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LIMITATIONS OF LIFTING-LINE THEORY FOR ESTIMATION OF AILERON HINGE-MOMENT CHARACTERISTICS

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LIMITATIONS OF LIFTING-LINE THEORY FOR ESTIMATION OF AILERON HINGE-MOMENT CHARACTERISTICS

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SUMMARY

Hinge-moment parameters for several typical ailerons were calculated from section data with the aspect-ratio correction as usually determined from lifting-line theory. The calculations showed that the agreement between experimental and calculated results was unsatisfactory. An additional aspect-ratio correction, calculated by the method of lifting-surface theory, was applied to the slope of the curve of hinge-moment coefficient against angle of attack at small angles of attack. This so-called streamline-curvature correction brought the calculated and experimental results into satisfactory agreement.

INTRODUCTION

The present report is one of a group being prepared by the NACA to present in compact form collections and analyses of lateral-control data. References 1 to 4 are parts of this group that have already been completed.

The purpose of the present investigation is to evaluate the limitations of lifting-line theory in determining the aspect-ratio correction usually applied in the computation of aileron hinge-moment characteristics. The results are presented for the few cases in which section and finite-span experimental data are most nearly comparable.

The experimental data are used to check an additional aspect-ratio correction, called the streamline-curvature correction, which was determined from lifting-surface theory. This additional correction is used to estimate the slope of the curve of hinge-moment coefficient against angle of attack at small angles of attack.

SYMBOLS

C_L lift coefficient

C_h hinge-moment coefficient

c_l section lift coefficient

c_{l_1} additional lift coefficient at a section caused by an angle-of-attack change over wing (denoted by c_{l_a} in reference 5)

c_h section hinge-moment coefficient

A aspect ratio

b wing span

b_a aileron span

c wing chord

c_a aileron chord

$\bar{c}_a$ root-mean-square aileron chord

α angle of attack, degrees

δ_a aileron deflection, degrees

$\frac{\partial \alpha}{\partial \delta_a}$ effectiveness parameter $\left(\left|\frac{\partial c_l / \partial \delta_a}{\partial c_l / \partial \alpha}\right|\right)$

y distance along span axis

$\frac{c_l c}{c_s \alpha}$ local lift parameter used to determine section lift for an aileron deflection (reference 6)

y_o spanwise location of outboard end of aileron

y_1 spanwise location of inboard end of aileron

λ taper ratio; ratio of tip chord to root chord

$\frac{c_s}{c}$ ratio of wing chord at plane of symmetry to wing chord at any section

METHOD OF CALCULATING AILERON HINGE MOMENTS FROM LIFTING-LINE THEORY

The total hinge-moment coefficient of an aileron is obtained by a spanwise integration of the experimental section hinge-moment coefficients at each section for a fixed aileron deflection and for an angle of attack that corresponds to the geometric angle minus the induced angle caused by the downwash.

The downwash at each point along the span is determined, in lifting-line theory, from the fundamental assumption that it is equal to one-half the value calculated for a point infinitely far downstream. This value of the downwash divided by the free-stream velocity is used as an increment of angle of attack to correct the section data.

In order to obtain the variation of the induced angle of attack across the span from the corresponding section lift coefficients, the span loading caused by an angle-of-attack change and the span loading resulting from an aileron deflection must be known. These load curves, which can be obtained from references 5 and 6, were computed from lifting-line theory and apply to wings with rounded tips. For wings with square tips, the load curves may be slightly modified. Span-load curves for wings or ailerons of other plan forms may be calculated by the procedure given in reference 7.

An alternate method, which is considerably shorter than the one just described, is to use the lifting-line-theory factors with the slopes of the hinge-moment-coefficient curves rather than with the actual hinge-moment coefficients at each angle of attack and aileron deflection. The section hinge-moment parameters may then be summed across the aileron to give the total hinge-moment parameters as follows:

$$\frac{\partial C_h}{\partial \alpha} = \frac{1}{\frac{b_a}{b/2}\,\overline{c}_a{}^2}\frac{\partial C_L}{\partial \alpha}\int \frac{\partial c_h}{\partial \alpha}\,\frac{\partial \alpha}{\partial c_l}\,\frac{c_{l_1}}{C_L}\,c_a{}^2\,d\left(\frac{y}{b/2}\right)$$

and

$$\frac{\partial C_h}{\partial \delta_a} = \frac{1}{\frac{b_a}{b/2}\,\overline{c}_a{}^2}\left[\int \frac{\partial c_h}{\partial \delta_a}\,c_a{}^2\,d\left(\frac{y}{b/2}\right) + \int \frac{\partial c_h}{\partial \alpha}\,\frac{\partial \alpha}{\partial \delta_a}\left(\frac{\partial \alpha}{\partial c_l}\,\frac{c_s}{c}\,\frac{c_l c}{c_s \alpha} - 1\right)c_a{}^2\,d\left(\frac{y}{b/2}\right)\right]$$

The values of c_{l_1}/C_L obtained from reference 5, are for the loading due to change in angle of attack; whereas the values of $\frac{c_l c}{c_s \alpha}$, obtained from the influence lines of reference 6, are for the loading caused by aileron deflection.

