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A PRELIMINARY THEORETICAL STUDY OF HELICOPTER-BLADE

FLUTTER INVOLVING DEPENDENCE UPON CONING ANGLE

AND PITCH SETTING

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SUMMARY

A preliminary analysis has been made of the conditions of stability of free oscillations of a hinged rotor in hovering flight. The case analyzed is a rotor with hinges allowing freedom in flapping and lagging and having a completely reversible cyclic pitch-control system, so that a twisting moment on a blade moves the control stick without hindrance from spring or friction constraint.

The principal results of this study are presented in the form of a stability chart in which the quantities specifying average coning angle, pitch setting, moments of inertia, chordwise and spanwise mass distribution are combined into generalized parameters in such a way that the stability condition can be plotted on a single chart.

The results show that the stability is extremely sensitive to chordwise center of mass, and that forward movement of the center of mass increases the stability. It is also shown that the stability depends upon coning angle and pitch setting. For all examples in which the coning angle is determined by a balance between lift forces and inertia forces, the condition for neutral stability corresponds to the same point of the chart.

Several lines of attack for further theoretical work are suggested, which would extend the generality and strengthen the validity of the analysis.

INTRODUCTION

It is important for the safety of helicopter flight that suitable precautions be taken to avoid the occurrence of self-excited vibrations such as blade flutter. The understanding of flutter phenomena is therefore important in the design of rotary wing aircraft. Although helicopter blade flutter has a certain similarity to airplane wing flutter, the analysis is somewhat different because of the occurrence of terms associated with the rotation of the blades.

Rosenberg in reference 1 has given an analysis of helicopter blade flutter that consists essentially of adding centrifugal stiffening terms to the standard wing flutter analysis. But rotation can also introduce inertia coupling between flapping, lagging, and feathering. These coupling terms, depending upon the average coning angle and pitch setting of the blades, have not previously been considered in flutter analysis.

The purpose of the present paper is to exhibit the simplest case that will point out these new effects. This is done by treating the extreme case where inertia forces and air forces are dominant in comparison with elastic forces. As this case is characterized by a low flutter frequency, the phenomenon is conveniently referred to as low frequency helicopter flutter.

In reference 1 the elastic forces are considered and certain inertia coupling terms are ignored, while in this paper these inertia coupling terms are considered and the elastic terms are ignored. The next logical step in the solution of the flutter problem now seems to be to combine the two theories into a single scheme of analysis that includes both elastic effects and coning and pitch-angle effects. This further development, however, has not been included in the present paper.

It should be mentioned that the present analysis takes no account of the phenomenon of stall flutter although pitch angle is also an important parameter in that problem.

ANALYSIS

If the cyclic pitch-control system of a rotor is considered to be completely reversible, so that a twisting moment on a blade moves the control stick without hindrance from spring or friction constraint then, except for inertia effects due to rotation, the

blade may be considered to have zero frequency in torsion. Also, if all the blades have the same physical characteristics, the stability of the rotor as a whole can be obtained from a study of the equations of motion for a single blade. If further, for purposes of vibration analysis, the hinge lines of flapping, lagging and feathering are assumed to pass through a common point on the spindle axis, then the blade may be treated as a rigid body with one point fixed. The well known Euler's equations of motion for this case are

$$A\dot{\omega}_x - (B - C)\omega_y\omega_z = L_x$$

$$B\dot{\omega}_y - (C - A)\omega_z\omega_x = L_y \qquad (1)$$

$$C\dot{\omega}_z - (A - B)\omega_x\omega_y = L_z$$

where

A, B, C   principal moments of inertia of blade about x, y, and z axes, respectively

$\omega_x, \omega_y, \omega_z$ components of angular velocity resolved along instantaneous directions of principal axes of blade

L_x, L_y, L_z components of external moments about the fixed point

x, y, z   subscripts referring to principal axes of inertia of blade. (See fig. 1.)

The average values of aerodynamic and inertia moments will be considered to be balanced by suitable constant torques applied at the hub by the drive shaft and, if necessary, by the control system. The deviations from the average will then appear in the equations governing the vibration.

The total external moment L_x, L_y, L_z is the sum of the aerodynamic moment L_x', L_y', L_z' and the hub torque Q_x, Q_y, Q_z.

