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METHOD OF MATCHING PERFORMANCE OF COMPRESSOR SYSTEMS WITH THAT OF AIRCRAFT POWER SECTIONS

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SUMMARY

A method is developed of easily determining the performance of a compressor system relative to that of the power section for a given altitude. Because compressors, reciprocating engines, and turbines are essentially flow devices, the performance of each of these power-plant components is presented in terms of similar dimensionless ratios. The pressure and temperature changes resulting from restrictions of the charge-air flow and from heat transfer in the ducts connecting the components of the power plant are also expressed by the same dimensionless ratios and the losses are included in the performance of the compressor.

The performance of a mechanically driven, single-stage compressor in relation to the performance of a conventional air-cooled engine operating at sea-level conditions is presented as an example of the application of the method.

INTRODUCTION

One of the principal problems in evaluating the performance of aircraft power plants for a given altitude is determining the performance of the compressor system in relation to that of the power section when the curves describing the performance of each have been separately established. The problem is illustrated by the relation between the performance of a reciprocating engine and that of a mechanically driven, single-stage compressor. For a given altitude, engine speed, and fuel-air ratio, the weight flow of charge air consumed by the engine is a function of the manifold pressure and temperature. The manifold pressure and temperature, however, depend on the pressure and the temperature developed by the compressor and are therefore functions of the inlet conditions, the speed, and the weight flow of charge air through the compressor.

An exact solution to this problem can be obtained by tedious trial-and-error calculations. When the compressor speed and the pressure and the temperature at the compressor inlet are known, the pressure and the temperature at the manifold of the engine can be calculated from an assumed weight flow of charge air and the curves describing the performance of the compressor. The resulting weight flow of charge air consumed by the engine can then be found from the calculated manifold pressure and temperature and the curves representing the performance of the engine. If this weight flow of charge air does not agree with the assumed value, the procedure must be repeated until the calculated weight flow of charge air equals that assumed. This solution is further complicated when pressure and temperature changes occur between the compressor outlet and the engine manifold.

A more convenient method that provides a rapid means of relating the performance of a compressor system to that of a power section is presented herein. This method is based on the premise that compressors and power sections are essentially flow devices and are susceptible to a common dimensional analysis. If the effects of heat transfer and viscosity are neglected, a relation may be shown to exist among the pressure ratio across any component, the Mach number of the flow, and the ratio of a characteristic speed of the component to the speed of sound at some point.

The complete solution of the problem requires the consideration of the pressure and temperature changes in the ducts connecting the components of a power plant. When heat transfer and viscosity are neglected, the pressure changes in a duct are a function of the Mach number of the flow and the effects of these changes can be incorporated by appropriately modifying the performance curves of either the compressor system or the power section. When heat transfer occurs in the ducts, the additional pressure and temperature changes are a function of the Mach number of the flow and the ratio of the heat transferred to the heat content of the charge air. For any given value of this heat ratio, if viscosity is neglected, the curves obtained without the effect of heat transfer may be corrected to include this effect. A reference point can be so chosen that the performance of the component upstream of this reference point is given by curves of pressure ratio against Mach number for various values of the ratio of the characteristic speed of the compressor to the speed of sound; the performance of the component downstream of the reference point is given by curves of pressure ratio against Mach number for various values of the ratio of the characteristic speed of the power section to the speed of sound.

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SYMBOLS

A area, square feet

a local speed of sound, feet per second

B gear ratio

D characteristic dimension, feet

D_r impeller or rotor diameter, feet

g acceleration of gravity, 32.174 feet per second per second

K factor for conversion of inches of mercury to pounds per square foot, 70.73

k_1, k_2 proportionality factors

N characteristic speed, rpm

n impeller or rotor speed, rps

p pressure, inches of mercury absolute

Δp pressure drop, inches of mercury

Q volume flow, cubic feet per second

R gas constant for normal air, 53.50 foot-pounds per pound °F

S speed ratio proportional to ratio of impeller or rotor tip speed to speed of sound $(ND/\sqrt{T})$

T temperature, °R

U tip speed of impeller or rotor, feet per second

V velocity of charge air in duct, feet per second

W weight flow of charge air, pounds per second

γ ratio of specific heats for normal air, 1.3947

η adiabatic compressor efficiency

η_e aftercooler effectiveness ratio

η_t adiabatic turbine efficiency

θ ratio of actual stagnation temperature at inlet to compressor or turbine and standard NACA sea-level temperature $(T_{t,1}/518.4)$

ρ density of air, slugs per cubic foot

Subscripts:

s static

t stagnation or total

1 compressor inlet (fig. 1)

2 compressor outlet (fig. 1)

3 power-section inlet (fig. 1)

4 power-section outlet (fig. 1)

