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AN INVESTIGATION OF AIRCRAFT HEATERS

VI - HEAT TRANSFER EQUATIONS FOR THE SINGLE PASS LONGITUDINAL EXCHANGER

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VI - HEAT TRANSFER EQUATIONS FOR THE SINGLE PASS LONGITUDINAL EXCHANGER

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SUMMARY

Presented herein is an analysis of parallel flow and contraflow single pass heat exchangers, together with charts which allow the direct evaluation of the thermal performance of such units without recourse to trial-and-error techniques. The use of the equations and charts is illustrated by several examples. The analysis indicates that one of the frequently stated restrictions on the use of the logarithmic mean temperature difference - that is, that the exchanger must be perfectly insulated - is not always necessary.

SYMBOLS

A area of heat transfer surface, ft²

c_{p_a} heat capacity of air at constant pressure, Btu/lb °F

c_{p_g} heat capacity of exhaust gas at constant pressure, Btu/lb °F

K_a ratio of the energy transferred through the heat transfer surface to that gained by fluid a

K_g ratio of the energy transferred through the heat transfer surface to that lost by fluid g

q rate of heat transfer through surface separating hot and cold fluid, Btu/hr

q_a rate of heat gain by gas a, Btu/hr

q_g rate of heat transfer from gas g, Btu/hr

q_{l_a} rate of heat transfer to surroundings from fluid a, Btu/hr

q_{lg} rate of heat transfer to surroundings from fluid g, Btu/hr

U over-all conductance, Btu/hr ft^2 °F

(UA) over-all conductance, Btu/hr °F

x coordinate measured along path of fluid flow, ft

τ_a mixed mean temperature of fluid a at any point x, °F

τ_{a1} mixed mean temperature of fluid a at point 1 of exchanger (see fig. 2), °F

τ_{a2} mixed mean temperature of fluid a at point 2 of exchanger (see fig. 2), °F

τ_g mixed mean temperature of fluid g at any point x, °F

τ_{g1} mixed mean temperature of fluid g at point 1 of exchanger, °F

τ_{g2} mixed mean temperature of fluid g at point 2 of exchanger, °F

Φ_c function of $\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}}$ and $\frac{UA}{K_a W_a c_{p_a}}$ for a contra-flow single pass exchanger, defined by equation (13).

Φ_p function of $\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}}$ and $\frac{UA}{K_a W_a c_{p_a}}$ for a parallel flow single pass exchanger, defined by equation (9).

DISCUSSION

Consider the diagram shown in figure 1 in which heat is being transferred from fluid g to fluid a. Heat is being transferred also to the surroundings from each fluid.

Let dq = rate of heat flow through surface separating the cool fluid (a) and the warm fluid (g)

dq_{la} = rate of heat transfer from fluid (a) to surroundings

dq_{lg} = rate of heat transfer from fluid (g) to surroundings

A heat balance yields:

$$dq = dq_{la} + W_a \, c_{p_a} \, d\,T_a \qquad (1)$$

$$dq = dq_{lg} - W_g \, c_{p_g} \, d\,T_g \qquad (2)$$

since $d\,T_g$ is a negative value.

The above equations may be written as:

$$dq = K_a \, W_a \, c_{p_a} \, d\,T_a \qquad (3)$$

where
$$K_a = \left(1 + \frac{dq_{la}}{W_a \, c_{p_a} \, d\,T_a}\right) \qquad (4)$$

(K_a is a number greater than or equal to 1.)

and
$$dq = - K_g \, W_g \, c_{p_g} \, d\,T_g \qquad (5)$$

where
$$K_g = \left(1 - \frac{dq_{lg}}{W_g \, c_{p_g} \, d\,T_g}\right) \qquad (6)$$

(K_g is a number greater than or equal to 1.)

The ratios K_a and K_g may be defined as follows:

K_a = the ratio of the energy transferred through the heat transfer surface to that gained by the cold fluid (a).

K_g = the ratio of the energy transferred through the heat transfer surface to that transferred by the hot fluid (g).

If the magnitudes of K_a and K_g are constant along the length of the heat exchanger; that is, the transfer of heat to the surroundings is a fixed fraction of the net rate of heat transfer to the corresponding fluid, the effect is exactly equivalent to changing the heat capacity of the fluids from c_{p_a} and c_{p_g} to $K_a c_{p_a}$ and $K_g c_{p_g}$, as may be seen by means of equations (3) and (5). If the ratios K_a and K_g do not differ too greatly from unity, even though they may vary with length to some extent, an average magnitude of K_a, K_g may be utilized with small error. For these conditions the logarithmic mean temperature difference may be used with confidence.