Expressions for the components of aerodynamic moment L_x', L_y', L_z' are obtained by assuming that each airfoil section has an aerodynamic force normal to its instantaneous velocity and equal in magnitude to the conventional expression for lift of an airfoil in steady motion (see fig. 2) thus:

$$F = \frac{1}{2}\rho V^2 ca_0 \sin \alpha \qquad (2)$$

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where

F   aerodynamic force per unit span

V   instantaneous velocity of section

c   chord of section

α instantaneous angle of attack

a_o slope of lift curve

The effect of induced velocity is considered only in the choice of numerical value for the slope of the lift curve. Drag forces have been ignored in this introductory treatment.

As the inertia terms in Euler's equations are expressed in terms of ω_x, ω_y, ω_z it is desirable to express the aerodynamic moments in terms of these same variables, thus:

$$V_y = V \cos \alpha = r\omega_z$$

$$V_z = V \sin \alpha = r\omega_y$$

where r is radial distance of blade element from hinge point.

then

$$F_z = F \cos \alpha = \tfrac{1}{2}\rho V^2 ca_o \sin \alpha \cos \alpha = \tfrac{1}{2}\rho ca_o r^2 \omega_y \omega_z$$

$$F_y = F \sin \alpha = \tfrac{1}{2}\rho V^2 ca_o \sin^2 \alpha = \tfrac{1}{2}\rho ca_o r^2 \omega_y^2$$

The aerodynamic moments can then be written:

$$\left.\begin{array}{l} L_x' = \displaystyle\int_o^R F_z s\,dr = \omega_y\omega_z \int_o^R \tfrac{1}{2}\rho ca_o s r^2\,dr \\[2em] L_y' = \displaystyle\int_o^R -F_z r\,dr = -\omega_y\omega_z \int_o^R \tfrac{1}{2}\rho ca_o r^3\,dr \\[2em] L_z' = \displaystyle\int_o^R F_y r\,dr = \omega_y^2 \int_o^R \tfrac{1}{2}\rho ca_o r^3\,dr \end{array}\right\} \quad (3)$$

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where s is the distance of the chordwise center of mass behind the aerodynamic center.

Euler's equations now become:

$$\left.\begin{array}{l} A\dot{\omega}_x - (B - C)\,\omega_y\omega_z = \omega_y\omega_z \displaystyle\int_0^R \tfrac{1}{2}\rho c a_o s r^2 dr + Q_x \\[2ex] B\dot{\omega}_y - (C - A)\omega_z\omega_x = -\omega_y\omega_z \displaystyle\int_0^R \tfrac{1}{2}\rho c a_o r^3 dr + Q_y \\[2ex] C\dot{\omega}_z - (A - B)\omega_x\omega_y = \omega_y{}^2 \displaystyle\int_0^R \tfrac{1}{2}\rho c a_o r3 dr + Q_z \end{array}\right\} \quad (4)$$

where Q_x, Q_y, Q_z are the components of constant torque required to balance average values of assumed aerodynamic and inertia moments. These torque components may represent such things as weight moment, constant structural bending moment, and constant bungee force in the control system.

In order to obtain solutions of equations (4) the deviations from the average values of ω_x, ω_y, ω_z are assumed to be small enough so that squares and products of deviations can be neglected.

Equations (4) are linearized by putting

$$\left.\begin{array}{l} \omega_x = \Omega_x + \omega_x' \\[2ex] \omega_y = \Omega_y + \omega_y' \\[2ex] \omega_z = \Omega_z + \omega_z' \end{array}\right\} \quad (5)$$

and with the assumption that Q_x, Q_y, Q_z are in equilibrium with terms not containing ω_x', ω_y', ω_z' the constant terms become

$$\left.\begin{array}{l} (I_1 - H_1)\Omega_y\Omega_z = \dfrac{Q_x}{A} \\[2ex] -I_2\Omega_z\Omega_x + H_2\Omega_y\Omega_z = \dfrac{Q_y}{B} \\[2ex] I_3\Omega_x\Omega_y - H_3\Omega_y{}^2 = \dfrac{Q_z}{C} \end{array}\right\} \quad (6)$$