DEVELOPMENT OF FUNCTIONS

FLOW DEVICES

When the effects of heat transfer and viscosity are neglected, a dimensional analysis, based on the method presented in reference 1, shows that the performance of a flow device can be expressed by a relation among three variables that are dimensionless or proportional to dimensionless quantities. The general relation among these variables may be expressed

$$\frac{p_2}{p_1} = F\left(\frac{nD}{a}, \frac{Q}{D^2 a}\right)$$

$$= f\left(\frac{ND}{\sqrt{T}}, \frac{Q}{\sqrt{T}}\right) \qquad (1)$$

where

p_2/p_1 ratio of pressures between which device operates

$ND/\sqrt{T}$ speed ratio proportional to ratio of characteristic speed to speed of sound

$Q/\sqrt{T}$ flow function proportional to Mach number of charge-air flow

Inasmuch as a compressor, a reciprocating engine, and a turbine are essentially flow devices, the performance of each can be expressed by these three quantities.
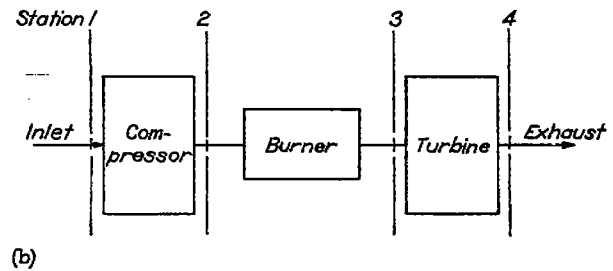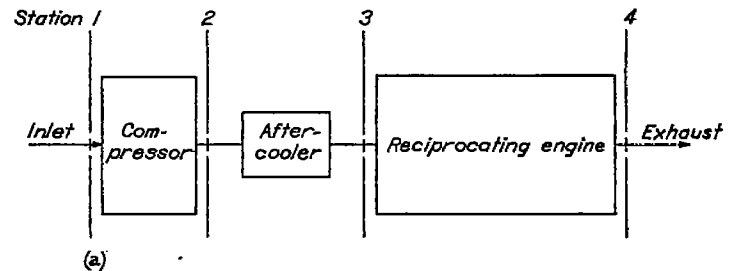
(a) Reciprocating power plant with single-stage compressor system.

(b) Compressor-turbine power plant.

FIGURE 1.—Schematic diagrams of charge-air passage through typical aircraft power plants.

Compressor.—When the performance of a compressor is expressed by the variables of equation (1), the dependent variable becomes the total-pressure ratio $p_{t,2}/p_{t,1}$ across the compressor. The speed ratio S is conveniently expressed as the quotient of the tip speed U of the impeller or rotor and the speed of sound at the inlet of the compressor. In order to eliminate the effect of the flow area, it is desirable to express the flow function as $Q_t/\sqrt{T_t}$, where Q_t is a fictitious total volume flow (the quotient of the mass flow of charge air and the stagnation density) and T_t is a stagnation temperature. The flow function is expressed as a function of the charge-air flow at the outlet of the compressor to facilitate the solution of the over-all problem. The appendix shows that, if the performance of a compressor can be expressed as a unique function of the load coefficient $Q_{t,1}/n$ and the speed ratio, it is also a unique function of the flow function at the outlet of the compressor $Q_{t,2}/\sqrt{T_{t,2}}$ and the speed ratio. The derivation of the flow of charge air or the load coefficient of the compressor is also presented.

The performance curves of typical centrifugal, mixed-flow, and axial-flow compressors are presented in figures 2, 3, and 4, respectively, as the variation of $p_{t,2}/p_{t,1}$ with the flow function at the outlet of the compressor $Q_{t,2}/\sqrt{T_{t,2}}$ and in terms of the standard flow parameter $Q_{t,1}/\sqrt{\theta}$ for various values of the speed ratio. The speed ratio is represented by the quantity $U/\sqrt{\theta}$ to conform with the standard notation. These curves were derived from tests made in accordance with the recommendations of reference 2. A comparison of the two sets of curves shows that plotting the performance of a compressor in terms of the flow function at the outlet $Q_{t,2}/\sqrt{T_{t,2}}$ tends to give closer vertical alinement to the peak values of the curves.

Conventional power section.—Because the flow function depends on the operating conditions of the engine, $Q/\sqrt{T}$ can be expressed as the dependent variable by rearranging the terms in equation (1).

