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AN INVESTIGATION OF AIRCRAFT HEATERS

X - MEASURED AND PREDICTED PERFORMANCE OF A FLUTED-TYPE  
EXHAUST GAS AND AIR HEAT EXCHANGER

By L. M. K. Boelter, H. G. Dennison,  
A. G. Guibert, and E. H. Morrin  
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WASHINGTON

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ADVANCE RESTRICTED REPORT

AN INVESTIGATION OF AIRCRAFT HEATERS

X - MEASURED AND PREDICTED PERFORMANCE OF A FLUTED-TYPE

EXHAUST GAS AND AIR HEAT EXCHANGER

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SUMMARY

Performance data on a small Airesearch exhaust gas and air heat exchanger are presented. Heat transfer rates were measured, using about 8000 pounds per hour of exhaust gas and about 4300 pounds per hour of ventilating air. The inlet exhaust gas temperature was maintained at approximately 1400° F; whereas the ventilating air temperature was about 95° F. Tests were made with the air shroud disposed in two different positions. Pressure drop measurements were made on both the exhaust gas side and the ventilating air side of the heat exchanger.

The maximum measured rate of heat transfer was 259,000 Btu per hour; whereas the maximum static pressure drops were 7.3 inches of water and 12.4 inches of water on the exhaust gas and ventilating air sides of the heater, respectively.

The measured thermal outputs and pressure drops are compared with predicted values and also with measured values obtained on a similar but larger Airesearch fluted-type heater.

Equations for the unit thermal conductance reported earlier are rewritten in terms of the L/D ratio of finned or unfinned heat exchangers.

INTRODUCTION

The heater, designated for purposes of this report as Airesearch heater no. 2, was tested on the large test

stand in the Mechanical Engineering Laboratories of the University of California. (See fig. 1; for a description of this test stand, see reference 1.)

The results of tests on a similar but larger Airesearch heater (designated as no. 1) have been given previously. (See reference 1.) These heaters are designed for use in the exhaust gas stream of aircraft engines for cabin, wing, and tail-surface heating systems.

The following data were obtained:

1. Weight rates of exhaust gas and of ventilating air through the two sides of the heat exchanger
2. Temperatures of ventilating air and of exhaust gas at entrance and exit of the heater
3. Temperatures of the heater surfaces
4. Static pressure drop measurements on the exhaust gas and ventilating air sides of the heater under both isothermal and non-isothermal flow conditions

These measurements were made with both of the air-shroud openings on the same side of the heater and were then repeated with the air inlet on the opposite side from the air outlet header. (See figs. 4 and 5.)

#### SYMBOLS

- A    area of heat transfer,  $\text{ft}^2$
- $C_1$     constant (defined in Appendix)
- $c_{p_a}$     heat capacity of air at constant pressure,  $\text{Btu/lb } ^\circ\text{F}$
- $c_{p_g}$     heat capacity of exhaust gas at constant pressure,  
           $\text{Btu/lb } ^\circ\text{F}$
- D    hydraulic diameter, ft
- $D_a$     hydraulic diameter on ventilating air side, ft
- $D_g$     hydraulic diameter on exhaust gas side, ft

- $f_c$  unit thermal convective conductance (average with length), Btu/hr ft<sup>2</sup> °F
- $f_{c_a}$  unit thermal convective conductance for the ventilating air (average with length), Btu/hr ft<sup>2</sup> °F
- $f_{c_D}$  unit thermal convective conductance in a duct for either fluid using the hydraulic diameter  $D$  as the significant dimension in equation (12) (average with length), Btu/hr ft<sup>2</sup> °F
- $f_{c_g}$  unit thermal convective conductance for the exhaust gas (average with length), Btu/hr ft<sup>2</sup> °F
- $f_{c_l}$  unit thermal convective conductance along a flat plate for either fluid, using the length of the flat plate in the direction of fluid flow as the significant dimension in equation (9) (average with length), Btu/hr ft<sup>2</sup> °F
- $g$  gravitational force per unit of mass, lb/(lb sec<sup>2</sup>/ft)
- $G$  weight rate per unit of area, lb/hr ft<sup>2</sup>
- $G_a$  weight rate per unit of area for ventilating air, lb/hr ft<sup>2</sup>
- $G_g$  weight rate per unit of area for exhaust gas, lb/hr ft<sup>2</sup>
- $l$  length of flat plate measured in direction of fluid flow, ft
- $L$  length of heat transfer surface, ft
- $P$  heat transfer perimeter, ft
- $q_a$  measured rate of enthalpy change of ventilating air, Btu/hr
- $q_g$  measured rate of enthalpy change of exhaust gas, Btu/hr
- $t_1$  arithmetic average temperature of heater surface at section defined by point 1, °F
- $t_2$  arithmetic average temperature of heater surface at section defined by point 2, °F

- $T_a$  arithmetic average mixed mean absolute temperature of either fluid =  $T_1 + T_2/2$ , in equation (8) only; otherwise arithmetic average mixed-mean absolute temperature of air =  $\frac{T_{a1} + T_{a2}}{2} + 460, ^\circ R$
- $T_g$  arithmetic average mixed-mean absolute temperature of exhaust gas =  $\frac{T_{g1} + T_{g2}}{2} + 460, ^\circ R$
- $T_1$  mixed-mean absolute temperature of fluid at point 1,  $^\circ R$
- $T_2$  mixed-mean absolute temperature of fluid at point 2,  $^\circ R$
- $T_{iso}$  mixed-mean absolute temperature of fluid for isothermal pressure-drop tests,  $^\circ R$
- $U$  over-all unit thermal conductance, Btu/hr ft<sup>2</sup>  $^\circ F$
- $UA$  over-all thermal conductance, Btu/hr  $^\circ F$
- $(UA)_{center}$  over-all thermal conductance for the center full-fluted section of the heater, Btu/hr  $^\circ F$
- $(UA)_{ends}$  over-all thermal conductance for the tapered ends of the heater, Btu/hr  $^\circ F$
- $W_a$  weight rate of air, lb/hr
- $W_g$  weight rate of exhaust gas, lb/hr
- $x$  length along a flat plate or duct measured in direction of fluid flow, ft
- $\gamma_1$  weight density of fluid at entrance to heating section (point 1), lb/ft<sup>3</sup>
- $\delta$  thickness of boundary layer, ft
- $\Delta P$  non-isothermal pressure drop along heater, lb/ft<sup>2</sup>
- $\Delta P_a$  pressure drop along heater on ventilating-air side, lb/ft<sup>2</sup>

- $\Delta P'_a$  pressure drop along heater on ventilating-air side, inches  $H_2O$
- $\Delta P_g$  pressure drop along heater on exhaust gas side, lb/ft<sup>2</sup>
- $\Delta P'_g$  pressure drop along heater on exhaust gas side, inches  $H_2O$
- $\Delta P_{T_{iso}}$  isothermal pressure drop due to friction along heater at temperature  $T_{iso}$
- $\Delta t_{lm}$  logarithmic mean temperature difference, °F
- $\mu$  viscosity of fluid, lb sec/ft<sup>2</sup>
- $\Delta T_a$  difference between mixed-mean temperatures of ventilating air at sections defined by points 1 and 2 =  $(T_{a2} - T_{a1})$ , °F
- $\Delta T_g$  difference between mixed-mean temperatures of exhaust gas at sections defined by points 1 and 2 =  $(T_{g1} - T_{g2})$ , °F
- $T_{a1}$  mixed-mean temperature of ventilating air at section defined by point 1, °F
- $T_{a2}$  mixed-mean temperature of ventilating air at section defined by point 2, °F
- $T_{g1}$  mixed-mean temperature of exhaust gas at section defined by point 1, °F
- $T_{g2}$  mixed-mean temperature of exhaust gas at section defined by point 2, °F
- Re Reynolds number =  $G D / 3600 g \mu$
- Point 1 refers to entrance end of heater
- Point 2 refers to exit end of heater

DESCRIPTION OF AIRESEARCH NO. 2 HEATER AND  
TESTING PROCEDURE

The Airesearch no. 2 unit is a parallel-flow, unfinned, fluted-type heater containing 40 alternate exhaust

gas and ventilating air passages. The over-all length of the heat transfer surface is  $20\frac{1}{2}$  inches. A sketch of this heater is shown in figure 6. Also, photographs of the heater are shown in figures 2 to 5.

The principal difference between Airesearch heater no. 2 and heater no. 1 (see reference 1 for a description of tests on Airesearch heater no. 1) is that the heat transfer surface of no. 2 is shorter ( $20\frac{1}{2}$  in. as against 26 in.). The diameter of the shorter heater (no. 2) at the center section is one-half inch less than that of the longer one ( $8\frac{1}{2}$  in. as against 9 in.), and the total number of passages for the no. 2 unit is also less (40 against 48). The tapered end sections are approximately the same on the two heaters. The same air shroud as was used for the Airesearch no. 1 heater tests (obtained from Ames Aeronautical Laboratory, Moffett Field, Calif.) was shortened to fit the smaller heater. (See figs. 4 and 5.) The inlet header of the air shroud contains vanes arranged to distribute the flow of air around the perimeter of the heater.

The weight rates of exhaust gas and ventilating air were obtained by means of calibrated square edge orifices.

The exhaust gas temperatures were measured at the inlet and outlet of the heater by means of shielded, traversing thermocouples.

A mixing device was used at the exit of the natural-gas furnace to give an approximately uniform temperature distribution at the entrance to the heater (the measured temperature distribution in deg F at both inlet and outlet ends of the heater was within  $\pm 5$  percent of complete uniformity). (See reference 1 for a description of the test stand and its instrumentation.)

The exhaust gas temperature traverses were made at points 22 inches upstream and 16 inches downstream from the heat transfer section of the heater.

Temperatures of the ventilating air before and after passage through the heater were determined from traverses made by unshielded thermocouples. The temperature traverses at the inlet end of the heater were uniform within  $\pm 1$  percent and those at the outlet end within  $\pm 2$  percent. These traverses were made at points  $22\frac{1}{2}$  inches before and  $39\frac{1}{2}$  inches after the air-shroud openings.



The heat loss to the surroundings was reduced to a negligible amount by wrapping the ducts with asbestos sheets.

Temperatures of the heater surfaces were measured at six points, three at each end of the heater. (See fig. 6.)

Static pressure drop measurements were made across the air and exhaust gas sides of the heater. Two taps, 180 degrees apart, were installed at each pressure measuring station. The pressure taps on the exhaust gas side were placed 12 inches before and  $12\frac{1}{2}$  inches after the heat transfer section of the heater in an 8-inch duct and those on the ventilating air side were placed 27 inches ahead of and 20 inches after the air-shroud openings in a 5-inch duct.

Heat transfer and pressure drop data were taken with the air-shroud openings on the same side of the heater (see fig. 4) and also with the air openings on opposite sides (see fig. 5).

## CALCULATIONS

### Heat Transfer

The thermal output of the heater was determined by the enthalpy change of the ventilating air:

$$q_a = W_a c_{p_a} (\tau_{a_2} - \tau_{a_1}) \quad (1)$$

in which  $c_{p_a}$  was evaluated at the arithmetic average ventilating-air temperature as a good approximation. A plot of  $q_a$  against  $W_a$  at constant values of the exhaust gas rate  $W_g$  is shown in figure 7.

On the exhaust gas side of the heater:

$$q_g = W_g c_{p_g} (\tau_{g_1} - \tau_{g_2}) \quad (2)$$

where  $c_{p_g}$  was evaluated for air at the arithmetic average exhaust gas temperature previously mentioned.

