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APPLICATION OF A NUMERICAL PROCEDURE TO STRESS ANALYSIS OF STRINGER-REINFORCED PANELS

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ADVANCE RESTRICTED REPORT

APPLICATION OF A NUMERICAL PROCEDURE TO STRESS ANALYSIS OF STRINGER-REINFORCED PANELS

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SUMMARY

A numerical procedure, as well as the underlying theory and assumptions, is presented for the calculation of the stringer stresses and shear stresses in reinforced panels. The method may be applied to all panel problems in which the loads may be considered acting in the plane of the sheet.

Examples are given to illustrate the use of the method for axially loaded panels with and without rectangular cut-outs and for the covers of box beams with and without rectangular cut-outs.

The results of this procedure are compared with the experimental data and the approximate engineering methods of analysis of previous NACA papers from which the problems are obtained.

INTRODUCTION

Several papers have been written on the stress analysis of sheet-stringer panels loaded axially or as the cover of box beams. (See references 1 and 2.) The solutions presented in these papers are generally sufficiently accurate for most practical cases of construction and loading but are not readily applicable to more general cases in which the cross section varies and the loads are arbitrarily distributed.

In the present paper a numerical procedure for the stress analysis of flat-sheet and stringer combinations of arbitrary construction and loading is presented and applied to axially loaded panels and to the reinforced

[redacted]

covers of box beams. The basic theory of the procedure was originally developed in reference 3. Comparisons are given of the results obtained by the numerical procedure of the present report and the results obtained by the approximate analyses and experimental results of references 1 and 2.

The numerical procedure parallels that of Southwell's relaxation method and Cross's moment-distribution method (references 4 and 5) but is so given in the present report that the reader need have no previous knowledge of these techniques. The equations obtained in reference 3 are solved by a relaxation procedure, whereas in the present paper a direct solution of simultaneous equations is used.

SYMBOLS

A, B, C,...	stringers; also used as subscripts
A_A, A_B,...	total effective cross-sectional area of stringers A, B,..., respectively, square inches
E	Young's modulus of elasticity, ksi
F	internal direct force in stringer, kips
G	shear modulus of elasticity, ksi
P	external applied load or force, kips
R	reaction at fixed ends of stringers, kips
S	shear force, kips
X	total internal force in x-direction, kips
a	length of panel unit, inches
b	width of panel unit, inches
t	average sheet thickness of panel unit, inches
u	displacement in x-direction, 10^{-5} inch

Δu elongation of stringer segment, 10^{-5} inch

x distance along stringer, inches

γ average shear strain

σ average stringer stress, ksi

τ average shear stress, ksi

Subscripts:

1, 2, 3,... transverse stations; also indicate structural unit when used with a and b

Forces acting on the structure and displacements of the structure in the positive x-direction are positive.

BASIC THEORY AND ASSUMPTIONS

Any structure may be considered composed of a number of smaller units and, if suitable expressions are obtained relating the deformations due to forces acting on these units, the deformations of the entire structure can be obtained by satisfying the conditions of static equilibrium and the continuity of the deformations of the units of the loaded structure. For a stringer-reinforced panel, the unit considered is a flat rectangular sheet bounded on its longitudinal edges by stringers and on its transverse edges by ribs. Such a unit is shown in figure 1(a). All the structures analyzed herein are symmetrical and are loaded in the direction of the axis of symmetry. For such problems the transverse displacements of the ribs can be neglected, which is equivalent to assuming that the ribs are rigid. As a consequence of this assumption, the ribs bounding the edges of the unit need not be those of the actual structure but can be fictitious ribs assumed to exist at the transverse edges of the units. The procedure is not limited to symmetrical structures but can be readily extended to more general problems which involve displacements of the ribs (reference 6).

The unit problem.- If the corner B_2 of the rectangular unit of figure 1(a) is displaced a distance u in the positive x-direction while the remaining corners are held fixed as shown in figure 1(b), internal forces are created that tend to restore the structural unit to its original rectangular shape. These restoring forces, which are assumed to be concentrated at the corners, act in the directions indicated by the arrows in figure 1(b). From consideration of static equilibrium, the sum of the forces at the fixed corners A_1, A_2, and B_1 must be equal to the force at the displaced corner B_2.

Because of the relative motion of B_2 with respect to B_1, a direct force is developed in stringer B and a shear force is developed in the sheet. The direct force in the stringer is, from Hooke's law,

$$F = \frac{EA_B}{a} u \tag{1}$$

where F is the force acting in stringer B in the positive x-direction and A_B and a are the total effective area and length, respectively, of stringer B.

The shear force in the sheet can be assumed equal to the product of the average shear stress and the sheet area and can be assumed equally divided between the points B_1 and B_2. From the static equilibrium of the sheet, an equal force must exist at the other stringer and can also be assumed equally divided between the points A_1 and A_2.

If for the unit considered the stress is assumed constant along a stringer, the average displacement of the points on stringer B is $u/2$ and the average shear strain is $u/2b$ where b is the width of the panel. If γ is the average shear strain in the sheet and G is the shear modulus of elasticity of the sheet, the average shear stress in the sheet is

$$\begin{aligned} \tau &= \gamma G \\ &= \frac{u}{2b} G \end{aligned} \tag{2}$$

Consequently, if the average sheet thickness is t, the total shear force U acting along stringer B is

$$U = \frac{u}{2b} Gta \tag{3}$$

The shear force assumed acting at each of the four corners is then

$$S = \pm \frac{Gta}{4b}u \tag{4}$$

The total forces X_{A_1}, X_{A_2}, X_{B_1}, and X_{B_2} are assumed to act on the four corners and are expressed as follows:

$$\left.\begin{aligned}
X_{A_1} &= X_{A_2} = S \\
&= \frac{Gta}{4b}u \\
X_{B_1} &= F - S \\
&= \left(\frac{A_B E}{a} - \frac{Gta}{4b}\right)u \\
X_{B_2} &= -(F + S) \\
&= -\left(\frac{A_B E}{a} + \frac{Gta}{4b}\right)u
\end{aligned}\right\} \tag{5}$$

Equations (5) constitute the solution of the unit problem.

Combinations of the unit problem.- The fundamental consideration in the unit problem and in the three combinations of it is the evaluation of the internal restoring forces, which are assumed to act at the corner points of the structural units, when one corner point is displaced a prescribed amount with all other corner points held fixed. In figure 2, a displacement u of any one of the corner points A_1, C_1, A_3, or C_3 with all other points held fixed results in elementary equations similar to those of equations (5). The three combinations of the unit problem are obtained as follows:

(1) If a point such as A_2 is moved while the other points are fixed, the direct forces induced in the two stringer segments A_1A_2 and A_2A_3 must be considered, as well as the shearing forces in the two fields adjacent to these segments

(2) When B_1 is displaced, a direct force occurs in one stringer segment B_1B_2 and shearing forces occur in the two fields adjacent to this segment

(3) The most general combination of the unit problem considered herein is that of the displacement of a point such as B_2 involving a direct force in the two stringer segments B_1B_2 and B_2B_3 and shearing forces in the four fields adjacent to these segments

