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COMPARISON OF AN APPROXIMATE AND AN EXACT METHOD OF SHEAR-LAG ANALYSIS

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SUMMARY

Numerical comparisons were made between the approximate substitute single-stringer method of shear-lag analysis and the exact solutions, which indicate that for beams of practical proportions the approximate method yields a value for the maximum root stress that exceeds the value obtained by use of the exact solution by less than 10 percent. Comparisons of the approximate solution with experimental results indicate that the actual difference may be less than 10 percent.

Similar comparisons made for axially loaded panels show that adequate values of design stresses for stringers and skin may be obtained by the approximate analysis. In particular, at the application of a concentrated load in the corner flange, a finite value for the shear stress in the sheet is obtained by the approximate method instead of the infinite value given by the exact method. This finite value agrees well with values obtained experimentally.

INTRODUCTION

The analysis of skin-stringer combinations used as axially loaded panels or as covers of box beams presents a problem to the stress analyst when shear deformation of the skin alters the stress distribution predicted by simple engineering theories. Younger (reference 1) presented the first exact solution of this problem for the cantilever beam. The solution as presented was limited to the cantilever beam of constant rectangular section with a cosine curve of bending-moment distribution and neglected the effect of the transverse strains in the cover.

In reference 2, Kuhn extended Younger's solution, by means of trigonometric series, to the cantilever beam of constant section with arbitrary distribution of the load. He suggested that the solutions for beams with covers unreinforced by stringers could be extended to beams with covers reinforced by many stringers by multiplying the ratio of Young's modulus to shear modulus E/G by the factor t_s/t, where t_s is the thickness of a fictitious sheet obtained by spreading the stringer area and the effective skin area across the width of the cover and t is the actual thickness of the cover sheet.

These series solutions were not well suited to numerical computation because they converged very slowly at points of maximum stress. Hildebrand (reference 3) has solved the same problems and, by more refined analysis, has expressed the solutions in the form of more rapidly convergent series, which are better suited for obtaining numerical results. These forms of the exact solutions, or slight approximations of them due to the presence of stringers, are used herein. In reference 4 a procedure for the approximate solution of such problems was presented and, although very much simplified, the theory agreed well with the test data.

In the present paper, numerical comparisons are made between the approximate solutions of reference 4 and the exact solutions obtained by the methods of reference 3. In addition, three NACA tests of beams that had been analyzed previously by the approximate method are reanalyzed by the exact method and tests of a new tension panel are analyzed by both methods.

SYMBOLS

A_F area of corner flange of beam or panel, square inches

A_L area of stringers and effective sheet in half width of beam or panel, square inches

E Young's modulus, kips per square inch

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G shear modulus, kips per square inch

$K^2 = \frac{Gt}{Eb_s}\left(\frac{1}{A_F} + \frac{1}{A_L}\right)$

L length of beam, inches

b half width of beam or panel, inches

b_s substitute half width of beam or panel, inches

t thickness of sheet, inches

$t_s = \frac{A_L}{b}$

x spanwise distance measured from tip of beam or panel, inches

y chordwise distance measured from center line of beam or panel, inches

λ_n nth positive root of equation

$$\tan \lambda_n + \frac{A_F}{A_L} \lambda_n = 0$$

$\mu_n = \sqrt{\frac{Gt}{Et_s}} \frac{L}{b} \lambda_n$

σ stress in cover of beam, kips per square inch

σ_F stress in corner flange of beam, kips per square inch

σ_o average stress (Mc/I stress) at root of beam, kips per square inch

COMPARISON OF THEORETICAL SOLUTIONS

Cantilever Beams

Most of the numerical comparisons between the theoretical solutions for the stress distribution in cantilever beams were made for the uniformly loaded cantilever beam of constant rectangular cross section. The solution for the corner-flange stress for this beam and loading, derived on the basis of the substitute single-stringer theory, is

$$\sigma_F = \sigma_o\left[\frac{x^2}{L^2} + \frac{2A_L}{A_F}\left(\frac{\sinh Kx}{KL \cosh KL} + \frac{\cosh [K(L - x)]}{K^2L^2 \cosh KL} - \frac{1}{K^2L^2}\right)\right]$$

In accordance with the procedure of reference 4, the value of the shear-lag parameter K was determined with a substitute width of beam equal to the spanwise average distance from the corner flange to the centroid of the forces in the stringers. The average stress in the stringers was computed by statics and was distributed chordwise among the stringers with a hyperbolic cosine curve of distribution.

The exact solution for the stresses in the cover of the uniformly loaded cantilever box beam, derived by the procedure suggested in reference 3 and extended with some approximation to the cover reinforced by many stringers, is

$$\sigma = \sigma_o\left\{\frac{x^2}{L^2} - \frac{Et_sb^2}{GtL^2}\left(\frac{3y^2}{b^2} - \frac{3\frac{A_F}{A_L} + 1}{\frac{A_F}{A_L} + 1}\right)\right.$$

$$\left. + 4\left(1 + \frac{A_F}{A_L}\right)\frac{b}{L}\sqrt{\frac{Et_s}{Gt}}\sum_{n=1}^{\infty}\left[\frac{\cos \lambda_n \cos \lambda_n\frac{y}{b}}{\lambda_n\left(1 + \frac{A_F}{A_L}\cos^2\lambda_n\right)}\right]\left[\frac{\sinh \mu_n\frac{x}{L}}{\cosh \mu_n} + \frac{\cosh \mu_n\left(1 - \frac{x}{L}\right)}{\mu_n \cosh \mu_n}\right]\right\}$$

in which the stresses in the corner flange are obtained by setting y equal to b.

In both solutions the origin of coordinates is at the intersection of the center line and the tip. Curves of corner-flange stress and beam center-line stress are given in figures 1 to 3 for values of A_F/A_L of 0.2 and 1.0 and values of KL of 3, 6, and 12.

Similar numerical comparisons were made between the solutions by the substitute single-stringer method and the exact solutions derived in reference 3 for two other beams: the tip-loaded cantilever beam of constant section and the uniformly loaded cantilever beam of constant depth and width in which the areas A_F and A_L decrease linearly from a maximum at the root to zero at the tip. In these comparisons only beams with an average value of KL of 6 were considered. Curves of flange stress and center-line stress for these beams are given in figures 4 and 5.

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A consideration of the numerical comparisons for all the beams and loadings considered indicates that the substitute single-stringer method yields conservative values for the maximum stress in the root section at the corner flange and that these values become more conservative as the ratio A_F/A_L decreases. Because a too conservative value of corner-flange stress results in a decrease in the average stringer stress, the stringer stresses in the root section predicted by the substitute single-stringer solution are too low. The reason for this prediction of too great a shear-lag effect at the root is the conservative method chosen for the evaluation of the substitute width. In the root region the value of the substitute width is less than the average value used in the approximate solution, which means that the stringers in this region are more effective than the approximate method indicates.

As a further study of the effect of varying the ratios A_F/A_L, computations of the corner-flange stress at the root were made by the exact and the approximate methods for ratios of A_F/A_L of 0.2, 0.4, and 1.0 and values of KL from 3 to 20. The ratios of the approximate to the exact solutions for corner-flange stress at the root for both tip and uniform load are given in figure 6. It is apparent that the ratio A_F/A_L is more important than the factor KL in the tendency of the approximate solution to predict greater corner-flange stresses at the root than the exact solution. In the practical range, KL greater than 6.0 and A_F/A_L greater than 0.4, the error is less than 10 percent.

