NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT 1145

A METHOD OF CALIBRATING AIRSPEED INSTALLATIONS ON AIRPLANES AT TRANSONIC AND SUPERSONIC SPEEDS BY THE USE OF ACCELEROMETER AND ATTITUDE-ANGLE MEASUREMENTS

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A METHOD OF CALIBRATING AIRSPEED INSTALLATIONS ON AIRPLANES AT TRANSONIC AND SUPERSONIC SPEEDS BY THE USE OF ACCELEROMETER AND ATTITUDE-ANGLE MEASUREMENTS [1]

By John A. Zalovcik, Lindsay J. Lina, and James P. Trant, Jr.

SUMMARY

A method is described for calibrating airspeed installations on airplanes at transonic and supersonic speeds in vertical-plane maneuvers in which use is made of measurements of normal and longitudinal accelerations and attitude angle. In this method, all the required instrumentation is carried within the airplane.

An analytical study of the effects of various sources of error on the accuracy of an airspeed calibration by the accelerometer method indicated that the required measurements can be made accurately enough to insure a satisfactory calibration. This conclusion was verified experimentally by a calibration of the airspeed installation of a jet fighter airplane. The tests included shallow dives up to a Mach number of about 0.80 with pull-ups of about 4, 3, and 2g normal acceleration. A calibration by the radar-phototheodolite method (NACA Rep. 985) was also made for comparison.

INTRODUCTION

A method of calibrating the static-pressure source of a pitot-static installation on an airplane at high speed and high altitude by the use of radar-phototheodolite tracking equipment was described in reference 1. In this method, the radar-phototheodolite equipment is used to establish the geometric altitude of the airplane in surveys of atmospheric pressure made at speeds for which the airspeed calibration is known and in maneuvers under conditions for which the calibration is desired. Although this method is precise, its use is limited since radar-phototheodolite equipment is not generally available. In order that airspeed calibrations may be performed universally with the added convenience of having instrumentation all within the airplane, a method, herein referred to as the accelerometer method, was devised at the Langley Aeronautical Laboratory of the National Advisory Committee for Aeronautics. The method makes use of measurements of total pressure, static pressure, temperature, acceleration, and attitude angle in vertical-plane maneuvers.

This report presents an analysis of the accelerometer method including a discussion of the required instrumentation, calibration procedures, and the effect of errors in measurement on the accuracy of a calibration. Also included is a comparison of airspeed calibrations of a jet fighter airplane evaluated by both the accelerometer and radar-phototheodolite methods. These calibrations were made simultaneously in shallow dives up to a Mach number of about 0.8.

SYMBOLS

p	free-stream static or atmospheric pressure
p_m	indicated static pressure
p_a	free-stream static pressure computed from equation (7) by using values of temperature and altitude computed from accelerometer measurements
p_r	free-stream static pressure computed from equation (7) by using values of temperature and altitude determined by radar
ρ	density
e_p	static-pressure error ratio, $\frac{p_m-p}{p}$
p_T	free-stream total pressure for subsonic flow and total pressure behind normal shock for supersonic flow
q_c'	indicated impact pressure, p_T-p_m
t	elapsed time
t_m	measured elapsed time
e_t	error in elapsed time, $\frac{t_m-t}{t}$
h	altitude
M	free-stream Mach number
M'	indicated free-stream Mach number
T	free-stream temperature, absolute units
T_m	measured temperature, absolute units
T'	temperature defined by equation (5)
e_T	error in free-stream temperature, $\frac{T'-T}{T}$
K	temperature recovery factor, $\frac{T_m-T}{0.2M^2T}$
a_x	longitudinal acceleration, positive forward along x-axis
a_z	normal acceleration, positive upward along z-axis
a_v	vertical acceleration, positive upward along vertical of earth (defined by eq. (10))
a_y	lateral acceleration, positive to right of y-axis
α	angle between rays of sun and longitudinal axis as measured by sun camera

[1] Supersedes NACA TN 2099, "A Method of Calibrating Airspeed Installations on Airplanes at Transonic and Supersonic Speeds by Use of Accelerometer and Attitude-Angle Measurements" by John A. Zalovcik, 1950, and NACA TN 2570, "Comparison of Airspeed Calibrations Evaluated by the Accelerometer and Radar Methods" by Lindsay J. Lina and James P. Trant, Jr., 1952.

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β longitude of airplane
γ ratio of specific heats, 1.4
ϵ declination of sun
λ elevation angle of sun
θ attitude angle, positive below horizon
τ latitude of airplane
ϕ angle of bank, positive right wing down
ψ angle of yaw, positive to right
ω Greenwich hour angle
v vertical velocity
R gas constant
s_β error in estimating latitude of airplane (distance along longitude β)
s_τ error in estimating longitude of airplane (distance along latitude τ)
g acceleration due to gravity
Δ prefix denoting error

Subscripts:

0 beginning of level-flight run or shallow dive preceding calibration maneuver or end of level-flight run or shallow dive following calibration maneuver
1 beginning of evaluation of maneuver

ANALYSIS OF METHOD

THEORY

The principal feature of most procedures for calibration of airspeed installations on airplanes is the determination of ambient or free-stream static pressure. In the method described herein, free-stream static pressure is obtained over a range of altitude with the aid of the relation that the rate of change of static pressure with altitude is equal to the density, or

$$\frac{dp}{dh}=-g\rho \tag{1}$$

Since

$$g\rho=\frac{p}{RT}$$

then

$$\frac{dp}{dh}=-\frac{p}{RT}$$

or

$$dp=-\frac{p}{RT}\,dh \tag{2}$$

Equation (2) may be integrated as

$$p=p_1e^{-\int_{h_1}^{h}\frac{dh}{RT}} \tag{3}$$

Therefore, if the pressure at one altitude is known, the pressure at other altitudes may be determined provided the ambient temperature and the change in altitude are determined. The ambient temperature T may be determined from the measured temperature T_m with the use of the relation

$$T=\frac{T_m}{1+\frac{\gamma-1}{2}KM^2} \tag{4}$$

Since only the indicated Mach number M' is known for flight conditions where no airspeed calibration exists, the ambient temperature is determined only approximately as

$$T'=\frac{T_m}{1+\frac{\gamma-1}{2}KM'^2} \tag{5}$$

The use of T' in equation (3) would result in a small error in p and, hence, two or more approximations may be necessary.

An alternative integral form of equation (2) is

$$\left(\frac{p}{p_1}\right)^n=1-n\int_{h_1}^{h}\left(\frac{p}{p_1}\right)^n\frac{dh}{RT} \tag{6}$$

After substitution of the measured pressure p_m and the temperature T' in the right side of equation (6), the equation becomes

$$\left(\frac{p}{p_1}\right)^n=1-n\int_{h_1}^{h}\left(\frac{p_m}{p_1}\right)^n\frac{dh}{RT'}$$

or

$$\left(\frac{p}{p_1}\right)^n=1-n\int_{h_1}^{h}\left(\frac{p_m}{p_1}\right)^n\frac{\left(1+\frac{\gamma-1}{2}KM'^2\right)}{RT_m}\,dh \tag{7}$$

The value of n may be selected for the flight conditions encountered so that only one approximation is required in the determination of p. (See appendix A.) For values of temperature recovery factor K of the thermometer near unity and for subsonic and low supersonic airspeeds, a value of n of $\frac{\gamma-1}{\gamma}$ or 0.286 gives satisfactory results.

The change in altitude dh in equation (7) may be determined from vertical velocity computed from measurements of pressure and temperature at an instant in the calibration run when the airspeed calibration or the static pressure is known and from vertical acceleration computed from measurements of longitudinal and normal acceleration and attitude angle, as

$$dh=\left(v_1+\int_{t_1}^{t}a_v\,dt\right)dt \tag{8}$$

where

$$v_1=\frac{-RT_1}{p_1}\left(\frac{dp}{dt}\right)_1 \tag{9}$$

and

$$a_v=a_z\cos\theta-a_x\sin\theta-g \tag{10}$$

Vertical velocity v_1 may also be determined as described in appendix B.

The relation given by equation (7) may also be used to advantage in the radar method of reference 1, particularly for tests which would not permit a survey of static pressure to be made over the desired range of altitude at flight conditions (Mach number and lift coefficient) at which the airspeed calibration was known. For such a flight condition, however, the airspeed calibration at least at one instant during the test must be known and the change in altitude from the altitude at that instant would, of course, be determined from radar measurements.

