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CHARTS FOR CALCULATION OF THE CRITICAL STRESS FOR LOCAL INSTABILITY OF COLUMNS WITH I-, Z-, CHANNEL, AND RECTANGULAR-TUBE SECTION

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CHARTS FOR CALCULATION OF THE CRITICAL STRESS FOR LOCAL INSTABILITY OF COLUMNS WITH I-, Z-, CHANNEL, AND RECTANGULAR-TUBE SECTION

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SUMMARY

Charts are presented for the calculation of the critical stress for local instability of columns with I-, Z-, channel, and rectangular-tube section. These charts are intended to replace the less complete charts published in NACA Technical Note No. 743. The values used in extending the charts are computed by moment-distribution methods that give somewhat more accurate values than the energy method previously used and also make it possible to determine theoretically which element of the cross section is primarily responsible for instability.

An experimental curve is included for use in taking into account the effect of stresses above the elastic range on the modulus of elasticity of 24S-T aluminum alloy.

A determination of the dimensions of a thin-metal column for maximum critical stress with certain given conditions is presented.

INTRODUCTION

One of the important requirements in the design of thin-metal columns for aircraft is the determination of the critical compressive stress at which local instability occurs. Local instability of a column is defined as any type of instability in which the cross sections are distorted in their own planes but are not translated or rotated.

The critical stress for local instability can usually be given in terms of the geometry of the section, the properties of the material, and a coefficient. Reference 1 presented charts for the determination of such coefficients for columns of I-, Z-, channel, and rectangular-tube sections. These charts, however, contained relatively few curves and in some cases required interpolation over a wide range.

In order to make the charts of reference 1 more nearly complete and to reduce the necessary range of interpolation, each chart has been extended to include eight intermediate curves. The values used in extending the charts are computed by moment-distribution methods that give somewhat more accurate values than the energy method previously used and also make it possible to determine theoretically which element of the cross section is primarily responsible for instability.

The present report includes the extended charts, along with tables of the values used in preparing the charts, and is intended to supersede reference 1. An experimental curve is included for use in taking into account the effect of stresses above the elastic range on the modulus of elasticity of 24S-T aluminum alloy. A determination of the dimensions of a thin-metal column for maximum critical stress with certain given conditions is presented.

SYMBOLS

A cross-sectional area

b width of end or narrower wall of rectangular tube or of plate element of I-, Z-, or channel section

$\bar{D}$ effective flexural stiffness of plate per unit length $\left[\frac{\eta E t^3}{12(1-\mu^2)}\right]$

E modulus of elasticity

h width of side or wider wall of rectangular tube

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k nondimensional coefficient dependent upon relative dimensions of cross section

k_{sec} section coefficient

t thickness

S^{III} stiffness in moment-distribution analysis for far edge free (no support and no restraint against rotation)

S^{IV} stiffness in moment-distribution analysis for far edge supported and subjected to sinusoidally distributed moment equal and opposite to moment applied at near edge

ϵ restraint coefficient, a measure of relative resistance to rotation of restraining element at edge of plate

λ half wave length of buckle

μ Poisson's ratio

σ_{cr} critical compressive stress

η nondimensional coefficient that takes into account reduction of modulus of elasticity for stresses above the elastic range. Within the elastic range, $\eta = 1$.

Subscripts:

F flange

W web

b end or narrower wall of rectangular tube

h side or wider wall of rectangular tube

FORMULAS FOR CRITICAL STRESS

For an I-, Z-, or channel section, either of two formulas given in reference 1 may be used for calculating the critical compressive stress. The two formulas are

$$\frac{\sigma_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1 - \mu^2) b_W^2} \qquad (1)$$

and

$$\frac{\sigma_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2} \qquad (2)$$

The corresponding formula for a rectangular-tube section is given in reference 1 as

$$\frac{\sigma_{cr}}{\eta} = \frac{k \pi^2 E t_h^2}{12(1 - \mu^2) h^2} \qquad (3)$$

In using formulas (1), (2), and (3) when the stresses are above the elastic range, σ_{cr}/η is first evaluated, and σ_{cr} is determined from this value by means of the curve of figure 6. The relationship between σ_{cr} and σ_{cr}/η will be further discussed in another section of this report.