Axially Loaded Panels

A problem closely related to the stress distribution in the cover of box beams without camber is the stress distribution in axially loaded panels reinforced with stringers. One case is the panel loaded by concentrated loads at the corner-flange tips and long enough that the longitudinal stresses at the far end may be considered uniformly distributed chordwise. The solution for such a panel of constant cross section by the substitute single-stringer method is given in reference 4 and the exact solution is given in reference 3. In figure 7 curves of stringer stress at the corner flange and center line of the panel are given for ratios of A_F/A_L of 0.2 and 1.0 and in figure 8 curves of shear stress at the corner flange are given.

It is apparent that the decrease of the stringer stresses from the known values at the tip to the average value in the panel takes place in a distance roughly equal to the full width of the panel. Of more importance in design is the prediction of the maximum shear stress in the sheet near the corner flange at the tip of the panel. The exact solution predicts an infinite shear stress which yields no design solution to the problem, for it is known that the give of the rivets and the finite spacing and size of the rivets produce a finite value of shear stress instead of the infinite stress given by the exact solution. How closely the substitute single-stringer solution estimates this stress can be indicated only by experiment.

Although the series forms of the exact solutions that were used for comparison for both beams and panels converge more rapidly than the series solutions that have been previously available, the exact method still requires more time and effort to use than does the substitute single-stringer method. A direct measure of the rate of convergence of these series is the ratio A_F/A_L. Five or six terms are sufficient to determine the maximum stringer stress in a beam or panel when the ratio A_F/A_L is greater than 0.5. For ratios less than 0.5, the number of terms required for an adequate solution for the maximum stress increases rapidly with decreasing values of A_F/A_L.

COMPARISON OF THEORETICAL SOLUTIONS WITH EXPERIMENTAL RESULTS

Cantilever Beams

As a further comparison between the exact method and the substitute single-stringer method for the solution of shear-lag problems, previous tests of three beams were

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reanalyzed by the exact method. One of the beams was beam 3 of reference 4 tested without distinct corner flanges, and the second was the same beam tested with distinct corner flanges. The dimensions of these beams are given in figure 9. The third beam, previously reported as beam 1 in reference 5, is shown in figure 10.

In figure 11, the measured spanwise stresses at the corner flange and at the center line of beam 3 without flanges are compared with those computed by the exact and the substitute single-stringer methods. At the root, the corner-flange stress computed by the exact method is about 2 percent less than the measured stress and by the approximate method is about 10 percent greater. At the center line of the beam the measured stresses are consistently lower than the stress computed by either method except in the root section where the rapid increase in corner-flange stress computed by the approximate method results in a sharp decrease in the computed center-line stress.

As shown by comparing figures 11 and 12, when the ratio A_F/A_L for beam 3 is increased from 0.087 to 0.356 by the addition of corner-flange reinforcement, the tendency of the approximate method to predict a larger corner stress is reduced. The exact method predicted the measured corner-flange stress very closely while the substitute single-stringer method gave a value 6 percent higher. The measured center-line stresses are again less than those predicted by either method except at the root where the values predicted by the approximate method are below the measured values.

In figure 13 the chordwise distribution of measured stresses at the root of beam 1 from reference 5 is given as well as the stresses computed by both methods. There is some approximation of the exact method due to the presence of stringers but it should be small because of their large number. The results, in general, do not differ from those obtained for the other beams. At the root the stress obtained by the exact method coincides with the lower of the two measured corner-flange stresses while the stress given by the approximate method almost coincides with the higher measured stress. At the center line the substitute single-stringer stresses are again lower than the experimental stresses, but the experimental stresses lie halfway between those predicted by the two methods.

Except where there is no distinct corner flange, the general conclusion that can be drawn from these tests is that the scatter of the experimental results is about equal to the difference between the stresses obtained by the two methods and, as a result, the values of the maximum stresses in the corner flange at the root section are not too conservatively predicted by the substitute single-stringer method.

Axially Loaded Panels

The tension panel tested was the one that had been used in reference 6. The rectangular cut-out at the center was enlarged until only the corner flanges remained continuous. The structure then consisted of two tension panels with concentrated loads in the corner flanges and with uniform loads at the far ends. A cross section of the panel is shown in figure 14. The results of the test of the tension panel are given in figures 15 and 16. In figure 15 the stresses in the corner flange and the two adjacent stringers are compared with those computed by both exact and substitute single-stringer solutions. In figure 16 the measured shear stresses for the three skin panels nearest the corner flange are compared with the computed values.

The approximate method gives directly only a solution for the corner-flange stresses, the average stresses in the stringers, and the shear stresses at the flange, which are the important values of stresses for design. For comparison with the measured stresses, values of stringer stresses and shear stresses within the panel were obtained as follows: The stringer stresses were distributed chordwise according to the hyperbolic cosine curve discussed in reference 4. The shear stresses were distributed in accordance with the procedure developed in reference 6 for the distribution of shear stresses around rectangular cut-outs. At the rib that reinforced the tip of the panel, the shear stresses were distributed chordwise according to the ordinates of a cubic parabola. At a station for which the distance from the rib station was equal to, or greater than, one-quarter of the full width of the panel, the chordwise variation of shear stress was assumed to be linear, decreasing from a maximum at the flange to zero at the center line. At intermediate stations, the shear stresses were determined by assuming a linear variation spanwise.

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In general, close agreement exists between the stringer and skin stresses obtained by the exact method and the experimental results, but at the points important in design the stresses obtained by the approximate method are adequate. The approximate method predicts the maximum stresses occurring in those stringers adjacent to the flange and indicates that these stresses may rise well above the average panel stress before they decrease to the average stress. Close to the rib the stringer stresses rise more sharply than the substitute single-stringer method or the exact method indicates, but this rise is of little consequence because these stringers would have to be designed for the maximum stresses. The shear stresses that occur in the skin panel adjacent to the corner flange are within 8 percent of the shear stresses computed by the approximate method and show no tendency to rise to the high value predicted by the exact method for the rib station.

CONCLUDING REMARKS

In beams of practical proportions the approximate theory of the substitute single stringer will predict a conservative value for the maximum corner-flange stress which, on the basis of the exact solutions, will be in error by less than 10 percent. Comparisons with experimental results indicate that the difference between the substitute single-stringer solution for the maximum corner flange and observed values is, in general, less than the value of 10 percent indicated by the exact method.

Adequate values of design stress for stringers and sheets of axially loaded panels can be obtained by the substitute single-stringer method, while the exact method greatly exaggerates the maximum shear stress.