The over-all thermal conductance UA was evaluated from the expression

$$q_a = (UA) \Delta t_{lm} \quad (3)$$

A plot of UA as a function of the ventilating air rate  $W_a$  at constant values of  $W_g$  is shown in figure 8.

A comparison of predicted and measured results for the larger Airesearch no. 1 fluted-type heater (see reference 1) reveals that an arbitrary selection of the heat transfer length L was not adequate to account for the different mechanism of heat transfer along the tapered end section as compared to that along the uniform full-fluted center section.

The rate of heat transfer through this uniform full-fluted center section can be predicted by means of the equation

$$UA = \frac{1}{\left(\frac{1}{f_c A}\right)_a + \left(\frac{1}{f_c A}\right)_g} \quad (4)$$

where

A heat transfer surface of the center full-fluted section

$f_{c_a}$ ,  $f_{c_g}$  the unit thermal conductances on the ventilating air and exhaust gas sides of the heater, respectively, are evaluated from the following equations:

$$f_{c_a} = 5.56 \times 10^{-4} T_a^{0.296} \frac{G_a^{0.8}}{D_a^{0.2}} \quad (5)$$

and

$$f_{c_g} = 5.56 \times 10^{-4} T_g^{0.296} \frac{G_g^{0.8}}{D_g^{0.2}} \quad (6)$$

in which  $D$  is the hydraulic diameter and the subscripts  $a$  and  $g$  denote ventilating air and exhaust gas sides, respectively. (See references 2 and 3 for derivation of equations (5) and (6).)

The rate of heat transfer through the tapered ends of the heater was obtained in a different manner. It is very difficult to calculate the rate of heat transfer through these end sections because of the nonuniformity of the fluid flow and of the geometry of the fluid passages. Therefore the difference between the predicted over-all thermal conductance through the uniform center section of the larger heater, using equations (4), (5), and (6) and the total measured conductance of the larger heater (Airesearch no. 1)(reference 1) was used to determine the conductance through the tapered end sections, which are about the same size and shape on both heaters.

The end correction for the larger heater (no. 1) was found to vary in the following manner:

$W_a$ (lb/hr)	$(UA)_{ends}$ (Btu/hr °F)
3000	64
5000	73
6000	77

The variation of  $(UA)_{ends}$  with the exhaust gas rate  $W_g$  was smaller in magnitude than with  $W_a$  and was somewhat inconsistent since the magnitudes of  $(UA)_{ends}$  at one value of  $W_g$  were smaller than those at a lower value of  $W_g$ .

Thus the predicted magnitudes of  $UA$ , for Airesearch no. 2 heater, as shown in figure 8, are evaluated as the sum of the  $UA$  calculated for the uniform full-fluted center section of heater no. 2 from equations (4), (5), and (6) and the  $(UA)_{ends}$  of heater no. 1 evaluated from its variation with  $W_a$  previously shown, according to the equation

$$q_a = (UA) \Delta t_{lm} = \left[ (UA)_{\text{center}} + (UA)_{\text{ends}} \right] \Delta t_{lm} \quad (7)$$

This equation is exact only when the temperature potential between the exhaust gas and the ventilating air is constant along the length of the heat exchanger. This is approximated for an exhaust gas and air heat exchanger in which the temperature potential is large and the  $\Delta t_{lm}$  is a mean value of this potential.

### Pressure Drop

The non-isothermal pressure drop of either fluid through the heater was predicted from isothermal measurements by means of the equation

$$\Delta P = \Delta P_{T_{\text{iso}}} \left( \frac{T_a}{T_{\text{iso}}} \right)^{1.13} + \left( \frac{G}{3600} \right)^2 \frac{1}{\gamma_1 g} \left( \frac{T_2}{T_1} - 1 \right) \quad (8)$$

where

$\Delta P_{T_{\text{iso}}}$  isothermal pressure drop due to friction at temperature  $T_{\text{iso}}$

$T_1$  and  $T_2$  mixed-mean absolute temperatures of fluid at the inlet and the exit of heater, respectively

$T_a$  arithmetic average of  $T_1$  and  $T_2$

$G$  fluid flow per unit of cross-sectional area

$\gamma_1$  unit weight of fluid at inlet to heater evaluated at temperature  $T_1$

The heat transfer and pressure drop data for Airesearch heater no. 2 obtained from tests with the air-shroud openings on the same side of the heater are presented in table I and those from tests with the air-shroud openings on opposite sides are given in table II.

A comparison of measured and predicted pressure drops along both sides of the heater is presented in table III and shown graphically in figures 9 to 11.

## DISCUSSION OF RESULTS ON AIRESEARCH NO. 2 HEATER

Since the accuracy of temperature measurements was much greater on the air side of the heater, the enthalpy change of the ventilating air was used to determine the thermal output of the heater.

The average measured temperature change of the exhaust gases during passage through the heater was about  $75^{\circ}$  F. A 1-percent error in the determination of either inlet or outlet exhaust gas temperature  $T_g \approx 1400^{\circ}$  F could cause an error of 19 percent in the determination of the temperature change of the gas. The poor heat balances obtained for these tests may be due to this error in measurement and also may be due partly to incomplete combustion of the exhaust gases.

At a ventilating air rate of 4000 pounds per hour the over-all thermal conductance UA was 14 percent lower for this heater (Airesearch no. 2) than for the larger (Airesearch no. 1) heater at an exhaust gas rate of 7300 pounds per hour and 11 percent lower at an exhaust gas rate of 4200 pounds per hour. (See reference 1 for results on Airesearch no. 1 heater.)

The isothermal pressure drop on the ventilating air side of the heater was 17 percent less and on the exhaust gas side 30 percent less for the smaller heater (Airesearch no. 2).

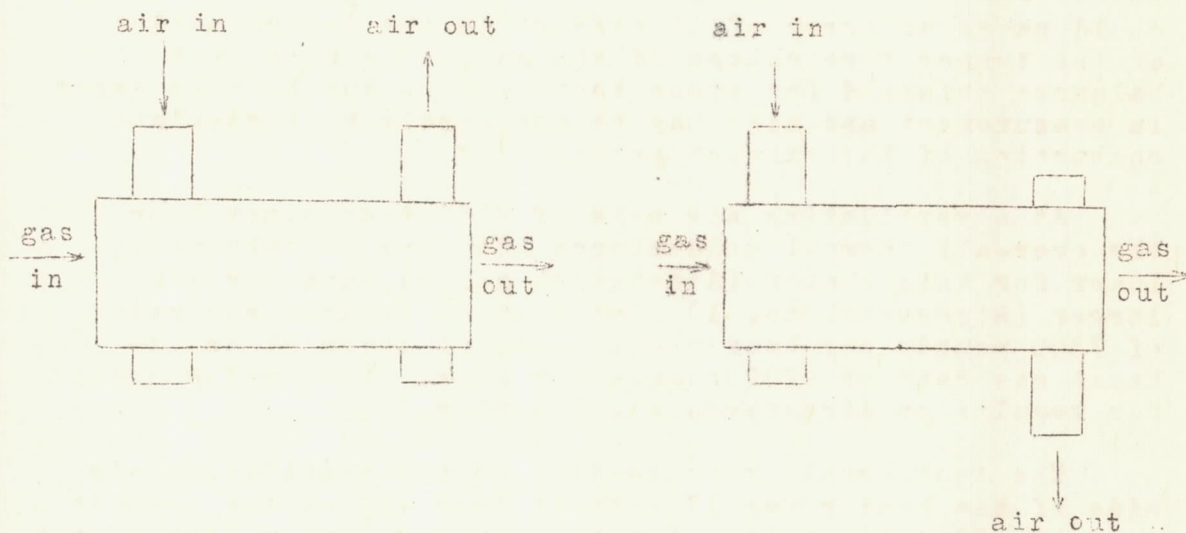
The heat transfer length was reduced by 21 percent and the diameter of the heater by  $5\frac{1}{2}$  percent for the smaller unit as compared with Airesearch heater no. 1.

The measured heater surface temperatures were about 18 percent lower for the smaller heater than for the larger heater at corresponding fluid rates and temperatures.

Reversing the position of the outlet ventilating air header, thus causing the air-side openings to be on opposite sides of the heater, did not appreciably affect the rate of heat transfer. (See fig. 7.) The corresponding isothermal pressure drops were about 8 percent greater with the reversed arrangement. The heater temperatures were from 10 to 15 percent higher at the outlet end of the heater for this reversed arrangement; whereas the heater temperatures

near the inlet end were about the same for the two positions of the air shroud.

The two air-shroud arrangements are pictured below:



Air openings on  
same side

Air openings on  
opposite sides

The predicted over-all thermal conductance UA is about 17 percent lower than the value derived from laboratory measurements at an exhaust gas rate of 7940 pounds per hour and about 3 percent lower at an exhaust gas rate of 4230 pounds per hour. (See fig. 8.)

The seemingly small variation in the predicted magnitudes of UA with the exhaust gas rate was due in part to the use of a value of  $(UA)_{ends}$  which is dependent only on the ventilating-air rate. (See discussion under Calculations.)

The predicted magnitude of the non-isothermal pressure drop derived from equation (8) compares well with the measured value when the air-shroud openings were on the same side of the heater. (See fig. 9.) The deviation between predicted and measured values for the reversed air-shroud arrangement may be due to the increased pressure drop caused by greater contraction, expansion, and eddy losses occasioned by the reversing of the ventilating air outlet-header duct. (See fig. 10.) The isothermal pressure-drop term  $\Delta P_{T_{iso}}$  used in equation (8) should be that due to friction alone.

The predicted non-isothermal pressure drops on the exhaust gas side of the heater are about 15 percent lower than those measured in the laboratory. (See fig. 11.)

An inspection of figures 9 to 11 reveals that the slope of the non-isothermal pressure drop curve is less than that of the isothermal one. At high weight rates the temperature change of the fluid is small; this causes the last term in equation (8)  $T_2/T_1 - 1$  to be small and thereby lowers the magnitude of the non-isothermal pressure drop. Also on the ventilating air side the non-isothermal pressure drop must coincide at an infinite weight rate of ventilating air with the isothermal pressure drop since the inlet and outlet air temperature would be equal - that is, isothermal.

#### CONCLUSIONS

1. The thermal performance of Airesearch heater no. 2 can be estimated to within 3 to 17 percent by use of the method described in this report.

2. The non-isothermal pressure drop due to friction can be approximated from measured isothermal pressure drop values by means of equation (8).

3. A reduction in length from Airesearch heater no. 1 by 21 percent and in diameter by  $5\frac{1}{2}$  percent reduced the thermal performance by only 10 to 14 percent. On the exhaust gas side the pressure drop was reduced by 30 percent and on the ventilating air side the pressure drop was reduced by about 17 percent.

4. The alternative arrangement of the outlet ventilating air header for which the air inlet and outlet openings were on opposite sides of the heater did not greatly affect the thermal or pressure drop characteristics of Airesearch heater no. 2 tested here.

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## APPENDIX

### TENTATIVE EQUATIONS FOR THE CALCULATION OF THE UNIT THERMAL CONVECTIVE CONDUCTANCE ALONG A FINNED OR UNFINNED HEAT EXCHANGER

#### INTRODUCTION

The flow of a fluid along a flat-plate fin or at the entrance of a tube or duct placed in a field of uniform velocity may be approximately described as follows. A laminar boundary layer of retarded velocity is initiated at the edge of the fin or tube and increases in thickness with distance from the entrance. If the turbulence in the fluid or the surface roughness, etc., is great enough, the flow in this boundary layer may become turbulent. As a first approximation the flow characteristics near the tube inlet may be considered to be those along the leading edge of a flat plate. At some point downstream from the entrance of the tube, however, the retarded layers along the walls may fill the tube completely. The same condition applies to flow between fins placed in close proximity. (See sketch on p. 21 and fig. 3 of reference 3.)