DISCUSSION OF CHARTS

All of the quantities on the right-hand side of equation (1), (2), or (3) are known except the value of the coefficient k_W, k_F, or k. This value may be read from the appropriate chart (figs. 1 to 5) after the necessary dimension ratios are computed and applies whenever the length of the column is greater than several (3 or 4) times the width of the widest plate element.

In general, when a column of I-, Z-, channel, or rectangular-tube section fails by local instability, one of the two elements (web and flange or end wall and side wall) of the cross section may be said to be primarily responsible for the instability; that is, as the load approaches its critical value, this one element is no longer capable in itself of supporting the loads imposed on it without buckling and requires a certain amount of restraint from the other element of the cross section in order to delay buckling until that load for which the cross section as a whole becomes unstable is reached. The charts show which element of the cross section is being restrained against buckling by the other element. A dashed line is drawn on each of the charts (figs. 1 to 5) connecting the points for which the two elements are equally responsible for the instability of the section. This line divides the chart into two regions: In one region the web (or side wall) is primarily responsible for instability and in the other region the flange (or end wall) is primarily responsible for instability. A column with a given cross section will fall into one of these two regions, depending on the values of the various dimension ratios.

RELATIONSHIP BETWEEN σ_{cr} AND σ_{cr}/η

Figure 6 shows the relationship between σ_{cr} and σ_{cr}/η as determined from tests of 24S-T aluminum-alloy columns of Z-, H-, and channel section, either formed from flat sheet or extruded. This figure was prepared by plotting the experimentally determined values of σ_{cr} as ordinates against the values of σ_{cr}/η as abscissas. The values of σ_{cr}/η were computed according to equation (1) and the chart of figure 1 or 3. The results of the tests are discussed in more detail in reference 2.

Similar experimental data for materials other than 24S-T aluminum alloy are not now available, and further study of this subject seems desirable.

METHOD OF PREPARING CHARTS

Values of k_W, k_F, and k used in preparing the charts (figs. 1 to 5) were computed by an application of the principles of moment distribution to the stability of thin plates. This method is presented in detail and one example of its application is given in reference 3.

An alternate procedure, which makes use of the charts presented in references 4 and 5, was used in computing some of the values for the charts of figures 1 to 5. An example of the application of this method follows:

It is desired to determine the value of the coefficient k_W for a column of the cross-sectional dimensions shown in figure 7. It is necessary to predict which element of the cross section will be primarily responsible for the instability. The calculations will then show whether this prediction is correct. For a section with a relatively wide flange, such as that shown in figure 7, the flange will probably be primarily responsible for failure. On this assumption, the detailed procedure for determining k_W or k_F is as follows:

1. Compute the ratios t_F/t_W and b_W/b_F.

2. Assume a value of λ/b_W, where λ is the half wave length of the buckle.

3. Compute λ/b_F from the equation $\frac{\lambda}{b_F} = \frac{\lambda}{b_W}\frac{b_W}{b_F}$.

4. Assume several values of k_W. In order to avoid the necessity for interpolation in the tables of reference 6, the values of k should be assumed for the part that is not primarily responsible for instability because these values are the ones that must be used to enter the tables. Unless previous experience has revealed the approximate range of such values to be assumed, this range may be estimated by the use of the relation between the values of k for the two parts of the cross section given in step 5 and by the fact that the value of K for the part which is primarily responsible for instability will be somewhere between that for simple support and that for fixed edges.

5. For each value of k_W, compute k_F from the equation

$$k_F = \frac{k_W}{\left(\frac{t_F}{t_W}\frac{b_W}{b_F}\right)^2}$$

which was obtained from equations (1) and (2) and the assumption that stress is uniform across the section.

6. Using the assumed values of λ/b_W and k_W, evaluate the quantity $S^{IV}{}_W/(\bar{D}/b)_W$ from the tables of reference 6, where

$$\bar{D}_W = \frac{\eta E t_W{}^3}{12(1 - \mu^2)}$$

7. Compute $\epsilon = \dfrac{4S^{IV}{}_W b_F}{\bar{D}_F}$ (see reference 4), where

$$\bar{D}_F = \frac{\eta E t_F{}^3}{12(1 - \mu^2)}$$

The formula is

$$\epsilon = 4 \frac{S^{IV}{}_W}{\left(\frac{\bar{D}}{b}\right)_W} \frac{\left(\frac{\bar{D}}{b}\right)_W}{\left(\frac{\bar{D}}{b}\right)_F} = \frac{S^{IV}{}_W}{\left(\frac{\bar{D}}{b}\right)_W} \times 4 \left(\frac{b_F}{b_W}\right)\left(\frac{t_W}{t_F}\right)^3$$

8. With the values of ϵ from step 7, determine k_F from the chart of figure 3, reference 4.

9. Plot k_F from step 5 and k_F from step 8 as ordinate against either of the two values as abscissa. The intersection of the two curves gives the correct value of k_F for the particular value of λ/b_F.

