REPORT 995

BLOCKAGE CORRECTIONS FOR THREE-DIMENSIONAL-FLOW CLOSED-THROAT WIND TUNNELS, WITH CONSIDERATION OF THE EFFECT OF COMPRESSIBILITY

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SUMMARY

Theoretical blockage corrections are presented for a body of revolution and for a three-dimensional unswept wing in a circular or rectangular wind tunnel. The theory takes account of the effects of the wake and of the compressibility of the fluid, and is based on the assumption that the dimensions of the model are small in comparison with those of the tunnel throat. Formulas are given for correcting a number of the quantities, such as dynamic pressure and Mach number, measured in wind-tunnel tests. The report presents a summary and unification of the existing literature on the subject.

INTRODUCTION

When a model is placed in a closed-throat wind tunnel there is an effective constriction or blockage of the flow at the throat of the tunnel. The effect of this blockage is to increase the velocity of the fluid flowing past the model; if the model is not too large relative to the tunnel throat, this velocity increment is approximately the same at all points of the model so that the model is effectively working in a uniform stream of fluid the velocity of which is, however, greater than the free-stream velocity observed at some distance upstream of the model. It is therefore necessary to correct the observed velocity, dynamic pressure, Mach number, and other measured quantities for the effect of this constriction. This correction is frequently called the correction for "solid blockage." In addition to this solid-blockage correction, a correction for "wake blockage" is also necessary if the true dynamic pressure and Mach number at the model are to be determined. This wake blockage arises because the fluid is slowed down in the wake and consequently must be speeded up outside the wake. A further effect of the wake is to produce a pressure gradient which must be considered in correcting the drag coefficient.

Formulas for the solid-blockage correction for a model mounted in a two-dimensional-flow wind tunnel are given in references 1, 2, 3, and 4. The first-order effects of the compressibility of the fluid on these corrections are given in reference 5, as well as in references 3 and 4. References 2, 3, and 4 consider also the wake-blockage correction, and reference 3 also considers the drag correction due to the pressure gradient caused by the wake. The formulas given in reference 3 for the solid- and wake-blockage corrections will usually be found most convenient whenever the engineer is confronted by a practical problem of determining the corrections for any configuration met in his experimental work.

The solid-blockage correction for a model mounted in a three-dimensional-flow wind tunnel has been given in a number of different forms by different authors. Not only do different authors give the correction for the same configuration in different forms but no one author gives formulas which are applicable to both fuselages and wings in tunnels of various shapes; for this reason, the engineer confronted with a correction problem may have to refer to several reports to get the complete solution of his problem. Moreover the modifications of the formulas for the first-order effects of fluid compressibility are given incorrectly in some cases. References 1 and 2 give a formula for the solid-blockage correction for a body of revolution in a circular or rectangular tunnel for the case of incompressible flow. In references 5 and 6 the effect of the compressibility of the fluid on this correction is discussed, but the result given is incorrect. Reference 4 gives a formula for the solid-blockage correction for a body of revolution and for a three-dimensional wing in a 7- by 10-foot wind tunnel. The modification for compressibility is correct for the wing but wrong for the body of revolution. Reference 7 gives a formula for the solid-blockage correction for a body of revolution in a circular tunnel, correctly taking account of the effect of the compressibility of the fluid. References 8, 9, and 10 give a formula for the solid-blockage correction for any body in a circular wind tunnel together with the appropriate constants for a body of revolution and for a rectangular wing having various span-to-diameter ratios; the modification of this formula to take account of compressibility is correctly given.

It is clear that no one report gives all the necessary formulas together with the appropriate constants and compressibility modifications to enable the engineer to calculate the solid-blockage correction for any case with which he may be confronted. Moreover, when the results of two or more reports overlap, the forms are frequently different so that it is not obvious whether the results are in agreement. It is the purpose of the present report to summarize and extend the results of the previously mentioned reports. Formulas are given for the calculation of the solid-blockage correction for a body of revolution or a three-dimensional unswept wing in a circular or rectangular tunnel. These formulas contain two constants, one depending on the shape of the body and the other on the shape of the tunnel and the ratio of wing span to tunnel breadth. (This ratio may be taken to be zero for a body of revolution.) Values of the first constant for various bodies of revolution and for a number of frequently encountered wing-profile sections are given. Values of the second constant for a circular tunnel and for rectangular tunnels of a number of commonly encountered breadth-to-height ratios are given for various wing-span-to-tunnel-breadth ratios. Some of these values have been taken from the previously mentioned reports; whereas others appear for the first time in the present report. The discussion is limited to bodies centrally located in the wind tunnel.

The wake-blockage correction for a model mounted in a three-dimensional-flow tunnel is given in references 4, 8, 9, and 10. For the case of incompressible flow the formulas are in agreement, but the modification to take account of compressibility given in reference 4 differs from that given in references 8, 9, and 10. This matter is discussed in the present report. In reference 11 the corrections for the pressure gradient due to the wake are given with the correct compressibility factors.

All the final correction formulas together with directions for their use are given in the final section entitled "Concluding Remarks." Mathematical symbols are defined as introduced in the text. For reference, a list of the more important symbols and their definitions is given in appendix B.

SOLID BLOCKAGE IN INCOMPRESSIBLE FLOW

In studying the flow over a thin airfoil of small camber at a small angle of attack it has been shown that the effects of camber and thickness may be considered independently. In treating the problem of wall interference, it is again convenient to consider the thickness and camber effects separately. The camber effect, as pointed out in reference 3, contributes nothing to the blockage correction. Consequently it suffices to determine the blockage correction for symmetrical bodies at zero angle of attack. This means that for wings it is necessary to consider only the base profile of the airfoil, the base profile being defined as the profile the airfoil would have if the camber were removed and the resulting symmetrical airfoil placed at zero angle of attack; bodies of revolution need be considered only at zero angle of attack.

RECTANGULAR TUNNEL

Three-dimensional wing.—The blockage correction for a three-dimensional wing in a rectangular tunnel is considered in reference 4. Numerical values are given only for a wing of 6-foot span in a 7- by 10-foot wind tunnel. The method may, however, be applied to any rectangular tunnel and any span-to-breadth ratio.

Consider a rectangular tunnel of height H and breadth B. Suppose that the wing span $2s$ is in the B-direction so that $2s/B$ is the span-to-breadth ratio. As in reference 4, let the wing be represented by a series of finite lines of sources and sinks. (See fig. 1.) The strengths of these sources and sinks are assumed to depend on the airfoil profile and the determination of these strengths, which is a two-dimensional problem, is explained presently. If each line retains the same strength from one end to the other, the wing section cannot be exactly constant. It will thin down at the extreme tip and the plan form will not be exactly rectangular, but neither of these features is such as to detract from its usefulness for the present purpose, which

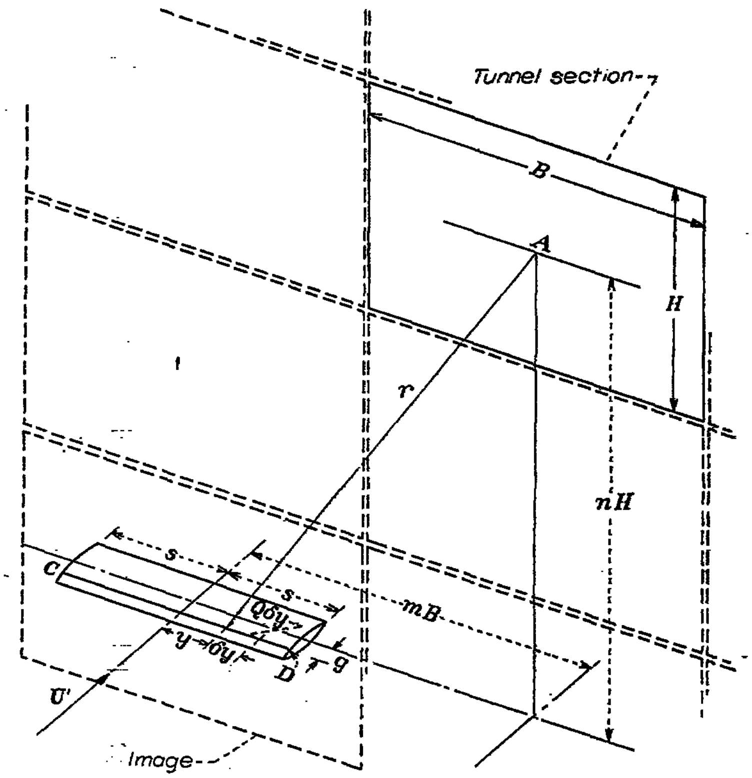


FIGURE 1.—Image system for three-dimensional wing in rectangular tunnel.

is to represent an actual wing sufficiently well to enable a calculation to be made of the velocity along the tunnel axis "induced" by images of the wing. It is this velocity induced by the images which represents the effect of the tunnel walls on the velocity at the model, and which is the solid-blockage correction.