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and the linear terms become

$$\dot{\omega}_x' + (I_1 - H_1)(\Omega_y \omega_z' + \Omega_z \omega_y') = 0$$

$$\dot{\omega}_y' - I_2(\Omega_z \omega_x' + \Omega_x \omega_z') + H_2(\Omega_y \omega_z + \Omega_z \omega_y') = 0 \qquad (7)$$

$$\dot{\omega}_z' + I_3(\Omega_x \omega_y' + \Omega_y \omega_x') - H_3 2\Omega_y \omega_y' = 0$$

where

$$I_1 = \frac{C - B}{A} \qquad\qquad H_1 = \frac{1}{A}\int_0^R \frac{1}{2}\rho c a_0 s r^2 dr$$

$$I_2 = \frac{C - A}{B} \qquad\qquad H_2 = \frac{1}{B}\int_0^R \frac{1}{2}\rho c a_0 r^3 dr$$

$$I_3 = \frac{B - A}{C} \qquad\qquad H_3 = \frac{1}{C}\int_0^R \frac{1}{2}\rho c a_0 r^3 dr$$

The quadratic terms are neglected.

The linear terms govern the vibrations. Their solution is of the form

$$\omega_x' = \omega_{xo}' \, e^{\lambda t}$$

$$\omega_y' = \omega_{yo}' \, e^{\lambda t} \qquad (8)$$

$$\omega_z' = \omega_{zo}' \, e^{\lambda t}$$

where λ is a root of the determinantal equation

$$\begin{vmatrix} \lambda & (I_1 - H_1)\Omega_z & (I_1 - H_1)\Omega_y \\ -I_2\Omega_z & \lambda + H_2\Omega_z & -I_2\Omega_x + H_2\Omega_y \\ I_3\Omega_y & I_3\Omega_x - 2H_3\Omega_y & \lambda \end{vmatrix} = 0 \qquad (9)$$

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This equation is a cubic in λ. When expanded in the form

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$

where

$a = H_2\Omega_z$

$b = (I_1 - H_1)(I_2\Omega_z^2 - I_3\Omega_y^2) + (I_2\Omega_x - H_2\Omega_y)(I_3\Omega_x - 2H_3\Omega_y)$

$c = 2I_2(I_1 - H_1)\Omega_z\Omega_y(-I_3\Omega_x + H_3\Omega_y)$

the well known conditions for stability are

$$a > 0$$

$$b > 0$$

$$c > 0$$

$$c < ab$$

The fourth condition is the important one for oscillatory instability.

For the critical condition corresponding to $c = ab$ the cubic equation becomes

$$(\lambda + a)(\lambda^2 + b) = 0$$

$$\lambda = \begin{cases} -a \\ \pm i\sqrt{b} \end{cases} \qquad (10)$$

This condition represents the border line between damped and self-excited vibration of the system. A chart has been devised to show the values of the pertinent parameters corresponding to this boundary between damped and self-excited vibrations.

The explicit equation corresponding to $c = ab$ is

$$\Omega_y(I_1 - H_1) 2I_2 (-I_3\Omega_x + H_3\Omega_y)$$
$$= H_2 \left[(I_1 - H_1)(I_2\Omega_z^2 - I_3\Omega_y^2) + (I_2\Omega_x - H_2\Omega_y)(I_3\Omega_x - 2H_3\Omega_y) \right] \qquad (11)$$

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which can be reduced to the form

$$\frac{H_1 - I_1}{I_3}\frac{\Omega_z^2}{\Omega_x^2} = \frac{\left(\frac{H_2}{I_2}\frac{\Omega_y}{\Omega_x} - 1\right)\left(2\frac{H_3}{I_3}\frac{\Omega_y}{\Omega_x} - 1\right)}{1 - \frac{\Omega_y^2}{\Omega_z^2}\left(\frac{2H_3}{H_2} + \frac{I_3}{I_2} - \frac{2I_3}{H_2}\frac{\Omega_x}{\Omega_y}\right)} \tag{12}$$

The average pitch setting θ and coning angle β are now introduced by the substitution

$$\left.\begin{array}{l} \Omega_x = \Omega \sin \beta \\[6pt] \Omega_y = \Omega \cos \beta \sin \theta \\[6pt] \Omega_z = \Omega \cos \beta \cos \theta \end{array}\right\} \tag{13}$$

In terms of β and θ equation (12) becomes

$$\frac{H_1 - I_1}{I_3}\frac{\cos^2 \theta}{\tan^2 \beta} = \frac{\left(\frac{H_2}{I_2}\frac{\sin \theta}{\tan \beta} - 1\right)\left(2\frac{H_3}{I_3}\frac{\sin \theta}{\tan \beta} - 1\right)}{1 - \tan^2 \theta\left(2\frac{H_3}{H_2} + \frac{I_3}{I_2} - \frac{2I_3}{H_2}\frac{\tan \beta}{\sin \theta}\right)} \tag{14}$$