REQUIRED EQUIPMENT

The airplane on which the pitot-static installation is to be calibrated should be equipped with instruments to record the following: static pressure measured by the static-pressure source; impact pressure, or the difference between total and static pressure, measured by the pitot-static installation; temperature; normal and longitudinal components of acceleration; attitude angle; and time. If the static-pressure recorder does not have the required sensitivity for accurately measuring changes in static pressure for the determination of vertical velocity, a statoscope with a sensitive differential-pressure recorder should also be included. The impact-pressure recorder, the static-pressure recorder, and the statoscope, if one is used, should be the only instruments connected to the static-pressure source and should be located as near as possible to it in order to minimize the lag of the pressure system. Provision must also be made so that the large volume of the statoscope is not continuously open to the static-pressure source because of lag considerations and so that the sensitive pressure cell is not subjected to pressures beyond its limit in order to prevent damage. The magnitude of the pressure lag of the airspeed installation may be determined by methods described in reference 2. Where the lag is appreciable, corrections must be made to the measured static pressure. The thermometer is used to determine free-stream temperature. A properly shielded thermometer with a high temperature recovery factor (approaching 1.0) is recommended since it should be least affected by position on the airplane. The thermometer should also have a low lag; corrections should be made if the lag is appreciable. The attitude recorder is used to measure the attitude of the airplane and may consist of a horizon camera, a sun camera, or an attitude gyroscope. A horizon camera shooting either forward or laterally is probably the most desirable attitude recorder in localities where the horizon is not obscured by haze. The attitude gyroscope and the sun camera, however, may be more generally used. The attitude gyroscope measures the change in attitude angle. The attitude angle at some instant during the calibration must therefore be determined from other measurements, perhaps from the statoscope measurements and from the estimated angle of attack. When a sun camera is used, the attitude angle is determined by subtracting the elevation angle of the sun from the angle recorded by the camera

$$\theta=\alpha-\lambda \tag{11}$$

The elevation angle of the sun may be found in navigational tables by use of the date of calibration, the time at the start or end of the calibration run, and the longitude and latitude of the airplane. The elevation angle may also be found from the expression

$$\sin \lambda=\sin \epsilon \sin \tau+\cos \epsilon \cos \tau \cos (\omega-\beta) \tag{12}$$

When the sun camera is used, the airplane should be equipped with an indicating device to enable the pilot to fly the airplane in a vertical plane with the lateral axis normal to the rays of the sun. One such device is a sundial.

CALIBRATION PROCEDURE

The calibration procedure described herein may be used for level flight, climb, dive, push-down, pull-out and any combination of these maneuvers in a vertical plane. A calibration may be obtained in a single run for some conditions and does not require any additional survey of static pressure and temperature as is necessary for the method of reference 1.

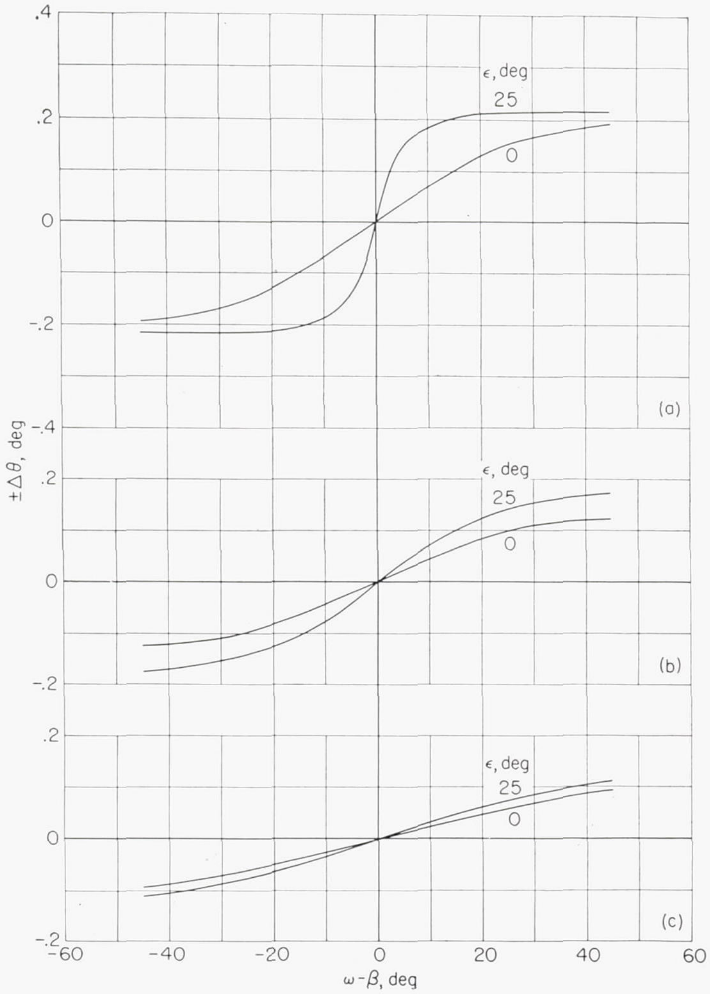
The calibration maneuver consists essentially of two parts. In one part the airplane is flown at a condition (lift coefficient and Mach number) for which the airspeed calibration is known. This part of the maneuver should preferably be made in or near level flight over an interval of time consistent with an accurate evaluation of vertical velocity. In the other part of the maneuver the airplane is flown at conditions for which the calibration is desired. This part of the maneuver should cover as short a time interval as practical in order to avoid the accumulation of errors in the evaluation. Either part of the maneuver may precede the other. Measurements, continuous throughout the maneuver, are made of impact pressure, static pressure, temperature, longitudinal acceleration, normal acceleration, attitude angle, and change in static pressure if a statoscope is used.

When a sun camera is used to obtain the attitude angle, the airplane must be flown with the lateral axis normal or very nearly normal to the rays of the sun. A simple sundial mounted ahead of the canopy may be used by the pilot as an indicator for keeping the lateral axis normal to the rays of the sun. The local time when the instruments are turned on should be determined accurately. The clock that is used for this purpose may be checked against radio time signal (National Bureau of Standards radio station WWV).

When a horizon camera or an attitude gyroscope is used, the airplane must be flown in a vertical plane. The airplane, however, is not restricted to any particular vertical plane as in the case with the sun camera.

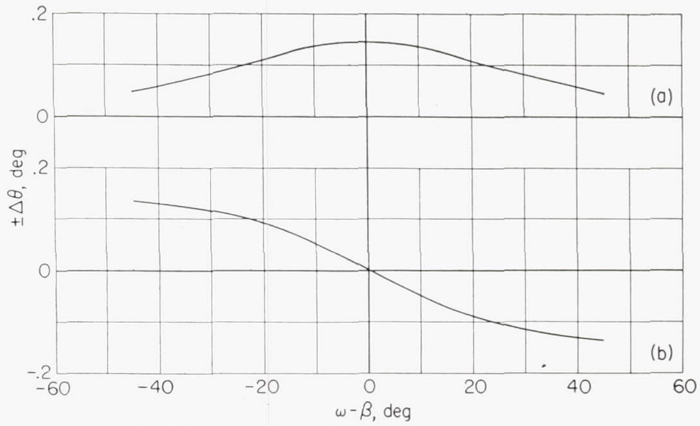
EFFECT OF ERRORS IN MEASUREMENT

The accuracy of the method depends principally on the accuracy of determining the attitude angle and the accuracy of the accelerometer measurements. The equations for the various errors are derived in appendix C, and the errors in the computed quantities due to assumed errors in measured quantities are presented in figures 1 to 11. When a sun camera is used to determine the attitude angle, errors in attitude angle may arise from an error in time of the calibration, an error in longitude and latitude of the airplane, and an error in measurement of angle with the sun camera. By measuring the time of the start or end of the calibration run to within a few seconds, the error in attitude angle due to an error in the measurement of the time of the calibration may be almost eliminated (see fig. 1). The clock should be checked against an accurate source of time, perhaps against radio time signal, before or after the calibration. Because of the speed of the airplane, and hence the distance covered in a calibration run, the latitude and longitude of the airplane may vary appreciably during the run. If, however, the pilot can estimate his position to within 10 miles, the error in attitude angle can be kept to within $\pm 0.1^{\circ}$ (fig. 2). The



(a) $\tau=30°$.
(b) $\tau=45°$.
(c) $\tau=60°$.

