REPORT 1006

ANALYSIS OF THRUST AUGMENTATION OF TURBOJET ENGINES BY WATER INJECTION AT COMPRESSOR INLET INCLUDING CHARTS FOR CALCULATING COMPRESSION PROCESSES WITH WATER INJECTION [1]

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SUMMARY

A psychrometric chart having total pressure (sum of partial pressures of air and water vapor) as a variable, a Mollier diagram for air saturated with water vapor, and charts showing the thermodynamic properties of various air–water vapor and exhaust gas–water vapor mixtures are presented as aids in calculating the thrust augmentation of a turbojet engine resulting from the injection of water at the compressor inlet.

Curves are presented that show the theoretical performance of the augmentation method for various amounts of water injected and the effects of varying flight Mach number, altitude, ambient-air temperature, ambient relative humidity, compressor pressure ratio, and inlet-diffuser efficiency. Numerical examples, illustrating the use of the psychrometric chart and the Mollier diagram in calculating both compressor-inlet and compressor-outlet conditions when water is injected at the compressor inlet, are presented.

For a typical turbojet engine operating at sea-level zero flight Mach number conditions, the maximum theoretical ratio of augmented to normal thrust, when a nominal decrease in compressor efficiency with water injection was assumed, was 1.29. The ratio of augmented liquid consumption to normal fuel flow for these conditions, assuming complete evaporation, was 5.01. Both the augmented thrust ratio and the augmented liquid ratio increased rapidly as the flight Mach number was increased and decreased as the altitude was increased. Although the thrust augmentation possible from saturating the compressor-inlet air is very low at low flight speeds, appreciable gains in thrust are possible at high flight Mach numbers. For standard sea-level atmospheric temperature, the relative humidity of the atmosphere had a small effect on the augmented thrust ratio for all flight speeds investigated. For sea-level zero flight Mach number conditions, the augmented thrust ratio increased as the atmospheric temperature increased for low values of atmospheric relative humidity. Water injection therefore tends to overcome the loss in take-off thrust normally occurring at high ambient temperatures. For very high atmospheric relative humidities, the ambient temperature had only a small effect on the augmented thrust ratio.

INTRODUCTION

One method of augmenting the thrust of turbojet engines is to inject water or a nonfreezing mixture of water and alcohol into the air entering the compressor of the engine. The evaporation of this injected liquid extracts heat from the air and results, for the same compressor work input, in a higher compressor pressure ratio. This increased pressure ratio is reflected throughout the engine and increases both the mass flow of gases through the engine and the exhaust-jet velocity; both factors increase the thrust produced by the engine.

An analysis of the evaporative cooling process can be conveniently divided into two phases: (1) The cooling occurring at constant pressure before the air enters the compressor, and (2) the additional cooling associated with further evaporation during the mechanical compression process. Analysis of both phases of the evaporative cooling process and the thrust augmentation produced for a typical turbojet engine were conducted at the NACA Lewis laboratory during 1947–48 and are described herein.

A specialized psychrometric chart that permits calculation of the constant-pressure evaporation process was developed and is presented. This psychrometric chart differs from the usual form, which is valid for only one value of total pressure (sum of partial pressures of air and water vapor), in that pressure is included as a variable and the temperature range has been greatly extended.

The evaporation of water during the mechanical compression process greatly complicates the calculation of this portion of the turbojet-engine cycle by conventional methods because of the trial-and-error solution involved. In order to provide a more convenient method for calculating a compression process during which evaporative cooling occurs, a Mollier diagram for air saturated with water vapor was developed. This Mollier diagram is similar to that presented in reference 1, but the ranges of temperature and pressure are extended to values suitable for use in turbojet-engine-compressor problems.

As a further aid in calculating the turbojet-engine cycle when water is injected at the compressor inlet, curves are presented giving the thermodynamic properties of the working fluid for both the compression and expansion processes. The use of both the psychrometric chart and the Mollier diagram is illustrated by means of numerical examples.

The performance of a turbojet engine utilizing thrust augmentation by water injection at the compressor inlet was evaluated over a range of flight Mach numbers and altitudes and for a range of water-injection rates. The effect on thrust augmentation of varying ambient-air temperature, relative humidity, compressor pressure ratio, and inlet-diffuser efficiency was also investigated. A comparison of the water-injection method of thrust augmentation with other thrust-augmentation methods is presented in references 2 and 3.

ASSUMPTIONS AND METHODS OF ANALYSIS

In evaluating the performance of a turbojet engine utilizing water injection at the compressor inlet, the psychrometric chart was used to obtain the compressor-inlet conditions, the Mollier diagram was used to obtain the conditions at the end of the evaporation process in the compressor, and the remainder of the turbojet-engine cycle was calculated by conventional step-by-step methods.

The following discussion is a description of (a) the assumptions and the methods used in deriving the psychrometric chart and the Mollier diagram, and (b) the assumed design constants and component efficiencies of the turbojet engine used to illustrate performance with water injection at the compressor inlet. All symbols used in the analysis are defined in appendix A and the derivations of the equations necessary for obtaining the psychrometric chart and the Mollier diagram are presented in appendix B.

PSYCHROMETRIC CHART

The familiar psychrometric chart used in air-conditioning calculations is generally limited to values of total pressure (sum of partial pressures of air and water vapor) very near standard sea-level total pressure and to relatively low temperatures, usually less than 300° F. Inasmuch as the conditions of temperature and pressure at the compressor inlet of a turbojet engine vary widely over ranges not included in the usual psychrometric chart, the chart described herein was constructed to include these ranges.

In deriving the psychrometric chart, the following assumptions were made:

(a) The air and the water vapor are always at the same temperature.

(b) Dalton's law of partial pressures is valid for the mixture.

(c) The thermodynamic properties of water vapor are functions only of temperature for the range of vapor pressures considered.

(d) Air is a perfect gas.

By means of these assumptions, the theory of mixtures, the thermodynamic data for air and water vapor contained in references 4 and 5, respectively, and the equations derived in appendix B, a psychrometric chart with pressure as a variable and for temperatures up to 1500° R was constructed.

MOLLIER DIAGRAM

Because of the somewhat involved and tedious trial-and-error methods generally used for calculating compression processes during which water is evaporated, the Mollier diagram described in this report was prepared to provide a more direct solution of this particular problem.

The data presented in the Mollier diagram were calculated using the following assumptions:

(a) All values presented on the diagram are for a saturated mixture containing 1 pound of air.

(b) The water vapor and the air in the mixture are at the same temperature.

(c) Dalton's law of partial pressures is valid for the mixture.

(d) Air is a perfect gas.

The data presented in the Mollier diagram were calculated using the aforementioned assumptions, the thermodynamic-properties data for air and water vapor presented in references 4 and 5, respectively, and the equations derived in appendix B.

Examples of the use of the psychrometric chart and the Mollier diagram are given in the subsequent section DISCUSSION OF CHARTS and in appendix C.

THRUST-AUGMENTATION CALCULATIONS

From an examination of experimental data of complete turbojet engines and of the compressor component alone, both operating with water injection at the compressor inlet, it is apparent that two prime factors cause deviation of the experimentally obtained results from the ideal process. These two factors are the decrease in compressor adiabatic efficiency that occurs with water injection and the failure of all the injected water to evaporate within the confines of the compressor. The decrease in compressor efficiency apparently is a function of the water-injection rate; however, the magnitude of the effect varies considerably with the particular compressor design. Because of the fundamental nature of the effect of compressor efficiency on the turbojet-engine cycle, its effect is relatively obscure and, for the present analysis, an arbitrary variation with water-injection rate was assumed that is representative of the effect experimentally observed in a current compressor. The theoretical results obtained are thus not expected to agree completely with experimental data obtained for engines embodying different compressor designs. The influence of this assumed decrease in compressor efficiency on the performance of the water-injection method of thrust augmentation will be indicated herein.

The degree of evaporation of water varies widely with the mode of injection and the engine operating conditions; however, insufficient data are available to determine the quantitative effects of these conditions on the degree of evaporation. Inasmuch as the thrust produced by the engine is primarily affected by the amount of water that is evaporated, the influence of incomplete evaporation can be evaluated simply by increasing the injected-water rate commensurate with the desired degree of evaporation. In the present analysis, all performance calculations have therefore been made by assuming complete evaporation.

The augmented thrust ratios experimentally obtained for engines embodying centrifugal-flow-type compressors agree quite well with those theoretically predicted herein for the condition of a saturated mixture throughout the compression process. For engines having axial-flow-type compressors, however, the efficacy of the water-injection method is considerably reduced because of centrifugal separation of the water from the air in passing through the compressor. In order to avoid the centrifugal-separation effect, the water must be injected at several stages throughout the compressor.

Inasmuch as considerable increase in thrust is attainable at high inlet temperatures or high flight Mach numbers merely by saturating the compressor-inlet air with water, the performance of the water-injection method has been determined for both the condition of saturation throughout the compression process and saturation only of the compressor-inlet air. The thrust increases obtained for the condition of saturation throughout the compression process are believed indicative of the increases possible for engines having centrifugal-flow compressors, whereas the thrust increases shown for the condition of saturated inlet air are more nearly equal to the possibilities of the water-injection method when applied to an engine having an axial-flow compressor and not utilizing interstage injection. Experimental investigations of interstage injection indicate that the thrust increases available for this method lie between those attainable for saturated inlet air and those obtained by saturation throughout the compression process. The decreased thrust augmentation obtainable with an engine having an axial-flow compressor utilizing interstage injection may be due to the increased sensitivity of the efficiency of this type compressor to water injection.

In order to determine the theoretical thrust augmentation made possible by evaporation of water during compression, step-by-step calculations of normal and augmented thrust for a typical tubojet engine operating under various flight conditions were made. The psychrometric chart was used to determine the compresor-inlet conditions and the Mollier diagram to calculate the conditions at the end of the evaporation process in the compressor with water injection. Equations used in the step-by-step calculations are given in appendix D. It was assumed that the inlet air was diffused to a sufficiently low velocity so that the static temperature at the compressor inlet did not appreciably differ from the total temperature.

The component efficiencies chosen for the typical turbojet engine were: inlet-diffuser adiabatic efficiency (except where otherwise noted), 0.85 up to a flight Mach number of 1.0, linearly decreasing to 0.75 at a flight Mach number of 2.0 (these values of diffuser efficiency correspond to total-pressure recovery ratios of 0.915 and 0.66 at flight Mach numbers of 1.0 and 2.0, respectively); compressor adiabatic efficiency, 0.80; turbine adiabatic efficiency, 0.85; and exhaust-nozzle adiabatic efficiency, 0.95. The compressor adiabatic efficiency is defined as the ratio of isentropic to actual work.

For the compression process with evaporative cooling, it must be recalled that a higher pressure ratio is obtained for a given isentropic work than is obtained with dry air. Inasmuch as experimental data of compressor performance with water injection indicate a decrease in compressor adiabatic efficiency, the compressor efficiency η_c was arbitrarily assumed to decrease according to the relation $\eta_c = 0.80 - \Delta X_c$, where ΔX_c is the change in water vapor–air ratio in the compressor. This decrease in compressor efficiency was assumed for all conditions of altitude and flight Mach number, except when it was desired to show the effect of this assumption on performance. A single-stage, centrifugal-flow compressor with a tip speed of 1500 feet per second

and a slip factor of 0.95 was assumed for all of the calculations except when it was desired to show the effect of normal compressor pressure ratio. For the assumed values of compressor tip speed, slip factor, and efficiency, the resulting compressor pressure ratio for sea-level zero flight Mach number conditions with no water injection was 4.61. The normal engine performance would be equally applicable to an engine having an axial-flow compressor with the same enthalpy rise, pressure ratio, and compressor efficiency. A combustion-chamber pressure loss of 3 percent and a turbine-inlet temperature of 2000° R were assumed. The ambient relative humidity was assumed to be 0.50 except when it was desired to show the effect of varying ambient relative humidity. All liquid water was assumed injected at the compressor inlet at a temperature of 519° R and the air was assumed to remain saturated as long as any liquid water was present. For conditions where all the injected water evaporated before the end of the compression process, that portion of the process occurring after evaporation was calculated from the thermodynamic relations for an adiabatic compression by using the appropriate thermodynamic properties of the resulting air–water vapor mixtures. The adiabatic efficiency of this portion of the compression process was assumed equal to that previously described for the compression process during which evaporation occurred. Calculations were made for flight Mach numbers up to 2.0 and for standard altitudes of sea level and 35,332 feet.