10. Repeat steps 2 to 9, assuming different values of λ/b_W.

11. Plot the values of k_F from step 9 against λ/b_F. The minimum of this curve gives the required value of k_F.

If the calculations indicate that S^{IV}_W is negative, the prediction that the flange is the primary cause of instability is wrong. In such a case, the calculation must be carried out with S^{III}_F instead of S^{IV}_W in step 6, and with the chart of figure 3, reference 5, in step 8. In addition, all the subscripts F will become W, and vice versa.

The results of the procedure outlined herein as applied to the problem of figure 7 are given in table I. The values of k_F in the last column of table I were determined according to step 9. If these values of k_F are plotted against λ/b_F, the minimum value is found to be about 0.73. The value of k_W can be computed from the formula given in step 5.

Tables II to VI give the minimum values of k_W, k_F, and k used in the preparation of figures 1 to 5. All of the values of k and k_W in these tables except those marked a were computed either by the method just outlined or by the moment-distribution method discussed in reference 3. The values of k_F were then computed by the equation given in step 5. The values marked a are those computed by the energy method and used in the preparation of the charts of reference 1.

DIMENSIONS OF THIN-METAL COLUMNS FOR MAXIMUM CRITICAL STRESS

Equation (1) gives the critical stress for an I-, Z-, or channel column in terms of the width and the thickness of the web. The effect of the presence of flanges is taken into account in the evaluation of the coefficient k_W. For the purpose of studying the dimensions that give maximum critical stress, the form of equation (1) is preserved but the concept of certain terms is generalized.

The ratio b/t of a plate may be called the aspect ratio of the plate. A corresponding quantity that expresses the "section aspect ratio" for a thin-metal column is the area of the section divided by the square of some thickness. If, therefore, equation (1) is written

$$\frac{\sigma_{cr}}{\eta} = k_{sec} \frac{\pi^2 E}{12(1 - \mu^2)} \left(\frac{t_W^2}{A}\right)^2 \qquad (4)$$

then the value of the section coefficient k_{sec} is a measure of the effect of the shape of the section b_F/b_W on σ_{cr}/η for a given section aspect ratio A/t_W^2 and a given value of t_W/t_F. In order to show that k_{sec} is dependent on only b_F/b_W and t_W/t_F, equation (1) is set equal to equation (4), with the result that

$$k_W \left(\frac{t_W}{b_W}\right)^2 = k_{sec} \left(\frac{t_W^2}{A}\right)^2 \qquad (5)$$

From the geometry of the section (Z- or channel), $A = b_W t_W + 2 b_F t_F$. If this value of A is substituted in equation (5) and the equation is solved for k_{sec}, the result is:

$$k_{sec} = k_W \left(1 + 2 \frac{b_F/b_W}{t_W/t_F}\right)^2 \qquad (6)$$

The value of k_W depends on only b_F/b_W and t_W/t_F, and the value of k_{sec} therefore also depends on only these two ratios.

In figure 8 the values of k_{sec} as determined by equation (6) are plotted for channel- and Z-section columns, and in figure 9 similar values are plotted for I-section columns.

A method exactly analogous to the foregoing method can be applied to rectangular tubes. In this case, equation (4) is written

$$\frac{\sigma_{cr}}{\eta} = k_{sec} \frac{\pi^2 E}{12(1 - \mu^2)} \left(\frac{th^2}{A}\right)^2 \qquad (7)$$

and the formula for k_{sec} becomes

$$k_{sec} = 4k\left(1 + \frac{b}{h}\frac{t_b}{t_h}\right)^2 \tag{8}$$

In figure 10 are plotted the values of k_{sec} for rectangular tubes, as determined by equation (8).