FIGURE 1.—Error in attitude angle due to error of ±60 seconds in determining local time of calibration when use is made of a sun camera.



(a) Due to error of ±10 miles in latitude.
(b) Due to error of ±10 miles in longitude.

FIGURE 2.—Error in attitude angle due to error in estimating position of airplane. $\epsilon=15°$; $\tau=40°$.

error in the angle of sun rays relative to the longitudinal axis as measured with the sun camera need not be more than about 0.1° for a properly designed sun camera. The probable maximum error in attitude angle should be of the order of 0.1°.

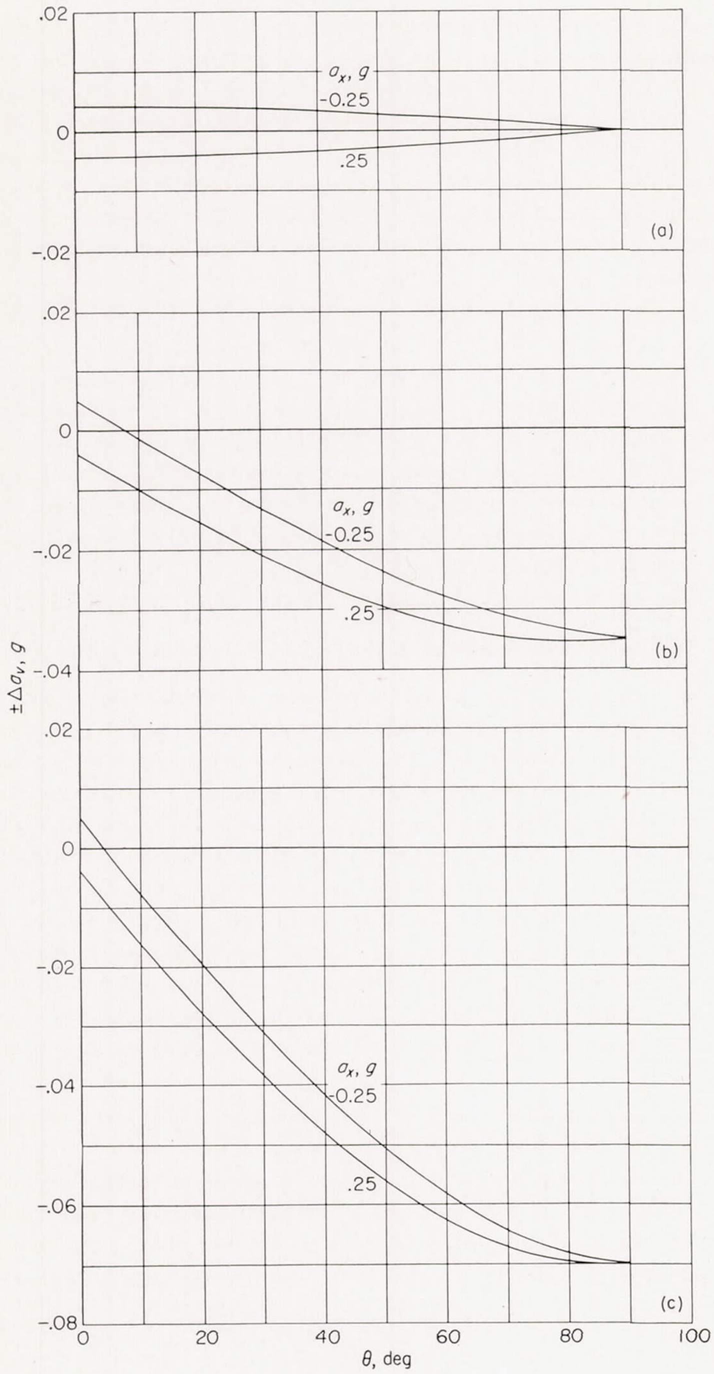
A properly designed horizon camera can probably measure attitude angle to within 0.1°. When the horizon camera is shooting forward, the correction for dip of the horizon can be estimated to within ±0.1°.

When the attitude angle is obtained from the flight-path angle and the angle of attack at one instant in the calibration run, together with the change in attitude angle as measured with an attitude gyroscope for other times during the calibration run, the error in attitude angle may be of a larger magnitude than that for the method with which the sun camera is used. The error in the flight-path angle arises from an error in determining vertical velocity. If the error in vertical velocity is 1 foot per second, the corresponding error in flight-path angle is about 0.1° at a flight Mach number of 0.6. The error in estimating the angle of attack at the instant when the flight-path angle is determined depends on the applicability and the accuracy of the information on which the estimate is based. The NACA attitude gyroscopes have a drift of about 3° per minute. For a calibration run lasting, for example, 20 seconds, the error in attitude angle due to the drift of the gyroscope would be of the order of 1° at the end of the calibration run. If the error in estimating the angle of attack is 0.3° and the error in determining attitude angle is 0.1°, the maximum error in attitude angle for a calibration run lasting 20 seconds would vary from 0.4° at the start to about 1.4° at the end.

The error in the vertical component of acceleration due to an error of ±1° in attitude angle as shown in figure 3 increases with increase in normal acceleration and with increase in attitude angle. In dives, normal acceleration would vary from about 1 g near level flight to 0 in a vertical dive. The error in vertical acceleration in a dive would probably be of the order of $0.01g$ for an error of ±1° in attitude angle. In a pull-out, the error in vertical acceleration would be larger but would probably occur only near the end of the calibration run.

The error in the vertical component of acceleration due to an error of $\pm 0.01g$ in the normal and longitudinal components of acceleration is of the order of $0.01g$ (fig. 4). A standard NACA recording accelerometer has an accuracy of $\frac{1}{4}$ percent of full range. A longitudinal accelerometer of $\pm\frac{1}{2}g$ range and a normal accelerometer of 0 to 1 g range would have an accuracy of $0.0025g$. A normal accelerometer of 0 to $4g$ range would have an accuracy of $0.01g$. In order to get improved accuracy for normal accelerations, a combination low-range and high-range normal accelerometer can be used. For example, in a calibration run involving a dive and pull-out, a low-range normal accelerometer can be evaluated for the dive and the high-range accelerometer for the pull-out. Errors in normal acceleration due to zero shift of the instrument may be largely eliminated by the method of appendix B.

The error in the vertical component of acceleration due to neglecting the angle of bank varies with attitude angle and normal acceleration (fig. 5). For 1 g normal acceleration, neglecting an angle of bank of 10° results in an error in vertical acceleration of $-0.015g$ at zero attitude angle and of $0g$ in a vertical dive.

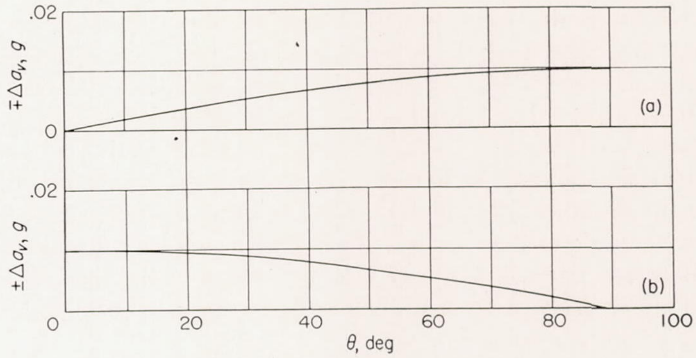


(a) $a_z=0$.
(b) $a_z=2.0g$.
(c) $a_z=4.0g$.

FIGURE 3.—Error in vertical component of acceleration due to $\pm 1°$ error in attitude angle.

