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AN INVESTIGATION OF AIRCRAFT HEATERS

IV - MEASURED AND PREDICTED PERFORMANCE OF LONGITUDINALLY FINNED TUBES

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AN INVESTIGATION OF AIRCRAFT HEATERS

IV - MEASURED AND PREDICTED PERFORMANCE OF LONGITUDINALLY FINNED TUBES

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INTRODUCTION

Two double tube, cylindrical, longitudinally finned heat exchangers in which hot exhaust gases pass through the annular space and ventilating air passes through the center tube have been tested to determine heat transfer and pressure drop performance.

One of the exchangers (57 in. long) was provided with eight longitudinal fins 52 inches in width* (fig. 1). The fins on the other tube were 6 inches wide, placed end to end, spaced 1/16 inch (fig. 2).

The tests were conducted in order

1. To establish a method of predicting the performance of longitudinally finned double tube heat exchangers

2. To compare the performance of the two finned tubes with reference to the effect of fin width*

3. To compare the performance of the finned tubes with the straight and dimpled tubes discussed in reference 1

4. To determine the pressure drop across the finned tubes under isothermal and non-isothermal conditions

EXPERIMENTAL EQUIPMENT

The natural gas test stand described in reference 1 was utilized in all the finned tube tests.

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*Throughout this report the term "width" refers to the fin dimension in the direction of fluid flow. The fin length refers to the dimension perpendicular to the tube wall, and fin thickness refers to the remaining dimension.

SYMBOLS

A	heat transfer area, ft^2
A_g	heat transfer area on gas side, ft^2
A_u	unfinned area of tube, ft^2
A_{u_a}	unfinned area of tube on air side, ft^2
A_{u_g}	unfinned area of tube on gas side, ft^2
A_w	surface area of outer tube of annulus, ft^2
B	experimental constant, $^\circ F$
C	experimental constant, $^\circ F/ft$
c_p	heat capacity at constant pressure, Btu/lb $^\circ F$
c_{p_a}	heat capacity of air at constant pressure, Btu/lb $^\circ F$
c_{p_g}	heat capacity of gas at constant pressure, Btu/lb $^\circ F$
D	diameter, ft
D_H	hydraulic diameter, ft
D_a	hydraulic diameter of finned tube on air side, ft
D_g	hydraulic diameter of gas annulus, ft
D_i	inner diameter of air tube, ft
D_o	outer diameter of air tube, ft
E	experimental constant, $^\circ F$
e_1, e_2	emissivities of inner and outer surfaces of annulus, respectively
f	unit thermal convective conductance, Btu/hr ft^2 $^\circ F$
$(fA)_e$	effective conductance of finned tube, Btu/hr $^\circ F$
$(fA)_{ea}$	effective conductance on air side, Btu/hr $^\circ F$
$(fA)_{eg}$	effective conductance on gas side, Btu/hr $^\circ F$

$(fA)_{fins}$ effective conductance for finned tube, Btu/hr °F

$(fA)_{smooth}$ effective conductance for unfinned smooth tube, Btu/hr °F

f_c unit thermal convective conductance along tube, Btu/hr ft² °F

f_{ca} unit thermal convective conductance along tube, on air side, Btu/hr ft² °F

f_{cg} unit thermal convective conductance along tube, on gas side, Btu/hr ft² °F

f_D unit thermal conductance based on hydraulic diameter, Btu/hr ft² °F

f_F unit thermal conductance along fin, Btu/hr ft² °F

f_{Fa} unit thermal conductance along fin on air side, Btu/hr ft² °F

f_{Fg} unit thermal conductance along fin on gas side, Btu/hr ft² °F

f_l unit thermal conductance based on width of fins, Btu/hr ft² °F

f_r equivalent unit thermal conductance for radiation, Btu/hr ft² °F

F experimental constant, °F/ft

F_A shape modulus, the factor in the radiation equation which allows for the geometrical position of the radiating surfaces

F_E emissivity modulus, the factor in the radiation equation which allows for the non-Planckian character of the radiation surfaces

g gravitational force per unit mass, lb/(lb sec²/ft)

G weight rate of flow per unit area, lb/hr ft²

G_a weight rate of flow per unit area for air, lb/hr ft²

G_g weight rate of flow per unit area for gas, lb/hr ft²

k thermal conductivity of fin material, Btu/hr ft^2 (°F/ft)

k_a thermal conductivity of air, Btu/hr ft^2 (°F/ft)

l width of fin in direction of fluid flow, ft

L length of fin measured perpendicular to tube surface, ft

L_a length of fin measured perpendicular to tube surface on air side, ft

L_g length of fin measured perpendicular to tube surface on gas side, ft

N distance between pressure taps, ft

n number of fins measured along circumference of tube

P perimeter of fin in appendix B, ft

P_1 pressure at entrance to heat exchanger, lb/ft^2

P_2 pressure at exit from heat exchanger, lb/ft^2

q rate of heat transfer, Btu/hr

q_a predicted rate of heat transfer, Btu/hr

q_M measured rate of heat transfer, Btu/hr

$q_{n=0}$ rate of heat transfer for unfinned tube, Btu/hr

s thickness of fin, ft

t_p average intensifier tube wall temperature, °F

t_w average temperature of outer wall of annulus, °F

T absolute temperature, °R

T_1 absolute mixed mean temperature of air entering heating section, °R

T_2 absolute mixed mean temperature of air leaving heating section, °R

T_a arithmetic average absolute temperature of air in heater, °R

T_g arithmetic average absolute temperature of gas in heater, °R

T_p average absolute temperature of intensifier tube wall, °R

T_w average absolute temperature of outer wall of annulus, °R

U over-all unit conductance, Btu/hr ft² °F

(UA) over-all conductance for any tube, Btu/hr °F

$(UA)_e$ over-all conductance for finned tube defined by equation (28), Btu/hr °F

$(UA)_u$ over-all conductance for unfinned tube defined by equation (25), Btu/hr °F

W fluid rate, lb/hr

W_a air rate, lb/hr

W_g exhaust gas rate, lb/hr

X defined in appendix B

x length of finned portion of tube, ft

γ weight density of air at any temperature and pressure, lb/ft^3

γ_1 weight density of air at entrance to heating section, lb/ft^3

ΔP pressure drop across pipe, lb/ft^2

ΔP_T isothermal pressure drop across pipe, at temperature T, lb/ft^2

ΔP_{T_1} isothermal pressure drop due to friction at temperature T_1, lb/ft^2

$\frac{\Delta P}{\Delta N}$ pressure drop per foot, $lb/ft^2/ft$

Δt_{lm} logarithmic mean temperature difference, °F

ζ isothermal friction factor

μ_a viscosity of air, lb sec/ft^2

τ_1 mixed-mean temperature of fluid at $x = o$ (appendix B), $^\circ$F

τ_{a1} mixed-mean temperature of air at entrance to heating section, $^\circ$F

τ_{a2} mixed-mean temperature of air leaving heating section, $^\circ$F

τ_{a_x} mixed-mean temperature of air at any point x, $^\circ$F

τ_{g1} mixed-mean temperature of gas at entrance to heating section, $^\circ$F

τ_{g2} mixed-mean temperature of gas leaving heating section, $^\circ$F

τ_{g_x} mixed-mean temperature of gas at any point x, $^\circ$F

$\tau_a = \dfrac{\tau_{a1} + \tau_{a2}}{2}$, $^\circ$F

$\tau_g = \dfrac{\tau_{g1} + \tau_{g2}}{2}$, $^\circ$F

τ_x mixed-mean temperature of fluid at any point x, $^\circ$F

$Pr = \dfrac{\mu_a \, c_{p_a} \, g}{k_a} \times 3600$, Prandtl modulus for air

$Re = \dfrac{G \, D_E}{3600 \, \mu_a \, g}$, Reynolds modulus for air (pipe)

$Re_p = \dfrac{G \, l}{3600 \, \mu_a \, g}$, Reynolds modulus for air (flat plate)

ANALYSIS OF THE MECHANISM OF HEAT TRANSFER IN A LONGITUDINALLY FINNED DOUBLE TUBE HEAT EXCHANGER

In appendix B of this report the following equation is derived for the rate of heat transfer to a fluid from a length dx of a longitudinally finned tube (neglecting the heat transfer from the ends of the fins):

$$dq = \left[n\sqrt{2skf_F}\ \tanh \sqrt{\frac{2f_F}{ks}}\ L + f_c(\pi D - ns) \right](t_p - \tau_x)dx \quad (1)$$

where

dq rate of heat transfer from length of tube dx, Btu/hr

n number of fins in length dx

s thickness of fin, ft

k thermal conductivity of fin material, Btu/hr ft^2 $\left(\frac{^\circ F}{ft}\right)$

f_F unit thermal conductance along fin, Btu/hr ft^2 $^\circ$F

L length of fin, measured from base outward, ft

f_c unit thermal conductance along tube surface, Btu/hr ft^2 $^\circ$F

D diameter of finned tube, ft

t_p temperature of tube wall at point x, $^\circ$F

τ_x mixed-mean temperature of gas at point x, $^\circ$F

x length of finned tube, ft

For purposes of analysis equation (1) may be equated to:

$$dq = (t_p - \tau_x)d(fA)_e \quad (2)$$

where the product $(fA)_e$ may be called the "effective conductance" of the finned tube.

Thus, equating (1) and (2), and integrating* the effective conductance of the finned tube becomes:

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*In the ideal system the bracketed expression in equation (1) is postulated as constant with x. In the actual system this is only approximately true, since for narrow fins f_F will be shown to be inversely proportional to the 1/5 power of the fin width (measured parallel to the direction of flow) in the turbulent flow region.

$$(fA)_e = \left[n\sqrt{2skf_F}\ \tanh\sqrt{\frac{2f_F}{ks}}\ L + f_c(\pi D - ns) \right] x \qquad (3)$$

The first term in the brackets represents the effective conductance of the fins proper, and the second term the conductance of the pipe surface not covered by fins.

It should be noted that, if either n or s becomes zero, equation (3) yields the conductance of a smooth unfinned pipe, that is, $f_c\ (\pi Dx)$.

If either f_F or k becomes zero, the conductance becomes $f_c(\pi D - ns)x$, that is, the fins act as insulators and reduce the effective heat transfer area of the tube.

Thus in some exceptional cases poorly designed fins may actually reduce the effectiveness of the tube. Under normal conditions, however, the heat transfer from the fins will more than offset the reduction in the heat transfer from the tube proper brought about by the addition of the fins.

The expression for the effective conductance of the fins proper (the first term in the bracketed expression of equation (3)) merits further analysis. If the hyperbolic tangent term is expanded in a power series (reference 2, p. 93),

$$\tanh\sqrt{\frac{2f_F}{ks}}\ L = \left(\frac{2f_F}{ks}\right)^{1/2} L - \frac{1}{3}\left(\frac{2f_F}{ks}\right)^{3/2} L^3 + \frac{2}{15}\left(\frac{2f_F}{ks}\right)^{5/2} L^5 + \ldots \qquad (4)$$

the term

$$(fA)_{fins} = \left[n\sqrt{2skf_F}\ \tanh\sqrt{\frac{2f_F}{ks}}\ L \right] x \qquad (5)$$

becomes:

$$(fA)_{fins} = f_F(2nLx) \left[1 - \frac{1}{3}\left(\frac{2f_F}{ks}\right) L^2 + \frac{2}{15}\left(\frac{2f_F}{ks}\right)^2 L^4 + \ldots \right] \quad (6)$$

If the term $\frac{2f_F L^2}{ks}$ is small (0.1 or less), it is apparent that the thermal conductivity of the fin material k and the fin thickness s are immaterial in determining the thermal performance of the fin.

For many fins utilized in practice, the term $\frac{2f_F L^2}{ks}$ is of the order of magnitude of 0.30 to 0.40. Even for these magnitudes, however, inspection of equation (6) reveals that the fin thickness and conductivity are of small importance.

If, on the other hand, the magnitude of $\frac{2 f_F L^2}{ks}$ is about 4.0 or greater, the hyperbolic tangent of $\sqrt{\frac{2f_F}{ks}}\, L$ approaches unity and the effective fin conductance becomes

$$(fA)_{fins} = n \sqrt{2skf_F}\; x \qquad (7)$$

In this case the thermal conductivity of the fin material and the fin thickness are of direct importance, but the magnitude of the fin length (measured perpendicular to the tube surface) becomes immaterial.

For intermediate magnitudes of $\frac{2f_F L^2}{ks}$ all the variables involved are effective.

Equation (3) cannot be applied directly to the experimental heat exchangers. Inspection of figure 1 and figure 2 reveals that the fins did not extend the whole distance along the central tube, but that $2\frac{1}{2}$ inches at each end of the tube were unfinned. The conductance of the unfinned area may be added to that of the finned portion of the tube, and as a first approximation, for simplicity, the unit conductance at the ends is assumed equal to that along the finned portion of the tube. Thus the total effective conductance for the experimental heat exchanger becomes:

$$(fA)_e = \left[n \sqrt{2skf_F}\, \tanh \sqrt{\frac{2f_F}{ks}}\, L + f_c\, (\pi D - ns)\right] x + f_c\, A_u \qquad (8)$$

It should be noted that in equation (8) neither the f nor the A in the product $(fA)_e$ can be evaluated, but the product $(fA)_e$, that is, the effective conductance of the tube depends on f_F, f_c, the fin arrangement, the fin size and material, and so forth. If the product $(fA)_e$ is assumed constant with length, it may be demonstrated readily (reference 3, p. XIV-3) that the thermal output of the finned heat exchanger becomes:

$$q_a = \frac{\Delta t_{lm}}{\left[\frac{1}{(fA)_{ea}} + \frac{1}{(fA)_{eg}}\right]} \qquad (9)$$

where

Δt_{lm} log mean temperature difference between exhaust gases and ventilating air, °F

The effective conductance on the air side is:

$$(fA)_{ea} = \left[n\sqrt{2skf_{Fa}}\tanh\sqrt{\frac{2f_{Fa}}{ks}}\,L_a + f_{ca}(\pi D_i - ns)\right]x + f_{ca}A_{ua} \qquad (10)$$

The effective conductance on the gas side is*

$$(fA)_{eg} = \left[n\sqrt{2sk(f_{Fg} + f_r)}\tanh\sqrt{\frac{2(f_{Fg} + f_r)}{ks}}\,L_g + (f_{cg} + f_r)(\pi D_o - ns)\right]x + A_{ug}(f_{cg} + f_r) \qquad (11)$$

The unit conductances f_r, f_{Fa}, f_{ca}, f_{Fg}, and f_{cg} must be evaluated before the thermal output of the heater can be predicted. The phenomenon of heat transfer from the

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*Addition of the equivalent radiation conductance to the unit convective conductance on the gas side is only an approximation, but does not introduce a very large error. (See reference 1.)

fins requires some elucidation before a method of analysis can be established.