Consider the image line source CD of strength Q per unit length. The velocity potential of a three-dimensional source of strength $Q\delta y$ (volume per unit time) is $-Q\delta y/4\pi r$. It follows that the component velocity along the tunnel center line induced at A by the source element $Q\delta y$ is

$$\delta u_A=\frac{Qg}{4\pi r^3}\,\delta y$$

where g and r are defined in figure 1. Putting $r^2=(mB-y)^2+n^2H^2+g^2$ and integrating from $-s$ to s gives

$$\Delta u_A=\frac{Qg}{4\pi(n^2H^2+g^2)}\left[\frac{mB+s}{\sqrt{n^2H^2+g^2+(mB+s)^2}}-\frac{mB-s}{\sqrt{n^2H^2+g^2+(mB-s)^2}}\right] \tag{1}$$

for the single line source CD. Now CD is one image of one finite line source used to represent the wing in the tunnel. In order to find the velocity induced by all the images of this particular line source, it is necessary to add the results obtained from equation (1) by giving m and n all positive and negative integral values except $m=n=0$. Thus for a single line source

$$u_A=\frac{Qg}{4\pi}\sum{}'\frac{1}{n^2H^2+g^2}\left[\frac{mB+s}{\sqrt{n^2H^2+g^2+(mB+s)^2}}-\frac{mB-s}{\sqrt{n^2H^2+g^2+(mB-s)^2}}\right] \tag{2}$$

where the prime denotes that the term $m=n=0$ is to be omitted from the summation. Equation (2) may be rewritten

$$u_A=\frac{Qgs}{2\pi H^3}\sigma\left(\frac{g}{H},\frac{s}{B},\frac{B}{H}\right)$$

where

$$\sigma\left(\frac{g}{H},\frac{s}{B},\frac{B}{H}\right)=\frac{B}{2s}\sum{}'\frac{1}{n^2+(g/H)^2}\left[\frac{m+s/B}{\sqrt{n^2+(g/H)^2+(m+s/B)^2(B/H)^2}}-\frac{m-s/B}{\sqrt{n^2+(g/H)^2+(m-s/B)^2(B/H)^2}}\right] \tag{3}$$

It is convenient to define

$$\tau\left(\frac{g}{H},\frac{s}{B},\frac{B}{H}\right)=\frac{1}{2}\left(\frac{B}{\pi H}\right)^{3/2}\sigma\left(\frac{g}{H},\frac{s}{B},\frac{B}{H}\right) \tag{4}$$

so that

$$u_A=\frac{Qgs}{(BH)^{3/2}}\pi^{1/2}\tau\left(\frac{g}{H},\frac{s}{B},\frac{B}{H}\right)$$

Now σ and τ depend only very slightly on g/H and so it usually suffices to take $g/H=0$ in the evaluation of these quantities. Any wing profile can be represented by a suitable distribution of sources and sinks along the chord. It follows that the total induced velocity due to the wing images is obtained by summing over this distribution and is approximately

$$\Delta_1 U'=\frac{s\Sigma Qg}{(BH)^{3/2}}\pi^{1/2}\tau\left(0,\frac{s}{B},\frac{B}{H}\right) \tag{5}$$

It may be noted here that setting $g/H=0$ in the evaluation of σ and τ is equivalent to representing the wing by a line doublet of strength ΣQg (analogous to references 1 and 2) instead of by a distribution of sources and sinks.

The quantity ΣQg of equation (5) can be determined approximately from the wing profile and this determination is a two-dimensional problem. From reference 2, but with the notation of reference 3, there is obtained

$$\Sigma Qg=\frac{\pi}{8}\Lambda c^2 U' \tag{6}$$

where

- c airfoil chord
- U' apparent free-stream velocity at airfoil as determined from measurements taken at a point far ahead of model
- Λ a factor dependent on shape of base profile

Substitution of equation (6) into equation (5) yields

$$\frac{\Delta_1 U'}{U'}=\frac{2sc^2}{(BH)^{3/2}}\frac{\pi^{3/2}}{16}\Lambda\tau\left(0,\frac{s}{B},\frac{B}{H}\right)$$

$$=\frac{2sct}{C^{3/2}}\frac{\pi^{3/2}}{16}\frac{\Lambda}{t/c}\tau\left(0,\frac{s}{B},\frac{B}{H}\right) \tag{7}$$

where

- t maximum thickness of airfoil
- C cross sectional area of tunnel

If V denotes the volume of the wing, then $V=2sct\kappa_1$ where $\kappa_1\leq 1$ depends on the shape of the base profile. Equation (7) may be rewritten

$$\frac{\Delta_1 U'}{U'}=\frac{K_1\tau V}{C^{3/2}} \tag{8}$$

or

$$\frac{\Delta_1 U'}{U'}=\frac{K_2\tau\, 2sct}{C^{3/2}} \tag{9}$$

where

$$\tau=\tau\left(0,\frac{s}{B},\frac{B}{H}\right) \tag{10}$$

$$K_1=\frac{\pi^{3/2}}{16}\frac{\Lambda}{t/c}\frac{1}{\kappa_1} \tag{11}$$

$$K_2=\frac{\pi^{3/2}}{16}\frac{\Lambda}{t/c} \tag{12}$$

It is clear that τ depends only on the tunnel shape and the wing-span-to-tunnel-breadth ratio; whereas K_1, K_2 depend only on the shape of the base profile.

The factor Λ can be determined for any base profile from the relation (references 2 and 3)

$$\Lambda=\frac{16}{\pi}\int_0^1\frac{y_t}{c}\sqrt{1-P}\sqrt{1+(dy_t/dx)^2}\,d\left(\frac{x}{c}\right) \tag{13}$$

where

- y_t ordinate of base profile at chordwise station x
- dy_t/dx slope of surface of base profile at x
- P base-profile pressure coefficient at x in an incompressible flow

Values of Λ for a number of base profiles are given in reference 3. Thus the value of K_2 can be calculated from equation (12). The value of κ_1 is immediately found from the area of the base profile which may be calculated, for example, by a numerical integration from the ordinates of the base profile. As soon as κ_1 is known, K_1 can then be calculated from equation (11). The evaluation of τ requires the summation of the infinite series in equation (3). The summation of this series is explained in Appendix A.

Values of τ for rectangular tunnels of various breadth-to-height ratios and for various wing-span-to-tunnel-breadth ratios are given in table I and figure 2. Values of K_1 and K_2 for various base profiles are given in tables II and III. The choice between the two formulas (8) and (9) is entirely a matter of convenience and should be decided in the light of the available data.

Body of revolution.—The blockage correction for a body of revolution in a rectangular tunnel is considered in references 1, 2, and 4. In reference 4 numerical values are given for some average streamline body of revolution in a 7- by 10-foot wind tunnel; whereas in references 1 and 2 numerical values are given for prolate spheroids and Rankine Ovoids in square and duplex (breadth equal to twice the height) wind tunnels. Either the method of reference 4 in which the body of revolution is represented by a suitable distribution of sources and sinks along its chord or the method of

TABLE I.—VALUES OF τ FOR VARIOUS TUNNEL SHAPES AND CONFIGURATIONS

Tunnel shape		Body of revolution	3-dimensional wing with span-to-breadth ratio $\frac{2s}{B}=0$	$\frac{2s}{B}=0.25$	$\frac{2s}{B}=0.50$	$\frac{2s}{B}=0.75$	$\frac{2s}{B}=1.00$
Circular		0.797	0.797	0.812	0.828	0.859	-----
Square		.812	.812	.818	.836	.874	0.951
Rectangular	$B/H=10/7$	.863	.863	.864	.866	.884	.916
	$B/H=7/4$	.946	.946	.941	.930	.923	.937
	$B/H=2$	1.028	1.028	1.017	.990	.967	.962
	$B/H=7/2$	1.729	1.729	1.630	1.436	1.204	1.160
	$B/H=2/7$	1.729	1.729	1.783	1.896	2.196	2.665
	$B/H=1/2$	1.028	1.028	-----	-----	-----	1.434
	$B/H=6/10$	.922	.922	.932	.977	1.063	1.230
	$B/H=7/10$	.863	.863	-----	-----	-----	1.110