Equations for the internal forces for the most general combination of the unit problem.- If B_2 is displaced a distance u_{B_2} in the positive x-direction, the following nine internal restoring forces X arise, which are assumed to act at the corners of the structural units indicated by the subscripts on X:

$$\left.\begin{aligned}
X_{A_1} &= \left(\frac{Gta_1}{4b_1}\right)u_{B_2} \\
X_{B_1} &= \left[\frac{EA_B}{a_1} - \left(\frac{Gta_1}{4b_1} + \frac{Gta_1}{4b_2}\right)\right]u_{B_2} \\
X_{C_1} &= \left(\frac{Gta_1}{4b_2}\right)u_{B_2} \\
X_{A_2} &= \left(\frac{Gta_1}{4b_1} + \frac{Gta_2}{4b_1}\right)u_{B_2} \\
X_{B_2} &= -\left(\frac{EA_B}{a_1} + \frac{EA_B}{a_2} + \frac{Gta_1}{4b_1} + \frac{Gta_1}{4b_2} + \frac{Gta_2}{4b_1} + \frac{Gta_2}{4b_2}\right)u_{B_2} \\
X_{C_2} &= \left(\frac{Gta_1}{4b_2} + \frac{Gta_2}{4b_2}\right)u_{B_2} \\
X_{A_3} &= \left(\frac{Gta_2}{4b_1}\right)u_{B_2} \\
X_{B_3} &= \left[\frac{EA_B}{a_2} - \left(\frac{Gta_2}{4b_1} + \frac{Gta_2}{4b_2}\right)\right]u_{B_2} \\
X_{C_3} &= \left(\frac{Gta_2}{4b_2}\right)u_{B_2}
\end{aligned}\right\} \quad (6)$$

Calculation of displacements, stresses, and reactions.- Consider the general sheet-stringer structure shown in figure 3. As each corner of the various structural units B_2, C_2..., B_3, C_3..., etc. is displaced a distance u_{B2}, u_{C2}...,u_{B3}, u_{C3}..., etc., respectively, in the positive x-direction with the remaining corners held fixed, the internal restoring forces that result are given by a set of equations similar to equations (6). The total internal restoring force caused by these displacements at any point such as B_2 is obtained by adding the values X_{B_2} given by the successive sets of equations. This force is therefore the sum of all forces at B_2 caused by the unknown displacement of B_2 and the points surrounding B_2 and can be conveniently obtained by use of Maxwell's reciprocal theorem. If the equations for the most general combination of the unit problem are written for any corner point and if the force and displacement subscripts are interchanged, all the internal restoring forces acting at the corner point considered are obtained; for example, the total internal restoring force at B_2 (fig. 3) can be obtained from equations (6) by interchanging subscripts on the X-force and u-displacement in each equation and adding the nine values of X_{B_2} that result. When this total internal restoring force is obtained for each corner point and equated to the load or force applied externally at that point in accordance with the principles of statics, a system of simultaneous equations is obtained that establishes the corner-point displacements. With the distorted shape of the structure known from the solution of the simultaneous equations for the displacements u, the stresses consistent with the distortion are readily obtained.

If u_{A_1} and u_{A_2} are the displacements obtained for adjacent points A_1 and A_2 on stringer A (see fig. 2), the stress in this stringer is

$$\sigma_{A_1A_2} = \left(u_{A_1} - u_{A_2}\right)\frac{E}{a_1} \tag{7}$$

The stress thus calculated is the average stress for the stringer segment A_1A_2. Also, if u_{A1} and u_{B1} are the displacements obtained for A_1 and B_1, which are adjacent points on the chordwise station 1, the shear stress τ in the panel at this station between stringers A and B is

$$\tau_{A_1B_1} = \left(u_{A_1} - u_{B_1}\right)\frac{G}{b_1} \qquad (8)$$

If the structure is fixed at one end, the reactions R at the ends of the stringers are obtained by finding the sum of the forces transmitted to the fixed points because of the displacements of the points surrounding the fixed points. If in figure 3 the station at 4 is fixed, then for point B_4,

$$R_{B_4} = \frac{Gta_3}{4b_1}u_{A_3} + \left(\frac{EA_B}{a_3} - \frac{Gta_3}{4b_1} - \frac{Gta_3}{4b_2}\right)u_{B_3} + \frac{Gta_3}{4b_2}u_{C_3} \qquad (9)$$

The stresses at the fixed ends are found by dividing each reaction by the stringer area at the reaction.

General remarks.- In order to apply the numerical procedure to a sheet-stringer panel, the structure is divided into a convenient number of units. The number of unknown displacements and equations is entirely dependent upon the number of units chosen. If many stringers are present, the combination of two or more into a substitute stringer will aid in the reduction of the unknowns. When the sheet thickness and/or stringer area varies, the elastic properties of the units EA/a and $Gta/4b$ are calculated with the average values for each unit. If the structure is divided into equal units and if the sheet or stringer dimensions do not vary, only one set of elastic constants need be calculated.

The displacement equations may be solved by two different methods: a relaxation procedure explained and utilized in reference 3 or a direct solution of simultaneous equations. A numerical example of the application of the procedure is given in appendix A and the displacement equations obtained are solved by a simple direct method in appendix B.

DESCRIPTION OF PANELS AND LOADINGS USED IN ANALYSIS

As a check of the applicability of the method of analysis to the more complex problems of stress distribution, four problems are solved by use of the numerical procedure.

Problems 1, 2, and 3.- The first three problems are concerned with the calculation of stresses in the panel with tapered stringers shown in figure 4(a). This panel was used in the analysis and experiments of reference 1. Because of the symmetry about the longitudinal axis, in all three cases only one-half of the structure was considered in the analysis. The distinguishing features of each problem are as follows:

Problem 1 - The end of the panel having the larger cross-sectional area was rigidly fixed, while at the other end two concentrated loads of 1.2 kips each acted on the two outer stringers.

Problem 2 - By the addition of shear webs and compression flanges, the panel was converted into the cover of a cantilever box beam, the cross section of which is shown in figure 4(b). The beam was loaded with four equally spaced loads of 0.225 kip each on each web as shown in figure 4(c). The end of the panel with the larger cross-sectional area was at the root of the beam.

Problem 3 - Two rectangular cut-outs were then made in the panel. These cut-outs were located symmetrically with respect to the longitudinal center line of the beam and extended from the flanges to the second stringer from the flanges. The ends of the cut-outs were 24 and 36 inches from the tip of the beam. A load of 0.6 kip was applied to the tip of each shear web.

Problem 4.- The fourth problem solved by the numerical procedure was the 16-stringer tension panel of reference 2. A transverse cross section of the panel is given in figure 4(d). The panel was 144 inches long and contained a rectangular cut-out at its center. The cut-out analyzed herein had a total length of 30 inches parallel to the longitudinal axis of the panel and cut four stringers on each side of this axis. The panel was axially loaded by a tensile force of 15 kips uniformly distributed to the ends of the stringers. Because of the double symmetry of the panel; only one-fourth of the structure was considered.

DETAILS OF ANALYSIS

In the four problems solved, the entire width of sheet was assumed effective in tension and therefore added to the stringer areas with the exception of a local region near the single concentrated load of problem 1.

Problem 1.- In order to apply the numerical procedure to the tapered-stringer tension panel the structure was assumed divided into six bays of equal length. For each of the twenty-four points resulting from the intersection of a station line and a stringer, a displacement equation was obtained. Because the stringers tapered, the area at the midpoint of each stringer segment was used to obtain the displacement coefficients. The half width of sheet adjacent to the loaded stringer in the structural unit nearest to the applied load was assumed ineffective in tension since this region was evidently too near to the concentrated load for a build-up of appreciable forces in it.