If a single fin is attached to a flat surface, the fin acts as a flat plate as far as its fluid-dynamic and thermal performance is concerned. It is well known (reference 4, vol. I, p. 50) that a laminar boundary layer builds up on such a plate, the thickness of which increases as the fluid proceeds down the plate away from the leading edge. The thickness of the boundary layer, the frictional drag, and the unit thermal conductance for this laminar condition are all functions of the square root of the Reynolds modulus for the plate (reference 4, vol. I, p. 50 and vol. II, p. 623).

At a certain point along the plate the boundary layer becomes turbulent, and from this point on (fig. 3) the thickness of the boundary layer, the drag, and the unit thermal conductance are all functions of the 1/5 power (reference 4, vol. II, p. 361 and reference 5, p. 174) of the Reynolds modulus for the plate. In most longitudinally finned air heat exchangers the latter type of flow occurs over the greatest portion of the fin unless the fins* are extremely narrow (reference 6, p. 489). (Only the turbulent condition will be considered in the remainder of the analysis.)

Thus, for a single fin along which a turbulent boundary layer has been established, the unit thermal conductance will be a function of the 1/5 power of the fin width (measured in the direction of fluid flow). When, however, several fins are placed close to each other inside of a closed tube, the simple picture presented above cannot exist. At the leading edge of a long fin the boundary layer will build up, but, after the fluid has passed a short distance down the fin, the other fins and the tube walls will begin to influence the flow. The boundary layer will not continue to build up, but will reach a constant thickness which will depend, not on the plate width (measured in the direction of fluid flow), but on the hydraulic diameter of the finned tube (reference 7, p. 142 and reference 8, p. 447).

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*The criterion for the type of flow existing over the plate is the Reynolds modulus for the plate (reference 4, vol. II, p. 366). If $Re_p > 50,000$, the flow is considered turbulent; if $Re_p < 50,000$, the flow is laminar.

The same type of phenomenon also takes place in unfinned tubes with no hydrodynamic calming section. (See reference 7, p. 142.) The entering section of such a tube can be regarded as a flat plate, since the velocity and temperature distributions in the fluid stream are affected only by the state of the tube wall immediately in contact with the fluid and are unaffected by the conditions at the center of the air stream. When the fluid has traversed a certain length of tube, however, steady state temperature and velocity distributions are established which are independent of tube length, for low rates of heat transfer, but depend on the tube diameter. Thus the unit conductance of a wide fin enclosed in a tube will not depend primarily on the width of the fin, but on the hydraulic radius of the tube. It will be shown that the unit convective conductance for this case varies inversely with the 1/5 power of the hydraulic diameter.

If, however, the enclosed fins are narrow enough, the thickness of the boundary layer built up about each fin will not become sufficiently great to interfere with the flow along the tube and along the other fins. These narrow fins may then be considered as isolated flat plates.

A sketch showing the three phenomena discussed above is shown in figure 3.

The terms "wide" and "narrow" fins have been used in the above discussion with no exact definition. The term "wide fin" is used in the sense of a fin along which the boundary layer does not build up completely, but is limited in its growth by the interference from other fins or adjacent tube walls. The term "narrow fin" is used in the sense of a fin along which the turbulent boundary layer is completely established. Thus the "width" of the fin depends on the presence and proximity of other solid boundaries. An approximate criterion of fin width may be determined by equating the unit thermal conductances based on fin width and on hydraulic diameter. The width of fin which makes these conductances equal will be the criterion for fin width.

It will be shown that the equation for the unit conductance along a fin based on fin width is

$$f_l = 9.36 \times 10^{-4}\ T^{0.296}\ \frac{G^{0.800}}{l^{0.200}} \qquad (12)$$

The unit conductance based on hydraulic diameter is:

$$f_D = 5.56 \times 10^{-4} T^{0.296} \frac{G^{0.800}}{D_H^{0.200}} \qquad (13)$$

Equating these two conductances yields

$$\frac{l}{D_H} = 13.4 \qquad (14)$$

Thus, if a fin is wider than 13.4 times the hydraulic diameter of the finned tube, it is a wide fin and the unit conductances for prediction of performance should be based on hydraulic diameter. If, on the other hand, the width of the fin is less than 13.4 times the hydraulic diameter of the finned tube, it is a narrow fin and its unit conductance should be based on fin width. The equations for unit convective conductances referred to in this discussion are presented below:

(a) Unit conductance for a tube based on the hydraulic diameter (See reference I, p. 11):

$$f_D = 5.56 \times 10^{-4} T^{0.296} \frac{G^{0.800}}{D_H^{0.200}} \qquad (15)$$

where

G weight rate of flow per unit area, lb/hr ft²

T absolute temperature of the gases, °R

D_H hydraulic diameter of tube, ft

$$\left(4 \times \frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$

(b) Unit conductance of a flat plate based on the width of the plate (measured in the direction of flow)

In reference 9 the equation for the unit conductance along a flat plate is presented, based on the approximation of Colburn (reference 5) to the analogy between heat and momentum transfer. (See reference 8, p. 447, and references 10 and 11.) The equation is

$$\frac{f_l}{c_{p_a} G} = 0.037 \, Pr^{-2/3} \left(\frac{Gl}{3600 \mu_a \, g}\right)^{-1/5} \qquad (16)$$

Equation (16) reduces to

$$f_l = 0.000160 \, (k_a^{0.667} \, \mu_a^{-0.467} \, c_{p_a}^{0.333}) \frac{G^{0.800}}{l^{0.200}} \qquad (17)$$

which for air* reduces to (reference 1, p. 11)

$$f_l = 9.36 \times 10^{-4} \, T^{0.298} \, \frac{G^{0.800}}{l^{0.200}} \qquad (18)$$

where

G weight rate of flow per unit area, lb/hr ft²

T absolute temperature of gas, °R

l width of flat plate measured in the direction of gas flow, ft

(c) Calculation of f_r (equivalent unit conductance for radiation)

If the gaseous radiation and absorption (reference 12, p. 531) in the exhaust gas annulus is neglected, the equivalent unit conductance for radiation may be calculated from (refererence 13, p. 61)

$$f_r = \frac{0.173 \, F_A \, F_E \left[\left(\frac{T_w}{100}\right)^4 - \left(\frac{T_p}{100}\right)^4\right]}{(t_g - t_p)} \qquad (19)$$

where

$$F_A = 1.00$$

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*The properties of air were utilized for the exhaust gas also, in the absence of more precise data. As discussed in reference 1, this approximation will probably not introduce a large error.

and as a first approximation

$$F_E = \frac{1}{\frac{1}{e_1} + \frac{A_g}{A_w}\left(\frac{1}{e_2} - 1\right)} \qquad (20)$$

Recapitulation

For the prediction of the performance of the finned double tube heat exchangers the following equations will be utilized:

(a)

$$q_a = \frac{\Delta t_{lm}}{\left(\frac{1}{fA}\right)_{ea} + \left(\frac{1}{fA}\right)_{eg}} \qquad (21)$$

The effective conductances $(fA)_{ea}$ and $(fA)_{eg}$ are given by equations (10) and (11), page 10.

(b)

$$f_D = 5.56 \times 10^{-4}\, T^{0.296}\, \frac{G^{0.80}}{D_H^{0.20}} \qquad (22)$$

The unit conductance f_D is utilized along the unfinned portion of the tube and for fins when

$\frac{l}{D_H} > 13.4$

(c)

$$f_l = 9.36 \times 10^{-4}\, T^{0.296}\, \frac{G^{0.80}}{l^{0.20}} \qquad (23)$$

The unit conductance f_l is utilized for the fins whenever $\frac{l}{D_H} < 13.4$

(d)

$$f_r = \frac{0.173\, F_A\, F_E\left[\left(\frac{T_w}{100}\right)^4 - \left(\frac{T_p}{100}\right)^4\right]}{(\tau_g - t_p)} \qquad (24)$$

SAMPLE CALCULATION

Run N-11 (52-in. fins)

From figure 1

Wetted perimeter (including fins) - on air side.	1.33 ft
Wetted perimeter (including fins) - on gas side.	1.66 ft
Cross-sectional area of flow - on air side . . .	0.01526 ft
Cross-sectional area of flow - on gas side . . .	0.0286 ft
Hydraulic diameter - on air side, D_a.	0.0460 ft
Hydraulic diameter - on gas side, D_g.	0.0688 ft
Width of fins parallel to flow - on air side, l_a	4.33 ft
Width of fins parallel to flow - on gas side, l_g	4.33 ft
Length of finned portion of tube, x	4.33 ft
Area of unfinned ends - air side A_{ua}.	0.19 ft
Area of unfinned ends - gas side A_{ug}.	0.22 ft
Length of fins perpendicular to tube - on air side, L_a	0.0537 ft
Length of fins perpendicular to tube - on gas side, L_g	0.0208 ft
Thickness of fins, s.	0.00521 ft
Inside diameter of ventilating air tube, D_i . .	0.1491 ft
Outside diameter of ventilating air tube, D_o. .	0.167 ft
Thermal conductivity of fin material (iron). . 23	$\frac{\text{Btu}}{\text{hr ft}^2 \left(\frac{^\circ F}{ft}\right)}$

From data:

Rate of air flow, W_a . . . . 199 lb/hr

Rate of gas flow, W_g . . . . 181 lb/hr

Air temperature entering, T_{a1} . . . . 98 °F

Air temperature (mixed mean) leaving, T_{a2} . . 552 °F

Gas temperature entering, T_{g1} . . . . .1496 °F

Gas temperature leaving, T_{g2} . . . . 848 °F

(a) Calculation of the log mean temperature difference

$$\Delta t_{lm} = \frac{(1496 - 98) - (848 - 552)}{\ln\left(\frac{1496 - 98}{848 - 552}\right)} = 710 \text{ °F}$$

(b) Calculation of f_{ca} (unit conductance at the tube surface; air side)

$$T_a = \frac{T_{a1} + T_{a2}}{2} + 460 = 785° \text{ R}$$

$$G_a = \frac{W_a}{0.01525} = 13{,}000 \text{ lb/hr ft}^2$$

$$D_a = 0.0460 \text{ ft}$$

$$f_{ca} = 5.56 \times 10^{-4} \, T_a^{0.298} \times \frac{G_a^{0.80}}{D_a^{0.20}}$$

$$= 5.56 \times 10^{-4} \, (785)^{0.298} \times \frac{(13{,}000)^{0.80}}{(0.046)^{0.20}}$$

$$f_{ca} = 14.5 \frac{\text{Btu}}{\text{hr ft}^2 \text{ °F}}$$

(c) Calculation of f_{Fa} (the unit thermal conductance along the fins on the air side)

For the 52-inch fins, $l_a = 4.33$ feet. The hydraulic diameter of the finned tube on the air side is 0.0460 foot. The ratio

$$\frac{l_a}{D_a} = \frac{4.33}{0.046} = 94.1$$

Thus the calculation of f_{Fa} should be based on the hydraulic radius.

$$T_a = 785^\circ \text{ R}$$

$$G_a = 13,000 \text{ lb/hr ft}^2$$

$$D_a = 0.0460 \text{ ft}$$

$$f_{Fa} = 5.56 \times 10^{-4} \; T_a^{0.296} \; \frac{G_a^{0.8}}{D_a^{0.2}}$$

$$f_{Fa} = 14.5 \text{ Btu/hr ft}^2 \; ^\circ\text{F}$$

(d) Calculation of f_{cg}

(Unit thermal conductance at the tube surface on the gas side)

$$T_g = \frac{T_{g1} + T_{g2}}{2} + 460 = 1632^\circ \text{ R}$$

$$G_g = \frac{W_g}{0.0286} = 6340 \text{ lb/hr ft}^2$$

$$D_g = 0.0688 \text{ ft}$$

$$f_{cg} = 5.56 \times 10^{-4} \; T_g^{0.296} \; \frac{G_g^{0.800}}{D_g^{0.200}}$$

$$f_{cg} = 9.2 \text{ Btu/hr ft}^2$$

(e) Calculation of f_{Fg} (the unit thermal conductance along the fins on the gas side)

The fin width l_g = 4.33 feet. The hydraulic diameter of the exhaust gas annulus D_g = 0.0688 feet. Thus

$\frac{l_g}{D_g} = 63.0$. The unit conductance f_{F_g} should be based on hydraulic diameter.

$$T_g = 1632^\circ \text{ R}$$

$$G_g = 6{,}340$$

$$D_g = 0.0688 \text{ ft}$$

$$f_{F_g} = 5.56 \times 10^{-4}\, T_g^{\,0.288} \times \frac{G_g^{\,0.80}}{D_g^{\,0.20}}$$

$$= 9.2 \text{ Btu/hr ft}^2 \, {}^\circ\text{F}$$

(f) Calculation of f_r (equivalent unit conductance for radiation)

In order to evaluate the equivalent unit conductance for radiation, the temperature of the outer surface of the inner tube must be calculated. This calculation must be one of trial and error. The rate of heat transfer from the gas to the outer wall of the tube is given by:

$$q_a = (fA)_{eg}\,(T_g - t_p) \qquad (25)$$

where

$$(fA)_{eg} = \left[n\sqrt{2sk(f_{F_g} + f_r)}\,\tanh\sqrt{\frac{2(f_F + f_r)}{ks}}\,L_g + (f_{cg} + f_r)(\pi D_o - ns)\right] x + (f_{cg} + f_r)A_{u_g} \qquad (26)$$

All magnitudes in equation (25) and in equation (26) are known except q_a, t_p, and f_r. The measured rate of heat transfer q_m is utilized for q_a. A magnitude of $f_r = 6$ may be assumed as a reasonable value and t_p calculated from equations (25) and (26). This procedure yields:

$$(fA)_{eg} = 56.8 \text{ Btu/hr } {}^\circ\text{F}$$

$$t_p = T_g - \frac{q_M}{(fA)_{eg}}$$

$$t_p = 1172 - \frac{21300}{56.8} = 797^\circ\ F$$

Experimental data for the finned tubes indicated that the average temperature along the outer surface of the exhaust gas annulus (t_w) was 175° F less than the arithmetic average exhaust gas temperature

Thus

$$t_w = T_g - 175 = 1172 - 175 = 997^\circ\ F$$

The emissivity of oxidized steel is approximately (reference 13, p. 46) equal to 0.79

$$F_E = \frac{1}{\frac{1}{e_1} + \frac{A_g}{A_w}\left(\frac{1}{e_2} - 1\right)} = \frac{1}{\frac{1}{0.79} + \frac{3.93}{5.87}\left(\frac{1}{0.79} - 1\right)} = 0.695$$

$$F_A = 1.00$$

Then

$$f_r = \frac{0.173 \times 0.695 \left[\left(\frac{997 + 460}{100}\right)^4 - \left(\frac{797 + 460}{100}\right)^4\right]}{(1172 - 797)} = 6.4 \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}}$$

This magnitude of f_r is practically equal to the value assumed (6.0). Thus the calculation need not be repeated.