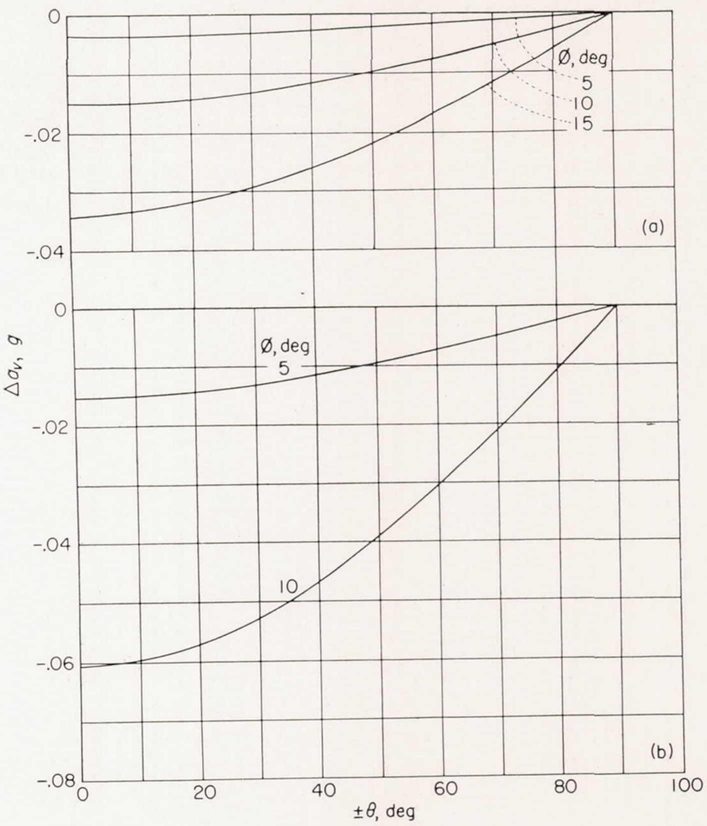
The error in the vertical component of acceleration due to neglecting the angle of yaw is shown in figure 6. For a 2° angle of yaw, the error in vertical acceleration is less than $0.01g$. For transonic and supersonic speeds, an angle of yaw of 2° probably would not be unintentionally exceeded and certainly would not be maintained.

The error in static pressure due to a constant error of $\pm 0.01g$ in vertical acceleration varies as the square of the time for the calibration run (fig. 7). For a calibration run lasting 20 seconds, the error, as percent of free-stream static pressure, is 0.3, and for 40 seconds, the error is 1.1.



(a) Due to error in longitudinal acceleration.
(b) Due to error in normal acceleration.

FIGURE 4.—Error in vertical component of acceleration due to an error of $\pm 0.01g$ in longitudinal and normal accelerations.



(a) $a_z=1.0g$.
(b) $a_z=4.0g$.

FIGURE 5.—Error in vertical component of acceleration due to neglecting angle of bank.

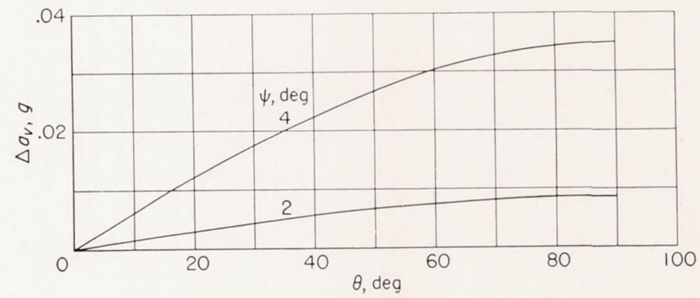


FIGURE 6.—Error in vertical component of acceleration due to neglect of angle of yaw. $a_y=0.25g$ and $0.50g$ for $\psi=2°$ and 4°, respectively. $a_x=0$.

Errors in the time rate of change of static pressure may arise not only from the recording instruments but also from the time rate of change of Mach number and of angle of attack when Mach number and angle of attack affect the airspeed calibration. These Mach number and angle-of-attack effects may be avoided by making the measurements for the determination of vertical velocity v_1 in approximately constant-speed level flight. This type of maneuver will also minimize errors in vertical velocity due to uncertainties in the static-pressure measurements if the method of appendix B (eq. (20)) is used.

The error in vertical velocity due to an error of ± 0.01 inch of water in the time rate of change of static pressure at 40,000 feet is 2.8 feet per second (fig. 8). This error in vertical velocity results in a static-pressure error which is shown as a function of time in figure 9. For calibration runs lasting 20 and 40 seconds the errors in static pressure are about 0.3 and 0.5, respectively.

The error in static pressure due to an error of ± 1 percent in measured temperature (or about $\pm 5°$ F) varies with the range of pressures covered in the calibration (fig. 10). For a range of static pressures from 0.7 to 3 times the initial static pressure, the error is within 1 percent of free-stream static pressure.

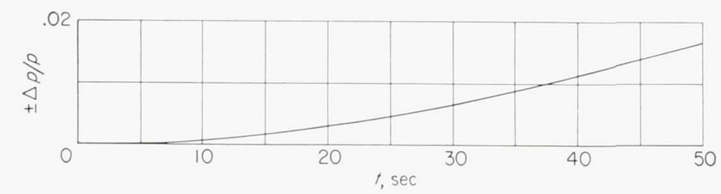


FIGURE 7.—Error in free-stream static pressure due to consistent error of $\pm 0.01g$ in vertical component of acceleration.

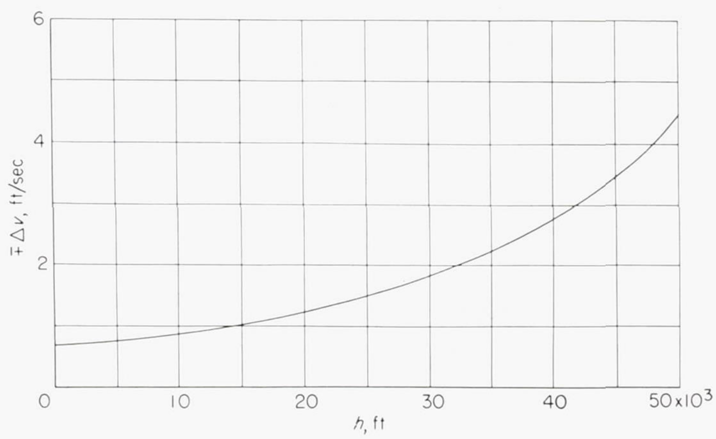


FIGURE 8.—Error in vertical velocity due to error of ± 0.01 inch of water per second in time rate of change of static pressure.

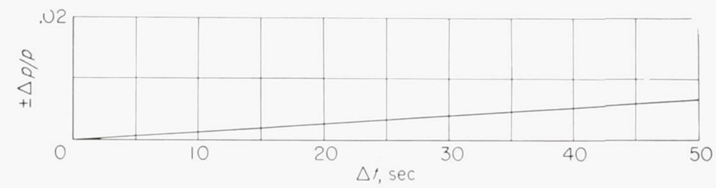


FIGURE 9.—Error in static pressure due to error of ± 0.01 inch of water per second in time rate of change of static pressure. $h \approx 40{,}000$ feet.

An error in total pressure results in an error in Mach number and, hence, in static pressure as determined from equation (7). Errors in total pressure due, for instance, to angularity of flow may be avoided by the use of properly designed pitot tubes. The error in free-stream static pressure due to an error of ± 1 percent of total pressure is seen in figure 10 to be within 0.3 percent of free-stream static pressure for a range of static pressures from 0.7 to 3 times the initial static pressure.

In evaluating the free-stream static pressure with the use of equation (6), the value of the gas constant may be taken to be 53.3, the value for dry air. The resulting error due to neglecting moisture content may be shown to be negligible. For example, if the air were saturated, the use of the gas constant for dry air would introduce an error of about 0.2 percent of free-stream static pressure in a dive from an altitude of 10,000 feet to sea level and of 0.01 percent in a dive from 30,000 feet to 20,000 feet.

The error in static pressure due to error in a standard NACA static-pressure recorder is $\pm\frac{1}{4}$ percent of the full-scale reading. For a static-pressure recorder covering a range from sea level to 50,000 feet, the error in static pressure is ± 1 inch of water or ± 1.3 percent of static pressure at an altitude of 40,000 feet. For a static-pressure recorder covering a range of altitudes above 30,000 feet, the error is ± 0.2 inch of water or ± 0.3 percent of static pressure at 40,000 feet. Further improvement in accuracy of the static-pressure measurements may be obtained with the use of a statoscope equipped with a differential-pressure recorder having a range to cover the change in static pressure over the desired range of altitudes. The error in Mach number due to an error of ± 1 percent of static pressure is shown in figure 11.