The relative thickness  $\delta/x$  of the boundary layer in the laminar regime varies approximately as  $Re^{-0.5}$  and the thickness  $\delta$  as  $x^{0.5}$ ; whereas the ratio  $\frac{f}{G c_p} = \frac{Nu}{Re Pr}$  varies as  $Re^{-0.5}$  or as  $x^{-0.5}$ , where  $x$  is the distance in the direction of fluid flow measured from the entrance of the tube (or from the edge of the flat-plate fin). If the boundary layer is



turbulent, its relative thickness  $\delta/x$  varies as  $Re^{-0.2}$  and the thickness  $\delta$  as  $x^{0.8}$ ; whereas the ratio  $\frac{f}{G c_p} = \frac{Nu}{Re Pr}$  varies as  $Re^{-0.2}$  or as  $x^{-0.2}$ .

The corresponding functions in the region of transition between laminar and turbulent flow are less well known. Downstream from the point where the fluid flow is fully developed (tube completely filled by boundary layers), the unit thermal conductance is independent of the distance from the tube entrance and depends on the hydraulic diameter of the tube. When the location of this point of fully developed flow is known, the equation for the unit thermal conductance along flat plates and for that in tubes may be used to approximate the conductance as a function of the heat exchanger length.

The conductance equations reported earlier (see references 2 and 3) may be rewritten in order to take into account the variation of the unit thermal conductance  $f_c$  with exchanger length by a tentative method derived as follows:

#### Derivation of Equations

For the turbulent regime the average unit thermal conductance for the length  $l$  along a flat plate (see reference 3) is

$$f_{c_l} = 9.36 \times 10^{-4} T^{0.256} l^{-0.2} G^{0.8} \quad (9)$$

The local or point value of  $f_c$  may be found from the expression (see reference 4, p. 38):

$$f_{c_x} = f_c (1+n) \quad (10)$$

where  $n$  is the exponent of  $l$  ( $-0.2$ ) in equation (9), so that

$$f_{c_x} = 0.8 f_{c_l} = 7.49 \times 10^{-4} T^{0.256} \frac{G^{0.8}}{X^{0.2}} \quad (11)$$

The average conductance in a long tube or duct of hydraulic diameter  $D$  (see references 2 and 3) is

$$f_{cD} = 5.56 \times 10^{-4} T^{0.296} \frac{G^{0.8}}{D^{0.2}} \quad (12)$$

Along a tubular heat exchanger or in the space between flat-plate fins the conductance near the entrance is given by equation (11) and that near the end of the passage by equation (12).

The correct mechanism of heat transfer at any point  $x$  may be characterized by the equation yielding the greater magnitude of  $f_c$  at that point. By equating the right sides of equations (11) and (12) the value of  $x/D$  for  $f_{cD} = f_{cx}$  is 4.4. Therefore at a distance of  $x = 4.4D$  from the entrance of the tubular heat exchanger or from the entrance to the space between two flat-plate fins the local values of  $f_c$  derived from equations (11) and (12) are equal and at this point it can be said that the flow is fully developed (that is, the tube or space is completely filled by the boundary layers).

Since equation (11) yields a higher value of  $f_c$  up to the point where  $x = 4.4D$ , this equation is used to evaluate the local value of  $f_{cx}$  up to this point. The average value of  $f_c$  for any length  $l$  up to the point  $x = 4.4D$  may be derived from equation (9).

For distances along a tube, channel, or fin greater than  $x = 4.4D$ , the local value of  $f_c$  from equation (12) is higher (furthermore it is constant with length). Therefore equation (12) is used to calculate the local  $f_c$  for all values of  $x$  between  $x = 4.4D$  and the end of the tube or fin.

The average value of  $f_c$  for a distance greater than  $x = 4.4D$  is a combination of equations (11) and (12). For a tube of hydraulic diameter  $D$  and length  $L \geq 4.4D$ , the average  $f_c$  would thus be

$$f_c = \frac{1}{L} \left[ \int_{x=0}^{x=4.4D} f_{cx} dx + \int_{x=4.4D}^{x=L} f_{cD} dx \right] \quad (13)$$

Let equation (12) be written as

$$f_{c_x} = 7.49 \times 10^{-4} T^{0.296} \frac{G^{0.8}}{x^{0.2}} = \frac{C_1}{x^{0.2}}$$

where

$$C_1 = 7.49 \times 10^{-4} T^{0.296} G^{0.8}$$

Equation (13) becomes

$$f_c = \frac{1}{L} \left[ \int_{x=0}^{x=4.4D} \frac{C_1}{x^{0.2}} dx + \int_{x=4.4D}^{x=L} f_{c_D} dx \right]$$

and, since

$$f_{c_D} \neq F(x)$$

$$f_c = \frac{1}{L} \left[ C_1 \frac{1}{0.8} (4.4D-0)^{0.8} + f_{c_D} (L-4.4D) \right]$$

or

$$f_c = \frac{1}{L} \left[ \frac{C_1}{0.8} (4.4D)^{0.8} + f_{c_D} (L-4.4D) \right]$$

and

$$C_1 = 7.49 \times 10^{-4} T^{0.296} G^{0.8}$$

Hence

$$f_c = \frac{1}{L} \left[ \frac{7.49}{0.8} \times 10^{-4} T^{0.296} G^{0.8} \frac{4.4D}{(4.4D)^{0.2}} + f_{c_D} (L-4.4D) \right] \quad (14)$$

It can be seen from equation (11) that the term

$$7.49 \times 10^{-4} T^{0.298} \frac{G^{0.8}}{(4.4D)^{0.2}}$$

is equal to  $f_{c_x}$  for  $x = 4.4D$  which was shown to be also equal to  $f_{c_D}$  at this value of  $x$ . So equation (14) can be rewritten as

$$\begin{aligned} f_c &= \frac{1}{L} \left[ \frac{f_{c_D}}{0.8} 4.4D + f_{c_D} (L - 4.4D) \right] \\ &= 1.25 \times 4.4 \frac{D}{L} f_{c_D} + f_{c_D} \frac{L}{L} - 4.4 \frac{D}{L} f_{c_D} \\ &= 0.25 \times 4.4 \frac{D}{L} f_{c_D} + f_{c_D} \\ f_c &= f_{c_D} \left[ 1 + 1.1 \frac{D}{L} \right] \end{aligned} \quad (15)$$

Hence for tubes or fins of length  $L \geq 4.4D$ , where  $D$  is the hydraulic diameter, the average conductance  $f_c$  is greater than that expressed by equation (12), using the hydraulic diameter as the significant dimension, by the multiplier  $\left[ 1 + 1.1 \frac{D}{L} \right]$ . For a circular tube in which  $D = 1$  inch and  $L = 12$  inches this factor is 1.092, indicating a 9.2 percent correction to equation (12).

The equations above hold for the system where a fin or tube is placed in a fluid of uniform velocity distribution and the transition of the boundary layer from laminar to turbulent flow occurs very near the origin of the boundary layer.

In the actual system where fins are attached to a surface over which a fluid is flowing, the leading edge of the fin or flat plate is subjected to a nonuniform distribution of fluid velocity since the velocity is zero at the wall or base of the attached fin.

Another deviation is found in the system where long continuous fins are cut or slotted at regular intervals. The velocity distribution may or may not be appropriate to initiate boundary layers at each slot if the slots are narrow.

The effect of placing cuts or slots of various sizes and spacings along flat-plate fins is being investigated at present in the laboratory. Experiments are also being planned to determine the unit thermal conductance as a function of the exchanger length near the inlet.

#### Recapitulation of Equations

The following criterion is presented tentatively (turbulent regime established in the boundary layers):

1. For  $0 < x \leq 4.4D$

(a) The local conductance  $f_{c_x}$  at the distance  $x$  is expressed by equation (11)

$$f_{c_x} = 7.49 \times 10^{-4} T^{0.296} \frac{G^{0.8}}{x^{0.2}} \quad (11)$$

(b) The average conductance  $f_{c_l}$  for the length  $x$  is given by equation (9)

$$f_{c_l} = 9.36 \times 10^{-4} T^{0.296} \frac{G^{0.8}}{x^{0.2}} \quad (9)$$

2. For  $x \geq 4.4D$

(a) The local constant conductance  $f_{c_D}$ , expressed by equation (12), is independent of the distance from the entrance of the heat exchanger. Thus

$$f_{c_D} = 5.56 \times 10^{-4} T^{0.296} \frac{G^{0.8}}{D^{0.2}} \quad (12)$$

- (b) The average value of  $f_c$  for a length greater than or equal to  $x = 4.4D$  is given by equation (15)

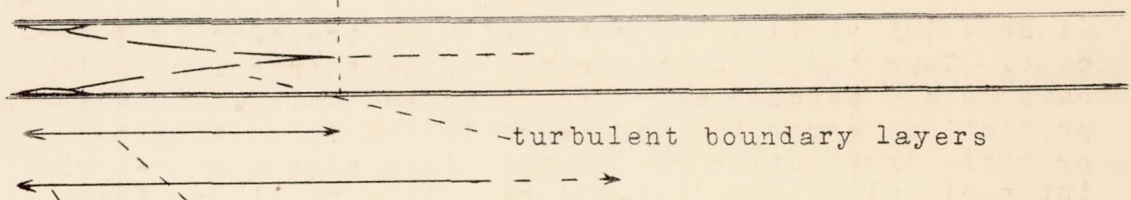
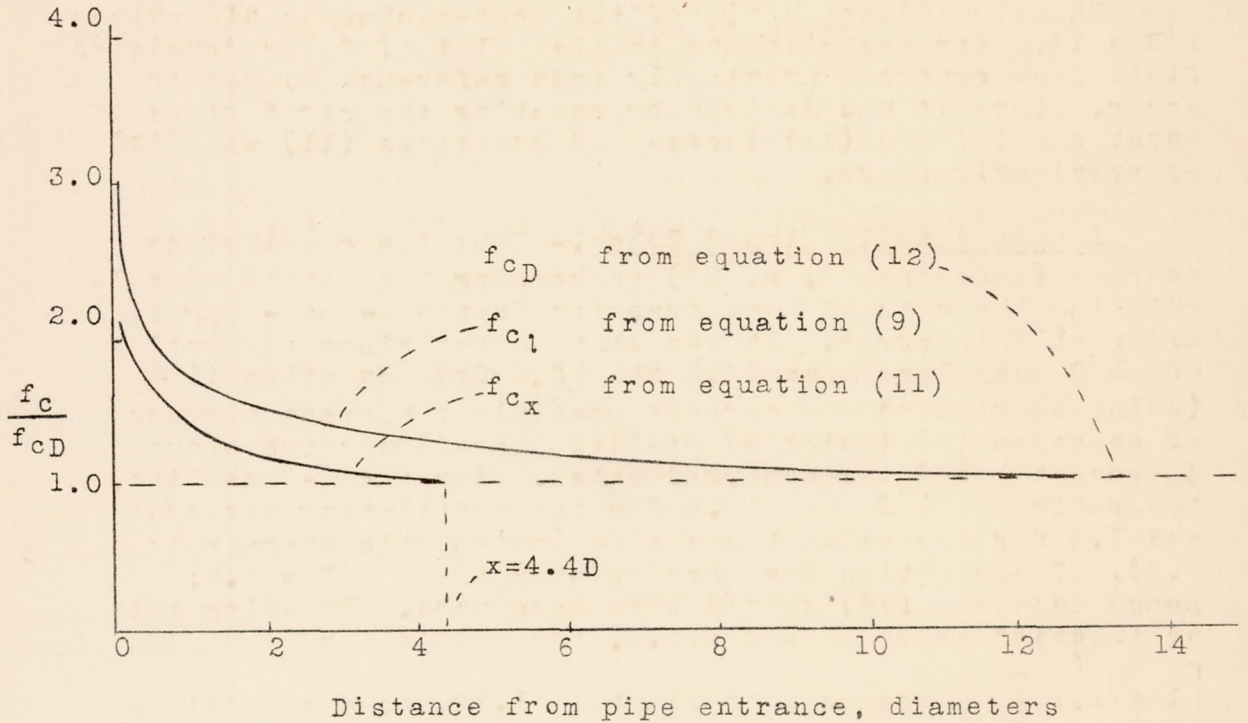
$$f_c = f_{c_D} \left[ 1 + 1.1 \frac{D}{L} \right] \quad (15)$$