At each point of the structure considered, an equation was written relating the internal and external forces. The solution of this system of 24 simultaneous equations gave the displacement of each point relative to its original position. From these displacements, the stresses and reactions of the structure were obtained. The spanwise stringer stress distribution as well as the stresses computed by the substitute single-stringer method and the experimental data of reference 1 is given in figure 5. In appendix A a simplified analysis of this problem involving but six equations is presented in detail.

Problem 2.- Except for those equations containing coefficients dependent upon the flange area, the equations used for the solution of the previous problem were utilized for the stress analysis of the box beam with four concentrated loads. For each of the six bays, the effective area of the shear web was added to the area of the flange of the tension panel. This additional area was the sum of one-sixth the area of the shear web and the area of the flanged portion of the web sheet (fig. 4(b)). The running shear in the web was assumed to act as loads concentrated at the six stations along the flange. In figure 6 the stringer stress distribution obtained from the numerical procedure is compared with the experimentally obtained stresses and with the stresses obtained from the substitute single-stringer method of analysis.

Problem 3.- The box beam with two rectangular cut-outs in its cover was divided into five bays, three 12-inch bays toward the tip and two 6-inch bays near the root. As in the preceding beam problem, the running shear in the web was assumed to act as concentrated loads at the points of intersection of the flange and station lines. The computed and the experimental stresses are plotted in figure 7.

Problem 4.- In order to reduce the number of equations required for the stress analysis of the uniformly loaded 16-stringer tension panel with cut-out, the actual structure was simplified. Instead of the full half-length of the panel, only 40 inches of the panel on either side of the center line of the cut-out was used and the external loads were assumed to be introduced at the new end station. In addition, the area of the outer stringer and the adjacent stringer were combined and the resultant substitute stringer placed at their centroid. The three stringers nearest the longitudinal center line of the panel were also combined. The simplified structure consisted then of five stringers instead of the eight of the actual structure. A unit 17.5 inches long was chosen at the tip. The length of each of the remaining three units along the span was 7.5 inches. The stringer stress distribution is plotted in figure 8, along with the experimental stresses and the stresses obtained from the methods of analysis of reference 2. It should be noted that the forces on the substitute stringers were assumed uniformly divided among the actual stringers comprising the substitute stringers.

DISCUSSION

Examination of figures 5 through 8 reveals that the stresses calculated by the numerical procedure are in good agreement with the experimental and computed stresses of references 1 and 2 for all conditions and loadings of the tapered-stringer panel as well as for the 16-stringer tension panel.

In figure 6 the stresses computed by the numerical procedure for the central stringers C, D, and E of the approximately uniformly loaded box beam are in better agreement with the stresses obtained from the experimental data than are the stresses found by the substitute single-stringer method of reference 1, particularly in the region near the root. Similar results are observed for the center stringer D of the tip-loaded box beam (fig. 7). Although for these cases the stresses near the longitudinal center line of the beam covers are of minor importance, for box beams with cambered covers they may be significant.

For the 16-stringer tension panel the assumption that the loads were introduced at a station 40 inches from the transverse center line of the structure caused discrepancies at this station between the results of the procedure and those of experiment. (See fig. 8.) These stresses, however, are of minor importance compared with the high stresses that exist near the cut-out. For the cut stringer nearest to the longitudinal edge of the cut-out, considerably better agreement with the experimental stresses is given by the numerical procedure than by the simplified three-stringer method.

If a very detailed stress analysis is not required, the use of a small number of stations and stringers is helpful in considerably reducing the number of equations needed. This reduction is readily accomplished if substitute stringers and large bays are used for the portions of the structure that are some distance from isolated concentrated loads or discontinuities. In this manner the more important stresses may be obtained with but a relatively small amount of computation (see appendix A).

Although the structures considered in the present paper had no chordwise variations in stringer area, no difficulties are encountered when the numerical procedure is applied to problems having such variations. This fact is in contrast to the method of reference 1 which is based upon the assumption of a reasonably uniform chordwise distribution of the stringer area.

Because all formulas utilized in the numerical procedure are elementary, no difficulty should be encountered in solving successfully panel problems similar to those discussed herein. The solution of the equations, which constitutes by far the largest part of the computations, can be readily made by a computer using a slide rule, if slide-rule accuracy is sufficient, or a calculating machine, if greater accuracy is desired.

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APPENDIX A

NUMERICAL EXAMPLE, PROBLEM 1

A detailed application of the numerical procedure as applied to the complete solution of problem 1 is given in this appendix. As mentioned previously in the discussion of problem 1, only half of the structure is considered because of its symmetry.

In order to simplify the actual structure for analysis, a substitute stringer composed of half the center-line stringer and the adjacent stringer was assumed to act at the centroid of the combination. Instead of considering six bays as in the section "Details of Analysis," only two were chosen: a 16-inch bay at the tip and a 32-inch bay at the root. The resulting simplified structure is shown in figure 9.

In table 1 the average effective areas of the stringer segments and the resulting elastic constants are tabulated. The areas are those at the center of the stringer segments and include stringer area, effective sheet area, and, for the loaded stringer, the small area of sheet to the left of the center line of this stringer. The effective area of sheet for the substitute stringer was equal to that of the original structure.

TABLE 1

ELASTIC CONSTANTS FOR STRINGERS

Bay	A_A	A_B	A_C	a	$\frac{EA_A}{a}$	$\frac{EA_B}{a}$	$\frac{EA_C}{a}$
1-2	0.184	0.210	0.315	16	124	142	213
2-3	.251	.275	.413	32	84.8	92.9	139

The value of G (4,320 ksi) used for the calculation of the shear coefficients was obtained by using values of E = 10,800 ksi and $\frac{G}{E} = 0.4$, which were the values used in reference 1. Because the value of $\frac{G}{E} = 0.4$ is an approximate value, the resulting value of G should be considered as a fictitious one that does not correspond to actual material properties.

The shear coefficients are

$$\left.\begin{aligned}
\frac{Gta_1}{4b_1} &= \frac{4.32 \times 10^3 \times 0.015 \times 16}{4 \times 4} \\
&= 64.8 \\
\frac{Gta_1}{4b_2} &= \frac{4.32 \times 10^3 \times 0.015 \times 16}{4 \times 5.20} \\
&= 49.8 \\
\frac{Gta_2}{4b_1} &= \frac{4.32 \times 10^3 \times 0.015 \times 32}{4 \times 4} \\
&= 129.6 \\
\frac{Gta_2}{4b_2} &= \frac{4.32 \times 10^3 \times 0.015 \times 32}{4 \times 5.20} \\
&= 99.6
\end{aligned}\right\} \qquad \text{(A1)}$$

The displacement equations can now be obtained; for example, if the equilibrium of the forces at point A_1 is considered, the following equation results:

$$-\left(\frac{EA_A}{a_1} + \frac{Gta_1}{4b_1}\right)u_{A1} + \frac{Gta_1}{4b_1}u_{B1} + \left(\frac{EA_A}{a_1} - \frac{Gta_1}{4b_1}\right)u_{A2} + \frac{Gta_1}{4b_1} u_{B2} + P = 0$$

which, upon substitution of the proper values for the coefficients from table 1 and equations (A1), yields the equation

$$- 188.8u_{A1} + 64.8u_{B1} + 59.2u_{A2} + 64.8u_{B2} + 1.2 \times 10^5 = 0$$

in which the displacements are in hundred-thousandths of an inch. This equation states that the sum of the internal forces at point A_1 due to the unknown displacements of the points, the motions of which directly affect the equilibrium at point A_1, and the external load acting at this point is equal to zero. If the equilibrium of each point is considered, the six equations for the solution of this problem are obtained and are as given in table 2.