$$\frac{Q_{t,3}}{\sqrt{T_{t,3}}} = f_1\left(\frac{p_{t,3}}{p_{t,4}}, \frac{ND}{\sqrt{T_{t,3}}}\right) \qquad (2)$$
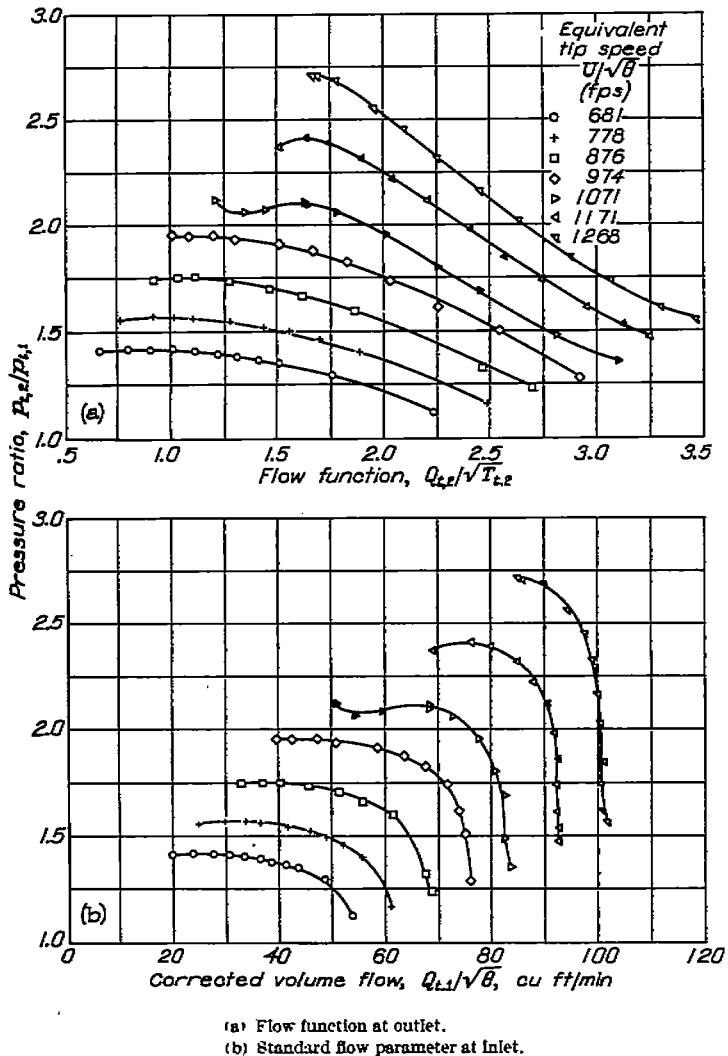
(a) Flow function at outlet.
(b) Standard flow parameter at inlet.

FIGURE 3.—Performance curves of typical mixed-flow compressor.

(a) Flow function at outlet.
(b) Standard flow parameter at inlet.

FIGURE 2.—Performance curves of typical centrifugal compressor.

(a) Flow function at outlet.
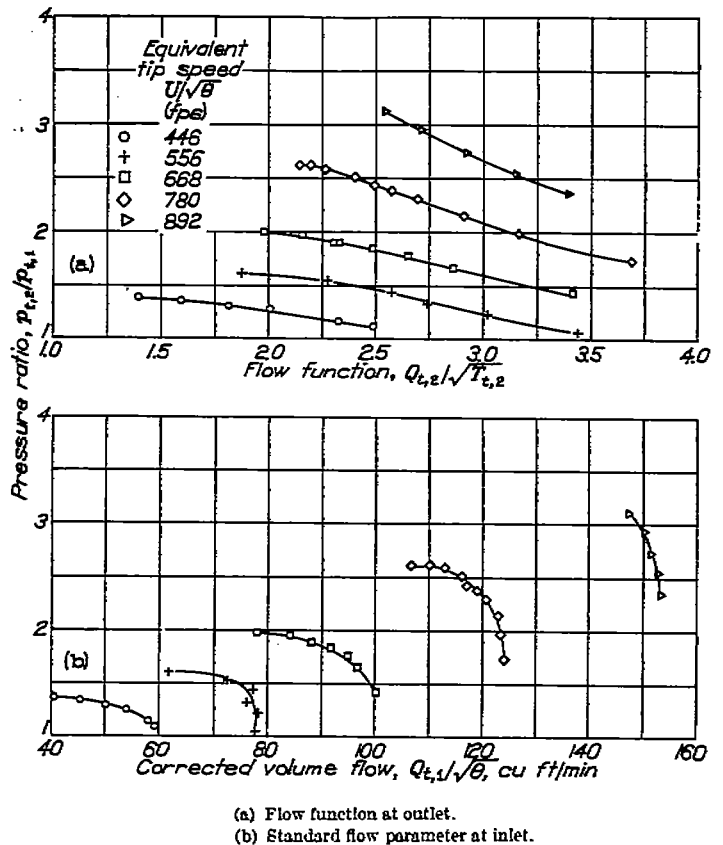(b) Standard flow parameter at inlet.