where  $L$  is the length of the fin or tube. Equations (5) and (6) can now be rewritten:

$$f_{ca} = 5.56 \times 10^{-4} T_a^{0.296} \frac{G_a^{0.8}}{D_a^{0.2}} \left[ 1 + 1.1 \frac{D_a}{L} \right] \quad (5a)$$

and

$$f_{cg} = 5.56 \times 10^{-4} T_g^{0.296} \frac{G_g^{0.8}}{D_g^{0.2}} \left[ 1 + 1.1 \frac{D_g}{L} \right] \quad (6a)$$



For  $0 < x \leq 4.4D$

$f_{c_x}$  from equation (11) (local value)

$f_{c_l}$  from equation (9) (average value)

For  $4.4D \leq x < \text{total length}$

$f_{c_x}$  from equation (12) (local value)

$f_c$  from equation (15) (average value)

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### Discussion of Previous Heater Results

The significant ratio of the heat-exchanger dimensions  $x/D = 13.4$  for the distance to the point of fully developed fluid flow reported previously (see reference 3) was in error, since it was derived by equating the right sides of equations (9) and (12) instead of equations (11) and (12) as previously shown.

Longitudinally Finned Tubes.— Thus the conclusions reached (reference 3, p. 23) concerning the method of calculating the rate of heat transfer from a 6-inch-long fin are partly in error. It was stated that since the ratio of  $L/D$  was less than 13.4 the  $f_c$  from equation (10) (using  $L$ ) yielded the correct heat flow; whereas the use of equation (9) (using  $D$ ) yielded heat-flow rates about 3 percent below the measured rates. For the 6-inch fins, the ratio of  $L/D$  was 10.9 for the ventilating air side and 7.8 for the exhaust gas side (arithmetic average is 9.3). These ratios then are greater than  $L/D = 4.4$ ; hence equation (15) should have been used. By using the arithmetic\* average  $L/D = 9.3$ , the multiplier

$\left[ 1 + 1.1 \frac{D}{L} \right]$  becomes approximately 1.12, or a correction of 12 percent over that which was obtained by using  $f_{cD}$  alone which yielded low rates of heat transfer by 3 percent. Thus the predicted rates of heat transfer would have been 9 percent above the measured value. That these predictions were high may be explained in part by the probable ineffectiveness of the slots placed at 6-inch intervals along the fins, since only a small gap (1/16 inch) was used. (See conclusion 4 on p. 28 of reference 3.)

The predicted results of the heat flow from the 52-inch fins (see reference 4) are not, however, appreciably affected, since the correction factor in equation (15) is only 1.014, which means only a 1.4-percent error.

Airesearch No. 2 heater.— The over-all thermal conductance of the Airesearch no. 2 heater described in this report has been recalculated using equation (15). Thus

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\*The correct result would be obtained by correcting the  $f_c$  on the air and gas sides by the corresponding values of  $L/D$  and recomputing the over-all conductance  $U$ .



equations (5) and (6) are rewritten as

$$f_{c_a} = 5.56 \times 10^{-4} T_a^{0.296} \frac{G_a^{0.8}}{D_a^{0.2}} \left( 1 + 1.1 \frac{D_a}{L} \right) \quad (5a)$$

and

$$f_{c_g} = 5.56 \times 10^{-4} T_g^{0.296} \frac{G_g^{0.8}}{D_g^{0.2}} \left( 1 + 1.1 \frac{D_g}{L} \right) \quad (6a)$$

The previous measurements of heat transfer on the larger Airesearch no. 1 heater were used to compute the thermal effect of the tapered end sections by subtracting the predicted over-all thermal conductance of the full-fluted center section from the total conductance derived from laboratory measurements. However, equations (5a) and (6a) were used to predict the conductances in the full-fluted center section instead of the analogous equations (5) and (6) without the multiplier  $(1 + 1.1 D/L)$ . The conductance of the ends, which are practically the same as those on the smaller Airesearch no. 2 unit, was found to vary in the following manner:

$W_a$ (lb/hr)	$(UA)_{ends}$ (Btu/hr °F)
3000	70
5000	81
6000	85

The over-all conductance of the Airesearch no. 2 heater was then found by adding the conductance for the uniform full-fluted center section predicted for this heater by means of equations (4), (5a), and (6a) to the value of  $(UA)_{ends}$  for the larger no. 1 heater evaluated at the corresponding air rate  $W_a$  from the table above, using the approximate equation (7)

$$q_a = (UA) \Delta t_{1m} = \left[ (UA)_{center} + (UA)_{ends} \right] \Delta t_{1m} \quad (7)$$

The resulting magnitudes of the over-all thermal conductance using equations (4), (5a), (6a), and (7) are within 1 or 2 percent of those derived from the use of equations (4), (5), (6), and (7), shown in fig. 8, which do not take into account the L/D effect in the full-fluted center section.

However, the use of equations (5a) and (6a) yields values of UA along the center full-fluted section\* which are 3.2 percent higher for the large Airesearch no. 1 heater and 6.4 percent higher for the smaller Airesearch no. 2 heater than is the value of UA obtained by use of equations (5) and (6). The method used in this report for establishing the heat flow through the tapered ends of the heater therefore masks the variations of UA caused by use of the two forms of the equations for  $f_c$  along the center sections of the heater. That is, the difference in the values of UA for the center section of the large no. 1 heater resulting from use of the two forms of the equations for  $f_c$  is absorbed in the tapered end correction (UA)<sub>ends</sub> and carried through to the calculation for the smaller no. 2 heater. This difference does not then appear in the final value of UA.

Airesearch no. 1 heater.— For the larger Airesearch no. 1 heater the use of equations (5a) and (6a) in the computation of UA (see reference 1) would have decreased the discrepancy between the measured and predicted values by about 3 percent. (The discrepancy reported was as much as 18 percent.)

Solar fluted-type heater.— For the Solar fluted-type heater (see also reference 1) the ratio of L/D was about 15.0; hence the multiplier  $1 + 1.1 \frac{D}{L} = 1.073$ . This factor used in equations (5a) and (6a) would have reduced the discrepancy between the predicted and measured magnitudes of UA from about 20 percent to about 13 percent.

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 \*For the full-fluted center sections of the Airesearch heaters the magnitude of L/D for the large no. 1 heater is 34.0 and for the smaller no. 2 heater it is 17.2.

## REFERENCES

1. Boelter, L. M. K., Miller, M. A., Sharp, W. H.,  
Morrin, E. H., Iverson, H. W., Mason, W. E.:  
An Investigation of Aircraft Heaters. IX -  
Measured and Predicted Performance of Two  
Exhaust Gas-Air Heat Exchangers and an  
Apparatus for Evaluating Exhaust Gas-Air Heat  
Exchangers. NACA A.R.R., March 1943.
2. Martinelli, R. C., Weinberg, E. B., Morrin, E. H.,  
and Boelter, L. M. K.: An Investigation of  
Aircraft Heaters. III - Measured and Predicted  
Performance of Double Tube Heat Exchangers. NACA  
A.R.R., Oct. 1942.
3. Martinelli, R. C., Weinberg, E. B., Morrin, E. H.,  
and Boelter, L. M. K.: An Investigation of Air-  
craft Heaters. IV - Measured and Predicted Per-  
formance of Longitudinally Finned Tubes. NACA  
A.R.R., Oct. 1942.
4. Martinelli, R. C., Tribus, M., and Boelter, L. M. K.:  
An Investigation of Aircraft Heaters. I-Elementary  
Heat Transfer Considerations in an Airplane. NACA  
A.R.R., Oct. 1942.

TABLE I.— AIRESEARCH FLUTED-TYPE HEATER \*2  
AIR-SHROUD OPENINGS ON SAME SIDE

Run No.	← AIR SIDE →						← EXHAUST-GAS SIDE →						$\frac{q_g}{q_a}$	HEATER TEMPS.		OVERALL PERFORMANCE	
	$T_{a_1}$ °F	$T_{a_2}$ °F	$\Delta T_a$ °F	$W_a$ lb/hr	$\Delta P'_a$ Inches H <sub>2</sub> O	$q_a$ KBtu/hr	$T_{g_1}$ °F	$T_{g_2}$ °F	$\Delta T_g$ °F	$W_g$ lb/hr	$\Delta P'_g$ Inches H <sub>2</sub> O	$q_g$ KBtu/hr		$t_1$ °F	$t_2$ °F	$\Delta t_{lm}$ °F	(UA) Btu/hr °F
1	97	358	261	4000	12.0	252	1411	1373	38	7690	7.08	80.0	0.32	404	616	1160	217
2	97	378	281	3400	9.00	231	1420	1385	35	7740	7.30	74.6	0.32	440	667	1160	199
3	97	410	313	2700	6.10	208	1411	1368	43	7690	7.30	91.0	0.45	484	718	1130	184
4	95	400	305	2750	6.10	203	1398	1355	43	7230	6.65	85.5	0.42	493	697	1130	182
5	95	358	263	3400	9.20	217	1394	1320	74	7230	6.50	147	0.68	413	624	1130	192
6	93	333	240	4100	12.3	238	1390	1360	74	7250	6.45	147	0.62	373	581	1140	209
7	89	319	230	4100	6.25	228	1403	1282	121	5940	4.45	197	0.87	346	541	1130	202
8	89	344	255	3350	8.75	207	1394	1325	69	5940	4.51	113	0.55	386	581	1140	182
9	89	377	288	2650	6.00	185	1385	1312	73	5940	4.60	119	0.65	431	641	1110	167
10	90	430	340	2000	3.67	164	1424	1368	56	5915	4.74	91.4	0.56	519	748	1120	146
11	93	395	302	1950	3.60	142	1420	1338	82	4230	2.70	95.4	0.67	448	654	1120	127
12	89	347	258	2640	5.90	165	1398	1312	86	4230	2.70	100	0.60	377	572	1130	146
13	86	308	222	3440	8.90	185	1385	1282	103	4230	2.60	120	0.65	328	502	1130	164
14	85	284	199	4200	12.4	203	1408	1312	96	4230	2.60	112	0.65	292	466	1175	173

\* Arithmetic average of three surface temps. measured near exhaust-gas inlet to heater. (see Fig. 6)  
\* " " " " " " " " " " outlet to heater. " " "