The elapsed time t may be measured to within 0.01 percent with the use of a tuning-fork timer. The static-pressure error corresponding to this error in time is (according to eq. (50)) within 0.05 percent of free-stream static pressure for a range of altitudes $(h-h_1)$ of 50,000 feet. Since all instruments can be of the continuous-recording type, no

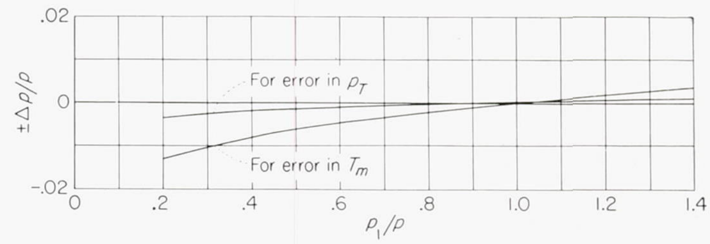


FIGURE 10.—Error in free-stream static pressure due to ± 1-percent error in measured temperature and ± 1-percent error in total pressure.

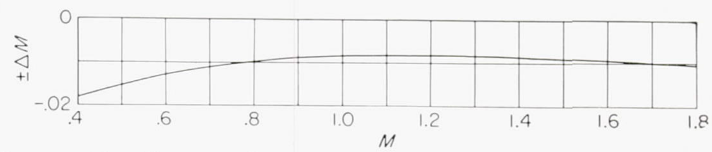


FIGURE 11.—Error in Mach number corresponding to ± 1-percent error in static pressure.

consistent error should result from the correlation of these records.

When a calibration run begins and ends near level flight at a speed for which the calibration is known, a check on the constant errors in calibration (due to errors in acceleration or attitude angle) is obtained in that the vertical velocity at the end of the calibration run should be equal to vertical velocity at the start plus the time integral of the vertical acceleration. A consistent error in the static-pressure recorder does not affect the determination of the static-pressure error since the consistent error would be included in the computed free-stream static pressure as well as in the measured pressure.

The effect of the various errors on a calibration was evaluated for a hypothetical maneuver involving a level-flight run of 4 seconds followed by a 30° dive lasting 30 seconds on the assumption that standard NACA instruments were used. This study indicated that the various quantities could be measured accurately enough with standard instruments to insure a satisfactory calibration.

FLIGHT EVALUATION OF METHOD

An experimental verification of the conclusions reached in the analytical study of the effect of errors on the calibrations was considered desirable. Flight tests were therefore made to obtain a calibration of an airspeed installation on a jet fighter airplane by the accelerometer method. A calibration was also made at the same time by the radar-phototheodolite method for comparison. These flight tests are described in the following sections.

EQUIPMENT

The jet-powered fighter airplane used for the calibration tests was equipped with a pitot-static tube mounted on a boom about 1 fuselage maximum diameter ahead of the fuselage nose. A resistance-type free-stream thermometer equipped with two radiation shields was mounted about $\frac{2}{3}$ fuselage maximum diameter ahead of the nose on the airspeed boom. The thermometer had a recovery factor very nearly 1.0 and a time lag of about $\frac{1}{10}$ second for the altitude and speed range at which the tests were made. The thermometer and pitot-static head are shown mounted on the boom in figure 12.

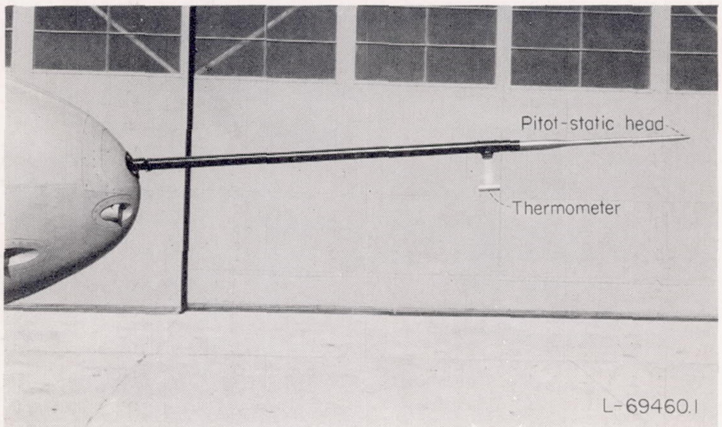


FIGURE 12.—Pitot-static head and thermometer mounted on the fuselage nose boom.

The instruments installed in the airplane and the range of each instrument are as follows:

Thermometer	−40° to 40° F
Static-pressure recorder	95 to 422 inches of water
Impact-pressure recorder	30 to 84 inches of water
Normal accelerometer	0 to 2g
Normal accelerometer	2 to 4g
Longitudinal accelerometer	0.5 to 0.2g
Sun camera	30°

All of these instruments recorded the measurements continuously on films which were synchronized by a $\frac{1}{10}$-second timer. An identification code, which synchronized the radar measurements with the measurements taken in the airplane, was transmitted to the ground radar station by the aircraft radio.

The sun camera was mounted in the airplane below an opening in the skin ahead of the pilot's canopy. The camera was designed to record continuously the attitude of the airplane relative to the sun. A simple sundial, shown in figure 13, was installed to aid the pilot in maintaining the lateral axis of the airplane normal to the rays of the sun.

The time of the start or end of a test was determined by means of an ordinary watch checked against time obtained from radio station WWV operated by the National Bureau of Standards.

The radar-phototheodolite ground equipment was the same as that described in reference 1.

ACCURACY OF RECORDING EQUIPMENT

In order to utilize adequately the accuracy of the instruments, much care was taken in the calibration of the instruments and in the reading of the film.

A flight test was made prior to the airspeed-calibration tests to check the measurements of free-stream temperature by using two thermometers and recording galvanometers of the same design. The recorders indicated an occasional difference in temperature of no more than $\frac{1}{2}$° F. The errors in temperature resulting from lag in the thermometer varied with the rate of change of measured temperature. The average error was only about −0.1° F in the dives; therefore, no corrections were applied since the errors were considered negligible.

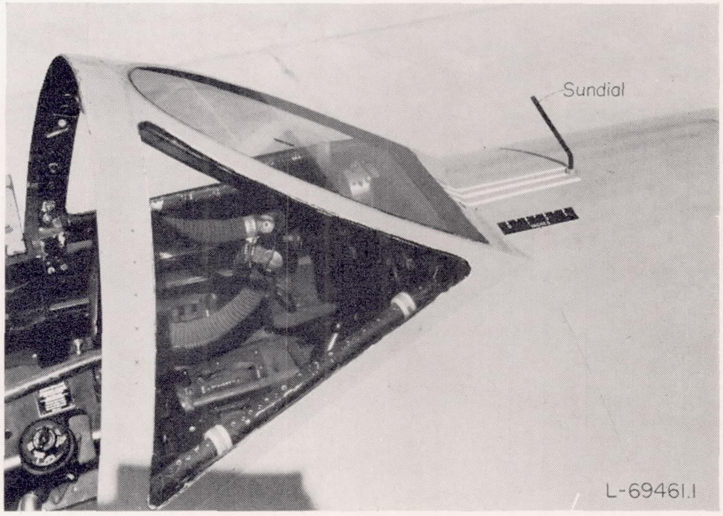


FIGURE 13.—Sundial used by the pilot as an aid in alining the airplane with the sun.

The static-pressure recorder was specially built and had a reading accuracy of about ± 0.05 inch of water. A calibration of the static-pressure recorder with increasing and then decreasing pressure indicated a hysteresis loop of about ± 0.5 inch of water. The accuracy of the static-pressure recorder, however, is believed to be better than this value indicates since the diaphragm was put in a rested state by applying and releasing a suction of about 350 inches of water several times immediately before flight and since the static-pressure recorder was calibrated immediately after flight by using a pressure-time sequence approximating the flight tests.

The impact-pressure recorder had an accuracy greater than about ± 0.1 inch of water. The accuracy of the recorder is well within the precision required by the accelerometer method since impact pressure affects only the ratio of static-pressure error to impact pressure, the determination of Mach number M', and the temperature T'.

The time lag in the static-pressure line connected to the static-pressure recorder was estimated to be 0.05 second for the altitude of the tests. Since this time lag corresponded to a lag in static pressure of less than 0.1 inch of water or 0.001 q_c' for the maximum rate of change of static pressure occurring in the maneuvers, no correction was applied. The time lag in the total-pressure line was estimated to be 0.03 second. The effect of the lag in the total-pressure and static-pressure lines on impact pressure was negligible.