TABLE II.—VALUES OF K_1 FOR VARIOUS BASE PROFILES

$\frac{t}{c}=0.XX$	Ellipse	Joukowski section	Conventional NACA sections 00XX	NACA low-drag sections 16–0XX	64–0XX	65–0XX	66–0XX
0.06	0.938	-----	0.941	0.938	0.962	0.965	0.987
.09	.965	0.991	.972	.962	.961	.967	.984
.12	.993	1.016	1.005	.997	.987	.989	1.006
.15	1.019	1.045	1.035	1.028	1.019	1.020	1.032
.18	1.046	1.068	1.063	1.062	1.047	1.051	1.057
.21	1.072	1.083	1.090	1.090	1.073	1.079	1.079
.25	1.108	-----	1.128	1.128	-----	-----	-----
.30	1.152	-----	-----	-----	-----	-----	-----
.35	1.196	-----	-----	-----	-----	-----	-----
.50	1.329	-----	-----	-----	-----	-----	-----
1.00	1.772	-----	-----	-----	-----	-----	-----

TABLE III.—VALUES OF K_2 FOR VARIOUS BASE PROFILES

$\frac{t}{c}=0.XX$	Rankine Oval	Ellipse	Joukowski section	Conventional NACA sections 00XX	NACA low-drag sections 16–0XX	64–0XX	65–0XX	66–0XX
0.06	-----	0.737	-----	0.644	0.690	0.615	0.632	0.679
.09	0.913	.758	0.599	.665	.708	.611	.630	.673
.12	.928	.780	.615	.687	.734	.624	.641	.684
.15	.935	.800	.633	.708	.756	.640	.657	.698
.18	.953	.822	.652	.727	.781	.654	.673	.712
.21	.961	.842	.670	.746	.802	.666	.686	.723
.25	.979	.870	.692	.771	.830	.678	.697	.732
.30	1.002	.905	.726	-----	-----	-----	-----	-----
.35	1.043	.940	.763	-----	-----	-----	-----	-----
.50	1.176	1.044	.876	-----	-----	-----	-----	-----
1.00	1.392	1.392	-----	-----	-----	-----	-----	-----

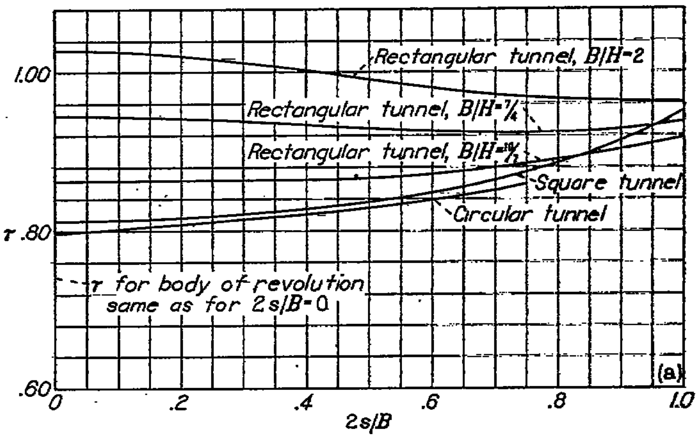
references 1 and 2 in which the body is represented by a doublet of suitable strength at its center may be used to obtain results for any streamline body of revolution in any rectangular tunnel. Since the method of reference 4 was used for the three-dimensional wing, it is instructive to use the doublet method of references 1 and 2 for the body of revolution case, although both methods give the same results.

Consider a body of revolution of maximum thickness t and length c centrally located in a rectangular tunnel of breadth B and height H. As in reference 2 the body may be represented by a doublet of strength μ given by the equation

$$\mu=\frac{\pi}{4}\lambda t^3 U' \tag{14}$$

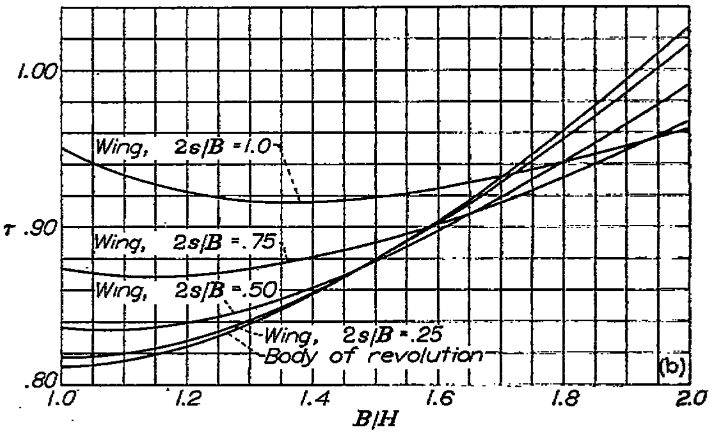
where λ is a constant depending only on the shape and fineness ratio of the body. The velocity induced at the model by the tunnel walls is the same as that induced by a doubly infinite array of images of the doublet and is given by

$$\Delta_1 U'=\frac{\mu}{4\pi}\sum{}' \frac{1}{(n^2H^2+m^2B^2)^{3/2}} \tag{15}$$



(a) Variation of τ with ratio of wing span to tunnel breadth, $2s/B$, for various tunnel shapes.

FIGURE 2.—Values of τ for various tunnel shapes and configurations.



(b) Variation of τ with ratio of rectangular tunnel breadth to height, B/H, for a body of revolution and for wings of various span-to-tunnel-breadth ratios.

FIGURE 2.—Concluded.

the summation being taken over all positive and negative integral values of m and n except $m=n=0$. If $g/H=0$ equation (3) may be rewritten

$$\sigma\left(0,\frac{s}{B},\frac{B}{H}\right)=\frac{B}{2s}\sum{}'\frac{1}{n^2}\left[\frac{(m+s/B)F_{mn}-(m-s/B)E_{mn}}{E_{mn}F_{mn}}\right]$$

$$=\sum{}'\frac{2m}{E_{mn}F_{mn}[(m+s/B)F_{mn}+(m-s/B)E_{mn}]} \tag{16}$$

where the quantities E_{mn} and F_{mn}, which are introduced for convenience, are defined by the equations

$$E_{mn}=\sqrt{n^2+(m+s/B)^2(B/H)^2}$$

$$F_{mn}=\sqrt{n^2+(m-s/B)^2(B/H)^2}$$

It follows that

$$\sigma\left(0,0,\frac{B}{H}\right)=\sum{}'\frac{1}{[n^2+m^2(B/H)^2]^{3/2}} \tag{17}$$

Substitution of equation (17) into equation (15) yields

$$\Delta_1 U'=\frac{\mu}{4\pi H^3}\,\sigma\left(0,0,\frac{B}{H}\right)$$

If μ is replaced by its value from equation (14) and equation (4) is used, there is obtained

$$\frac{\Delta_1 U'}{U'}=\frac{\pi^{3/2}}{8}\frac{\lambda t^3}{(BH)^{3/2}}\tau\left(0,0,\frac{B}{H}\right)=\frac{ct^2}{C^{3/2}}\frac{\pi^{3/2}}{8}\frac{\lambda t}{c}\tau\left(0,0,\frac{B}{H}\right) \quad (18)$$

If V denotes the volume of the body of revolution, then $V=\kappa_2 ct^2$ where $\kappa_2 \leq \pi/4$ depends on the shape of the meridian section of the body. (For a right circular cylinder whose meridian section is clearly a rectangle $\kappa_2=\pi/4$.) Equation (18) may be rewritten

$$\frac{\Delta_1 U'}{U'}=\frac{K_3 \tau V}{C^{3/2}} \quad (19)$$

or

$$\frac{\Delta_1 U'}{U'}=\frac{K_4 \tau ct^2}{C^{3/2}} \quad (20)$$

where

$$\tau=\tau\left(0,0,\frac{B}{H}\right) \quad (21)$$

$$K_3=\frac{\pi^{3/2}}{8}\frac{\lambda t}{c}\frac{1}{\kappa_2} \quad (22)$$

$$K_4=\frac{\pi^{3/2}}{8}\frac{\lambda t}{c} \quad (23)$$

TABLE IV.—VALUES OF K_3 FOR VARIOUS BODIES OF REVOLUTION

$\frac{t}{c}$	Prolate spheroid	Rankine Ovoid	NACA 111	NACA 133	Fuhrmann source-sink body	Symmetric Fuhrmann source-sink body	(3, 6, −1, 0) body of reference 14
0.08	0.900	0.913	0.902	-----	-----	-----	-----
.10	.905	.923	.909	-----	-----	-----	-----
.12	.910	.932	.916	-----	-----	-----	-----
.14	.917	.941	.924	-----	-----	-----	-----
.16	.924	.949	.931	-----	-----	-----	-----
.18	.931	.957	.939	-----	0.933	0.932	0.925
.20	.938	.964	.948	-----	-----	-----	-----
.24	.954	.979	.964	-----	-----	-----	-----
.30	.980	1.000	-----	0.983	-----	-----	-----
.40	1.025	-----	-----	-----	-----	-----	-----
.50	1.072	-----	-----	-----	-----	-----	-----
1.00	1.329	-----	-----	-----	-----	-----	-----