TABLE II.- AIRESEARCH FLUTED-TYPE HEATER #2  
AIR-SHROUD OPENINGS ON OPPOSITE SIDES

Run No.	← AIR SIDE →						← EXHAUST-GAS SIDE →						HEATER TEMPS.		OVERALL PERFORMANCE		
	$T_{a1}$ °F	$T_{a2}$ °F	$\Delta T_a$ °F	$W_a$ $\frac{\text{lbs}}{\text{hr}}$	$\Delta P_a'$ Inches H <sub>2</sub> O	$q_a$ $\frac{\text{KBtu}}{\text{hr}}$	$T_{g1}$ °F	$T_{g2}$ °F	$\Delta T_g$ °F	$W_g$ $\frac{\text{lbs}}{\text{hr}}$	$\Delta P_g'$ Inches H <sub>2</sub> O	$q_g$ $\frac{\text{KBtu}}{\text{hr}}$	$\frac{q_g}{q_a}$	$t_1^*$ °F	$t_2^{\dagger}$ °F	$\Delta t_{lm}$ °F	(UA) $\frac{\text{Btu}}{\text{hr}^\circ\text{F}}$
29	104	362	258	4150	12.3	259	1407	1368	39	7910	7.00	85.0	0.33	391	672	1150	225
30	104	380	276	3580	9.65	239	1415	1368	47	7940	7.00	103	0.43	424	717	1145	208
31	100	422	322	2770	6.20	216	1424	1364	60	7940	7.35	131	0.61	493	815	1125	192
32	102	373	271	2770	5.95	182	1420	1360	60	4490	2.60	74.1	0.41	400	674	1145	159
33	98	332	234	3560	9.05	202	1415	1334	81	4490	2.50	100	0.50	341	589	1160	174
34	96	311	215	4290	12.2	223	1403	1316	87	4490	2.50	108	0.48	301	527	1150	194

\* Arithmetic average of three surface temperatures measured near exhaust-gas inlet to heater. (see Fig. 6)

† " " " " " " " " " " " " outlet " " " " "

TABLE III  
AIRESEARCH FLUTED-TYPE HEATER NO. 2  
PRESSURE DROP DATA

Run	W (lb/hr)	G (lb/hr) sq ft	Measured isothermal pressure drop $\Delta P_{T_{iso}}$ (lb/sq ft) ( $T_{iso}=560^{\circ}R$ )	Predicted non- isothermal pressure drop $\Delta P$ (lb/sq ft)	Measured non- isothermal pressure drop $\Delta P$ (lb/sq ft)	$T_1$ ( $^{\circ}R$ )	$T_2$ ( $^{\circ}R$ )	$T_a$ ( $^{\circ}R$ )
-----	--------------	-----------------------	--	---	--	--------------------------	--------------------------	--------------------------

Air Side (air-shroud openings on same side)

14	4200	23,300	47.5	62.6	64.2	545	744	645
5	3400	18,900	33.5	48.0	47.8	550	811	680
3	2700	15,000	21.6	32.8	31.5	557	870	713
10	2000	11,100	12.3	19.1	19.0	550	890	720

Air Side (air-shroud openings on opposite sides)

34	4290	23,800	54.5	74.1	63.5	556	771	663
33	3560	19,800	39.0	53.9	47.0	558	792	675
31	2770	15,400	24.6	37.8	32.2	560	882	721

Exhaust Gas Side

3	7690	35,900	9.50	33.4	37.9	1871	1828	1849
6	7250	33,900	8.60	27.3	33.3	1850	1776	1813
10	5915	27,600	6.10	21.2	24.8	1884	1828	1856
14	4230	19,800	3.50	10.3	13.5	1868	1772	1820

These entries are taken from plot of  $\Delta P_a$  against  $W_a$  or  $\Delta P_g$  against  $W_g$ , since actual isothermal measurements were at slightly different fluid rates.

$$\Delta P = \Delta P_{T_{iso}} \left( \frac{T_a}{T_{iso}} \right)^{1.13} + \frac{G^2}{(3600)^2 \gamma_1 g} \left( \frac{T_2}{T_1} - 1 \right) \quad (8)$$

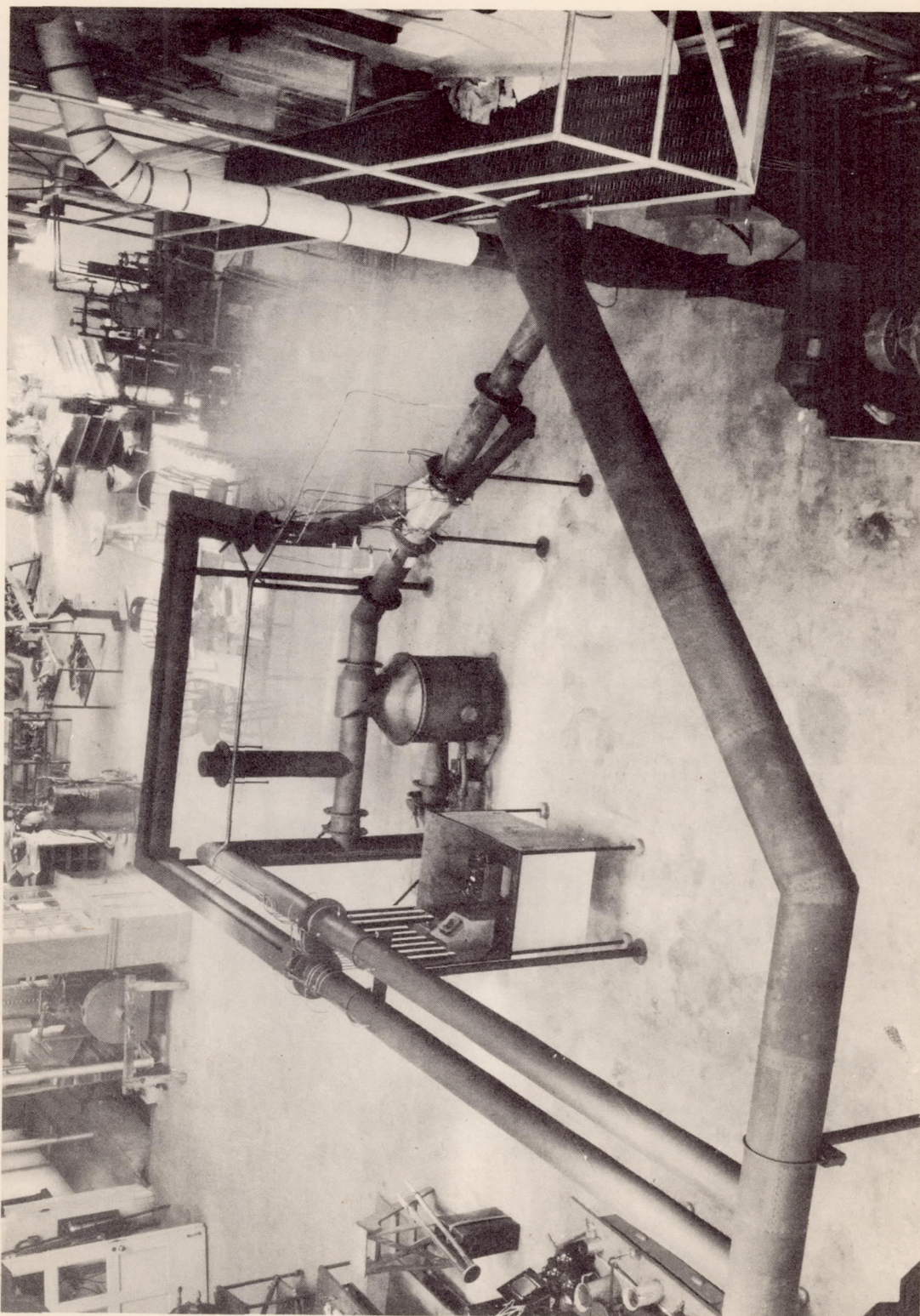


Figure 1.- Photograph of heater test stand.

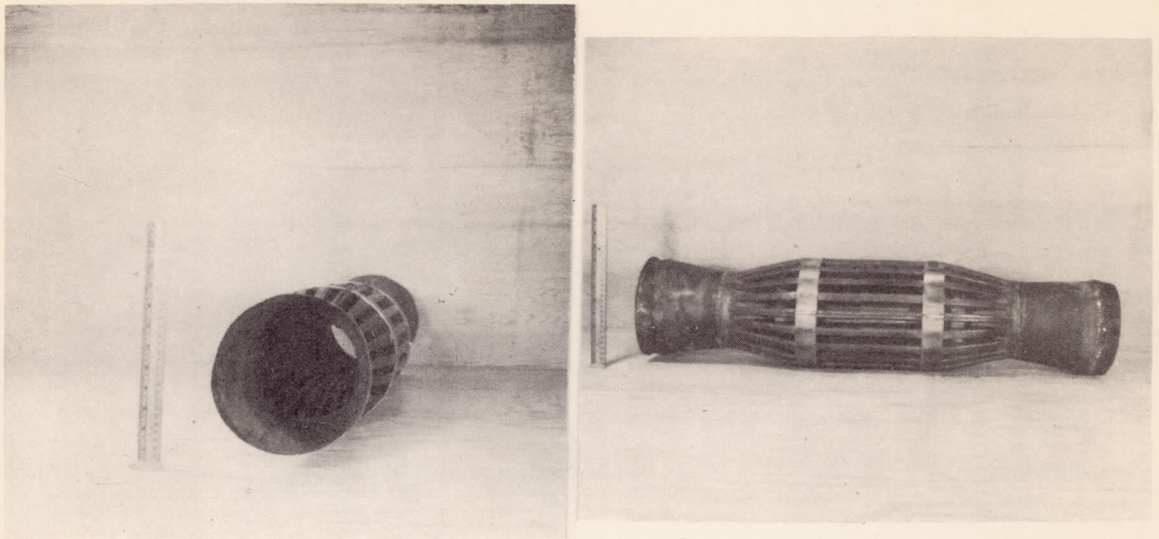
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Front of building







Figures 2 and 3.- Photograph of Airesearch No.2 heater.

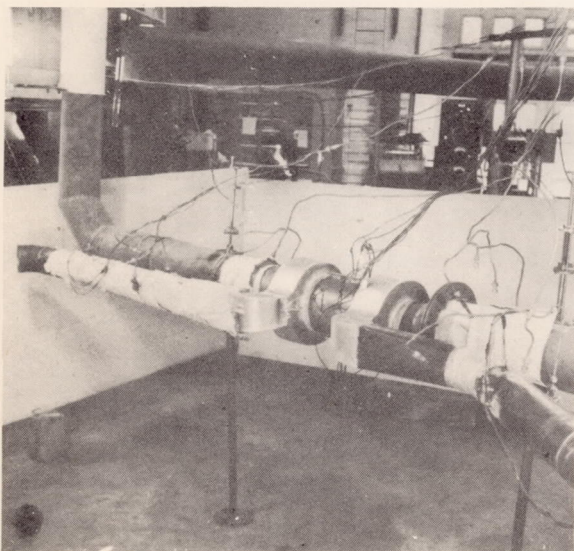


Figure 4.- Photograph of heater in test stand with air-shroud headers in normal position.