Nesselman (reference 1) has treated this problem by considering the heat transfer to the surroundings to be independent of exchanger length the magnitude of which is added to or subtracted from the net rate of heat transfer of the corresponding fluid. The resulting equations are somewhat complex.

Expressing equations (3) and (5) in terms of the over-all thermal conductance (UA) the following equation. (see references 2, 3, and 4) is obtained:

$$-U\left(\frac{1}{K_a\ W_a\ c_{p_a}} + \frac{1}{K_g\ W_g\ c_{p_g}}\right) dA = \frac{d(\tau_g - \tau_a)}{(\tau_g - \tau_a)}$$

When this expression is integrated with respect to A and $(\tau_g - \tau_a)$, one obtains the exponential function of the temperature differences at the entrance to the heat exchanger (point 1) and at the exit end (point 2)

$$\frac{\tau_{g2} - \tau_{a2}}{\tau_{g1} - \tau_{a1}} = e^{-\left(\frac{1}{K_a\ W_a\ c_{p_a}} + \frac{1}{K_g\ W_g\ c_{p_g}}\right)(UA)}$$

From this equation the rate of heat transfer through the surface separating fluids a and g can be written

$$\dot{q} = K_a\ W_a\ c_{p_a}(\tau_{g1} - \tau_{a1}) \left[\frac{1 - e^{-\left(1 + \frac{K_a\ W_a\ c_{p_a}}{K_g\ W_g\ c_{p_g}}\right)\frac{UA}{K_a\ W_a\ c_{p_a}}}}{\left(1 + \frac{K_a\ W_a\ c_{p_a}}{K_g\ W_g\ c_{p_g}}\right)}\right] \tag{7}$$

where the specific heats c_{p_a}, c_{p_g} in equation (B17) of reference 2 have been replaced by $K_a\ c_{p_a}$ and $K_g\ c_{p_g}$.

The above equation may be written as:

$$\dot{q} = K_a\ W_a\ c_{p_a}(\tau_{g1} - \tau_{a1})\ \Phi_p\left(\frac{K_a\ W_a\ c_{p_a}}{K_g\ W_g\ c_{p_g}}\right)\frac{UA}{K_a\ W_a\ c_{p_a}} \tag{8}$$

The function (references 3 and 4)

$$\Phi_p = \left[\frac{1 - e^{-\left(1 + \frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}}\right)\frac{UA}{K_a W_a c_{p_a}}}}{\left(1 + \frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}}\right)}\right] \quad (9)$$

is plotted in figure 3.

It may be demonstrated readily that equation (7) is exactly equivalent to the well-known form:

$$q = UA \left[\frac{(\tau_{g1} - \tau_{a1}) - (\tau_{g2} - \tau_{a2})}{ln\left(\frac{\tau_{g1} - \tau_{a1}}{\tau_{g2} - \tau_{a2}}\right)}\right] = UA\ \Delta t_{lm} \quad (10)$$

A procedure similar to that outlined above yields the equivalent expressions for contraflow, single pass exchangers. These are:

$$q = K_a W_a c_{p_a}(\tau_{g1} - \tau_{a1}) \left[\frac{1 - e^{-\left(\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - 1\right)\frac{UA}{K_a W_a c_{p_a}}}}{\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - e^{-\left(\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - 1\right)\frac{UA}{K_a W_a c_{p_a}}}}\right] \quad (11)$$

$$= K_a W_a c_{p_a}(\tau_{g1} - \tau_{a1})\ \Phi_c\left(\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}}, \frac{UA}{K_a W_a c_{p_a}}\right) \quad (12)$$

where

$$\Phi_c = \left[\frac{1 - e^{-\left(\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - 1\right)\frac{UA}{K_a W_a c_{p_a}}}}{\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - e^{-\left(\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} - 1\right)\frac{UA}{K_a W_a c_{p_a}}}}\right] \quad (13)$$

Equation (13) is plotted in figure 4. It may be readily shown again that equation (11) is exactly equivalent to the expression utilizing the log mean temperature difference (equation (10)).

Equations (7) and (11) have the great advantage of being explicit solutions for q, while equation (10) requires a trial-and-error solution.

The net thermal energy gained by fluid a and transferred from fluid g is readily obtainable from the amount which is transferred through the surface which separates fluid a from fluid g:

$$q_a = \frac{q}{K_a} \tag{14}$$

$$q_g = \frac{q}{K_g} \tag{15}$$

In double tube, single pass, gas-air heat exchangers, if the air is in the central tube, $K_a = 1$, $K_g \geq 1$. If the air is in the annular space, $K_g = 1$, $K_a \geq 1$.