STABILITY CHART

A chart has been plotted in which all variables of equation (14) have been included subject only to the restriction

$$\begin{array}{l} H_2 = H_3 \\ I_2 = I_3 \end{array} \tag{15}$$

which is a very good approximation for a long thin body like a rotor blade. In terms of variables defined by

$$X = \frac{H_1 - I_1}{I_3}\frac{\cos^2 \theta}{\tan^2 \beta}$$

$$Y = \frac{H_2}{I_2}\frac{\sin \theta}{\tan \beta}$$

equation (14) becomes

$$X = \frac{(Y - 1)(2Y - 1)}{1 - \tan^2 \theta \left(3 - \frac{2}{Y}\right)} \qquad (16)$$

The chart, figure 3, is then a plot of Y against X for constant values of θ.

DISCUSSION OF RESULTS

The variables X and Y of equation (16) have been defined in such a way as to make the chart nearly a universal single curve. A sight dependence upon θ indicated by the different curves of the chart, shows that it is, in reality, a family of curves; but that all the curves are close together for any reasonable range of values of θ. The main dependence of the flutter condition upon the physcial characteristics and the operating condition of the blade is consequently implied in the form of expression defining X and Y.

The most important blade characteristic is the chordwise position of center of mass with respect to aerodynamic center. This distance s appears in the definition of H_1 and consequently in X. For typical parameters, changing the center of mass from 1 percent ahead to 1 percent behind the aerodynamic center will change the value of X from -15 to 15 if $I_1 = 0$ and from -45 to -15 if $I_1 = 1$. The value of I_1 would be close to zero for a blade with a heavy spar of circular cross section. It would be close to 1 for a blade with its mass well distributed in the chordwise direction.

The variable Y is equal to the ratio of the average aerodynamic moment to the average inertia moment about the axis parallel to the blade chord.

For

average aerodynamic moment, $L_y' = -\Omega_y \Omega_z BH_2$

average inertia moment, $M_y = (C - A)\Omega_z \Omega_x$

$$-\frac{L_y'}{M_y} = \frac{H_2 B \Omega_y}{(C - A) \Omega_x} = \frac{H_2 \sin \theta}{I_2 \tan \beta} = Y$$

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The value of Y for blades with a flapping hinge will therefore
normally be close to 1, so that the critical point for nearly all
typical applications will be close to Y = 1, X = 0. Departures
from the value · Y = 1 will be associated with such things as
gravity moment or structural bending moment in the blade, or
possibly transient flapping conditions as in gusts. These effects
may thus make the blade flutter if they increase the coning angle
or make it more stable if they decrease the coning angle. It
seems, for example, as though the gravity moment would tend to make
the blade more stable. Cases could thus be imagined that, for a
constant pitch setting were stable at very low rpm, where gravity
has a large relative effect, and unstable at operating rpm, where
a typical value of Y is 1.04. A model tested upside down would
be expected to show the opposite effect.

FURTHER REFINEMENTS OF THEORY

This treatment is to be considered as only a preliminary
study of blade flutter. Many simplifying assumptions have been
made in order to obtain a simple stability chart and for this reason
the results should be applied with caution. This analysis can now
be used as a starting point for further refinements. Some effects
which might be considered are:

1. drag forces

2. case where $I_2 \neq I_3$

3. elastic and friction forces

4. unsteady-lift functions

5. constraints such as completely irreversible controls

6. induced velocities and flight velocity

It is hoped that the present treatment, which is based upon
linearized Euler's equations, may suggest interesting alternatives
to the more common hinge angle representation in other problems of
rotor dynamics.

CONCLUSIONS

1. For a helicopter blade in which the restoring forces in vibration are due to inertia effects rather than springs or structural stiffness, a simplified theory leads to conditions for the occurrence of flutter that can be represented by a single generalized chart. For all examples in which the coning angle is determined by a balance between lift forces and inertia forces the critical condition corresponds to the same point on the chart.

2. The theory indicates the dependence of flutter instability upon the physical characteristics and the coning angle and pitch setting of the blade.

3. The results show that the stability is extremely sensitive to chordwise position of center of mass with respect to aerodynamic center. Forward movement of the center of mass increases the stability.

4. This paper is to be considered as only a preliminary study of blade flutter and as a starting point for further developments.

