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AN INVESTIGATION OF AIRCRAFT HEATERS

I - ELEMENTARY HEAT TRANSFER CONSIDERATIONS IN AN AIRPLANE

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NACA

WASHINGTON

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I - ELEMENTARY HEAT TRANSFER CONSIDERATIONS IN AN AIRPLANE

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INTRODUCTION

No exact data exist on heat losses from aircraft and aircraft personnel during flight. It is imperative, however, for the designer to have knowledge of the magnitude of these losses and the physical factors which control them, in order properly to design cabin heaters, seating arrangements, insulation, and so forth.

The following report was written in an attempt to clarify the basic factors of heat transfer applied specifically to aircraft heating and to guide the experimenter in planning experiments to obtain further data. In the absence of exact test results, the authors have studied the literature and tentatively present equations and charts for the various thermal resistances encountered in aircraft work. These recommendations will be changed as more is known about the various components of the system under consideration.

In many cases, from necessity, the actual conditions are excessively idealized: namely, a man is considered as a vertical cylinder in order to establish the convective loss from his body. The order of magnitude of the quantity involved, however, may be determined readily, and what is more important, the effect of the variables involved becomes apparent. Thus, numerical calculations based on the data presented will be a first approximation only, but they will show the designer the relative effect of any changes which he may propose and will aid the experimenter to determine what further data are necessary.

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SYMBOLS

a the coefficient of absorption (radiant energy), ft^{-1}

A area of heat transfer perpendicular to direction of heat flow, ft^2

A_1 area of smaller radiating surface, ft^2

A_2 area of larger radiating surface, ft^2

c experimental constant (dimensionless)

C_p unit heat capacity of fluid at constant pressure, Btu/lb °F

d thickness of the absorbing material, ft

D diameter, ft

e_1 emissivity of the hot surface

e_2 emissivity of the cold surface

E voltage, volts

f_c unit thermal conductance for convection, Btu/hr ft^2 °F

f_{cx} unit thermal conductance at any point x from leading edge of flat plate, Btu/hr ft^2 °F

f_r equivalent unit conductance for radiation, Btu/hr ft^2 °F

F function

F_A shape modulus, a factor in the radiation equation which allows for the relative geometrical position of the radiating surfaces

F_E emissivity modulus, a factor in the radiation equation which allows for the non-Planckian character of the radiating surfaces

g gravitational force per unit mass, lb/(lb sec^2/ft)

G_o solar irradiation at any altitude, Btu/hr ft^2

H height of plate, ft

i electrical current, amperes

k thermal conductivity (of solid for conductance equations and of fluid for convection equations, Btu/hr ft^2(°F/ft)

l length of plate in direction of air flow, ft

n exponent (secs. XI and XVI); number of radiation shields (sec. VIII)

P atmospheric pressure at any altitude, lb/in^2

P_0 atmospheric pressure at ground level, lb/in^2

q rate of heat transfer, Btu/hr

q_a rate of absorption of solar energy by an opaque body, Btu/hr

q_{ab} rate of absorption of solar energy by a translucent body, Btu/hr

q_t rate of energy transmission through a translucent body, Btu/hr

R electrical resistance, ohms

R_1 thermal resistance of skin, °F/Btu/hr

R_2 convective thermal resistance of air gap, °F/Btu/hr

R_3 evaporative thermal resistance of air gap, °F/Btu/hr

R_4 radiant thermal resistance of air gap, °F/Btu/hr

R_5 thermal resistance of clothing, °F/Btu/hr

R_6 convective thermal resistance between outer surface of clothing and cabin air, °F/Btu/hr

R_7 thermal resistance through which the sun's irradiation flows to cabin interior, °F/Btu/hr

R_8 radiant thermal resistance between outer surface of clothing and cabin walls, °F/Btu/hr

R_9 thermal resistance through which respiratory heat loss flows, °F/Btu/hr

R_{10} thermal resistance through which leakage heat loss flows, °F/Btu/hr

R_{11} convective thermal resistance between cabin air and cabin walls, °F/Btu/hr

R_{12} thermal resistance of cabin wall, °F/Btu/hr

R_{13} radiant thermal resistance between outside of plane and surrounding environment, °F/Btu/hr

R_{13B} R_{13T} same definition as R_{13} except for bottom and top of plane, respectively

R_{14} thermal resistance through which heat is transferred by irradiation, °F/Btu/hr

R_{15} thermal resistance through which heat is transferred by friction, °F/Btu/hr

R_{16} convective thermal resistance between outside of cabin wall and outside air, °F/Btu/hr

t_1 skin temperature, °F

t_2 temperature of inner surface of clothing, °F

t_4 temperature of outer surface of clothing, skin in contact with cabin air, °F

t_{earth} effective earth temperature, °F

t_{sky} effective sky temperature, °F

t_w temperature of inner surface of cabin wall, °F

t_{w2} temperature of outer surface of cabin wall, °F

T_1 skin temperature, °R

T_2 temperature of inner surface of clothing, $^\circ R$

T_4 same as t_4 except in absolute units, $^\circ R$

T_A hot body temperature, $^\circ R$

T_B cold body temperature, $^\circ R$

T_r transmittance of plexiglas

T_w temperature of inner surface of cabin wall, $^\circ R$

T_{ws} temperature of outer surface of cabin wall, $^\circ R$

T_s temperature of radiation shield, $^\circ R$

U_m velocity of fluid, ft/sec

W rate of fluid flow, lb/hr

x thickness of solid in direction of heat flow; distance along flat plate (sec. XVI) ft

α absorptance of surface

β coefficient of expansion of fluid, $^\circ R^{-1}$

γ weight density of fluid, lb/ft^3

γ_0 weight density of fluid at 70° F and 1 atmosphere, lb/ft^3

δ thickness of air gap, ft

ϵ base of Naperian logarithms

Δt temperature difference causing heat flow, $^\circ F$

φ angle between normal to surface and line connecting surface and sun

μ absolute viscosity of fluid, lb sec/ft^2

ν kinematic viscosity of fluid, ft^2/sec

σ Stefan-Boltzman radiation constant = 0.173×10^{-8}, Btu/ft^2 hr $^\circ R^4$

T_3 temperature of air between skin and clothes, °F

T_a temperature of cabin air, °F

T_e temperature of air when exhaled, °F

T_i temperature of air when inhaled, °F

T_o temperature of air outside cabin, °F

$$Gr = \frac{g\ \beta\ \Delta t\ H^3}{\nu^2}, \quad \text{Grashof modulus}$$

$$Nu = \frac{f_c\ H}{k}, \quad \text{Nusselt modulus}$$

$$Pr = \frac{3600\ \mu\ C_p\ g}{k}, \quad \text{Prandtl modulus}$$

$$Re = \frac{U_m\ l\ \gamma}{\mu\ g}, \quad \text{Reynolds modulus for flat plate}$$

THERMAL CIRCUITS

(Temperature not a Function of Time)

The analysis of the flow of heat from a body to the surrounding environment is simplified by utilizing the circuit concept. The thermal circuit may be solved in the same manner as the corresponding electrical circuit. In the steady state the elements of the thermal circuits are resistances and lumped unidirectional heat sources.

Certain thermal quantities and corresponding electrical quantities for one analogous circuit are illustrated in table I. The table entries are limited to unidirectional steady state variables and elements. Distributed parameters and thermal capacities have not been included.

TABLE I

Mechanism	Relation	Equivalent current	Equivalent voltage	Equivalent resistance
Electrical conduction	$i = \frac{E}{R}$	i amps	E volts	R ohms
		(Btu/hr)	(°F)	$\left(\frac{°F}{Btu/hr}\right)$
Thermal conduction	$q = kA\frac{\Delta t}{x}$ or $q = \frac{\Delta t}{\left(\frac{x}{kA}\right)}$	q	Δt	$\frac{x}{kA}$
Thermal convection	$q = f_c A\ \Delta t$ $q = \frac{\Delta t}{\left(\frac{1}{f_c A}\right)}$	q	Δt	$\frac{1}{f_c A}$
Thermal* radiation	$q = f_r A\ \Delta t$	q	Δt	$\frac{1}{f_r A}$
Flow of gases	$q = WC_p(\tau_1 - \tau_2)$	q	**Thermal resistance proportional to $1/WC_p$	

*The term f_r is an equivalent conductance for radiation. (See secs. IV, VIII, and XIII for a more detailed description.)

**See sec. X for discussion.

W weight rate of flow, lb/hr

C_p heat capacity at constant pressure, Btu/lb °F

x thickness in the direction of flow, ft

k thermal conductivity, Btu/hr ft^2 ($^\circ$F/ft)

A area of heat transfer perpendicular to the direction of flow, ft^2

f_c unit conductance for convection, Btu/hr ft^2 $^\circ$F

f_r equivalent unit conductance for radiation, Btu/hr ft^2 $^\circ$F

Δt temperature difference effective across resistance, $^\circ$F

$\tau_1 - \tau_2$ change in mixed mean temperature of flowing gas, $^\circ$F

Inspection of table I reveals that, for the circuits under consideration (in which the heat flow does not depend on time), the rate of heat transfer is equivalent to the current and the temperature drop equivalent to the voltage drop in the particular electrical circuit. The evaluation of the equivalent thermal resistances is usually the most difficult part of the analysis. Methods of computing these resistances are presented in the body of this report. These are recommended only until better data are available.

EXAMPLES OF THERMAL CIRCUITS

As examples of the above technique applied directly to the problem of aircraft heating, four thermal circuits are arranged in figures 1, 2, 3, and 4. All circuits represent the case of heat transfer independent of time and correspond to:

Figure 1.- The flow of heat from a clothed man in the cabin of an airplane.

Figure 2.- The flow of heat from an unclothed portion of a man inside the cabin of an airplane.

Figure 3.- The flow of heat from an inanimate object inside the cabin of an airplane.

Figure 4.- The flow of heat from the plane as a unit to the external environment (heat balance on plane).

Referring to figure 1, a large fraction of the heat lost by the human body either passes through the skin or is given to the air which is breathed. The latter quantity need not be considered when the flow through the skin and clothes is calculated.

The rate of heat transfer through the skin is determined by the magnitudes of all the other resistances in the circuit and must be such that the skin temperature does not fall below a certain minimum value. After passing through the skin the heat passes to the inner surface of the clothing in three parallel paths, that is, by conduction and/or convection, by radiation, and by evaporation.

Practically all of this heat then passes through the clothing. In loose fitting clothing there is usually a certain amount of air circulation. Some of the heat may be carried directly into the cabin air by these air currents. This leakage is as yet an indeterminate amount and is represented by resistances 2a and 3a.

At the outer surface of the clothing there are three paths for the flow of the heat. The body may gain by convection from the relatively warm cabin air, lose heat by radiation to the cabin walls, and perhaps gain heat as a result of irradiation by the sun transmitted through the cabin windows. In the following sections of this paper each resistance is discussed in some detail. By knowing the manner of variation of the resistances an estimate can be made of the effect of changes in the system on the rate of heat transfer from the body. Two qualitative examples will be given: (Refer to fig. 1.)

1. Contrast the heat flow from the portion of a man in contact with a cold metal seat to that from a standing man. The resistance 2 is reduced considerably; 5 also is diminished. Resistance 6 is materially reduced and in addition the temperature in the lower end of this resistance is decreased from the air temperature to that of the cold metal seat. The radiation resistance 8 is somewhat increased, and the irradiation resistance 7 becomes infinite.

2. Next consider the heat loss from the bare skin of a man. In this case resistances 2, 4, and 5 become zero and a diagram such as is shown in figure 2 results.

Figure 3 shows the diagram for an inanimate object in the cabin. Such an object normally will not be a source of heat; so the sum of the gain of heat by convection and irradiation must just equal the loss of heat by radiation to the colder cabin walls. A convenient method for calculating the equilibrium temperature for such a body is discussed and presented in section XVII.

Figure 4 illustrates the manner in which heat is lost from the aircraft to the outside air. Heat is gained by the plane through the heater (an exhaust gas type is shown in the figure) and from the heat losses of the men in the plane. The heat may be lost by air leakage and by passing through the cabin wall, first passing through a convective resistance on the inside surface of the wall. The heat then flows through the cabin wall and finally by convection and radiation to the outside. Heat may be gained on the outside surface by irradiation from the sun and by frictional heating (the latter at high velocities only). A numerical evaluation of such a heat balance is given in section XVII.

I. RESISTANCE 1

THE RESISTANCE OF THE HUMAN BODY

Hardy (reference 1), in his papers on the heat losses from human bodies, presents the data for an unclothed man; these are given in figure 5. Dr. Hardy's experiments were performed on naked subjects placed in a calorimeter. For this reason, some of his results do not properly apply to clothed men. However, they serve to indicate the trends in the influence of temperature on the body mechanism. Hardy (reference 2) makes the following statements:

"Measurements of thermal gradients in man indicate a depth of 2 to 3 cm is involved in the thermal gradient from the internal tissue to the skin surface. Assuming an average depth of 2 cm for the whole body, the thermal conductivity

$$k = 0.00048 \text{ gram cal/cm}^2 \text{ sec } (^{\circ}\text{C/cm})$$

$$k = 0.116 \text{ Btu/hr ft}^2 \text{ } (^{\circ}\text{F/ft})$$

"Lefèvre (reference 3) in 1911 arrived at a value of 0.00066 gram cal/cm^2 sec $\frac{^{\circ}\text{C}}{\text{cm}}$. Comparing this value with other substances shows that from substances which have considerable air space, such as wool, hair, felt, etc., few materials have higher insulating ability than living tissue. Leather, for example, has a value of 0.0004, paper 0.0003, cork 0.0007. It would thus appear that living tissue could hardly increase its thermal insulating ability if it had no circulation of blood and that the heat transferred through the tissue in this environmental range is due to pure conduction. This would account for the constant value of the tissue conductance from 82.4° F (28° C)* down. The body in this region may be compared to a cylinder of the same surface area as a man, wrapped with a layer of paper 1 cm thick, the internal temperature of which is maintained at 98.6° F (37° C)".

The thermal conductance referred to was obtained by dividing the thermal losses through the skin by the average temperature difference between the skin and the internal tissue. As shown in figure 6, this ratio remains constant below a calorimeter temperature of 82.4° F.

The increase in the thermal conductance above 82.4° F (28° C) is due to the increased circulation of blood. This effect may be induced also by muscular activity or chills. In attempting to apply these values to calculations it must be borne in mind that a criterion of comfort is the skin temperature. If the subject indulges in great muscular activity, he can stand a much greater heat loss without discomfort than if he remains quiet. The governing factor is not the heat load on the man, but the skin temperature.

The temperature may vary greatly from one portion of the body to another. In one experiment it was found that the temperature of the skin on the forehead was 89.6° F (32° C) while the temperature at the feet was 80.1° F (26.7° C) (reference 4). The values shown in the figures are average temperatures.

If the skin temperature drops below 86° F (30° C), shivering will generally occur. In considering the design of heating equipment, it will be advisable probably to specify that the average skin temperature of a man dressed in specified clothing shall not be below a certain temperature, probably 88° F.

The heat loss through the man's skin by conduction is, therefore,

$$q = k \frac{A}{x} (98.6 - t_1)$$

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*Calorimeter temperature.

where

A surface area of the man (about 20 ft^2)

x depth of conducting layer (about 2 cm = 0.0662 ft)

q heat loss, Btu/hr

t_1 skin temperature, °F

k thermal conductivity of living tissue, 0.116 Btu/hr $ft^2 \frac{°F}{ft}$

Resistance 1 is, therefore,

$$R_1 = 0.566/A \frac{°F}{Btu/hr}$$

Whenever perspiration is induced at a rate which exceeds the rate of evaporation and/or absorption by clothing, and additional resistance due to the liquid layer on the skin must be added to resistance 1. This resistance is equal to the thickness of the liquid layer divided by the thermal conductivity and surface area.

II. RESISTANCE 2

THE RESISTANCE BETWEEN THE SKIN AND THE CLOTHING

The quantity of heat which flows from the skin to the clothing by means of conduction and convection depends mainly on the thickness of the air space between the clothes and the skin. Fishenden and Saunders (reference 5, p. 115) and Ten Bosch (reference 6) discuss the mechanism of heat transfer by conduction and convection across air spaces. The conclusion is reached that for air gaps less than 0.10 inch thick, convection currents are negligible and the transfer of heat is by pure conduction across the stagnant air layer. When the air gap is thicker than 0.10 inch, free convection currents become more effective, until for gaps of about 2 inches the transfer is largely by free convection.*

Ten Bosch (reference 6) illustrates the convective currents which will exist in an air gap, as shown in figure 7. The fluid velocity is zero at the hot surface, increases rapidly a short

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*Radiation across the air gap is discussed under resistance 4.

distance from the wall, and falls to zero at the center of the air gap. The fluid falls next to the cold surface. These convective currents transfer the heat from one vertical surface to the other.

Quantitative Data

1. Gaps less than 0.10 inch. - The thermal resistance for small air gaps is given by the conduction equation:

$$R_2 = \frac{\delta}{kA}, \quad \frac{^\circ F}{(Btu/hr)} \tag{2a}$$

where

δ air gap thickness, ft

k thermal conductivity of air, $\dfrac{Btu}{hr\ ft^2\left(\frac{^\circ F}{ft}\right)}$

A area through which heat transfer is taking place, ft^2

2. Gaps greater than 2 inches. - Gaps greater than 2 inches generally will not exist between the skin and clothing of a man, but it is of interest to know the mechanism of heat transfer across such air gaps. The air gap will be considered vertical, for purposes of analysis. The relations which are discussed under Resistance 11 will apply directly, that is, the mode of heat transfer is by free convection. Thus from the hot surface to the ambient air, if the flow is laminar:*

$$f_c = 0.30\ (t_1 - \tau_3)^{1/4}\left(\frac{P}{P_o}\right)^{1/2}, \quad \frac{Btu}{hr\ ft^2\ ^\circ F} \tag{2b}$$

and from the ambient air to the cold surface

$$f_c = 0.30\ (\tau_3 - t_2)^{1/4}\left(\frac{P}{P_o}\right)^{1/2} \tag{2c}$$

As a first approximation, the temperature of the ambient air is equal to

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*More precisely, as is discussed under Resistance 11,

$$f_c = 0.30\ (t_1 - \tau_3)^{1/4}\left(\frac{\gamma}{\gamma_o}\right)^{1/2}$$

$$T_3 = \frac{t_1 + t_2}{2}$$

Thus, both (2b) and (2c) may be written as

$$f_c = \frac{0.30}{\sqrt[4]{2}} (t_1 - t_2)^{1/4} \left(\frac{P}{P_0}\right)^{1/2}$$

$$= 0.25 \ (t_1 - t_2)^{1/4} \left(\frac{P}{P_0}\right)^{1/2} \qquad (2d)$$

The total resistance across the air gap is twice the resistance from one surface to the ambient air. Thus

$$R_2 = \frac{2}{f_c A} = \frac{1}{0.125(t_1 - t_2)^{1/4} \left(\frac{P}{P_0}\right)^{1/2} A} \qquad (2e)$$

where

t_1 temperature of hot surface, $^{\circ}F$

t_2 temperature of cold surface, $^{\circ}F$

P cabin pressure at any altitude, lb/sq in.