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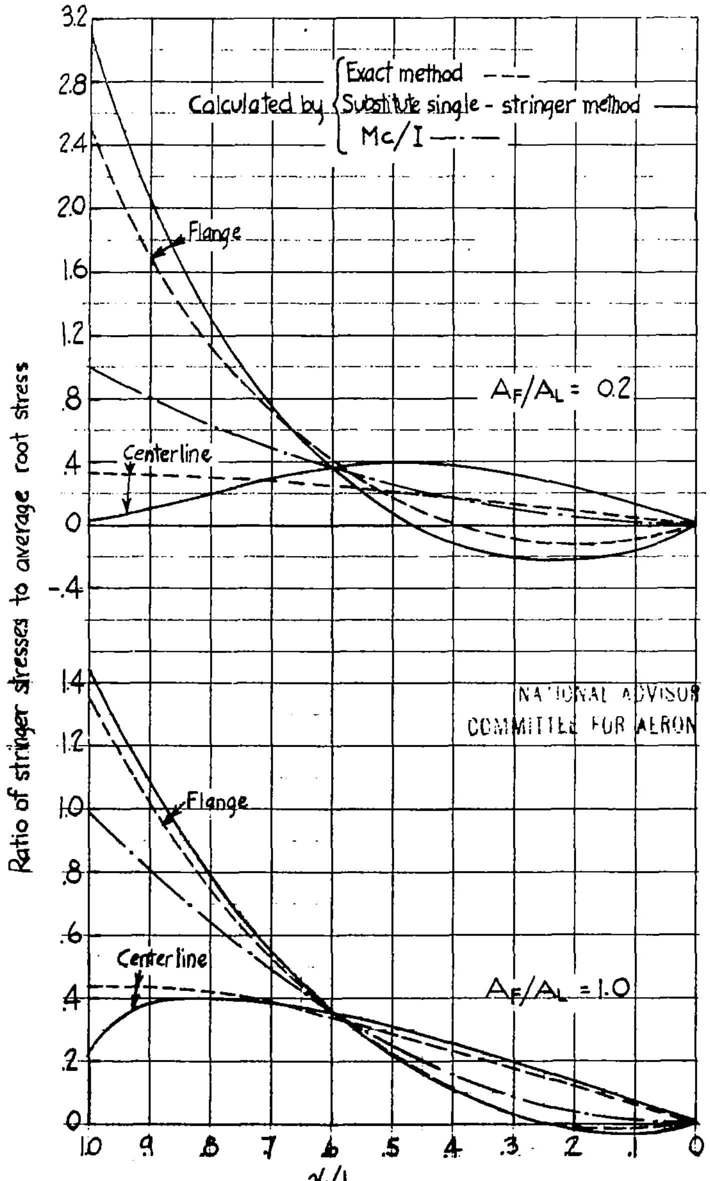

Figure 1.–
Stringer stresses for cantilever beam of constant section, uniformly loaded. KL = 3.

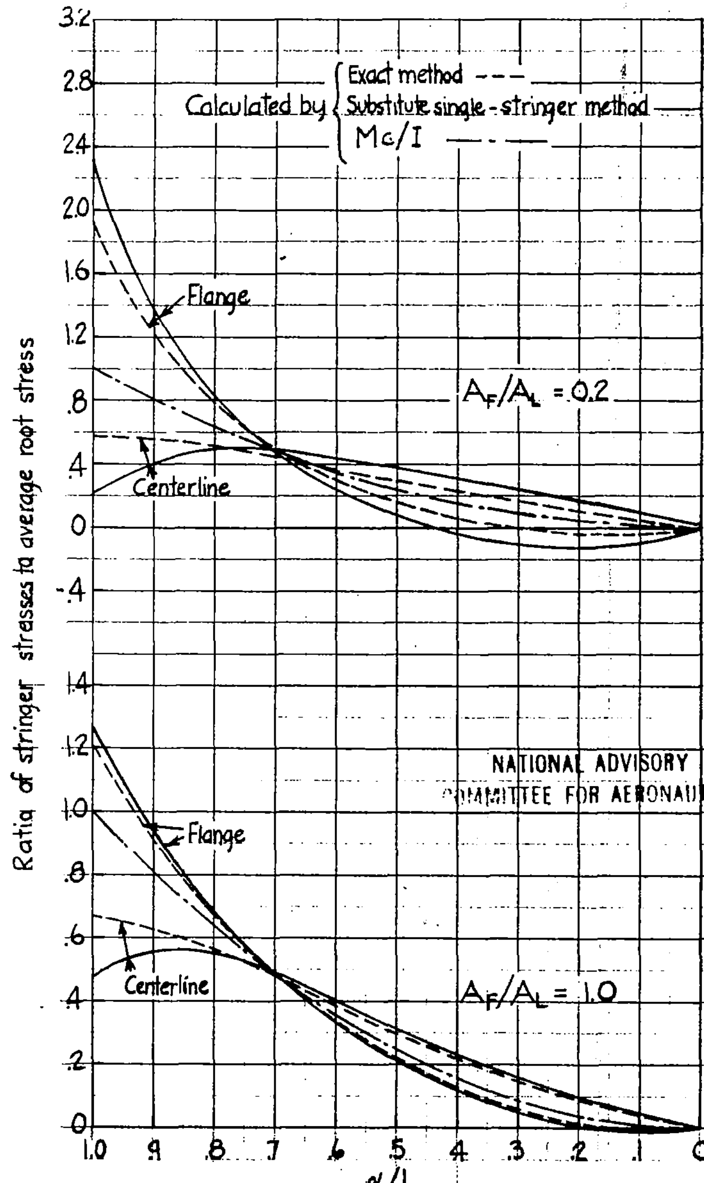

Figure 2.–
Stringer stresses for cantilever beam of constant section, uniformly loaded. KL = 6.

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(1 block = 10 divisions on 1/30 Engr. scale)

Figs. 1,2

(1 block = 10/30")

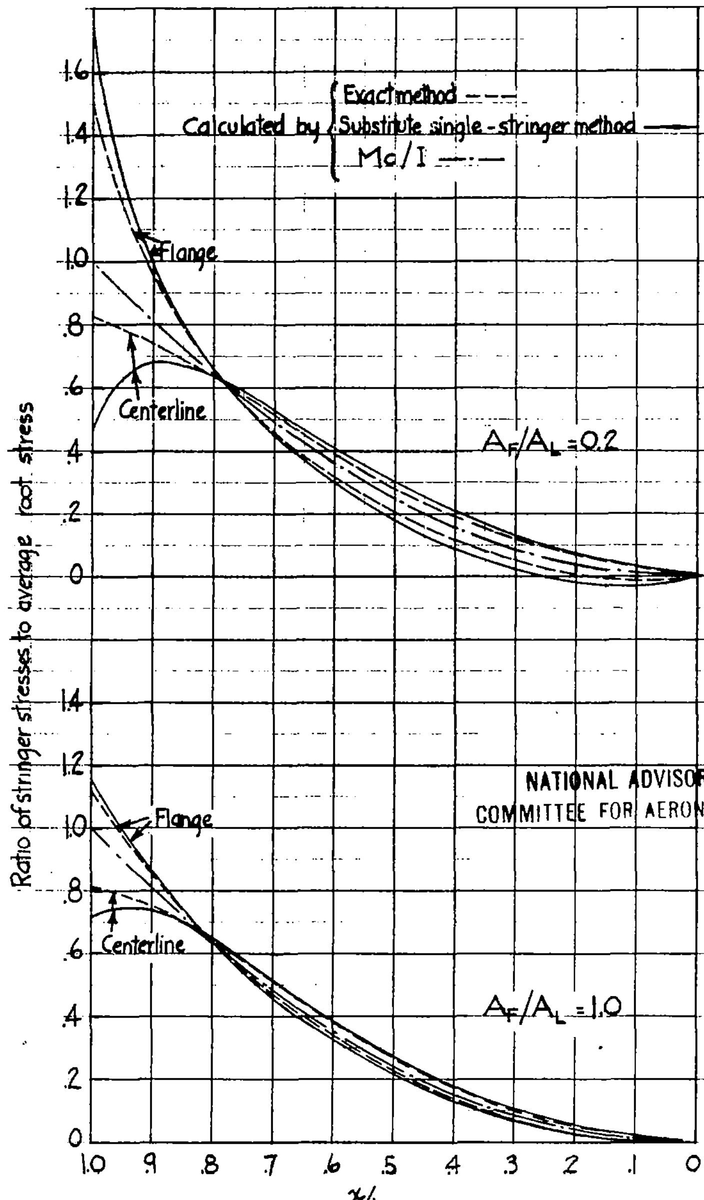


Figure 3.-

Stringer stresses for cantilever beam of constant section, uniformly loaded. KL=12.