DATA

The geometric characteristics of the wings and ailerons, which were tested as reflection-plane models in the LMAL 7- by 10-foot tunnel, are given in table I. The section data used for the computations were obtained from tests in the LMAL 7- by 10-foot tunnel and the NACA 4- by 6-foot vertical tunnel (reference 8 and unpublished). These data were used because the model construction features and the air-flow conditions are very similar in the two tunnels. No corrections for tunnel-wall effect had been applied to the hinge-moment coefficients in reference 8. These corrections were computed and applied, by the methods of reference 9, before the section data were used for the calculations of the present investigation. Tunnel-wall corrections were also applied to the results of the tests of reflection-plane models. The computations were made only for sealed unbalanced ailerons because that arrangement presented the minimum number of variables and because the available section data were most nearly comparable.

The section and finite-span hinge-moment parameters are given in table II. No values of $(\partial C_h/\partial\alpha)_{\alpha=0^\circ}$ are included for ailerons D and E because the data are considered unreliable. The tests were run at only a few rather widely separated angles of attack. The curves were nonlinear; consequently, the measured values of $(\partial C_h/\partial\alpha)_{\alpha=0^\circ}$ are questionable.

DISCUSSION OF LIMITATIONS OF LIFTING-LINE THEORY

The values of the hinge-moment parameters calculated from lifting-line theory and obtained experimentally are shown in table II. It will be noted that in all cases the parameters calculated from lifting-line theory are more negative than the experimental values. The calculated values of $(\partial C_h/\partial\alpha)_{\alpha=0^\circ}$ are only about half way between the section values and the experimental values; and the calculated values of $\partial C_h/\partial\delta_a$ are only about one-fourth the way between section and experimental values. Effects other than that taken care of by the lifting-line correction must therefore be exceedingly important.

A preliminary investigation was started to examine the limitations of lifting-line theory. The lifting-surface-theory calculations in reference 10 showed that the distribution of lift assumed by lifting-line theory induces a downwash which is not constant along the wing chord, as is implied by the assumptions upon which lifting-line theory is based. The wing must thus be cambered by the amount of the curvature of the streamlines in order to sustain the load assumed by lifting-line theory. Because the wing shape is fixed for this case, an additional load that is not accounted for by lifting-line theory must be acting on the wing. The direction and magnitude of this additional load, which results from the streamline curvature, causes a large decrease in the aileron hinge moments. It may be reasonably assumed that the curvature of the streamlines is approximately circular. Formulas for the corrections to the hinge-moment coefficients for circularly curved streamlines are presented in reference 9.

Some computations have been made by Cohen (reference 10 and unpublished) of the amount of streamline curvature for elliptical wings of aspect ratios 3 and 6. The theoretical hinge-moment corrections for streamline curvature $\Delta(\partial C_h/\partial \alpha)_{\alpha=0^o}$ for these wings were calculated by the methods of reference 9 and are presented in figure 1. The experimental values of the corrections for models A, B, and C are also presented in figure 1, along with the interpolated values of the theoretical corrections for each of the particular models. The theoretical and experimental streamline-curvature corrections are in good agreement; in fact, the agreement is somewhat better than might be expected in view of the following considerations:

(1) In reference 10, the downwash was calculated for elliptical wings.

(2) The assumption of circular camber may not be justified.

(3) The hinge-moment corrections for circular camber presented in reference 9 were based on thin-airfoil theory, and no account was thus taken of wing thickness or of the effects of viscosity.

The effects of viscosity in reducing the computed corrections are, however, partly counteracted by the effect of airfoil thickness in increasing the corrections. An examination of some experimental pressure distributions of cambered airfoils at an angle of attack of 0^o indicates that the hinge-moment corrections of reference 9 may be about 10 percent too large for the airfoils and ailerons considered in the present report.