P_0 cabin pressure at sea level, lb/sq in.

A area for heat transfer, ft^2

3. Intermediate gap thickness. - The values of $\frac{1}{R_2 A}$ for intermediate gap thickness are shown in figure 8 for atmospheric pressure conditions and a $(t_1 - t_2)$ of 30° F. These values were obtained from Fishenden and Saunders (reference 5). For other atmospheric pressures and other temperature differences $(t_1 - t_2)$ the magnitudes of $1/R_2A$ for gaps less than 0.1 inch will not change appreciably; whereas for gaps greater than 2 inches $1/R_2A$ may be computed from equation (2e). Interpolation between these magnitudes, using the curves shown in figure 8 as a guide will allow prediction of the thermal resistance of gaps of intermediate widths.

III. RESISTANCE 3

EVAPORATION OF MOISTURE FROM THE SKIN

The rate of evaporation of moisture from the skin will depend on the humidity and the temperature of the air in contact with the skin and the skin temperature. It also will depend on the rate of perspiration of the subject, or, more exactly, the concentration of water vapor on the skin. Moisture removed from the skin is not necessarily evaporated. Moisture absorbed into the clothing, for example, has no cooling effect other than its lowering of the resistance of the clothing to heat transfer. (See resistance 5.)

At high altitudes the air in the cabin will be generally of low humidity. This will result in the drying out of the skin wherever it is exposed to the air, and experience similar to that experienced by mountain climbers. Under the clothing, however, the layer of air will quickly become rather humid as there is very little circulation of air inside the clothing. DuBois (reference 4) has shown that at low temperatures in air of average humidity the amount of heat lost by the body through evaporation from nude subjects is not very great. This is due largely to the decrease of the vapor pressure of water with temperature and to the lowered activity of the perspiratory organs. The latter effect will depend somewhat on the mental state of the subject. The data for nude subjects in a calorimeter at various temperatures is shown in figure 9. These data cannot be applied directly to the problem of clothed persons but should be used only to indicate a trend. The fraction of the total heat loss due to evaporation in the region below 78.8° F (calorimeter temperature) is about 1/10. This, however, is for nude persons. For clothed persons it should be somewhat less, because the clothing will tend to absorb any perspiration and may even become negligible.

This portion of the thermal circuit will require further research. For the present, it may be assumed that the heat lost through this means is negligible and the resistance 3 is infinite.

IV. RESISTANCE 4

RADIATION FROM SKIN TO CLOTHING

Thermal energy is radiated from the skin to the inner surface of the clothes. The discussion presented under Resistance 8 applied directly to this case. The energy is transferred approximately in accordance with the fourth power law. However, the water vapor in the air in the space between the skin and clothes will take part in the radiation process in some wavelengths. Until this fraction has been more definitely established, the fourth power law will be utilized without correction (reference 7). Thus the rate of energy transfer is given by

$$q = 0.173\ A \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] F_A\ F_E \tag{4a}$$

Since (reference 8, p. 54) the clothes completely enclose the body, $F_A = 1$. Further, the areas of the body and the clothes are practically equal, so that

$$F_E = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \tag{4b}$$

The emissivity of the skin (reference 9) is very close to 1.00 and the emissivity of clothing at long wavelengths (corresponding to a low temperature radiation) is approximately (reference 10) 0.95, regardless of color.

Thus $F_E = 0.95$

Expanding the equation for q as shown for resistance 6 yields

$$f_r = 0.173 \times 10^{-8}\ (T_1^2 + T_2^2)(T_1 + T_2)\ F_A\ F_E \tag{4c}$$

Thus

$$R_4 = \frac{1}{f_r\ A} = \frac{5.78 \times 10^8}{(T_1^2 + T_2^2)(T_1 + T_2) F_A\ F_E},\ \frac{{}^\circ F}{\text{Btu/hr}}$$

The quantity $\frac{f_r}{F_A F_E}$ is shown plotted in figure 16 against T_1 and T_s. If $F_A F_E = 0.95$, f_r and R_4 may be readily calculated.

V. RESISTANCE 5

THE RESISTANCE OF THE CLOTHING

Scant results of work on the effectiveness of clothing can be found in the literature.

The resistance of the clothing is the sum of the resistance of each layer of cloth and the contact resistances between them. Contact resistances are treated in section II. In table II is given the thermal conductivity and the density of various materials.

TABLE II

Material	Conductivity* $\left(\text{Btu/hr ft}^2\left(\frac{^\circ F}{ft}\right)\right)$	Density (lb/ft^3)
Carbon dioxide	0.0088	0.12
Air	.014	.08
Kapok	.020	.88
Hair felt	.021	17
Wool	.022	6.9
Mineral wool	.022	9.4
Balsa wood	.023	2.2
Cotton wool	.024	5.06
Cork	.025	9.0
Silk	.026	6.3
Earth (loose)	.026	10.6
Felt (wool)	.030	20.6
Sawdust	.030	12.0
Charcoal (loose)	.030	15.2
Cotton	.032	5.0
Wood shavings	.034	8.8
Earth (powder)	.036	20.0
Lampblack	.038	10.0
Gypsum (loose)	.046	30.0
Linen	.05	
Asbestos	.089	29
Sand	.19	94.6

*See references 11 and 12.

If, for the materials listed in the table, the conductivity is plotted against the density, a rough correlation may be observed. This correlation exists because the insulating value of cloth (or other material containing air gaps) depends to the first approximation not on the cloth itself, but on the air spaces in the cloth. The best insulator would have an infinite number of air spaces, infinitely small. A line has been drawn through the points in figure 10 and made to coincide with the value for pure air. The lower line represents the probable value of the conductivities were the materials saturated with carbon dioxide instead of air.

The effects of compressing the clothing (when sitting down or bending the elbow, for instance) are threefold. First, the thermal conductivity of the cloth is increased since the air gaps are made smaller. Secondly, the contact resistances between clothes are made much smaller, and lastly, the total thickness of the cloth is decreased. The effect of compression on the thermal conductivity of rock wool is shown in figure 10a (reference 13).

The resistance of the cloth to heat transfer will depend greatly on the vapor content of the cloth. In figure 11 the effect of water vapor on the thermal conductivity of rag felt is shown (reference 14).

Flying clothes may be hung in moisture-free containers when not in use and then kept dry. Pilots moving from the hangars to the ships on foggy or rainy nights will absorb large quantities of water which will not only cool the clothing as the clothes dry, but will lower the resistance of the cloth as well.

The same precautions should be taken concerning the underclothes. A waterproof garment worn between the skin and clothing or between the underclothes and the next layers of clothing might prevent perspiration from seeping through the clothing and lowering its resistance. The physiological effects of such a garment except for short periods may preclude its use, however.

The heat flow through the clothing will be given by

$$q = (t_3 - t_4)/R_5$$

R_5 = summation of cloth resistance + contact resistances

For approximate calculations it will probably suffice to decrease the thermal conductivity of the clothing material by a third to account for the contact resistances and use this value as the over-all conductivity of the total thickness of clothing. (See sample calculations in sec. XVII.)

VI. RESISTANCE 6

CONVECTIVE RESISTANCE ON OUTER SURFACE OF CLOTHING

The convection from the outside of the man's body is quite complex, because of its irregular geometrical shape. As a first approximation, a standing man may be considered as a vertical cylinder about 14 inches in diameter and 6 feet tall. Heat is lost from this "cylinder" by free convection, which causes vertical air currents, and by forced convection which on the average is caused by air flow perpendicular to the cylinder axis. It is difficult to combine the effect of these two modes of transfer, but it is suggested as a preliminary step that both unit conductances be computed and the larger of the two chosen for calculation.

Forced Convection

The data presented by McAdams (reference 8, pp. 219-220) for forced flow past right circular cylinders about 3.75 inches diameter, may be expressed as:

$$\frac{f_c D}{k} = 0.35\left(\frac{D\ U_m \gamma}{\mu\ g}\right)^{0.56} \tag{6a}$$

substituting the values of μ, k for air at 50° F yields

$$f_c = 2.90\ \frac{(U_m\ \gamma)^{0.56}}{D^{0.44}},\quad \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}} \tag{6b}$$

U_m air velocity, ft/sec

γ air density, lb/ft^3

D cylinder diameter, ft

For a diameter of 14 inches (man)

$$f_c = 2.70(U_m\ \gamma)^{0.56}, \quad \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}} \tag{6c}$$

Free Convection

As in the case of resistance 11, the free convection from the man may be expressed as:

$$f_c = 0.30\ (t_4 - \tau_a)^{1/4} \left(\frac{P}{P_o}\right)^{1/2}, \quad \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}} \tag{6d}$$

This equation is a reasonable approximation and expresses the probable effect of temperature difference and cabin pressure on the rate of heat flow by free convection.

Recapitulation

For free convection

$$R_6 = \frac{1}{0.30(t_4 - \tau_a)^{1/4} \left(\frac{P}{P_o}\right)^{1/2} A} \tag{6e}$$

and for forced convection

$$R_6 = \frac{1}{2.70(U_m\ \gamma)^{0.56} A} \tag{6f}$$

The smaller of the two resistances (larger f_c) is to be used.

Since, in general, the cabin air will be warmer than the man's clothing, an attempt should be made to reduce R_6 to as small a value as possible. This may be done by the use of fans or the proper location of air vents. Care should be taken to avoid excessive drafts at the cabin wall to prevent R_{11} from becoming too small, thereby lowering the cabin air temperature.

Equations (6c) and (6d) are presented graphically in figures 12 and 13.

VII. RESISTANCE 7

RADIATION FROM THE SUN THROUGH THE WINDOWS

The transmittance of plexiglas has been measured as a function of the wavelength of the incident energy. The transmittance is defined as the ratio of the transmitted energy to the incident energy.

$$T_r = \frac{\text{transmitted radiant energy}}{\text{incident radiant energy}}$$

In the wavelengths represented in sunlight, it was found (reference 15) that the average transmittance varied with the plexiglas thickness shown in figure 14.

For the wavelengths emitted by bodies at moderate temperatures, such as the walls of a cabin or panels heated to 400° F, the transmittance of plexiglas is so low as to render it opaque to radiant energy. The conclusion to be drawn from this phenomenon is that any energy that gets into the plane from sunlight will not be reradiated outward by the man. At night, no matter how cold the objects which his body can "see" may be, he will not radiate to them through the plexiglas windows.

Solar irradiation as a function of altitude is known and can be used to compute the rate of heat flow through the resistance shown as 7. If G_o is the irradiation at the altitude under consideration (Btu/hr ft^2) and T_r the transmittance, A the area, the energy reaching the man is

$$q_t = T_r G_o A \cos \varphi \tag{7a}$$

where φ is the angle between the normal to the glass surface and the line connecting the surface and the sun.

No numerical value need be assigned to resistance 7, since the rate of head flow may be readily determined from equation (7a). Since the heat flow is essentially in one direction, it is not necessary to include it in an analysis of the amount of heat necessary to keep a man comfortable. If he is comfortable when no receiving sunlight, it will not be a difficult matter to make him comfortable when he is.

For numerical values of the irradiation at various altitudes, see section XIV.

VIII. RESISTANCE 8

RADIATION FROM THE MAN TO WALLS

The total radiation from a perfect or Planckian radiator is given by the Stefan-Boltzmann equation:

$$q = \sigma A T^4 \tag{8a}$$

where

q heat loss, Btu/hr

T absolute temperature, °R

A area, ft²

σ constant, Btu/hr ft² °R⁴ 0.173 × 10⁻⁸

The net energy interchange between two parallel bodies is then given by (reference 16)

$$q = 0.173\ A \left[\left(\frac{T_A}{100}\right)^4 - \left(\frac{T_B}{100}\right)^4\right] \tag{8b}$$

If the bodies are not perfect radiators, a correction factor (emissivity modulus) F_E must be inserted. If the bodies are so situated with respect to one another that all the radiation from one does not strike the other, a correction factor (shape modulus) F_A must be included as a multiplier. The equation then becomes:

$$q = 0.173\ A \left[\left(\frac{T_A}{100}\right)^4 - \left(\frac{T_B}{100}\right)^4\right] F_A\ F_E \tag{8c}$$

Hottel (reference 7) has tabulated values of F_A and F_E for various systems. The multiplier $F_A\ F_E$ is not always separable into the two components. Moreover, the moisture and CO_2 content of the air between surfaces A and B will change the net transfer, as water vapor and CO_2 enter into the radiation process in the longer wavelengths. (See references 7 and 17.)

For a body completely enclosed in another and with no reentrant angles

$$F_A = 1 \quad \text{and} \quad F_E = \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

When A_1 is very much smaller than A_2, F_E will be a function of e_1 only. If A_1 is nearly the same as A_2, the energy interchange will be a sensitive function of both e_1 and e_2. This is shown in figure 15.

where

e_1 emissivity of inner body

e_2 emissivity of outer body

A_1 area of inner body, ft^2

A_2 area of outer body, ft^2

The emissivity of most clothing will be about 0.95 and is usually not susceptible to change. The upper horizontal line in figure 15 represents the case of one man in a large cabin; the lower one many men in a small section. For an average case, where $A_1/A_2 = 0.1$, changing the emissivity of the walls from 0.9 (painted aluminum surface) to 0.055 (unpainted) (reference 8, p. 45) will change the heat loss by about 80 percent. For the case of many men in a section the lowering will be even greater.

The "equivalent unit conductance for radiation" (reference 16, ch. XVIII) is defined as follows:

$$q = f_r A(T_A - T_B) = 0.173 \times 10^{-8}(T_A^4 - T_B^4)F_A F_E$$

$$f_r = 0.173 \times 10^{-8}(T_A^2 + T_B^2)(T_A + T_B)F_A F_E \qquad (8d)$$

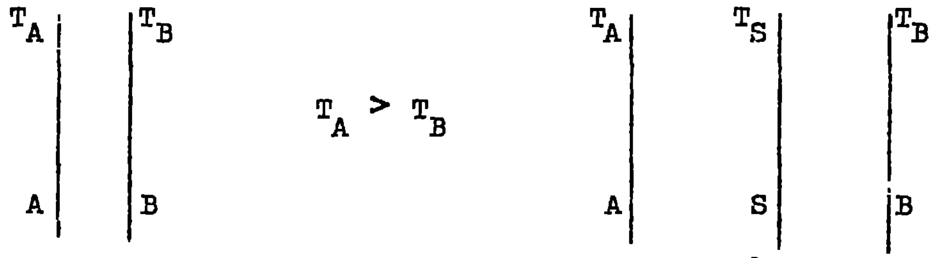
For a man radiating to the cabin walls, $T_A = T_4$ and $T_B = T_W$. In general, T_W will be known, but T_4 will not. As a first approximation it will suffice to assume T_4 is the same as the equilibrium temperature of an inanimate object in the same position. This problem is treated in an example in section XVII. The resistance for this method of heat transfer is therefore

$$R_s = \frac{1}{f_r A} \; \frac{{}^{\circ}F}{Btu/hr}$$

The Use of Radiation Shields

The radiant energy transfer can be decreased tremendously by the use of radiation shields. Even a radiation shield of infinite thermal conductivity (approximated by a very thin sheet of metal) and an emissivity of unity (blackened aluminum) will decrease the energy interchange by 50 percent. This fact is demonstrated as follows:

Consider two bodies at temperatures T_A and T_B. Assume that they are placed so that their shape modulus is unity and further assume that they are both black body radiators (fig. a). A radiation shield of unit emissivity and infinite thermal conductivity is interposed (fig. b). Neglecting the effects of convection to the surrounding air, the energy transferred from A to the shield will be



a.   b.

$$\frac{q}{A} = \sigma(T_A{}^4 - T_S{}^4) \quad \frac{Btu}{hr\ ft^2} \tag{8e}$$

That from the shield to B will be:

$$\frac{q}{A} = \sigma(T_S{}^4 - T_B{}^4)$$

Since there can be no storage of energy, the two rates must be equal.

$$T_A^4 - T_S^4 = T_S^4 - T_B^4$$

$$T_S^4 = \frac{T_A^4 + T_B^4}{2} \tag{8f}$$

The heat flow from A without the shield is:

$$\frac{q}{A} = \sigma(T_A^4 - T_B^4) \tag{8g}$$

The heat flow with the shield is:

$$\frac{q}{A} = \sigma\left[T_A^4 - \left(\frac{T_A^4 + T_B^4}{2}\right)\right] = \frac{1}{2}\ \sigma(T_A^4 - T_B^4) \tag{8h}$$

The shield, therefore, reduces the heat loss by 50 percent.

By similar reasoning it can be shown (reference 16, ch. XVIII) that the presence of n radiation shields of varying emissivities, e_1, e_2 ....., e_1', e_2', e_3'..... will reduce the radiant heat transfer by a factor

$$\frac{\dfrac{1}{e_A} + \dfrac{1}{e_B} - 1}{\dfrac{1}{e_A} + \dfrac{1}{e_B} - 1 + \dfrac{1}{e_1} + \dfrac{1}{e_2} + \dfrac{1}{e_3} + \dfrac{1}{e_4} + \ldots + \dfrac{1}{e_1'} + \dfrac{1}{e_2'} + \dfrac{1}{e_3'} + \ldots - n} \tag{8i}$$

where

e_1 emissivity of one side of shield 1

e_1' emissivity of other side of shield 1

e_A emissivity of surface A

e_B emissivity of surface B

n number of shields

IX. RESISTANCE 9

RESPIRATION

To complete the heat balance on the man, it is necessary to take into account the fact that he warms the air he breathes. This heat loss, unless the air is uncomfortably cold, will not affect his sensation of warmth because it will not affect his skin temperature. If the air is very cold, the man will experience difficulty in breathing. Long before this happens, however, other factors probably will have acted to make him uncomfortable.