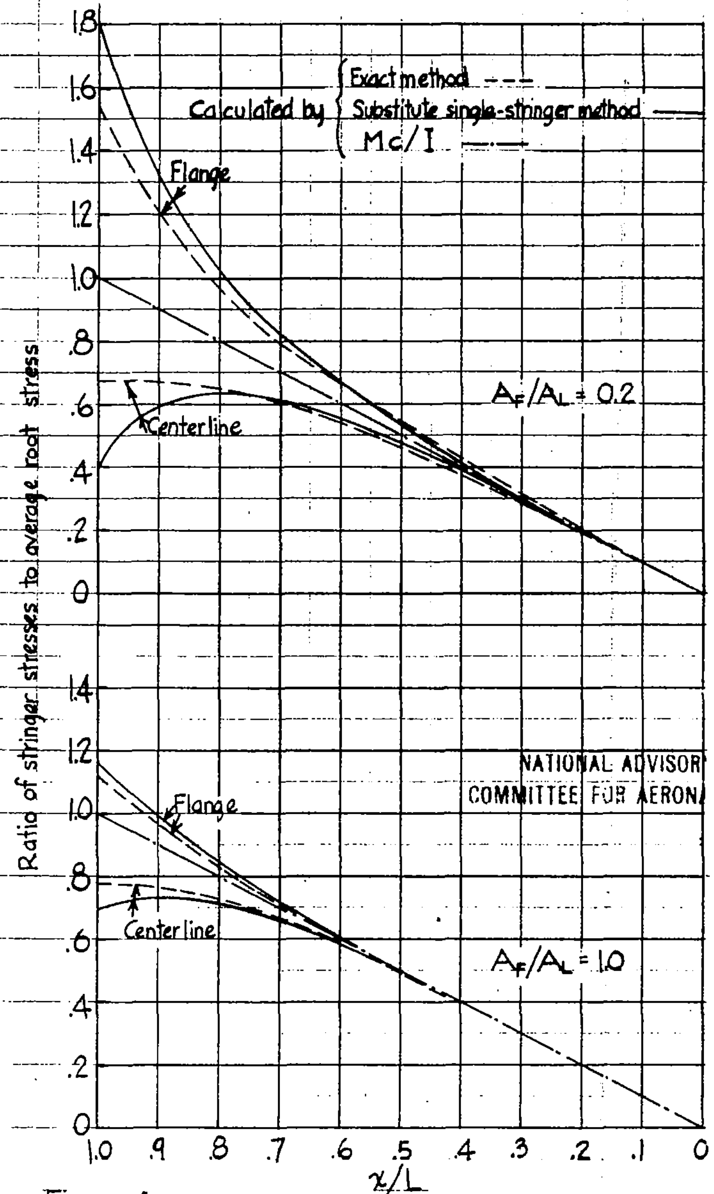


Figure 4.-

Stringer stresses for cantilever beam of constant section, tip loaded. KL=6.

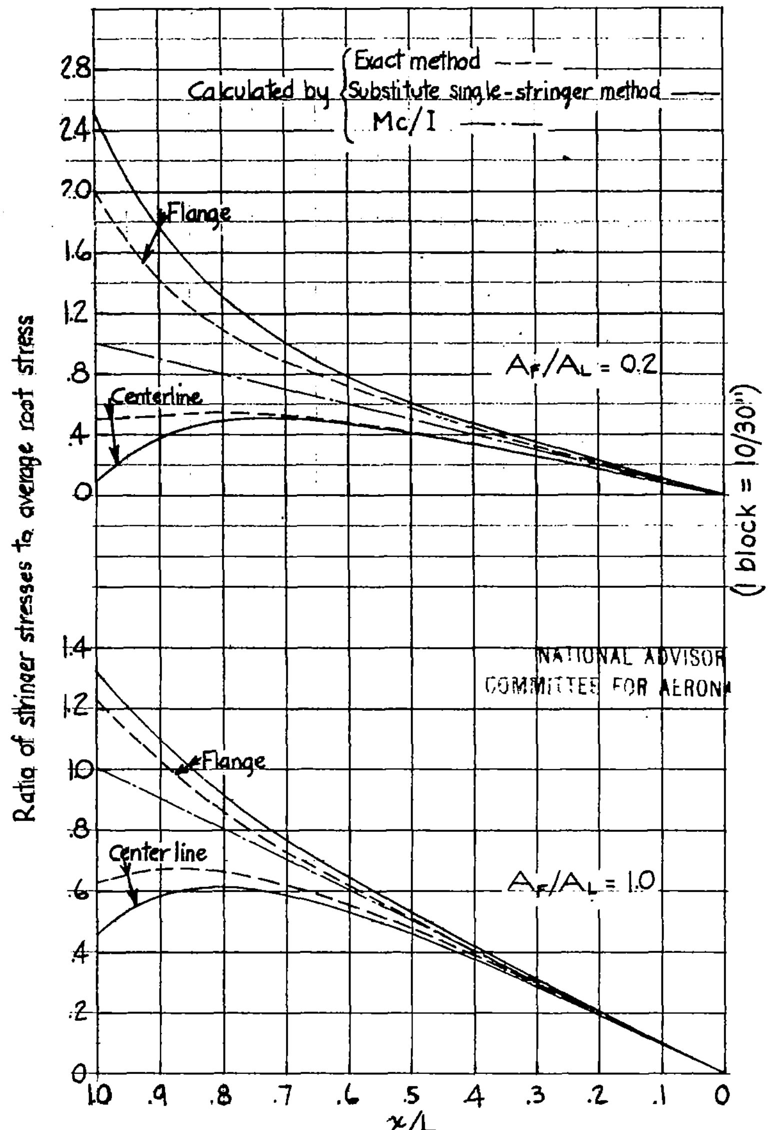


Figure 5.-
Stringer stresses for cantilever beam with tapered cover, uniformly loaded. KL=6.

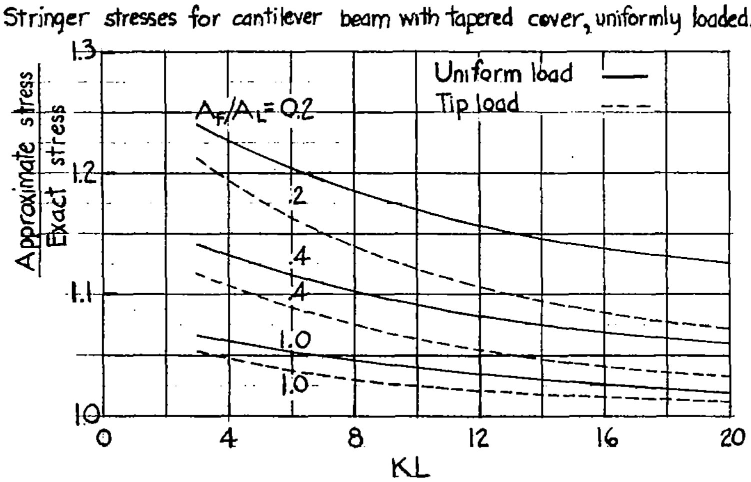


Figure 6.- Effect of the variation of beam dimensions on the ratio of the maximum flange stress at the root computed by the approximate method to that computed by the exact method.