In order to simplify the calculations of the adiabatic compression occurring after the evaporation of all the injected water and the expansion processes occurring in the turbine and the exhaust nozzle, curves of the thermodynamic properties of air–water vapor and exhaust gas–water vapor mixtures were obtained. These curves were obtained from the theory of mixtures and the thermodynamic data for air, water vapor, and exhaust gases.

Because of the low temperatures occurring at high altitudes, a nonfreezing mixture of water and alcohol rather than water alone is usually employed for compressor-inlet injection. Although all the results presented herein are based on the injection of only water at the compressor inlet, they are probably equally applicable to the injection of water-alcohol mixtures because of the close agreement experimentally obtained between results for water- and for water-alcohol-mixture injection.

DISCUSSION OF CHARTS

PSYCHROMETRIC CHART

In the psychrometric chart (fig. 1), total enthalpy is plotted against dry-bulb temperature for various values of water-air ratio. Also included are curves of the ratio of relative humidity to relative pressure, where relative pressure is the ratio of static pressure to standard sea-level static pressure. Inasmuch as evaporation occurs at constant pressure, the value of relative pressure is unchanged by water injection. The curves of constant water-air ratio are approximately straight lines having a positive slope and the curves of constant ratio of relative humidity to relative
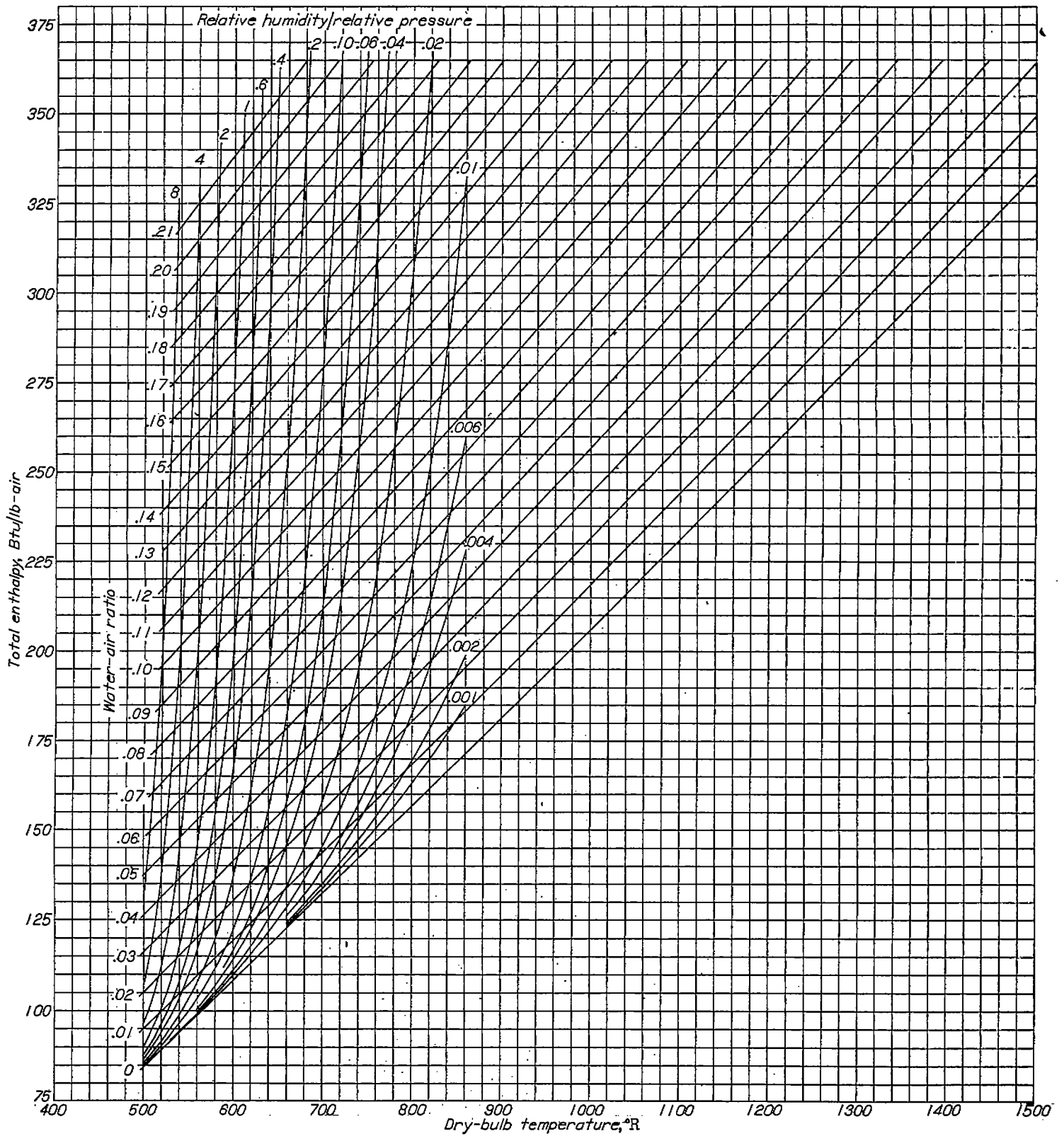
FIGURE 1.—Psychrometric chart.   (A 23- by 25-inch print of this figure is available from NACA upon request.)

pressure are curved upward. The enthalpy scale was arbitrarily chosen so that the enthalpy of a saturated mixture of air and water vapor at a temperature of 519° R and a pressure of 14.696 pounds per square inch was 100 Btu per pound of air.

The temperature resulting after water evaporation may be obtained from figure 1 for a wide range of initial air temperatures and pressures. The temperature can be determined either for the evaporation of a given amount of water or for any final relative humidity.

In the construction of figure 1, the enthalpy of liquid water was assumed to be zero at 519° R. The chart values are thus directly applicable to cases where the water is injected at this temperature. For temperatures of liquid water only slightly different from 519° R, the errors involved in the chart are negligible; however, for extremely accurate work or for temperatures of injected water greatly different from 519° R, corrections must be made for the enthalpy of liquid water. These corrections are the same as the enthalpy corrections for the Mollier diagram discussed in appendix C, and consist in adding the enthalpy of liquid water at the start of the process and, if any liquid remains at the end of the process, subtracting the enthalpy of the remaining liquid.

The method of using the chart is indicated by the following illustrative cases:

Example 1.—For initially dry air at a temperature of 1260° R, assume that sufficient water is injected to give a final water-air ratio of 0.05. From figure 1, the total enthalpy is 271.6 Btu per pound of air for a temperature of 1260° R and a water-air ratio of 0. (For this example and all following examples, the values have been read from the large prints of figs. 1 or 2, which are available from the NACA on request and are more convenient to use when a high degree of accuracy is desirable.) Inasmuch as the evaporation occurs at constant total enthalpy, the temperature after evaporation can be determined from the initial total enthalpy and the final water-air ratio. For a total enthalpy of 271.6 Btu per pound of air and a water-air ratio of 0.05, the temperature is 1008° R. The temperature drop is then 252 F° for these particular conditions.

Example 2.—In order to illustrate the use of the chart in obtaining the temperature and the water-air ratio necessary to saturate air from a given set of initial conditions, initially dry air at a temperature of 1060° R and at twice standard sea-level static pressure is assumed. The enthalpy corresponding to a temperature of 1060° R and a water-air ratio of 0 is 221.0 Btu per pound of air. The value of relative humidity for saturation is 1.0 and for these conditions the value of relative pressure δ is 2.0. The temperature and the water-air ratio are 613° R and 0.0995, respectively, at an enthalpy of 221.0 Btu per pound of air and a ratio of relative humidity to relative pressure φ/δ of 0.5. For these conditions, the initial mixture may be cooled 447 F° by injecting 0.0995 pound of water per pound of air.

Example 3.—The use of the chart in obtaining temperatures resulting from water evaporation when some water is initially present in the air is the same as that previously described, except that the starting enthalpy is found from the initial water-air ratio and temperature. For example, for the conditions of temperature and pressure in example 2 and an initial water-air ratio of 0.02, the initial enthalpy is 247.0 Btu per pound of air. The saturation temperature for an enthalpy of 247.0 Btu per pound and a value of φ/δ of 0.5 is 620° R and the final water-air ratio is 0.122. For these conditions, cooling the initial mixture 440 F° is therefore possible by evaporating 0.102 pound of water per pound of air.

The effect of initial temperature and pressure on the amount of cooling possible from saturation can be readily seen from figure 1. For a constant pressure and consequent constant value of φ/δ ($\varphi=1$), the amount of cooling possible increases as the initial temperature is raised. For a constant temperature, increasing the pressure decreases the value of φ/δ and, from figure 1, decreasing the value of φ/δ decreases the amount of cooling obtained. In general, when some water vapor is initially present, the cooling possible from saturation and the amount of water that must be injected to obtain saturation are somewhat less than those for dry air. In some instances, however, depending on the particular initial conditions, the amount of water that must be injected to obtain saturation is greater when some water vapor is initially present than for dry air, as illustrated in examples 2 and 3.

Example 4.—In order to illustrate the method of using the psychrometric chart when water is injected at a temperature other than 519° R and the enthalpy of the injected water is to be considered, assume that it is desired to saturate initially dry air at a temperature of 910° R and standard sea-level pressure with water injected at a temperature of 619° R. From figure 1, which is correct for water injected at 519° R, the initial enthalpy of the air at a temperature of 910° R is 183.5 Btu per pound. For a value of relative humidity divided by relative pressure of 1.0 (saturation at standard sea-level pressure) and an enthalpy of 183.5 Btu per pound, the temperature is 577° R and the water-air ratio is 0.075. If the water were injected at 519° R, cooling the air 333 F° would be possible by injecting sufficient water to obtain a water-air ratio of 0.075. For water not injected at 519° R, an estimate must be made of the final water-air ratio in order to determine the starting condition. For the present example, assume that the final water-air ratio is 0.08. From the insert of figure 2, the enthalpy of liquid water at a temperature of 619° R is 100 Btu per pound of water. The new starting enthalpy is now 183.5 plus 8.0 (water-air ratio, 0.08) or 191.5 Btu per pound. For a value of φ/δ of 1.0 and an enthalpy of 191.5 Btu per pound, the temperature is 579° R and the water-air ratio is 0.081. If water were injected at a temperature of 619° R, the cooling obtained from saturation would therefore be 331 F° and the required water-air ratio would be 0.081.
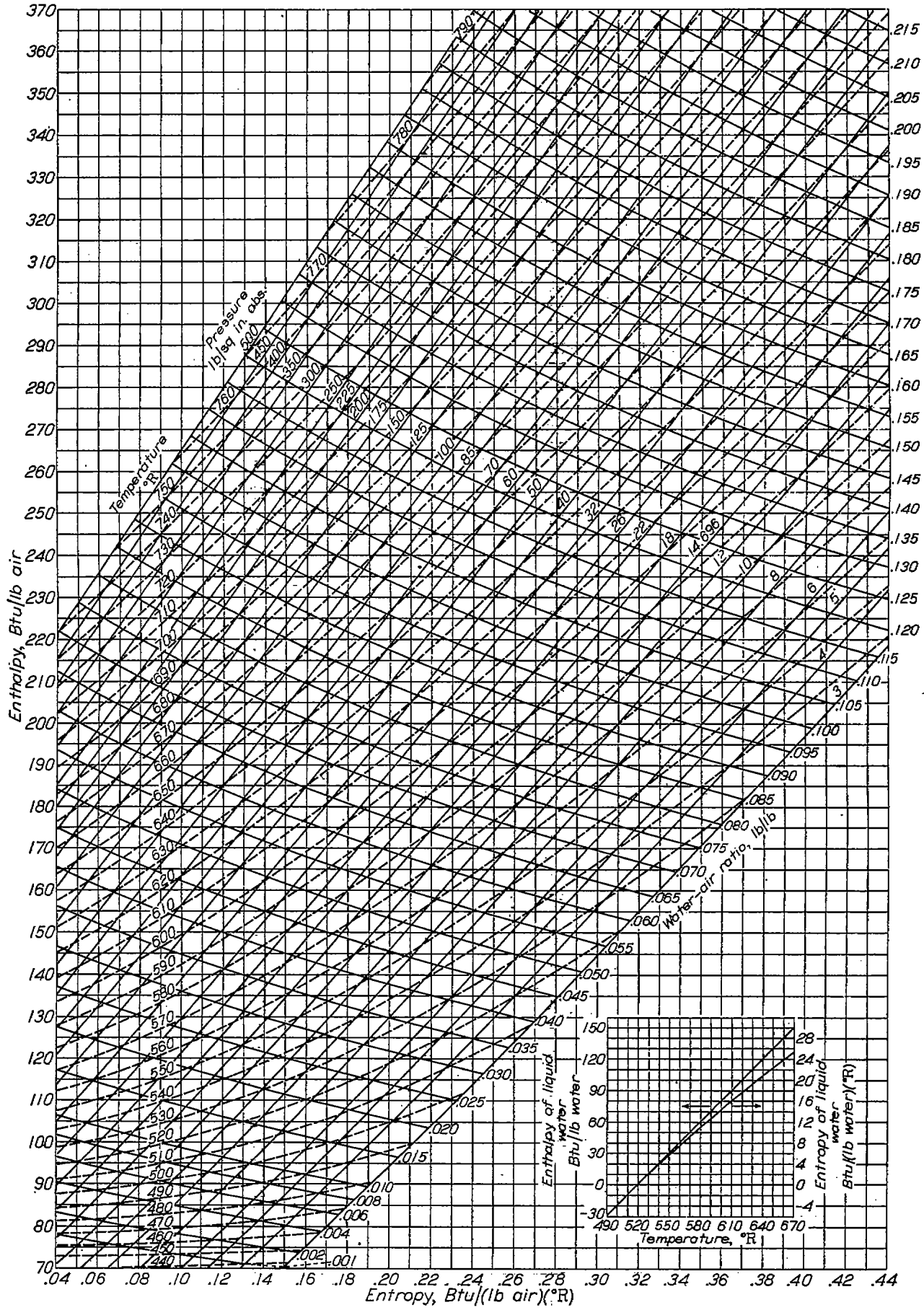
FIGURE 2.—Mollier diagram. (Ten 16- by 21-inch expanded sectional prints available from NACA upon request.)