It is not proposed that the energy lost through respiration be taken into account in determining whether a man will be comfortable. This resistance is shown only to complete the exact circuit as it exists. If it is desired to include it in making a total heat balance, the heat lost will be given by:

$$q = W\, C_p\, (\tau_e - \tau_i) \tag{9a}$$

where

q heat lost, Btu/hr

W weight rate of respiration, pounds of air/hr

C_p heat capacity of the air, Btu/lb $^\circ$F

τ_e temperature at which the air is exhaled, $^\circ$F

τ_i temperature at which the air is inhaled, $^\circ$F

No number need be assigned to this resistance, for it may be treated as a portion of the thermal circuit in which the current flow is known.

X. RESISTANCE 10

AIR LEAKAGE

If the amount of air that escapes from the cabin is W pounds per hour, the enthalpy that will be lost from the cabin is

$$q = W\, C_p\, (\tau_a - \tau_o) \text{ Btu/hr} \tag{10a}$$

where

C_p specific heat of air, Btu/lb °F

T_a temperature of the cabin air, °F

T_o temperature of the outside air, °F

If there are leaks in the cabin and the cabin is slightly pressurized, the leakage will be augmented by the decrease in external pressure as the airplane gains altitude. Such leaks also generate drafts which increase the heat losses through the sides of the airplane, although such drafts under certain environmental conditions may prove beneficial to the men by helping to warm them.

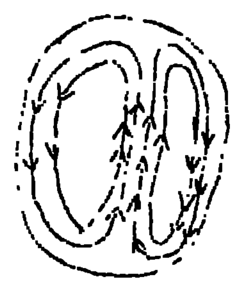
If resistance 10 is low, it is evident from figure 4 that it will function almost as a direct short circuit. In many airplanes the heat lost by air leakage is far greater than the heat transmitted through the cabin walls. Thus the amount of air leakage must be known before the necessary capacity of the cabin heater for an airplane can be established.

No number need be assigned to this resistance, for it may be treated as a portion of the thermal circuit in which the current flow is known.

XI. RESISTANCE 11

CONVECTIVE RESISTANCE AT THE INSIDE SURFACE OF THE CABIN WALL

Heat flows from the cabin air to the cabin walls by convection. If there are no drafts, the mode of heat transfer is by free convection. The warm air will flow down the cold cabin walls and up the center of the cabin. In addition, if there is a draft down the center of the cabin, heat transfer by forced convection will take place. It is difficult to combine the two modes of transfer, and, as an approximation, both unit conductances should be calculated and the larger of the two used.



Free Convection

It may be shown readily (references 18, 19, and 20) that the unit thermal conductance for free convection depends on the Grashof modulus and the Prandtl modulus. In general

$$Nu = F\ (Gr,\ Pr) \tag{11a}$$

Experimentally it is found that $Nu = c(Gr \times Pr)^n$, approximately. The exponent n is constant for a narrow range of the variables.

For vertical plates

$$Nu = \frac{f_c\ H}{k},\ \text{Nusselt modulus}$$

$$Gr = \frac{g\ \beta\ \Delta t\ H^3}{\nu^2},\ \text{Grashof modulus}$$

$$Pr = \frac{\mu\ C_p\ g\ (3600)}{k},\ \text{Prandtl modulus}$$

where

f_c unit thermal conductance for free convection, Btu/hr ft^2 $^{\circ}$F

k thermal conductivity of the gas in contact with the plate, Btu/hr ft^2 $\frac{^{\circ}F}{ft}$

c experimental constant (dimensionless)

n experimental exponent,* $1/4 < n < 1/3$

H height of the plate, ft

g 32.2 ft/sec^2

β coefficient of expansion of gas, 1/$^{\circ}$R

Δt difference in temperature between plate and ambient gas, $^{\circ}$F

---

*The 1/4 power corresponds to $10^5 < Gr\ Pr < 10^9$ for which the flow is viscous. The 1/3 power corresponds to $10^9 < Gr\ Pr < 10^{12}$ for which the flow is turbulent (reference 8, p. 248).

ν kinematic viscosity of gas, ft^2/sec

μ absolute viscosity of gas, lb sec/ft^2

C_p unit heat capacity of gas at constant pressure, Btu/lb °F

For air, evaluating the various physical properties, the above expression may be written approximately (references 5, p. 107, and 16, p. XII-4) as

$$f_c = 0.30\ \Delta t^{1/4}\left(\frac{\gamma}{\gamma_o}\right)^{1/2}, \quad \text{Btu/hr ft}^2\ {}^\circ\text{F} \tag{11b}$$

where

γ density of gas at the arithmetic mean of the plate temperature and the ambient gas temperature at the pressure of the ambient gas

γ_o density of the gas at 70° F and 1 atmosphere

For cases where the arithmetic average of the plate temperature and the ambient gas temperature is close to 70° F, the equation may be written as

$$f_c = 0.30\ \Delta t^{1/4}\left(\frac{P}{P_o}\right)^{1/2}, \quad \text{Btu/hr ft}^2\ {}^\circ\text{F} \tag{11c}$$

where

P ambient gas pressure at any altitude, lb/in^2

P_o atmospheric pressure at sea level, lb/in^2

Forced Convection

When a fluid flows along a flat plate of length l (measured in the direction of flow), the flow is termed "viscous" if Reynolds number $\frac{U_m\, l\, \gamma}{\mu\, g}$ is less than 50,000 and "turbulent" (reference 21, p. 366) if it is greater than 50,000. As a very close approximation to more exact analyses (references 22 to 25), Colburn (reference 26) has shown that the average unit conductance for a plate l feet long in the viscous region may be expressed as:

$$\frac{f_c}{3600\ C_p\ U_m\ \gamma}\ (Pr)^{2/3} = 0.66\left(\frac{U_m\ l\ \gamma}{\mu\ g}\right)^{-1/2} \tag{11d}$$

Substituting the properties of air at 50° F in the above expression yields:

$$f_c = 2.45\left(\frac{U_m\ \gamma}{l}\right)^{0.5} \tag{11e}$$

This expression may be used for values of $Re = \dfrac{U_m\ l\ \gamma}{\mu\ g}$ less than 50,000; that is, for air at 50° F the product $U_m < 16$ ft²/sec.

If the Reynolds number is greater than 50,000, Colburn (reference 26) gives

$$\frac{f_c}{3600\ C_p\ U_m\ \gamma}\ (Pr)^{2/3} = 0.036\left(\frac{U_m\ l\ \gamma}{\mu\ g}\right)^{-1/5} \tag{11f}$$

which checks closely the value presented by others (reference 27). For air at 50° F this reduces to

$$f_c = 4.25\ \left(\frac{U_m\ \gamma}{l^{0.25}}\right)^{0.8} \tag{11g}$$

This expression for the average unit conductance over a flat plate l feet long applies for values of

$$Re > 50{,}000,\quad \text{that is,}\quad U_m\ l > 16\ \ \text{ft}^2/\text{sec}$$

Recapitulation

1. If free convection controls

$$R_{11} = \frac{1}{0.30\ (\tau_a - t_w)^{1/4}\ \left(\dfrac{p}{p_o}\right)^{1/2} A} \tag{11h}$$

2. If $U_m l < 16$ ft^2/sec

$$R_{11} = \frac{1}{2.45 \left(\frac{U_m \gamma}{l}\right)^{0.5} A} \tag{11i}$$

3. If $U_m l > 16$ ft^2/sec

$$R_{11} = \frac{1}{4.25 \left(\frac{U_m \gamma}{l^{0.25}}\right)^{0.8} A} \tag{11j}$$

where

t_a cabin air temperature, °F

t_w cabin wall temperature, °F

P cabin air pressure, lb/in^2

P_o atmospheric pressure at sea level, lb/in^2

U air velocity in cabin due to drafts, ft/sec

l effective cabin length measured in direction of forced air flow, ft

A area of heat transfer perpendicular to direction of heat flow, ft^2

Plots of these equations are shown in figure 13, 17, and 18.

The cabin of an airplane is not a single flat plate; so the use of equations for a flat plate of length l is only an approximation. However, since l enters these equations only as the 1/2 and the 1/5 power, a reasonable estimate of f_c will result even though the exact magnitude of the length l may not be known. Since R_{11} is so large, it usually will be the major resistance determining the rate of heat transfer through the cabin walls. Efforts to decrease heat losses from the airplane may be aimed at increasing R_{11} (decreasing f_c).

XII. RESISTANCE 12

THE RESISTANCE OF THE WALL

The resistance of the walls of most military craft of present design can be shown to be negligible. The wall thickness in such craft is about 0.032 inch. The thermal conductivity of aluminum is about 117 Btu/hr ft^2 $\frac{^\circ F}{ft}$. The heat lost through such a wall will be:

$$q = \frac{k\,A}{x}\,(t_w - t_{w2})$$

where

q heat flow, Btu/hr

k thermal conductivity of the aluminum, $\frac{Btu}{hr\ ft^2\,(^\circ F/ft)}$

A area of heat transfer, perpendicular to heat flow, ft^2

x thickness of wall in direction of heat flow, ft

t_w temperature of the inside surface, $^\circ$F

t_{w2} temperature of the outside surface, $^\circ$F

The resistance is:

(For an aluminum wall, uninsulated)

$$R_{12} = \frac{x}{kA} = \frac{0.032}{12 \times 117A} = 2.28 \times 10^{-5}/A \quad \frac{^\circ F}{Btu/hr}$$

This resistance can be neglected in comparison to the others and the inside temperature of the wall assumed to be that of the outside wall surface. This will have many unfavorable effects beyond the immediate loss of heat through the walls. The surfaces which receive the radiant energy from the men inside the airplane will be colder, increasing the radiant loss. The temperature difference between the inside cabin walls and the cabin air will be greater, resulting in increased free convection and lowering resistance 11. Finally, the walls may be so cold that it will be impossible to touch them without injury. Equipment fastened to the walls may be rendered useless at high altitudes by the excessively low temperatures.

If the cabin walls are insulated, the resistance to heat transfer will be that of the insulation alone, since the metal has such a small resistance. If the thermal conductivity and the thickness of the insulation are known, the calculations are treated exactly as above. Any air spaces introduced by adding the insulation are treated as described under resistance 2.

As pointed out in the discussion of resistance 5, the resistance of the men's clothing, the presence of moisture in insulating materials may affect the heat transfer tremendously. The liquid or ice layer on the inside metal wall will increase the resistance of this portion of the circuit. The cabin insulation should be sealed wherever possible since, in normal use, there is a tendency for moisture to condense in the insulation and not be removed. The rate of moisture condensation is appreciable if the windshields fog rapidly during flight. Unlike the windshield, however, the walls are usually not defrosted nor is any attempt made to dry them. If it is desired to maintain them at their maximum efficiency, therefore, it is necessary to take special precautions to keep them dry.

The possibility of saturating the insulation with carbon dioxide gas should not be overlooked, for it is a much poorer thermal conductor than air.

XIII. RESISTANCE 13

RADIATION FROM OUTER SURFACE OF AIRPLANE TO SURROUNDINGS

In level flight the surfaces of the airplane on the under side see the earth, the upper surfaces see outer space, and the sides see both. But the atmosphere is not perfectly transparent to all wavelengths, for it contains water vapor, carbon dioxide, ozone, dust, and liquid water (clouds). These constituents absorb and radiate in wavelengths in the infrared region which correspond to reasonably large magnitudes of the monochromatic emissive power at the temperatures under consideration.

Several methods are available for the calculation of the irradiation of a surface by the constituents of the atmosphere. Reference to the pertinent articles will be made here. A simple method has been devised by Andersen (reference 16, pp. 53-59) which is based on the earlier interpretations by Elsasser of spectroscopic data on water vapor. The procedure of Andersen will account only for the effect of water vapor; dry bulb and wet bulb data are necessary as a function of altitude in order to accomplish a numerical solution. Elsasser (reference 28) has recently proposed another graphical technique which is based on more recent interpretations of spectroscopic data; also, a correction is made for the carbon dioxide content of the atmosphere.

Hottel and Egbert (reference 17, pp. 297-307) present the data for the emissivity of water vapor and carbon dioxide on a large range of temperatures down to as low as 0° F. In particular, the effect of path length, the effect of superposed water vapor, and carbon dioxide radiation and formulas to be employed are discussed. Brooks (reference 7) presents various data for the transmissivity of water vapor and he also includes an analysis of the data of other investigators.

A cloud bank may be considered as a radiation shield as a first approximation. If the temperature of the side of the cloud which sees the airplane is known, it may be conceived of as a gray body and it will replace the earth or "outer space" depending on the location of the cloud bank in respect to the airplane. The cooling which clouds experience is discussed by Elsasser (reference 29).

In the illustrative example which follows later in this section, the absorption and radiation of the water vapor and other constituents of the atmosphere are omitted. These corrections will be the subject of a later contribution. In the illustrative example, the fourth power radiation law, between surfaces was utilized with outer space presumed to be at absolute zero. The numerical work also was carried through on the presumption that the airplane lost or gained no energy by radiant exchange. These calculations are presented in section XVII.

XIV. RESISTANCE 14

SOLAR IRRADIATION

The solar irradiation on a horizontal plane in Btu/hr ft^2 may be obtained at the station level for certain localities from the reports of the Weather Bureau. Abbott (reference 30, p. 192) presents information on the solar constant, that is, the irradiation at the outer atmosphere. The solar energy absorbed by an opaque surface of an airplane flying in the upper atmosphere may be calculated from:

$$q_a = \alpha\, G_o\, A \cos \Phi$$

where

G_o the solar irradiation at the altitude of the airplane, Btu/hr ft^2

α absorptance of the surface

Φ angle between normal to the surface and line connecting the surface and the sun

A area of absorbing surface, ft^2

(In the absence of irradiation data, G_o may be taken as the irradiation at the outer atmosphere.)

If the sun strikes a transparent or a translucent surface, the energy transmitted is

$$q_t = T_r\, G_o\, A \cos \Phi$$

where T_r is transmittance.

The transmittance depends on the angle of incidence and is relatively constant for small angles with respect to the normal. Finally, the absorption of a transparent or translucent body is:

$$q_{ab} = (1 - \epsilon^{-a\, d})\, G_c\, A \cos \Phi$$

where

d thickness of the absorbing body, ft

a coefficient of absorption, ft^{-1}

ϵ base of Naperian logarithms

XV. RESISTANCE 15

FRICTIONAL HEATING

The drag of the air on the airplane structure will generate heat which must be dissipated to the air stream and/or inward through the skin to the interior of the cabin, depending on the sign of the temperature gradient. Of the several references on this subject, two will be mentioned which deal with the temperature of the skin of an airplane on the assumption that no heat flows to or from it (references 31 and 32).

For an airplane traveling at 550 feet per second in air, the increase in temperature will be approximately 15° F with the air temperature equal to -60° F. This increase will result only if the surface is insulated and after such time of flight that the steady state obtains. Experimental evidence of the increase in temperatures of wires in a cross and parallel stream is presented by Eckert and Weise (reference 33). Finally, an expression for the thermal conductance from a gas to the solid boundary including the effect of frictional heating is presented by Schirokow (reference 34).

XVI. RESISTANCE 16

CONVECTION FROM OUTER SURFACE OF AN AIRPLANE

In the absence of better data, the heat loss by convection from the outside surface of the airplane may be estimated by calculating the heat loss from a large flat plate with the same over-all length as the portion of the airplane under consideration. Investigations (references 35 and 36) of heat losses from airfoil shapes seem to indicate that the heat loss from a streamline body in which separation is not serious may be calculated with some accuracy on the basis of flat plate data.

For a short distance from the leading edge of an airfoil shape, "viscous" flow will exist, but for most of the airplane under flight conditions the Reynolds number $\frac{U_m \, l \, \gamma}{\mu \, g}$ will be greater than 50,000.

Thus equation (11c), presented in the discussion of resistance 11, will apply to the calculation of f_c from the fuselage, and wings of the airplane.

This equation is:

$$\frac{f_c}{3600\ C_p\ U_m\ \gamma}\ Pr^{2/3} = 0.036\left(\frac{U_m\ l\ \gamma}{\mu\ g}\right)^{-1/5} \tag{16a}$$

For air temperatures of 70° F and -60° F the equation reduces to:

$$f_c = 4.25\left(\frac{U_m\ \gamma}{l^{0.25}}\right)^{0.8}\ \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}} \quad \text{at } 70^\circ\ \text{F} \tag{16b}$$

$$f_c = 4.03\left(\frac{U_m\ \gamma}{l^{0.25}}\right)^{0.8} \quad \text{at } -60^\circ\ \text{F} \tag{16c}$$

respectively.

The above equations express the average unit conductance for l feet of flat plate, l being measured in the direction of air flow. If the unit conductance at any point (f_{cx}) along the plate is required, it may be obtained by the following method:

By definition
$$f_c = \frac{\int_0^x f_{cx}\, dx}{\int_0^x dx} \tag{16d}$$

or

$$f_c x = \int_0^x f_{cx}\, dx$$

but if $f_c = c\ x^n$, substituting in the equation above yields:

$$c\ x^{n+1} = \int_0^x f_{cx}\, dx$$

Differentiating

$$c(n + 1)x^n = f_{cx}$$

or

$$f_{cx} = f_c(n + 1) \tag{16e}$$

For the flat plate in the turbulent region $n = -0.20$

Thus $f_{cx} = 0.8\ f_c$ (16f)

The thermal conductance at any point along the flat plate is 0.80 times the average f_c up to that point. It should be noted that f_{cx} is extremely high at small values of x. A plot of f_c against $\left(\frac{U_m\ \gamma}{l^{0.85}}\right)$ is shown in figure 19. Inspection of the curve shows that under normal flight conditions f_c is quite large, thus making R_{15} quite small. However, since γ decreases with altitude, R_{15} will increase as altitude is increased.

$$R_{15} = \frac{1}{f_c\ A}\ \frac{{}^{\circ}F}{Btu/hr}$$

(See fig. 19 for f_c.)