(g) Calculation of the effective conductance of the finned tube on the air side

From equation (10)

$$(fA)_{ea} = \left[n\sqrt{skf_{Fa}}\ 2\ \tanh\sqrt{\frac{2f_{Fa}}{ks}}\ L_a + f_{p_a}(\pi D_i - ns)\right]x + f_{p_a} A_{ua}$$

Substituting the magnitudes which have been obtained

$$n\sqrt{2skf_{Fa}}\ x = 8\sqrt{2 \times 0.00521 \times 23 \times 14.5}\ (4.33)$$

$$= 64.6\ \text{Btu/hr}\ {}^\circ\text{F}$$

$$\sqrt{\frac{2f_{F_a}}{ks}}\, L_a = \sqrt{\frac{2 \times 14.5}{23 \times 0.00521}}\,(0.0537) = 0.835 \text{ (dimensionless)}$$

$$f_{ca}(\pi D - ns)x = 14.5(3.14 \times 0.1491 - 8 \times 0.00521)\, 4.33$$

$$= 26.9 \text{ Btu/hr } ^{o}F$$

$$f_{ca}\, A_{ua} = (14.5)(0.19) = 2.7 \text{ Btu/hr } ^{o}F$$

$$(fA)_{ea} = 64.6 \tanh 0.835 + 26.9 + 2.7$$

$$= (44.2) + (26.9) + (2.7) = 73.8 \text{ Btu/hr } ^{o}F$$

Thus it is noted that on the air side the fins contribute 60 percent of the effective conductance, and the unfinned portion 40 percent. (This calculation indicates the effect of the fins per se. The indirect effects of the fins which also tend to increase heater output are discussed on p. 23.)

(h) Calculation of the effective conductance of the finned tube on the gas side.

In exactly the same manner as was shown for the air side,

$$n\sqrt{2\, s\, k(f_{F_g} + f_r)}\,x = 67.0 \text{ Btu/hr } ^{o}F$$

$$\sqrt{\frac{2(f_{F_g} + f_r)}{k\, s}}\, L_g = 0.335 \text{ (dimensionless)}$$

$$(f_r + f_{cg})(\pi D - ns)x = 32.9 \text{ Btu/hr } ^{o}F$$

$$(f_r + f_{cg})A_{ug} = 3.4 \text{ Btu/hr } ^{o}F$$

$$(fA)_{eg} = 67.0 \tanh(0.335) + 32.9 + 3.4$$

$$= (21.6) + (32.9) + (3.4) = 57.9$$

On the gas side, therefore, owing to the shorter fins, the contribution of the fins to the effective conductance is only 37.4 percent; whereas the unfinned portion of tube contributes 62.6 percent.

(i) Calculation of q_a

$$q_a = \frac{\Delta t_{lm}}{\left(\frac{1}{fA}\right)_{eg} + \left(\frac{1}{fA}\right)_{ea}} = \frac{710}{\left(\frac{1}{57.9}\right) + \left(\frac{1}{73.8}\right)} = 23{,}000 \text{ Btu/hr}$$

The measured rate of heat transfer is

$$q_M = W_a \, c_p \, (T_{a2} - T_{a1})$$

$$= (199)\,(0.241)\,(552 - 98)$$

$$\doteq 21{,}800 \text{ Btu/hr}$$

Ratio of $\frac{q_a}{q_M} = 1.055$

DISCUSSION OF RESULTS

Inspection of table I reveals that the average of the ratio of predicted magnitudes of heat output to the measured magnitudes for the exchanger with the 52-inch fins is 1.00,* with an average deviation of ±4 percent, when the unit conductances along the fins are calculated on the basis of the hydraulic diameter of the finned tube. A complete calculation in which the unit conductances along the fin are based on the fin width (measured parallel to the direction of gas flow) is also shown in table I, and indicates an underestimation of the predicted output of 10 percent. Thus the results for the 52-inch fins verify the conclusion reached in an earlier paragraph, that is, since the 52-inch fins are greater than 13.4 times the hydraulic diameter of the finned tube, calculation of unit conductances along the fin should be based on hydraulic diameter.

Inspection of table II reveals that the average of the ratio of predicted magnitudes of heat output to measured magnitudes for the exchangers with the 6-inch fins is 1.00,* with an average deviation of ±5 percent, when the

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*During the runs with the finned tubes, a leak developed in the exhaust line between the heater and the venturi meter. This leak, unfortunately, was not discovered until the runs were completed. The venturi, therefore, indicated a greater rate of exhaust gas flow than was actually passing through the heating section. In order to estimate what the gas flow through the heater was, the heat loss from the heating section to the ambient air was estimated (10,000 Btu/hr), added to the heat gained by the air, and the weight rate of gas necessary to yield this quantity of heat calculated. These are the values of W_g tabulated in tables I and II. This procedure is justified, since calculation shows that an error as large as ±5000 Btu/hr in the heat loss from the heating section to the ambient air changes the predicted heater output only ±3 percent. The accuracy of the prediction shown in tables I and II is in some measure fortuitous, therefore, but is nevertheless accurate to about ±5 percent.

unit conductances along the fins are calculated on the basis of fin width (measured parallel to the direction of air flow). A tabulation of a complete calculation in which the unit conductances along the fins is based on the hydraulic diameter of the finned tube also is shown in table II. The latter calculation indicates a slight underestimation (3 percent) of the heat output of the exchanger. This slight difference in the predicted magnitudes tends to verify the conclusion that for the 6-inch fins (which are slightly narrower than 13.4 times the hydraulic diameter of the finned tube) the fin width should be utilized in calculating the unit conductance along the fins.

The difference in the two methods of prediction for the 6-inch fins is so small, however, that a series of tests on a much shorter fin (say, 3 in.) should be performed in order further to test the proposed method. These tests are being planned and will be reported in a later section.

It is important, of course, to determine the effect of the fins on the heat output of the exchanger. The effect of longitudinal fins on the rate of heat transfer is fourfold:

(1) The fins increase the effective conductance of the tube as shown by equation (3).

(2) Next, the fins increase the unit thermal conductance along the tube surface by decreasing the cross-sectional area of the tube and thus increasing G for a given weight rate of flow.

(3) Further, the fins increase the wetted perimeter, as well as decrease the cross-sectional area of the tube, thus decreasing the hydraulic diameter of the tube and further increasing the unit thermal conductances.

(4) Lastly, the fins decrease the log mean temperature difference for a fixed temperature difference at the entrance to the heat exchanger, owing to the greater rate of heat transfer.

The total effectiveness of the fins, which considers all four of the above points, may be obtained by dividing the thermal output of the finned heater by the thermal output of the smooth unfinned tube, for the same rates of gas and air flow, and for a fixed temperature difference between gas and air temperatures at the entrance to the exchanger.

It has been shown in appendix B that the thermal output of a parallel flow heat exchanger may be calculated from the equation:

$$q_a = W_a c_{p_a}(\tau_{g1} - \tau_{a1}) \left[\frac{1 - e^{-\left(1+\frac{W_a c_{p_a}}{W_g c_{p_g}}\right)\left(\frac{UA}{W_a c_{p_a}}\right)}}{1 + \frac{W_a c_{p_a}}{W_g c_{p_g}}} \right] \tag{27}$$

Equation (27) accounts for all of the four effects of the fins discussed. The product (UA) for the finned tube is calculated from;

$$(UA)_e = \frac{1}{\left(\frac{1}{fA}\right)_{ea} + \left(\frac{1}{fA}\right)_{eg}} \tag{28}$$

where $(fA)_{ea}$ and $(fA)_{eg}$ are given by equations (10) and (11).

Equation (27) may be utilized directly for the smooth unfinned tube, also, if the product (UA) is calculated from:

$$(UA)_u = \frac{1}{\frac{1}{(f_{cg} + f_r)A_g} + \frac{1}{f_{ca} A_a}} \tag{29}$$

Equation (29) was utilized in reference 1.

By means of equations (27), (28), and (29), the thermal output of the finned and unfinned heat exchangers was calculated for the following conditions:

Average air temperature . . . . 230° F

Average gas temperature . . . . 1300° F

Difference between gas and air temperature at entrance to heating section ($T_{g1} - T_{a1}$) . . . . 1450° F

Equivalent unit conductance for radiation (f_r). . . . . 7 Btu/hr ft² °F

The resulting curves are shown in figure 4 for several gas weight rates, together with several experimental points for a gas rate of 230 pounds per hour. The effectiveness of the finned tubes at any air weight rate may be obtained readily from figure 4 by dividing the magnitudes of the thermal output of the finned tubes by that of the smooth, unfinned tube at equal magnitudes of gas weight rate.* Analytically the total effectiveness of the finned tube may be calculated readily from equations (27), (28), and (29).

$$\text{Effectiveness} = \frac{1 - e^{-\left(1 + \frac{W_a\, c_{p_a}}{W_g\, c_{p_g}}\right)\frac{(UA)_e}{W_a\, c_{p_a}}}}{1 - e^{-\left(1 + \frac{W_a\, c_{p_a}}{W_g\, c_{p_a}}\right)\frac{(UA)_u}{W_a\, c_{p_a}}}} \qquad (30)$$

where $(UA)_e$ is the over-all effective conductance for the finned tube (equation (28)) and $(UA)_u$ is the over-all conductance for the unfinned tube (equation (29)). Such a calculation of the effectiveness, for the finned tubes tested, indicated a range of effectiveness from 1.6 to 1.9, the high values corresponding to the higher weight rates of gas flow. The improvement in total effectiveness at higher gas rates is to be expected, since the relatively short fins on the exhaust gas side become more effective as the gas rate (and thus f_{F_g}) is increased.

In the design of a finned tube heat exchanger the effective conductances on the two sides of the tube should be made as nearly equal as possible, unless the fins are being utilized to lower the temperature of the heat transfer surface. In the latter case the effective conductance on the cool gas side should be larger than that on the hot gas side.

*Since the convective unit thermal conductances depend on the fluid weight rate per unit area, the better criterion for comparison of heat exchangers is G, the fluid rate per unit area, but, in order to illustrate the total effect of the fins the gas weight rates are shown in fig. 4.

PRESSURE DROP

As was discussed in reference 1, the pressure drop along a tube of uniform diameter in which a compressible fluid (gas) is being heated or cooled will not be simply the frictional loss occurring along the tube, but will also include the effect of fluid acceleration. It was shown in reference 1 (p. 22) that the non-isothermal pressure loss is related to the isothermal pressure loss by the expression:

$$(P_1 - P_2) = \Delta P_{T_{iso}} \left(\frac{T_a}{T_{iso}}\right)^{1.13} + \frac{G^2}{(3600)^2 \gamma_1 g} \left(\frac{T_2}{T_1} - 1\right) \quad (31)$$

where

$P_1 - P_2$	non-isothermal pressure loss, lb/ft^2
$\Delta P_{T_{iso}}$	isothermal pressure loss at temperature T_{iso}, lb/ft^2
T_a	arithmetic average gas temperature in tube, ^oR
T_1	gas temperature entering heater, ^oR
T_2	gas temperature leaving heater, ^oR
γ_1	density of gas entering heater, lb/ft^3
G	weight rate of gas flow per unit area, $lb/hr\ ft^2$
g	$32.2\ ft/sec^2$

This equation was applied to the non-isothermal pressure drop data obtained for the two finned tubes. The calculated pressure drop $\Delta P_{T_{iso}}$ was then corrected to 78° F as outlined on page 23 of reference 1. These calculated isothermal pressure drops as shown in figures 5 and 6 check the measured isothermal pressure drop data very closely (less than 10 percent discrepancy). These data are further proof of the applicability of equation (31) to the prediction of non-isothermal pressure drops in gas-air heat exchangers.*

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*Actually equation (31) is only an approximation, since it is based on a linear increase in gas temperature with length. The increase is more nearly exponential.

In addition to changing the magnitude of the pressure drop, equation (31) changes the slope of the $\Delta P/\Delta N$ against W_a curve as shown in figure 5. The slope of the non-isothermal pressure drop curve is less than the isothermal one. This is to be expected for lower air rates, since the temperature rise of the air passing through the heater is large, thus causing a larger pressure drop owing to the greater exit velocity of the warm air.

The isothermal friction factor for the finned tubes was calculated, utilizing the hydraulic radius of the finned tube as the significant dimension in the Reynolds modulus and in the equation defining the friction factor. Then

$$Re = \frac{G\ D_a}{3600\ g\ \mu_a} \tag{32}$$

and

$$\frac{\Delta P_T}{\gamma_T} = \zeta \left(\frac{N}{D_a}\right) \frac{G^2}{2g(3600)^2 \gamma_T^2} \tag{33}$$

where D_a = the hydraulic diameter of the finned tube.

Satisfactory correlation between the isothermal friction factor for the finned 52-inch and 6-inch tubes and the smooth tube discussed in reference 1 resulted. The points for all three tubes fell within 15 percent of the recommended curve for "commercial" pipe (reference 3, p. XI-8) (fig. 7). Thus it appears that, for longitudinally finned tubes with wide fins, the pressure drop may be satisfactorily predicted by means of the normal friction factor data. The increased pressure drop for the finned tube is due mainly to its increased wetted perimeter.

When narrow fins are utilized, however, the above conclusions probably will no longer apply, owing to the increased pressure drop produced by eddies, and so forth, on the downstream side of each narrow fin.*

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*The 6-in. fin is so close to being a wide fin that it is considered as such in the pressure drop discussion. Actually the ratio of l/D_H for the 6-inch fin is 10.9, thus causing it to fall in the narrow fin specification as discussed on p. 12.

CONCLUSIONS

1. The effect of longitudinal fins may be predicted successfully (±5 percent) by utilizing equation (8) to calculate the effective conductance of the finned tube.

2. If the ratio $\frac{2 f_F L^2}{k s}$ for a fin is of the order of magnitude of 0.10, the thermal conductivity of the fin material and its thickness are unimportant in establishing the fin performance. If the ratio $\frac{2 f_F L^2}{k s}$ is of the order of magnitude of 4.0, fin length measured perpendicularly to the tube surface becomes unimportant.

3. It appears, although it has not been conclusively proved,* that if a longitudinal fin is wider (measured in the direction of fluid flow) than about 13.4 times the hydraulic diameter of the finned tube, the unit conductances along the fin surface should be based on the hydraulic diameter (equation (13)). If the fin is narrower than 13.4 times the hydraulic diameter, the unit conductance should be based on the fin width (equation (12)).

4. The 6-inch fins indicated a slight advantage (3 percent) over the 52-inch fins. Conclusion 3 predicts that the use of still narrower fins would produce greater improvement in heat-transfer performance. The almost negligible improvement for the 6-inch fins, however, may be due in part to the very small gap between fins (1/16 in.). A greater gap or staggering of the fins would perhaps improve the fin performance.

5. For the same weight rates of flow, the finned tube provided about 1.75 times the thermal output of the smooth, unfinned tube. The improvement in output is, of course, a function of the fin design and this particular increase of output by the use of fins applies only to the unit tested.