A calibration of the normal accelerometer indicated effects of longitudinal acceleration and of temperature for which corrections were applied. Consistent errors in normal acceleration due to zero shift in the instrument are believed to be eliminated by use of the method described in appendix B. The normal accelerometer had a constant uncertainty of about ± 0.2 percent of the change of normal acceleration from 1 g. Uncertainty of the longitudinal accelerometer zero is believed to be about $\pm 0.002g$.

The sun camera had a reading accuracy of about 0.07° and its setting relative to the axes of the accelerometers could be measured to within about 0.2°. Although the solar time was determined to within 5 seconds, the time was taken at the midpoint of each maneuver since the resulting error in the elevation angle of the sun at the beginning and end of the maneuver was estimated to be small (less than 0.1°).

TESTS

The tests consisted of three shallow dives from an altitude of 31,000 feet to an altitude of about 26,000 feet. Two dives of about $\frac{1}{2}$-minute duration covered a Mach number range from about 0.6 to 0.8 with 2 and 4g pull-ups, and the third dive of about $1\frac{3}{4}$-minutes duration covered a range of Mach numbers from about 0.40 to 0.80 with a 3g pull-up. Prior to each of the short dives, a survey of atmospheric pressure for the radar-phototheodolite method was made in a climb at an airplane Mach number of about 0.45 and records were taken about every 500 feet between altitudes of 23,000 and 31,000 feet. Continuous measurements were made during the dives and pull-ups. Radar-phototheodolite equipment was not used for the third dive. The pilot attempted to hold the lateral axis of the airplane normal to the sun's rays through the use of the sundial.

EVALUATION OF MEASUREMENTS

The calibration of the airspeed installation by the radar-phototheodolite method was made as described in reference 1. Free-stream static pressure was determined from surveys of atmospheric pressure by using the static-pressure error determined in previous tests with a trailing airspeed head up to a Mach number of about 0.40. The surveys were made at a Mach number of about 0.45; therefore, extrapolation of the static-pressure error obtained from the tests with trailing airspeed head for the approximate Mach number range from 0.40 to 0.45 was necessary.

For the purpose of evaluation, the dives were divided into two parts. Data from the first part of each dive were used to determine the vertical velocity v_1 at the beginning of the last part of the dive as described in appendix B. The static-pressure error was evaluated only for the last part of the dive.

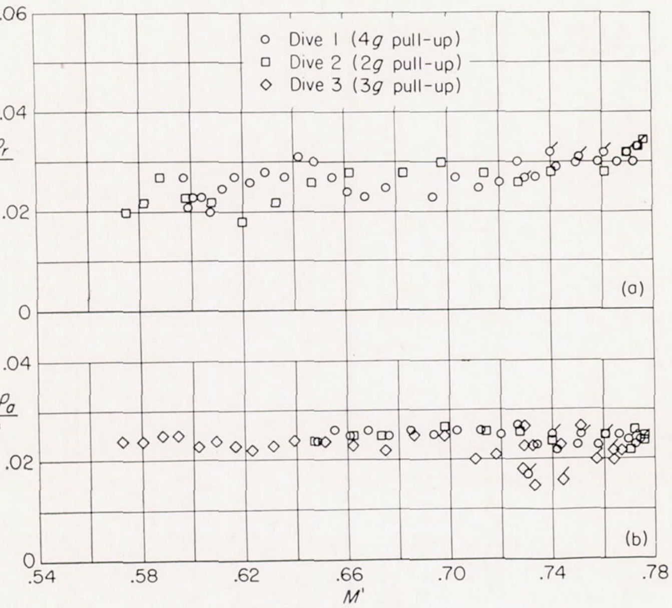
The static-pressure error determined for corresponding flight conditions from results obtained with the trailing airspeed head and the radar method was used in computing the vertical velocity v_1 from data taken during the first 12 seconds of dive 1, the first 13 seconds of dive 2, and the first 45 seconds of dive 3.

The free-stream static pressure p_1 in equation (7) was obtained from the static pressure measured at the beginning of the last part of the dives by using the static-pressure error as determined from the calibration by the radar method. Since the results of the radar calibration were obtained from Mach numbers of 0.57 to 0.78, the results of dive 3 were evaluated by the accelerometer method starting at the time at which a Mach number of 0.57 was attained (57 sec from end of maneuver).

By using vertical velocity v_1 and static pressure p_1 thus determined, the static-pressure error was evaluated for the last 22, 24, and 57 seconds of dives 1, 2, and 3, respectively.

RESULTS AND DISCUSSION

The results of the calibrations by both methods are presented as plots of $\frac{p_m - p}{q_c'}$ against indicated Mach number M' in figure 14. The static-pressure error determined by the accelerometer method for dives 1 and 2 and subsequent pull-outs was nearly constant at about 2.5 percent of impact pressure with very little scatter of data over the range of Mach numbers used in the evaluation (0.65 to 0.78). The results of dive 3, evaluated over a much larger time interval and Mach number range (0.57 to 0.78), agreed closely with the results of the other dives up to the start of the pull-out, after which the results for dive 3 showed a few points that were lower than the average for the other dives by as much as 1 percent of impact pressure. Because of the absence of similar effects in pull-outs following dives 1 and 2, this deviation is not considered to be the effect of airplane lift coefficient on the calibration. The deviation is, however, within the accuracy accepted for most calibrations. Uncertainties of the airspeed calibration, due to the estimated errors of several sources, varied from zero near the beginning of dive 3 to maximum values near the end of the dive. These maximum values are shown in table I. Since the calculations for two of the sources were necessarily probable errors and for



(a) Radar method.
(b) Accelerometer method.

FIGURE 14.—Airspeed calibration as evaluated by the radar and accelerometer methods. Flagged symbols are in the pull-out where $a_z \geqq 2g$.

the remaining sources were necessarily nominal maximum possible errors, no attempt was made to compare the data with any combination of these uncertainties.

The static-pressure error as determined by the radar method increased from about 2.5 percent of impact pressure at a Mach number of 0.57 to a little over 3.0 percent at a Mach number of 0.78. The scatter of about ± 0.5 percent at the low Mach numbers and ± 0.2 percent at the high Mach numbers is about $\frac{1}{2}$ the maximum possible scatter due to inaccuracies of measuring static pressure and height by radar (± 45 ft for slant range and ± 0.2 mil for elevation angle). On the basis of the faired data, the results of the radar-method calibration show the greatest difference (0.5 percent) from the accelerometer-method calibration at the high Mach numbers. Differences of this magnitude have been noted between two tests for a radar calibration in reference 1, although in the present tests there was no consistent difference between the two successive dives. It should be noted that the maximum uncertainty in the accelerometer method, due to the estimated possible error in static pressure (shown in table I), is about the same magnitude.

The calibration, as determined by both the radar and accelerometer methods, is typical of nose-boom installations inasmuch as there was little effect of either Mach number or lift coefficient over the ranges covered in these tests.

TABLE I.—ESTIMATED ERRORS IN AIRSPEED CALIBRATION NEAR THE END OF DIVE 3 DUE TO VARIOUS SOURCES OF CONSTANT ERROR IN THE EVALUATION BY THE ACCELEROMETER METHOD

Source	Source error	Error in airspeed calibration (percent impact pressure)
Initial velocity v_0 determined by least-squares method	± 0.09 ft/sec (probable error)	± 0.04 (probable)
Zero of normal accelerometer determined by least-squares method	± 0.0053 ft/sec^2 (probable error)	± 0.21 (probable)
Sensitivity of the normal accelerometer	± 0.002 $(a_z - g)$	± 0.11
Zero of longitudinal accelerometer	± 0.064 ft/sec^2	∓ 0.08
Attitude angle	$\pm 0.2°$	∓ 0.02
Temperature	$\pm \frac{1}{2}°$ F	± 0.25
Static pressure	$\pm \frac{1}{4}$ in. of water	∓ 0.46

CONCLUDING REMARKS

A method is described for calibrating airspeed installations on airplanes at transonic and supersonic speeds in vertical-plane maneuvers in which use is made of measurements of normal and longitudinal accelerations and attitude angle. The method involves starting or ending a calibration run near level flight at a speed for which the airspeed calibration is known and, hence, for which the free-stream static pressure may be determined. Integration of the vertical acceleration computed from the normal and longitudinal accelerations and the attitude angle determines the change in altitude which, when combined with the temperature measurements, gives the change in static pressure from the start or end of the calibration run and, hence, the variation of free-stream static pressure during the calibration run. The static-pressure error is then obtained at any instant during the calibration run by subtracting the free-stream static pressure from the indicated static pressure.