TABLE 2

SIMULTANEOUS EQUATIONS AND DISPLACEMENTS

Coefficients of displacements						
u_{A_1}	u_{B_1}	u_{C_1}	u_{A_2}	u_{B_2}	u_{C_2}	Constants
-188.8	64.8		59.2	64.8		120,000
64.8	-256.6	49.8	64.8	27.4	49.8	0
	49.8	-262.8		49.8	163.2	0
59.2	64.8		-403.2	194.4		0
64.8	27.4	49.8	194.4	-578.7	149.4	0
	49.8	163.2		149.4	-501.4	0
Displacements						
1120	575.0	376.8	457.9	417.1	304.1	

A discussion of the method of solution with its application to these equations is given in appendix B.

With the displacements from table 2, the elastic constants in table 1, and the shear coefficients calculated in equations (A1), the loads at the fixed end may be calculated from equations similar to equation (9). Thus,

$$R_{A_3} = (84.8 - 129.6)(457.9 \times 10^{-5}) + 129.6\,(417.1 \times 10^{-5})$$

$$= -0.205 + 0.540$$

$$= 0.335 \text{ kip}$$

$$R_{B_3} = 129.6\,(457.9 \times 10^{-5}) + (92.9 - 129.6 - 99.6)(417.1 \times 10^{-5})$$

$$+ 99.6\,(304.1 \times 10^{-5})$$

$$= 0.594 - 0.569 + 0.303$$

$$= 0.328 \text{ kip}$$

$$R_{C_3} = 99.6\,(417.1 \times 10^{-5}) + (139.0 - 99.6)(304.1 \times 10^{-5})$$

$$= 0.416 + 0.120$$

$$= 0.536 \text{ kip}$$

The corresponding stresses at the reactions are

$$\sigma_{A_3} = \frac{0.335}{0.295} = 1.136 \text{ ksi}$$

$$\sigma_{B_3} = \frac{0.328}{0.318} = 1.031 \text{ ksi}$$

$$\sigma_{C_3} = \frac{0.536}{0.477} = 1.124 \text{ ksi}$$

The stringer stresses at the midpoints of the stringer segments are obtained by the use of equations similar to equation (7) and are computed in table 3.

TABLE 3

CALCULATION OF STRINGER STRESSES

Station	Displacement u	Elongation of segment Δu	Average stress σ (ksi)
Stringer A			
1	1120		
		662.1	4.470
2	457.9		
		457.9	1.550
3	0		
Stringer B			
1	575.0		
		157.9	1.070
2	417.1		
		417.1	1.410
3	0		
Stringer C			
1	376.8		
		72.9	0.491
2	304.1		
		304.1	1.030
3	0		

The shear stresses are computed in table 4 with the aid of equations similar to equation (8).

TABLE 4

CALCULATION OF SHEAR STRESSES

Station	Relative displacement of stringers A and B $(u_A - u_B)$	Shear stress between stringers A and B τ (ksi)	Relative displacement of stringers B and C $(u_B - u_C)$	Shear stress between stringers B and C τ (ksi)
1	544.6	5.880	198.2	1.650
2	40.8	0.433	113.0	0.939
3	0	0	0	0

If the stringer stresses computed in table 3 are compared with the curves of figure 6, which were obtained from the solution of 24 equations, it is apparent that there is little difference between the simple 6-point solution and the more detailed 24-point solution. Evidently for the problem of the simple-tension panel only a small number of equations is required to compute the maximum stresses.

APPENDIX B

SOLUTION OF EQUATIONS

Of the several numerical methods available for the solution of simultaneous algebraic equations, the method which appears to be most satisfactorily applied to the equations arising from the numerical procedure is Doolittle's method as given in reference 7. This method for the solution of systems of normal linear equations (linear simultaneous equations symmetrical about the principal diagonal) is the Gaussian substitution method shortened by taking advantage of the symmetrical distribution of the coefficients in the equations.

The solution of the six simultaneous equations obtained in table 2 of appendix A is given in table 5.. Only those coefficients to the right of the principal diagonal are given in the equations in rows (1) to (6) of the table. The numbers in the column at the extreme right of each row are the algebraic sums of all the coefficients and the constant terms that appear in the actual equations contained in the rows and are used to provide a continuous arithmetic check. Because of the symmetrical form of the original equations, these summations (including the terms not written) can be obtained by adding the numbers from right to left in the rows as far as the main diagonal and then continuing the addition upward. In each row the same arithmetical operations are performed on the summation terms as are performed on the actual terms of the equations and the summation therefore provides a continual check on the arithmetical work.

The equations are solved systematically in the following manner, as indicated by the operations given at the right in table 5: The first equation is entered in row (7) and the coefficients of the displacements other than the first one are placed in brackets to facilitate reference. It should be noted that the summation term is also entered. In the next step (row (8)) the equation is divided through by the negative of the u_{A_1} coefficient, giving in effect a solution of u_{A_1} in terms of the remaining unknowns and the constant.

A double line is drawn to indicate that the equation is in its modified form. Evidently the summation term checks the arithmetic for it is equal to the sum of the quantities to its left. The second equation is written in row (9). In order to represent u_{B_1} in terms of the remaining variables, it is now necessary to eliminate the u_{A_1} term from this equation. Because of the form of the equations, this elimination is readily accomplished by multiplying the coefficients in row (8) by the coefficient of u_{B_1} in row (7) and adding the products to the equation represented by row (9) in order to obtain row (11). The heavy horizontal line indicates that row (11) is the result of adding rows (9) and (10). In row (12) the displacement u_{B_1} is given in terms of the remaining variables and a constant. A check on the preceding calculations is obtained by comparing the summation term with the sum of all the values to its left.

Each cycle generally consists of bringing down the next equation to be considered and eliminating from it the unknown displacements previously considered. The elimination is accomplished systematically by adding to this equation the products of the bracketed terms in the column above the first term that appears in the equation and the numbers in the row immediately below and to the right of each bracketed number. By dividing the sums by the negative coefficient of the first number in the row of sums, an equation is obtained that in effect expresses the dominant term of the equation considered in terms of the remaining variables and a constant. This process is continued until the last unknown u_{C_2} is determined in terms of a constant only. The remaining unknowns may be computed by substitution in the double-underlined equations. In rows (37) to (43) this substitution is done systematically. The terms from left to right in row (37) are the constants in the double-underlined equations obtained by starting at row (6) and going down to row (36). The quantities in row (38) are the products of u_{C_2} and the coefficients of u_{C_2} from the double-underlined equations and are entered in a manner similar to the constant terms. The value of u_{B_2} is obtained by adding the numbers in rows (37) and (38) in the u_{B2} column. With u_{B2} available the values in row (39) are calculated as were those in row (38). The process is continued until the last unknown is determined as in row (43).

The solution of the six equations indicates that the computations may be carried out readily on a slide rule if slide-rule accuracy is sufficient. In addition the practically mechanical procedure and the constant check ensure a rapid and accurate solution of simultaneous equations. If the system of simultaneous equations is to be solved by a computer using a calculating machine, more rapid solutions can be obtained by using the Crout method which is described in detail in reference 8.