The effects of viscosity in reducing the magnitude of the calculated hinge-moment corrections are smaller for changes in circular camber than for changes in angle of attack because the pressure gradient is less adversely affected. The streamline-curvature correction is therefore more accurately applied as an increment correction, as done herein, than as a percentage correction (percent of the thin-airfoil-theory values).

The streamline-curvature corrections presented in figure 1 were determined for small angles of attack. The effect of the vertical spacing and inclination of the trailing vortices must be considered in order to obtain a nonlinear wing theory (reference 11) which may account for the fact that greater values of $\partial C_h/\partial\alpha$ are usually obtained at high angles of attack (even though the section hinge-moment curves are very nearly linear) than at low angles of attack.

Further solutions by the methods of lifting-surface theory are necessary to determine numerical values of the corrections to the hinge-moment characteristics for various plan forms, aileron arrangements, and high angles of attack. Until these solutions have been found, it is suggested that an arbitrary streamline-curvature correction of about the magnitude given in figure 1 be applied to $(\partial C_h/\partial\alpha)_{\alpha=0^o}$ and that a correction of about the magnitude indicated by the data of table II be applied to $\partial C_h/\partial\delta_a$.

CONCLUDING REMARKS

The calculations of the present investigation indicated that lifting-line theory is inadequate for determining aileron hinge moments from section data. The limitations of lifting-line theory have been examined and it is believed that the use of lifting-surface theory will largely account for the differences between the experimental and calculated results, provided account is taken of the vertical spacing and inclination of the trailing vortices. The theoretical streamline-curvature correction calculated for elliptical wings and given herein is recommended for use in determining the slipe of the curve of hinge-moment coefficient against angle of attack at small angles of attack.

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TABLE I. – GEOMETRIC CHARACTERISTICS OF WINGS AND AILERONS USED IN PRESENT INVESTIGATION

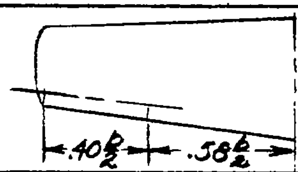
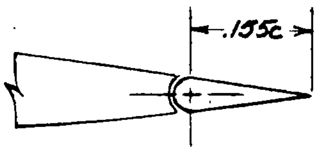
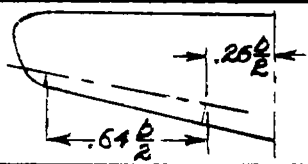
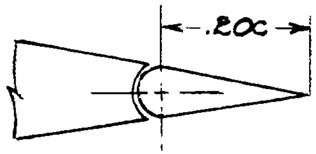
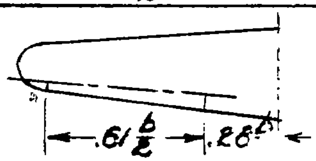
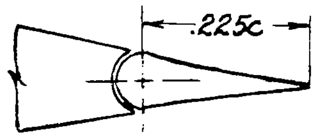
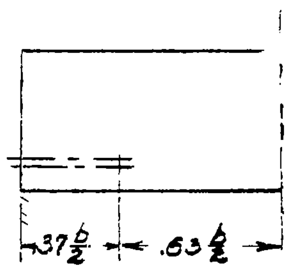
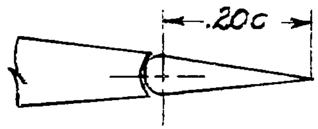
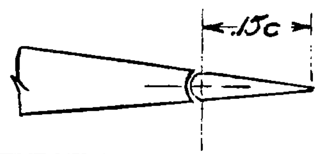
Model	Plan form of surface	Typical aileron section	Airfoil section Root	Airfoil section Tip	A	λ	Reference
A			NACA 23015.5 (approx.)	NACA 23008.25 (approx.)	5.6	0.60	13
B			NACA low drag		5.7	0.52	Unpublished
C			NACA low drag		8.1	0.49	Unpublished
D			NACA 23012		4.0	1.00	4
E			NACA 23012		4.0	1.00	4

TABLE II

HINGE-MOMENT PARAMETERS

Model	Section data $\left(\frac{\partial c_h}{\partial \alpha}\right)_{\alpha=0^\circ}$	Finite-span data $\left(\frac{\partial c_h}{\partial \alpha}\right)_{\alpha=0^\circ}$			Section data $\frac{\partial c_h}{\partial \delta_a}$	Finite-span data $\frac{\partial c_h}{\partial \delta_a}$	
		Calculated from lifting-line theory	Calculated with correction from fig. 1	Experimental		Calculated from lifting-line theory	Experimental
A	-0.0035	-0.0023	-0.0011	-0.0013	-0.0100	-0.0093	-0.0070
B	-.0040	-.0031	-.0023	-.0022	-.0110	-.0102	-.0070
C	-.0058	-.0049	-.0042	-.0042	-.0108	-.0097	-.0075
D	-------	-------	-------	-------	-.0098	-.0085	-.0049
E	-------	-------	-------	-------	-.0089	-.0081	-.0055

