour., vol. 41, no. 3, Sept. 1937, pp. 388-392.
7. Rollin, Vern G., and Ellerbrock, Herman H., Jr.: Pressure Drop across Finned Cylinders Enclosed in a Jacket. T. N. No. 621, NACA, 1937.
8. Pinkel, Benjamin: Heat-Transfer Processes in Air-Cooled Engine Cylinders. Rep. No. 612, NACA, 1938.
9. Campbell, Kenneth: Cylinder Cooling and Drag of Radial Engine Installations. SAE Jour., vol. 43, no. 6, Dec. 1938, pp. 515-527.
10. Brevoort, Maurice J.: The Effect of Air-Passage Length on the Optimum Fin Spacing for Maximum Cooling. T. N. No. 649, NACA, 1938.