Inspection of figures 3 and 4 reveals that, for the usual range of variables found in exhaust gas-air heat exchangers, there is little superiority of the contraflow arrangement over the parallel flow arrangement.

The over-all conductance UA for a certain unfinned, parallel flow, single pass, double tube heat exchanger is 250 Btu/hr °F. The over-all conductances UA for smooth tubes may be determined from equations (14), (15), (16), and (17), or from chart B of reference 5. It should be mentioned here that the equations for finned tubes presented in reference 2 assume that the velocity of the fluid past the fins may be obtained by dividing the rate of flow (cu ft per sec) by the net cross-sectional area of the tube (sq ft). This result may be far from exact in the region of the fins if the fins are very close together and occupy a small percentage of the total cross-sectional area, because more of the air will flow through the space not occupied by the fins.

The following three examples illustrate how figures 3 and 4 may be employed to determine the heat transfer in single pass, parallel flow or contraflow heat exchangers:

The rate of ventilating air flow, W_a = 2000 lb/hr

The rate of exhaust gas flow, W_g = 6000 lb/hr

Temperature of exhaust gas entering exchanger, $\tau_{g_1} = 1600^\circ$ F

Temperature of ventilating air entering exchanger, $\tau_{a_1} = 0^\circ$ F

The air flows in the annular space, and it is estimated that 10 percent* of the heat transferred through the heat transfer surface is transferred to the surroundings: that is, $K_a \frac{1}{0.90} = 1.11$ and $K_g = 1$.

Determine:

1. The rate of heat flow through the transfer surface
2. The rate at which thermal energy is gained by the air
3. The answers to questions 1 and 2 for a perfectly insulated heater
4. The answers to questions 1, 2, and 3 for contraflow conditions

Solutions:

(a) The ratio

$$\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} = \frac{1.11 \times 2000 \times 0.241}{1 \times 6000 \times 0.267} = 0.335 \text{ (dimensionless)}$$

(b) The ratio

$$\frac{UA}{K_a W_a c_{p_a}} = \frac{250}{1.11 \times 2000 \times 0.241} = 0.466 \text{ (dimensionless)}$$

(c) The product

$$(\tau_{g_1} - \tau_{a_1}) K_a W_a c_{p_a} = 1600 \times 1.11 \times 2000 \times 0.241$$
$$= 855{,}000 \text{ Btu/hr}$$

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*This amount usually can be calculated from a consideration of the appropriate resistances.

From figure 3

$$\Phi_p = 0.346$$

Thus $q = 0.346 \times 855{,}000 = 296{,}000$ Btu/hr transferred through the heat transfer surface.

The air gains heat at a rate equal to:

$$q_a = \frac{292000}{1.11} = 266{,}000 \text{ Btu/hr}$$

If no heat transfer to the surroundings had occurred, $K_a = 1$, instead of 1.11, and

$$\frac{K_a W_a c_{p_a}}{K_g W_g c_{p_g}} = 0.302$$

$$\frac{UA}{K_a W_a c_{p_a}} = 0.518$$

Thus

$$\Phi_p = 0.378$$

$$q = q_a = 1600 \times 2000 \times 0.241 \times 0.378 = 292{,}000 \text{ Btu/hr}$$

The rate of heat transfer through the surface separating the two fluids is practically independent of small rates of heat transfer to the surroundings. The slight decrease in q (2 percent) in the case of the adiabatic exchanger follows from the reduction in log mean temperature difference resulting from the absence of heat transfer to the surroundings.

For a contraflow exchanger from figure 4

Φ_c with heat transfer to surroundings = 0.357

Thus, $q = 0.357 \times 855{,}000 = 306{,}000$ Btu/hr

$$q_a = 275{,}000 \text{ Btu/hr}$$

Φ_c with no heat transfer to surroundings = 0.389

$$q = q_a = 300{,}000 \text{ Btu/hr}$$

Thus the increase in thermal output due to contraflow, over parallel flow, is about 3 percent, for the conditions stated.

The mixed mean temperature of the fluids leaving the exchanger for any of the conditions stated, of course, can be readily calculated.

CONCLUSIONS

1. The use of the logarithmic mean temperature difference as the heat flow potential in exchangers which have heat flow to the surroundings is justified when this flow of heat is constant along the length of the exchanger or when it is variable with length but is small.

2. Heat transfer in single pass, parallel flow or contraflow heat exchangers may be computed directly with the aid of the equations and curves given.

3. For the case of a heat exchanger utilizing the hot exhaust gases from an airplane engine, very little advantage is gained by using the contraflow arrangement.