FIGURE 4.—Performance curves of typical axial-flow compressor.
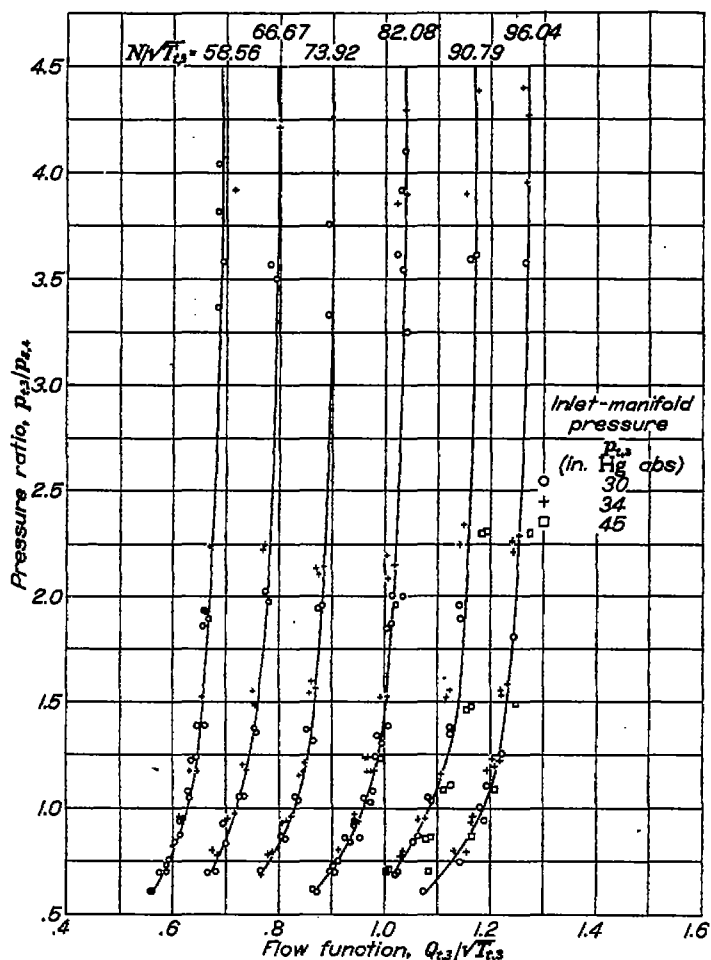
FIGURE 5.—Relation between flow function and pressure ratio for air-cooled reciprocating aircraft engine at sea-level conditions.

The flow function $Q_{t,3}/\sqrt{T_{t,3}}$ and the pressure ratio between the manifold and the exhaust $p_{t,3}/p_{s,4}$ are related in figure 5 for several values of the speed ratio $N/\sqrt{T_{t,3}}$ for an air-cooled reciprocating engine. For a given engine, the omission of the constant characteristic dimension D does not alter the significance of the speed ratio; in this case, the crankshaft speed in rpm is used as the characteristic speed.

Turbine.—When the variables governing the performance of a flow device are applied to the performance of a turbine, equation (1) may be written

$$\frac{Q_{t,3}}{\sqrt{T_{t,3}}} = f_2\left(\frac{p_{t,3}}{p_{s,4}}, \frac{ND}{\sqrt{T_{t,3}}}\right) \tag{3}$$

where the subscripts 3 and 4 denote the inlet and the outlet of the turbine, respectively, as shown in figure 1 (b).

The speed ratio $ND/\sqrt{T_{t,3}}$ of a turbine can be expressed by the quantity $N/\sqrt{\theta}$. Inasmuch as the efficiency of operation is not readily obtained from a plot of the variation of $Q_t/\sqrt{T_t}$ with $p_{t,3}/p_{s,4}$ for various values of $N/\sqrt{\theta}$, the variation of the turbine efficiency η_t (based on the over-all pressure ratio)

with $p_{t,3}/p_{t,4}$ for given values of $N/\sqrt{\theta}$ may be included to clarify the performance characteristics. These two plots can be conveniently combined to show the performance of a conventional exhaust-gas impulse turbine by using the variation of the pressure ratio as a common ordinate (fig. 6).

CORRECTIONS FOR LOSSES IN CONNECTING DUCTS

The effects of losses in the ducts connecting the components of a power plant expressed in terms of the flow function can be incorporated in the performance of the compressor, which, in effect, converts the actual compressor and duct into an equivalent compressor in which no additional losses occur.

Pressure drop.—When the temperature in the duct connecting the compressor to the power section remains constant but a pressure drop occurs, the loss in stagnation pressure can be expressed

$$\Delta p_t = k_1 \frac{V_2^2}{g} \tag{4}$$

where V_2 is the velocity of the charge air at the inlet to the duct. Inasmuch as the temperature remains fixed, the velocity is proportional to the Mach number of the flow at the same point. The flow function, however, is also related to the Mach number of the flow at the inlet to the duct with the result that, in terms of the flow function and the stagnation pressure at the inlet of the connecting duct, equation (4) can be written

$$\frac{\Delta p_t}{p_t} = k_2\left(\frac{Q_{t,2}}{\sqrt{T_{t,2}}}\right)^2 \tag{5}$$