6. Equation (31) may be used to predict the non-isothermal pressure drop across longitudinally finned double tube heat exchangers, if the isothermal drop is

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*Tests are planned on narrow fins (3 in.) to test further the applicability of the flat plate equation for narrow fins.

known. The isothermal and non-isothermal pressure drops differ greatly because of the large difference in velocity of the gases entering and leaving the exchanger.

7. The isothermal friction factor for the longitudinally finned tubes tested compared satisfactorily with commercial pipe friction factor data.

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APPENDIX A

MIXING CHAMBER

In order to obtain an accurate mixed mean temperature of the air leaving the heat exchanger (reference 1, p. 18), a mixing chamber was designed which produced a very uniform temperature across the mixing chamber outlet. This mixing chamber, composed of a set of perforated baffles, produced an excessive pressure drop which, in turn, reduced the air blower capacity. To remedy this, the original mixing chamber was progressively stripped of baffles until a satisfactory compromise was reached between the pressure drop and the effectiveness of mixing. The final design is shown in figure 8. A similar design was used by Colburn and Coghlan (reference 14). The pressure drop across the unit is about 2 in. CCl at a weight rate of air of 400 lb/hr.

The mixing chamber shown in fig. 8 allowed the determination of the mixed mean fluid temperature by means of a single thermocouple placed at the center of the stream with a maximum error in the temperature rise of the fluid passing through the heater of about 1 percent for the finned tubes, 2 percent with the dimpled tubes, and 5 percent with the smooth, unfinned tube.

As discussed above, a more precise mixing chamber was available but was not utilized because it introduced too great a pressure drop. A maximum error of 5 percent was considered permissible for the tests. In reference 1 concerning the smooth and dimpled tubes, it was shown that the omission of a mixing chamber introduced errors in the temperature rise of the fluid as high as 50 percent.

APPENDIX B

HEAT TRANSFER TO A FLUID FROM A LONGITUDINALLY FINNED TUBE

INTRODUCTION

An analytical expression for the heat transfer from a longitudinally finned tube is presented below:

Case I

Heat Transfer Equations for Longitudinally Finned Tubes when the Tube Wall Temperature is Known

The following system is defined:

1. There is no heat flow in the fins in the direction of fluid flow.

2. The heat transfer through the ends of the fins into the air stream is negligible.

3. The tube wall temperature is invariable with x and the base of the fin is at the tube wall temperature.

4. The unit conductance for heat transfer is the same over the fin as over the unfinned surface of the tube wall, and constant with length and temperature.*

5. Radiant heat transfer is zero.**

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*The unit convective thermal conductance may not always be independent of length, but may vary inversely with a fractional power of the fin width in the direction of fluid flow. However, the utilization of an average magnitude of thermal conductance in the length x will not introduce serious errors.

**Unless this postulate is made, the equation derived below cannot be integrated readily because of the manner in which radiant heat transfer varies with temperature. However, at the rates of fluid flow met in practice, the radiant heat transfer may be either neglected as postulated here or included in the magnitude of the unit thermal conductance by utilizing an equivalent thermal conductance for radiation.

6. The thickness of a fin is small compared to its length.

In a length of tube dx the change in enthalpy of the air will be:

$$dq = W\, c_p\, d\, \tau_x \qquad (B1)$$

The heat flow from the tube will be that from the fins plus that from the unfinned surface of the tube (reference 3, p. II-26)

$$dq = n\sqrt{f\ k\ dP\ dA}\ (t_p - \tau_x)\ \tanh\sqrt{\frac{f}{k}\frac{dP}{dA}}\ L$$

$$+ f(t_p - \tau_x)(\pi D - ns)dx \qquad (B2)$$

but

$$dP \cong 2dx$$

$$dA \cong s dx$$

$$dq = \left[n\sqrt{2skf}\ \tanh\sqrt{\frac{2f}{ks}}\ L + f(\pi D - ns)\right](t_p - \tau_x)dx \qquad (B2a)$$

Equating (B1) and (B2a), and defining X as:

$$X \equiv n\sqrt{2skf}\ \tanh\sqrt{\frac{2f}{ks}}\ L + f(\pi D - ns)$$

gives:

$$\frac{d\tau_x}{(t_p - \tau_x)} = \frac{X}{W\ c_p}\ dx \qquad (B3)$$

The integrated form is:

$$t_p - \tau_x = (t_p - \tau_1)e^{-\frac{X}{W\ c_p}x} \qquad (B4)$$

which may be substituted into equation B2a and integrated to yield:

$$q = W c_p (t_p - \tau_1) \left(1 - e^{-\frac{X}{W c_p} x}\right) \qquad (B5)$$

This is the general equation for the heat flow from a longitudinally finned tube of length x.

The term $W c_p(t_p - \tau_1) = q_{max}$ is the maximum possible heat exchange from tube to fluid, since the fluid cannot exceed the tube wall temperature.

Equation (5) may be expressed:

$$\frac{q}{q_{max}} = \left(1 - e^{-\frac{X}{W c_p} x}\right) = \left(1 - \frac{t_p - \tau_x}{t_p - \tau_1}\right) = \left(\frac{\tau_x - \tau_1}{t_p - \tau_1}\right) \qquad (B5a)$$

This equation is plotted in figs. 10, 11, and 12.

When $n = 0$, the equation represents the heat transfer from an unfinned tube.

$$q_{n=o} = W c_p (t_p - \tau_1) \left(1 - e^{-\frac{X_{n=o}}{W c_p} x}\right) \qquad (B6)$$

where

$$X_{n=o} = f\pi D$$

The effectiveness of the fins is denoted by

$$\frac{q}{q_{n=o}} = \frac{\left(1 - e^{-\frac{X}{W c_p} x}\right)}{\left(1 - e^{-\frac{X_{n=o}}{W c_p} x}\right)} \qquad (B7)$$

and is plotted in figs. 13 and 14.

Similar analyses were performed for the case for which the temperature at the base of the fin t_w was postulated to vary linearly and exponentially with length x.

Expressing the temperature at the base of the fin as a linear function of the distance,

$$t_p = C x + B$$

yields

$$q = W c_p \left[\left(B - C \frac{W c_p}{X} \right) \left(1 - e^{- \frac{X}{W c_p} x} \right) + C x \right] \quad (B8)$$

The constants B and C are evaluated by plotting the temperature distribution as a function of length. The constant B is the value of the intercept and C is the slope of the temperature-distance line.

If the temperature of the base is an exponential function of the distance along the tube,

$$t_p = E e^{-Fx}$$

yields

$$q = \left(X E \Big/ \frac{X}{W c_p} - F \right) \left(e^{-Fx} - e^{- \frac{X}{W c_p} x} \right) \quad (B9)$$

The constants E and F are evaluated from a semi-logarithmic plot of the temperature distribution.

DISCUSSION

Equation (B5a) has been plotted in three different forms, the ratio q/q_{max} and its equivalent $\frac{T_x - T_1}{t_p - T_1}$ against $\frac{X}{W c_p} x$, x, and n. Each plot exhibits sufficiently different characteristics to warrant separate consideration. The equation in generalized dimensionless form has been presented in fig. 10. This plot may be used to determine the manner in which variation in fin material and dimensions, tube dimensions, and rate of gas flow affects the performance of the finned tube.

Fig. 11 is a replot of fig. 10 showing q/q_{max} against x, with $\frac{X}{W c_p}$ as parameter. This plot enables one to observe that two short tubes in parallel will give a higher heat flow than one tube of twice the length with the same number of fins, but the temperature rise of the air will be less.

From fig. 11: For a 3-ft tube $q/q_{max} = 0.36$
For a 6-ft tube $q/q_{max} = 0.59$

A 20-percent increase in the rate of heat transfer is thus accomplished with two 3-ft tubes in parallel, as compared with one 6-ft tube. The temperature rise, however, is 60 percent of the value obtained by using one 6-ft tube.

A plot of q/q_{max} against n (fig. 12) based on fig. 10 allows the observation that two tubes with half the number of fins permit a higher heat transfer rate than one tube with twice the number of fins for the same length.

From fig. 12: At $x = 4.75$ ft

4 fins $q/q_{max} = 0.51$

8 fins $q/q_{max} = 0.64$

A 60.6-percent increase in the rate of heat transfer is thus obtained with two tubes having four fins each. Here, again, the temperature rise is 80.3 percent of the value for one tube.

In order to plot the equations, average values of data from laboratory tests were used.

The advantage of a finned tube over a tube without fins is obtained upon inspection of figs. 13 and 14, plots of equation (B7). From fig. 13 a plot of $q/q_{n=o}$ against x reveals that for a given length the effectiveness is increased as the number of fins increases. This is particularly true for short lengths. For long lengths the effectiveness is changed little for large increases in the number of fins.

Fig. 14, $q/q_{n=o}$ against n, indicates the manner in which the effectiveness decreases as the tube increases in length with the same number of fins.

A careful inspection of the equations and figures does not indicate any practical value of the optimum number of fins or length of tube. It should be emphasized, however, that other, more important considerations govern the selection of the number of fins and the length of tube, such as pressure drop, specified lengths, temperature rise, and quantity of fluid.

Case II

Heat Transfer Equations for Finned or Unfinned Heat Exchangers when the Tube Wall Temperature Is not Known and the Heat Loss to the Surroundings is Zero

The heat absorbed by the cool air at any point x is given by

$$d\,q_a = W_a\,c_{p_a}\,d\,\tau_{a_x} = (U dA)(\tau_{g_x} - \tau_{a_x}) \qquad \text{(B10)}$$

The heat given up by the hot gases at any point x is given by

$$d\,q_g = -W_g\,c_{p_g}\,d\,\tau_{g_x} = (U dA)\,(\tau_{g_x} - \tau_{a_x}) \qquad \text{(B11)}$$

Equations (B10) and (B11) may be rewritten as

$$\frac{d\,\tau_{a_x}}{(\tau_{g_x} - \tau_{a_x})} = \frac{U\,dA}{W_a\,c_{p_a}} \qquad \text{(B12)}$$

and

$$\frac{d\,\tau_{g_x}}{(\tau_{g_x} - \tau_{a_x})} = -\frac{U\,dA}{W_g\,c_{p_g}} \qquad \text{(B13)}$$

Subtracting equation (B12) from (B13) gives the following expression:

$$-U\left(\frac{1}{W_g\,c_{p_g}} + \frac{1}{W_a\,c_{p_a}}\right) dA = \frac{d(\tau_{g_x} - \tau_{a_x})}{(\tau_{g_x} - \tau_{a_x})} \qquad \text{(B14)}$$

When U and c_p are constant along the heat exchanger, equation (B14) may be integrated along its length from point 1 at entrance to point 2 at the exit end of the exchanger. The result is

$$\ln\left(\frac{\tau_{g_2} - \tau_{a_2}}{\tau_{g_1} - \tau_{a_1}}\right) = -\left(\frac{1}{W_g\,c_{p_g}} + \frac{1}{W_a\,c_{p_a}}\right)(UA)$$

This equation may be written as

$$\frac{T_{g2} - T_{a2}}{T_{g1} - T_{a1}} = e^{-\left(\frac{1}{W_g\ c_{p_g}} + \frac{1}{W_a\ c_{p_a}}\right)(UA)} \qquad (B15)$$

When both sides of equation (B15) are subtracted from unity and multiplied by $(T_{g1} - T_{a1})$, the result is

$$T_{g1} - T_{a1} - T_{g2} + T_{a2} = (T_{g1} - T_{a1})\left(1 - e^{-\left(\frac{1}{W_g\ c_{p_g}} + \frac{1}{W_a\ c_{p_a}}\right)(UA)}\right) \qquad (B16)$$

When the heat loss to the surroundings is zero, $q_a = q_g$ and separate integration of equations (B10) and (B11) yields

$$\frac{T_{g1} - T_{g2}}{T_{a1} - T_{a2}} = -\frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}$$

Substituting this last equation into equation (B16) and rearranging terms

$$(T_{a2} - T_{a1}) = \frac{T_{g1} - T_{a1}}{1 + \frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}}\left[1 - e^{-\left(\frac{1}{W_g\ c_{p_g}} + \frac{1}{W_a\ c_{p_a}}\right)(UA)}\right]$$

Multiplying both sides of this equation by $W_a\ c_{p_a}$ and rearranging the exponential term gives equation (27).

$$q_a = W_a\ c_{p_a}(T_{g1} - T_{a1})\left[\frac{1 - e^{-\left(1 + \frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}\right)\frac{UA}{W_a\ c_{p_a}}}}{1 + \frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}}\right] \qquad (B17)$$

Equation (B17) will yield exactly the same results as equation (9); in equation (B17) the log mean temperature difference has been evaluated in terms of known quantities, thereby allowing an explicit solution for q_a.

For a smooth unfinned tube (UA) is given by equation (29).

$$(UA)_u = \frac{1}{\dfrac{1}{(f_{c_g} + f_r)A_g} + \dfrac{1}{f_{c_a}\, A_a}} \qquad \text{(B18)}$$

while for a longitudinally finned tube, (UA) is given by equation (28).

$$(UA)_e = \frac{1}{\dfrac{1}{(fA)_{ea}} + \dfrac{1}{(fA)_{eg}}} \qquad \text{(B19)}$$

where $(fA)_{ea}$ and $(fA)_{eg}$ are given by equations (10) and (11).

The ratio of the rate of heat transfer from a longitudinally finned tube to that of a smooth unfinned tube is obtained by substituting equations (28) and (29) in equation (27) to yield equation (30).

$$\text{Effectiveness} = \frac{q}{q_{n=o}} = \frac{q_{a(\text{fins})}}{q_{a(\text{no fins})}}$$

$$= \frac{1 - e^{-\left(1 + \frac{W_a\, c_{p_a}}{W_g\, c_{p_g}}\right)\frac{(UA)_e}{W_a\, c_{p_a}}}}{1 - e^{-\left(1 + \frac{W_a\, c_{p_a}}{W_g\, c_{p_g}}\right)\frac{(UA)_u}{W_a\, c_{p_a}}}} \qquad \text{(B20)}$$

It should be noted that equation (B7) is a special case of the more general equation (B20). The former equation evaluates the effectiveness of the fins based on a constant tube wall temperature.

The term $(X\ x)$ in equation (B7) corresponds to $\left(1 + \frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}\right)(UA)_e$ in equation (B20) and the term $(X_{n=o}\ x)$ corresponds to $\left(1 + \frac{W_a\ c_{p_a}}{W_g\ c_{p_g}}\right)(UA)_u$.

The foregoing equations are applicable to parallel flow in double tube heat exchangers. Similar equations for contraflow of fluids are given in reference 3 (p. XIV-4).

The authors gratefully acknowledge the contributions of Messrs. M. Tribus, D. DuBain, M. A. Miller, F. Hamaker, and C. Buchter in the performance of the tests and the preparation of the report, and of Messrs. H. Eagles and H. Poeland in the construction of the equipment.

REFERENCES

1. Martinelli, R. C., Weinberg, E. B., Morrin, E. H., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. III - Measured and Predicted Performance of Double Tube Heat Exchangers.