MOLLIER DIAGRAM

The Mollier diagram (fig. 2) is so constructed that each point represents values for 1 pound of air plus the water vapor necessary for saturation. The abscissa is entropy (in Btu/(lb air) (°R)) and the ordinate is enthalpy (in Btu/lb air). The solid slanting lines with positive slope are lines of constant pressure in pounds per square inch. The broken lines with positive slope are lines of constant temperature (in °R). The solid lines of negative slope are lines of constant weight ratio of water vapor to air. Data are presented for pressures from 3 to 500 pounds per square inch and temperatures from 440° to 790° R. The entropy and the enthalpy of liquid water were assumed to be zero at 519° R and the bases for enthalpy and entropy of air were so fixed that at a temperature of 519° R and a pressure of 14.696 pounds per square inch the enthalpy of the saturated mixture is 100 Btu per pound of air and the entropy is 0.10 Btu/(lb air)(°R). When liquid water is present at a temperature other than 519° R, corrections for enthalpy and entropy must be made. These corrections are discussed in appendix B and a chart giving enthalpy and entropy of liquid water at various temperatures to be used for these corrections is shown on the Mollier diagram.

In order to utilize the Mollier diagram to calculate compressor-outlet conditions with evaporation of water during compression, knowledge of the conditions of the saturated air at the compressor inlet and the actual and isentropic work of the compression process is necessary. The conditions at the end of the saturated compression process may be obtained as follows:

A point is found on the Mollier diagram corresponding to the conditions of saturated air at the compressor inlet. An isentropic compression process is then followed on the diagram and a second point determined by moving vertically along a line of constant entropy until a value of enthalpy equal to the original enthalpy plus the enthalpy of the isentropic work of compression per pound of air (actual work/lb of air multiplied by adiabatic efficiency) is reached. The pressure read from the diagram at this second point is the pressure at the end of the compression process. The final temperature and water-air ratio for the compression process are read from the diagram at a third point, which is obtained by following a constant-pressure line from the second point until a value of enthalpy equal to the original enthalpy plus the enthalpy of the actual work of compression per pound of air is reached.

For a centrifugal-flow compressor, all the water to be evaporated during compression is added at the compressor inlet and the compressor enthalpy rise may be calculated per unit mass flow of the mixture of air and water vapor at the compressor outlet from considerations of tip speed and slip factor. Inasmuch as all values on the Mollier diagram are per unit mass flow of air, the value of enthalpy rise per unit mass flow of mixture at the compressor outlet must be converted to enthalpy rise per unit mass flow of air in order to use the chart. For an axial-flow compressor, if all the water is injected at the compressor inlet, centrifugal separation of the air and water greatly reduces the effects of water

injection and therefore injection of the water in several stages along the compressor length is desirable. If calculations for interstage water injection of this type are desired, the total enthalpy rise must be divided among the sections of the compressor between injection points and calculations similar to those for the centrifugal-flow compressor must be made for each section in order to allow for the change in mass flow from section to section and the corresponding change in enthalpy rise per unit mass flow of air.

When less water is injected than is necessary to saturate the air at the compressor outlet, the compression process is divided into two parts: (1) an adiabatic compression of a saturated mixture, which may be calculated by use of the Mollier diagram, and (2) further adiabatic compression of the resulting mixture of air and water vapor.

The following numerical example is presented to illustrate the method of using the Mollier diagram to calculate the compressor-outlet conditions for a saturated compression process:

Assume that a compressor having an adiabatic efficiency of 0.80 and imparting an enthalpy rise of 80 Btu per pound of air to the working fluid is operated with sufficient water injected at the compressor inlet at a temperature of 519° R to maintain saturation at all times. Further, assume that the inlet air has a relative humidity of 0.50 at a temperature of 530° R and a pressure of 14.7 pounds per square inch and that it is desired to find the compressor-outlet pressure P_3, -outlet temperature T_3, -outlet water vapor–air ratio X_3, and the amount of water evaporated during the process.

The conditions at the compressor inlet after saturation can be obtained from the psychrometric chart (fig. 1). For the initial condition of ambient relative humidity φ of 0.50, temperature T_0 of 530° R, and pressure P_0 of 14.7 pounds per square inch, the water vapor–air ratio X_0 is 0.0077 and the enthalpy H_0 is 100 Btu per pound of air.

After saturation at constant pressure, from figure 1, the temperature T_1 is 519° R, the water vapor–air ratio X_1 is 0.0106, and the enthalpy H_1 is 100 Btu per pound of air.

From the Mollier diagram (fig. 2), for a temperature T_1 of 519° R and a pressure P_1 of 14.7 pounds per square inch, the entropy S_1 is 0.10 Btu/(lb air) (°R).

By increasing the enthalpy at constant entropy by an amount equal to the ideal work of compression, the actual pressure at the compressor outlet may be read from the diagram. The ideal work of compression is

$$\Delta H_{c,i} = \eta_c \Delta H_c = 0.80 \times 80 = 64 \text{(Btu/lb air)}$$

and

$$H_{3,i} = H_1 + \Delta H_{c,i} = 100 + 64 = 164 \text{(Btu/lb air)}$$

The value of pressure read from the diagram at an enthalpy of 164 Btu per pound of air and an entropy of 0.10 Btu/(lb air)(°R) is

$$P_3 = 70.7 \text{(lb/sq in.)}$$

The enthalpy at the compressor outlet is equal to the enthalpy at the inlet plus the actual enthalpy change, or

$$H_3 = H_1 + \Delta H_c = 100 + 80 = 180 \text{ (Btu/lb air)}$$

From the Mollier diagram at a pressure of 70.7 pounds per square inch and an enthalpy of 180 Btu per pound of air, the values of temperature and water vapor–air ratio at the compressor outlet are

$$T_3 = 630° \text{ R and } X_3 = 0.0583$$

The amount of water evaporated is

$$X_3 - X_0 = (X_1 - X_0) + (X_3 - X_1) = (0.0106 - 0.0077) +$$

$$(0.0583 - 0.0106) = 0.0506 \text{ (lb water/lb air)}$$

The amount of water vapor represented by $X_1 - X_0$ is that evaporated at the inlet prior to compression and the amount $X_3 - X_1$ represents that water evaporated during the mechanical compression process.

Additional examples presented in appendix C illustrate the use of the Mollier diagram in calculating a compression process when the water is injected at a temperature other than 519° R and when a given amount of water (less than the amount required to saturate the compressor-outlet air) is injected.
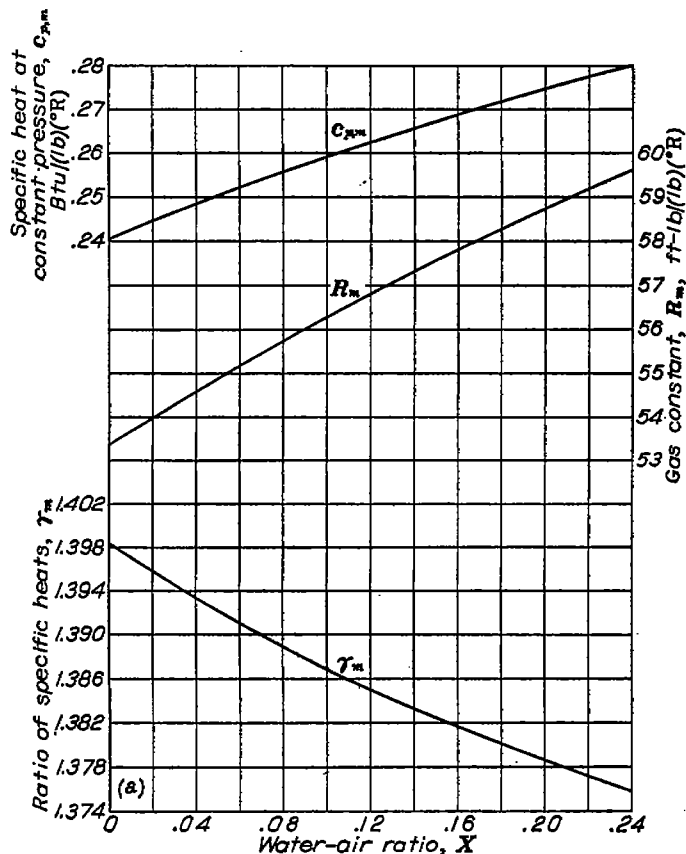
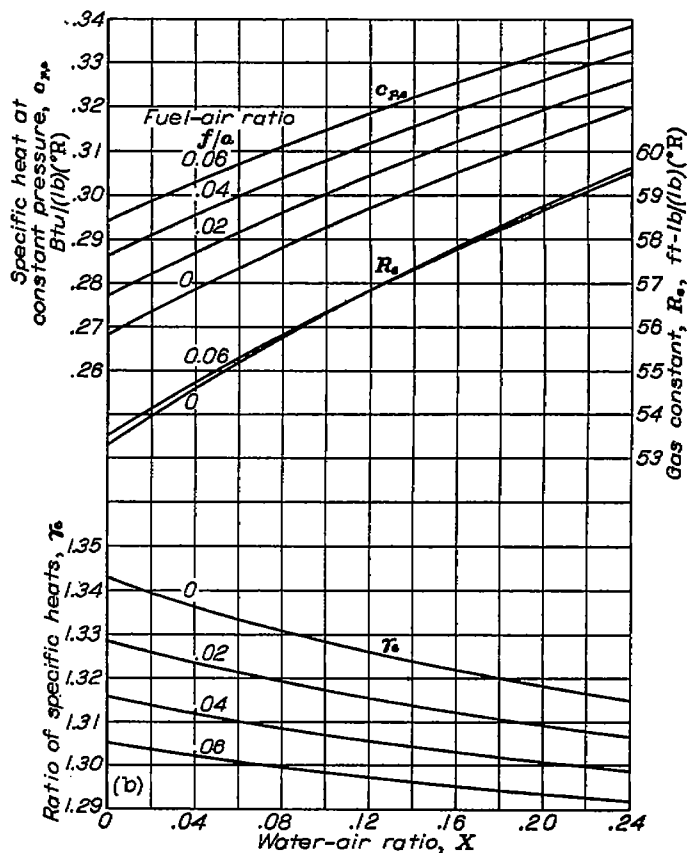(a) Air-water vapor mixture. Temperature, 600° R.

FIGURE 3.—Thermodynamic properties of mixtures at pressure of 0 pound per square inch absolute.