XVII. TYPICAL CALCULATIONS

1. A Heat Balance on an Inanimate Object in the Cabin

If the object in consideration is insulated from the walls and the floor and has reached equilibrium, the heat lost to the walls by radiation will be equal to the heat carried in by the air.

$$f_c\ A(t_4 - T_a) = 0.173 \times 10^{-8}\ (T_4^{\ 4} - T_w^{\ 4})\ A\ F_A\ F_E$$

$$(T_4^{\ 4} - T_w^{\ 4}) = \left(\frac{f_c}{F_A\ F_E}\right)\left(\frac{1}{0.173 \times 10^{-8}}\right)\ (t_4 - T_a)$$

If there are no drafts in the cabin, the temperature of the object will drop until the free convection induced by the temperature difference $t_4 - \tau_a$ becomes great enough to balance the radiant loss. As shown in section VI, the free convection can be represented by

$$f_c = 0.30(t_4 - \tau_a)^{1/4} (P/P_o)^{1/2}$$

Simultaneous solutions of the above equations substituting the appropriate values for the air and wall temperatures will yield the minimum temperature which an object will attain. These equations have been solved and are plotted in figures 20, 21, 22, 23, and 24. The dotted lines represent the minimum temperatures for free convection only, at various pressures. The solid lines are lines of constant cabin air temperature along which the equilibrium temperatures are plotted as a function of the ratio $f_c/F_A\ F_E$.

If there is forced convection, a value for f_c must be calculated separately by means of the equations presented in section VI and divided by the appropriate value for $F_A\ F_E$ (see sec. VIII). This ratio $f_c/F_A\ F_E$ is used to enter the chart and the equilibrium temperature is read directly on the cabin air temperature curve.

Example:

A machine gun is mounted in a cabin.

The cabin wall temperature is -60°.

The cabin air temperature is 70°.

The emissivity of the walls and the gun is 0.95.

The cabin is pressurized at 0.8 atmosphere.

Using figure 20, enter on the line marked $\tau_a = 70°$ F until it intersects the line marked 0.8 atmosphere. Opposite this point read the temperature at equilibrium, 13° F.

Assume now that the air temperature is 50° F and that an electric fan is used which blows the air across the machine gun with a velocity of 10 feet per second (utilizing the unit conductance for a standing man as a first approximation).

$$f_c = 2.7(U_m\ \gamma)^{0.58} \quad \text{(sec. VI, fig. 12)}$$

$$\gamma = \text{air density, lb/ft}^3$$

$$U_m = \text{air velocity, ft/sec}$$

$$f_c = 2.7(0.061 \times 10)^{0.58}$$

$$f_c = 2.04, \text{ Btu/hr ft}^2\ ^\circ\text{F}$$

$$T_4 = 485^\circ\ \text{R} = 25^\circ\ \text{F}$$

By using an electric fan to produce flow across the gun, it is possible to maintain a higher object temperature with 50° F cabin air than with stationary 70° air.

50° air, 10 ft/sec	25° F
70° air, 0 ft/sec	13° F

2. Heat Balance on a Man in a Cabin

Thickness of clothing	1 inch
Air gap, skin to cloth	0.1 inch
Conductivity of clothing	0.022 Btu/hr ft² (°F/ft)
Area of man	20 ft²
Area of cabin walls	200 ft²
Emissivity of clothing	0.95
Emissivity of walls	0.90
Temperature of walls	-60° F
Temperature of cabin air	70° F
Radiation from plexiglas windows	None

It is desired to know:

(a) The total heat loss from the man exclusive of respiration

(b) The temperature of the man's skin in order to determine whether he will feel cold

(c) The effect of placing an electric fan so that a 4 ft/sec breeze is blown over him

(d) The effect of polishing the cabin walls until the emissivity is equal to 0.1 (rough plate aluminum)

(e) The result of the combination of (c) and (d) above

The man is standing so that all his body can radiate to the walls and also receive heat by convection from the air. The cabin is to be draft free and all convection is free convection only. The cabin is pressurized at 0.8 atmosphere. There is no solar radiation into the cabin.

1(a) Refer to figure 1:

<u>The resistance between t_1 and t_4</u>

This resistance, $R_{1-5} = R_1 + R_5 + \dfrac{1}{1/R_2 + 1/R_4}, \dfrac{{}^{\circ}F}{Btu/hr}$

$R_1 = 0.566/20$ (sec. I)

$R_2 = 1/1.8A = 0.555/20$ (sec. II)

$R_4 = 1/1.15A = 0.870/20$ (sec. IV)

$R_5 = 1/12 \times 20 \times 0.022 = 3.79/20$ (sec. V)
(See sec. III concerning R_3)

$\therefore R_{1-5} = 0.2354 \dfrac{{}^{\circ}F}{Btu/hr}$

<u>The resistance between t_4 and the cabin walls (t_w)</u>

$R_8 = 1/f_r A$ (sec. VIII) $f_r = 0.173 \times 10^{-8}(T_4^{\,2} + T_w^{\,2}) \times (T_4 + T_w)\ F_A\ F_E$

As a first approximation the temperature of the man's jacket surface t_4 can be assumed to be about that of an inanimate object in the same position. From figure 20 this temperature is about 15° F.

$$f_r = 0.173 \times 10^{-8} \, (4.75^2 + 4.0^2) \, (4.75 + 4.0) \, 0.95$$

$$= 0.553 \quad \text{Btu/hr ft}^2 \, {}^{\circ}\text{F} \quad \text{(fig. 16)}$$

$$R_8 = 0.0903 \; \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

This value of R_8 is a function of the temperature t_4. If the value of t_4 calculated differs by more than a few degrees from the value (15° F) chosen, a new value for R_8 must be calculated.

<u>The resistance between t_4 and the cabin air (τ_a)</u>

$$R_6 = 1/f_c A \text{ (sec. VI) } f_c = 0.3 \, (\tau_a - t_4)^{1/4} \, (P/P_o)^{1/2}$$

$$= 0.3 \times 55^{1/4} \times 0.8^{1/2}$$

$$= 0.728 \quad \text{Btu/hr ft}^2 \, {}^{\circ}\text{F}$$

$$R_6 = 0.0686 \; \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

Since there is no thermal current in R_7, the heat flow through R_5 plus that through R_6 must equal that through R_8

$$q_5 + q_6 = q_8$$

$$\frac{98.6 - t_4}{0.2354} + \frac{70 - t_4}{0.0686} = \frac{t_4 - (-60)}{0.0903}$$

$$t_4 = 25.7^{\circ} \text{ F}$$

Using this temperature to recalculate resistances 6 and 8 yields:

$$R_6 = 0.0654 \qquad R_8 = 0.0872$$

$$\underline{t_4 = 25.5^{\circ} \text{ F}}$$

The temperature drop from the body to the outside of the clothes is 98.6 - 25.5 = 72.1° F.

1(a) The temperature drop through the skin is 72.5 $\times \frac{0.0283}{0.235} = 8.9$; therefore the temperature of the skin is 89.5° F.

1(b) The total heat loss is $\frac{72.1}{0.235} = 308$ Btu/hr, excluding respiration. This figure compares favorably with the data given in the Heating, Ventilating and Air Conditioning Guide (1942), p.48.

1(c) Forced convection, 4 ft/sec wind from an electric fan.

$$f_c = 2.70\ (U_m\ \gamma)^{0.58} \quad \text{(sec. VI)}$$

$$= 2.70 \times (4.0 \times 0.076 \times 0.8)^{0.58}$$

$$f_c = 1.22\ \text{Btu/hr ft}^2\ {}^\circ\text{F}$$

$$R_s = 0.041\ \frac{{}^\circ\text{F}}{\text{Btu/hr}}$$

From the chart (fig. 20) the jacket temperature, $T_4 = 487°$ R (27° F)

$$f_r = 0.173 \times 10^{-8}\ (4.87^8 + 4.0^8)\ (4.87 + 4)\ 0.95$$

$$= 0.577\ \text{Btu/hr ft}^2\ {}^\circ\text{F (fig. 16)}$$

$$R_s = 0.0865\ \frac{{}^\circ\text{F}}{\text{Btu/hr}}$$

$$\frac{98 - t_4}{0.235} + \frac{70 - t_4}{0.041} = \frac{t_4 - (-60)}{0.0865}$$

$$t_4 = 57.5°\ \text{F}$$

Using this new temperature, R_s is recalculated:

$$f_r = 0.173 \times 10^{-8}\ (5.175^8 + 4.0^8)\ (5.175 + 4.0)\ 0.95$$

$$= 0.646\ \text{Btu/hr ft}^2\ {}^\circ\text{F (fig. 16)}$$

$$R_s = 0.0774 \ \frac{^{\circ}F}{Btu/hr}$$

$$t_4 = 32.4^{\circ}\ F$$

Recalculating R_s again:

$$f_r = 0.173 \times 10^{-2}\ (4.92^2 + 4.0^2)\ (4.92 + 4.0)\ 0.95$$

$$= 0.588\ \ Btu/hr\ ft^2\ ^{\circ}F$$

$$R_s = 0.0850\ \ \frac{^{\circ}F}{Btu/hr}$$

$$t_4 = \underline{35^{\circ}\ F}$$

Substituting t_4 into the above equations yields the same temperature; hence it is shown that the use of a fan will raise the outer temperature of the clothes to 35° F, a rise of 10° F.

The skin temperature will now be 91.2° F.

1(d) If the walls of the cabin are polished until their emissivity is equal to 0.1, the product $F_A\ F_E$ will be lowered from 0.95 to 0.55. The calculations are initiated for the conditions calculated in 1(a) of this example.

$$R_{1-4} = 0.235,\ \ R_6 = 0.0686\ \ R_s = 0.0872 \times \frac{0.95}{0.55} = 0.151\ \ \frac{^{\circ}F}{Btu/hr}$$

$$\frac{98 - t_4}{0.235} + \frac{70 - t_4}{0.0686} = \frac{t_4 - (-60)}{0.151}$$

Using this temperature, 40.7° F, R_6 and R_s are recalculated:

$$f_c = 0.30\ \ [(70 - 40.7)^{1/4}\ (P/P_o)^{1/2}]$$

$$= 0.30 \times 29.3^{1/4} \times 0.8^{1/2}$$

$$= 0.624\ \ Btu/hr\ ft^2\ ^{\circ}F\ \ \text{(fig. 13)}$$

$$R_6 = 0.0802$$

$$f_r = 0.173 \times 10^{-8} (5.0^2 + 4.0^2)(5.0 + 4.0)\ 0.55$$

$$= 0.352 \frac{\text{Btu}}{\text{hr ft}^2\ {}^{\circ}\text{F}} \quad \text{(fig. 16)}$$

$$R_s = 0.142$$

$$t_4 = 36.6^{\circ}\ \text{F}$$

Using this temperature in the recalculation:

$R_6 = 0.0835$, $R_s = 0.144$, and $\underline{t_4 = 36.2^{\circ}\ \text{F}}$

1(e) If the walls are polished and the fan is used:

$R_{1-4} = 0.235 \quad R_6 = 0.041$

Assuming that the outer temperature of the clothes is 40° F

$f_r = 0.352$ (See calculations above.)

$$R_s = 0.142$$

$$t_4 = 46.5^{\circ}\ \text{F}$$

Using this temperature to calculate the outer temperature of the clothes again:

$$R_s = 0.144$$

$$\underline{t_4 = 47.5^{\circ}\ \text{F}}$$

Summarizing:

Condition	Outer temperature of clothes (°F)	Skin temperature (°F)
No draft, painted walls	25.5	89.8
4 ft/sec draft, painted walls	35	91.2
No draft, polished walls	36.6	91.2
4 ft/sec draft, polished walls	47.5	92.6

3. Heat Balance on an Airplane*

The evaluation of the heat balance on an airplane flying in air at night at various altitudes will be undertaken. The airspeed is assumed to be 250 miles per hour. The cabin walls are assumed to be uninsulated 0.032 inch thick aluminum.

Cabin length:	50 ft	Cabin air temperature, 70° F (at all altitudes)
Cabin height:	6 ft	Cabin air pressure, 0.8 atm
Cabin width:	8 ft	Earth temperature, 10° F

To show the effect of drafts, the cases in which the motion of the cabin air is such as to cause a 5 ft/sec and 10 ft/sec draft along the cabin wall will be calculated, and the case of quiescent air will be considered.

Resistance 10 (sec. X)

No reasonable figures have been advanced for use in calculating this resistance. Measurements of leakage rates in aircraft are necessary before any numerical magnitudes can be assigned. The heat loss from the airplane will be calculated, neglecting the loss due to leakage. The resulting magnitudes for heat loss therefore will be much smaller than the heat which must be supplied by the cabin heater.

Resistance 11 (sec. XI)

(a) Quiescent Air.

(1) Airplane flying at 12,000 ft

As a first approximation it can be assumed that the temperature of the inside wall of the cabin is at the same temperature as the outside air. This will be only approximately true. If in the final stages of the calculation it becomes evident that there is a serious discrepancy between this assumed temperature and the one calculated, it will be necessary to repeat the process using the new temperature. One recalculation will usually suffice.

---

*References to resistances are to fig. 4.

At 12,000 feet the mean air temperature is 16° F (reference 37). From section XI,

$$f_c = 0.30 \quad [(70 - 16)^{1/4} \quad (0.8)^{1/2}]$$

$$= 0.727 \quad \text{Btu/hr ft}^2 \ {}^{\circ}\text{F} \quad \text{(fig. 13)}$$

The surface area of the airplane is about 2500 ft² (excluding wings).

$$R_{11} = 1/(f_c A) = 5.5 \times 10^{-4} \ \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

(2) Airplane flying at 24,000 ft

The mean air temperature will be -26° F

$$f_c = 0.30 \quad [(70 + 26)^{1/4} \quad (0.8)^{1/2}]$$

$$= 0.840 \quad \text{Btu/hr ft}^2 \ {}^{\circ}\text{F} \ \text{(fig. 13)}$$

$$R_{11} = 4.76 \times 10^{-4} \ \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

(3) At 36,000 ft, the mean air temperature is -67° F

$$f_c = 0.914$$

$$R_{11} = 4.37 \times 10^{-4} \ \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

(b) Draft of 5 ft/sec

Assuming that the cabin walls are sectionized every 2.37 ft and using the equation presented in section XI, since $U_m \, l < 16 \ \text{ft}^2/\text{sec}$

$$f_c = 2.45 \quad [(5 \times 0.076 \times 0.8)/2.37]^{0.5}$$

$$= 0.876$$

$$R_{11} = 4.52 \times 10^{-4} \ \frac{{}^{\circ}\text{F}}{\text{Btu/hr}}$$

At 36,000 ft the 5 ft/sec draft will have little effect since the free convection forces are so great. At this altitude the resistance calculated from a consideration of the free convection alone should be used since it is the smaller of the two calculated.

(c) Draft of 10 ft/sec

$$f_c = 1.25 \quad \text{Btu/hr ft}^2\ {}^\circ\text{F}$$

$$R_{11} = 3.2 \times 10^{-4} \quad \frac{{}^\circ\text{F}}{\text{Btu/hr}}$$

Resistance 12 (See sec. XII.)

$$R_{12} = 0.228 \times 10^{-4}/2500 = 0.912 \times 10^{-8} \quad \frac{{}^\circ\text{F}}{\text{Btu/hr}}$$

Resistance 13

Consider the worst possible case for this resistance. The temperature of the surface of the ship can be shown to differ slightly from that of the ambient air. Choose the earth emissivity as unity and assume that intervening water vapor and carbon dioxide have a negligible effect on the radiant energy interchange. This is not true, but their effect is such as to decrease the radiation loss. The sky temperature is postulated to be zero degree Rankine.

For level flight the top half of the airplane radiates to the sky while the bottom half radiates to or receives radiation energy from the earth. For this case resistance 13 is a combination resistance composed of two resistances in parallel.

The top half radiates outward to absolute zero;

$$f_r = 0.173 \times 10^{-8} \, (T_{w2}^3)$$

$$R_{13_T} = 1/(0.173 \times 10^{-8} \times T_{w2}^3 \times 1250)$$

The bottom half radiates downward;

$$f_r = 0.173 \times 10^{-8} \, (470^2 + T_{w2}^2) \, (470 + T_{w2}) \, F_A \, F_E$$

$$F_A \, F_E = 1$$

$$f_r = 0.173 \times 10^{-8} \, (T_{w2}^2 + 470^2) \, (T_{w2} + 470)$$

Altitude (ft)	Temperature (°F)	f_r (bottom half)	R_{13_b}	
12,000	16	0.729	1.10×10^{-3}	$\frac{°F}{Btu/hr}$
24,000	-26	.638	1.25	
36,000	-67	.557	1.44	
		$T^3_{w_s}$	R_{13_t}	
12,000	16	106×10^6	4.38×10^{-3}	$\frac{°F}{Btu/hr}$
24,000	-26	80.9	5.75	
36,000	-67	60.1	7.74	

Resistance 14

Since there is no solar irradiation at night, there will be no heat flow through this resistance. This resistance is treated as infinite in this calculation.

Resistance 15

At a flying speed of less than 300 miles per hour the frictional heating of the surface of the airplane is negligible. The resistance is also treated as infinite.

Resistance 16

The equivalent conductance on the outside of the ship is calculated from the equation presented in section XVI, equation (16c), or figure 19.

$$f_c = 4.03 \left(\frac{U_m \gamma}{l^{0.25}}\right)^{0.8}$$

$$U_m = \frac{250 \times 5280}{3600} = 367 \text{ ft/sec}$$

$$l^{1/4} = 50^{1/4} = 2.66 \quad \text{ft}^{1/4}$$

$$\gamma = 0.053 \text{ lb/ft}^3 \text{ at } 12{,}000 \text{ ft} \quad \text{(reference 37)}$$

$$\gamma = 0.0355 \text{ lb/ft}^3 \text{ at } 24{,}000$$

$$\gamma = 0.02265 \text{ lb/ft}^3 \text{ at } 36{,}000$$

Altitude (ft)	$\frac{U_m \gamma}{l^{0.25}}$	f_c	R_{16}	
12,000	7.30	19.75	2.06×10^{-5}	$\frac{^\circ F}{Btu/hr}$
24,000	4.90	14.4	2.78	
36,000	3.12	10.1	3.98	

Having evaluated all the resistances involved at the various altitudes, the validity of the assumption that the outside surface temperature is essentially the same as the ambient air temperature can be checked.