where, for a first approximation, the value of k_2 can be assumed constant for a given duct.
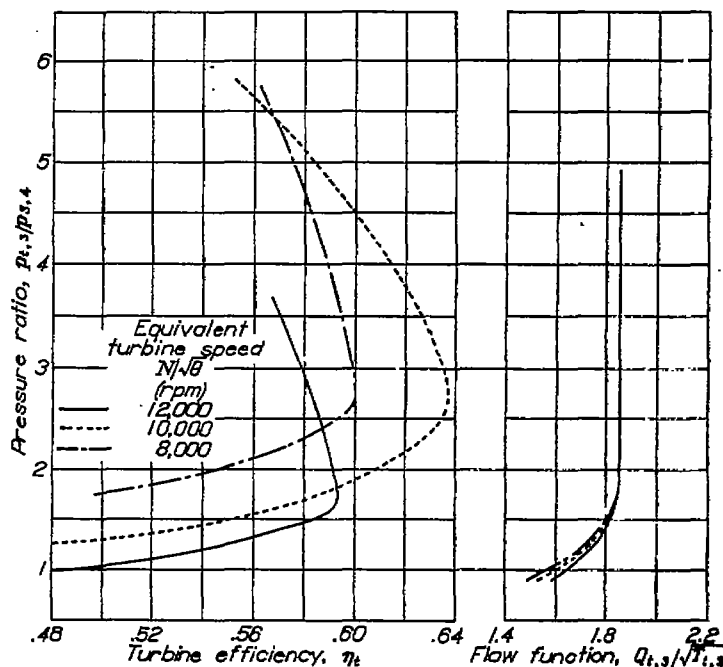

FIGURE 6.—Performance characteristics of exhaust-gas impulse turbine.

When the pressure drop in a connecting duct is included in the performance of a compressor, the outlet of this duct is coincident with the inlet of the power section, and the over-all pressure ratio between this point and the inlet of the compressor can be found from

$$\frac{p_{t,3}}{p_{t,1}}=\frac{p_{t,2}}{p_{t,1}}\left[1-k_2\left(\frac{Q_{t,2}}{\sqrt{T_{t,2}}}\right)^2\right]$$ (6)

The flow function must now be adjusted for the pressure drop between the same points to complete the conversion of the performance of the compressor.. When the effects of heat transfer are negligible, the temperature ratio across the duct is unity and the flow function at the outlet of the duct can be evaluated from that at the outlet of the compressor by the relation

$$\frac{Q_{t,3}}{\sqrt{T_{t,3}}}=\frac{Q_{t,2}}{\sqrt{T_{t,2}}}\frac{p_{t,2}}{p_{t,3}}=\frac{Q_{t,2}}{\sqrt{T_{t,2}}}\frac{1}{1-k_2(Q_{t,2}/\sqrt{T_{t,2}})^2}$$ (7)

Heat transfer.—In many power-plant installations, heat transfer in the connecting ducts cannot be ignored because of charge-air coolers or, in the case of compressor-turbine power plants, combustion. A first approximation of the effect of heat transfer through a charge-air cooler can be obtained from the effectiveness ratio η_e of the cooler. The temperature ratio across the cooler resulting from the loss of heat to the coolant can then be expressed

$$\frac{T_{t,3}}{T_{t,2}}=1-\eta_e+\eta_e\frac{T_{t,1}}{T_{t,2}}$$ (8)

and the over-all temperature ratio of the compressor including the effect of the heat transfer in the cooler can be found from

$$\frac{T_{t,3}}{T_{t,1}}=\frac{T_{t,2}}{T_{t,1}}-\eta_e\left(\frac{T_{t,2}}{T_{t,1}}-1\right)$$ (9)

The pressure ratio across the compressor and the cooler can be determined from an expression similar to equation (7), and the flow function at the inlet to the power section can be found by substituting the value of the temperature ratio across the cooler in equation (14) of the appendix

$$\frac{Q_{t,3}}{T_{t,3}}=\frac{Q_{t,2}}{T_{t,2}}\left[\frac{1}{1-k_2(Q_{t,2}/\sqrt{T_{t,2}})^2}\right]\sqrt{1-\eta_e+\eta_e\frac{T_{t,1}}{T_{t,2}}}$$ (10)