As a practical problem in the determination of the dimensions of a thin-metal column for the development of maximum critical stress, consider a flat strip of metal of constant thickness which is to be formed into a Z- or channel section. In the formed section, $t_W/t_F = 1$. The section aspect ratio A/t_W^2 is equal to the width of this strip, or the developed length of the final cross section, divided by the thickness. When bent to form a channel- or Z-section column, this strip of metal of constant thickness develops the highest σ_{cr}/η for local instability if the bends are so located that the ratio of flange width to web width b_F/b_W is equal to about 0.41, which is the maximum of the curve for $t_W/t_F = 1$ in figure 8.

Regardless of the thickness used in the definition of the section aspect ratio, the maximum value of σ_{cr}/η for a given value of the section aspect ratio will occur at the same values of b_F/b_W for a particular value of t_W/t_F. The maximum for each t_W/t_F ratio therefore reveals the shape - that is, the value of b_F/b_W - that the I-, Z-, or channel section should have if maximum σ_{cr}/η is desired. The same reasoning holds for the rectangular tube. (See fig. 10.)

Equations (1) to (3) and figures 1 to 5 are probably more useful to practical designers than the more general equations (4) and (7) and figures 8 to 10. The curves of figures 1, 3, and 5 have therefore been redrawn in figures 11 to 13 with dashed lines added to show the percentage of the maximum value of σ_{cr}/η that can be developed for given values of t_W/t_F and A/t_W^2 when b_F/b_W is varied. The position of these lines is independent of the thickness used in the definition of the section aspect ratio. It is of interest to observe, by comparison of figures 2, 4, and

5 with figures 11 to 13, that the line of maximum values bears no apparent relation to the line that shows the dimension ratios for which the web and flange (or end wall and side wall) are equally responsible for the instability of the section.

CONCLUSIONS

1. The critical compressive stress at which cross-sectional distortion begins in a thin-wall column of I-, Z-, or channel section is given by either of the following formulas:

$$\frac{\sigma_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1 - \mu^2) b_W^2}$$

or

$$\frac{\sigma_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2}$$

where

b_W width of web

b_F half width of flange for I-section, total width of flange for Z- and channel section

k_W and k_F nondimensional coefficients read from the appropriate chart

E and μ Young's modulus and Poisson's ratio for the material, respectively

t_W and t_F thickness of web and flange, respectively

η nondimensional coefficient that takes into account reduction of modulus of elasticity for stresses above the elastic range. Within the elastic range, $\eta = 1$.

For a rectangular-tube section

$$\frac{\sigma_{cr}}{\eta} = \frac{k \pi^2 E t_h^{\,2}}{12(1 - \mu^2)h^2}$$

where

k nondimensional coefficient read from appropriate chart

h and t_h width and thickness, respectively, of side or wider wall of rectangular tube

2. For stresses above the elastic range, the critical compressive stress is determined from a curve that gives the relationship between σ_{cr} and σ_{cr}/η for 24S-T aluminum alloy.

3. The charts of values of k are divided into two regions: In one region the web or side wall is primarily responsible for instability and in the other region the flange or end wall is primarily responsible for instability.

4. The equations for critical stress are also presented in general form with the ratio b/t replaced by the section aspect ratio A/t^2, where b is the width and t the thickness of an element of the cross section, and A is the area of the cross section. From these general equations, charts have been prepared that reveal the effect of shape alone on the critical stress for local instability. The shapes that give maximum critical stress bear no apparent relation to the proportions for which the web and flange (or end wall and side wall) are equally responsible for the instability of the section.

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REFERENCES

1. Stowell, Elbridge Z., and Lundquist, Eugene E.: Local Instability of Columns with I-, Z-, Channel, and Rectangular-Tube Sections. NACA TN No. 743, 1939.

2. Heimerl, George J., and Roy, J. Albert: Preliminary Report on Tests of 24S-T Aluminum-Alloy Columns of Z-, Channel, and H-Section That Develop Local Instability. NACA RB No. 3J27, Oct. 1943.

3. Lundquist, Eugene E., Stowell, Elbridge Z., and Schuette, Evan H.: Principles of Moment Distribution Applied to Stability of Structures Composed of Bars or Plates. NACA ARR No. 3K06, 1943.