2. Peirce, B. O.: A Short Table of Integrals. Ginn and Co., 1910, p. 93.

3. Boelter, L. M. K., Cherry, V. H., and Johnson, H. A.: Supplementary Heat Transfer Notes. Univ. of Calif. Press, 3d ed., 1942.

4. Goldstein, S., Ed.: Modern Developments in Fluid Dynamics. Oxford Univ. Press, 1938.

5. Colburn, Allan P.: A Method of Correlating Forced Convection Heat Transfer Data and a Comparison with Fluid Friction. Trans., Am. Inst. of Chem. Eng., vol. XXIX, 1933, pp. 174-210.

6. Norris, R. H., and Spofford, W. A.: High-Performance Fins for Heat Transfer. Trans., A.S.M.E., vol. 64, no. 5, July 1942, pp. 489-496.

7. ten Bosch, M.: Die Wärmeübertragung. Julius Springer (Berlin), 1936.

8. Boelter, L. M. K., Martinelli, R. C., and Jonassen, Finn: Remarks on the Analogy between Heat Transfer and Momentum Transfer. Trans., A.S.M.E., vol. 63, July 1941, pp. 447-455.

9. Martinelli, R. C., Tribus, M., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. I - Elementary Heat Transfer Considerations in an Airplane. NACA ARR, Oct. 1942.

10. Mattioli, G. D.: Theorie der Wärmeübertragung in glatten und rauhen Rohren. Forschung auf dem Gebiete des Ingenieurwesens, Ausgabe A, Bd. 11, no. 4, July-Aug. 1940, pp. 149-158.

11. Reichardt, H.: Die Wärmeübertragung in turbulenten Reibungschichten. Z.f.a.M.M., Bd. 20, no. 6, Dec. 1940, pp. 297-328.

12. Hottel, H. C., and Egbert, R. B.: Radiant Heat Transmission from Water Vapor. Trans., Am. Inst. Chem. Eng., vol. 38, June 25, 1942, p. 531.

13. McAdams, W. H.: Heat Transmission. McGraw-Hill Book Co., Inc., 1933.

14. Colburn, A. P., and Coghlan, C. A.: Heat Transfer to Hydrogen-Nitrogen Mixtures inside Tubes. Trans., A.S.M.E., vol. 63, no. 7, Oct. 1941, p. 563.

TABLE I. TABULATED RESULTS AND DATA (52" Fins).

Run No.	G_a $\left(\frac{lb}{hr\ ft^2}\right)$	G_g $\left(\frac{lb}{hr\ ft^2}\right)$	τ_{a1} (°F)	τ_{a2} (°F)	τ_{g1} (°F)	τ_{g2} (°F)	t_p (°F)	f_{cg} ←	f_{ca}	f_{Fg} $\left(\frac{BTU}{hr\ ft^2\ °F}\right)$	f_{Fa}	f_r →	q_a $\left(\frac{BTU}{hr}\right)$	q_M $\left(\frac{BTU}{hr}\right)$	Ratio $\frac{q_a}{q_M}$
							1. Unit Conductances Calculated on Basis of Hydraulic Diameter								
N-2	19,300	8,300	101	563	1579	900	795	11.7	19.9	11.7	19.9	8.3	32,800	32,900	1.00
-3	23,000	8,850	106	526	1547	876	730	12.2	22.8	12.2	22.8	7.7	34,600	35,400	0.98
-5	22,600	8,050	112	480	1460	809	765	11.3	22.0	11.3	22.0	7.5	30,400	30,000	1.01
-7	29,400	6,750	124	440	1621	774	695	9.9	27.1	9.9	27.1	7.5	34,300	33,700	1.02
-8	23,100	7,100	118	490	1628	835	750	10.3	22.6	10.3	22.6	8.2	33,000	32,500	1.02
-9	19,700	6,850	110	519	1620	863	810	10.0	20.2	10.0	20.2	8.5	32,000	29,600	1.08
-10	14,300	6,400	99	576	1630	913	890	9.6	15.6	9.6	15.6	8.3	28,400	25,100	1.13
-11	13,000	6,350	98	552	1494	848	800	9.2	14.5	9.2	14.5	6.4	23,000	21,800	1.05
-12	19,400	6,600	106	471	1492	780	700	9.6	19.6	9.6	19.6	6.5	25,900	25,800	1.00
-13	22,200	7,000	111	443	1490	758	675	10.0	21.7	10.0	21.7	6.3	27,600	27,100	1.02
-14	29,200	6,600	130	404	1497	714	600	9.6	27.0	9.6	27.0	6.0	27,900	29,400	0.95
-15	14,500	7,750	112	490	1299	778	670	10.7	15.7	10.7	15.7	4.5	20,400	20,800	0.98
-16	20,200	7,850	112	434	1302	735	590	10.8	20.0	10.8	20.0	4.6	23,200	24,000	0.97
-17	23,500	7,900	115	409	1301	713	575	10.8	22.5	10.8	22.5	4.7	24,700	25,400	0.97
-18	30,500	6,800	121	347	1301	618	465	9.5	27.5	9.5	27.5	4.2	22,500	25,300	0.89
							2. Unit Conductances Calculated on Basis of Fin Width								
N-2	19,300	8,300	101	563	1579	900	795	11.7	19.9	8.8	13.5	8.3	29,500	32,900	0.91
-3	23,000	8,850	106	526	1547	876	730	12.2	22.8	9.1	15.3	7.7	30,000	35,400	0.85
-5	22,600	8,050	112	480	1460	809	765	11.3	22.0	8.5	15.0	7.5	27,400	30,000	0.91
-7	29,400	6,750	124	440	1621	774	695	9.9	27.1	7.4	18.4	7.5	31.300	33,700	0.93
-8	23,100	7,100	118	490	1628	835	750	10.3	22.6	7.7	15.2	8.2	30,100	32,500	0.92
-9	19,700	6,850	110	519	1620	863	810	10.0	20.2	7.5	13.6	8.5	28,700	29,600	0.97
-10	14,300	6,400	99	576	1630	913	890	9.6	15.6	7.2	10.6	8.3	25,200	25,100	1.00
-11	13,000	6,350	98	552	1494	848	800	9.2	14.5	6.9	9.8	6.4	20,300	21,800	0.93
-12	19,400	6,600	106	471	1492	780	700	9.6	19.6	7.2	13.2	6.5	23,400	25,800	0.91
-13	22,200	7,000	111	443	1490	758	675	10.0	21.7	7.5	14.6	6.3	25,000	27,100	0.92
-14	29,200	6,600	130	404	1497	714	600	9.6	27.0	7.2	18.2	6.0	25,600	29,400	0.87
-15	14,500	7,750	112	490	1299	778	670	10.7	15.7	8.0	10.6	4.5	18,100	20,800	0.86
-16	20,200	7,850	112	434	1302	735	590	10.8	20.0	8.1	13.5	4.6	21,000	24,000	0.87
-17	23,500	7,900	115	409	1301	713	575	10.8	22.5	8.1	15.3	4.7	22,100	25,000	0.87
-18	30,500	6,800	121	347	1301	618	465	9.5	27.5	7.1	18.7	4.2	20,500	25,300	0.81

TABLE II. TABULATED RESULTS AND DATA (6" Fins).

Run No.	G_a $\left(\frac{lb}{hr\ ft}\right)$	G_g $\left(\frac{lb}{hr\ ft}\right)$	τ_{a1} (°F)	τ_{a2} (°F)	τ_{g1} (°F)	τ_{g2} (°F)	t_p (°F)	f_{cg} ←	f_{ca}	f_{Fg} $\left(\frac{BTU}{hr\ ft\ °F}\right)$	f_{Fa}	f_r →	q_a $\left(\frac{BTU}{hr}\right)$	q_M $\left(\frac{BTU}{hr}\right)$	Ratio $\frac{q_a}{q_M}$
					1. Unit Conductances Calculated on Basis of Fin Width										
J -1	19,000	8,450	89	538	1528	881	760	11.7	19.5	13.5	20.4	7.6	32,200	31,600	1.02
-2	28,600	8,650	106	444	1542	819	650	11.8	27.0	13.6	28.5	7.2	37,200	37,100	1.00
-3	20,400	8,800	98	523	1505	878	750	12.1	20.5	14.0	21.5	7.5	33,200	32,100	1.03
-4	19,600	8,650	102	453	1310	770	630	11.8	19.6	13.6	19.7	4.9	25,800	25,600	1.01
-5	21,200	8,500	105	444	1323	760	625	11.5	20.8	13.3	21.8	5.2	26,700	26,600	1.00
-6	30,300	8,150	120	379	1328	699	525	11.2	27.5	12.9	28.6	4.9	28,400	29,100	0.98
-7	18,800	8,300	104	579	1617	949	860	11.7	19.4	13.5	20.4	8.8	35,500	32,100	1.11
-8	22,000	9,300	108	567	1623	956	825	12.8	22.0	14.8	23.0	9.1	39,500	37,400	1.06
-9	28,900	9,500	117	504	1616	904	730	12.9	27.3	14.9	28.6	8.5	43,800	41,500	1.05
-10	34,500	8,200	84	323	1325	678	510	11.2	30.2	12.9	31.0	4.7	31,200	30,600	1.02
-11	41,700	8,350	84	294	1317	652	420	11.2	34.5	12.9	36.0	4.3	30,800	32,400	0.95
-12	36,400	8,100	76	360	1516	725	525	11.2	31.5	12.9	32.8	6.1	35,900	38,200	0.94
-13	40,000	8,200	73	335	1516	706	500	11.3	33.8	13.0	35.3	5.9	36,800	38,800	0.95
-14	35,500	9,000	78	414	1611	817	580	11.9	31.2	13.7	32.5	7.1	41,000	44,300	0.93
-15	39,300	8,900	74	389	1611	799	555	12.2	33.5	14.1	35.2	6.9	42,000	45,800	0.92
					2. Unit Conductances Calculated on Basis of Hydraulic Diameter										
J -1	19,000	8,450	89	538	1528	881	760	11.7	19.5	11.7	19.5	7.6	31,400	31,600	0.99
-2	28,600	8,550	106	444	1542	819	650	11.8	27.0	11.8	27.0	7.2	36,000	37,100	0.97
-3	20,400	8,800	98	523	1505	878	750	12.1	20.5	12.1	20.5	7.5	32,400	32,100	1.01
-4	19,600	8,650	102	453	1310	770	630	11.8	19.6	11.8	19.6	4.9	25,200	25,600	0.98
-5	21,200	8,500	105	444	1323	760	625	11.5	20.8	11.5	20.8	5.2	26,000	26,600	0.98
-6	30,300	8,150	120	379	1328	699	525	11.2	27.5	11.2	27.5	4.9	27,600	29,100	0.95
-7	18,800	8,300	104	579	1617	949	860	11.7	19.4	11.7	19.4	8.8	34,400	32,100	1.07
-8	22,000	9,300	108	567	1623	956	825	12.8	22.0	12.8	22.0	9.1	38,500	37,400	1.03
-9	28,900	9,500	117	504	1616	904	730	12.9	27.3	12.9	27.3	8.5	40,200	41,500	0.97
-10	34,500	8,200	84	323	1325	678	510	11.2	30.2	11.2	30.2	4.7	30,600	30,600	1.00
-11	41,700	8,350	84	294	1317	652	420	11.2	34.5	11.2	34.5	4.3	29,800	32,400	0.92
-12	36,400	8,100	76	360	1516	725	525	11.2	31.5	11.2	31.5	6.1	35,100	38,200	0.92
-13	40,000	8,200	73	335	1516	706	500	11.3	33.8	11.3	33.8	5.9	35,700	38,800	0.92
-14	35,500	9,000	78	414	1611	817	580	11.9	31.2	11.9	31.2	7.1	39,900	44,300	0.90
-15	39,300	8,900	74	389	1611	799	555	12.2	33.5	12.2	33.5	6.9	41,100	45,800	0.90

TABLE III.-ISOTHERMAL PRESSURE DROP - AIR SIDE

Run	W_a (lb/hr)	G_a (lb/hr ft^2)	T_1 (°F)	Measured pressure drop* (lb/ft^2/ft)	Reynolds modulus*	Friction factor*
Finned Tube (52-in. fins) (distance between pressure taps, 6.25 ft)						
KIF-1	375	24,600	78	6.35	25,400	0.0296
-2	416	27,200	78	7.68	28,100	.0294
-3	584	38,300	78	13.4	39,300	.0256
-4	536	35,100	78	11.7	36,200	.0268
-5	589	38,300	78	12.5	39,300	.0242
-6	640	42,000	78	15.8	43,300	.0254
Finned Tube (6-in. fins) (distance between pressure taps, 6.20 ft)						
LI-1	573	37,500	83	13.6	38,500	0.0270
-2	502	32,900	83	10.5	33,800	.0270
-3	428	28,000	83	8.29	28,800	.0294
-4	338	22,100	83	5.69	22,700	.0323

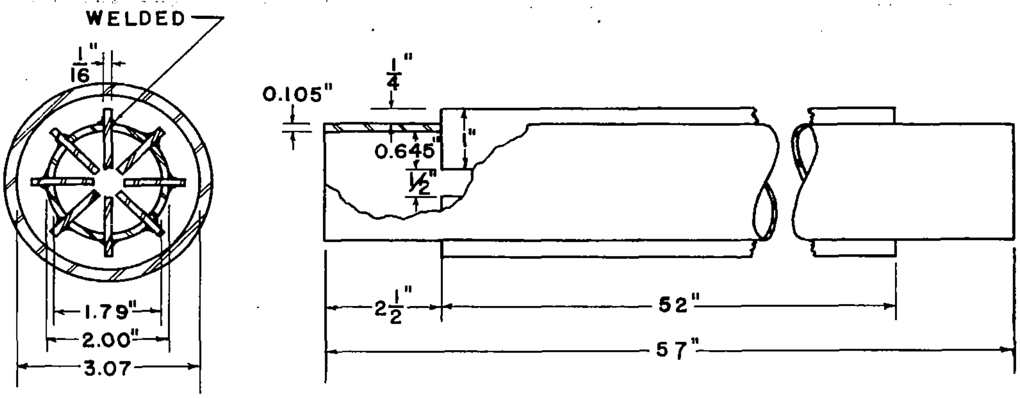
*Corrected to 78° F. (See equation (33) of this report and equation (25) of reference 1.)

