JAN 30 1947

ACR Aug. 1941

copy 2

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED
August 1941 as
Advance Confidential Report

FORMULAS FOR USE IN BOUNDARY-LAYER CALCULATIONS ON LOW-DRAG WINGS

By E. N. Jacobs and A. E. von Doenhoff

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NACA

WASHINGTON

L - 319

L-319

FORMULAS FOR USE IN BOUNDARY-LAYER CALCULATIONS ON LOW-DRAG WINGS

By E. N. Jacobs and A. E. von Doenhoff

INTRODUCTION

This report has been prepared in response to frequent requests received from aircraft companies for information concerning methods of computing on low-drag wings the

(1) Transition point

(2) Velocity distribution in laminar and turbulent boundary layers

(3) Thickness of the boundary layer, both laminar and turbulent

Location of Transition Point

The best estimate of the transition point that we can make at the present time is based on the data obtained from tests of a glove on the B-18 airplane in flight. Under one set of conditions values of R_δ in the laminar boundary layer were observed between 8000 and 9500, where $R_\delta = V\delta/\nu$ and V is the velocity outside the boundary layer at the point in question, δ the distance normal to the surface from the surface to the point in the boundary layer where the velocity head is one-half its local value outside the boundary layer ($u = 0.707\ V$; see fig. 1), ν the kinematic viscosity. On the new low-drag sections the laminar separation point is so close behind the point of minimum pressure that transition is assumed to occur at the minimum pressure point if R_δ is less than the previously mentioned value at the minimum pressure point.

We have not made any transition measurements on smooth streamline bodies under completely turbulence-free conditions. It seems reasonable to suppose, however, that the condition for transition is not very different from that for two-dimensional bodies.

Velocity Distribution in Laminar and Turbulent Boundary Layers

Boundary-layer velocity distribution measurements have shown that in a favorable pressure gradient the shape of the laminar distribution is very closely approximated by the Blasius distribution for a flat plate. An approximate relation for this distribution, due to Pohlhausen, is given by the formula

$$\frac{u}{V} = 0.8144\left(\frac{y}{\delta}\right) - 0.1350\left(\frac{y}{\delta}\right)^3 + 0.0275\left(\frac{y}{\delta}\right)^4$$

The outer limit of the boundary layer corresponds to $\left(\frac{y}{\delta}\right) = 2.4558$. In most cases laminar separation occurs so soon in a region of adverse pressure gradient that this region is of little importance. The velocity distributions in this region can, however, be calculated by the methods described in reference 1, although this is usually a very lengthy process.

In a turbulent boundary layer, the 1/7-power law is a fair approximation to the shape of the velocity profile, that is, $\frac{u}{V} = \left(\frac{y}{11.33\ \delta}\right)^{1/7}$ when the pressure gradient is favorable. In an unfavorable gradient there is a gradual departure from this shape, the departure increasing with the distance behind the point of minimum pressure. The problem of finding the shape of the turbulent boundary-layer velocity distribution as a function of pressure gradient has not yet been solved.

Thickness of Laminar and Turbulent Boundary Layers

The thickness of the laminar layer in a region of favorable pressure gradients is computed by assuming that the shape of the velocity distribution is the same as the Blasius distribution and then integrating the Karman momentum relation. The result may be given in the following form for two-dimensional flow

$$\frac{R_\delta^{\,2}}{R_c} = (2.3)^2\left(\frac{V_0}{V_1}\right)^{7.17}\int_0^{s/c}\left(\frac{V}{V_0}\right)^{8.17} d\,\frac{s}{c}$$

L-319

or $$\delta^2 = 5.3\,\frac{c\nu}{V_1}\left(\frac{V_o}{V_1}\right)^{8.17}\int_0^{s/c}\left(\frac{V}{V_o}\right)^{8.17} d\,\frac{s}{c}$$

where $R_\delta = \dfrac{V_1\delta}{\nu}$

$R_c = \dfrac{V_o c}{\nu}$ wing Reynolds number

s distance along surface from leading edge

c chord

V_o reference velocity, usually taken as the free stream velocity

V velocity outside boundary layer at any value of s/c

V_1 velocity outside boundary layer at point for which boundary layer is being computed

The corresponding relation for bodies of revolution is

$$\frac{R_\delta^{\,2}}{R_L} = (2.3)^2\left(\frac{L}{r_1}\right)^2\left(\frac{V_o}{V_1}\right)^{7.17}\int_0^{s/L}\left(\frac{V}{V_o}\right)^{8.17}\left(\frac{r}{L}\right)^2 d\,\frac{s}{L}$$

or

$$\delta^2 = 5.3\,\frac{L\nu}{V_1}\left(\frac{L}{r_1}\right)^2\left(\frac{V_o}{V_1}\right)^{8.17}\int_0^{s/L}\left(\frac{V}{V_o}\right)^{8.17}\left(\frac{r}{L}\right)^2 d\,\frac{s}{L}$$

where $R_L = \dfrac{V_o L}{\nu}$

L length of body

r distance of surface from axis of revolution at any value of s/L

r_1 distance of surface from axis of revolution at point for which boundary layer is being computed