where the correction for the pressure drop includes that through the cooler and all the ducting between the outlet of the compressor and the inlet to the power section. Figure 7 shows the results of applying the corrections of equations (6) and (10) to the variables governing the performance of a centrifugal compressor. Lines connecting the points where the adiabatic efficiency is 90, 95, and 100 percent of the maximum value for each speed ratio are also shown. For these curves, the values of k_2 and η_e from equation (10) were assumed to be 0.015 and 0.45, respectively.
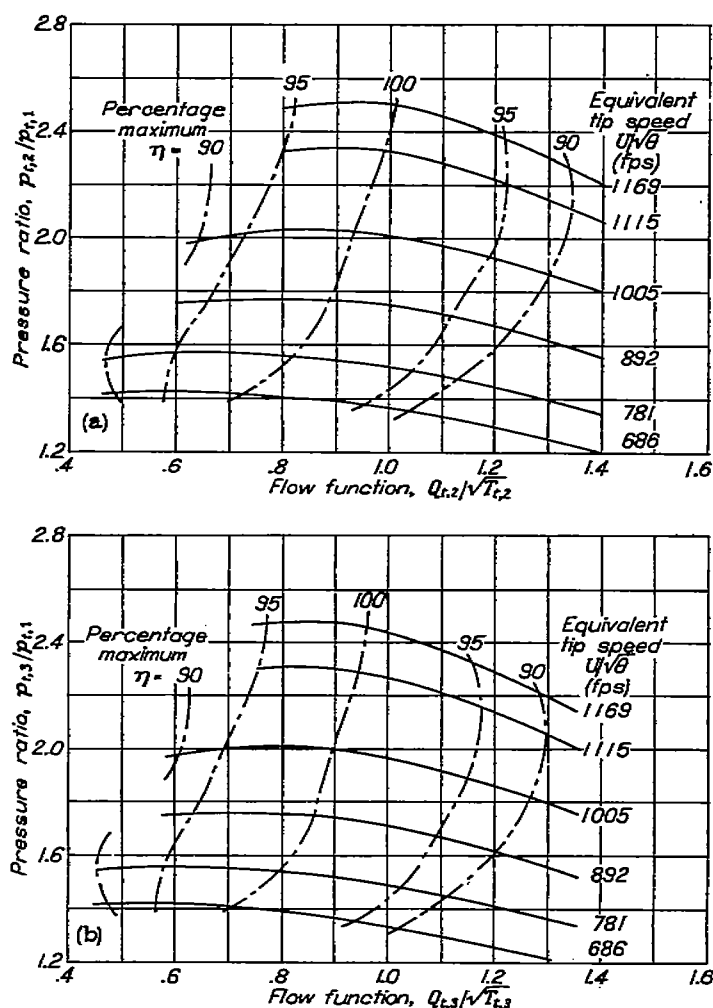
(a) Experimental performance.
(b) Performance corrected for pressure and temperature changes through aftercooler; k_2, 0.015; η_e, 0.45.

FIGURE 7.—Comparison of experimental performance of centrifugal compressor and performance corrected for pressure and temperature changes through aftercooler.

The effects of heat transfer are greatly increased by the combustion of fuel in the burners located between the two units in a power plant composed of a compressor and a turbine. Although the change in temperature is the principal effect of heat transfer, a change in pressure also occurs, but it is beyond the scope of this report to evaluate the factors governing this change in pressure. When the variations of pressure and temperature are established, however, the procedures previously outlined may be directly applied to the problem.

APPLICATION OF FUNCTIONS

The general description of the performance of the compressor in relation to that of the power section can be rapidly determined when the pressure ratio of each component has been established in terms of the flow function at a common reference point between them.

The performance of a compressor in relation to that of a power section can be analyzed by superimposing the curves as shown in figure 8. The performance of the power section is represented by the solid curves, which indicate the variation of the pressure ratio $p_{t,3}/p_{t,1}$ with the flow function $Q_{t,3}/\sqrt{T_{t,3}}$ for constant values of $N/\sqrt{T_{t,3}}$. These curves are similar to the curves of figure 5, which show the performance of an air-cooled engine at sea-level conditions. The performance of a centrifugal compressor including the effects of an aftercooler (fig. 7 (b)) is shown by the dashed curves. Also shown are lines of constant percentage of