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TABLE 5

SOLUTION OF EQUATIONS

Row	u_{A_1}	u_{B_1}	u_{C_1}	u_{A_2}	u_{B_2}	u_{C_2}	Constants	$\sum$	Operation
(1)	-188.8	64.8		59.2	64.8		120,000	120,000	
(2)		-256.6	49.8	64.8	27.4	49.8	0	0	
(3)			-262.8		49.8	163.2	0	0	
(4)				-403.2	194.4		0	-84.8	
(5)					-578.7	149.4	0	-92.9	
(6)						-501.4	0	-139.0	
(7)	-188.8	[64.8]		[59.2]	[64.8]		120,000	120,000	(1)
(8)	(-1)	0.3432		0.3136	0.3432		635.6	635.6	(7) ÷ -(-188.8)
(9)		-256.6	49.8	64.8	27.4	49.8	0	0	(2)
(10)		22.2		20.3	22.2		41,190	41,190	(8) × [64.8] from (7)
(11)		-234.4	[49.8]	[85.1]	[49.6]	[49.8]	41,190	41,190	(9) + (10)
(12)		(-1)	0.2125	0.3631	0.2116	0.2125	175.7	175.7	(11) ÷ -(-234.4)
(13)			-262.8		49.8	163.2	0	0	(3)
(14)			10.6	18.08	10.5	10.6	8,750	8,750	(12) × [49.8] from (11)
(15)			-252.2	[18.08]	[60.3]	[173.8]	8,750	8,750	(13) + (14)
(16)			(-1)	0.0717	0.2391	0.6891	34.69	34.69	(15) ÷ -(-252.2)
(17)				-403.2	194.4		0	-84.8	(4)
(18)				18.6	20.3		37,630	37,630	(8) × [59.2] from (7)
(19)				30.9	18.0	18.08	14,950	14,950	(12) × [85.1] from (11)
(20)				1.3	4.3	12.46	627	627	(16) × [18.08] from (15)
(21)				-352.4	[237.0]	[30.54]	53,210	53,120	(17) + (18) + (19) + (20)
(22)				(-1)	0.6725	0.0867	151.0	150.7	(21) ÷ -(-352.4)
(23)					-578.7	149.4	0	-92.9	(5)
(24)					22.2		41,190	41,190	(8) × [64.8] from (7)
(25)					10.5	10.5	8,715	8,715	(12) × [49.6] from (11)
(26)					14.4	41.6	2,092	2,092	(16) × [60.3] from (15)
(27)					159.4	20.5	35,780	35,720	(22) × [237.0] from (21)
(28)					-372.3	[222.0]	87,780	87,620	(23) + (24) + (25) + (26) + (27)
(29)					(-1)	0.5963	235.8	235.3	(28) ÷ -(-372.3)
(30)						-501.4	0	-139.0	(6)
(31)						10.6	8,750	8,750	(12) × [49.8] from (11)
(32)						119.8	6,029	6,029	(16) × [173.8] from (15)
(33)						2.7	4,611	4,602	(22) × [30.54] from (21)
(34)						132.4	52,350	52,240	(29) × [222.0] from (28)
(35)						-235.9	71,740	71,480	(30) + (31) + (32) + (33) + (34)
(36)						(-1)	304.1	303	(35) ÷ -(-235.9)
(37)	635.6	175.7	34.69	151.0	235.8	304.1			Constants from (8), (12), (16), (22), (29), (36)
(38)		64.6	209.6	26.4	181.3	304.1 = u_{C_2}			u_{C_2} × Coefficient of u_{C_2} from (12), (16), (22), (29)
(39)	143.1	88.3	99.7	280.5	417.1 = u_{B_2}				u_{B_2} × Coefficient of u_{B_2} from (8), (12), (16), (22)
(40)	143.6	166.3	32.8	457.9 = u_{A_2}					u_{A_2} × Coefficient of u_{A_2} from (8), (12), (16)
(41)		80.1	376.8 = u_{C_1}						u_{C_1} × Coefficient of u_{C_1} from (12)
(42)	19.73	575.0 = u_{B_1}							u_{B_1} × Coefficient of u_{B_1} from (8)
(43)	1,120 = u_{A_1}								

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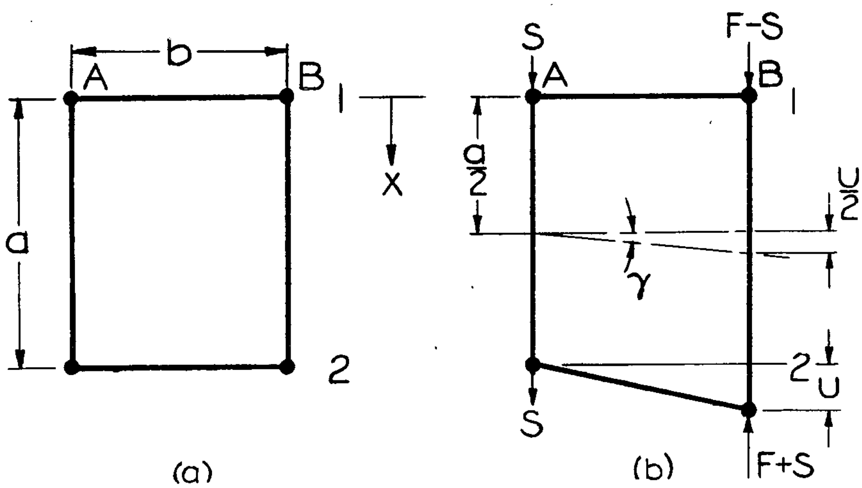

Figure 1.- Unit structure.

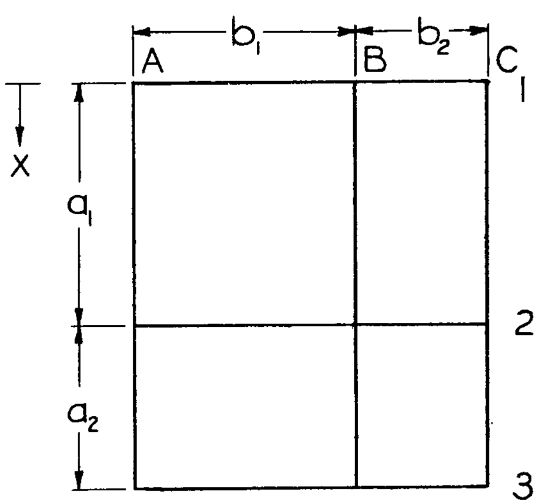

Figure 2.- General combination of unit structures.

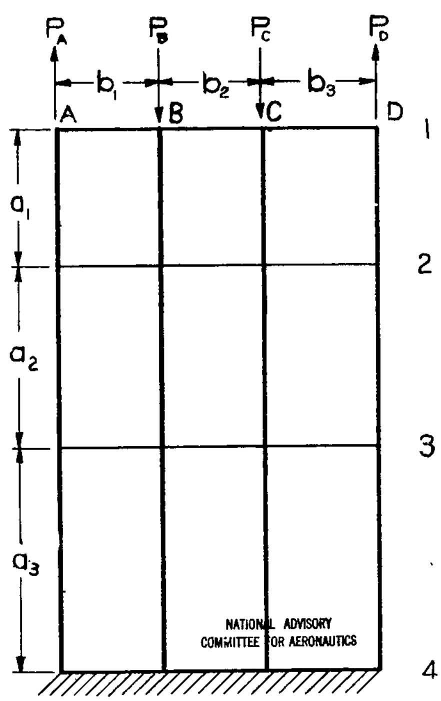

Figure 3.- Panel with one end fixed.