TABLE V.—VALUES OF K_4 FOR VARIOUS BODIES OF REVOLUTION

$\frac{t}{c}$	Prolate spheroid	Rankine Ovoid	NACA 111	NACA 133	Fuhrmann source-sink body	Symmetric Fuhrmann source-sink body	(3, 6, −1,0) body of reference 14
0.08	0.472	0.670	0.405				
.10	.474	.668	.409				
.12	.477	.665	.413				
.14	.480	.662	.417				
.16	.484	.659	.422				
.18	.488	.655	.428		0.417	0.414	0.436
.20	.491	.650	.433				
.24	.500	.640	.445				
.30	.514	.624		0.462			
.40	.537	.607					
.50	.562	.609					
1.00	.696	.696					

It should be noted that τ for the case of the body of revolution is the same as for the limiting case of a wing when the span approaches zero. As pointed out in reference 2, λ may be calculated for any body of revolution whose pressure distribution is known. The necessary formula is

$$\lambda=4\left(\frac{c}{t}\right)^3\int_0^1\left(\frac{y}{c}\right)^2\sqrt{1-P}\sqrt{1+(dy/dx)^2}\,d\left(\frac{x}{c}\right) \quad (24)$$

where

y radius of body at chordwise station x

dy/dx slope of meridian section at x

P pressure coefficient at x in incompressible flow

Values of λ for prolate spheroids and Rankine Ovoids are given in references 1 and 2. As soon as λ is known for a body, K_4 can be calculated at once from equation (23). The value of κ_2 may be found from the volume of the body of revolution which may be calculated for example by a numer-

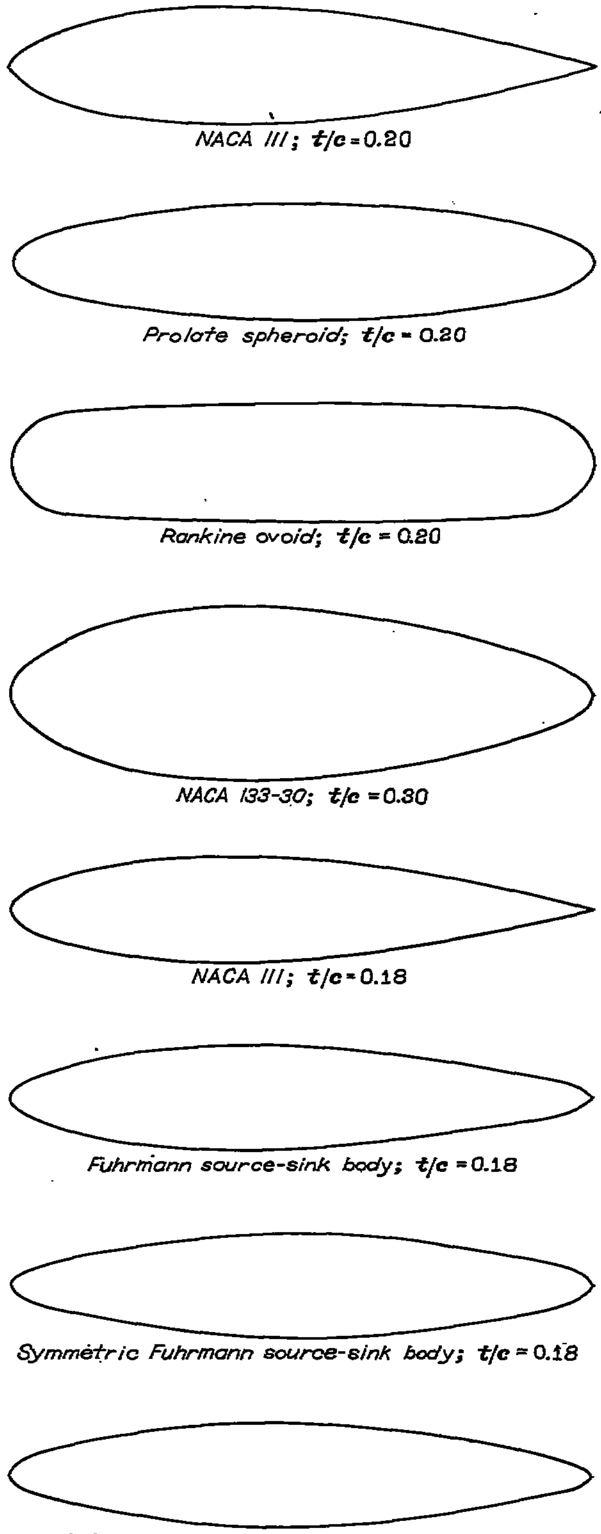


FIGURE 3.—Sample bodies of revolution.

ical integration, and K_3 can be calculated from equation (22) as soon as κ_2 is known.

Values of τ for rectangular tunnels of various breadth-to-height ratios are given in table I and figure 2. Values of K_3 and K_4 for various bodies of revolution are given in tables IV and V. Some of these bodies are drawn in figure 3. Unfortunately these tables are rather incomplete and moreover fuselage shapes used in practice are extremely varied. However, in table IV it is observed that the values of K_3 do not depend very strongly on the shape of the body, although they do depend on the thickness ratio. For this reason it appears that for most fuselages it will be sufficiently accurate to use the values of K_3 given for the NACA 111 bodies. As the values of K_4 given in table V are more dependent on the body shape, it is recommended that equation (19) and table IV be used in preference to equation (20) and table V whenever possible.

CIRCULAR TUNNEL

Body of revolution.—It is convenient to consider the case of a body of revolution before considering the case of the three-dimensional wing because more attention has been given to the former by other authors. (See references 1, 2, 7, 8, 9, and 10.) It will now be shown that the blockage correction is again given by equations (19) and (20) where τ has a value appropriate to a circular tunnel. Since K_3 and K_4 depend on the model and not on the tunnel, they are still given by equations (22) and (23).

The most convenient starting point is the formula of references 1 and 2, namely,

$$\frac{\Delta_1 U'}{U'}=\tau\lambda\left(\frac{S_m}{C}\right)^{3/2} \tag{25}$$

where S_m is the maximum cross-sectional area of the model. It is only necessary to note that

$$S_m=\pi t^2/4$$

$$V=\kappa_2 ct^2$$

It follows at once that

$$\frac{\Delta_1 U'}{U'}=\tau\lambda\frac{\pi^{3/2}}{8}\frac{t^3}{C^{3/2}}=\tau\frac{\lambda t}{c}\frac{\pi^{3/2}}{8}\frac{ct^2}{C^{3/2}} \tag{26}$$

Substitution from equations (22) and (23) reduces this to equations (19) and (20), respectively. It should be noted that τ of references 1 and 2 is identical with τ of the present report.

The value of τ is given in table I and figure 2. Values of K_3 and K_4 are given in tables IV and V. Again equation (19) is preferable to equation (20).

The result of reference 7 fails to take into consideration the body shape and so it is not very useful except for less exact calculations. The result of references 8, 9, and 10 is presented in a different form, namely,

$$\frac{\Delta_1 U'}{U'}=\lambda_V\tau_V\frac{V}{B^3} \tag{27}$$

where

λ_V factor depending on model shape
τ_V factor depending on tunnel shape
V volume of model
B diameter of wind tunnel

It is of interest to show that equations (19) and (20) can be deduced also from equation (27) showing the latter to be equivalent to equation (25). From reference 8, there is obtained

$$\lambda_V=\frac{\pi}{4}\frac{t^3}{V}\lambda$$

$$\tau_V=\frac{4}{\pi}\tau$$

Since also $C=\pi B^2/4$ equation (27) yields at once

$$\frac{\Delta_1 U'}{U'}=\left(\frac{\pi}{4}\frac{t^3}{V}\lambda\right)\left(\frac{4}{\pi}\tau\right)V\left(\frac{\pi^{3/2}}{8}\frac{1}{C^{3/2}}\right)=\tau\frac{\lambda t}{c}\frac{\pi^{3/2}}{8}\frac{ct^2}{C^{3/2}}$$

This is the same as equation (26) which yields equations (19) and (20) directly.