In the method described herein the required instrumentation is carried within the airplane. Should the airplane at any time enter a previously unexplored flight condition in a vertical-plane maneuver, a calibration may be readily obtained.

An analytical study of the effects of various sources of error on the accuracy of an airspeed calibration by the accelerometer method indicated that the required measurements can be made accurately enough to insure a satisfactory calibration.

In order to obtain an experimental verification of the analytical study, flight tests were made and an airspeed calibration of a jet fighter airplane was evaluated using the accelerometer method and, for comparison, the radar-phototheodolite method. The tests included shallow dives up to a Mach number of about 0.80 with pull-ups of about 4, 3, and $2g$ normal acceleration. The calibrations of the dives by the two methods are typical of a nose-boom installation inasmuch as there was little effect of either Mach number or lift coefficient over the ranges covered in these tests. The static-pressure error as determined by the radar-phototheodolite method increased from about 2.5 percent of impact pressure at a Mach number of 0.57 to a little over 3.0 percent at a Mach number of 0.78. The static-pressure error determined by the accelerometer method was nearly constant at about 2.5 percent of impact pressure over the same Mach number range. From the results of the tests it appears that, for vertical-plane maneuvers, the accelerometer method may be used as an alternate to the radar-phototheodolite method.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *Dec. 5, 1952.*

APPENDIX A

CALCULATION OF SUITABLE VALUES OF n FOR USE IN EQUATION (7)

The value of n that yields zero or nearly zero error in the computed free-stream static pressure as a result of using p_m and M' in equation (7) may be found by first differentiating equation (6) as

$$\frac{\Delta p}{p}=-\frac{1}{p^n}\int\frac{p^n}{RT}(ne_p-e_T)\,dh \tag{13}$$

where e_p in the integral is the error in the static-pressure source and $\Delta p/p$ on the left side of the equation is the error in the computed free-stream static pressure due to use of p_m and M'. Also in the integral e_T is the error in T due to the use of M'. For zero error in computing free-stream static pressure

$$ne_p=e_T$$

or

$$n=\frac{e_T}{e_p}$$

For $M\leqq 1.0$, the value of $\frac{e_T}{e_p}$ may be determined from equation (4) and the equation

$$p_T=p\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{14}$$

Differentiating each equation and combining results in the following expression:

$$n=\frac{e_T}{e_p}=\frac{p}{T}\frac{dT}{dp}=\frac{\gamma-1}{\gamma}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}KM^2}\right)K \tag{15}$$

For $M\geqq 1.0$ the value of n may be similarly computed from equation (4) and the equation

$$p_T=\frac{\gamma+1}{2}M^2p\left[\frac{(\gamma+1)^2M^2}{4\gamma M^2-2(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \tag{16}$$

or

$$n=\frac{\gamma-1}{2\gamma}\,\frac{2\gamma M^2-(\gamma-1)}{2M^2-1}\,\frac{M^2K}{1+\frac{\gamma-1}{2}KM^2} \tag{17}$$

The values of n for $K=0.9$ and $K=1.0$ are tabulated for various Mach numbers as follows:

M	n	
	$K=0.9$	$K=1.0$
0.5	0.258	0.286
1.0	.261	.286
1.5	.347	.374
2.0	.461	.490
3.0	.644	.670
4.0	.759	.779

Since, in the general case, a range of Mach number would be covered in a calibration test, a mean value of n may be taken. The possible error in the computed free-stream static pressure resulting from the use of the mean value for a range of Mach numbers from $M=1.0$ to $M=2.0$ was estimated to be less than 3 percent of the static-pressure error of the airspeed installation.

APPENDIX B

DETERMINATION OF INITIAL VERTICAL VELOCITY AND ZERO SHIFT IN A NORMAL ACCELEROMETER

If a run in, or near, level flight is made prior to, and continuous with, the calibration maneuver, the initial vertical velocity v_0 at the start of this run may be determined from integrations of the accelerometer measurements and change in geometric height computed from pressure and temperature measurements by using the equation

$$-\int_{p_0}^{p}\frac{RT}{p}dp=v_0t+\int_0^t\int_0^t a_v\,dt\,dt \tag{18}$$

This method of determining vertical velocity v_0, however, may introduce errors due to errors in vertical acceleration. An appreciable source of error in the airspeed calibration by the accelerometer method may be the zero shift of the normal accelerometer. This error can be corrected at the same time that v_0 is determined. The vertical acceleration may be written as

$$a_v=a_v'+\Delta a_v \tag{19}$$

where a_v' is indicated vertical acceleration and Δa_v is a constant error in vertical acceleration. In, or near, level flight

$$\Delta a_v\approx\Delta a_z \tag{20}$$

Equation (18) may therefore be rewritten as

$$-\int_{p_0}^{p}\frac{RT}{p}dp-\int_0^t\int_0^t a_v'\,dt\,dt=v_0t+\Delta a_z\frac{t^2}{2} \tag{21}$$

A solution of this equation which contains two unknowns, v_0 and Δa_z, may be determined by satisfying the equation over two time intervals. A better approach is to use the method of least squares with a large number of time intervals.

Once v_0 (and Δa_z) is determined, the vertical velocity v_1 at the start of the calibration maneuver may be determined as

$$v_1=v_0+\int_0^{t_1}a_v\,dt \tag{22}$$

Although the preceding discussion refers specifically to a maneuver starting in, or near, level flight, the method is also applicable to a maneuver ending in, or near, level flight.

APPENDIX C

CALCULATION OF ERRORS

ERROR IN ATTITUDE ANGLE DUE TO ERROR IN TIME OF CALIBRATION

An error in the time of calibration results in an error in the elevation angle of the sun and, hence, in the attitude angle. The error in the elevation angle of the sun is, after differentiation of equation (12),

$$\Delta\lambda = -\frac{\cos\epsilon\cos\tau}{\cos\lambda}\sin(\omega-\beta)\Delta\omega \tag{23}$$

or

$$\Delta\lambda = -\frac{\cos\epsilon\cos\tau}{\cos\lambda}\sin(\omega-\beta)\frac{15}{3600}\Delta t \tag{24}$$

where

$$\Delta\omega = \frac{15}{3600}\Delta t$$

and Δt is the error in time in seconds.

Since

$$\Delta\lambda = -\Delta\theta$$

$$\Delta\theta = 0.00416\frac{\cos\epsilon\cos\tau}{\cos\lambda}\sin(\omega-\beta)\Delta t \tag{25}$$

The error in attitude angle due to an error of 60 seconds in time of the calibration is plotted in figure 1.

ERROR IN ATTITUDE ANGLE DUE TO ERROR IN LATITUDE AND LONGITUDE OF THE AIRPLANE

The error in attitude angle due to error in the latitude of the airplane is

$$\Delta\theta = -\Delta\lambda = \left[\frac{-\sin\epsilon\cos\tau + \cos\epsilon\sin\tau\cos(\omega-\beta)}{\cos\lambda}\right]\Delta\tau \tag{26}$$

or

$$\Delta\theta = \left[\frac{-\sin\epsilon\cos\tau + \cos\epsilon\sin\tau\cos(\omega-\beta)}{\cos\lambda}\right]\frac{57.3}{4000}s_\beta \tag{27}$$

where

$$\Delta\tau = \frac{57.3}{4000}s_\beta$$

and s_β is in miles.

Similarly, the error in attitude angle due to error in the longitude of the airplane is

$$\Delta\theta = \frac{-\cos\epsilon\sin(\omega-\beta)}{\cos\lambda}\frac{57.3}{4000}s_\tau \tag{28}$$

where s_τ is in miles.

The error in attitude angle due to error in the latitude and longitude of the airplane is plotted in figure 2.

ERROR IN VERTICAL COMPONENT OF ACCELERATION DUE TO ERROR IN ATTITUDE ANGLE

The error in vertical acceleration due to an error in attitude angle is, after differentiation of equation (10),

$$\Delta a_v = -(a_z\sin\theta + a_x\cos\theta)\Delta\theta \tag{20}$$

where $\Delta\theta$ is in radians. The error in the vertical component of acceleration for a $\pm 1°$ error in attitude angle is shown in figure 3.