MIXTURE PROPERTIES FOR COMPRESSOR

A chart of the thermodynamic properties of water vapor–air mixtures is shown in figure 3(a). The variation of specific heat at constant pressure with temperature and pressure is small in the range of interest for compressor calculations; figure 3(a) was therefore based on an average temperature of 600° R and zero pressure. Values of the ratio of specific heats γ_m, gas constant R_m, and specific heat at

constant pressure $c_{p,m}$ are plotted for various values of the water vapor–air ratio X. The data presented in this chart were used in calculating compressor-outlet conditions for those cases where less water was injected than the amount necessary to saturate the air at the compressor outlet. The use of figure 3(a) for a case of this type is illustrated in Sample Calculation II (appendix C). The thermodynamic data for air and water vapor used in obtaining the data of figure 3(a) were obtained from references 4 and 5, respectively, and the values presented are for 1 pound of mixture of air and water vapor.

(b) Exhaust gas-water vapor mixture. Temperature, 1650° R; hydrogen-carbon ratio, 0.175.

FIGURE 3.—Concluded. Thermodynamic properties of mixtures at pressure of 0 pound per square inch absolute.

MIXTURE PROPERTIES FOR TURBINE AND JET

A chart of the thermodynamic properties of water vapor–exhaust gas mixtures for a temperature of 1650° R is shown in figure 3(b). Values for the ratio of specific heats γ_e, gas constant R_e, and specific heat at constant pressure $c_{p,e}$ are plotted for various values of the water vapor–air ratio X and the fuel-air ratio f/a. The thermodynamic data for water vapor and exhaust gases necessary for obtaining the data presented in figure 3(b) were obtained from references 5 and 6, respectively, and the values presented are for 1 pound of mixture of exhaust gas and water vapor. (A hydrogen-carbon ratio of the fuel of 0.175 was assumed in preparing these curves.) Although the actual values of γ_e and $c_{p,e}$ vary with temperature, the values given in figure 3(b) can be used for an appreciable temperature range with negligible error because of the compensating manner in which γ_e and $c_{p,e}$ enter the equations for an expansion process. For more accurate work, the method

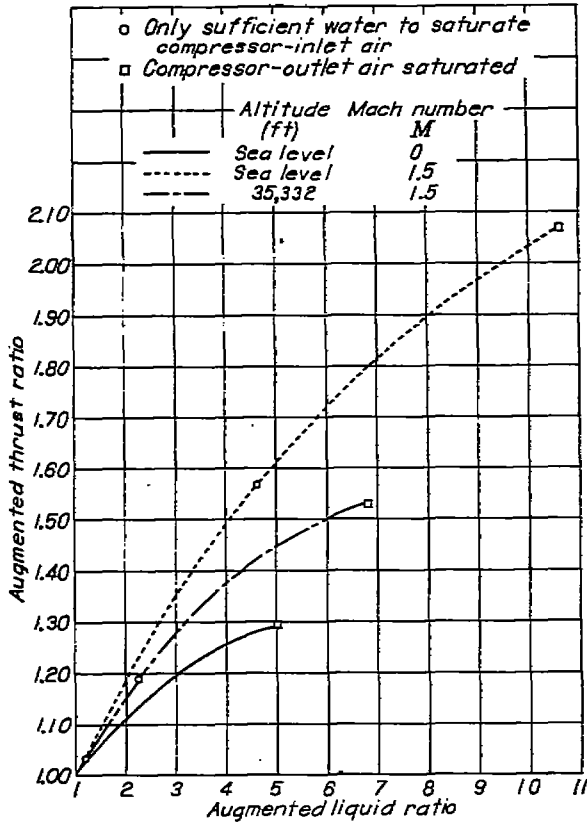presented in reference 7 may be used for calculating the expansion process.

FIGURE 4.—Effect of flight conditions and ratio of augmented to normal liquid flow on thrust augmentation.

DISCUSSION OF AUGMENTATION RESULTS

EFFECT OF WATER-INJECTION RATE

In figure 4, the augmented thrust ratio (ratio of augmented thrust to normal thrust) is shown as a function of the augmented liquid ratio (ratio of augmented total liquid flow to normal fuel flow) for three different operating conditions: flight Mach numbers of 0 and 1.5 at sea-level altitude and a flight Mach number of 1.5 at an altitude of 35,332 feet. The points on the curves indicated by circles represent the case where only sufficient water to saturate the compressor-inlet air is injected with no evaporation occurring during mechanical compression. The end points on the curves represent the condition where the air at the compressor outlet is saturated and indicate the maximum amounts of augmentation theoretically possible by evaporation of water before and during compression in the engine previously described and for the given flight conditions. As the augmented liquid ratio is increased by increasing the amount of injected water, the augmented thrust ratio increases but at a slightly decreasing rate. At sea-level, zero flight Mach number conditions for saturation only up to the compressor inlet, the augmented thrust ratio is 1.035 and the augmented liquid ratio is 1.18. As the augmented liquid ratio is increased by evaporating more water during mechanical compression, the augmented thrust ratio increases to 1.29 at an augmented liquid ratio of 5.01 for compressor-outlet saturation. For a given altitude, increasing the flight Mach number increases the compressor-inlet temperature, which permits the evaporation of more water and thereby increases the maximum

augmented thrust and augmented liquid ratios as well as increasing the augmented thrust ratio for a given augmented liquid ratio. At a sea-level flight Mach number of 1.5 for saturation of the compressor-outlet air, the augmented thrust ratio is 2.07 and the augmented liquid ratio is 10.66. Increasing the altitude for a given flight Mach number results in a decreased compressor-inlet temperature, thereby reducing the amount of water that may be evaporated. At an altitude of 35,332 feet and a flight Mach number of 1.5 for compressor-outlet saturation, the augmented thrust ratio is 1.53 at an augmented liquid ratio of 6.85.

EFFECT OF OPERATING CONDITIONS FOR SATURATED COMPRESSOR-INLET AND COMPRESSOR-OUTLET CONDITIONS

Effect of flight Mach number, altitude, and inlet-diffuser efficiency.—The augmented thrust and augmented liquid ratios are shown as functions of flight Mach number for altitudes of sea level and 35,332 feet and for various values of inlet-diffuser efficiency in figure 5 for compressor-inlet

(a) Augmented thrust ratios.
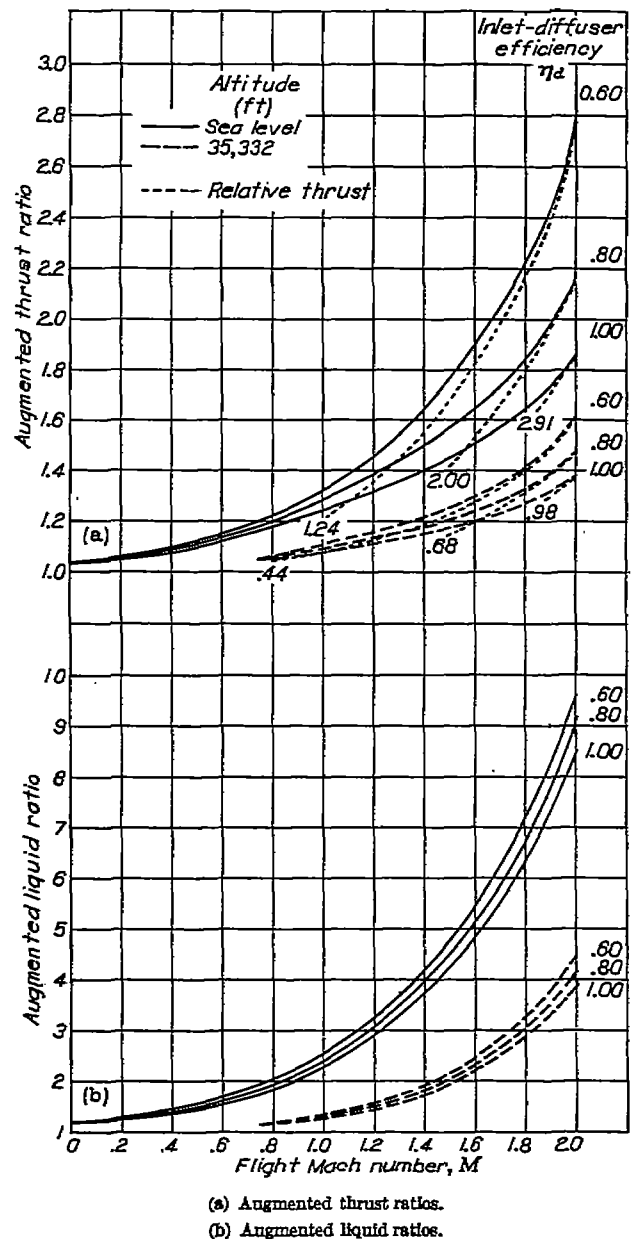(b) Augmented liquid ratios.

FIGURE 5.—Effect of flight Mach number and altitude on augmented thrust and augmented liquid ratios for various values of inlet-diffuser efficiency. Sufficient water injected to saturate compressor-inlet air.

(a) Augmented thrust ratios.
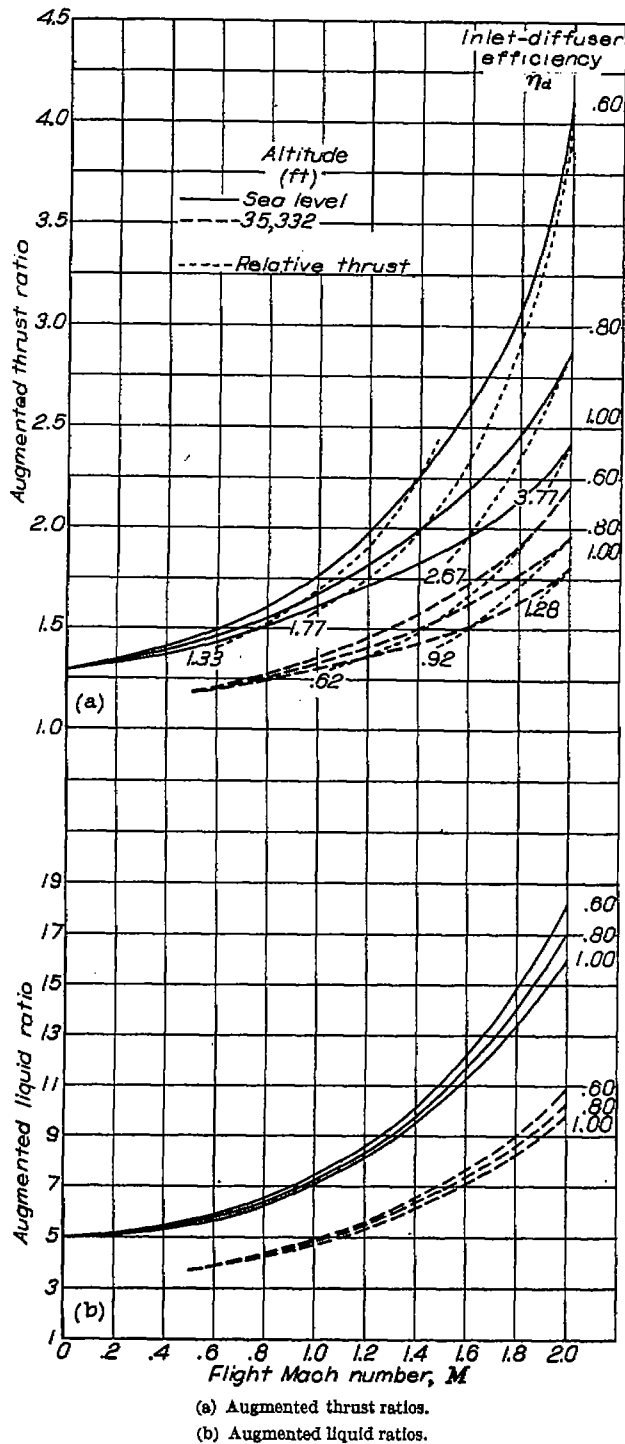
(b) Augmented liquid ratios.

FIGURE 6.—Effect of flight Mach number and altitude on augmented thrust and augmented liquid ratios for various values of inlet-diffuser efficiency. Sufficient water injected to saturate compressor-outlet air.

saturation, and in figure 6 for compressor-outlet saturation. In order to indicate the magnitude of the thrust available, relative-thrust curves (constant values of the ratio of thrust to thrust available at sea-level, zero flight Mach number conditions for operation without water injection) are superimposed on the curves of figures 5(a) and 6(a). Both the augmented thrust ratio and the augmented liquid ratio increase rapidly as the flight Mach number is increased and decrease as the altitude is increased. Although the augmented thrust ratio available at a given flight Mach number increases at decreased values of inlet-diffuser efficiency, the

actual thrust available decreases because of the deterioration of the engine performance without water injection at the decreased values of inlet-diffuser efficiency.