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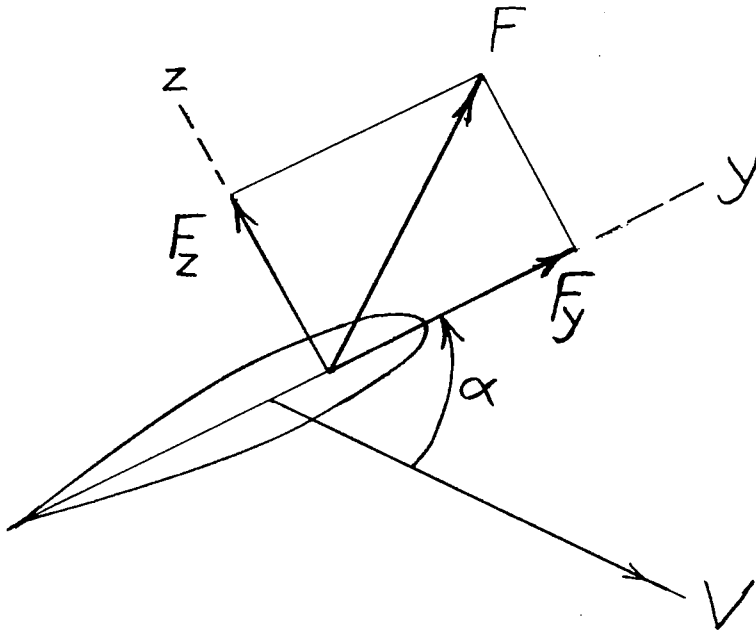   Langley Field, Va.