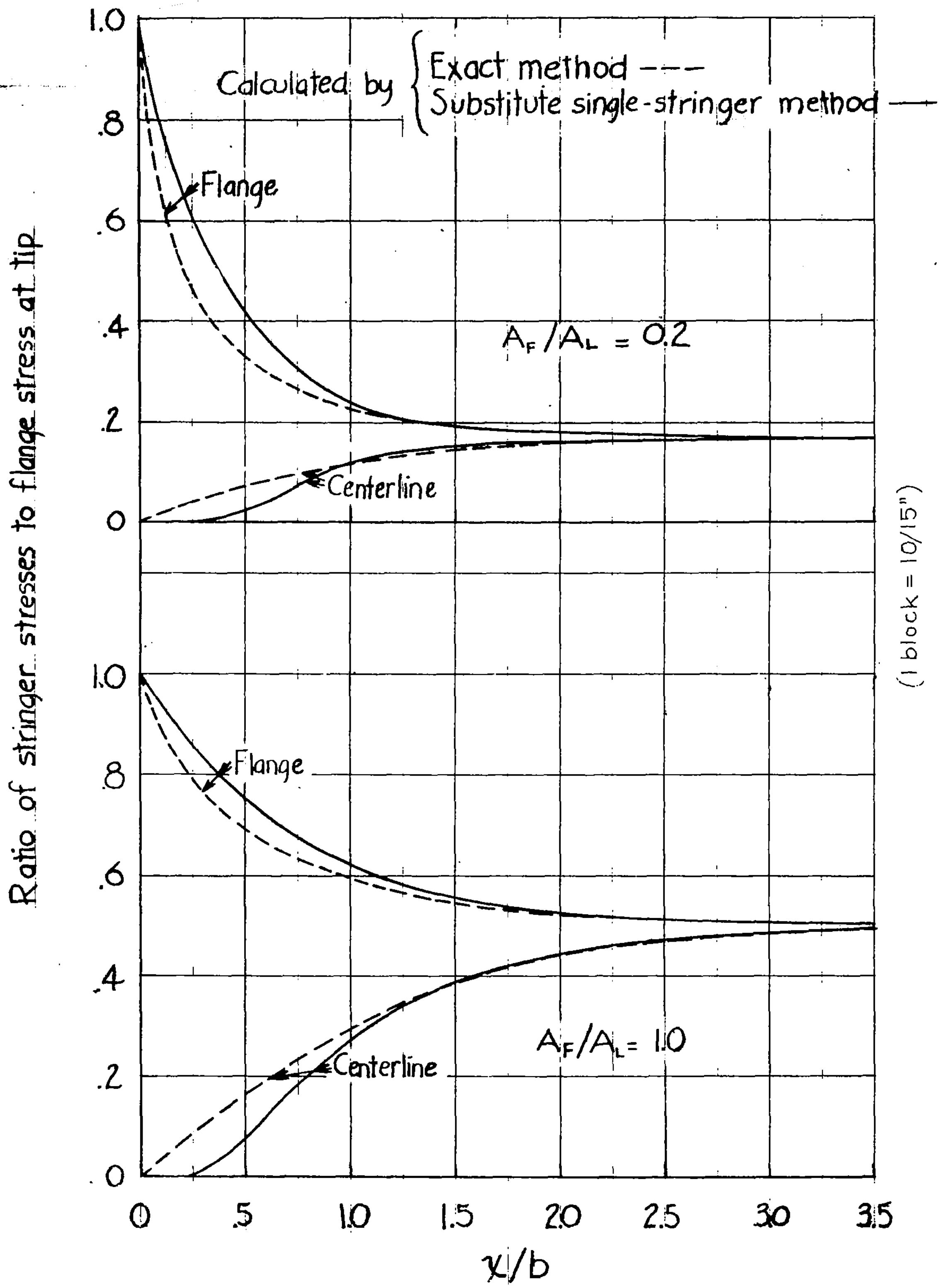


Figure 7.- Stringer stresses in a tension panel loaded in flange at tip.