For saturated compressor-inlet conditions (fig. 5(a)) and an inlet-diffuser adiabatic efficiency of 0.80, the augmented thrust ratio for sea-level zero flight Mach number is 1.035 and increases with increasing flight Mach number, reaching a value of 1.28 at a flight Mach number of 1.0 and 2.16 at a flight Mach number of 2.0. The corresponding augmented liquid ratios (fig. 5(b)) are 1.18, 2.40, and 9.20 for sea-level flight Mach numbers of 0, 1.0, and 2.0, respectively. For a flight Mach number of 2.0, increasing the altitude from sea level to 35,332 feet reduces the augmented thrust ratio from 2.16 to 1.48. The corresponding reduction in augmented liquid ratio is from 9.20 to 4.20. Although the thrust increases available from compressor-inlet saturation are very low at low flight Mach numbers, appreciable gains in thrust are possible at high flight speeds. Thrust augmentation by compressor-inlet-air saturation can therefore be used to good advantage not only for centrifugal-flow engines but also for axial-flow engines that require special injection systems to avoid centrifugal separation at higher injection rates.

For saturated compressor-outlet conditions and an inlet-diffuser adiabatic efficiency of 0.80, the augmented thrust ratio (fig. 6(a)) is 1.29, 1.67, and 2.88 for sea-level flight Mach numbers of 0, 1.0, and 2.0, respectively. The corresponding augmented liquid ratios (fig. 6(b)) are 5.01, 7.22, and 17.10, respectively. At a flight Mach number of 2.0, increasing the altitude from sea level to 35,332 feet reduces the augmented thrust ratio from 2.88 to 1.95 with a corresponding reduction in augmented liquid ratio from 17.10 to 10.30.
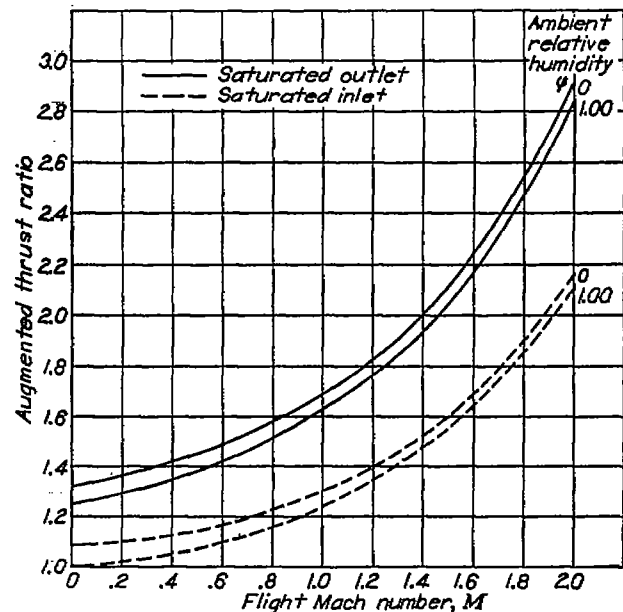

FIGURE 7.—Effect of atmospheric relative humidity on augmented thrust ratio for various flight Mach numbers. Altitude, sea level; inlet-diffuser efficiency, 0.80.

Effect of ambient relative humidity.—The augmented thrust ratio is shown in figure 7 as a function of flight Mach number for atmospheric relative humidities of 0 and 100 percent. The data shown are for standard sea-level condi-

Figure 8(a):

Ambient relative humidity φ

— Saturated outlet
- - - Saturated inlet
-------- Relative thrust

Augmented thrust ratio (y-axis): 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.1, 1.0, .9

Values on curves: 1.37, 1.29, 1.19, 1.07, .93, 1.12, 1.04, .94, .83, 74, .64

Relative humidity labels: 0, .50, 0, 1.00, 77, .50, 1.00

Ambient-air temperature, t_o, °R (x-axis): 490, 510, 530, 550, 570, 590, 610
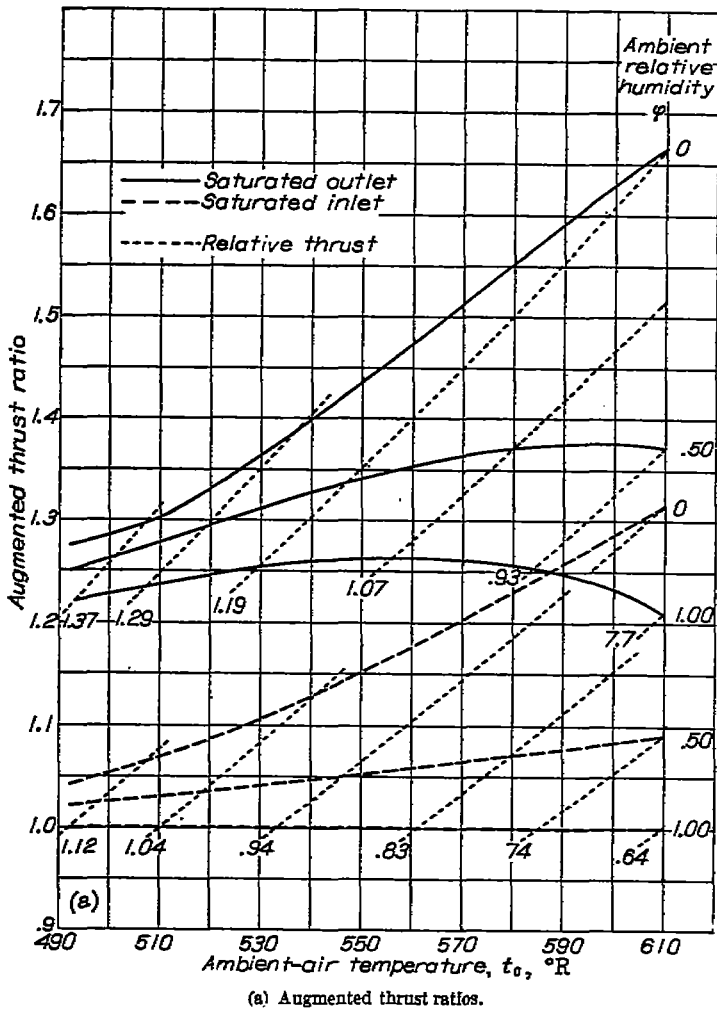
(a) Augmented thrust ratios.

Figure 8.—Effect of ambient temperature on augmented thrust and augmented liquid ratios for various ambient relative humidities. Sea-level pressure; zero flight Mach number.

Figure 8(b):

Ambient relative humidity φ

— Saturated outlet
- - - Saturated inlet

Augmented liquid ratio (y-axis): 8, 7, 6, 5, 4, 3, 2, 1

Relative humidity labels: 0, .50, 1.00, 0, .50, 1.00

Ambient air temperature, t_e, °R (x-axis): 490, 510, 530, 550, 570, 590, 610
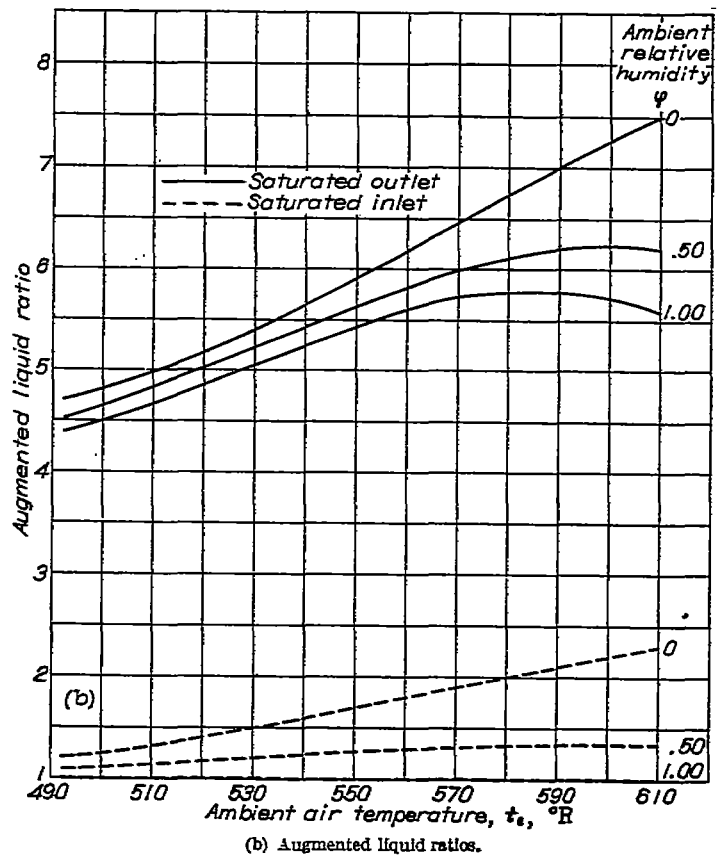
(b) Augmented liquid ratios.

Figure 8.—Concluded. Effect of ambient temperature on augmented thrust and augmented liquid ratios for various ambient relative humidities. Sea-level pressure; zero flight Mach number.

tions of temperature and pressure and an inlet-diffuser efficiency of 0.80. Results are presented for the case in which sufficient water is injected to saturate the compressor-inlet air and for the case in which the amount of water required to saturate the compressor-outlet air is injected. The ambient humidity has a relatively small effect on the augmented thrust ratio, especially at high flight Mach numbers.

Effect of ambient-air temperature for various relative humidities.—The effect of ambient temperature on augmented thrust and augmented liquid ratios for various ambient relative humidities is shown in figures 8(a) and 8(b), respectively. The data shown are for zero flight Mach number and standard sea-level atmospheric pressure. Results are shown for saturated compressor-inlet and -outlet conditions and relative-thrust curves showing constant values of the ratio of augmented thrust available to the thrust available from the engine operating without water injection át an atmospheric temperature of 519° R are included.

For an atmospheric relative humidity of 100 percent, no thrust augmentation may be obtained from compressor-inlet saturation inasmuch as no evaporative cooling is possible. As atmospheric relative humidity is decreased, the augmentation increases to about 8 percent for dry air at standard

sea-level temperature of 519° R. For low relative humidities, more evaporative cooling becomes possible as the ambient-air temperature is increased and at a temperature comparable to extreme summer heat (for example, 580° R) an augmented thrust ratio of 1.23 is available for dry air.

For saturated compressor-outlet air, the augmented thrust ratio for an atmospheric relative humidity of 1.00 is relatively unaffected by changes in atmospheric temperature. For an atmospheric relative humidity of 0.50, the augmented thrust ratio increases slightly and, for zero atmospheric relative humidity, a marked increase in the augmented thrust ratio occurs as the atmospheric temperature is increased. For an atmospheric temperature of 519° R, the augmented thrust ratio decreases from 1.33 to 1.24 as the atmospheric relative humidity increases from 0 to 1.00. At a temperature of 580° R, the decrease in augmented thrust ratio is from 1.55 to 1.26 for the same increase in atmospheric relative humidity.

As indicated by the relative-thrust curves in figure 8(a), the actual augmented thrust decreases as the atmospheric temperature is increased for all values of atmospheric relative humidity because of the marked decrease in normal engine thrust at increased compressor-inlet temperatures. The data of figure 8(a) indicate that thrust augmentation by water injection is one method for overcoming the marked decrease in take-off thrust accompanying high ambient-air temperatures. The augmented liquid ratios shown in figure 8(b) follow the same general trends as the augmented thrust ratios.