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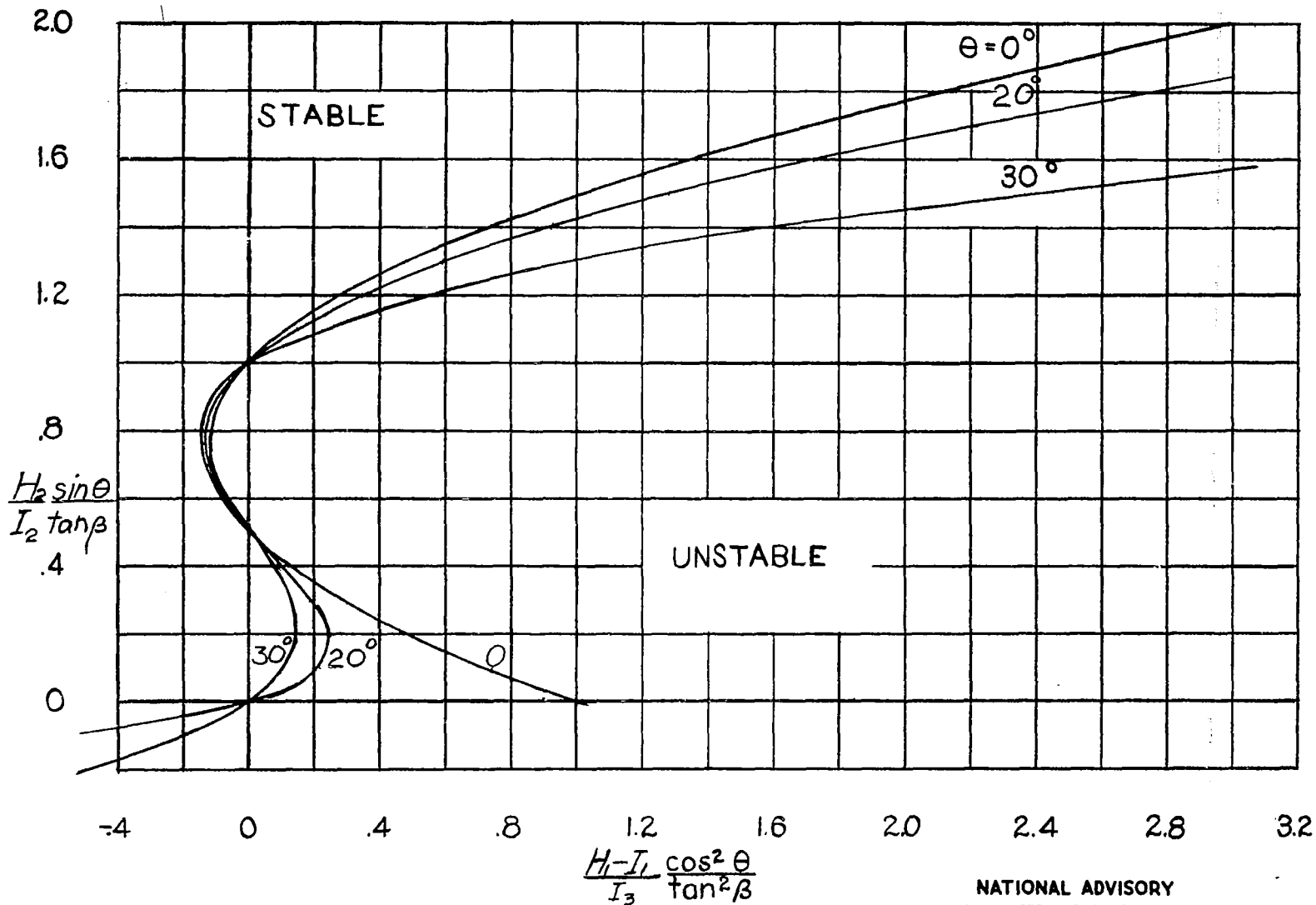   Lifting Rotor Blades. Jour. Aero. Sci., vol. 11, no. 4
    Oct. 1944, p. 361.
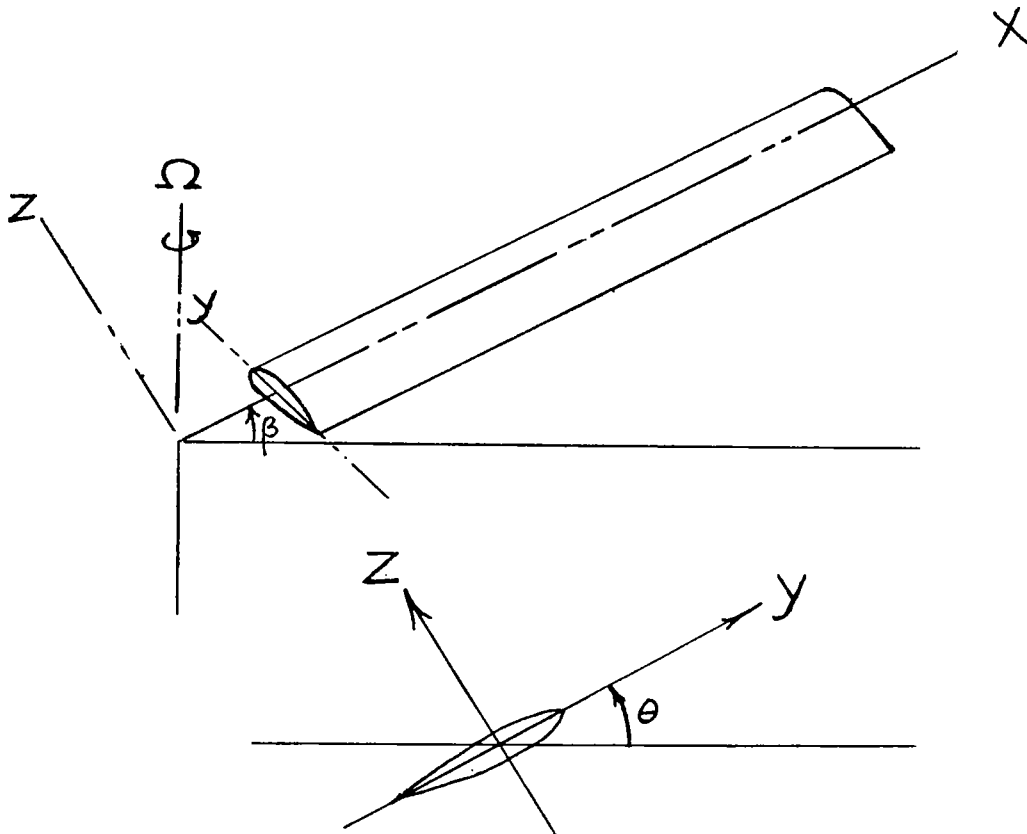
Figure 1.— Principal Axes of Inertia of a Blade.

Figure 2.— Assumed Instantaneous Force on Airfoil.

STABLE

$\theta = 0°$

$20°$

$30°$

UNSTABLE

$\dfrac{H_2 \sin\theta}{I_2 \tan\beta}$

$30°$ $20°$ 0

$\dfrac{H_1 - I_1}{I_3}\dfrac{\cos^2\theta}{\tan^2\beta}$

Figure 3.- Stability Chart.