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REFERENCES

1. Nesselmann, K.: Der Einfluss der Wärmeverluste auf Doppelrohrwärmeaustauscher. Zeitschr. für die Gesamte Kälte-Industrie, April 1928.

2. Martinelli, R. C., Weinberg, E. B., Morrin, E. H., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. IV - Measured and Predicted Performance of Longitudinally Finned Tubes. NACA ARR, Oct. 1942.

3. Grober, H.: Wärmeübertragung. Julius Springer, (Berlin), 1926, p. 112.

4. Boelter, L. M. K., Cherry, V. H., and Johnson, H. A.: Supplementary Heat Transfer Notes. Univ. of Calif. Press, Berkeley, Calif., 3d ed., 1942, p. XIV-3.

5. Martinelli, R. C., Weinberg, E. B., Morrin, E. H., and Boelter, L. M. K.: An Investigation of Aircraft Heaters. III - Measured and Predicted Performance of Double Tube Heat Exchangers. NACA ARR, Oct. 1942.

BIBLIOGRAPHY

Maker, F. L., and Thornburg, M. W.: Economic Features of Heat Exchanger Design. Mech. Eng., vol. 46, no. 12, Dec. 1924, p. 891.

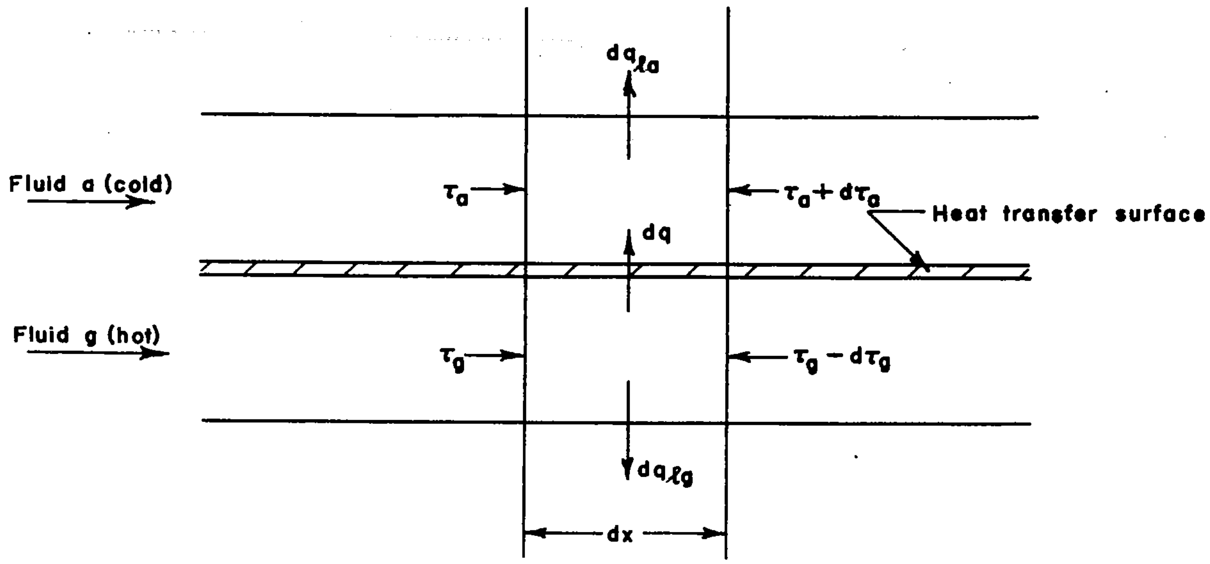

FIGURE 1. HEAT BALANCE

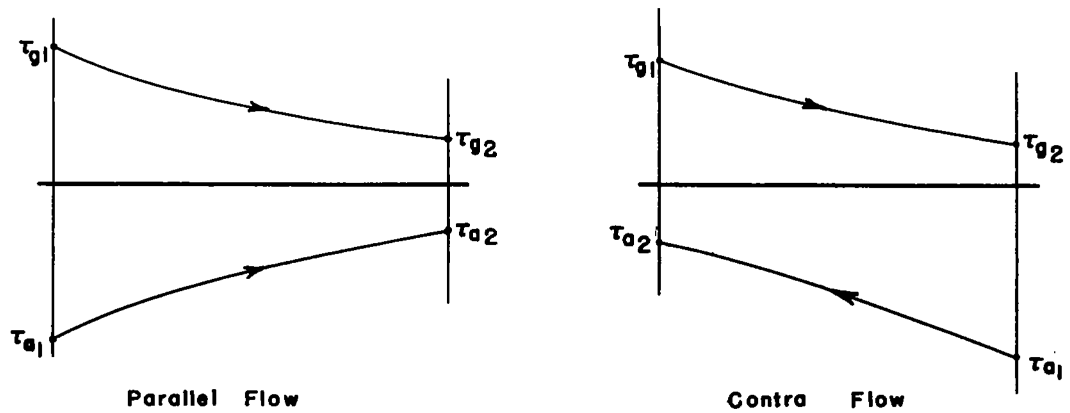

FIGURE 2. FLOW DIAGRAMS

FIGURE 3. FUNCTION Φ_p FOR SINGLE PASS PARALLEL FLOW HEAT EXCHANGERS

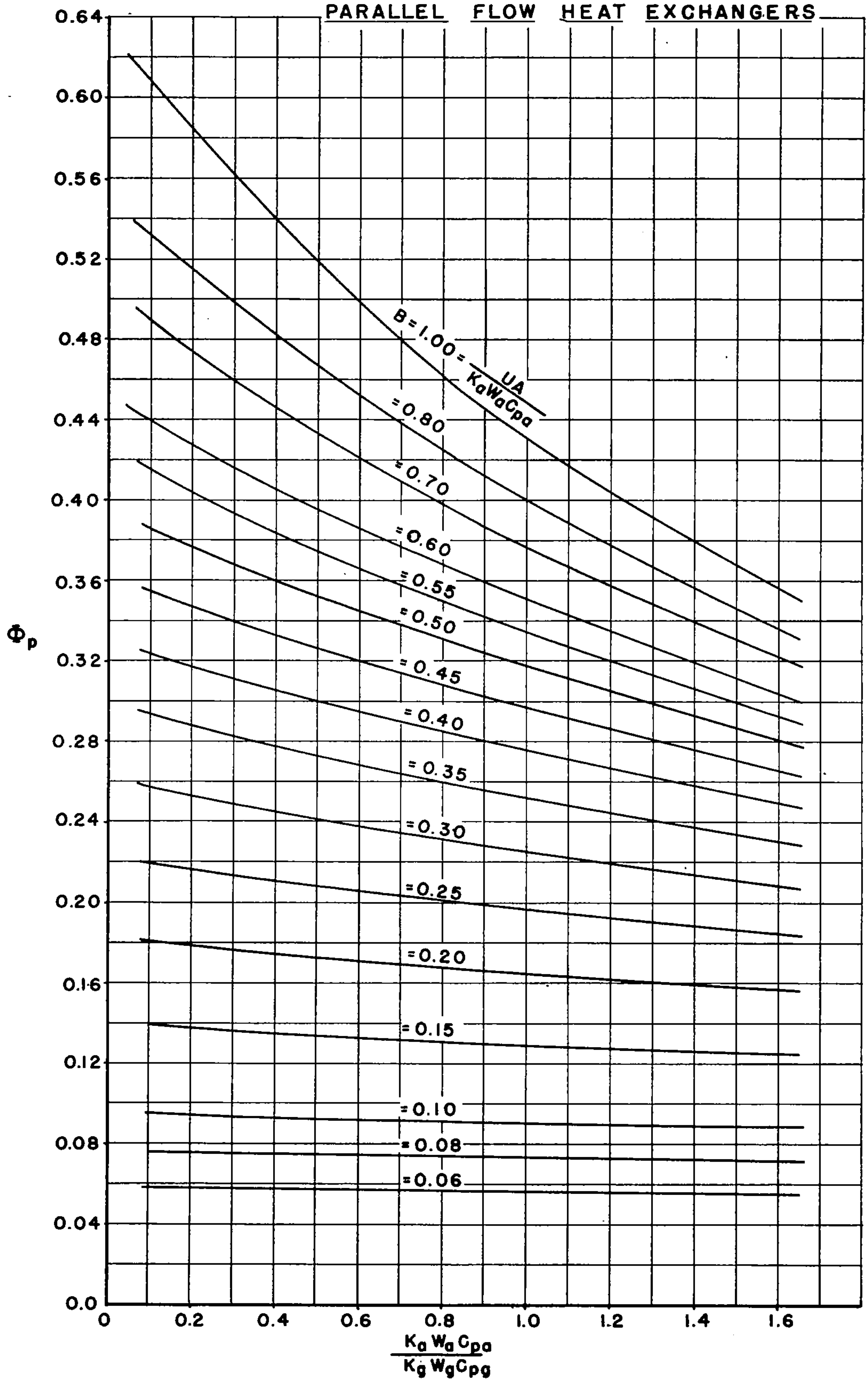

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FIGURE 4. FUNCTION Φ_c FOR SINGLE PASS CONTRA FLOW HEAT EXCHANGERS