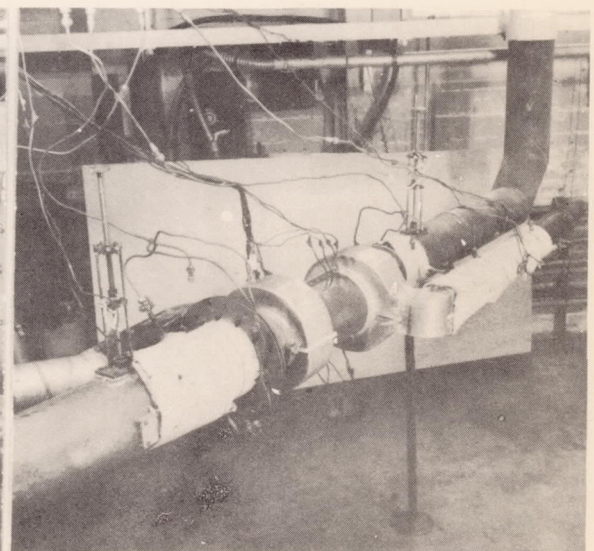
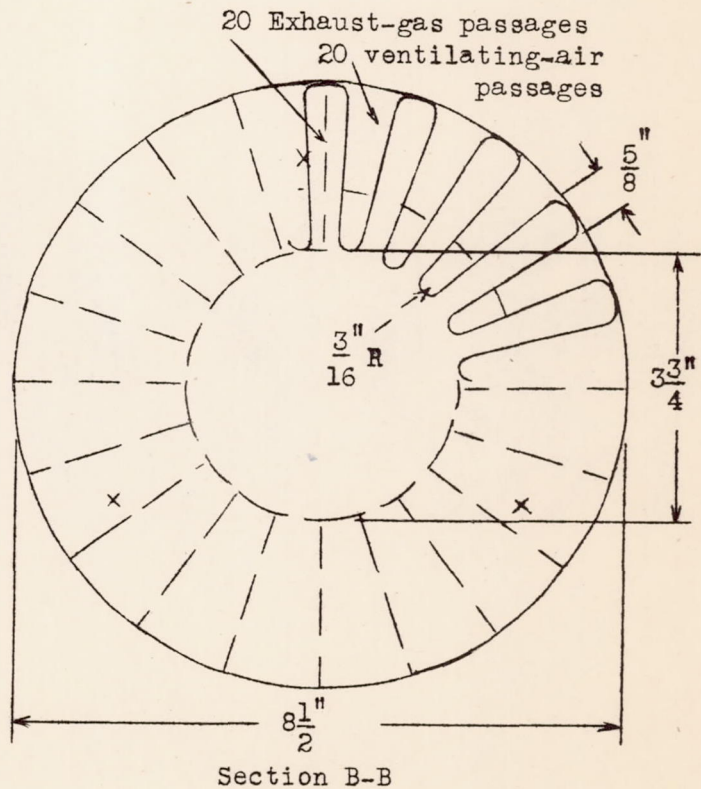
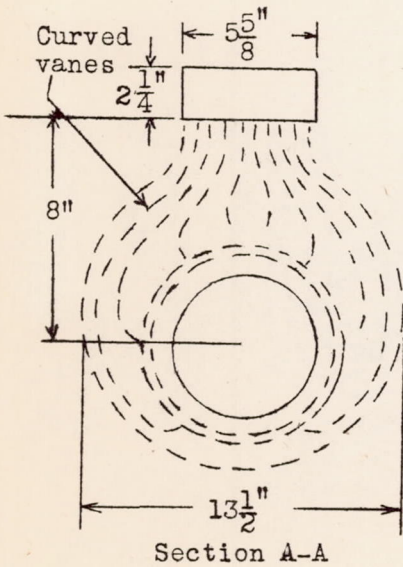
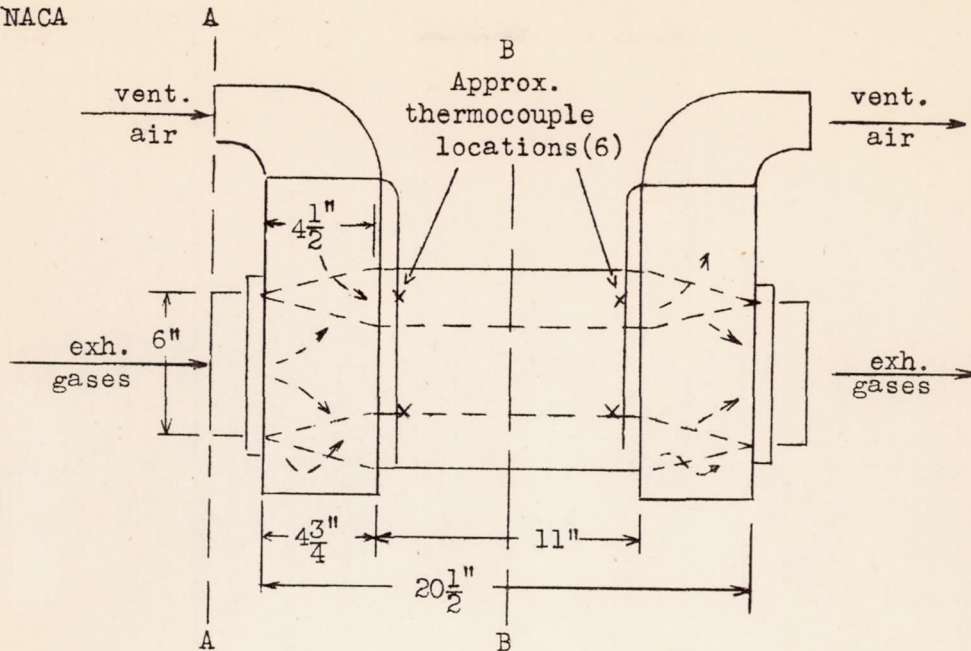


Figure 5.- Photograph of heater in test stand with air-shroud headers in reversed position.

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The diagram illustrates the relationship between the variables shown in the figure. It is a technical drawing that likely represents a specific process or system. The details are too faint to be accurately transcribed, but it appears to be a schematic or a map of some kind.



	Air side	Gas side
Section B-B		
Cross-section area, ft <sup>2</sup>	0.180	0.214
Wetted perimeter, ft	9.88	9.72
Heat transfer perimeter, ft	8.66	8.66
Weight of heater - $22\frac{1}{2}$ lb, shroud - $10\frac{1}{2}$ lb		

Figure 6.- Schematic diagram of Aircsearch No. 2 heater and air shroud.

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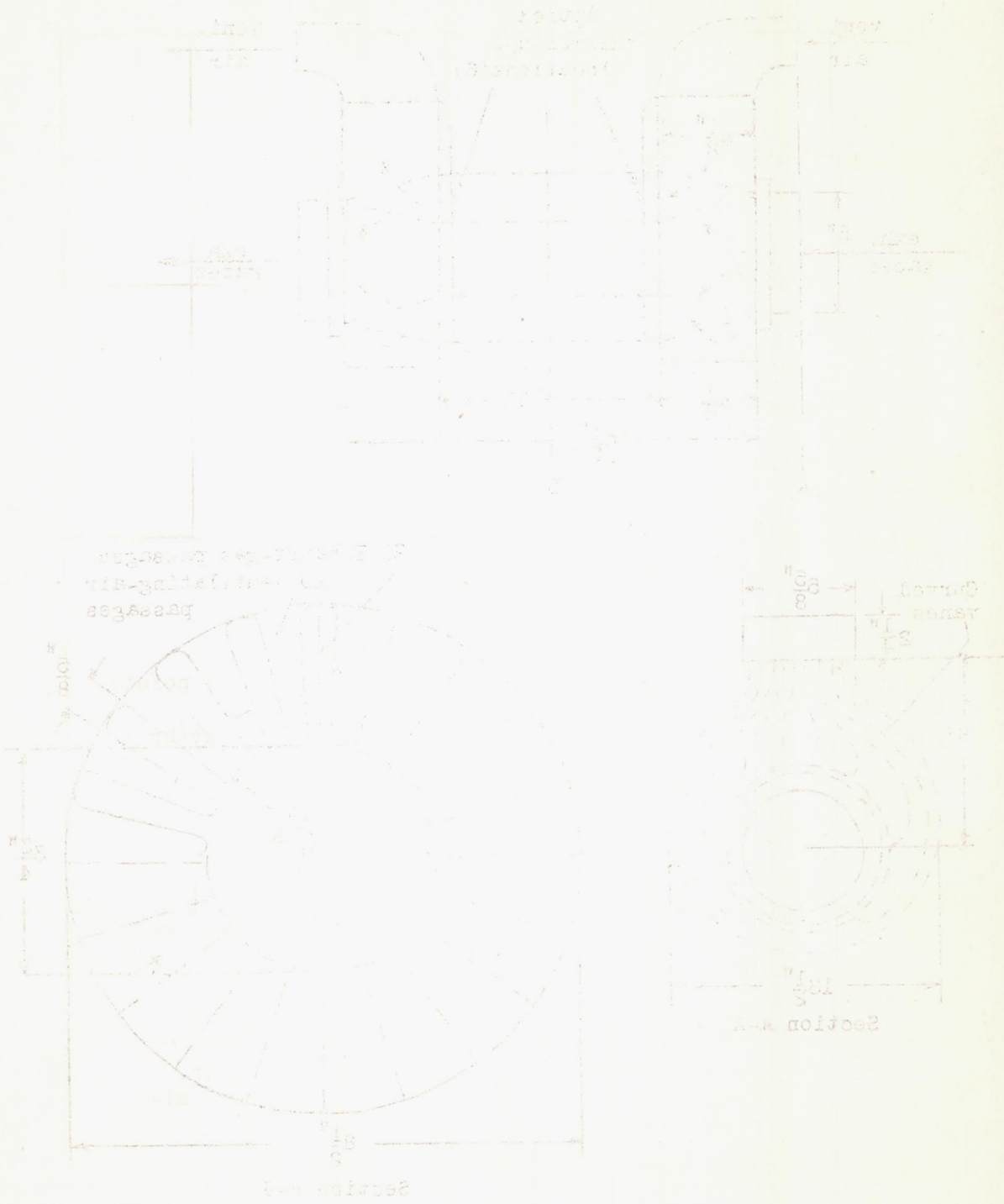


Figure 8. Schematic diagram of Altescon No. 2 heater and air shroud.