4. Lundquist, Eugene E., and Stowell, Elbridge Z.: Critical Compressive Stress for Outstanding Flanges. NACA Rep. No. 734, 1942.

5. Lundquist, Eugene E., and Stowell, Elbridge Z.: Critical Compressive Stress for Flat Rectangular Plates Supported along all Edges and Elastically Restrained against Rotation along the Unloaded Edges. NACA Rep. No. 733, 1942.

6. Kroll, W. D.: Tables of Stiffness and Carry-Over Factor for Flat Rectangular Plates under Compression. NACA ARR No. 3K27, 1943.

TABLE I

SOLUTION FOR k_F FOR CROSS SECTION SHOWN IN FIGURE 7

k_W	k_F (Step 5)	$\frac{S^{IV}_W}{(\bar{D}/b)_W}$	ϵ	k_F (Step 8)	k_F (Step 9)
$\frac{\lambda}{b_W} = 0.5$, $\frac{\lambda}{b_F} = 1.0$					
5.0	1.250	1.6751	3.350	1.487	1.455
5.5	1.375	1.2276	2.455	1.470	
6.0	1.500	.5272	1.054	1.440	
6.2	1.550	.1193	.239	1.410	
$\frac{\lambda}{b_W} = 1.0$, $\frac{\lambda}{b_F} = 2.0$					
2.5	0.625	0.7031	1.406	0.814	0.773
3.0	.750	.5093	1.019	.781	
3.5	.875	.2789	-.558	.736	
4.0	1.000	0	0	.670	
$\frac{\lambda}{b_W} = 1.5$, $\frac{\lambda}{b_F} = 3.0$					
2.5	0.625	0.4942	0.988	0.766	0.734
3.0	.750	.3977	.795	.728	
3.5	.875	.2928	.586	.683	
4.0	1.000	.1781	.356	.629	
$\frac{\lambda}{b_W} = 2.0$, $\frac{\lambda}{b_F} = 4.0$					
2.5	0.625	0.4767	0.953	0.882	0.821
3.0	.750	.4230	.846	.847	
3.5	.875	.3667	.733	.801	
4.0	1.000	.3076	.615	.757	

TABLE II

CALCULATED MINIMUM VALUES OF k_W FOR I-SECTIONS

b_F/b_W \ t_W/t_F	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0	[a]4.00	----	----	----	----	[a]4.00	----	----	----	----	[a]4.00
.05	[a]6.05	----	----	----	----	[a]4.49	----	----	----	----	[a]4.06
.10	[a]6.46	----	----	----	----	[a]4.82	----	----	----	----	[a]4.09
.15	[a]6.61	----	----	----	----	[a]4.98	----	----	----	----	[a]4.04
.20	[a]6.68	----	----	----	----	[a]5.01	----	----	----	----	[a]3.89
.25	[a]6.72	6.46	6.12	5.71	5.26	4.86	4.32	3.99	3.79	3.66	3.57
.30	6.72	6.49	6.15	5.59	5.05	4.62	3.94	3.59	3.34	3.17	2.97
.35	-----	6.51	6.09	5.31	4.64	4.12	3.49	3.07	2.78	2.54	2.23
.40	6.74	6.52	5.74	4.74	4.07	3.58	2.97	2.55	2.25	1.98	1.72
.45	-----	6.46	5.04	4.16	3.52	-----	----	----	----	----	-----
.475	-----	5.96	----	----	----	-----	----	----	----	----	-----
.50	6.75	5.51	4.31	3.54	3.02	2.61	2.12	1.76	1.50	1.30	1.10
.525	6.75	----	----	----	----	-----	----	----	----	----	-----
.55	6.39	----	----	----	----	-----	----	----	----	----	-----
.56	6.21	----	----	----	----	-----	----	----	----	----	-----
.575	5.95	----	----	----	----	-----	----	----	----	----	-----
.60	5.54	4.11	3.22	2.63	2.23	1.95	1.54	1.26	1.07	.90	.76
.70	4.23	3.12	2.46	2.02	1.71	1.48	1.17	.96	.78	.66	-----
.80	3.30	2.47	1.94	1.58	1.35	1.18	.91	.75	.61	.51	.44
1.00	2.19	1.65	1.29	1.07	.90	.78	.61	.49	.40	.33	.28

[a]Computed by energy solution (reference 1).