ERROR IN VERTICAL COMPONENT OF ACCELERATION DUE TO ERROR IN LONGITUDINAL AND NORMAL COMPONENTS OF ACCELERATION

The error in the vertical component of acceleration due to error in the longitudinal component of acceleration is, after differentiation of equation (10),

$$\Delta a_v = -\Delta a_x\sin\theta \tag{30}$$

Similarly, for an error in the normal component of acceleration

$$\Delta a_v = \Delta a_z\cos\theta \tag{31}$$

The error in the vertical component of acceleration due to an error of $\pm 0.01g$ in the longitudinal and normal components of acceleration is presented in figure 4.

ERROR IN VERTICAL COMPONENT OF ACCELERATION DUE TO ANGLE OF BANK

The expression for vertical acceleration, including the angle of bank, is

$$a_v = a_z\cos\theta\cos\phi - a_x\sin\theta - g \tag{32}$$

where ϕ is the angle of bank.

If the angle of bank is neglected, the error is

$$\Delta a_v = a_z\cos\theta(\cos\phi - 1) \tag{33}$$

The error in the vertical component of acceleration is plotted in figure 5 for angles of bank of 5°, 10°, and 15°.

ERROR IN VERTICAL COMPONENT OF ACCELERATION DUE TO ANGLE OF YAW

The vertical component of acceleration, including the effect of yaw, is

$$a_v = a_z\cos\theta - a_x\sin\theta\cos\psi + a_y\sin\psi\sin\theta - g \tag{34}$$

where ψ is the angle of yaw and a_y is the lateral acceleration due to yaw.

If the angle of yaw is neglected, the error is

$$\Delta a_v = a_x \sin\theta\,(1-\cos\psi) + a_y \sin\psi \sin\theta$$
$$= [a_x(1-\cos\psi) + a_y \sin\psi] \sin\theta \tag{35}$$

This error in the vertical component of acceleration is shown in figure 6 for 2° and 4° of yaw.

ERROR IN FREE-STREAM STATIC PRESSURE DUE TO ERROR IN DETERMINING VERTICAL COMPONENT OF ACCELERATION

The error in free-stream static pressure due to error in the vertical component of acceleration is found by substituting equation (8) into equation (7) and then differentiating the resulting equation, or

$$\frac{\Delta p}{p} = -\frac{1}{p^n}\int_{t_1}^{t}\left[\frac{p^n(1+0.2KM^2)}{RT_m}\int_{t_1}^{t}\Delta a_v\,dt\right]dt \tag{36}$$

In the integral Δa_v, p, M, and T_m may vary with time. In order to obtain the order of magnitude of the error, however, it is sufficient to assume these quantities as constant. Then

$$\frac{\Delta p}{p} = -\frac{\Delta a_v}{2RT}\,t^2 \tag{37}$$

The variation of the error in static pressure with time is shown in figure 7 for an error in vertical acceleration of $\pm 0.01g$.

ERROR IN VERTICAL VELOCITY DUE TO ERROR IN DETERMINING THE TIME RATE OF CHANGE OF STATIC PRESSURE

The error in vertical velocity due to error in determining the time rate of change of static pressure is obtained from equation (9) as

$$\Delta v = -\frac{RT}{p}\,\Delta\left(\frac{dp}{dt}\right) \tag{38}$$

The error in vertical velocity due to an error of 0.01 inch of water per second in the time rate of change of static pressure is shown in figure 8 for various altitudes.

ERROR IN FREE-STREAM STATIC PRESSURE DUE TO ERROR IN DETERMINING VERTICAL VELOCITY OR THE TIME RATE OF CHANGE OF STATIC PRESSURE

The error in free-stream static pressure due to error in determining vertical velocity is, after substitution of equation (8) into equation (7) and differentiation of the resulting equation,

$$\frac{\Delta p}{p} = -\frac{1}{p^n}\int_{t_1}^{t}\frac{p^n(1+0.2KM^2)}{RT_m}\,\Delta v_1\,dt \tag{39}$$

In evaluating the order of magnitude of error in static pressure, T_m, M, and p may be assumed as constant in the integral and therefore

$$\frac{\Delta p}{p} = -\frac{\Delta v t}{RT} \tag{40}$$

In terms of error in time rate of change of static pressure, the error in static pressure is, after substitution for Δv from equation (38),

$$\frac{\Delta p}{p} = \Delta\left(\frac{dp}{dt}\right)\frac{t}{p} \tag{41}$$

This static-pressure error is plotted in figure 9 as a function of time for an altitude of about 40,000 feet and an error of ± 0.01 inch of water in the time rate of change of static pressure.

ERROR IN FREE-STREAM STATIC PRESSURE DUE TO ERROR IN MEASURING T_m

The error in free-stream static pressure due to error in measuring T_m is, after differentiation of equation (7),

$$\frac{\Delta p}{p} = \frac{1}{p^n}\int_{p_1}^{p}\frac{p^n(1+0.2KM^2)}{RT_m}\,\frac{\Delta T_m}{T_m}\,dh \tag{42}$$

For $\frac{\Delta T_m}{T_m}$=Constant, equation (42) reduces to

$$\frac{\Delta p}{p} = \frac{\Delta T_m}{nT_m}\left[\left(\frac{p_1}{p}\right)^n - 1\right] \tag{43}$$

The error in free-stream static pressure due to an error of ± 1 percent in T_m is plotted in figure 10 against the ratio of initial pressure p_1 to pressure p at any time during the calibration. A value of n of $\frac{\gamma-1}{\gamma}$ or 0.286 was assumed.

ERROR IN FREE-STREAM STATIC PRESSURE DUE TO ERROR IN MEASURING TOTAL PRESSURE

The error in total pressure introduces an error in the computation of temperature and, hence, in the computation of free-stream static pressure. The static-pressure error may be found by differentiating equation (6) as

$$\frac{\Delta p}{p} = \frac{1}{p^n}\int_{h_1}^{h}\frac{p^n}{RT}\,\frac{\Delta T}{T}\,dh \tag{44}$$

If $\frac{\Delta T}{T}$ is assumed to be a constant and is related to $\frac{\Delta p_T}{p_T}$ through the use of equations (4), (14), and (16), the static-pressure error is found to be

$$\frac{\Delta p}{p} = -\frac{\Delta p_T}{p_T}\left[\left(\frac{p_1}{p}\right)^n - 1\right] \tag{45}$$

The error in static pressure due to ± 1-percent error in total pressure is plotted in figure 10 against the ratio of initial pressure p_1 to pressure p at any time during the calibration for $n=\frac{\gamma-1}{\gamma}$.

ERROR IN FREE-STREAM STATIC PRESSURE DUE TO ERROR IN ELAPSED TIME

Integration of equation (8) between the limits of h and h_1 yields

$$h - h_1 = v_1 t + \int_0^t\int_0^t a_v\,dt\,dt \tag{46}$$

If the measured elapsed time t_m is substituted into equation (46), the equation becomes

$$h_m - h_1 = v_1 t_m + \int_0^{t_m}\int_0^{t_m} a_v \, dt_m \, dt_m \tag{47}$$

where h_m is computed altitude corresponding to measured elapsed time t_m. If the error in elapsed time is defined as

$$e_t = \frac{t_m - t}{t}$$

the error in altitude determined from equations (46) and (47) becomes

$$\Delta h = e_t [2(h - h_1) - v_1 t] \tag{48}$$

The corresponding error in free-stream static pressure is

$$\frac{\Delta p}{p} = \frac{e_t}{RT} [v_1 t - 2(h - h_1)] \tag{49}$$

The error in free-stream static pressure is a maximum if v_1 is zero or if v_1 has a direction opposite to the resultant change in altitude. For $v_1 = 0$,

$$\frac{\Delta p}{p} = \frac{2e_t(h_1 - h)}{RT} \tag{50}$$

ERROR IN MACH NUMBER DUE TO ERROR IN FREE-STREAM STATIC PRESSURE

The error in Mach number due to an error in static pressure is, after differentiation of equation (14) for $M \leqq 1.0$,

$$\Delta M = -\frac{1+0.2M^2}{1.4M} \frac{\Delta p}{p} \tag{51}$$

and, after differentiation of equation (16) for $M \geqq 1.0$,

$$\Delta M = -\frac{M}{7} \frac{7M^2 - 1}{2M^2 - 1} \frac{\Delta p}{p} \tag{52}$$

The error in Mach number due to a ± 1-percent error in free-stream static pressure is shown in figure 11.

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