TABLE IV.-NON-ISOTHERMAL PRESSURE DROP - AIR SIDE

Run	W_a (lb/hr)	G_a $\left(\frac{lb}{hr\ ft^2}\right)$	T_1 (°R)	T_2 (°R)	T_a (°R)	Measured non-isothermal pressure drop per ft ($lb/ft^2/ft$)	Isothermal pressure drop calculated from non-isothermal pressure drop* ($lb/ft^2/ft$)
Finned Tube (52-in. fins)(distance between pressure taps, 6.25 ft)							
M-1	514	33,600	538	785	667	15.2	9.82
-2	378	24,800	542	798	670	10.3	6.92
-3	538	35,200	540	757	649	15.9	10.7
-4	459	30,000	542	770	656	12.9	8.80
-5	393	25,700	546	824	695	10.5	6.58
-6	323	21,100	545	858	701	8.38	5.20
-7	599	39,200	550	763	656	20.4	13.8
Finned Tube (6-in. fins)(distance between pressure taps, 6.20 ft)							
LL-1	586	38,300	549	780	664	23.6	15.9
-2	520	34,100	553	790	671	18.7	12.4
-3	331	21,700	546	872	709	10.0	6.26
-4	556	36,400	546	797	671	20.5	13.5

*Corrected to 78° F. (See equation (31) and footnote on table III of this report.)

FINNED TUBE (52")



CROSS – SECTIONAL AREAS

AIR SIDE = 0 01526 ft.^{2}

GAS SIDE = 0.0286 ft.^{2}

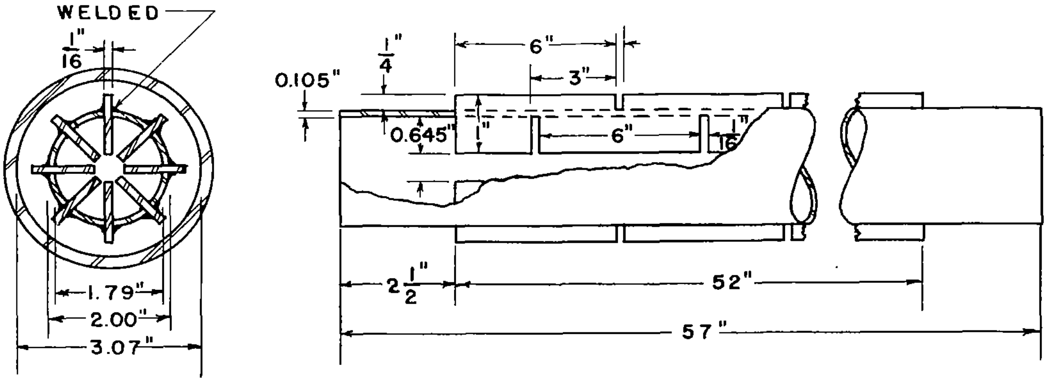
HYDRAULIC DIAMETERS

AIR SIDE = 0.0460 ft.

GAS SIDE = 0.0688 ft.

fig. 1

FINNED TUBE (6")



CROSS–SECTIONAL AREAS

AIR SIDE = 0.01526 ft.^{2}

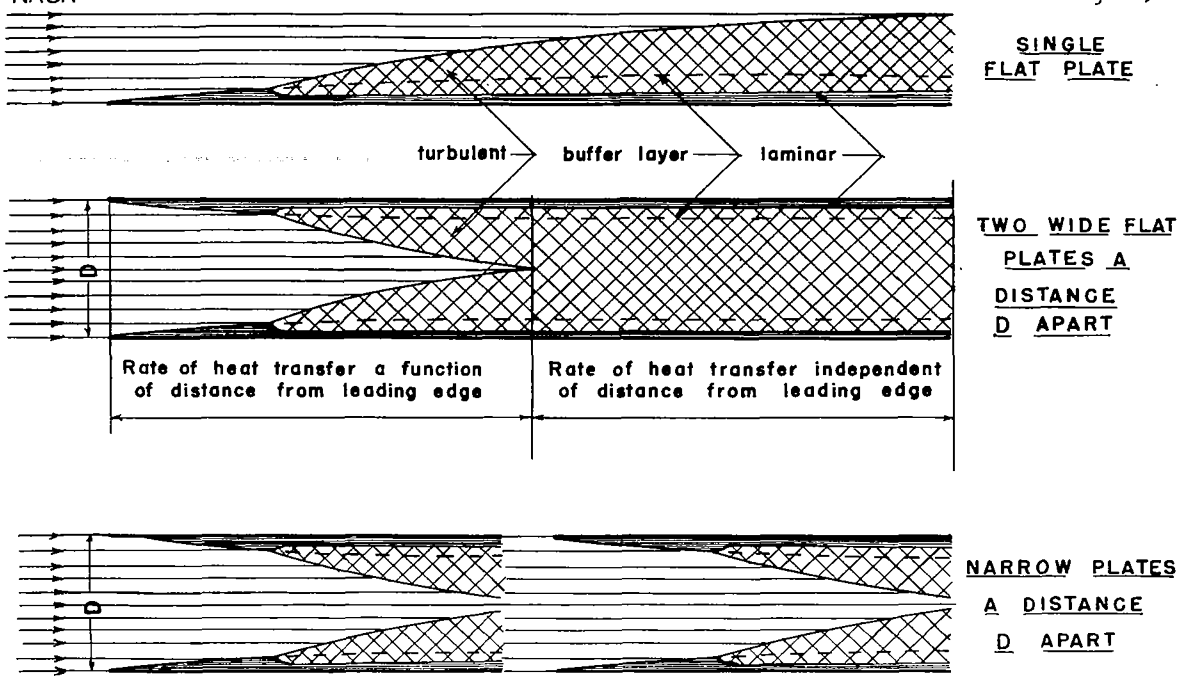
GAS SIDE = 0.0286 ft.^{2}

HYDRAULIC DIAMETERS

AIR SIDE = 0.0460 ft.

GAS SIDE = 0.0688 ft.

fig. 2



IDEALIZED FLOW ALONG FLAT PLATE FINS

fig. 3

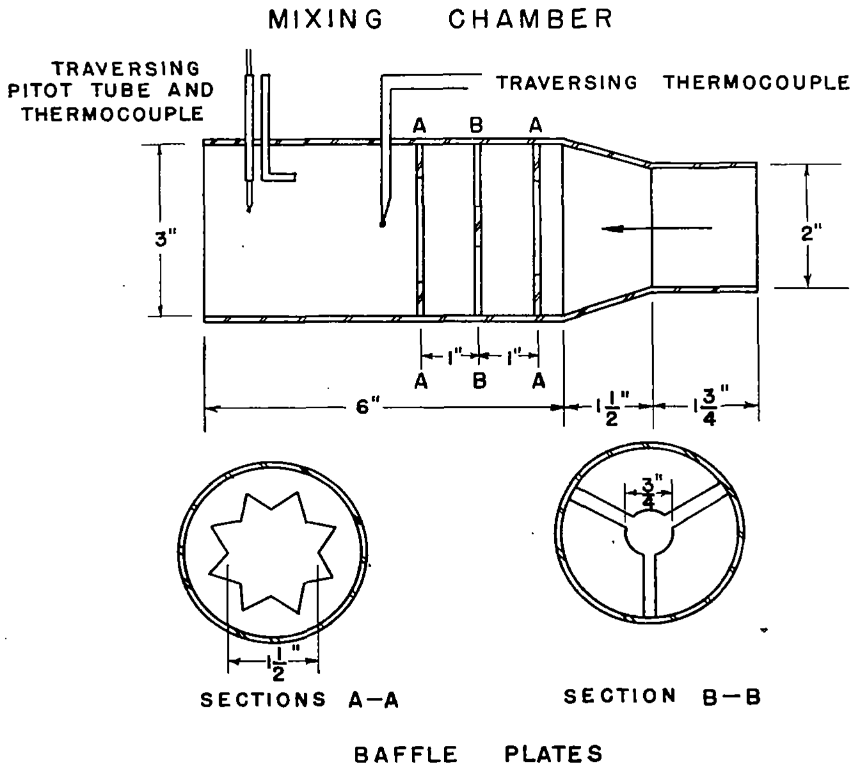


fig. 8

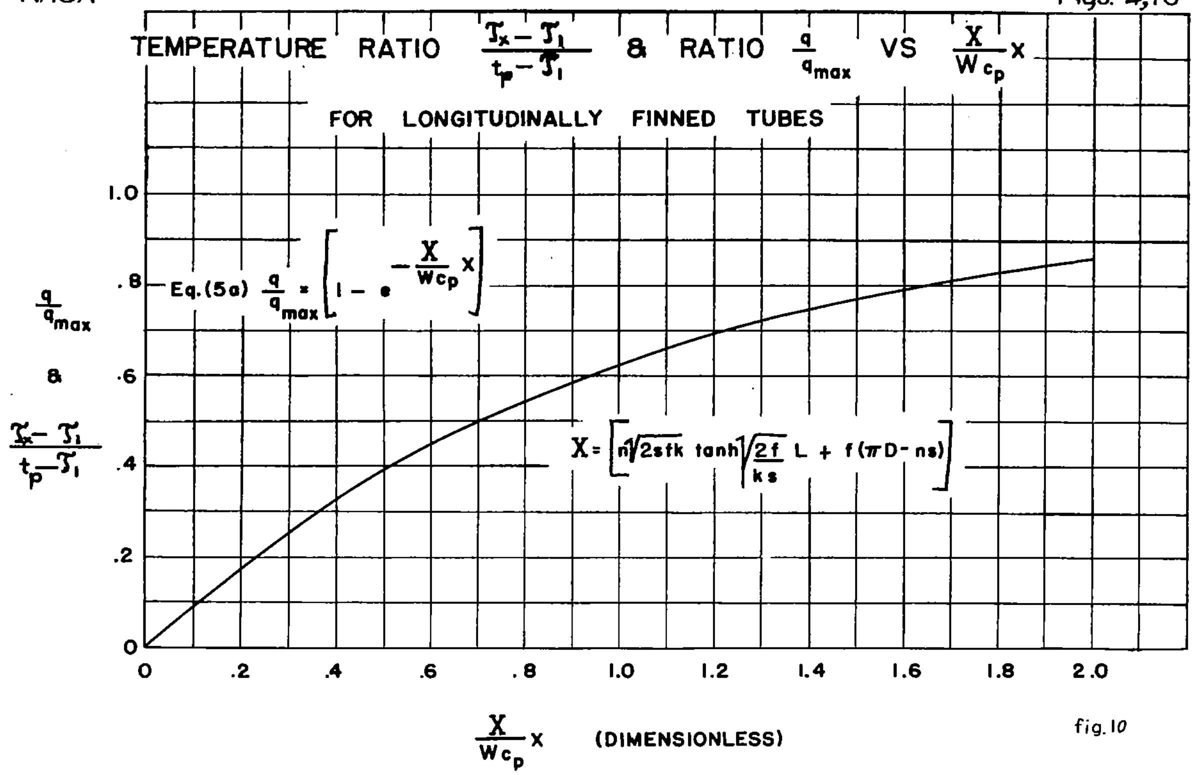

TEMPERATURE RATIO $\frac{\mathcal{T}_x - \mathcal{T}_1}{t_p - \mathcal{T}_1}$ & RATIO $\frac{q}{q_{max}}$ VS $\frac{X}{Wc_p}x$
FOR LONGITUDINALLY FINNED TUBES
Eq.(5a) $\frac{q}{q_{max}} = \left[1 - e^{-\frac{X}{Wc_p}x}\right]$
$X = \left[n\sqrt{2sfk}\,\tanh\sqrt{\frac{2f}{ks}}\,L + f(\pi D - ns)\right]$
$\frac{q}{q_{max}}$ & $\frac{\mathcal{T}_x - \mathcal{T}_1}{t_p - \mathcal{T}_1}$
0
.2
.4
.6
.8
1.0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
$\frac{X}{Wc_p}x$ (DIMENSIONLESS)