Effect of compressor efficiency.—As has been previously pointed out, the compressor efficiency has been decreased in

calculating the augmented engine performance with evaporation during mechanical compression in order to obtain closer agreement with experimental compressor performance with water injection. The effect of this decrease in compressor efficiency on the augmented thrust and augmented liquid ratios at various flight Mach numbers and for altitudes of sea level and 35,332 feet is shown in figure 9 for the condition of saturated compressor outlet. The results are shown for a constant value of compressor efficiency of 0.8 and for a value of compressor efficiency that has been decreased as indicated by the following relation:

$$\eta_c = 0.80 - \Delta X_c$$

(a) Augmented thrust ratios.
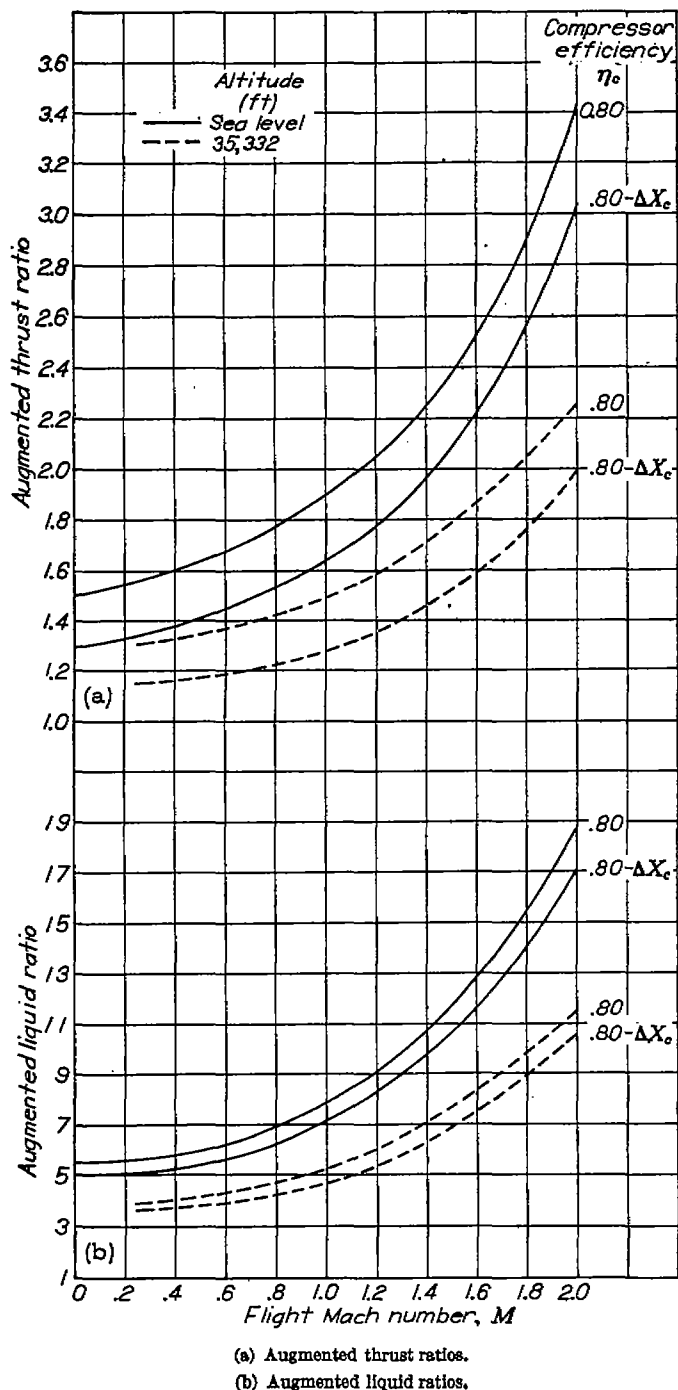(b) Augmented liquid ratios.

FIGURE 9.—Effect of compressor-efficiency assumption on augmented thrust and augmented liquid ratios for various flight conditions. Sufficient water injected to saturate compressor-outlet air.

where ΔX_c is equal to the change in water vapor-air ratio in the compressor.

From figure 9(a), it can be seen that the assumption of a constant compressor efficiency results in values of augmented thrust ratio considerably greater than the assumption of a compressor efficiency decreasing with the change in water vapor-air ratio in the compressor. For sea-level zero flight Mach number, the augmented thrust ratio is 1.29 when a decreased value of compressor efficiency is assumed, as compared with an augmented thrust ratio of 1.50 for the assumption of a constant compressor efficiency of 0.80. For a sea-level flight Mach number of 2.0, the corresponding values of augmented thrust ratio are 3.04 and 3.44. Similar results are obtained at an altitude of 35,332 feet. Additional calculations made for a constant compressor efficiency of 0.85 gave thrust ratios approximately equal to those for a constant efficiency of 0.80, which indicates that the magnitude of compressor efficiency has little effect on thrust augmentation if the efficiency for the normal and the augmented case is assumed to be the same.

The assumption of a constant value of compressor efficiency is shown in figure 9(b) to result in values of augmented liquid ratio somewhat higher than those obtained for the assumption of a decreased value of compressor efficiency.
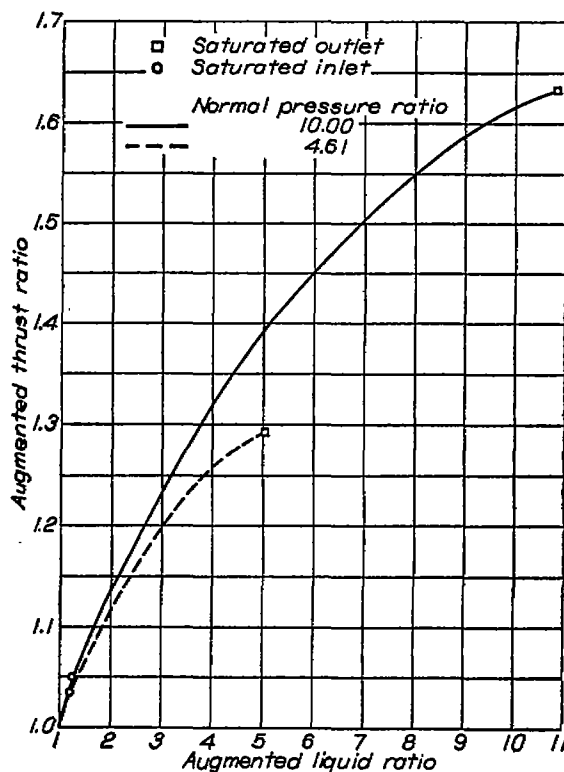

FIGURE 10.—Augmented thrust ratio as function of augmented liquid ratio for two normal compressor pressure ratios. Sea-level zero flight Mach number conditions.

Effect of normal compressor pressure ratio.—The augmented thrust ratio as a function of augmented liquid ratio for normal compressor pressure ratios of 4.61 and 10 is shown in figure 10. The data shown are for sea-level zero flight Mach number conditions. For any augmented liquid ratio, the engine having the high-pressure-ratio compressor gave the greatest augmented thrust ratio, with the difference

between the augmented thrust ratios for the low- and high-pressure-ratio engines increasing as the augmented liquid ratio increased. The use of a high-pressure-ratio compressor also permits operation at increased values of augmented liquid ratio and hence greater values of augmented thrust ratio. For a compressor pressure ratio of 4.61, the maximum value of augmented thrust ratio is 1.29 at an augmented liquid ratio of 5.01. For a compressor pressure ratio of 10, the maximum value of augmented thrust ratio is increased to 1.63 at an augmented liquid ratio of 10.84. The effect of compressor pressure ratio shown is based on a compressor efficiency reduced as a function of the amount of water evaporated, as previously mentioned. Because of the larger amounts of water that may be evaporated at the higher normal compressor pressure ratio, the reductions in compressor efficiency are correspondingly higher. The compressor efficiency was reduced from 0.80 for the normal condition to 0.70 for the full-augmented condition for the high-pressure-ratio compressor. Only experimental investigations of the effect of water injection on the efficiency of compressors having various pressure ratios will indicate the validity of this assumption. As previously mentioned, however, investigations of compressors having normal pressure ratios approximating that of the low-pressure-ratio compressor assumed in this investigation indicate the dependency of the compressor efficiency on the amount of water injected.

SUMMARY OF RESULTS

A psychrometric chart having total pressure as a variable and a Mollier diagram for a saturated mixture of air and water vapor were developed as aids in calculating the performance of compressors when water is injected at the compressor inlet. By use of the psychrometric chart and the Mollier diagram to calculate compressor performance, the performance of the water-injection method of thrust augmentation was evaluated over a range of flight and atmospheric conditions for a typical turbojet engine. The following results were obtained for an engine having a normal compressor pressure ratio of 4.61 at sea-level zero flight Mach number conditions when a nominal decrease in compressor efficiency with water injection was assumed:

1. For constant flight conditions, increasing the amount of water evaporated increased the ratio of augmented thrust to normal thrust and the ratio of augmented liquid flow to normal fuel flow. For sea-level zero flight Mach number conditions and an atmospheric relative humidity of 0.50, increasing the amount of injected water from the amount required to saturate the compressor-inlet air to the amount required to saturate the compressor-outlet air increased the augmented thrust ratio from 1.035 to 1.29. The corresponding increase in augmented liquid ratio was from 1.18 to 5.01.

2. The augmented thrust and augmented liquid ratios increased rapidly as the flight Mach number was increased and decreased as the altitude increased. Although the thrust increases obtainable by compressor-inlet-air saturation were very small at low flight speeds, appreciable gains in thrust were possible at high flight Mach numbers. Thrust augmentation by compressor-inlet-air saturation can therefore be used to good advantage at high flight Mach numbers not only for centrifugal-flow engines but also for axial-flow engines that require special injection systems to avoid centrifugal separation at higher injection rates.

3. For standard atmospheric temperatures, the ambient relative humidity had only a small effect on the augmented thrust ratios produced at all flight speeds investigated.

4. For sea-level zero flight Mach number conditions and low ambient relative humidities, the augmented thrust ratio increased as the ambient-air temperature was increased. Thrust augmentation by water injection was therefore shown to be desirable for take-off use inasmuch as water injection tends to overcome the loss in take-off thrust normally occurring at high ambient temperatures. For very high atmospheric relative humidities, the effect of ambient temperature on augmented thrust ratio was small.

5. The augmented thrust ratio produced for a given set of conditions increased as the inlet-diffuser efficiency was decreased; however, because of the rapid deterioration of normal engine performance at decreased values of inlet-diffuser efficiency, the actual value of augmented thrust was decreased.

6. Increasing the normal engine compressor pressure ratio increased the maximum possible value of augmented liquid ratio and, hence, the maximum possible value of augmented thrust ratio.

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  NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
    CLEVELAND, OHIO, *June 1, 1950*.

APPENDIX A

SYMBOLS

A area, sq in.

c_p specific heat at constant pressure, Btu/(lb)(°R)

F net thrust, lb

f/a fuel-air ratio, lb fuel/lb air

g acceleration due to gravity, 32.2 ft/sec^2

H enthalpy, Btu/lb

H_{corr} corrected enthalpy for saturated mixture of air and water vapor plus liquid water not at 519° R, Btu/lb air

ΔH enthalpy rise, Btu/lb

h_a enthalpy of air as given in reference 4, Btu/lb

h_f enthalpy of liquid water as given in reference 5, Btu/lb

h_g enthalpy of water vapor as given in reference 5, Btu/lb

J mechanical equivalent of heat, 778 ft-lb/Btu

K compressor slip factor

M flight Mach number

P total pressure, lb/sq in. absolute

p static and partial pressure, lb/sq in. absolute

p_m sum of partial pressures of air and water vapor, lb/sq in.

R gas constant, ft-lb/(lb)(°R)

S entropy, Btu/(lb)(°R)

S_{corr} corrected entropy for saturated mixture of air and water vapor plus liquid water not at 519° R, Btu/(lb air)(°R)

s_f entropy of liquid water as given in reference 5, Btu/(lb)(°R)

s_g entropy of water vapor as given in reference 5, Btu/(lb)(°R)

T total temperature, °R

ΔT total-temperature rise, F°

t static temperature, °R

Δt static-temperature rise, F°

V flight velocity, ft/sec

V_j jet velocity, ft/sec

V_T compressor-rotor-tip velocity, ft/sec

v specific volume, cu ft/lb

v_g specific volume of water vapor as given in reference 5 cu ft/lb

X ratio of weight of water vapor to weight of air in mixture, lb water/lb air

ΔX total amount of injected liquid water, lb water/lb air

W weight flow, lb/sec

γ ratio of specific heats

δ ratio of total pressure of mixture to standard sea-level static pressure

η adiabatic efficiency

$\theta = \dfrac{t_0}{519}$

φ ambient relative humidity

$\phi_a = \int c_p \, dt/t$ for air, as given in reference 4, Btu/(lb)(°R)

Subscripts:

a air

c compressor

d diffuser

e exhaust gas—water vapor mixture

f liquid water

g water vapor

i ideal (isentropic)

m air—water vapor mixture

n exhaust nozzle

s saturation

t turbine

0 ambient air

1 compressor inlet

2 end of portion of compression during which air remains saturated and after which no water is evaporated

3 compressor outlet

4 turbine inlet

5 turbine outlet

6 exhaust-nozzle outlet

$(-)$ between stations enclosed

Primed symbols are approximate values.