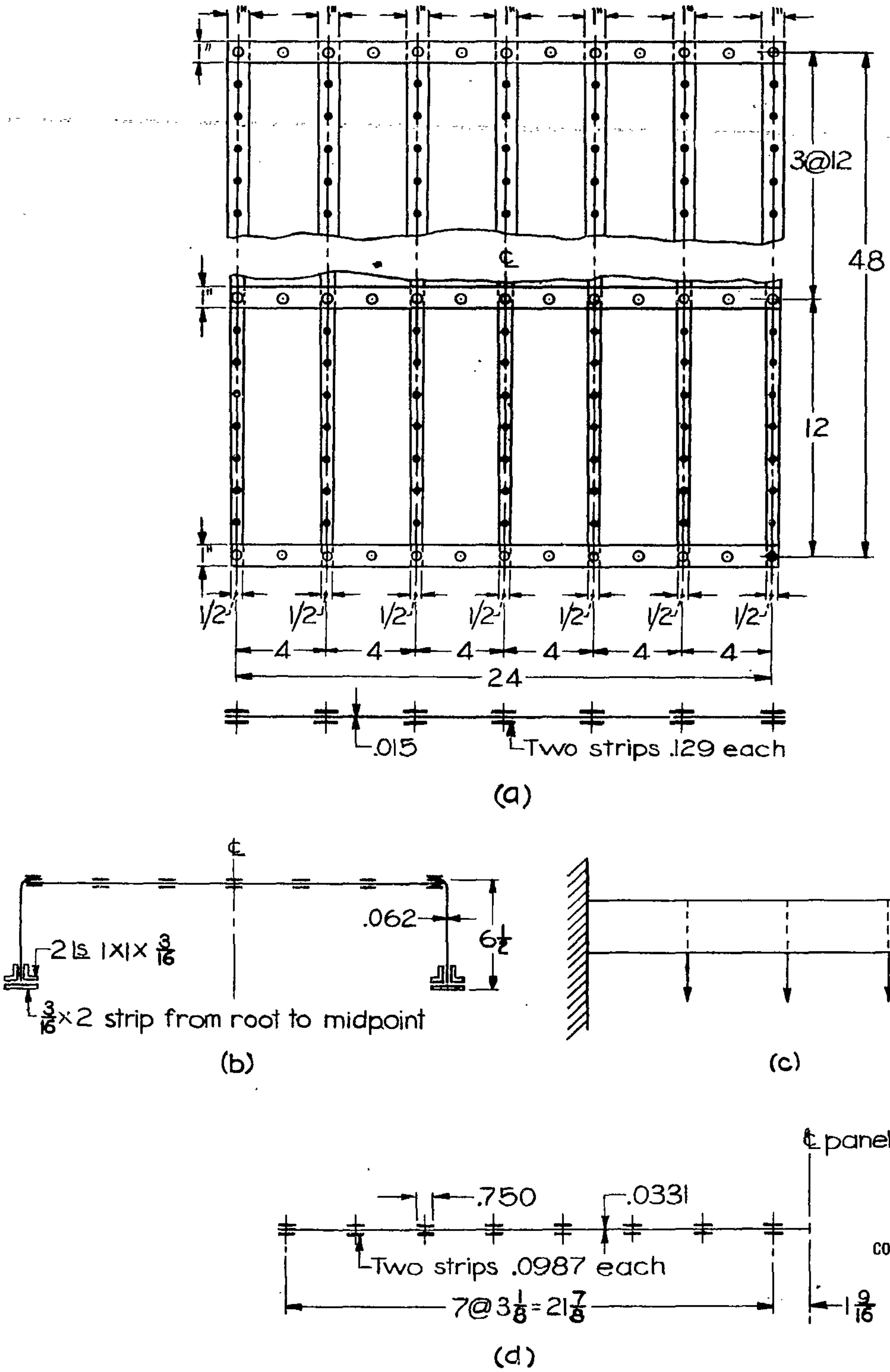

Figure 4.- Details of structures analyzed.

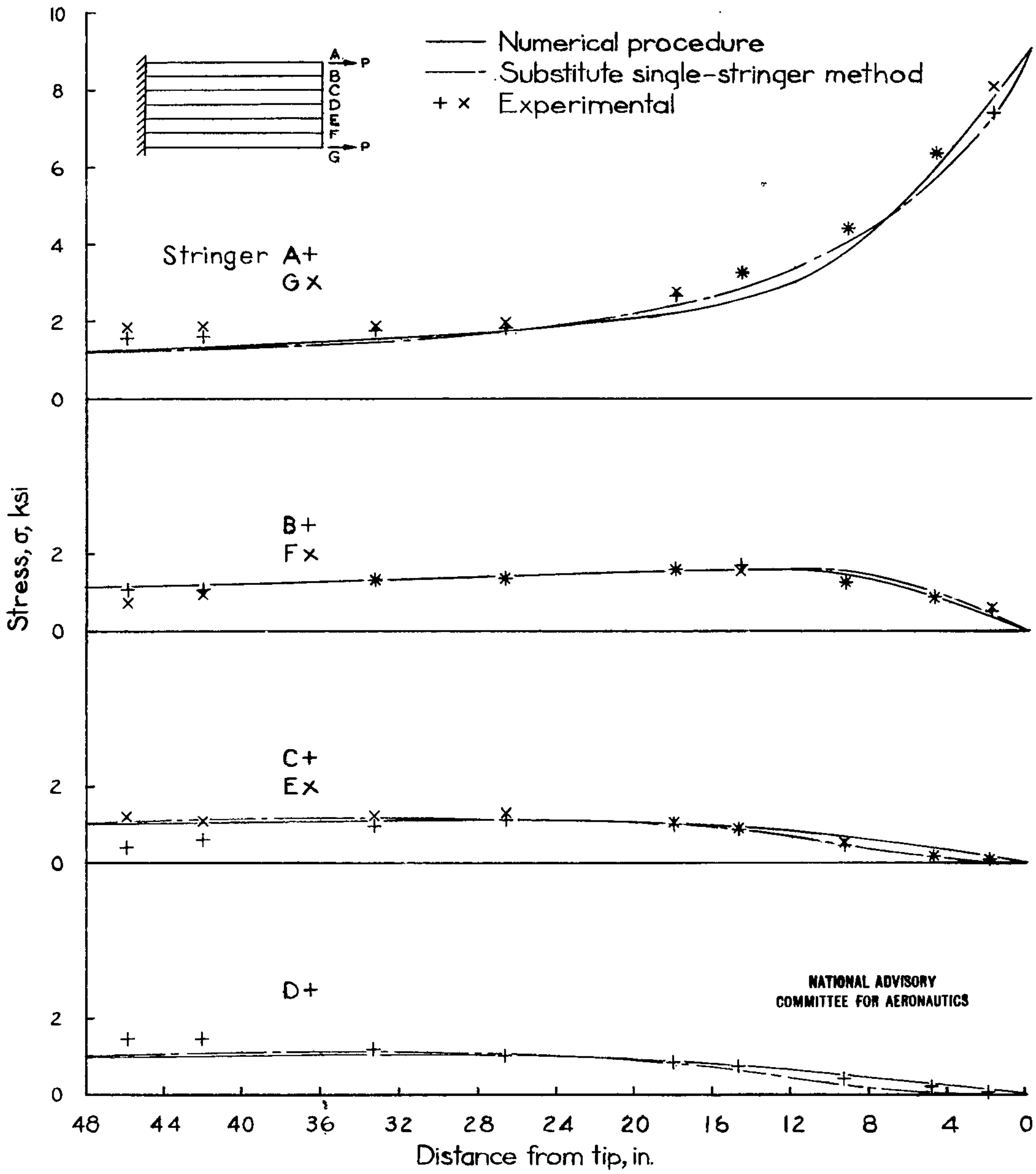

Figure 5.- Comparisons between calculated and experimental stresses for tension panel with concentrated loads; P=1.2 kips. (Test data and results of substitute single-stringer method from reference 1.)