fig.10

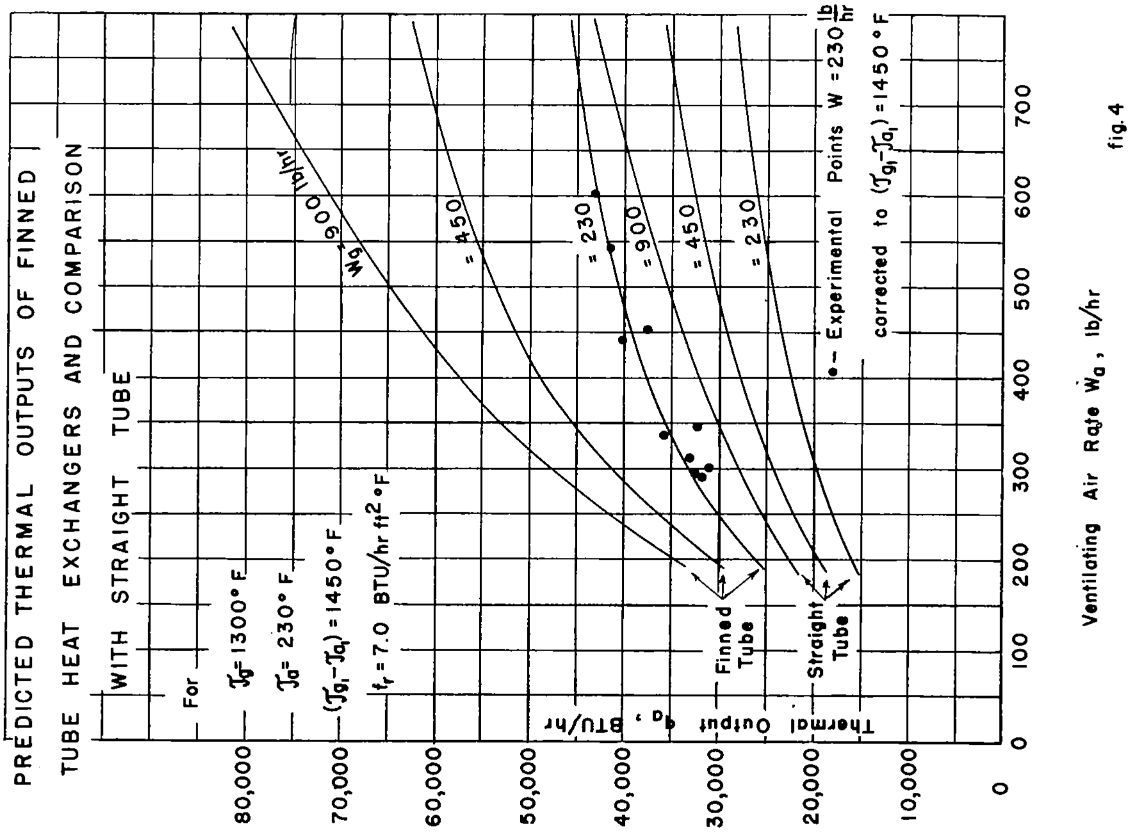
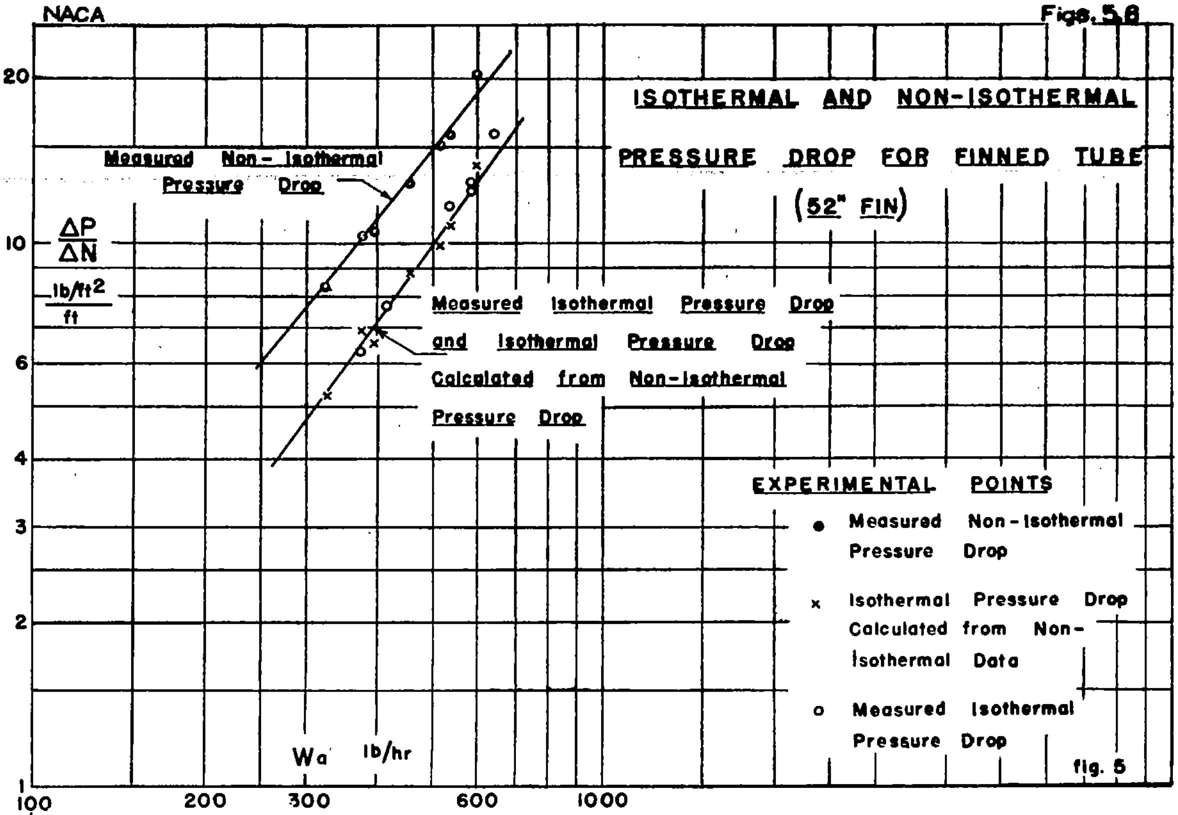
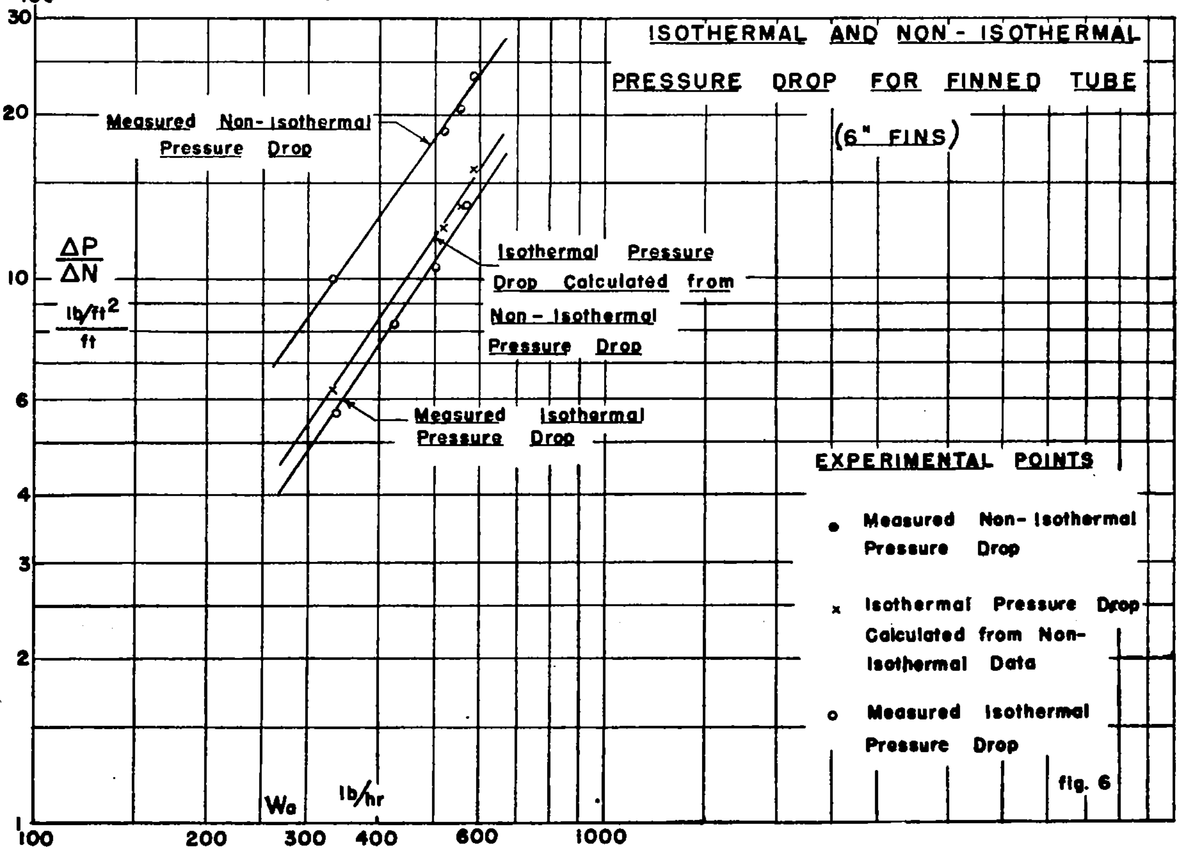
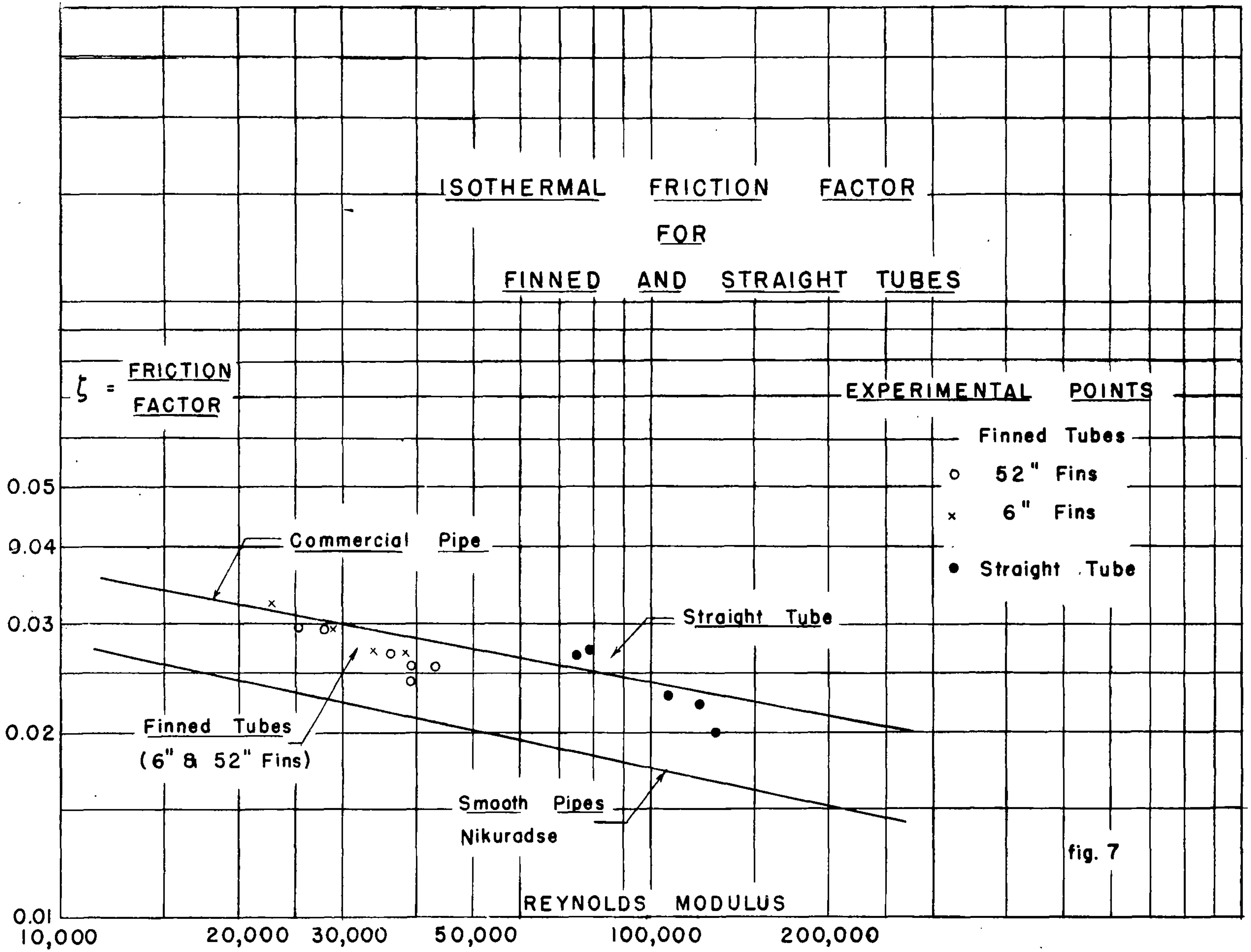

PREDICTED THERMAL OUTPUTS OF FINNED TUBE HEAT EXCHANGERS AND COMPARISON WITH STRAIGHT TUBE
For
$\mathcal{T}_g = 1300°F$
$\mathcal{T}_a = 230°F$
$(\mathcal{T}_{g_1} - \mathcal{T}_{a_1}) = 1450°F$
$f_r = 7.0$ BTU/hr ft² °F
$W_g = 900$ lb/hr
= 450
= 230
= 900
= 450
= 230
• – Experimental Points W = 230 lb/hr
corrected to $(\mathcal{T}_{g_1} - \mathcal{T}_{a_1}) = 1450°F$
Finned Tube
Straight Tube
Thermal Output q_a, BTU/hr
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
0
100
200
300
400
500
600
700
Ventilating Air Rate W_a, lb/hr

fig.4


ISOTHERMAL AND NON-ISOTHERMAL
PRESSURE DROP FOR FINNED TUBE
(52" FIN)
Measured Non-Isothermal Pressure Drop
Measured Isothermal Pressure Drop and Isothermal Pressure Drop Calculated from Non-Isothermal Pressure Drop
ΔP/ΔN lb/ft2/ft
Wa lb/hr
EXPERIMENTAL POINTS
● Measured Non-Isothermal Pressure Drop
x Isothermal Pressure Drop Calculated from Non-Isothermal Data
o Measured Isothermal Pressure Drop
1
2
3
4
6
10
20
100
200
300
600
1000
fig. 5


ISOTHERMAL AND NON-ISOTHERMAL
PRESSURE DROP FOR FINNED TUBE
(6" FINS)
Measured Non-Isothermal Pressure Drop
Isothermal Pressure Drop Calculated from Non-Isothermal Pressure Drop
Measured Isothermal Pressure Drop
ΔP/ΔN lb/ft2/ft
Wa lb/hr
EXPERIMENTAL POINTS
● Measured Non-Isothermal Pressure Drop
x Isothermal Pressure Drop Calculated from Non-Isothermal Data
o Measured Isothermal Pressure Drop
1
2
3
4
6
10
20
30
100
200
300
400
600
1000
fig. 6


ISOTHERMAL FRICTION FACTOR
FOR
FINNED AND STRAIGHT TUBES
ζ = FRICTION FACTOR
EXPERIMENTAL POINTS
Finned Tubes
52" Fins
6" Fins
Straight Tube
Commercial Pipe
Straight Tube
Finned Tubes
(6" & 52" Fins)
Smooth Pipes
Nikuradse
fig. 7
REYNOLDS MODULUS
0.05
0.04
0.03
0.02
0.01
10,000
20,000
30,000
50,000
100,000
200,000

NACA

Fig. 7

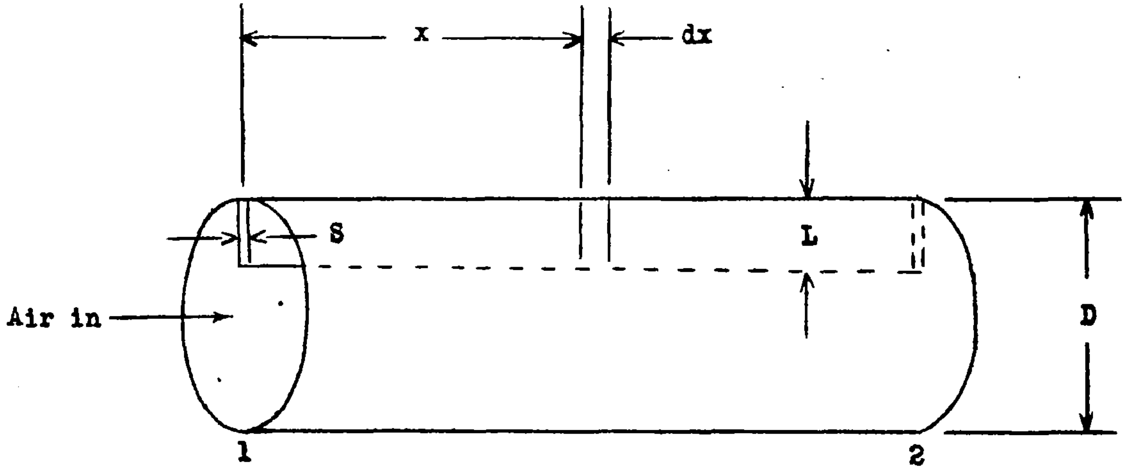


Figure 9.- Longitudinally finned tube.