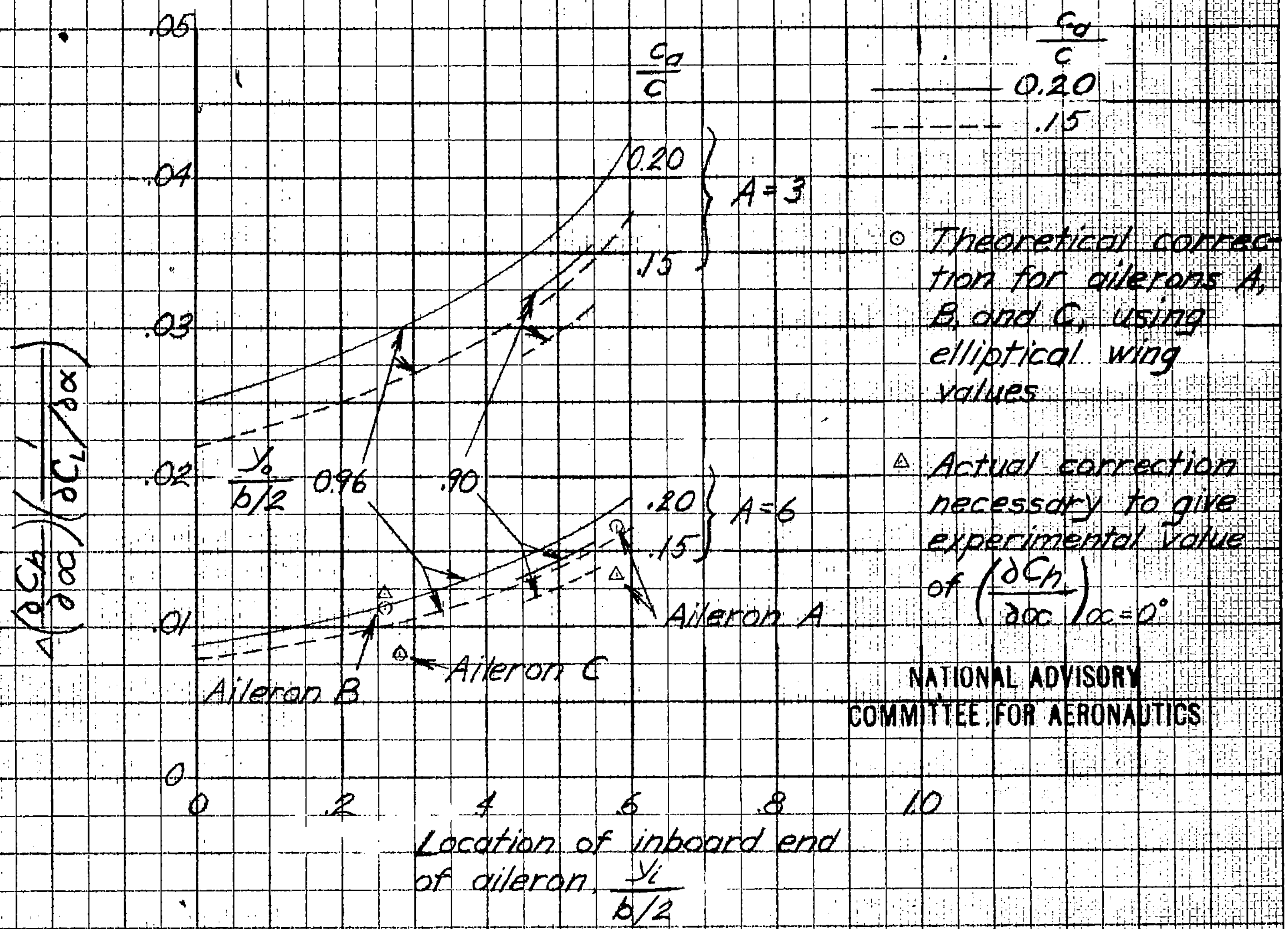


Figure 1.- Theoretical streamline-curvature correction for hinge moments of ailerons having a constant ratio of aileron chord to wing chord on elliptical wings.