The thermal current flow to the point t_{w2} (fig. 4) through resistances 12 and 16 must exactly balance the losses through resistances R_{13t} and R_{13b} (shown as one resistance, resistance 13, on the diagram; subscripts t and b refer to top and bottom, respectively).

$$\frac{70 - t_{w2}}{R_{11} + R_{12}} + \frac{T_0 - t_{w2}}{R_{16}} = \frac{t_{w2} - t_{sky}}{R_{13t}} + \frac{t_{w2} - t_{earth}}{R_{13b}}$$

NO LEAKAGE

(1) Quiescent Air

Altitude (ft)	Air temperature (°F)	$R_{11} + R_{12}$	R_{16}	R_{13t}	R_{13b}	t_{ws} (°F)
12,000	16	0.00055	0.0000206	0.00438	0.0011	15.65
24,000	-26	.000476	.0000278	.00575	.00125	-22.3
36,000	-67	.000438	.0000398	.00774	.00144	-55.6

(2) 5 ft/sec Air Velocity

12,000	16	.000452	.0000206	.00438	.0011	-16.1
24,000	-26	.000452	.0000278	.00575	.00125	-21.8
36,000	-67	.000438	.0000398	.00774	.00144	-55.6

(3) 10 ft/sec Air Velocity

12,000	16	.00032	.0000206	.00438	.0011	17
24,000	-26	.00032	.0000278	.00575	.00125	-19.65
36,000	-67	.00032	.0000398	.00774	.00144	-52.2

Since the calculated wall temperatures were not the same as the ambient air temperatures, but differed from them considerably in some cases, it is necessary to use these wall temperatures in recalculating the various resistances. Resistances 16 and 12 will not change since they are independent of the assumed wall temperature.

A tabulation of the recalculated resistances follows:

Altitude (ft)	Air temperature (°F)	$R_{11} + R_{12}$	R_{16}	R_{13t}	R_{13b}	t_{wa} (°F)
12,000	16	As before				15.7
24,000	-26	0.000516	0.0000278	0.00551	0.00124	-22.5
36,000	-67	.000448	.0000398	.00679	.00138	-56
		Draft to produce a 5 ft/sec wind				
12,000	16	As before				16.1
24,000	-26	.000452	.0000278	.00549	.00124	-21.8
36,000	-67	.000438	.0000398	.00697	.00138	-55.9
		Draft to produce a 10 ft/sec wind				
12,000	16	As before				17.0
24,000	-26	.00032	.0000278	.00546	.00124	-19.75
36,000	-67	.00032	.0000398	.00581	.00136	-52.6

Since these new temperatures agree very closely with the previously calculated ones, it is not necessary to recalculate them.

If the net radiant energy exchange for the surroundings is set equal to zero; that is, assume R_{13} to be infinite, the resultant calculated wall temperatures will be not very different from those calculated previously, as shown on page 53.

Altitude (ft)	Cabin air velocity (ft/sec)	Calculated wall temperature: Neglecting radiation (°F)	Calculated wall temperature: Including radiation (°F)	Error introduced in heat loss through walls (percent)
12,000	0	18.0	15.7	4.2
12,000	5	18.0	16.1	3.8
12,000	10	19.5	17.0	4.7
24,000	0	-20.9	-22.5	2.6
24,000	5	-20.4	-21.8	1.5
24,000	10	-18.4	-19.75	1.5
36,000	0	-56.0	-53.0	3.3
36,000	5	-55.9	-55.5	.3
36,000	10	-52.6	-51.7	.7

The errors shown should be even less since the radiation effects calculated were the maximum that could occur under the given circumstances. The radiant energy from the bottom of the airplane must pass through a layer of water vapor, carbon dioxide, ozone, and so forth, which acts, to a certain degree, as a radiation shield. Further, the top half does not radiate to absolute zero, but to an effective temperature somewhat higher. These two effects will serve to bring even closer together the two sets of calculated temperatures.

The total heat load on the airplane under the nine cases discussed above is the sum of the leakage losses and the losses through the sides of the ship. This total will represent the heat requirement on any cabin heater used to maintain the air temperature at 70° F at these three altitudes. Below are tabulated the heat losses through the cabin walls for the airplane discussed in the example. Data on leakage rates must be available before the leakage loss can be calculated. Preliminary estimates show this heat loss may be as large and in some airplanes larger than the loss through the cabin walls.

LOSSES THROUGH THE CABIN WALLS

Altitude (ft)	Cabin air velocity (ft/sec)	$R_{11} + R_{12}$ $\left(\frac{^{\circ}F}{Btu/hr}\right)$	$(T_a - t_{ws})$ ($^{\circ}F$)	$q = (T_a - t_{ws})/(R_{11} + R_{12})$ (Btu/hr)
12,000	0	0.000550	54.3	98,800
12,000	5	.000452	53.9	119,100
12,000	10	.000320	53.0	165,500
24,000	0	.000516	92.5	179,200
24,000	5	.000452	91.8	203,500
24,000	10	.000320	89.75	280,000
36,000	0	.000448	126.0	281,000
36,000	5	.000438	125.9	287,000
36,000	10	.000320	122.6	383,000

It should be noted that the cabin in the above example was considered pressurized at 0.8 atmosphere. If the cabin pressure had been taken equal to that of the outside air, the heat losses through the cabin walls would have been lower than those shown in the table. This lower heat loss follows from the decrease of the f_c at the inside surface of the cabin wall due to the lower cabin pressure.

It may be possible that a combination of this phenomenon, and the lowered leakage losses at low atmospheric pressures in some cases may result in a decreased total heat load at high altitudes in unpressurized cabins.

CONCLUSION

The foregoing discussion, presentation of conductances, and numerical illustrations are not to be construed as other than a "first contribution" which must be constantly augmented and improved. Many readers will no doubt have available more adequate data relative to some of the resistances and the requirements. The authors trust that test flights will yield confirmation, rejection, and augmentation of the various statements. In time a revision and an amplification of this material will be undertaken.

Recommended experiments include:

1. Test data in flight on temperatures throughout the aircraft, rates of heat transfer through the fuselage and air leakage rates.

2. Measurements of the emissivity, transmissivity, and conductivity of the various materials used in aircraft.

3. Confirmation of the values of the various resistances presented in this discussion by measurements of the temperatures of clothing, walls, air, and so forth, in airplanes during flight.

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THERMAL CIRCUIT FOR A CLOTHED MAN

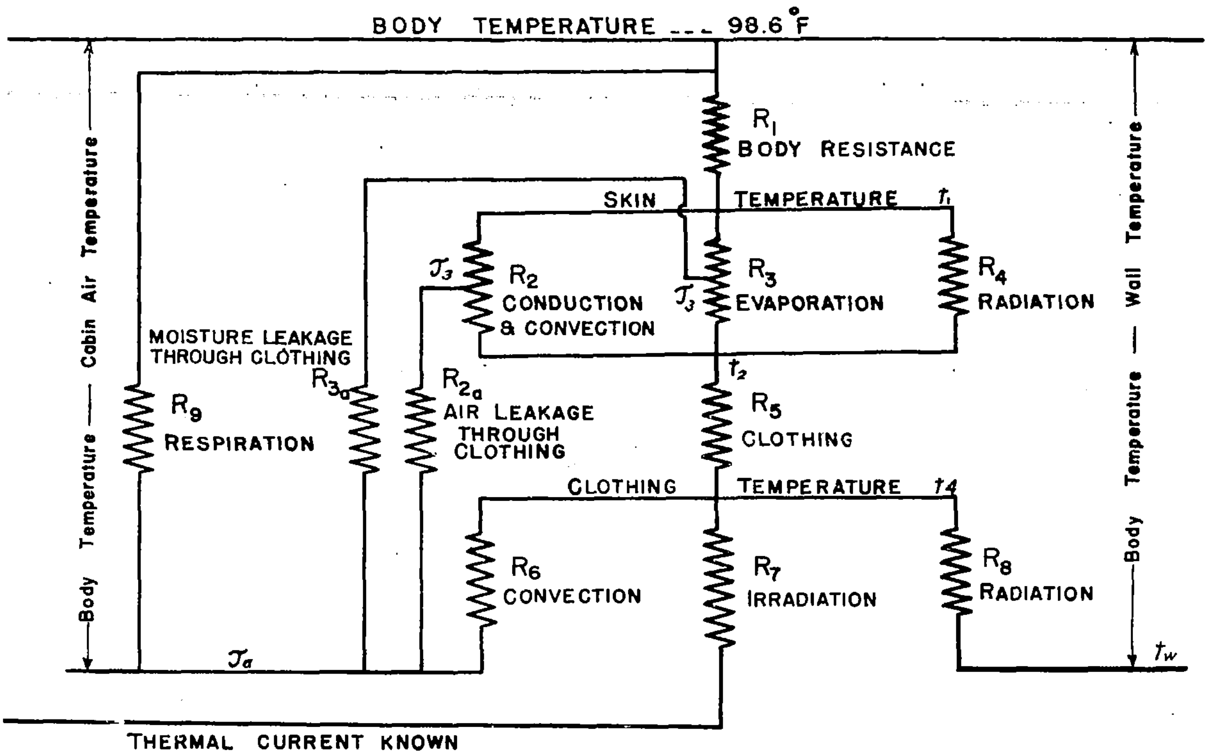


fig. 1

THERMAL CIRCUIT FOR UNCLOTHED PORTIONS OF A MAN

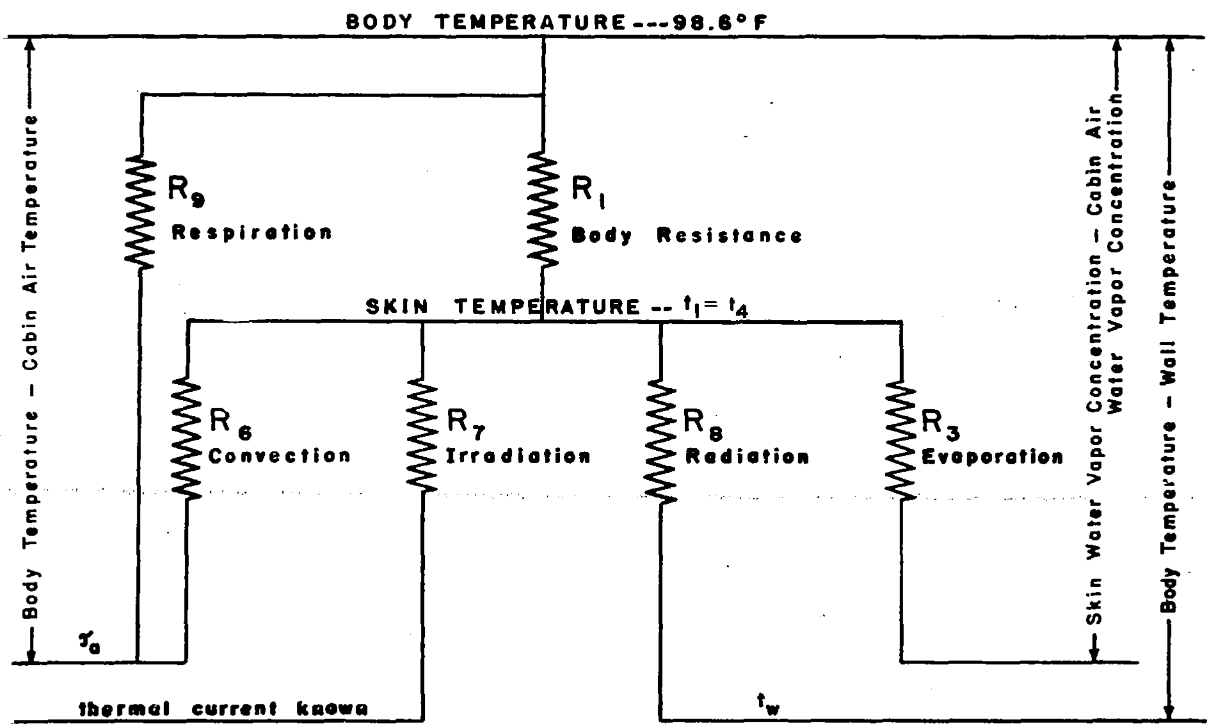


fig. 2

THERMAL CIRCUIT FOR AN INANIMATE OBJECT

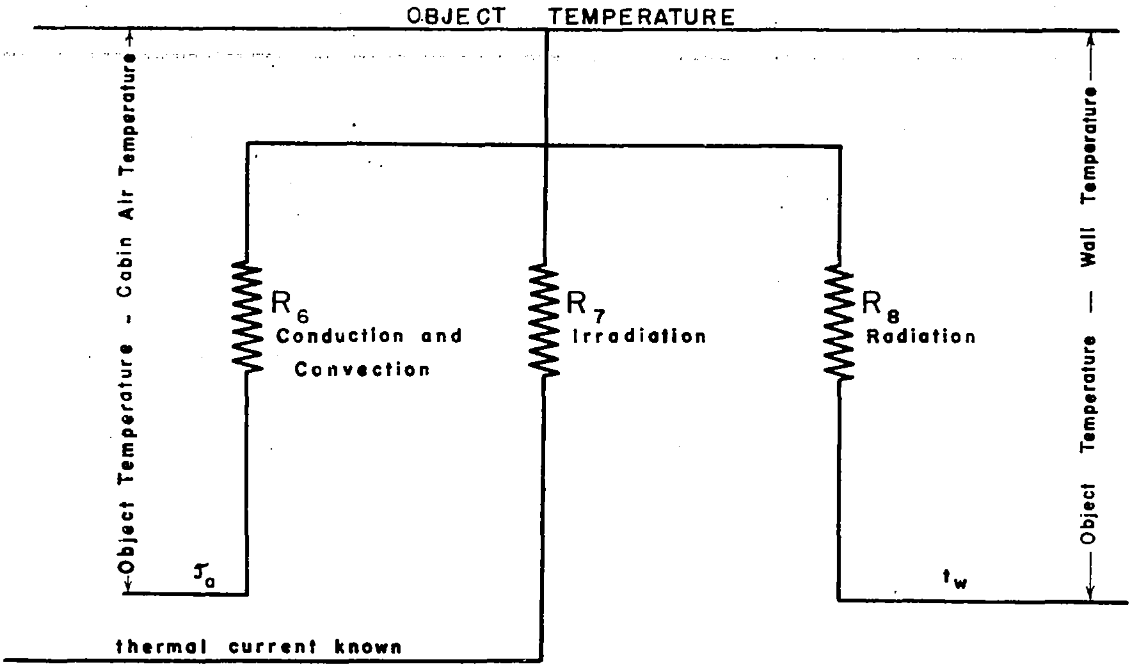


fig. 3

THERMAL CIRCUIT FOR AN AIRPLANE CABIN

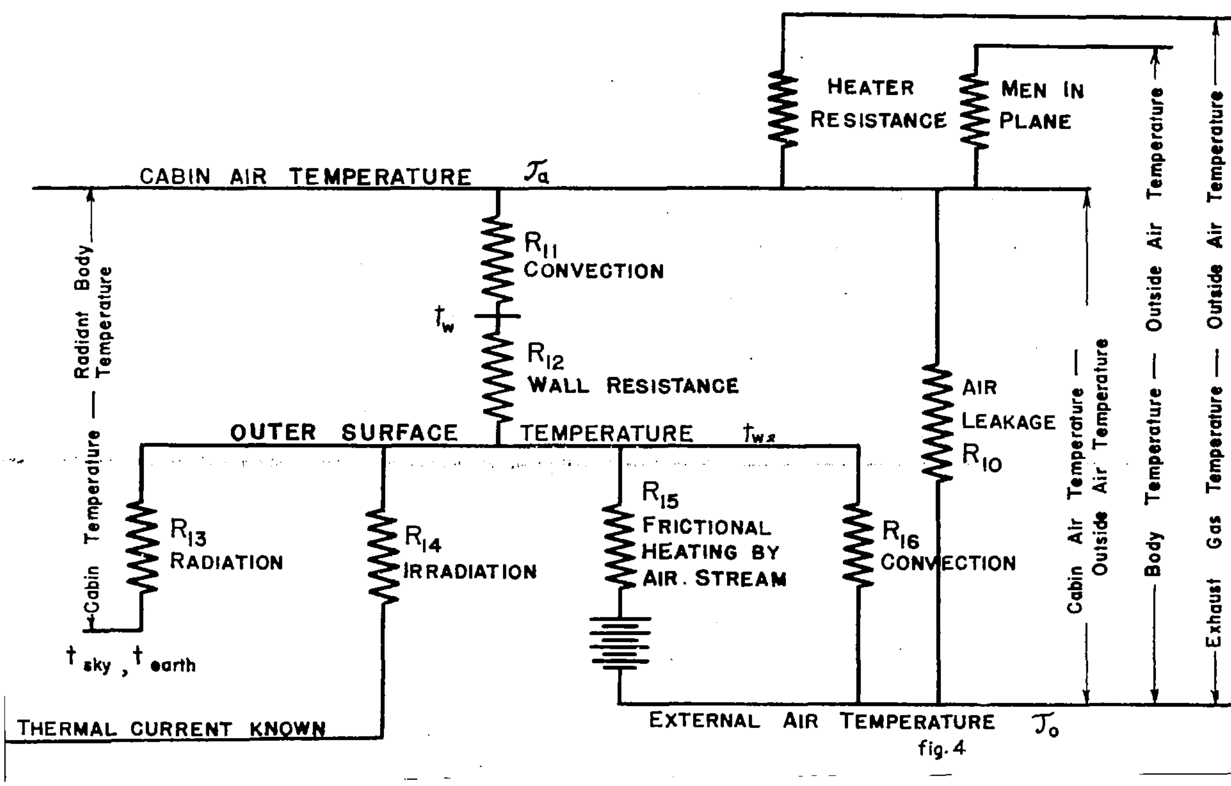


fig. 4

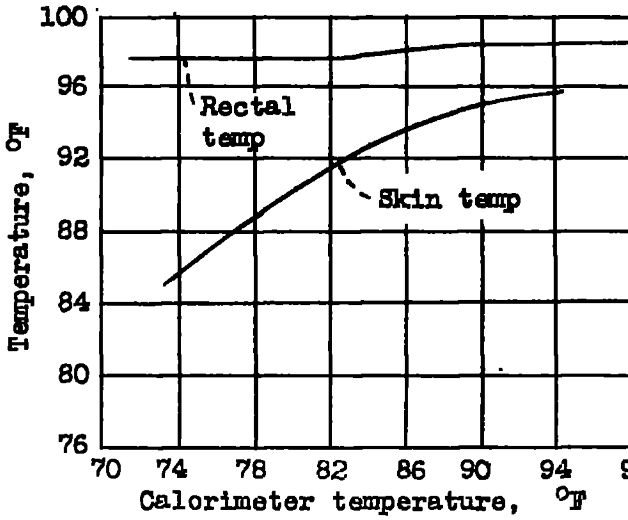


Figure 5.- Skin and rectal temperatures of average subjects.