DERIVATION OF EQUATIONS FOR PSYCHROMETRIC CHART AND MOLLIER DIAGRAM

PSYCHROMETRIC CHART

The equations necessary for obtaining a psychometric chart having total pressure (sum of partial pressures of air and water vapor) as a variable may be derived in the following manner:

From the general energy equation, the enthalpy of a mixture of air, liquid water, and water vapor per pound of air is

$$H_{a,0}+X_0 H_{g,0}+(X_1-X_0)\,H_{f,0}=H_{a,1}+X_1 H_{g,1} \tag{B1}$$

When only the equilibrium between air and water vapor is considered, the term involving the enthalpy of liquid water $H_{f,a}$ disappears; in order to apply equation (B1) to a particular process, however, some assumption must be made as to the temperature for which the enthalpy of liquid water is zero. For the present analysis, the enthalpy of liquid water was assumed to be zero at 519° R. The results are therefore accurate for water injected at this temperature, with only small errors resulting for water injected at a somewhat different temperature. When it is assumed that the enthalpy of liquid water is zero, equation (B1) may be stated as

$$H_{a,0}+X_0 H_{g,0}=H_{a,1}+X_1 H_{g,1} \tag{B2}$$

The total enthalpy H for condition 0 or 1 is defined as

$$H=H_a+X H_g \tag{B3}$$

The water-air ratio X is evaluated as follows:

$$X=\frac{v_a}{v_g}=\frac{p_g}{R_g t_g}\frac{R_a t_a}{p_a} \tag{B4}$$

If the temperature of the air and vaporized water are equal and Dalton's law of partial pressure is valid, equation (B4) becomes

$$X=\frac{R_a}{R_g}\frac{p_g}{p_m-p_g} \tag{B5}$$

From the definition of relative humidity φ, relative pressure δ, and gas constant R,

$$\varphi=\frac{p_g}{p_{g,s}} \tag{B6}$$

$$\delta=\frac{p_m}{14.696} \tag{B7}$$

and

$$R_g=\frac{p_g v_g}{t_g} \tag{B8}$$

Substituting equations (B6) to (B8) in equation (B5) gives

$$X=\frac{R_a}{\left(\dfrac{p_g v_g}{t_g}\right)}\frac{\dfrac{\varphi}{\delta}\dfrac{p_{g,s}}{14.696}}{\left(1-\dfrac{\varphi}{\delta}\dfrac{p_{g,s}}{14.696}\right)} \tag{B9}$$

When the water-air ratio X is replaced in equation (B3) by the value expressed in equation (B9), the expression for total enthalpy H becomes

$$H=H_a+H_g\frac{R_a}{\left(\dfrac{p_g v_g}{t_g}\right)}\frac{\dfrac{\varphi}{\delta}\dfrac{p_{g,s}}{14.696}}{\left(1-\dfrac{\varphi}{\delta}\dfrac{p_{g,s}}{14.696}\right)} \tag{B10}$$

The data presented in figure 1 were obtained by means of equations (B3), (B9), (B10), and the thermodynamic data for air and water vapor contained in references 4 and 5, respectively. For convenience, the enthalpy of the saturated mixture of air and water vapor at a temperature of 519° R and a pressure of 14.696 pounds per square inch was arbitrarily fixed at 100 Btu per pound of air. In using equation (B10), the enthalpies of air and water vapor obtained from references 4 and 5, respectively, must be corrected in order to satisfy the conditions of zero enthalpy of liquid water at 519° R and the condition of an enthalpy of 100 Btu per pound of air for a saturated mixture of air and water vapor at a temperature of 519° R and a pressure of 14.696 pounds per square inch. The value of enthalpy of water vapor obtained from reference 5 must be decreased by 27.06 Btu per pound in order to satisfy the first condition, and the enthalpy of air obtained from reference 4 must be increased by 60.24 Btu per pound in order to satisfy the second condition. For very accurate work, the injection of water at a temperature other than 519° R can be corrected for simply by adding the enthalpy of liquid water to find the starting point of the process and, when liquid water remains, subtracting the enthalpy of this liquid water at the end of the process. These corrections are the same as the enthalpy corrections to the Mollier diagram discussed in appendix C.

The enthalpy and the specific volume of water vapor are functions of the vapor pressure of water as well as of the temperature. In the range of temperatures and pressures encountered at the compressor inlet of a turbojet engine, however, the enthalpy of water vapor and the product of vapor pressure and specific volume can be considered as functions only of temperature for all practical purposes. For any value of temperature, the enthalpy and the specific volume were evaluated for the maximum pressure that would exist for that temperature at the compressor inlet of a turbojet engine. This maximum pressure is the pressure that would be obtained by ideal ram compression from standard sea-level temperature and pressure to the desired temperature.

MOLLIER DIAGRAM

The general energy equation for a compression of 1 pound of air saturated with water vapor when no liquid is present at the end of compression becomes, similar to equation (B1),

$$H_{a,1}+X_1 H_{g,1}+(X_2-X_1)H_{f,1}+\Delta H_c=H_{a,2}+X_2 H_{g,2} \tag{B11}$$

For the ideal conditions, the entropy remains constant; therefore

$$S_{a,1}+X_1 S_{g,1}+(X_2-X_1)S_{f,1}=S_{a,2}+X_2 S_{g,2} \tag{B12}$$

If the enthalpy and the entropy of the excess liquid water are assigned values of zero at a particular temperature and the liquid water is introduced at this temperature, equations (B11) and (B12) become

$$H_{a,1}+X_1 H_{g,1}+\Delta H_c = H_{a,2}+X_2 H_{g,2} \qquad (B13)$$

and

$$S_{a,1}+X_1 S_{g,1}=S_{a,2}+X_2 S_{g,2} \qquad (B14)$$

In a saturated mixture of air and water vapor, the enthalpy per pound of air is

$$H=H_a+XH_g \qquad (B15)$$

and the entropy per pound of air is

$$S=S_a+XS_g \qquad (B16)$$

If equations (B15) and (B16) are plotted for a range of temperatures and pressures, the conditions of the mixture resulting from a compression of 1 pound of air that is continually saturated with water vapor may be determined. Equations (B15) and (B16) are the basis for the Mollier diagram presented. The base temperature for enthalpy and entropy of liquid water was chosen as 519° R. Thus, if any liquid water present is assumed to be at 519° R, no correction need be made for the enthalpy and the entropy of the liquid when using the Mollier diagram. The values of enthalpy and entropy were so adjusted that at standard sea-level conditions of temperature of 519° R and pressure of 14.696 pounds per square inch the enthalpy of the saturated mixture of air and water vapor is 100 Btu per pound of air and the entropy is 0.10 Btu/(lb air) (°R). The modified versions of equations (B15) and (B16), which were used in plotting the Mollier diagram, are

$$H=h_a+60.24+X(h_g-27.06) \qquad (B17)$$

and

$$S=\phi_a-\frac{R_a}{J}\log_e\frac{(p_m-p_g)}{18.189}+X(s_g-0.0536) \qquad (B18)$$

Values for h_a and ϕ_a were obtained from reference 4 and

are for a base temperature of 400° R; values for h_g and s_g were obtained from reference 5 and are for a base temperature of 32° F (492° R).

The weight ratio of water vapor to air in a saturated mixture may be determined by the relation

$$X=\frac{v_a}{v_g} \qquad (B19)$$

When the equation of state and Dalton's law of partial pressures are used, equation (B19) may be written

$$X=\frac{R_a t_a}{(p_m-p_g)(144 v_g)} \qquad (B20)$$

Equation (B20) was used for determining values of X for equations (B17) and (B18).

If liquid water is injected at any temperature other than 519° R, the enthalpy and the entropy of the water will not be zero and an error will be introduced when using the Mollier diagram. This error can be corrected by adjusting the initial enthalpy and entropy, as read from the diagram, to include the enthalpy and the entropy of the injected liquid water. The corrected enthalpy and entropy for the initial condition will be

$$H_{corr}=H+(h_f-27.06)\Delta X \qquad (B21)$$

and

$$S_{corr}=S+(s_f-0.0536)\Delta X \qquad (B22)$$

A similar correction, made by subtracting the enthalpy and the entropy of the liquid, can be performed at the end of the process if any liquid water remains. Values of H_f, which is $h_f-27.06$, and S_f, which is $s_f-0.0536$, are plotted in an insert on the Mollier diagram for various temperatures.

If liquid water is injected at a temperature different from the initial temperature of the saturated air, a slight increase in entropy occurs because of the mixing of the constituents as a new temperature equilibrium is reached. This increase in entropy may be neglected, however, without introducing appreciable error when using the Mollier diagram.

APPENDIX C

SAMPLE CALCULATIONS

The use of the Mollier diagram in calculating saturated compression processes is illustrated by the following sample calculations:

SAMPLE CALCULATION I

In calculating a compression process for an actual engine, the work input to the compressor is usually obtained on the basis of 1 pound of working fluid passing through the compressor. In order to use the Mollier diagram for a saturated compression process, the work input must be found on a basis of 1 pound of air (the unit on which the Mollier diagram is based). Corrections must also be made for the enthalpy and the entropy of the liquid water when the water injected is not at a temperature of 519° R if it is desired to maintain a high degree of accuracy in using the Mollier diagram. In order to illustrate these refinements, the following example is presented for an engine with a single-stage centrifugal compressor. The following conditions are assumed:

Ambient static temperature, t_0, °R	519
Ambient static pressure, p_0, lb/sq in.	14.7
Flight Mach number, M	0.85
Flight velocity, V, ft/sec	949
Diffuser adiabatic efficiency, η_d	0.85
Ambient relative humidity, φ	0.50
Compressor-rotor-tip velocity, V_T, ft/sec	1500
Compressor slip factor, K	0.95
Compressor adiabatic efficiency, η_c	0.80

Enough liquid water is injected at the compressor inlet at a temperature of 540° R to saturate the air at this point and to keep the air saturated during the entire compression process. Find P_2, T_2, X_2, and the amount of water evaporated.

The water-air ratio for saturated conditions at an ambient temperature of 519° R and a pressure of 14.7 pounds per square inch is, from the Mollier diagram,

$$X = 0.0106$$

For a relative humidity of 0.50, the value of X for the ambient air is

$$X_0 = \frac{0.0106}{2} = 0.0053$$

The static-temperature rise in the inlet diffuser due to ram compression is (value of $c_{p,m}$ obtained from fig. 3(a))

$$\Delta t_{(0-1)} = \frac{V^2}{2gJc_{p,m}} = \frac{(949)^2}{(2)(32.2)(778)(0.2416)} = 74.4 \text{ F}°$$

and

$$T_1 = 519 + 74.4 = 593.4° \text{ R}$$

When equation (B17) is evaluated for 593.4° R, the enthalpy at the compressor inlet without taking into account the enthalpy of the liquid water that is injected at this point is

$$H_1' = h_a + 60.24 + X(h_g - 27.06)$$

$$= 46.31 + 60.24 + 0.0053(1119.1 - 27.06)$$

$$= 112.3 \text{ Btu/lb air}$$

The compressor-inlet pressure is (γ_m obtained from fig. 3(a))

$$P_1 = p_0 \left[\frac{\eta_d \Delta t_{(0-1)}}{t_0} + 1 \right]^{\frac{\gamma_m}{\gamma_m - 1}}$$

$$= 14.7 \left[\frac{(0.85)(74.4)}{519} + 1 \right]^{\frac{1.398}{0.398}}$$

$$= 22.0 \text{ lb/sq in.}$$

When liquid water is injected at the compressor inlet, the temperature decreases but the pressure remains constant, as does the enthalpy if the enthalpy of the injected water is neglected. From the Mollier diagram at an enthalpy of 112.3 Btu per pound of air and a pressure of 22.0 pounds per square inch, the entropy at the compressor inlet, not considering the entropy of the liquid water, is

$$S_1' = 0.0949 \text{ Btu/(lb air) (°R)}$$

This point on the Mollier diagram could also have been determined from P_1 and T_1 where the value of T_1 is obtained from figure 1.