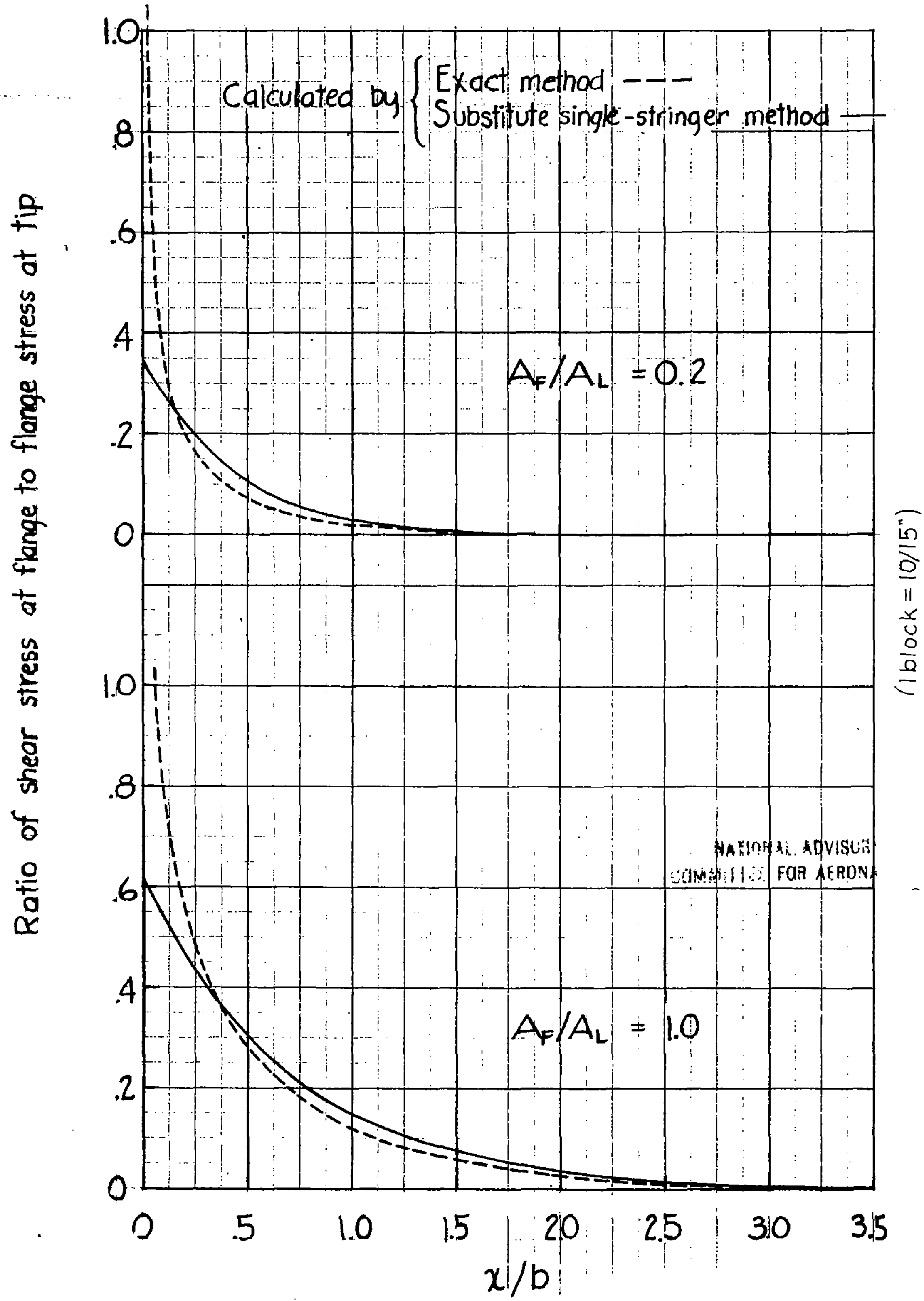
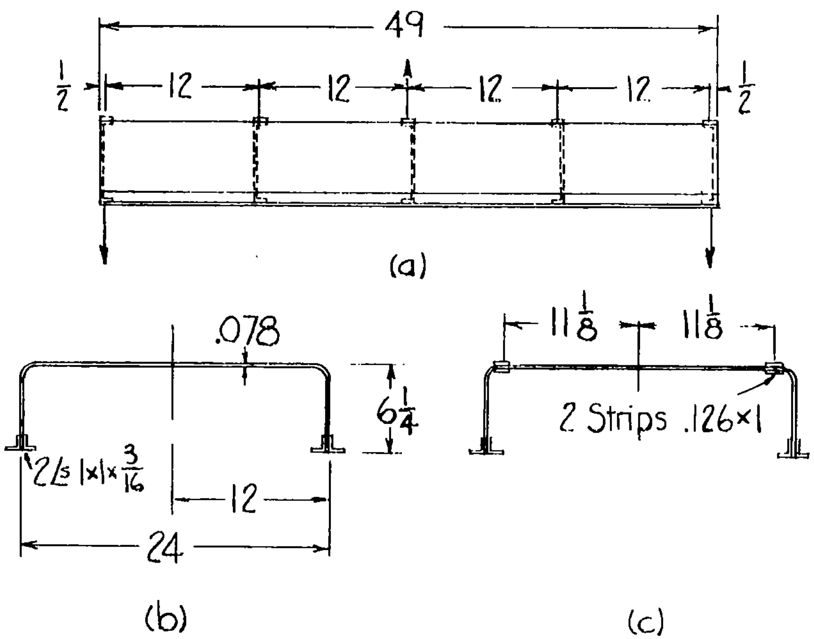
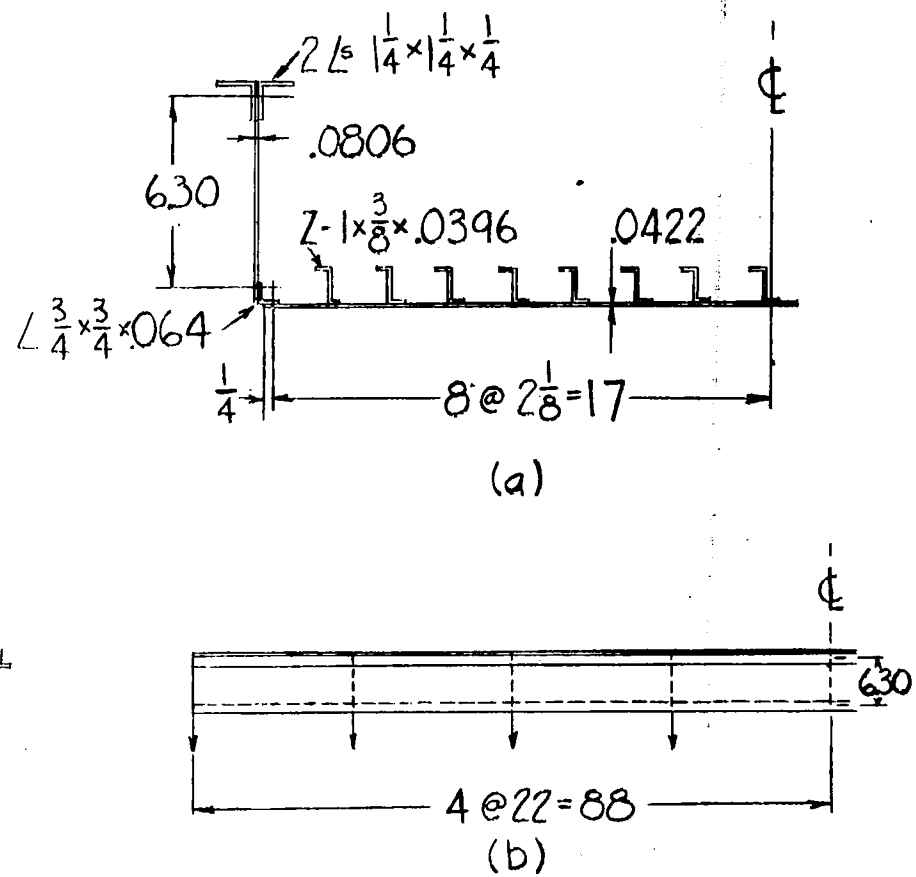


Figure 8.- Shear stresses at flange in a tension panel loaded in flange at tip.



(a) Side view.
(b) Original cross section.
(c) Modified cross section.

Figure 9.- Beam 3 of reference 4.



(a) Cross section.
(b) Half span.

Figure 10.- Beam 1 of reference 5.