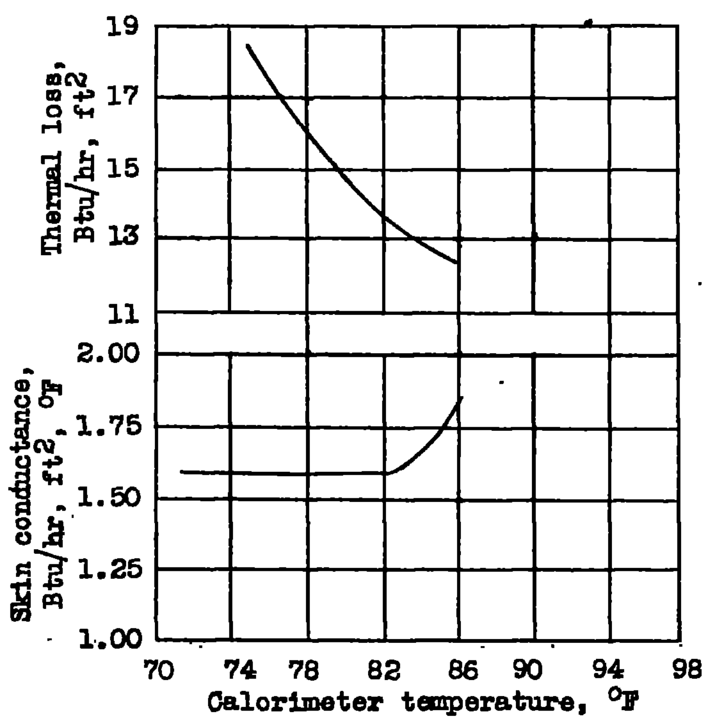


Figure 6.- Conductance of skin.

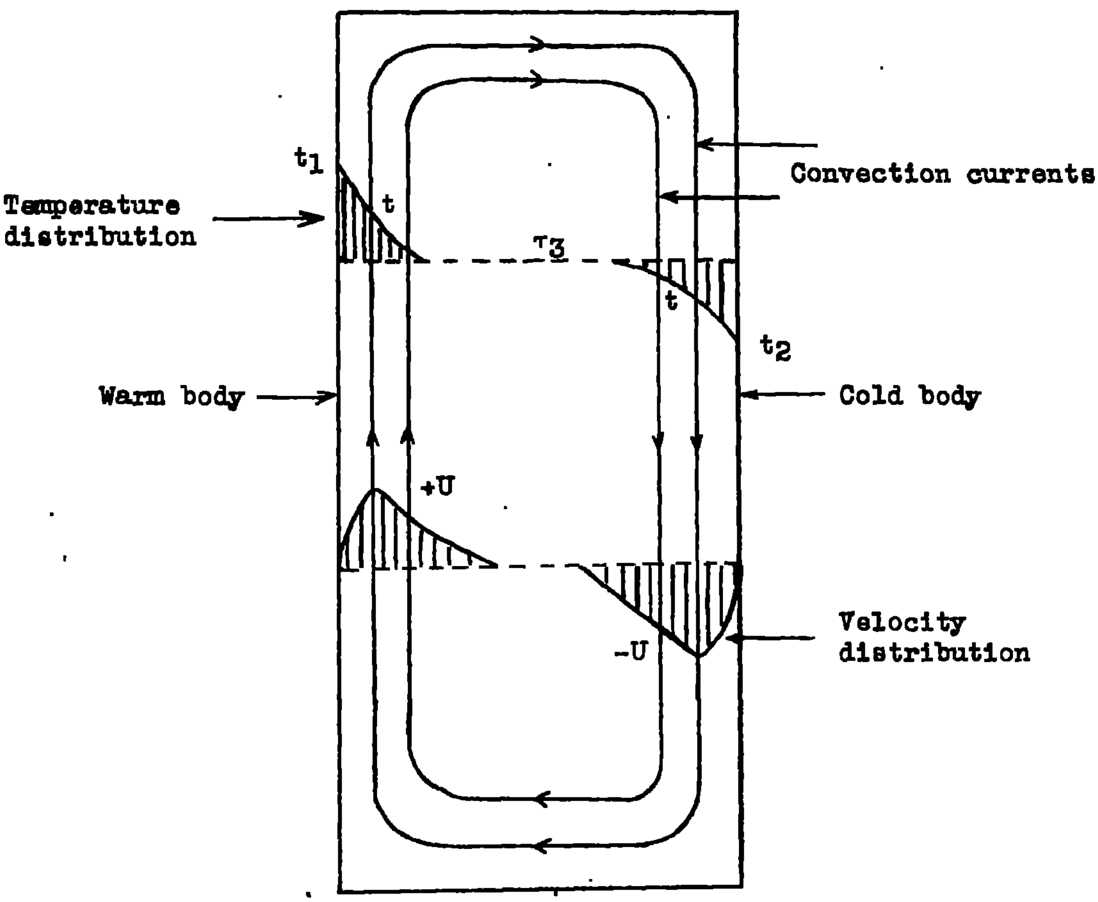


Figure 7.- Convective currents in air gaps.

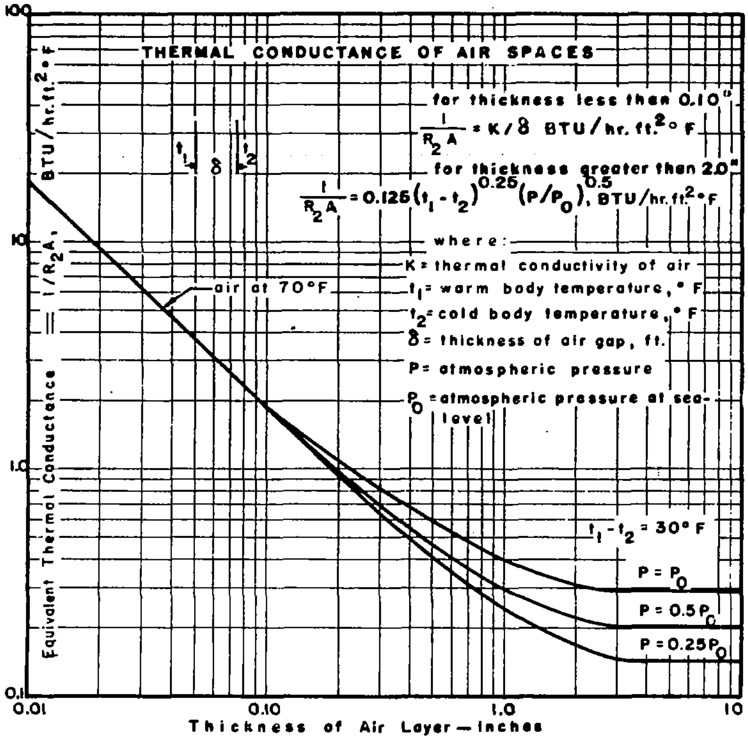

THERMAL CONDUCTANCE OF AIR SPACES
for thickness less than 0.10"
$\frac{1}{R_2 A} = K/\delta$ BTU/hr. ft.^{2} °F
for thickness greater than 2.0"
$\frac{1}{R_2 A} = 0.125(t_1 - t_2)^{0.25}(P/P_0)^{0.5}$, BTU/hr. ft.^{2} °F
where:
K = thermal conductivity of air
t_1 = warm body temperature, °F
t_2 = cold body temperature, °F
δ = thickness of air gap, ft.
P = atmospheric pressure
P_0 = atmospheric pressure at sea-level
t_1 δ t_2
air at 70°F
$t_1 - t_2 = 30$°F
$P = P_0$
$P = 0.5P_0$
$P = 0.25P_0$
Equivalent Thermal Conductance = $1/R_2A$, BTU/hr. ft.^{2} °F
Thickness of Air Layer — inches
100
10
1.0
0.1
0.01
0.10
1.0
10

fig. 8

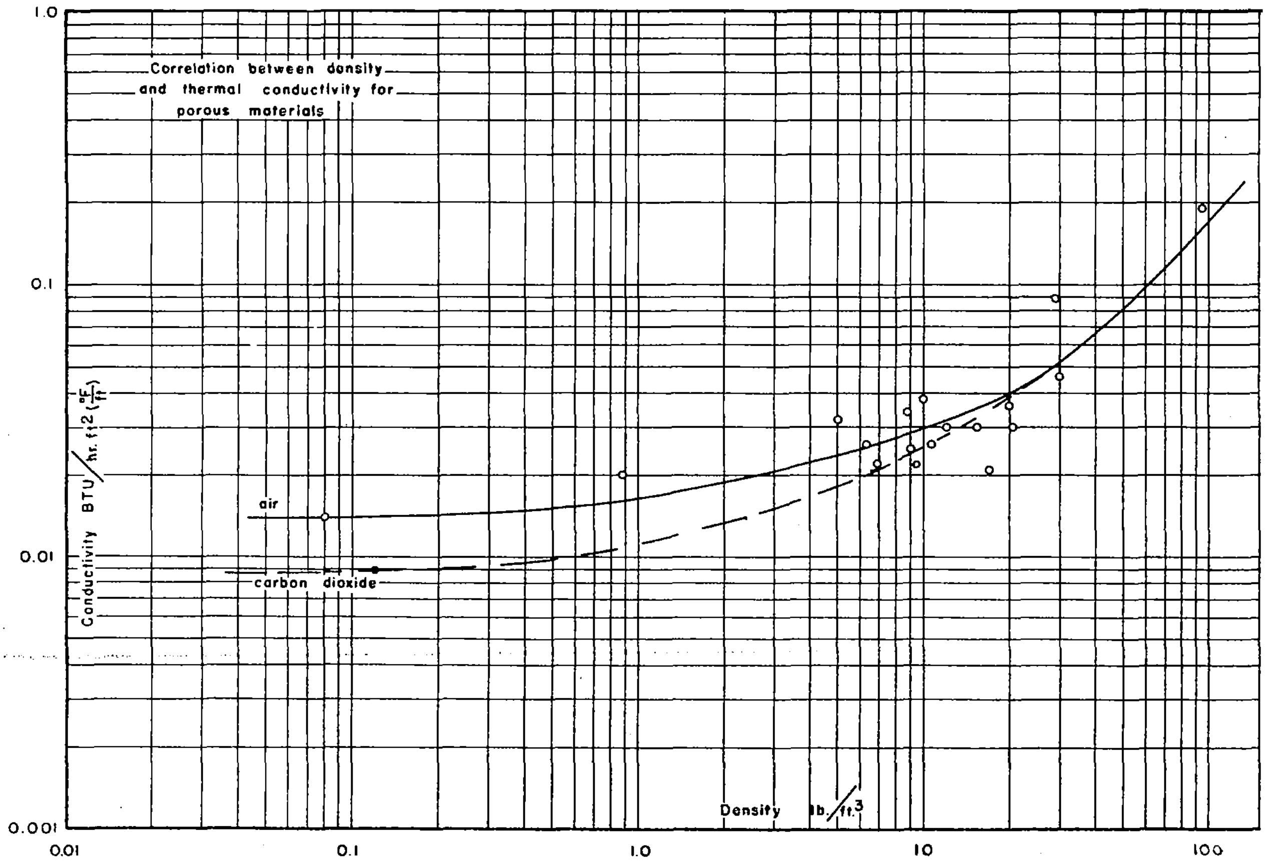

Correlation between density and thermal conductivity for porous materials
air
carbon dioxide
Conductivity BTU/hr. ft^2 (°F/ft)
Density lb./ft.^{3}
1.0
0.1
0.01
0.001
0.01
0.1
1.0
10
100

fig. 10

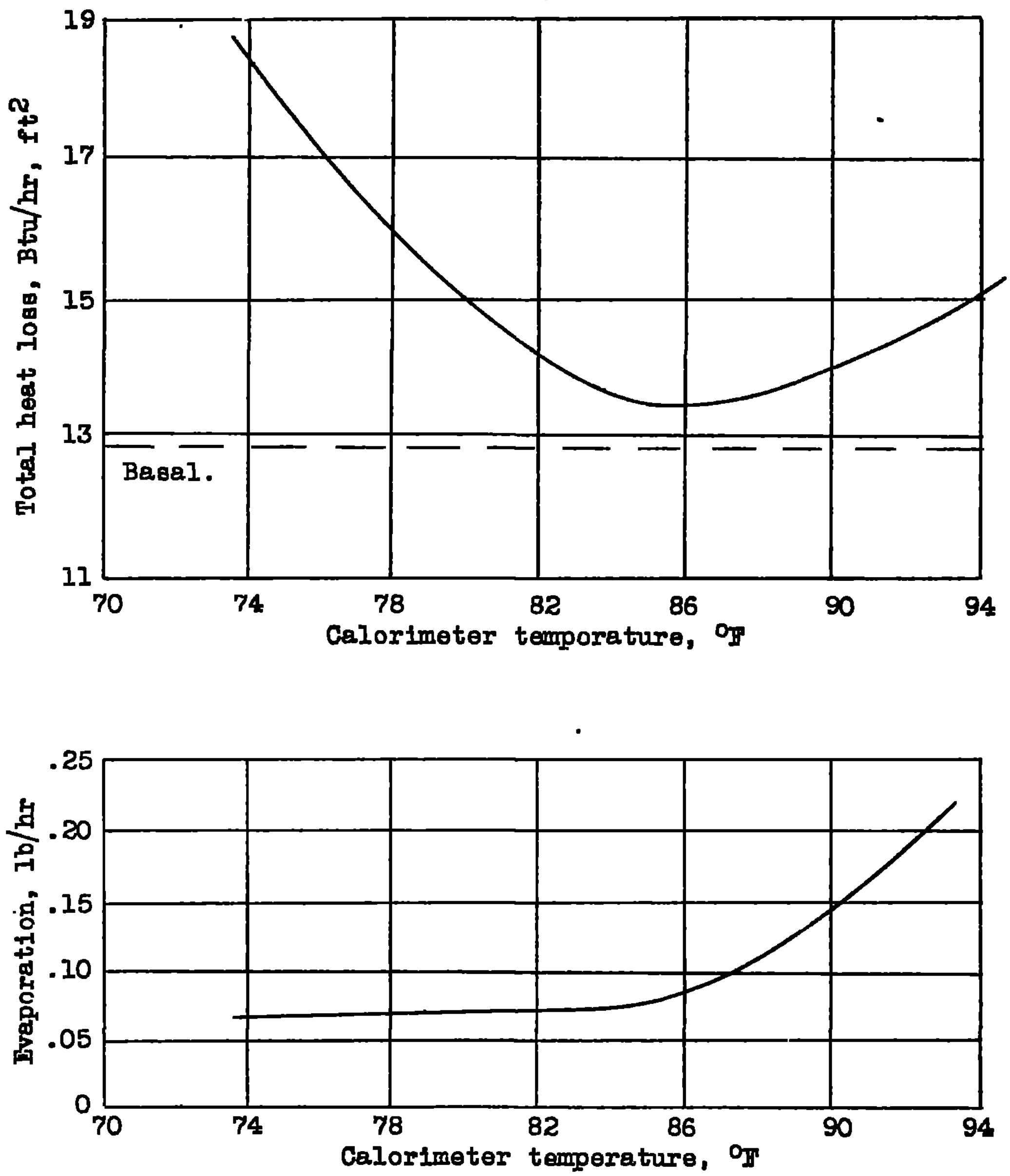


Figure 9.- Evaporation losses from body.

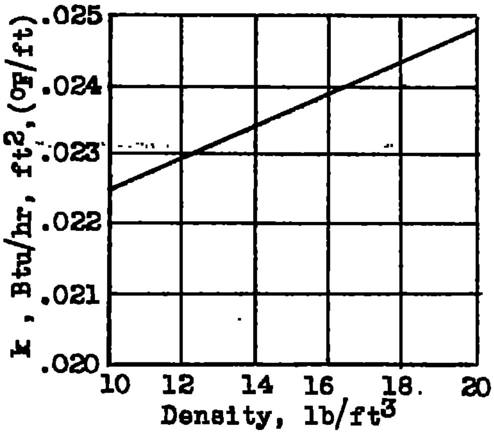


Figure 10a.- Effect of compression on thermal conductivity.

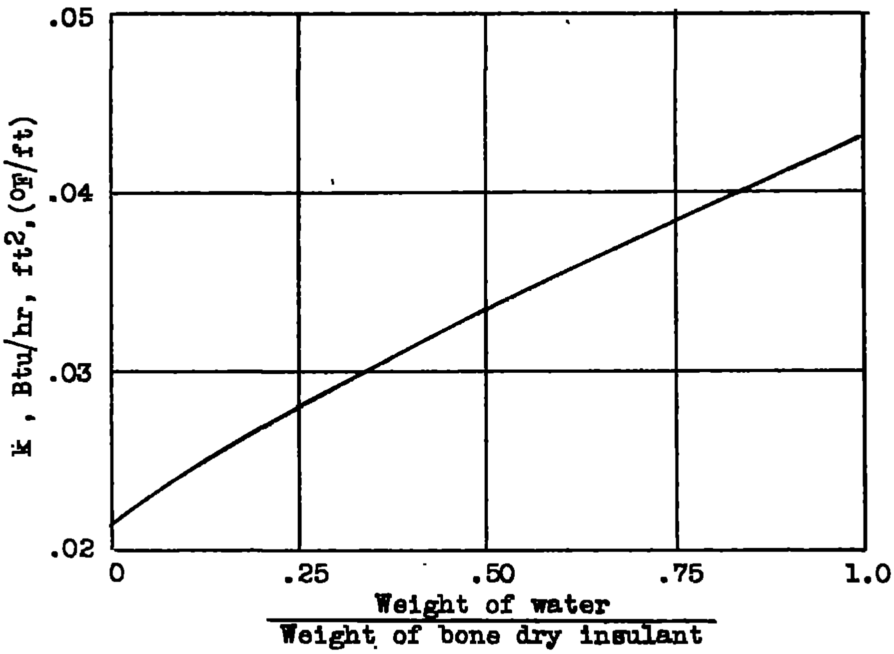


Figure 11.- Effect of moisture on thermal conductivity.