Three-dimensional wing.—The blockage correction for a three-dimensional wing in a circular tunnel is given in references 8, 9, and 10, the formula being of the same form as for a body of revolution in a circular tunnel, namely, equation (27) where τ_V and λ_V have values appropriate to the three-dimensional wing. From reference 8 (using the notation of reference 3 instead of reference 2) there is obtained

$$\lambda_V=\frac{\pi}{8}\frac{2sc^2}{V}\Lambda \tag{28}$$

$$\tau_V=\frac{4}{\pi}\tau \tag{29}$$

Since also $C=B\pi^2/4$ equation (27) yields at once

$$\frac{\Delta_1 U'}{U'}=\left(\frac{\pi}{8}\frac{2sc^2}{V}\Lambda\right)\left(\frac{4}{\pi}\tau\right)V\left(\frac{\pi^{3/2}}{8}\frac{1}{C^{3/2}}\right)=\frac{2sct}{C^{3/2}}\frac{\Lambda}{t/c}\frac{\pi^{3/2}}{16}\tau$$

This is the same as equation (7) which leads directly to equations (8) and (9) with the same definitions of K_1 and K_2 given in equations (11) and (12); τ is, however, given by equation (29) where τ_V is obtained from references 8, 9, or 10.

Thus equations (8) and (9) may also be used for circular tunnels provided only that the appropriate values of τ are used. Values of τ are given in table I and figure 2. Values of K_1 and K_2 are given in tables II and III.

SOLID BLOCKAGE IN COMPRESSIBLE FLOW

In the preceding section the solid-blockage corrections have been determined under the assumption that the fluid is incompressible. It is now necessary to determine the modifications required in these formulas to take account of the effects of the compressibility of the fluid. The methods of references 12 and 13 are very convenient for this purpose. As the required modifications are given incorrectly in references 5 and 6, partially incorrectly in reference 4, and correctly in references 7, 8, 9, and 10, it appears worth while to give some discussion of the matter.

For the purpose of deducing the properties of a compressible flow from those of a corresponding incompressible flow the so-called "Extension of the Prandtl Rule," which was first given in reference 12 and repeated as Method IV in reference 13, is probably of most general application; but the other methods of reference 13 are sometimes more convenient for certain problems. The Extension of the Prandtl Rule may be expressed in the following manner:

The streamline pattern of a compressible flow to be calculated can be compared with the streamline pattern of an incompressible flow which results from the contraction of the y and z axes including the profile contour by the factor $\sqrt{1-M^2}$ (M=free-stream Mach number) (x axis in the direction of the free stream). In the compressible flow the pressure coefficient as well as the increase in the longitudinal velocity are greater in the ratio $1/(1-M^2)$ and the streamline slopes greater in the ratio $1/\sqrt{1-M^2}$ than those at the corresponding points of the equivalent incompressible flow.

Since formulas (8) and (19) for the three-dimensional wing and the body of revolution have the same form and also apply to both rectangular and circular tunnels, it suffices to determine the modification due to compressibility for them. Let the subscript c refer to compressible flow and the subscript i to the corresponding incompressible flow. If V_c is the volume of the model in the compressible flow, then V_i, the volume of the model in the corresponding incompressible flow, is given by

$$V_i=[1-(M')^2]V_c$$

where M' is the apparent free-stream Mach number at the model as determined from measurements taken at a point far ahead of the model. Also if C_c is the cross-sectional area of the tunnel in the compressible flow, then C_i the cross-sectional area of the tunnel in the incompressible flow is given by

$$C_i=[1-(M')^2]\,C_c$$

τ is unaffected by the transformation and the effect on K_1 and K_3 is sufficiently small that it may be neglected. From the Extension of the Prandtl Rule it follows that

$$\left(\frac{\Delta_1 U'}{U'}\right)_c=\frac{1}{1-(M')^2}\left(\frac{\Delta_1 U'}{U'}\right)_i=\frac{1}{1-(M')^2}\,\frac{K_j\tau V_i}{C_i^{3/2}}=\frac{1}{[1-(M')^2]^{3/2}}\,\frac{K_j\tau V_c}{C_c^{3/2}}$$

where j denotes the numbers 1 or 3. Since equations (8) and (9) are equivalent as are also equations (19) and (20), it follows that in all cases it is only necessary to multiply the blockage corrections given by these formulas by $[1-(M')^2]^{-3/2}$ in order to take account of the compressibility of the fluid.

As a check it is useful to determine the compressibility modification for the case of a body of revolution in a circular tunnel by Method II of reference 13, since the derivation is so simple by this method. Both the body shape and the longitudinal velocities are the same in the corresponding compressible and incompressible flows; only the tunnel dimensions are altered by the factor $\sqrt{1-(M')^2}$ so that $C_i=[1-(M')^2]C_c$. There is obtained

$$\left(\frac{\Delta_1 U'}{U'}\right)_c=\left(\frac{\Delta_1 U'}{U'}\right)_i=\frac{K_3\tau V_i}{C^{3/2}}=\frac{1}{[1-(M')^2]^{3/2}}\,\frac{K_3\tau V_c}{C_c^{3/2}}$$

as before.

WAKE BLOCKAGE IN COMPRESSIBLE FLOW

The blockage due to the wake of a model in a two-dimensional-flow tunnel is discussed in some detail in reference 3. Much of this discussion is applicable without change to the case of a three-dimensional flow tunnel. The fundamental idea of replacing the model and its wake by a source (in this case a three-dimensional source) of suitable strength located at the position of the model can be used again and in fact the determination of the source strength Q can be carried out in exactly the same manner. The result is identical with that of reference 3, namely,

$$Q=\frac{\rho' U' C_D' S}{2}\,[1+(\gamma-1)(M')^2] \tag{30}$$

where

- Q mass flow of source rather than volume flow as used previously
- ρ' mass density of fluid at point far upstream
- C_D' uncorrected drag coefficient referred to apparent dynamic pressure q'
- S area on which drag coefficient is based
- γ ratio of specific heat of gas at constant pressure to specific heat at constant volume $\left(\frac{c_p}{c_v}\right)$

In equation (30) powers of M' higher than $(M')^2$ have been neglected.

Consider now a rectagular tunnel of height H and breadth B. The tunnel walls are replaced by a doubly infinite array of sources of strength Q at distances mB to the side and nH above and below the position of the model.

By making use of Method II of reference 13, it is readily shown that a three-dimensional source of strength Q (mass per unit time) in a uniform flow of compressible fluid will induce at the point the coordinates of which are x, y, z relative to the source a streamwise velocity.

$$\Delta u=\frac{Q}{4\pi\rho}\,\frac{x}{[x^2+(1-M^2)(y^2+z^2)]^{3/2}}$$

where the uniform flow is in the x direction and ρ and M are the density and Mach number, respectively, of the undisturbed stream.