The most reliable estimates of the momentum thickness of the turbulent boundary layer in two-dimensional flow may be found from the relations given in reference 2. These relations, which are valid in those cases where there is no danger of turbulent separation, are as follows:

$$\frac{V\theta}{\nu} = 0.2454e^{0.3914\zeta}$$

or

$$\theta = \frac{\nu}{V} 0.2454e^{0.3914\zeta} \tag{1}$$

$$\frac{d\zeta}{d\left(\frac{s}{c}\right)} = R_c \frac{V}{V_o} \frac{10.411}{\zeta^2} e^{-0.3914\zeta} - 6.13 \frac{V_o}{V} \frac{d\left(\frac{V}{V_o}\right)}{d\left(\frac{s}{c}\right)} \tag{2}$$

The quantity θ is the momentum thickness of the boundary layer $\theta = \frac{1}{V^2}\int_0^\infty u(V-u)dy$. For the Blasius distribution $\theta = 0.289\ \delta$. The initial value of θ for the turbulent layer is taken the same as that for the laminar boundary layer at the transition point. The corresponding value of ζ for the turbulent boundary layer is then found from equation (1). The variation of ζ, and hence by equation (1) of θ, along the surface is then found from equation (2), which is solved by a step-by-step process. In more detail, if ζ_n is the known value of ζ at any point s/c along the surface,

$$\zeta_{n+1} = \zeta_n + \left[\frac{d\zeta}{d\left(\frac{s}{c}\right)}\right]_n \Delta\left(\frac{s}{c}\right)$$

where ζ_{n+1} is the value of ζ at $(s/c) + \Delta(s/c)$. The process is then repeated, using ζ_{n+1} as the new value of ζ_n. The value of $\Delta(s/c)$ is arbitrary within limits.

L-319

If $\frac{d\zeta}{d\left(\frac{s}{c}\right)}$ is not varying rapidly with s/c, $\Delta(s/c)$ may be taken larger than when rapid variations in $\frac{d\zeta}{d\left(\frac{s}{c}\right)}$ occur. In most computations the value of $\Delta(s/c)$ may be taken as 0.01.

The quantity ζ is related to a skin friction coefficient

$$\zeta^2 = \frac{\rho V^2}{\tau_o}$$

where τ_o shearing stress at surface

ρ density

The profile drag of the airfoil can be obtained from the values of θ at the trailing edge on the upper and lower surfaces from the following equation

$$c_d = \frac{2(\theta_U + \theta_L)}{c}\left(\frac{V_t}{V_o}\right)^{3.2}$$

where θ_U value of θ at the trailing edge, upper surface

θ_L value of θ at the trailing edge, lower surface

V_t velocity outside the boundary layer at the trailing edge

For bodies of revolution the relations given in reference 3 may be used. The 1/7-power law for the velocity distribution in the turbulent boundary layer seems to be as good as any simple assumption but, of course, cannot be relied upon in the presence of a strongly adverse pressure gradient or condition approaching those of turbulent separation. The equation is as follows:

$$\frac{R_\delta}{R_L^{\frac{4}{5}}} = 0.0327\left(\frac{V_o}{V}\right)^{\frac{16}{7}} \frac{L}{r_1}\left[\int_{\frac{s_c}{L}}^{\frac{s}{L}}\left(\frac{V}{V_o}\right)^{\frac{27}{7}}\left(\frac{r}{L}\right)^{\frac{5}{4}} d\left(\frac{s}{L}\right) + 71.8\ R_L^{\frac{1}{4}}\left(\frac{\delta_c}{L}\right)^{\frac{5}{4}}\left(\frac{V_c}{V_o}\right)^{\frac{115}{28}}\left(\frac{r_c}{L}\right)^{\frac{5}{4}}\right]^{\frac{4}{5}}$$

where the subscript c denotes the value of the quantity at the transition point.

CONCLUSION

The greatest uncertainty about boundary-layer calculations such as those outlined is usually associated with our lack of knowledge about where transition will occur. A criterion suggested, R_δ = 8000 to 9500 , may be considered only a very rough estimate under favorable conditions. It is possible that higher values may be reached in flight. Premature transition resulting in lower maximum values of R_δ may be brought about by air-stream turbulence in wind-tunnel work and by vibration and disturbances due to surface irregularities in flight. To avoid the roughness effects, it has been specified that no roughness detectable to the finger tips or no waviness that may be felt or that might noticeably affect the pressure distribution should be allowed, although recent tests have indicated that these requirements may be too severe in some cases.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.