figure 8 (a). The compressor operates at or near 100 percent of maximum adiabatic efficiency at an approximate value of $N/\sqrt{T_{t,3}}$ of 73.92 for the engine and for pressure ratios between 1.50 and 2.47. The compressor, however, will not develop pressure ratios greater than 2.48 and will not operate efficiently at engine speed ratios much greater than 73.92. Efficient operation from the compressor at engine speed ratios greater than about 78 must be obtained by increasing the capacity of the compressor; moreover, if pressure ratios greater than 2.47 are required, a second compressor is needed.
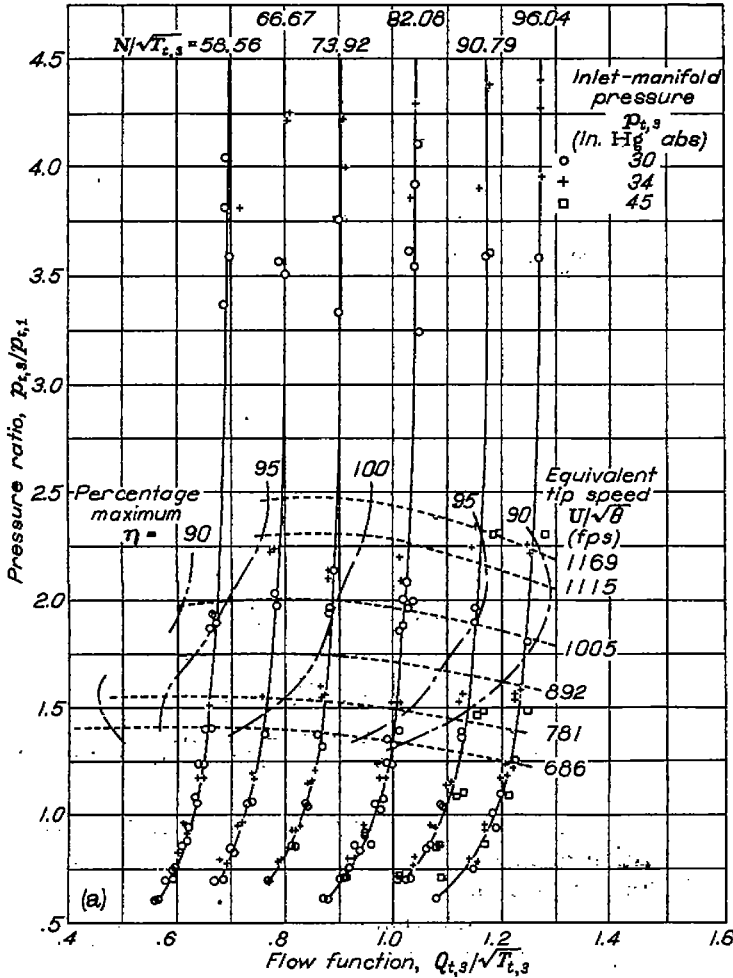
(a) Compressor performance for constant values of $U/\sqrt{\theta}$.
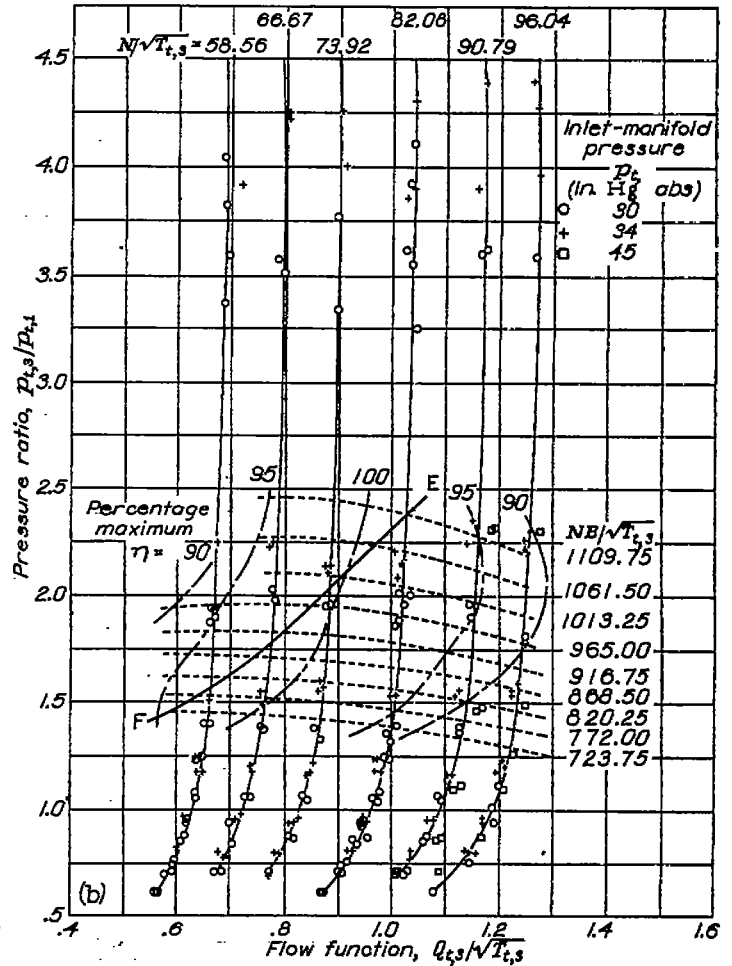
(b) Compressor performance for constant values of $NB/\sqrt{T_{t,3}}$; $B=13.52$.

FIGURE 8.—Performance of centrifugal compressor in relation to that of air-cooled reciprocating engine at sea-level conditions.

maximum adiabatic efficiency for the compressor. In a practical analysis, the increase in pressure and temperature due to ram at the inlet to the compressor would be charged to the performance of the compressor. The pressure ratio across the compressor and the cooler would therefore be the same as that across the engine and the effect of the aftercooler would be more indicative of actual flight conditions at a given altitude. For the present purposes, however, the effects of ram have been neglected.

A general description of the relation between the performance of the compressor and that of the engine is presented in

A more detailed analysis can be made by expressing the performance of the compressor in terms of constant values of $NB/\sqrt{T_{t,3}}$ when the compressor is shown to satisfy the general requirements of the engine. (See fig. 8 (b).) For this particular case, which involved the combined performance of a compressor and an aftercooler, the value of $NB/\sqrt{T_{t,3}}$ was found from equation (9) and the relation

$$\frac{U}{\sqrt{T_{t,3}}} = \frac{U}{\sqrt{T_{t,1}}} \sqrt{\frac{T_{t,1}}{T_{t,3}}}$$

The performance of each component in terms of variables common to both is shown in figure 8 (b), when the performance of the compressor is represented by curves of constant $NB/\sqrt{T_{t,2}}$. The gear ratio required to drive the compressor is given by the ratio of the values of $NB/\sqrt{T_{t,3}}$ for the compressor and of $N/\sqrt{T_{t,2}}$ for the engine at the intersection of the curves representing constant values of these quantities. The variation of the performance of the compressor can therefore be examined for a given gear ratio and various values of $N/\sqrt{T_{t,2}}$ for the engine. This variation is shown for an arbitrary gear ratio of 13.52. The heavy solid line E–F shows the change in the performance of the compressor as the speed ratio of the engine is reduced from 82.08 to 58.56. The compressor develops a pressure ratio of approximately 2.10 at 100 percent of maximum efficiency when the engine is operating at a speed ratio of about 75.55. Additional lines of constant gear ratio may be added to indicate the performance of the compressor for various operating conditions of the engine.