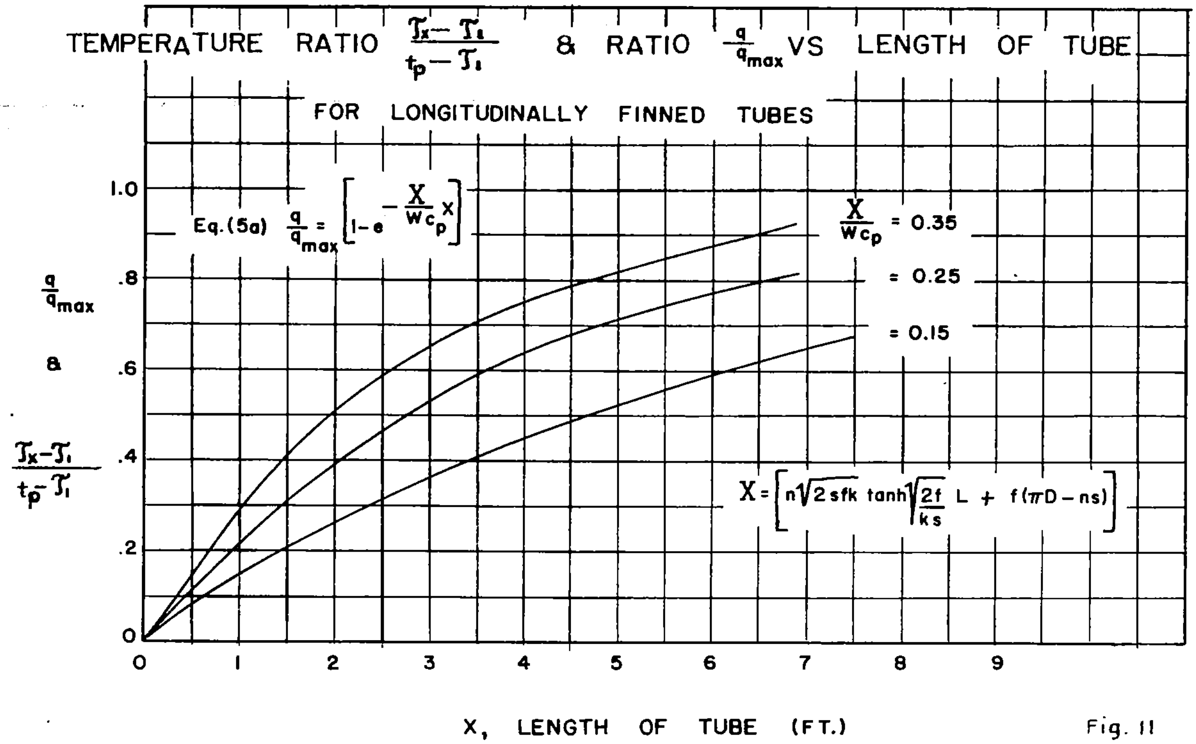

TEMPERATURE RATIO $\frac{T_x - T_1}{t_p - T_1}$ & RATIO $\frac{q}{q_{max}}$ VS LENGTH OF TUBE
FOR LONGITUDINALLY FINNED TUBES
Eq. (5a) $\frac{q}{q_{max}} = \left[1 - e^{-\frac{X}{Wc_p}x}\right]$
$\frac{X}{Wc_p}$ = 0.35
= 0.25
= 0.15
$X = \left[n\sqrt{2sfk}\tanh\sqrt{\frac{2f}{ks}}L + f(\pi D - ns)\right]$
$\frac{q}{q_{max}}$ & $\frac{T_x - T_1}{t_p - T_1}$
0
.2
.4
.6
.8
1.0
0
1
2
3
4
5
6
7
8
9
X, LENGTH OF TUBE (FT.)

Fig. 11

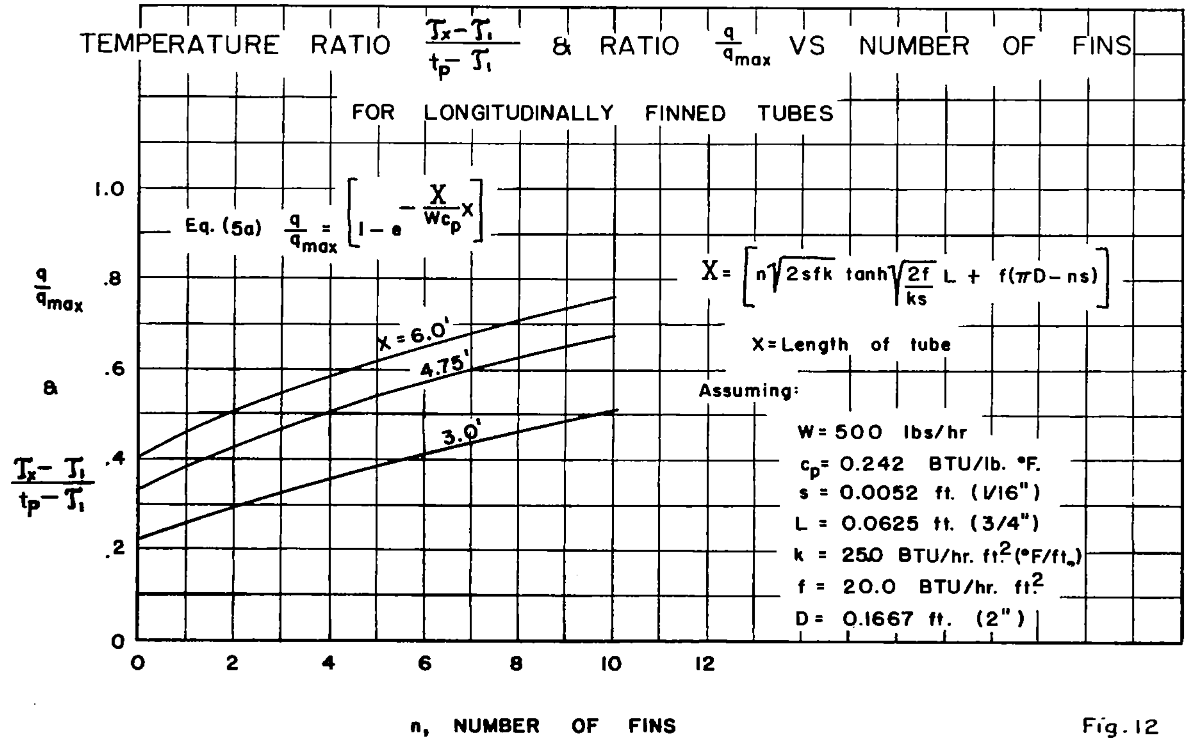

TEMPERATURE RATIO $\frac{T_x - T_1}{t_p - T_1}$ & RATIO $\frac{q}{q_{max}}$ VS NUMBER OF FINS
FOR LONGITUDINALLY FINNED TUBES
Eq. (5a) $\frac{q}{q_{max}} = \left[1 - e^{-\frac{X}{Wc_p}x}\right]$
$X = \left[n\sqrt{2sfk}\tanh\sqrt{\frac{2f}{ks}}L + f(\pi D - ns)\right]$
X = Length of tube
Assuming:
W = 500 lbs/hr
c_p = 0.242 BTU/lb. °F.
s = 0.0052 ft. (1/16")
L = 0.0625 ft. (3/4")
k = 25.0 BTU/hr. ft.^{2} (°F/ft.)
f = 20.0 BTU/hr. ft.^{2}
D = 0.1667 ft. (2")
x = 6.0'
4.75'
3.0'
$\frac{q}{q_{max}}$ & $\frac{T_x - T_1}{t_p - T_1}$
0
.2
.4
.6
.8
1.0
0
2
4
6
8
10
12
n, NUMBER OF FINS

Fig. 12

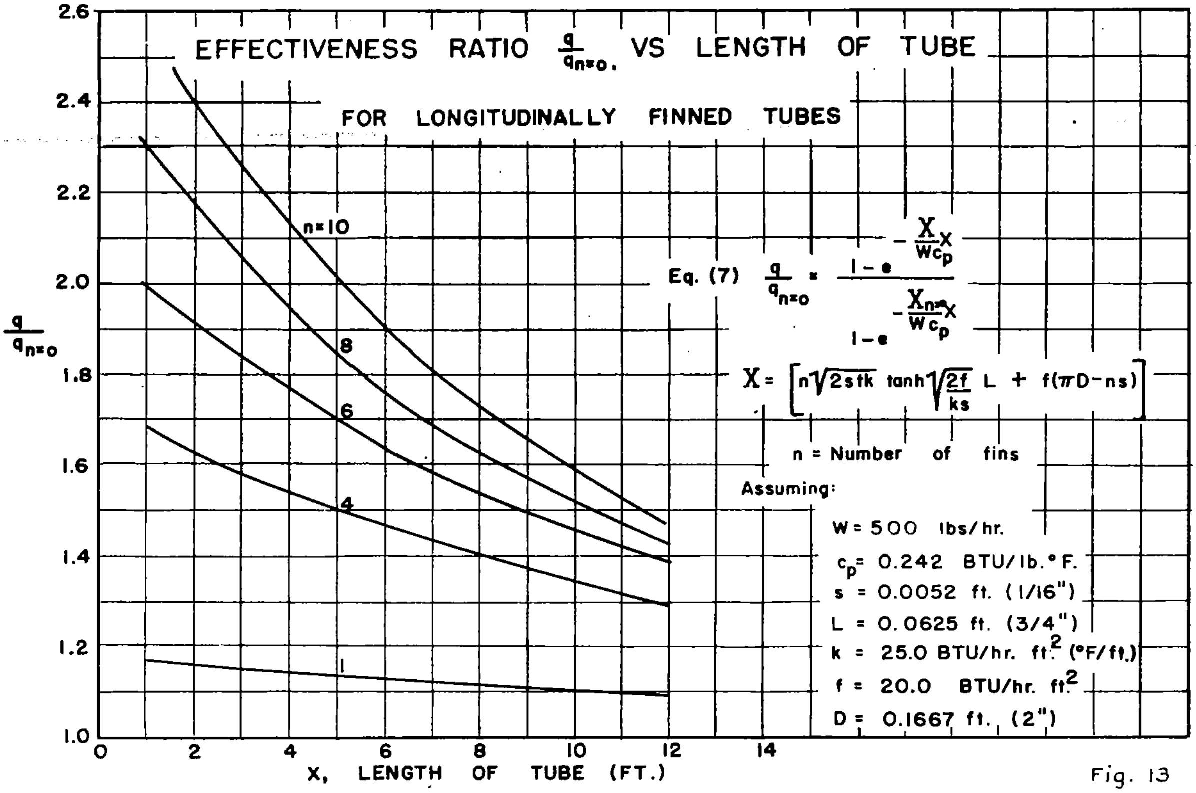

EFFECTIVENESS RATIO $\frac{q}{q_{n=o}}$ VS LENGTH OF TUBE
FOR LONGITUDINALLY FINNED TUBES
Eq. (7) $\frac{q}{q_{n=o}} = \frac{1-e^{-\frac{X}{Wc_p}x}}{1-e^{-\frac{X_{n=o}}{Wc_p}x}}$
$X = \left[n\sqrt{2sfk}\tanh\sqrt{\frac{2f}{ks}}L + f(\pi D - ns)\right]$
n = Number of fins
Assuming:
W = 500 lbs/hr.
c_p = 0.242 BTU/lb. °F.
s = 0.0052 ft. (1/16")
L = 0.0625 ft. (3/4")
k = 25.0 BTU/hr. ft.^{2} (°F/ft.)
f = 20.0 BTU/hr. ft.^{2}
D = 0.1667 ft. (2")
n=10
8
6
4
1
$\frac{q}{q_{n=o}}$
X, LENGTH OF TUBE (FT.)

Fig. 13

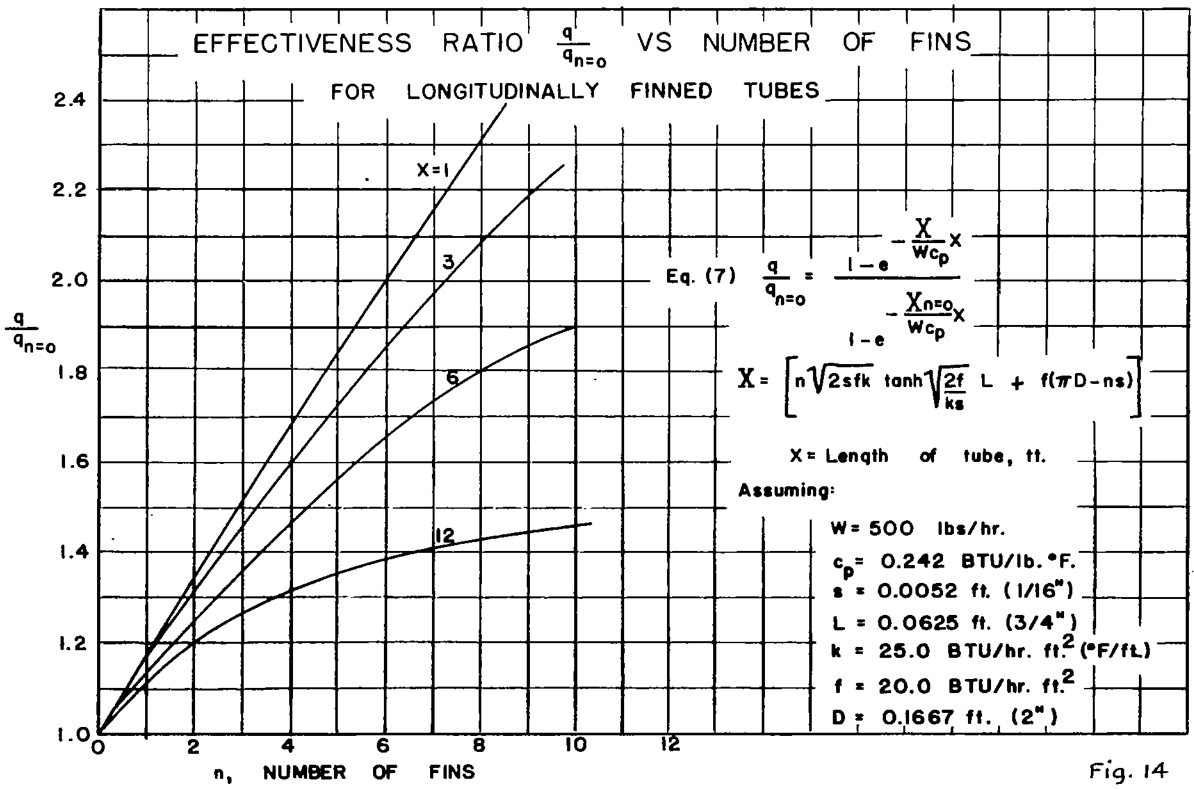

EFFECTIVENESS RATIO $\frac{q}{q_{n=o}}$ VS NUMBER OF FINS
FOR LONGITUDINALLY FINNED TUBES
Eq. (7) $\frac{q}{q_{n=o}} = \frac{1-e^{-\frac{X}{Wc_p}x}}{1-e^{-\frac{X_{n=o}}{Wc_p}x}}$
$X = \left[n\sqrt{2sfk}\tanh\sqrt{\frac{2f}{ks}}L + f(\pi D - ns)\right]$
X = Length of tube, ft.
Assuming:
W = 500 lbs/hr.
c_p = 0.242 BTU/lb. °F.
s = 0.0052 ft. (1/16")
L = 0.0625 ft. (3/4")
k = 25.0 BTU/hr. ft.^{2} (°F/ft.)
f = 20.0 BTU/hr. ft.^{2}
D = 0.1667 ft. (2")
X=1
3
6
12
$\frac{q}{q_{n=o}}$
n, NUMBER OF FINS

Fig. 14