It follows that the streamwise velocity $\Delta_2 U'$ induced at a point on the center line of the tunnel by the entire system of images is

$$\Delta_2 U'=\frac{Q}{4\pi\rho'}\sum{}'\,\frac{x}{\{x^2+[1-(M')^2](m^2B^2+n^2H^2)\}^{3/2}}$$

where ρ' and M' are the density and Mach number of the undisturbed flow in the tunnel and the summation is taken over all positive and negative integral values of m and n except $m=n=0$. The velocity induced by the image sources at an infinite distance upstream is

$$(\Delta_2 U')_{-\infty}=\lim_{x\to-\infty}\frac{Q}{4\pi\rho'}\sum{}'\,\frac{x}{\{x^2+[1-(M')^2](m^2B^2+n^2H^2)\}^{3/2}}$$

But conditions far upstream must remain unchanged and to achieve this it is necessary to counterbalance this velocity by superposition of a uniform flow of equal magnitude but opposite sign. The addition of this flow at all points in the field will result in a speeding up of the general flow at the position of the airfoil by the amount

$$\Delta_3 U' = \lim_{x\to\infty} \frac{Q}{4\pi\rho'} \sum{}' \frac{x}{\{x^2+[1-(M')^2](m^2B^2+n^2H^2)\}^{3/2}} \qquad (31)$$

The summation of this series can be obtained by the following artifice. Substitution of equation (30) into equation (31) and setting $M'=0$ gives

$$(\Delta_3 U')_i = \lim_{x\to\infty} \frac{C_D' S U'}{8\pi} \sum{}' \frac{x}{(x^2+m^2B^2+n^2H^2)^{3/2}} \qquad (32)$$

But according to references 4, 8, 9, and 10

$$\left(\frac{\Delta_3 U'}{U'}\right)_i = \frac{1}{4}\frac{C_D' S}{BH} \qquad (33)$$

Comparison of equations (32) and (33) shows that

$$\lim_{x\to\infty} \sum{}' \frac{x}{(x^2+m^2B^2+n^2H^2)^{3/2}} = \frac{2\pi}{BH} \qquad (34)$$

If in equation (34) B and H are replaced by $\sqrt{1-(M')^2}\,B$ and $\sqrt{1-(M')^2}\,H$, respectively, there is obtained

$$\lim_{x\to\infty} \sum{}' \frac{x}{\{x^2+[1-(M')^2](m^2B^2+n^2H^2)\}^{3/2}} = \frac{2\pi}{[1-(M')^2]BH} \qquad (35)$$

Substitution of equations (35) and (30) into equation (31) yields

$$\frac{\Delta_3 U'}{U'} = \frac{1+(\gamma-1)(M')^2}{1-(M')^2}\frac{C_D' S}{4BH} = \frac{1+0.4(M')^2}{1-(M')^2}\frac{C_D' S}{4C} \qquad (36)$$

on setting $\gamma=1.4$. The preceding discussion is for a rectangular tunnel, but in references 8, 9, and 10 the same formula is given for a tunnel of any shape for the case $M'=0$. Consequently, equation (36) may be taken to hold generally for a tunnel of any shape.

In reference 4 the wake-blockage correction is given correctly for incompressible flow but the modification to take account of compressibility is given incorrectly, as it is determined from reference 5 which, as pointed out in reference 13, is incorrect. The effect of compressibility on the wake-blockage correction is determined in references 8, 9, and 10 by means of the correct form of the Extension of the Prandtl Rule but there is some question whether this rule is applicable to this problem. It appears that in using this method to determine the effect of compressibility on the blockage correction, the effect of compressibility on the source strength Q given by equation (30) is overlooked. This explains the discrepancy between equation (36) and the formula of references 8, 9, and 10.

WAKE PRESSURE GRADIENT IN COMPRESSIBLE FLOW

The effect of the pressure gradient caused by the wake is discussed in reference 3 for compressible flow in a two-dimensional wind tunnel and in reference 11 for compressible flow in a circular wind tunnel. The method of reference 11 is equally applicable to the case of a rectangular wind tunnel if the appropriate value of τ (table I) is used.

The longitudinal velocity increment due to the effect of the tunnel boundaries on the source used to simulate the wake has an approximately linear gradient in the stream direction at the model location. This linear gradient in the velocity is equivalent to a linear gradient in the pressure as is easily seen from the approximate relation.

$$\Delta p' = -2q' \frac{\Delta_2 U'}{U'}$$

In reference 11 it is shown that the gradient in $\Delta_2 U'$ for a source in a wind tunnel is identically equal to the value of $\Delta_2 U'$ for a doublet in a wind tunnel. In the notation of the present report there is obtained

$$\left(\frac{dp'}{dx}\right)_i = -q' \sqrt{\frac{\pi}{4}}\,\tau\,\frac{C_D' S}{C^{3/2}}$$

For compressible flow this becomes

$$\frac{dp'}{dx} = -q' \frac{1+(\gamma-1)(M')^2}{[1-(M')^2]^{3/2}} \sqrt{\frac{\pi}{4}}\,\tau\,\frac{C_D' S}{C^{3/2}}$$

$$= -q' \frac{1+0.4(M')^2}{[1-(M')^2]^{3/2}} \sqrt{\frac{\pi}{4}}\,\tau\,\frac{C_D' S}{C^{3/2}}$$

The increase in the drag resulting from this pressure gradient is equal to the product of the pressure gradient by the sum of the actual model volume and the virtual volume (reference 2). It should be noted that the constants K_1 and K_3 of tables II and IV are equal to $\sqrt{(\pi/4)}$ times the ratio of the sum of the actual model volume and the virtual volume to the actual model volume, these quantities being calculated under the assumption of incompressible flow. Thus the increase in drag coefficient caused by the pressure gradient is given by

$$\Delta C_{D_w}' = \frac{1+0.4(M')^2}{[1-(M')^2]^{3/2}} \frac{K_1 \tau V_w C_D'}{C^{3/2}} \qquad (37)$$

for the wing, and

$$\Delta C_{D_b}' = \frac{1+0.4(M')^2}{[1-(M')^2]^{3/2}} \frac{K_3 \tau V_b C_D'}{C^{3/2}} \qquad (38)$$

for the body of revolution.

It is pointed out in reference 11 that the virtual volume is altered by the compressibility of the fluid. Thus equations (37) and (38) can be slightly improved if K_1 and K_3 appearing therein are corrected to take account of this effect. The necessary modification is made by replacing K_1 and K_3 in these equations by K_{1p} and K_{3p}, respectively, where

$$K_{jp} = \sqrt{\frac{\pi}{4}}\left[1+h(M')\left(\sqrt{\frac{4}{\pi}}K_j - 1\right)\right],\ j=1,3 \qquad (39)$$

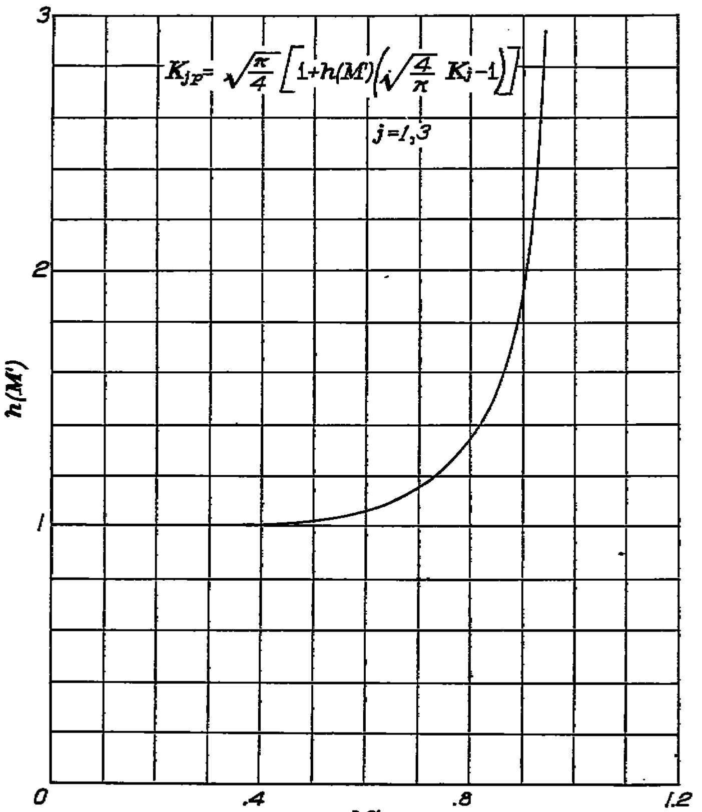


FIGURE 4.—Linear compressibility correction factor for virtual volume. From reference 11.

where $h(M')$ is given by figure 4 which is reproduced from reference 11. It should be noted that the improvement in modifying K_1 and K_3 for compressibility will be small especially in the case of K_3 for the body of revolution. It is important only for quite high Mach numbers. This additional refinement was not made in reference 3.

It appears that a similar compressibility modification of K_1 and K_3 in the formulas for the solid blockage should be made, but the exact modification required is not known. However, it is believed to be small.