Heat transfer parameter, $h$	1.86
Water perimeter, $P_w$	9.88
Cross-section area, $A_c$	0.180
Air film	
Use air	
0.234	
4.72	
8.58	

Water of heater - 37.5 lb. shroud - 10.5 lb

2/11/47

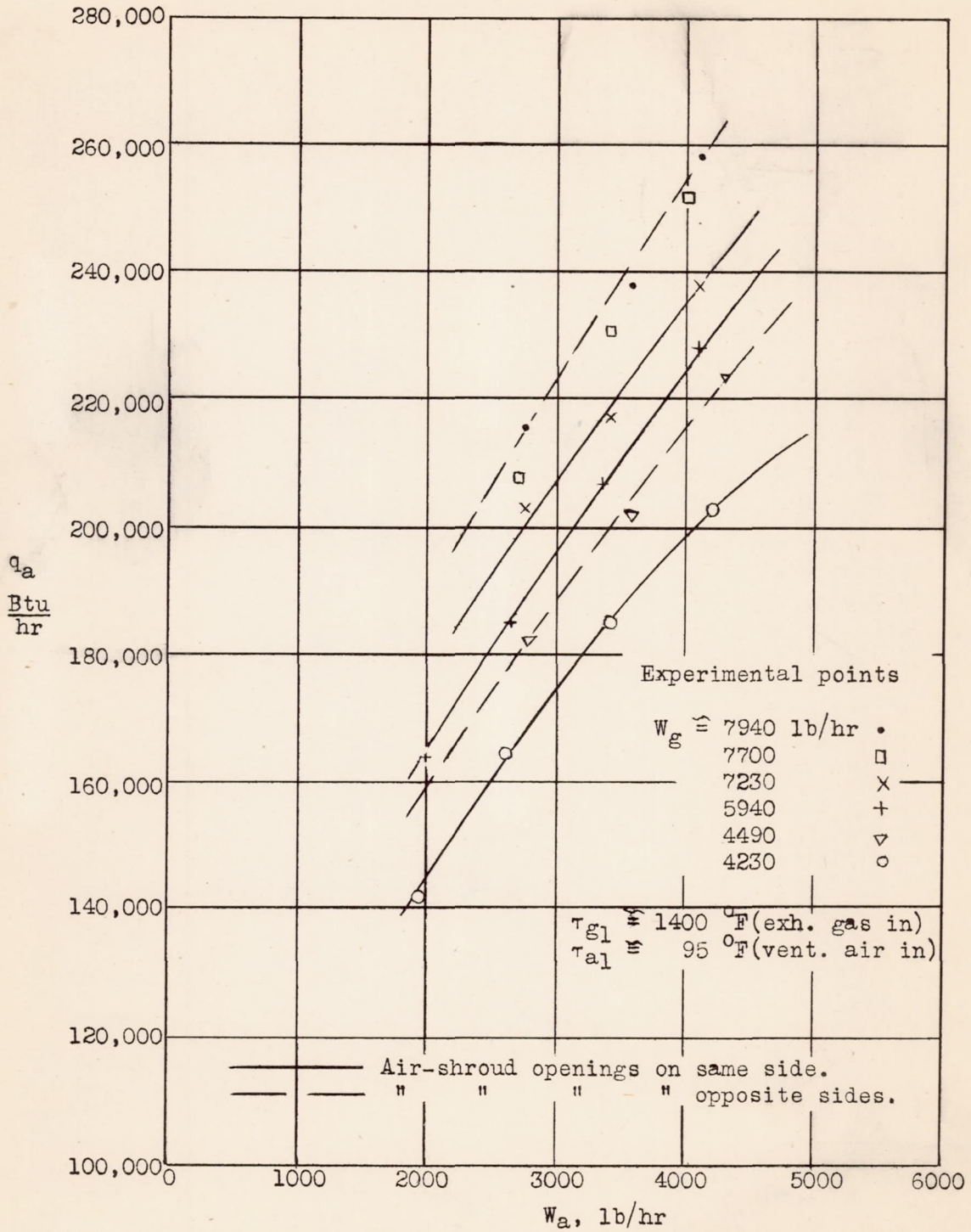


Figure 7.- Thermal output of Airesearch No.2 fluted-type heater as a function of ventilating-air rate.

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130,000  
140,000  
150,000  
160,000  
170,000  
180,000

Figure 1 - A line graph showing the relationship between the variables on the x and y axes. The y-axis represents the dependent variable, and the x-axis represents the independent variable. The data points are connected by a line, and a vertical dashed line is drawn at the 4th x-axis tick mark.

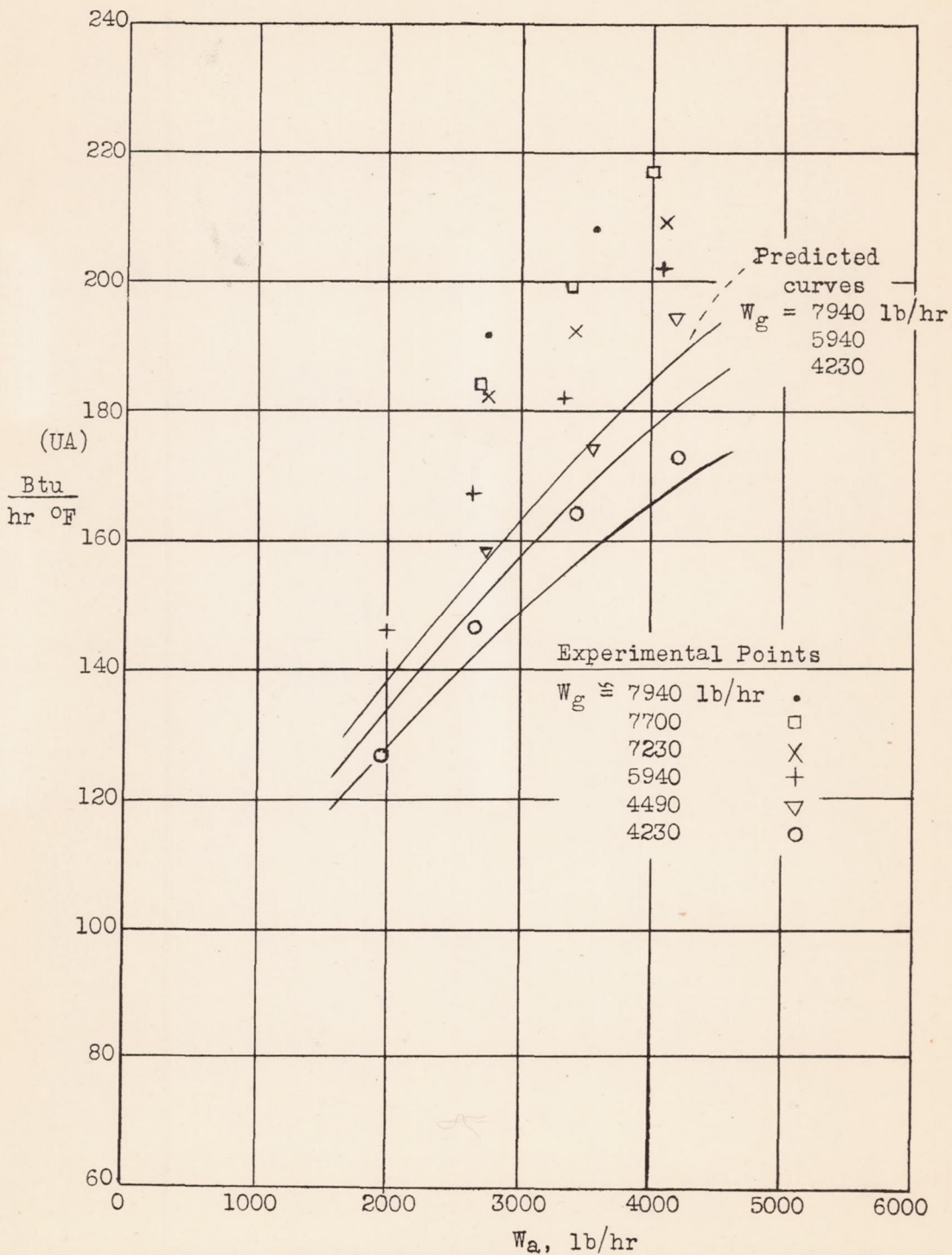
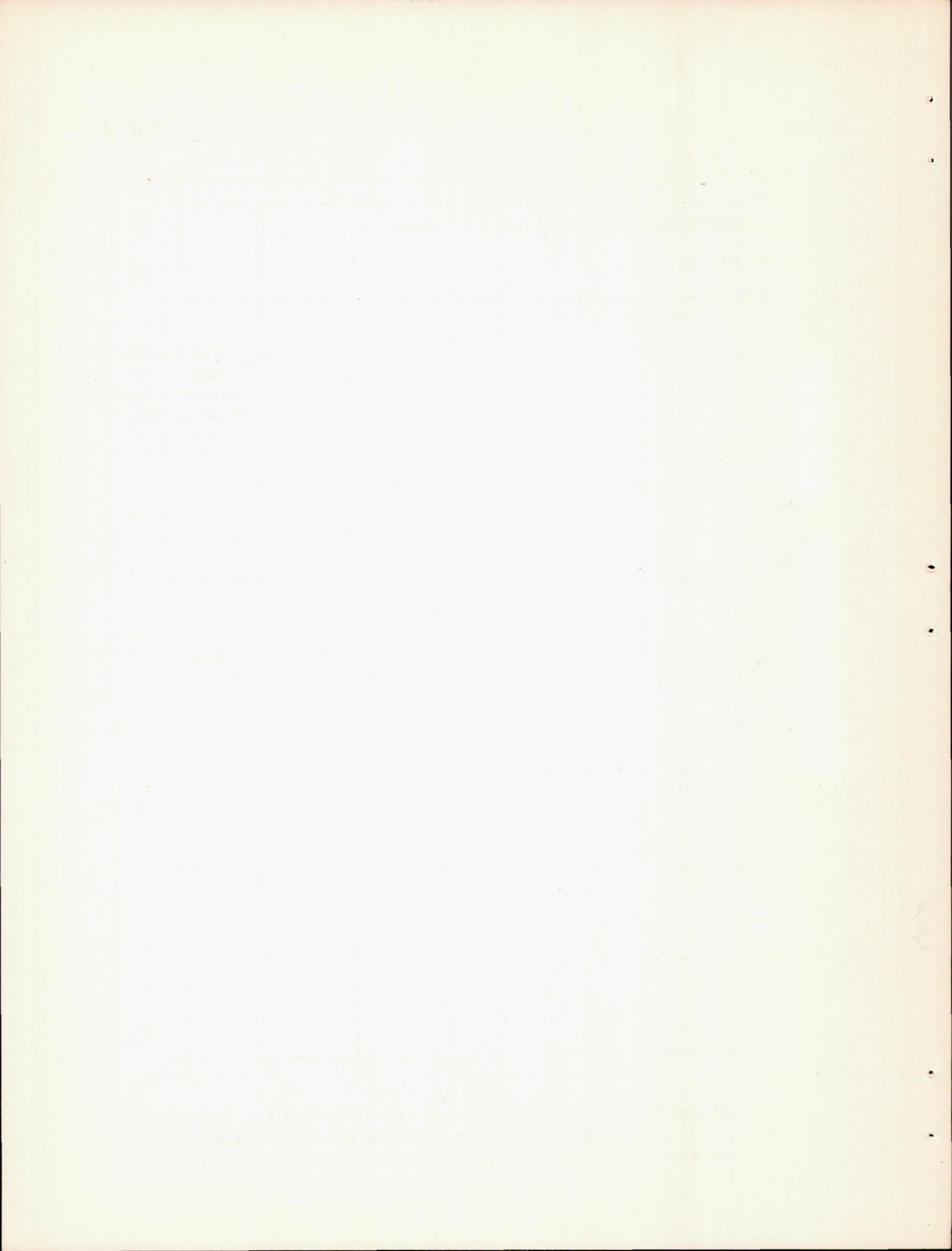


Figure 8.- Overall conductance of Airesearch No. 2 fluted-type heater as a function of ventilating-air rate.