CONCLUDING REMARKS

The method presented for obtaining the relative performance of a compressor system for a specified altitude derives its advantage from the use of three variables—the flow function, the speed ratio, and the total-pressure ratio—which not only define the performance of a compressor but also that of a power section. A reference point can be established to make the values of these variables representing a given operating point of the compressor equal to those of the corresponding variables defining an operating point of the power section. This procedure allows the use of a composite plot that clearly shows the performance of the compressor in relation to that of the power section.

Pressure and temperature changes occurring in the ducts connecting the compressor and the power section can also be expressed as functions of these same variables and included in the performance of the compressor. In addition, the use of these three variables to define the performance of a compressor facilitates the use of the characteristic curves and results in closer vertical alinement of the peaks of the curves.

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    NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
    CLEVELAND, OHIO, *November 15, 1945.*

APPENDIX

RELATIONS AMONG $Q/\sqrt{T}$, W, AND Q/n

The flow function $Q_t/\sqrt{T_t}$ can be expressed as a function of either the weight flow of charge air or the load coefficient $Q_{s,1}/n$ and the speed ratio S of the compressor. Inasmuch as the weight flow of charge air between the inlet and the outlet of a power plant is nearly constant, the flow function at any point in the system must depend on the weight flow

of charge air in order to maintain continuity. This relation can be derived from the statement that

$$\rho_t g Q_t = W$$

from which

$$\frac{Q_t}{\sqrt{T_t}} = \frac{WR\sqrt{T_t}}{Kp_t} \tag{11}$$

When the compressor load coefficient $Q_{s,1}/n$ and the speed ratio S are given, an expression for a flow function is

$$\frac{Q_{s,1}}{\sqrt{T_{s,1}}} = \frac{Q_{s,1}}{n} \frac{S\sqrt{\gamma gR}}{\pi D_r} \tag{12}$$

If the performance of a compressor is uniquely determined by the variables $Q_{s,1}/n$ and S, it is therefore also uniquely determined by the variables $Q_{s,1}/\sqrt{T_{s,1}}$ and S.

In order to avoid the effect of changes in flow area on the actual volume flow, it is frequently convenient in engine analysis to use the stagnation temperature $T_{t,1}$ together with the fictitious volume flow $Q_{t,1}$, which is the quotient of the mass flow and the total density. The flow function at the inlet $Q_{t,1}/\sqrt{T_{t,1}}$ can be derived from $Q_{s,1}/\sqrt{T_{s,1}}$ by use of the general energy equation and the adiabatic relations between the temperatures and the densities for static and stagnation conditions at the compressor inlet. These expressions can be combined and simplified to obtain

$$\frac{Q_{t,1}}{\sqrt{T_{t,1}}} = \frac{Q_{s,1}}{\sqrt{T_{s,1}}} \left[\frac{1}{1 + \dfrac{\gamma-1}{2\gamma gA_1^2 R}\left(\dfrac{Q_{s,1}}{\sqrt{T_{s,1}}}\right)^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{13}$$

The flow function at the outlet of the compressor $Q_{t,2}/\sqrt{T_{t,2}}$ may also be used with S to define the performance of the compressor. This function can be found from $Q_{t,1}/\sqrt{T_{t,1}}$ by means of the perfect gas law and is expressed

$$\frac{Q_{t,2}}{\sqrt{T_{t,2}}} = \frac{Q_{t,1}}{\sqrt{T_{t,1}}} \frac{p_{t,1}}{p_{t,2}} \sqrt{\frac{T_{t,2}}{T_{t,1}}} \tag{14}$$

Substitution of equations (12) and (13) in equation (14) results in

$$\frac{Q_{t,2}}{\sqrt{T_{t,2}}} = \frac{Q_{s,1}}{n} \frac{S\sqrt{\gamma gR}}{\pi D_r} \left[\frac{1}{1 + \dfrac{\gamma-1}{2}\dfrac{S^2}{\pi^2 D_r^2 A_1^2}\left(\dfrac{Q_{s,1}}{n}\right)^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \frac{p_{t,1}}{p_{t,2}} \frac{T_{t,2}}{T_{t,1}} \tag{15}$$

Inasmuch as $p_{t,2}/p_{t,1}$ and $T_{t,2}/T_{t,1}$ are functions of $Q_{s,1}/n$ and S, the value of $Q_{t,2}/\sqrt{T_{t,2}}$ is also a function of $Q_{1,1}/n$. If the performance of a compressor is a unique function of $Q_{s,1}/n$ and S, it is therefore a unique function of $Q_{t,2}/\sqrt{T_{t,2}}$ and S for a given compressor.

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