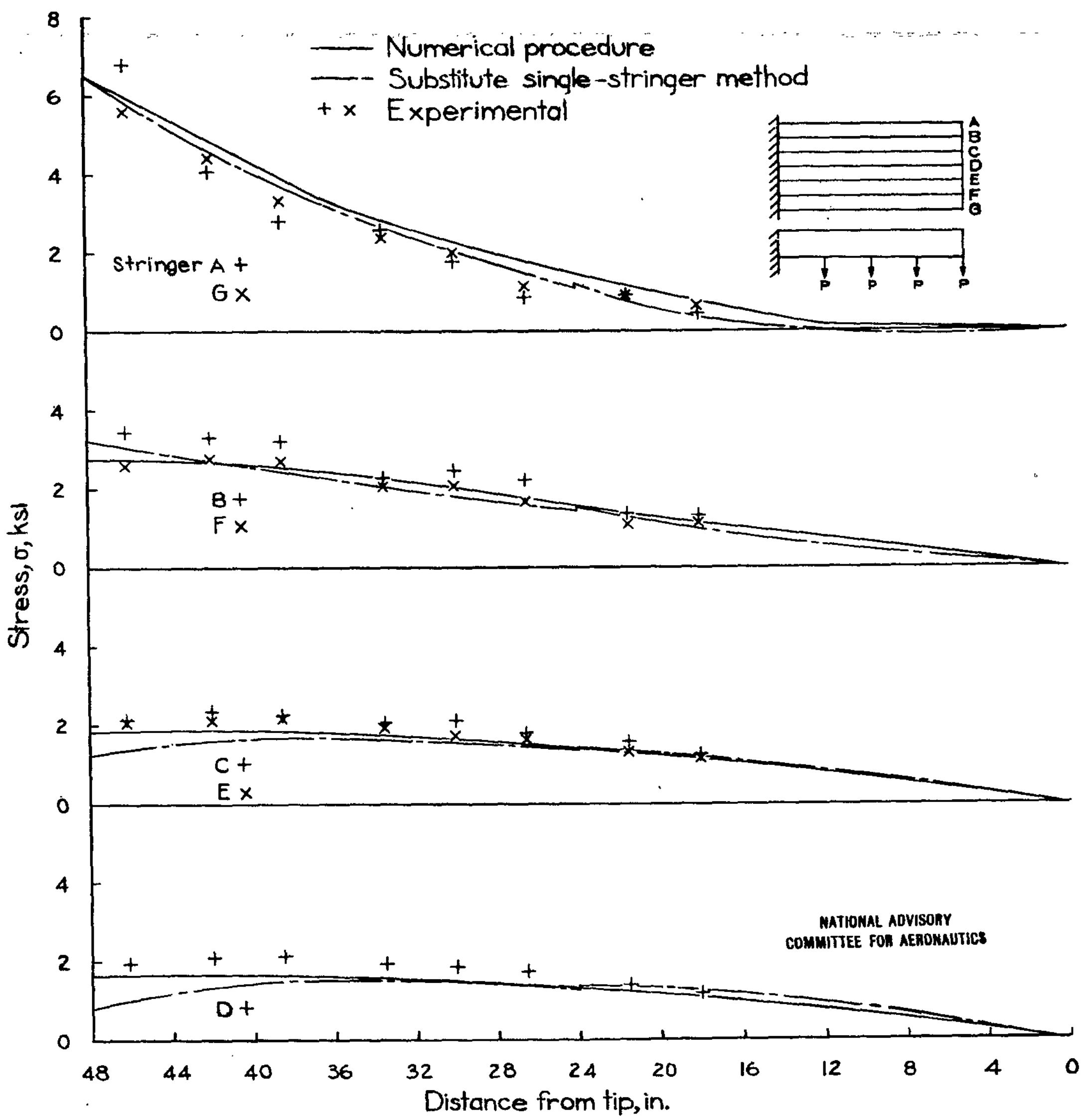

Figure 6.- Comparisons between calculated and experimental stresses for approximately uniformly loaded box beam; P=0.225 kips. (Test data and results of substitute single-stringer method from reference 1.)

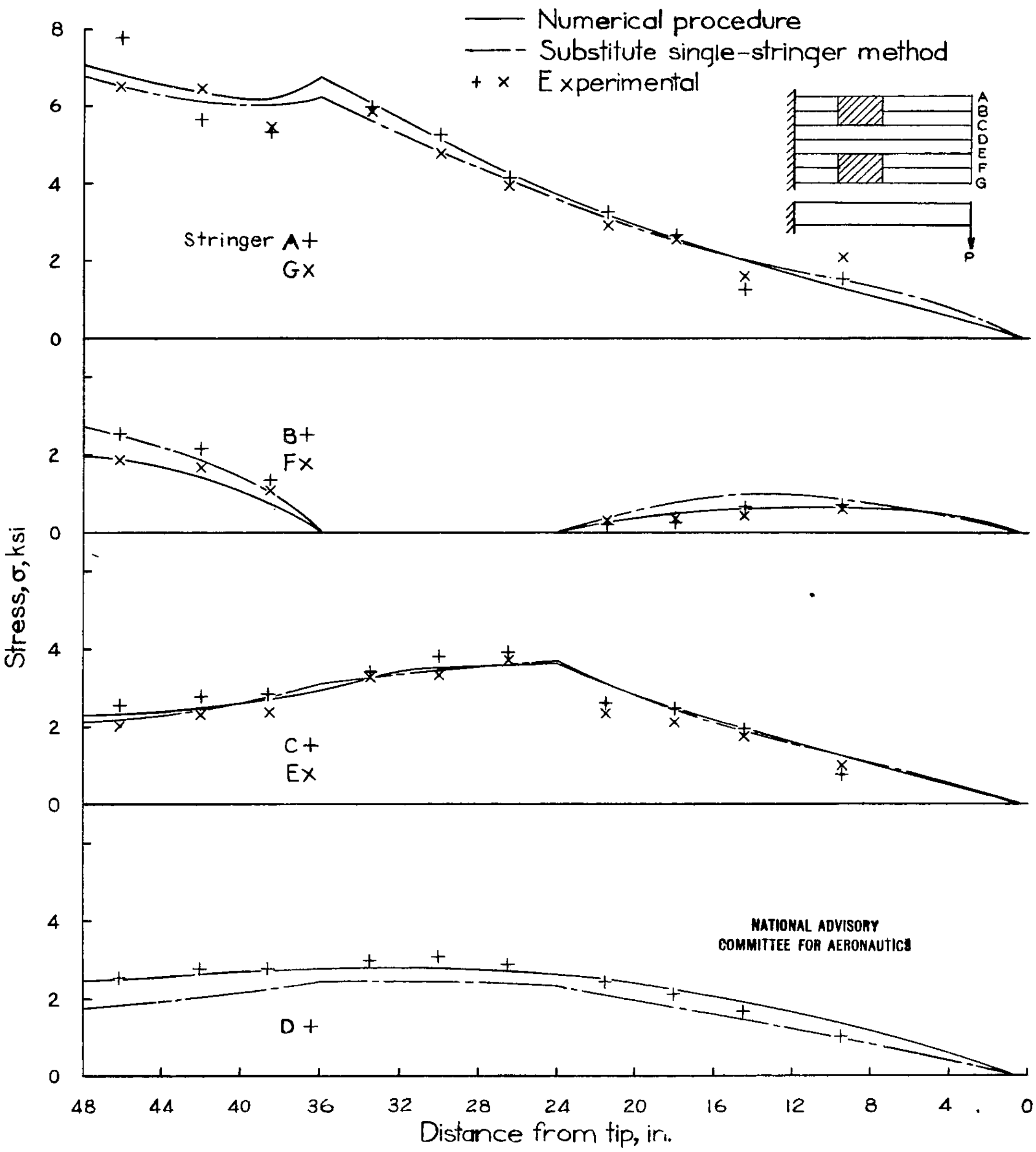

Figure 7 - Comparisons between calculated and experimental stresses for tip-loaded box beam with cut-outs; P = 0.6 kip. (Test data and results of substitute single-stringer method from reference 1)