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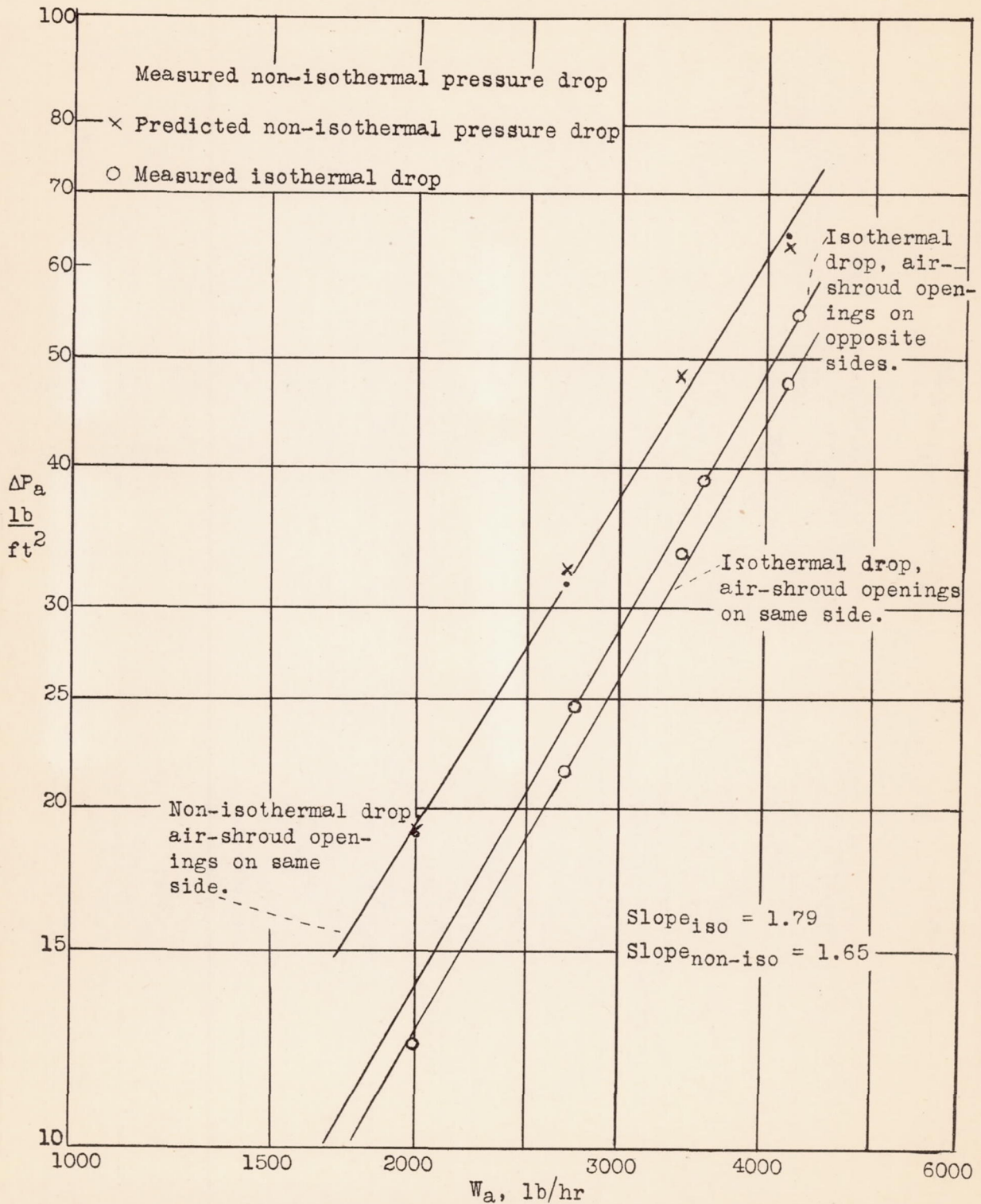


Figure 9.-- Pressure drop on air side of Airesearch No. 2 heater as a function of ventilating-air rate.

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1000	1000	1000	1000	1000	1000
1000	1000	1000	1000	1000	1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

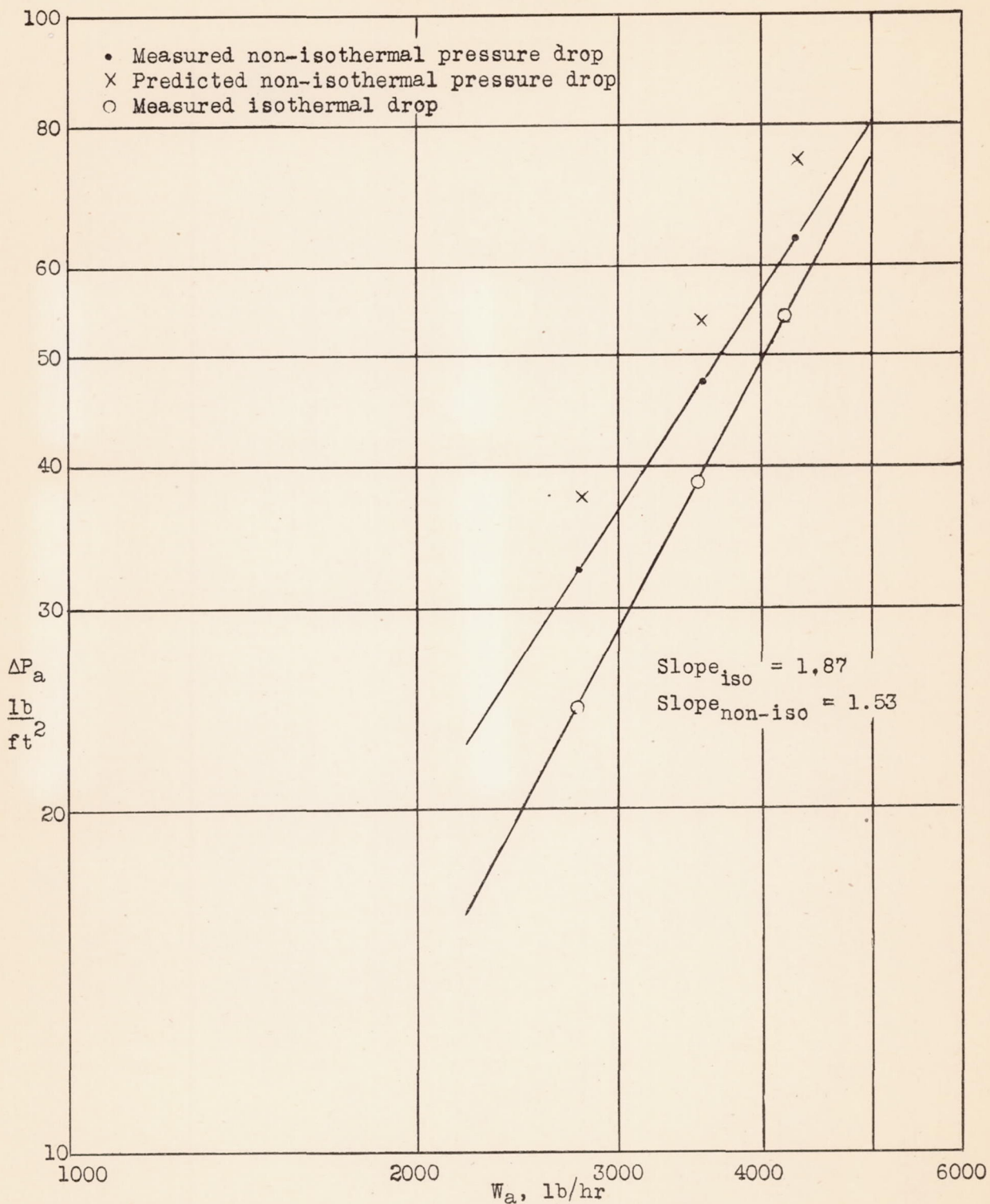


Figure 10.- Pressure drop on air side of Airesearch No. 2 heater as a function of ventilating-air rate. (Air-shroud openings on opposite sides.)

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Date		Description		Amount	
10/1					
10/2					
10/3					
10/4					
10/5					
10/6					
10/7					
10/8					
10/9					
10/10					
10/11					
10/12					
10/13					
10/14					
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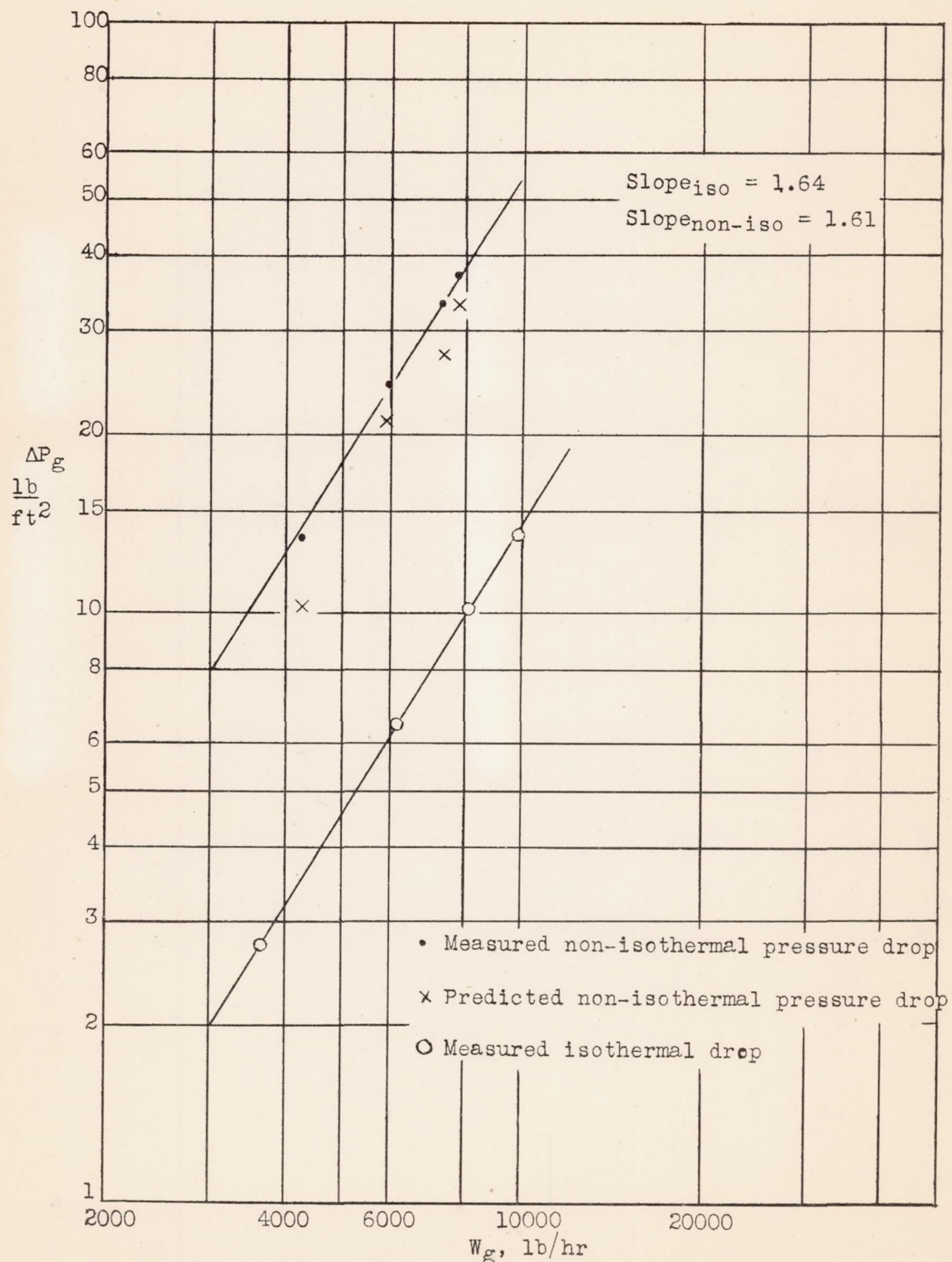


Figure 11.- Pressure drop on exhaust-gas side of Airesearch No. 2 heater as a function of exhaust-gas rate,

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