CORRECTION OF MEASURED QUANTITIES

The true velocity U at a model consisting of a body of revolution and wing can be obtained from the apparent velocity U' by applying the solid-blockage and wake-blockage corrections. The true velocity may be written in the form

$$U=U'(1+K) \tag{40}$$

where

$$K=K_w+K_b+K_{wk} \tag{41}$$

In equation (38) K_w is the solid-blockage correction due to the wing and is given by

$$K_w=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_1\tau V_w}{C^{3/2}} \tag{42}$$

or

$$K_w=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_2\tau 2sc_wt_w}{C^{3/2}} \tag{43}$$

K_b is the solid-blockage correction due to the body of revolution, being given by

$$K_b=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_3\tau V_b}{C^{3/2}} \tag{44}$$

or

$$K_b=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_4\tau c_bt_b^2}{C^{3/2}} \tag{45}$$

K_{wk} is the wake-blockage correction and is given by

$$K_{wk}=\frac{1+0.4(M')^2}{1-(M')^2}\frac{C_D'S}{4C} \tag{46}$$

It is evident that a correction to the apparent velocity in a compressible flow implies corrections also to the apparent density, dynamic pressure, Reynolds number, and Mach

TABLE VI.—COMPRESSIBILITY FACTORS FOR CORRECTION EQUATIONS

M'	$\frac{1}{1-(M')^2}$	$\frac{1}{[1-(M')^2]^{3/2}}$	$\frac{2-(M')^2}{[1-(M')^2]^{3/2}}$	$\frac{1-0.7(M')^2}{[1-(M')^2]^{3/2}}$	$\frac{1+0.2(M')^2}{[1-(M')^2]^{3/2}}$	$\frac{1+0.4(M')^2}{1-(M')^2}$	$\frac{[2-(M')^2][1+0.4(M')^2]}{1-(M')^2}$	$\frac{[1-0.7(M')^2][1+0.4(M')^2]}{1-(M')^2}$	$\frac{[1+0.2(M')^2][1+0.4(M')^2]}{1-(M')^2}$	$(M')^2$	$2-(M')^2$	$1-0.7(M')^2$	$1+0.2(M')^2$
0.200	1.042	1.063	2.034	1.033	1.072	1.058	2.074	1.029	1.067	0.0400	1.960	0.9720	1.008
.300	1.099	1.152	2.200	1.079	1.173	1.138	2.174	1.067	1.159	.0900	1.910	.9370	1.018
.400	1.191	1.299	2.390	1.154	1.341	1.267	2.331	1.125	1.307	.1600	1.840	.8880	1.032
.500	1.333	1.540	2.694	1.270	1.617	1.467	2.566	1.210	1.540	.2500	1.750	.8250	1.050
.550	1.434	1.717	2.914	1.353	1.821	1.607	2.728	1.267	1.705	.3025	1.698	.7882	1.060
.600	1.563	1.953	3.203	1.461	2.094	1.788	2.931	1.337	1.916	.3600	1.640	.7480	1.072
.625	1.641	2.102	3.383	1.527	2.266	1.897	3.054	1.379	2.046	.3906	1.609	.7266	1.078
.650	1.732	2.279	3.595	1.605	2.471	2.024	3.193	1.426	2.195	.4225	1.578	.7042	1.084
.675	1.837	2.489	3.845	1.696	2.716	2.172	3.354	1.479	2.369	.4556	1.544	.6811	1.091
.700	1.961	2.746	4.146	1.804	3.015	2.345	3.541	1.541	2.575	.4900	1.510	.6570	1.098
.725	2.108	3.061	4.513	1.935	3.382	2.551	3.761	1.613	2.819	.5256	1.474	.6321	1.105
.750	2.286	3.456	4.968	2.095	3.845	2.800	4.025	1.697	3.115	.5625	1.438	.6062	1.112
.775	2.504	3.962	5.545	2.297	4.438	3.105	4.346	1.800	3.478	.6006	1.399	.5796	1.120
.800	2.778	4.630	6.296	2.556	5.222	3.489	4.745	1.926	3.936	.6400	1.360	.5520	1.128
.820	3.052	5.333	7.080	2.823	6.050	3.874	5.142	2.050	4.395	.6724	1.328	.5293	1.134
.840	3.397	6.260	8.103	3.168	7.143	4.355	5.637	2.204	4.970	.7056	1.294	.5061	1.141
.860	3.840	7.526	9.486	3.630	8.639	4.976	6.272	2.400	5.712	.7396	1.260	.4823	1.148
.880	4.433	9.333	11.44	4.273	10.78	5.806	7.116	2.659	6.705	.7744	1.226	.4579	1.155
.900	5.263	12.08	14.37	5.229	14.03	6.968	8.292	3.018	8.098	.8100	1.190	.4330	1.162
.910	5.817	14.03	16.44	5.897	16.36	7.744	9.076	3.255	9.026	.8281	1.172	.4203	1.166
.920	6.510	16.61	19.16	6.770	19.43	8.715	10.05	3.552	10.19	.8464	1.154	.4075	1.169
930	7.402	20.14	22.86	7.946	23.62	9.963	11.31	3.931	11.69	.8649	1.135	.3946	1.173

number. These corrections are readily obtained on the basis of the usual assumption that the flow is adiabatic. It is assumed that the correction terms are small compared with unity, so that squares and products of these terms may be neglected. The analysis follows the lines of reference 3, and it is not necessary to repeat the details here. The following equations are obtained:

$$\rho=\rho'[1-(M')^2K] \tag{47}$$

$$q=q'\{1+[2-(M')^2]K\} \tag{48}$$

$$R=R'\{1+[1-0.7(M')^2]K\} \tag{49}$$

$$M=M'\{1+[1+0.2(M')^2]K\} \tag{50}$$

The drag coefficient must be corrected for the effect of the pressure gradient due to the wake as well as to refer it to the correct dynamic pressure. There is thus obtained

$$C_D=C_D'\{1-[2-(M')^2]K\}-\Delta C_{D_w}'-\Delta C_{D_b}' \tag{51}$$

where $\Delta C_{D_w}'$ and $\Delta C_{D_b}'$ are given by equations (37) and (38).

Numerical values of the functions of M' which appear in these equations are given in table VI.

CONCLUDING REMARKS

Data obtained from tests of three-dimensional models, which are small relative to the wind-tunnel dimensions, can be corrected for solid and wake blockage and for the pressure gradient due to the wake by means of the following equations:

$$U=U'(1+K) \tag{40}$$

$$q=q'\{1+[2-(M')^2]K\} \tag{48}$$

$$R=R'\{1+[1-0.7(M')^2]K\} \tag{49}$$

$$M=M'\{1+[1+0.2(M')^2]K\} \tag{50}$$

$$C_D=C_D'\{1-[2-(M')^2]K\}-\Delta C_{D_w}'-\Delta C_{D_b}' \tag{51}$$

In the preceding equations K is obtained from the following:

$$K=K_w+K_b+K_{wk} \tag{41}$$

where

$$K_w=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_1\tau V_w}{C^{3/2}} \tag{42}$$

or

$$K_w=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_2\tau 2sc_wt_w}{C^{3/2}} \tag{43}$$

$$K_b=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_3\tau V_b}{C^{3/2}} \tag{44}$$

or

$$K_b=\frac{1}{[1-(M')^2]^{3/2}}\frac{K_4\tau c_bt_b^2}{C^{3/2}} \tag{45}$$

$$K_{wk}=\frac{1+0.4(M')^2}{1-(M')^2}\frac{C_D'S}{4C} \tag{46}$$

$$\Delta C_{D_w}'=\frac{1+0.4(M')^2}{[1-(M')^2]^{3/2}}\frac{K_1\tau V_wC_D'}{C^{3/2}} \tag{37}$$

$$\Delta C_{D_b}'=\frac{1+0.4(M')^2}{[1-(M')^2]^{3/2}}\frac{K_3\tau V_bC_D'}{C^{3/2}} \tag{38}$$

In these equations τ is a factor which for a body of revolution depends only on the shape of the tunnel; whereas for a three-dimensional wing τ depends on the ratio of the wing span to the tunnel breadth as well as on the tunnel shape. (The wing span is in the direction of the tunnel breadth.) Values of τ are given in table I and figure 2. The constants K_1 and K_2 for the three-dimensional wing depend only on the wing base profile shape and can be calculated by means of equations (11), (12), and (13). Values of K_1 and K_2 for a number of wing profiles are given in tables II and III. The constants K_3 and K_4 for the body of revolution depend only on the shape of the body and can be calculated by means of equations (22), (23), and (24). Values of these constants for a number of body shapes are given in tables IV and V; some of these shapes are drawn in figure 3. Since these tables are incomplete and fuselage forms are not standardized as are wing sections, it is recommended that the values of K_3 given in table IV for the NACA 111 series of shapes be used for any fuselage shape which does not differ too greatly from an NACA 111 shape. This implies that equation (19) and table IV should be used in preference to equation (20) and table V whenever possible. Numerical values of the functions of M' which appear in the correction equations are given in table VI.