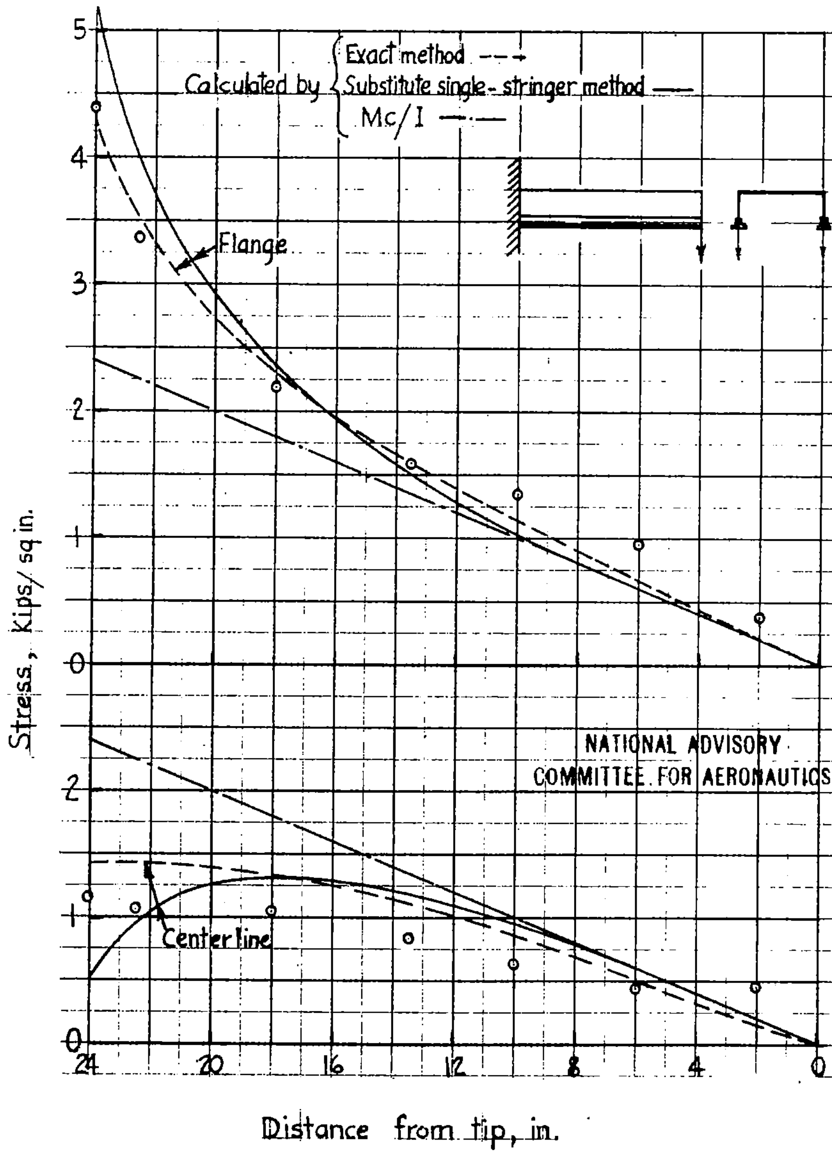


Figure 11.- Comparison between calculated and observed stresses in beam 3 of reference 4, no distinct flange.

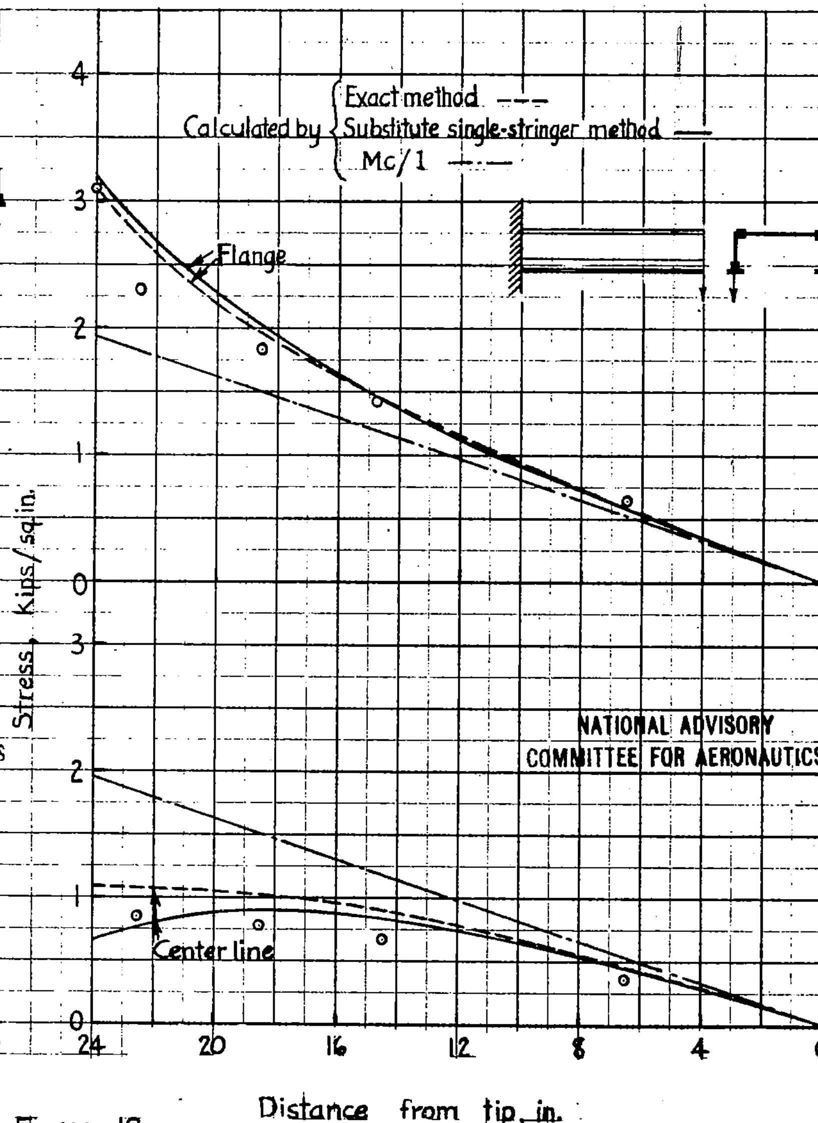
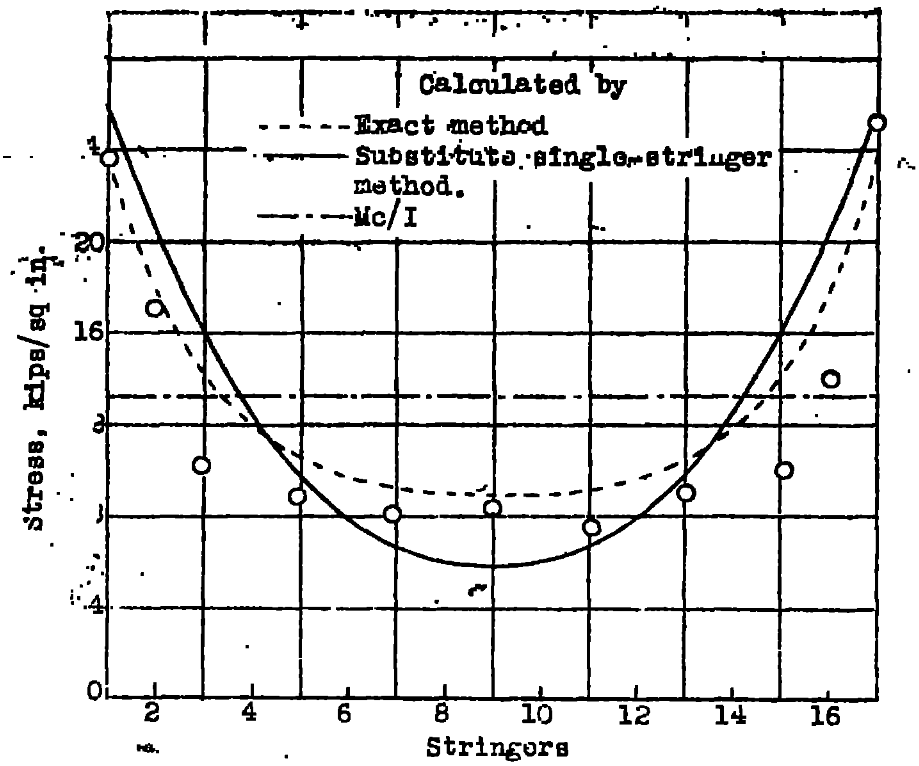


Figure 12.- Comparison between calculated and observed stresses in beam 3 of reference 4; flange added.

(1 block = 10/30")



13.- Comparison between observed and calculated chordwise distribution of root sses in beam 1 of reference 5.