In order to obtain the enthalpy of the mixture in Btu per pound of air at the compressor inlet, correction must be made for the enthalpy of the liquid water that is injected at 540° R, which requires that the amount of water injected for saturation at the compressor outlet be known. Because X_2 is unknown, an approximate value X_2' must be assumed. The following approximation for sea-level pressure was obtained by inspection of calculations for a wide range of ambient-air temperatures, flight Mach numbers, and compressor pressure ratios and may be used to determine X_2' in order to eliminate or to simplify successive approximation methods:

$$X_2' = X_0 + 0.00065 \, \Delta H_{m,(0-2)} \theta^2$$

The enthalpy rise in the compressor is

$$\Delta H_{m,(1-2)} = \frac{KV_T^2}{gJ}$$

$$= \frac{(0.95)(1500)^2}{(32.2)(778)}$$

$$= 85.3 \text{ Btu/lb mixture}$$

and the enthalpy rise in the inlet diffuser is

$$\Delta H_{(0-1)} = c_{p,a} \Delta t_{(0-1)} (1 + X_0) = (0.2416)(74.4)(1.0053)$$

$$= 18.1 \text{ Btu/lb air}$$

Therefore

$$X_2' = 0.0053 + 0.00065(18.1 + 85.3)(1)^2 = 0.0725$$

and

$$\Delta X'_{(0-2)} = 0.0725 - 0.0053 = 0.0672$$

These values will be used for X_2 and $\Delta X_{(0-2)}$ for the rest of the calculations.

From the insert on the Mollier diagram, the enthalpy and the entropy of the liquid water at 540° R are

$$H_f = 21.0 \text{ Btu/lb water}$$

and

$$S_f = 0.04 \text{ Btu/(lb water)(°R)}$$

The enthalpy and the entropy due to the liquid water are then

$$H_{f,1} = \Delta X_{(0-2)} H_f = (0.0672)(21.0) = 1.4 \text{ Btu/lb air}$$

and

$$S_{f,1} = \Delta X_{(0-2)} S_f = (0.0672)(0.04) = 0.0027 \text{ Btu/(lb air)(°R)}$$

The values for enthalpy and entropy at the compressor inlet corrected for the liquid water at a temperature of 540° R are

$$H_{1,corr} = H_1' + H_{f,1} = 112.3 + 1.4 = 113.7 \text{ Btu/lb air}$$

and

$$S_{1,corr} = S_1' + S_{f,1} = 0.0949 + 0.0027 = 0.0976 \text{ Btu/(lb air)(°R)}$$

When the enthalpy rise per pound of mixture in the compressor is known, it must be converted to enthalpy rise per pound of air in order to use the Mollier diagram. This conversion also requires a value for the water-air ratio at the compressor outlet. The ideal enthalpy rise per pound of air in the compressor is

$$\Delta H_{i,(1-2)} = \Delta H_{m,(1-2)}(1+X_2)\eta_c = (85.3)(1.0725)(0.80)$$

$$= 73.2 \text{ Btu/lb air}$$

and the final enthalpy for an isentropic compression is

$$H_{2,i} = H_{1,corr} + \Delta H_{i,(1-2)} = 113.7 + 73.2 = 186.9 \text{ Btu/lb air}$$

The compressor-outlet pressure P_2 read from the Mollier diagram at an enthalpy of 186.9 Btu per pound of air and an entropy of 0.0976 Btu/(lb air) (°R) is

$$P_2 = 118.7 \text{ lb/sq in.}$$

The actual enthalpy per pound of air at the compressor outlet is

$$H_2 = H_1 + \Delta H_{m,(1-2)} (1+X_2) = 113.7 +$$
$$(85.3 \times 1.0725) = 205.2 \text{ Btu/lb air}$$

When the Mollier diagram is read at an enthalpy of 205.2 Btu per pound of air and a pressure of 118.7 pounds per square inch, the temperature is

$$T_2 = 663.2° \text{ R}$$

and

$$X_2 = 0.0730$$

This value of X_2 is very close to the approximate value that was used for X_2 and therefore no further correction is necessary. If this value of X_2 differed greatly from the assumed approximate value, it would be advisable to repeat the calculations using the value of X_2 just obtained.

The amount of water injected at the compressor inlet is

$$\Delta X_{(0-2)} = 0.0730 - 0.0053 = 0.0677 \text{ lb water/lb air}$$

In order to show the effect of corrections for liquid-water temperatures other than 519° R, the following table presents values of P_2, T_2, and X_2 that were obtained for liquid-water-injection temperatures of 519° R (no correction), 540° R (the sample calculation just completed), and for 620° R; all other conditions remained the same as in the sample calculation:

Temperature of injected water (°R)	Pressure P_2 (lb/sq in.)	Temperature T_2 (°R)	Water-air ratio X_2
519	119.2	662.7	0.0718
540	118.7	663.2	.0730
620	118.0	666.0	.0782

This table shows that, for most limits of accuracy for turbojet-cycle calculations, the enthalpy and the entropy of the liquid water may be neglected in the probable range of injected-water temperature.

SAMPLE CALCULATION II

The following example illustrates the calculation of compressor-outlet conditions when an amount of water less than the amount required for outlet saturation is injected at the compressor inlet.

The following conditions are assumed:

Ambient static temperature, t_0, °R _____ 519
Ambient static pressure, p_0, lb/sq in. _____ 14.7
Flight velocity, V, ft/sec _____ 0
Ambient relative humidity, φ _____ 0.50
Compressor adiabatic efficiency, η_c _____ 0.80
Compressor enthalpy rise, ΔH_c, Btu/lb mixture _____ 85.3
Air weight flow, W_a, lb/sec _____ 80.5
Water-injection rate at 519° R, lb/sec _____ 4.0

From Sample Calculation I,

$$X_0 = 0.0053$$

and by evaluating equation (B17) for 519° R,

$$H_0 = h_a + 60.24 \, X_0(h_g - 27.06)$$
$$= 28.53 + 60.24 + 0.0053(1087.6 - 27.06$$
$$= 94.4 \text{ Btu/lb air}$$

From the Mollier diagram at an enthalpy of 94.4 Btu per pound of air and a pressure of 14.7 pounds per square inch, the entropy is

$$S_0 = 0.0890 \text{ Btu/(lb air) (°R)}$$

The change in water-air ratio ΔX due to the addition of the liquid water is

$$\Delta X_{(0-2)} = \frac{4}{80.5} = 0.0497$$

and

$$X_2 = X_0 + \Delta X_{(0-2)} = 0.0053 + 0.0497 = 0.0550$$

If the approximation from Sample Calculation II is used,

$$\Delta H_{m,\,(0-2)}' = \frac{\Delta X_{(0-2)}}{(0.00065)\theta^2} = \frac{0.0497}{(0.00065)1} = 76.5 \text{ Btu/lb mixture}$$

Using this approximate value of $\Delta H_{m,\,(0-2)}$ yields

$$\Delta H_{(0-2)}' = \Delta H_{m,\,(0-2)}'(1+X_2) = (76.5)(1.0550) = 80.7 \text{ Btu/lb air}$$

Then

$$H_2' = H_0 + \Delta H_{(0-2)}' = 94.4 + 80.7 = 175.1 \text{ Btu/lb air}$$

and

$$H_{2,i}' = H_0 + \Delta H_{(0-2)}'\eta_c = 94.4 + (80.7)(0.80) = 159.0 \text{ Btu/lb air}$$

From the Mollier diagram at an enthalpy of 159.0 Btu per pound and an entropy of 0.0890 Btu per pound per °R,

$$P_2' = 73.5 \text{ lb/sq in.}$$

From the Mollier diagram at a pressure of 73.5 pounds per square inch and an enthalpy of 175.1 Btu per pound air,

$$X_2' = 0.0542$$

This value of X_2' is lower than the desired value. If the value of $\Delta H_{m(0-2)}'$ is increased to 77.7 Btu per pound of air and the calculation repeated, the values obtained are

$$P_2 = 75.0 \text{ lb/sq in.}$$

$$X_2 = 0.0550$$

and

$$T_2 = 630.5° \text{ R}$$

The rest of the compression process may be calculated as an adiabatic compression of the mixture of air and water vapor using the thermodynamic properties given in figure 3(a). From figure 3(a) at $X = 0.0550$,

$$c_{p,m} = 0.2513 \text{ Btu/(lb)(°R)}$$

and

$$\gamma_m = 1.3916$$

Then

$$\Delta T_{(2-3)} = \frac{\Delta H_{m(2-3)}}{c_{p,m}} = \frac{(85.3 - 77.7)}{0.2513} = 30.2 \text{ F°}$$

and

$$\frac{P_3}{P_2} = \left[\left(\frac{\Delta T_{(2-3)}}{T_2}\right)\eta_c + 1\right]^{\frac{\gamma_m}{\gamma_m - 1}} = \left[\left(\frac{30.2}{630.5}\right)0.80 + 1\right]^{\frac{1.3916}{0.3916}} = 1.143$$

The conditions at the compressor outlet are

$$P_3 = \left(\frac{P_3}{P_2}\right)P_2 = (1.143)(75.0) = 85.7 \text{ lb/sq in.}$$

$$T_3 = T_2 + \Delta T_{(2-3)} = 630.5 + 30.2 = 660.7° \text{ R}$$

and

$$X_3 = X_2 = 0.0550$$

For a comparison of the effect of the amount of water evaporated on pressure and temperature at the compressor outlet, the results of Sample Calculation I (air saturated up to the compressor outlet for a water-injection temperature of 519° R), and of the cases of air saturated during one-half of the work input to the compressor, of air saturated up to the compressor inlet, and of no injection are shown in the following table:

$(M=0.85; \Delta H_e=85.3 \text{ Btu/lb mixture}; \eta_c=0.80)$

Condition	Amount of water evaporated (lb water/lb air)	Outlet pressure P_3 (lb/sq in.)	Outlet temperature T_3 (°R)
No injection	0	86.5	947
Air saturated up to compressor inlet	.0113	95.4	895
Air saturated through one-half of work input	.0384	108.0	778
Air saturated up to compressor outlet	.0665	119.2	663

APPENDIX D

EQUATIONS USED IN AUGMENTATION CALCULATIONS

The method of calculating compressor-outlet conditions when water is evaporated during compression is given in the section DISCUSSION OF CHARTS and illustrated in appendix C. The other equations used for the step-by-step calculations of engine performance are:

Inlet diffuser,

$$\frac{P_1}{p_0} = \left(\frac{\eta_d V^2}{2 J g c_{p,m} t_0} + 1\right)^{\frac{\gamma_m}{\gamma_m - 1}}$$

Compressor (with no evaporation during compression),

$$\frac{P_3}{P_1} = \left(\frac{\eta_c K V_T^2}{T_1 g J c_{p,m}} + 1\right)^{\frac{\gamma_m}{\gamma_m - 1}}$$

Turbine,

$$\frac{P_5}{P_4} = \left[1 - \frac{\Delta H_{m,c}(1 + X_3)}{\eta_t T_4 (1 + X_3 + f/a) c_{p,c}}\right]^{\frac{\gamma_e}{\gamma_e - 1}}$$

Thrust (for sonic jet velocity),

$$\frac{F}{A_t} = \frac{V_j W_t}{g A_t} + \frac{A_n}{A_t}(p_6 - p_0) - \frac{V W_a}{g A_t}(1 + X_0)$$

where

$$V_j = \sqrt{g \gamma_e R_e t_6}$$

$$t_6 = T_5 \left(\frac{2}{\gamma_e + 1}\right)$$

$$\frac{W_t}{A_t} = \left(\frac{P_4}{\sqrt{T_4 R_e / g \gamma_e}}\right)\left(\frac{2}{1 + \gamma_e}\right)^{\frac{\gamma_e + 1}{2(\gamma_e - 1)}}$$

$$\frac{A_n}{A_t} = \frac{W_t}{A_t} \frac{R_e t_6}{V_j p_6}$$

$$\frac{p_6}{P_5} = \left[1 - \left(\frac{\gamma_e - 1}{\gamma_e + 1}\right)\frac{1}{\eta_n}\right]^{\frac{\gamma_e}{\gamma_e - 1}}$$

and

$$\frac{W_a}{A_t} = \frac{W_t}{A_t}\left(\frac{1}{1 + X_3 + f/a}\right)$$

Thrust (for subsonic jet velocity),

$$\frac{F}{A_t} = V_j \frac{W_t}{g A_t} - \frac{V W_a}{g A_t}(1 + X_0)$$

where

$$V_j = \sqrt{2 \eta_n g R_e \frac{\gamma_e}{\gamma_e - 1} T_5 \left[1 - \left(\frac{p_0}{P_5}\right)^{\frac{\gamma_e - 1}{\gamma_e}}\right]}$$

and W_t/A_t and W_a/A_t are the same as for the case of sonic jet velocity. Values for $c_{p,m}$ and γ_m were obtained from figure 3(a) and values of $c_{p,e}$, R_e, and γ_e from figure 3(b). Values for f/a were obtained from reference 8.

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