The constants K_1 and K_3 appearing in equations (37) and (38) may be modified for compressibility by means of figure 4. The modified values of K_1 and K_3 are to be used in equations (37) and (38) only and not in equations (42) and (44).

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APPENDIX A

SUMMATION OF THE INFINITE SERIES FOR $\sigma(0, s/B, B/H)$

From equations (3) and (16) it is seen that $\sigma(0, s/B, B/H)$ may be written in the alternative forms

$$\sigma\left(0, \frac{s}{B}, \frac{B}{H}\right)=\frac{B}{2s}\sum{}' \frac{1}{n^2}\left(\frac{m+s/B}{E_{mn}}-\frac{m-s/B}{F_{mn}}\right) \quad \text{(A1)}$$

or

$$\sigma\left(0, \frac{s}{B}, \frac{B}{H}\right)=\sum{}' \frac{2m}{E_{mn}F_{mn}[(m+s/B)F_{mn}+(m-s/B)E_{mn}]} \quad \text{(A2)}$$

where the summation is taken for all positive and negative integral values of m and n except $m=n=0$ and the quantities E_{mn} and F_{mn}, which are introduced for convenience, are defined by the equations

$$E_{mn}=\sqrt{n^2+(m+s/B)^2(B/H)^2}$$
$$F_{mn}=\sqrt{n^2+(m-s/B)^2(B/H)^2}$$

It is possible to sum the series for $\sigma(0, s/B, B/H)$ exactly only when $s/B=\frac{1}{2}$. In this case equations (A1) and (A2) yield

$$\sigma\left(0, \frac{1}{2}, \frac{B}{H}\right)=2\sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{m=-\infty}^{\infty}\left[\frac{m+\frac{1}{2}}{\sqrt{n^2+\left(m+\frac{1}{2}\right)^2(B/H)^2}}-\frac{m-\frac{1}{2}}{\sqrt{n^2+\left(m-\frac{1}{2}\right)^2(B/H)^2}}\right]+2\sum_{m=1}^{\infty}\frac{m}{(m^2-1/4)^2(B/H)^3}$$

$$=2\sum_{n=1}^{\infty}\frac{1}{n^2}\lim_{m\to\infty}\frac{2\left(m+\frac{1}{2}\right)}{\sqrt{n^2+\left(m+\frac{1}{2}\right)^2(B/H)^2}}+\frac{2}{(B/H)^3}\sum_{m=1}^{\infty}\frac{1}{2}\left[\frac{1}{\left(m-\frac{1}{2}\right)^2}-\frac{1}{\left(m+\frac{1}{2}\right)^2}\right]$$

$$=\frac{4}{B/H}\sum_{n=1}^{\infty}\frac{1}{n^2}+\frac{4}{(B/H)^3}=\frac{4}{B/H}\left[\frac{\pi^2}{6}+\frac{1}{(B/H)^2}\right]$$

For other values of s/B it is necessary to sum the series numerically. The series may be rewritten

$$\sigma\left(0, \frac{s}{B}, \frac{B}{H}\right)=8\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{m}{E_{mn}F_{mn}[(m+s/B)F_{mn}+(m-s/B)E_{mn}]}+2\sum_{n=1}^{N}\frac{1}{n^2\sqrt{n^2+(s/H)^2}}+\frac{2}{(B/H)^3}\sum_{m=1}^{N}\frac{m}{[m^2-(s/B)^2]^2}+R_N\left(0, \frac{s}{B}, \frac{B}{H}\right) \quad \text{(A3)}$$

where N is an arbitrary positive integer and $R_N(0, s/B, B/H)$ denotes a remainder term. In evaluating $\sigma(0, s/B, B/H)$ in the present report, N was usually taken equal to 7 and the summations indicated in equation (A3) were carried out exactly; whereas an approximate value was used for the remainder term. An approximate formula for $R_N(0, s/B, B/H)$ is

$$R_N\left(0, \frac{s}{B}\frac{B}{H}\right)=\frac{1}{(B/H)^3\left(N+\frac{1}{2}\right)^2}+\frac{1}{\left(N+\frac{1}{2}\right)^2}+4\left[\frac{2}{B/H}+\frac{2}{(B/H)^2}-\frac{2\sqrt{(B/H)^2\left(N+\frac{1}{2}\right)^2+1/4}}{(B/H)^2\left(N+\frac{1}{2}\right)}-\frac{2\sqrt{(B/H)^2(1/4)+\left(N+\frac{1}{2}\right)^2}}{(B/H)^2\left(N+\frac{1}{2}\right)}+\frac{\sqrt{(B/H)^2+1}}{(B/H)^2\left(N+\frac{1}{2}\right)}\right] \quad \text{(A4)}$$

It will now be shown how formula (A4) is obtained. It is easily verified that

$$\int_b^{\infty}dy\int_a^{\infty}\frac{dx}{[y^2+(B/H)^2x^2]^{3/2}}=\frac{1}{(B/H)b}+\frac{1}{(B/H)^2a}-\frac{\sqrt{(B/H)^2a^2+b^2}}{(B/H)^2ab} \quad \text{(A5)}$$

and that

$$\int_a^{\infty}\frac{dx}{x^3}=\frac{1}{2a^2} \quad \text{(A6)}$$

provided a, $b>0$. Now $R_N(0, s/B, B/H)$ is approximately equal to $R_N(0,0,B/H)$ and it is easily found that

$$R_N(0,0,B/H)=4\left(\sum_{n=1}^{\infty}\sum_{m=N+1}^{\infty}+\sum_{n=N+1}^{\infty}\sum_{m=1}^{N}\right)\frac{1}{[n^2+m^2(B/H)^2]^{3/2}}+$$

$$2\sum_{n=N+1}^{\infty}\frac{1}{n^3}+\frac{2}{(B/H)^3}\sum_{m=N+1}^{\infty}\frac{1}{m^3}$$

These summations may be approximated by suitable integrals so that approximately

$$R_N\left(0,\frac{s}{B},\frac{B}{H}\right)=4\left(\int_{\frac{1}{2}}^{\infty}dy\int_{N+\frac{1}{2}}^{\infty}dx+\int_{N+\frac{1}{2}}^{\infty}dy\int_{\frac{1}{2}}^{\infty}dx-\right.$$

$$\left.\int_{N+\frac{1}{2}}^{\infty}dy\int_{N+\frac{1}{2}}^{\infty}dx\right)\frac{1}{[y^2+(B/H)^2x^2]^{3/2}}+$$

$$2\left[1+\frac{1}{(B/H)^3}\right]\int_{N+\frac{1}{2}}^{\infty}\frac{dx}{x^3} \tag{A7}$$

If the integrals of equation (A7) are evaluated by means of equations (A5) and (A6), then equation (A4) is obtained.

APPENDIX B

LIST OF IMPORTANT SYMBOLS

B — tunnel breadth or diameter
H — tunnel height
C — tunnel cross-sectional area
c — chord of airfoil or body of revolution
t — maximum thickness of airfoil or body of revolution
V — volume of wing or body of revolution
S_m — maximum cross-sectional area of body of revolution
s — half span of wing
C_D — drag coefficient
S — model area on which drag coefficient is based
U — stream velocity
M — Mach number
R — Reynolds number
γ — ratio of specific heat of gas at constant pressure to specific heat at constant volume (c_p/c_v)
ρ — mass density
q — dynamic pressure
Λ, K_1, K_2 — factors depending on shape of airfoil base profile (See equations (11), (12), and (13) and tables II and III.)
λ, K_3, K_4 — factors depending on shape of body of revolution (See equations (22), (23), and (24) and tables IV and V.)
$h(M')$ — linear compressibility correction factor for virtual volume (reference 11)
τ — factor depending on tunnel shape and wing-span-to-tunnel-breadth ratio (See equations (10) and (21) and table I.)
K — total blockage correction (See equation (41).)
K_w — wing-blockage correction (See equations (42) and (43).)
K_b — body-blockage correction (See equations (44) and (45).)
K_{wk} — wake-blockage correction (See equation (46).)

Superscript:

$(')$ — when pertaining to fluid properties, denotes values existing in tunnel far upstream from model; when pertaining to airfoil characteristics, denotes values in tunnel, coefficients being referred to apparent dynamic pressure q'

Subscripts:

(Used only when necessary to avoid ambiguity)

c — denotes values in compressible fluid
i — denotes values in incompressible fluid
w — denotes values for wing
b — denotes values for body of revolution
wk — denotes values for wake.

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