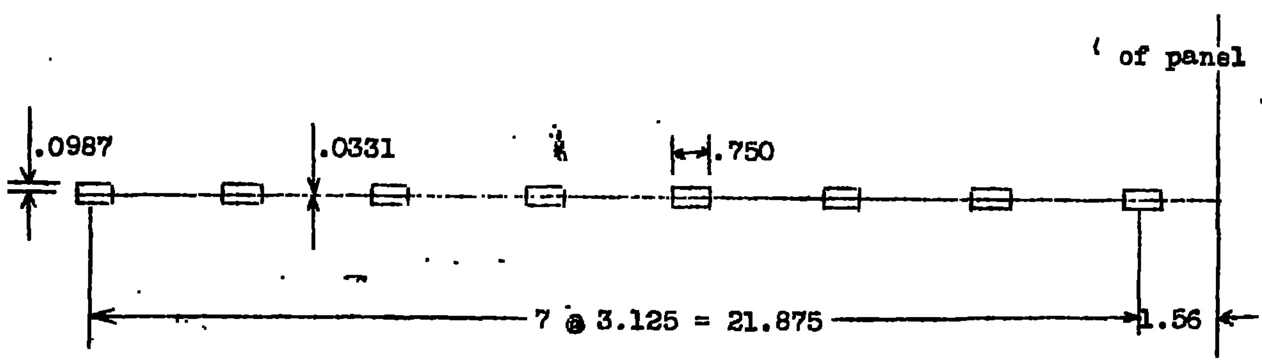


Figure 14.- Cross section of panel of reference 6.

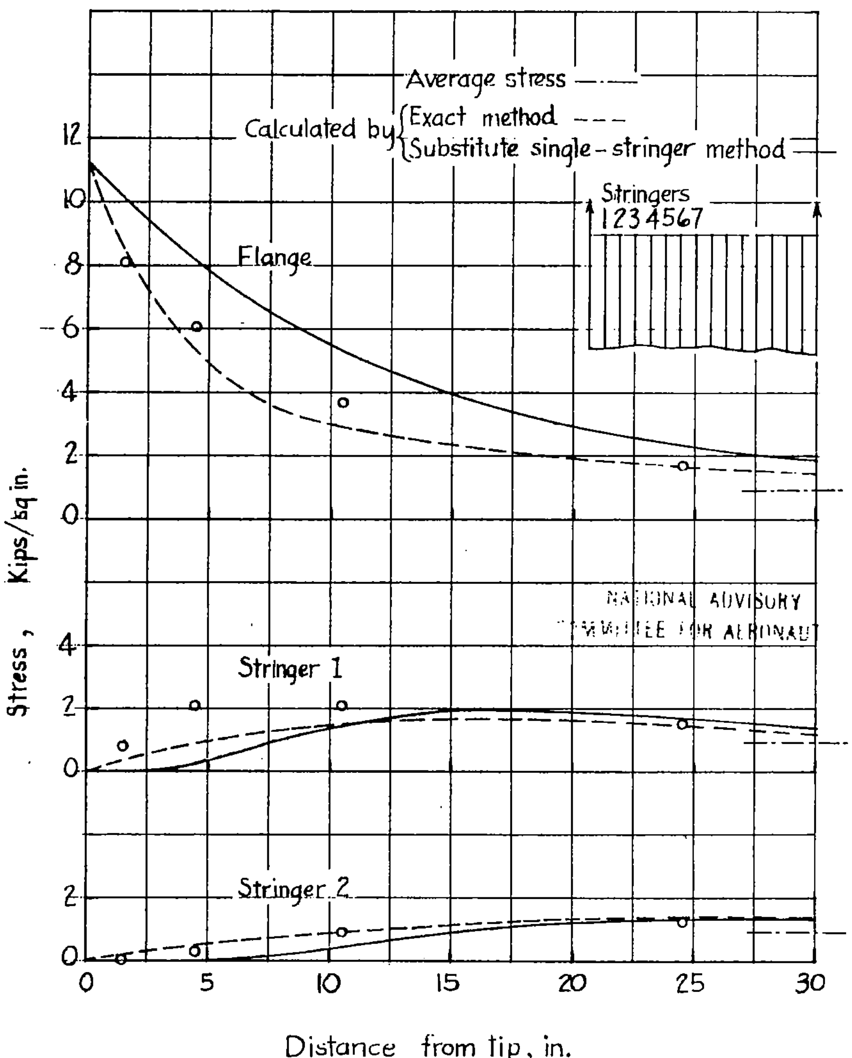


Figure 15.- Comparison of observed and calculated stringer stresses in tension panel.

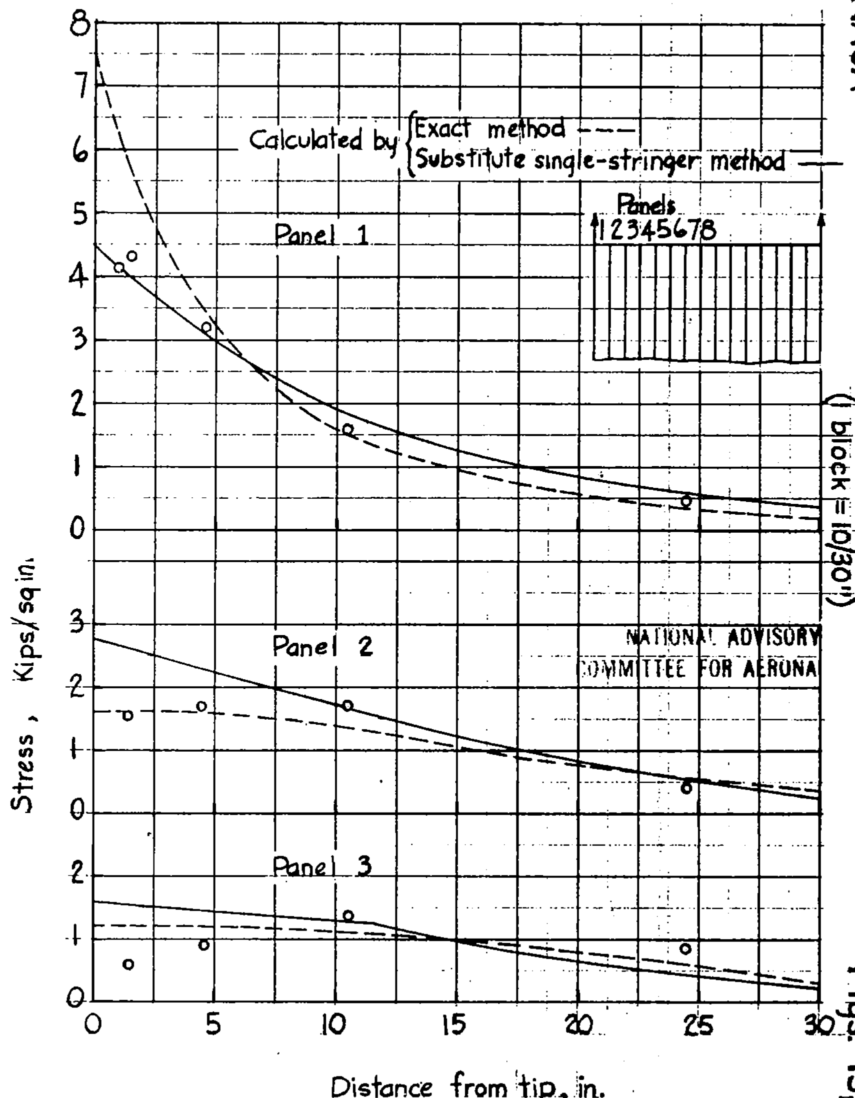


Figure 16.- Comparison of observed and calculated shear stresses in tension panel.