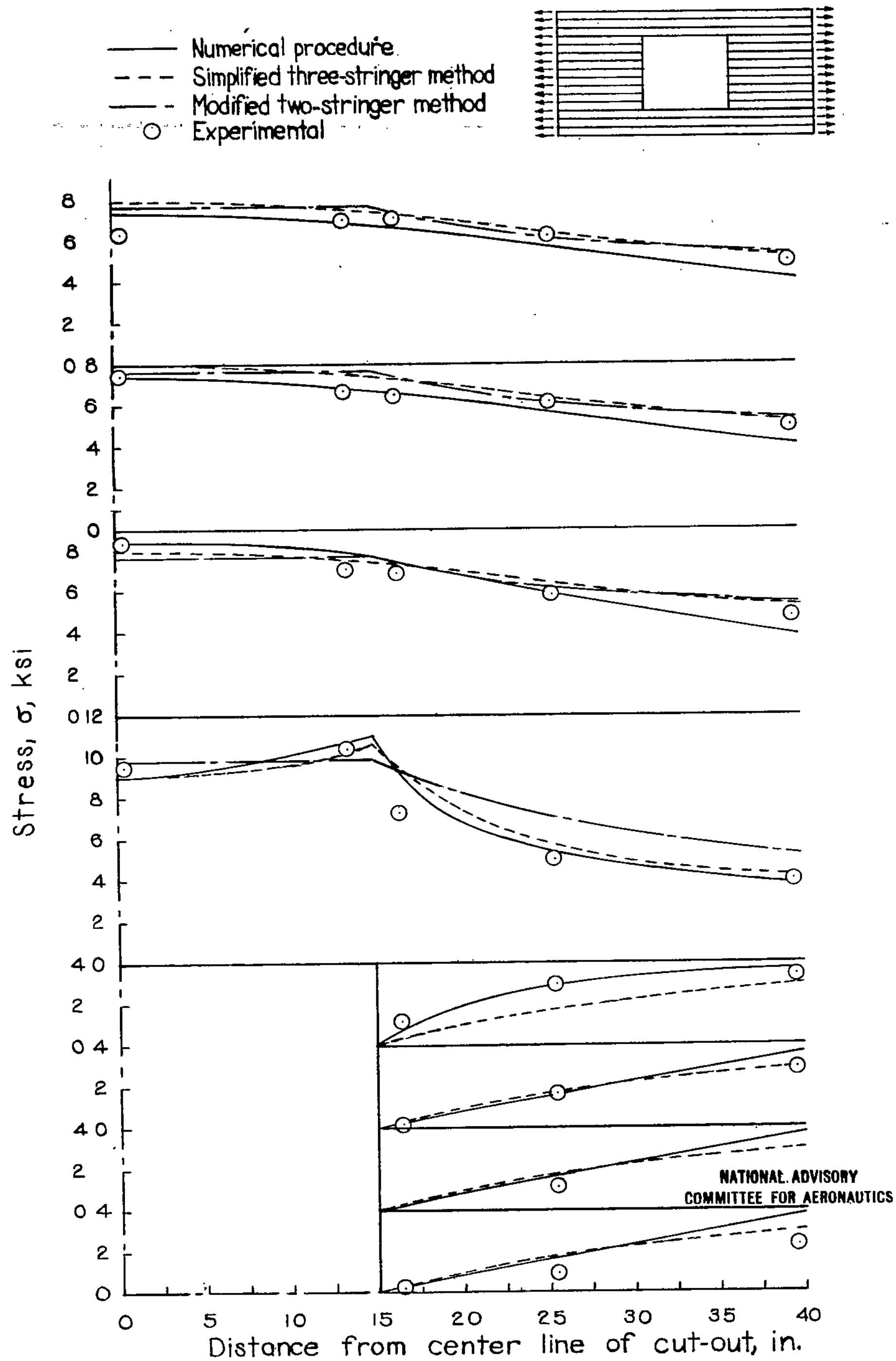

Figure 8.-Comparisons between calculated and experimental stresses for uniformly loaded tension panel with cut-out; total load = 15 kips. (Test data and results of simplified three-stringer method and modified two-stringer method from reference 2.)

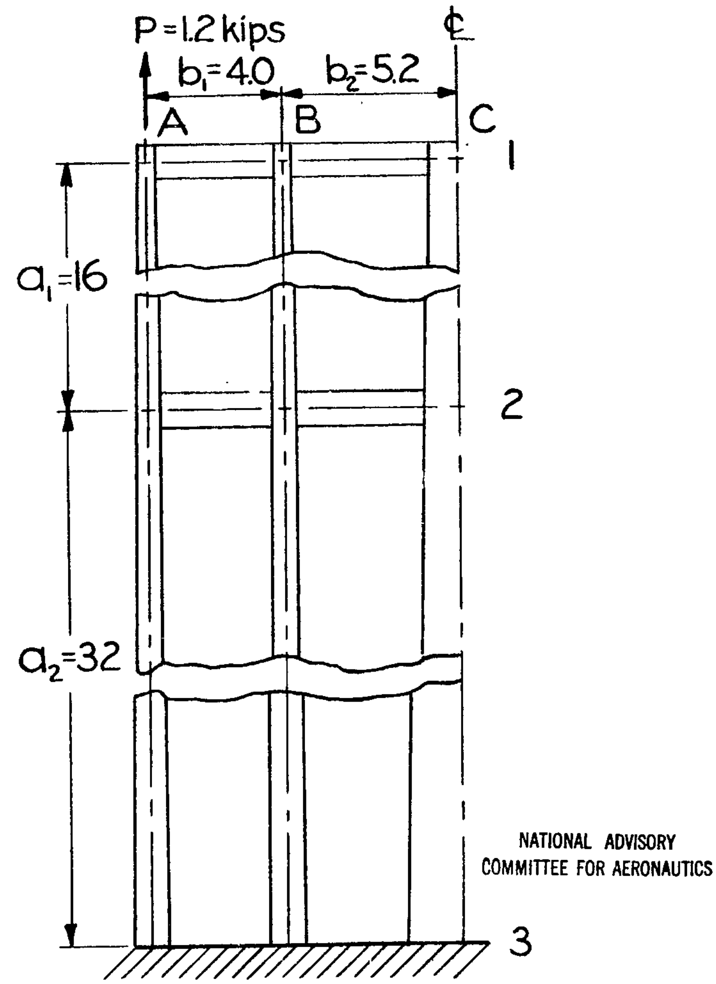

Figure 9.- Simplified tension panel used for analysis in appendix A.