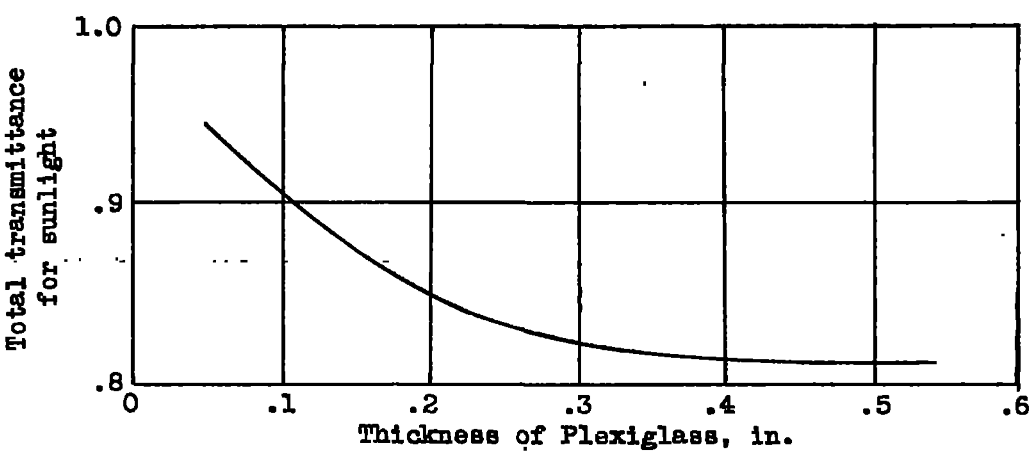


Figure 14.- Transmittance of Plexiglass.

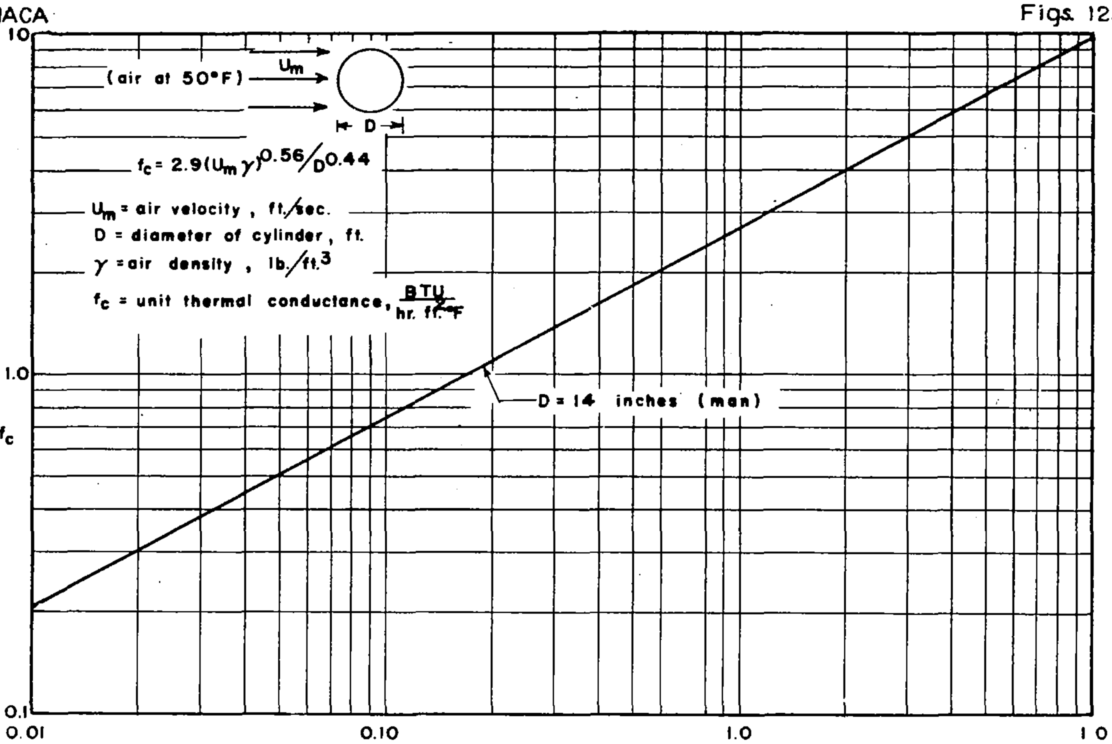

(air at 50°F)
U_m
D
$f_c = 2.9(U_m\gamma)^{0.56}/D^{0.44}$
U_m = air velocity, ft./sec.
D = diameter of cylinder, ft.
γ = air density, lb./ft.^{3}
f_c = unit thermal conductance, $\frac{BTU}{hr.\ ft.^2\ °F}$
D = 14 inches (man)
f_c
$U_m\gamma$
10
1.0
0.1
0.01
0.10
1.0
1 0

fig. 12

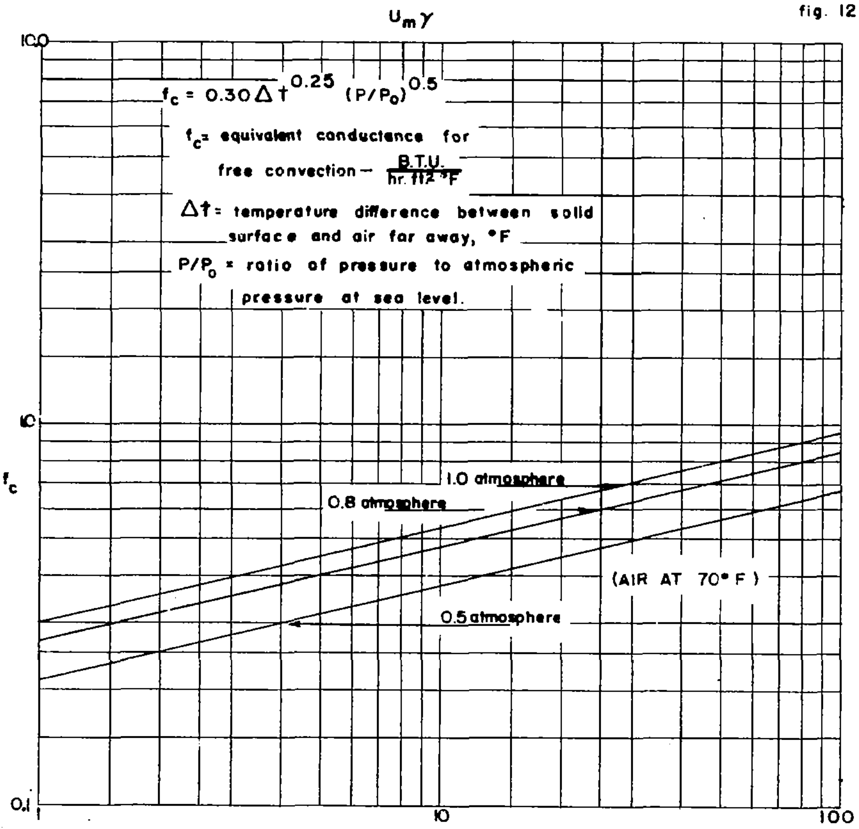

$f_c = 0.30\Delta t^{0.25}(P/P_0)^{0.5}$
f_c = equivalent conductance for free convection — $\frac{B.T.U.}{hr.\ ft^2\ °F}$
Δt = temperature difference between solid surface and air far away, °F
P/P_0 = ratio of pressure to atmospheric pressure at sea level.
1.0 atmosphere
0.8 atmosphere
(AIR AT 70° F)
0.5 atmosphere
f_c
Δt
10.0
1.0
0.1
1
10
100

fig. 13

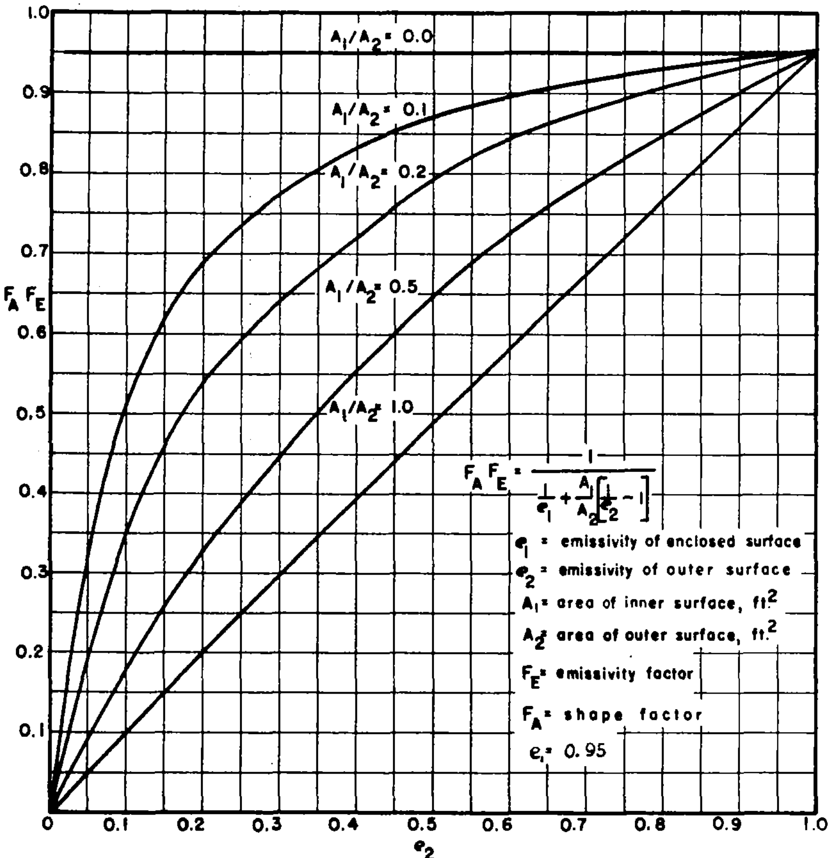


fig. 15

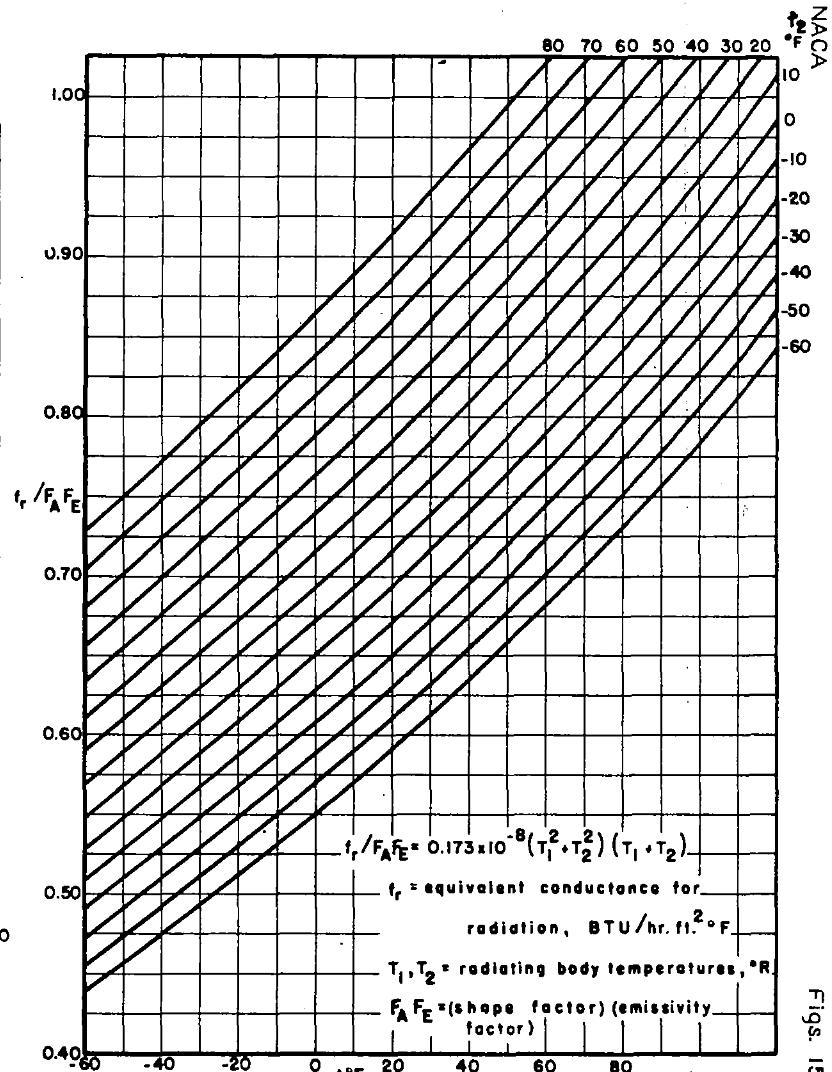


fig. 16

NACA

Figs. 15,16

NACA

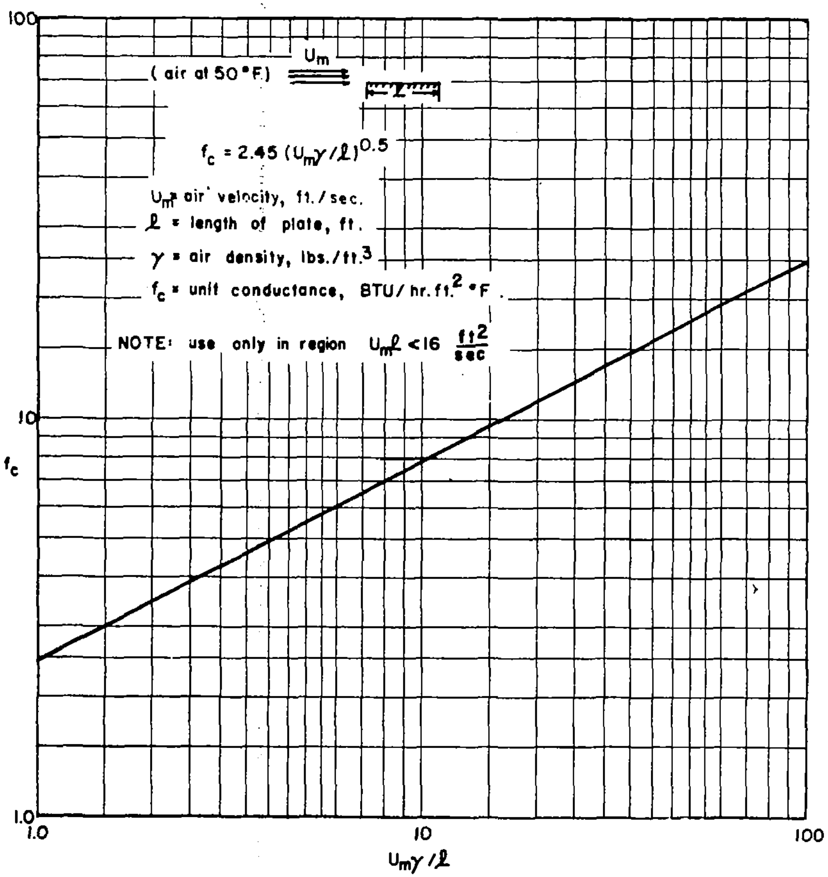


fig. 17

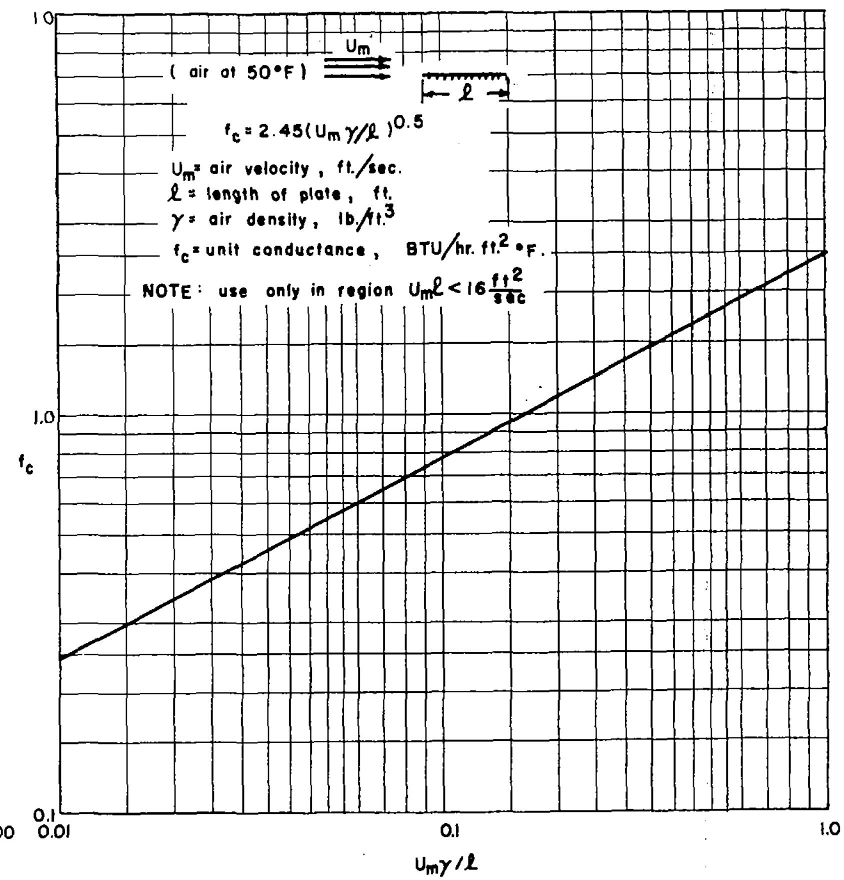
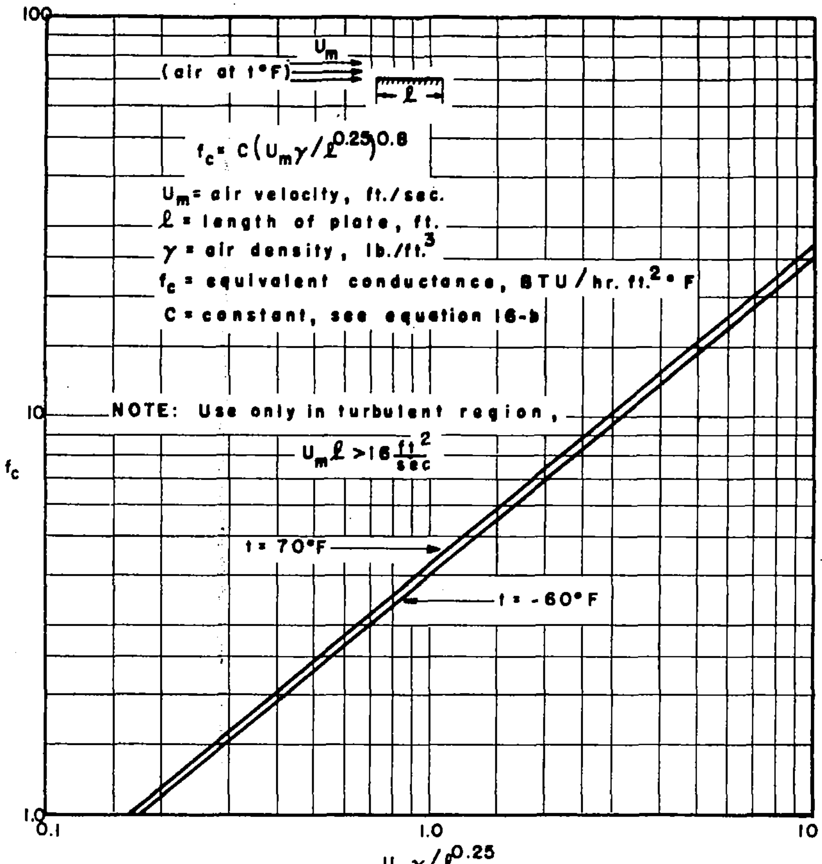


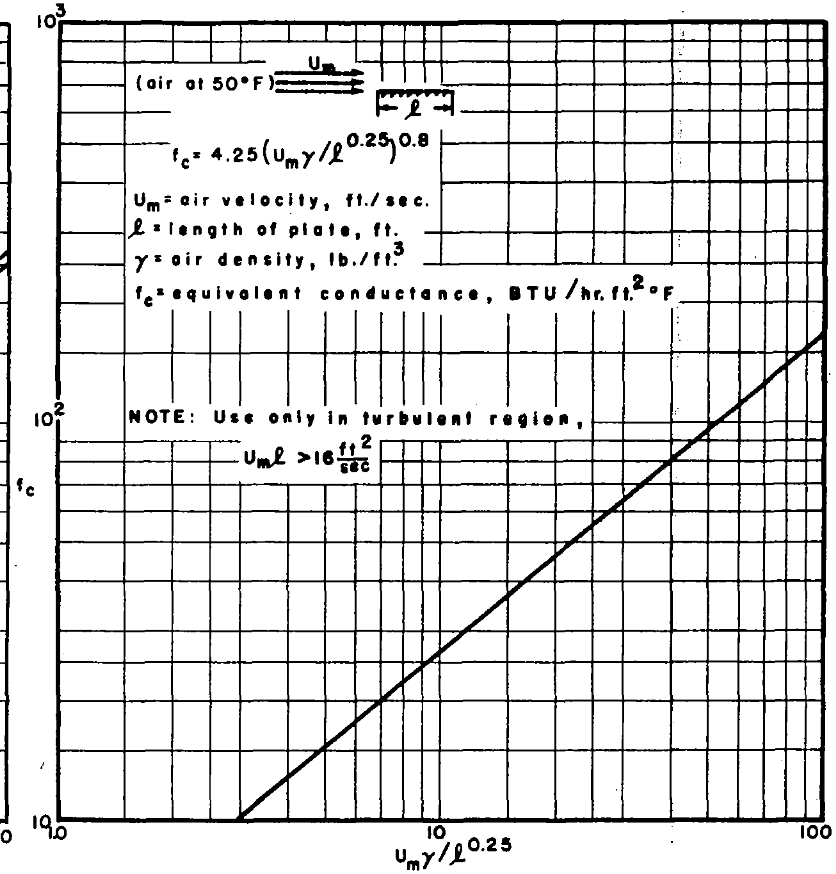
fig. 17 a.

Fig. 17



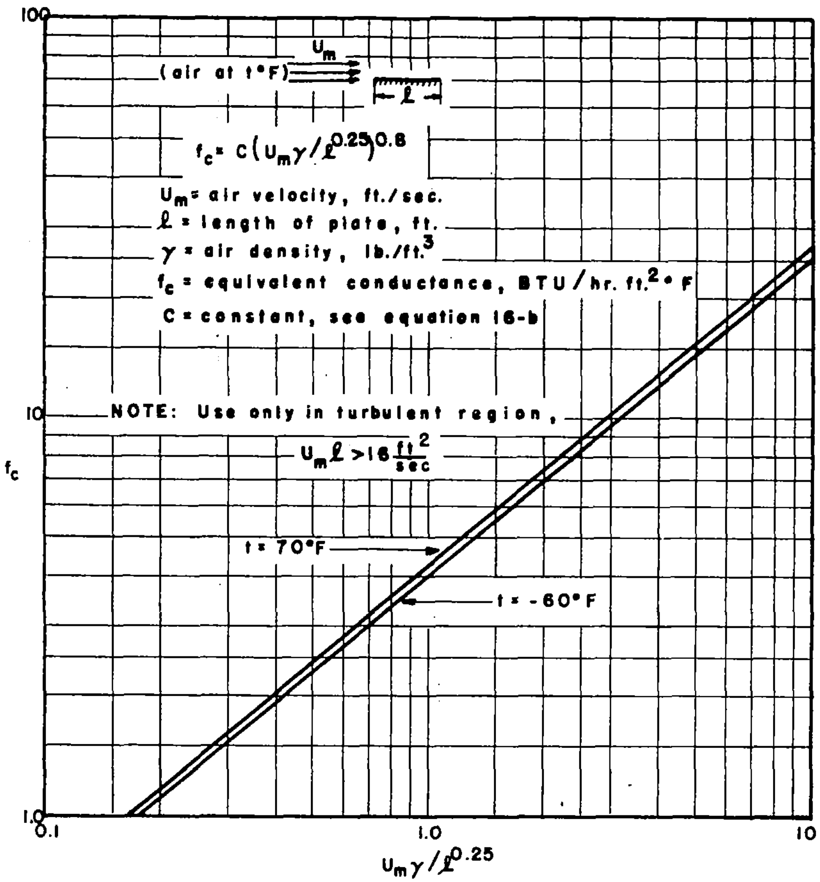
(air at t°F)
U_m
ℓ
$f_c = C(U_m\gamma/\ell^{0.25})^{0.8}$
U_m = air velocity, ft./sec.
ℓ = length of plate, ft.
γ = air density, lb./ft.^{3}
f_c = equivalent conductance, BTU/hr. ft.^{2} °F
C = constant, see equation 16-b
NOTE: Use only in turbulent region,
$U_m\ell > 16 \frac{ft^2}{sec}$
t = 70°F
t = -60°F
f_c
100
10
1.0
0.1
1.0
10
$U_m\gamma/\ell^{0.25}$

fig. 18



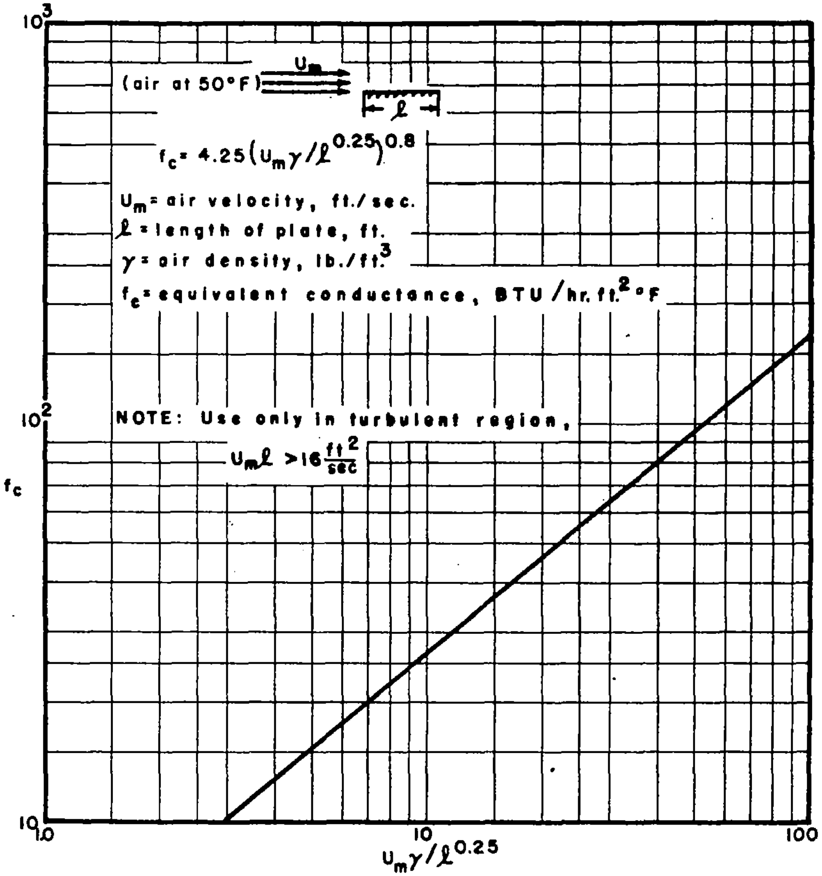
(air at 50°F)
U_m
ℓ
$f_c = 4.25(U_m\gamma/\ell^{0.25})^{0.8}$
U_m = air velocity, ft./sec.
ℓ = length of plate, ft.
γ = air density, lb./ft.^{3}
f_c = equivalent conductance, BTU/hr. ft.^{2} °F
NOTE: Use only in turbulent region,
$U_m\ell > 16 \frac{ft^2}{sec}$
f_c
10^3
10^2
10
1.0
10
100
$U_m\gamma/\ell^{0.25}$

fig. 19



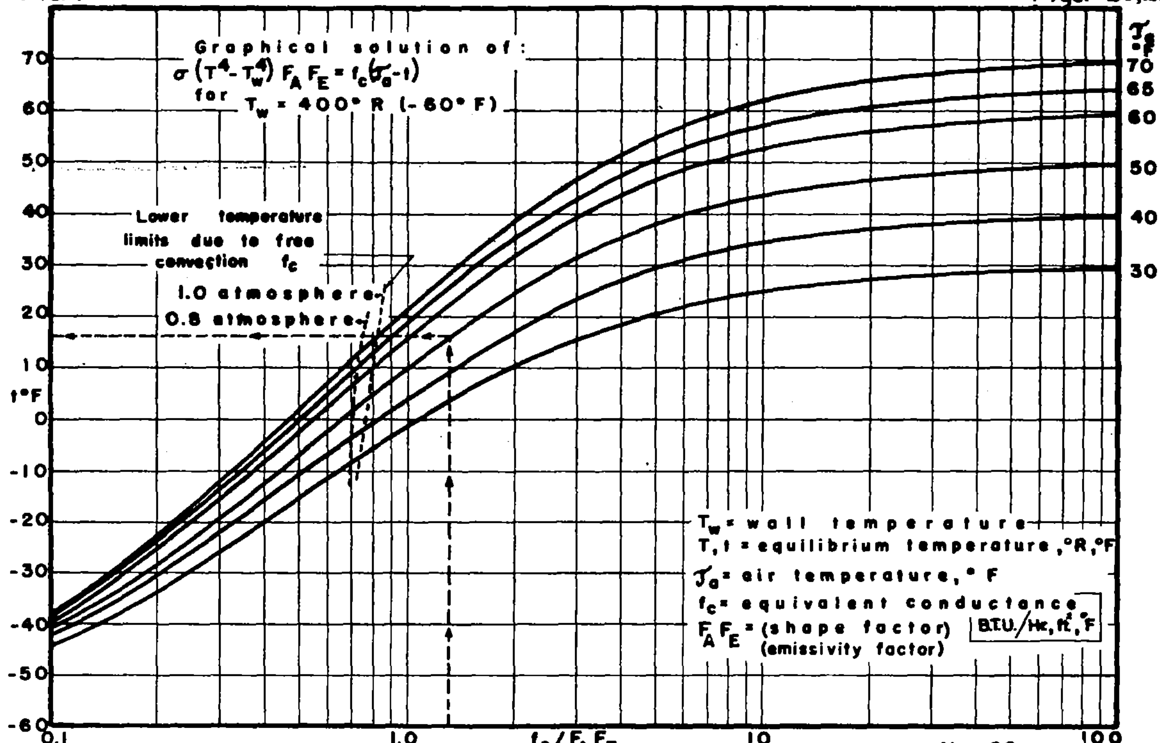
U_m
(air at t°F)
ℓ
$f_c = C(U_m\gamma/\ell^{0.25})^{0.8}$
U_m = air velocity, ft./sec.
ℓ = length of plate, ft.
γ = air density, lb./ft.^{3}
f_c = equivalent conductance, BTU/hr. ft.^{2} °F
C = constant, see equation 16-b
NOTE: Use only in turbulent region,
$U_m\ell > 16\ \frac{ft^2}{sec}$
t = 70°F
t = -60°F
f_c
100
10
1.0
0.1
1.0
10
$U_m\gamma/\ell^{0.25}$

fig. 18



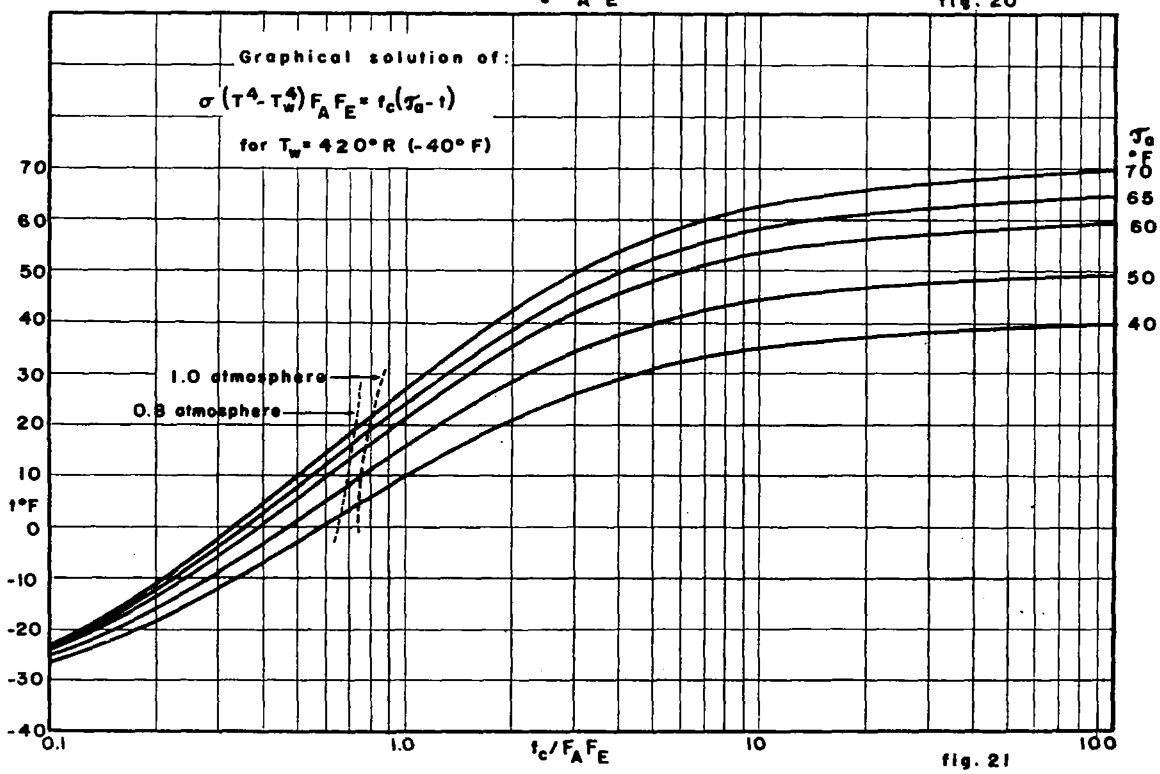
U_m
(air at 50°F)
ℓ
$f_c = 4.25(U_m\gamma/\ell^{0.25})^{0.8}$
U_m = air velocity, ft./sec.
ℓ = length of plate, ft.
γ = air density, lb./ft.^{3}
f_c = equivalent conductance, BTU/hr. ft.^{2} °F
NOTE: Use only in turbulent region,
$U_m\ell > 16\ \frac{ft^2}{sec}$
f_c
10^3
10^2
10
1.0
10
100
$U_m\gamma/\ell^{0.25}$

fig. 19



Graphical solution of:
$\sigma(T^4 - T_w^4) F_A F_E = f_c(\mathcal{T}_a - t)$
for T_w = 400° R (-60° F)
Lower temperature limits due to free convection f_c
1.0 atmosphere
0.8 atmosphere
T_w = wall temperature
T, t = equilibrium temperature, °R, °F
$\mathcal{T}_a$ = air temperature, ° F
f_c = equivalent conductance
$F_A F_E$ = (shape factor) (emissivity factor)
B.T.U./Hr, ft², °F
t°F
$\mathcal{T}_a$ °F
$f_c / F_A F_E$

fig. 20



Graphical solution of:
$\sigma(T^4 - T_w^4) F_A F_E = f_c(\mathcal{T}_a - t)$
for T_w = 420° R (-40° F)
1.0 atmosphere
0.8 atmosphere
t°F
$\mathcal{T}_a$ °F
$f_c / F_A F_E$

fig. 21