4.08

TABLE III

CALCULATED MINIMUM VALUES OF k_F FOR I-SECTIONS

b_W/b_F \ t_W/t_F	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0.0	[a]1.288	-----	-----	-----	-----	[a]1.288	-----	-----	-----	-----	[a]1.288
.2	------	-----	-----	-----	-----	[a]1.019	-----	-----	-----	-----	------
.4	[a].623	-----	-----	-----	-----	------	-----	-----	-----	-----	[a]1.193
.6	------	-----	-----	-----	-----	[a].852	-----	-----	-----	-----	------
.8	[a].567	-----	-----	-----	-----	------	-----	-----	-----	-----	[a]1.147
1.0	.547	0.594	0.634	0.685	0.729	.777	0.883	0.960	1.029	1.075	1.120
1.25	.527	.568	.607	.646	.699	.752	.840	.937	1.004	1.064	1.119
1.429	.518	.550	.590	.633	.678	.725	.823	.920	.982	1.054	------
1.667	.498	.532	.568	.605	.651	.703	.798	.886	.985	1.044	1.096
1.739	.492	-----	-----	-----	-----	------	-----	-----	-----	-----	------
1.786	.487	-----	-----	-----	-----	------	-----	-----	-----	-----	------
1.818	.484	-----	-----	-----	-----	------	-----	-----	-----	-----	------
1.905	.465	-----	-----	-----	-----	------	-----	-----	-----	-----	------
2.000	.422	.495	.528	.567	.611	.653	.762	.862	.962	1.049	1.098
2.105	------	.484	-----	-----	-----	------	-----	-----	-----	-----	------
2.222	------	.471	.500	.539	.577	------	-----	-----	-----	-----	------
2.500	.269	.375	.450	.485	.528	.573	.685	.799	.920	1.025	1.101
2.857	------	.287	.366	.416	.460	.505	.615	.737	.871	1.007	1.095
3.333	.151	.210	.271	.322	.368	.416	.511	.633	.769	.923	1.070
4.000	------	.145	.187	.228	.266	.304	.388	.489	.606	.741	.892
5.000	[a].067	-----	-----	-----	-----	[a].200	-----	-----	-----	-----	[a].623
6.667	[a].037	-----	-----	-----	-----	[a].112	-----	-----	-----	-----	[a].364

[a]Computed by energy solution (reference 1).

TABLE IV

CALCULATED MINIMUM VALUES OF k_W FOR CHANNEL AND Z-SECTIONS

b_F/b_W \ t_W/t_F	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0	[a]4.00	----	----	----	----	[a]4.00	----	----	----	----	[a]4.00
.050	[a]5.46	----	----	----	----	[a]4.26	----	----	----	----	[a]4.03
.100	[a]6.02	----	----	----	----	[a]4.45	----	----	----	----	[a]4.04
.130	[a]6.19	----	----	----	----	-----	----	----	----	----	-----
.167	[a]6.31	----	----	----	----	[a]4.58	----	----	----	----	[a]4.00
.179	-----	----	----	----	----	[a]4.59	----	----	----	----	[a]3.98
.192	[a]6.38	----	----	----	----	[a]4.60	----	----	----	----	[a]3.97
.208	-----	----	----	----	----	[a]4.59	----	----	----	----	[a]3.92
.227	[a]6.43	----	----	----	----	[a]4.58	----	----	----	----	[a]3.86
.250	-----	6.03	5.59	5.15	4.79	4.58	4.19	4.00	3.88	3.81	3.76
.300	6.50	6.10	5.59	5.12	4.69	4.41	3.98	3.76	3.59	3.48	3.26
.350	-----	6.13	5.55	4.92	4.46	4.11	3.65	3.36	3.13	2.78	2.41
.400	6.53	6.10	5.36	4.62	4.09	3.74	3.25	2.89	2.56	2.18	1.87
.450	-----	6.06	4.93	4.19	3.70	-----	----	----	----	----	-----
.475	-----	5.86	----	----	----	-----	----	----	----	----	-----
.500	6.54	5.46	4.43	3.74	3.26	2.90	2.40	2.02	1.69	1.42	1.19
.525	6.54	----	----	----	----	-----	----	----	----	----	-----
.550	6.46	----	----	----	----	-----	----	----	----	----	-----
.560	6.40	----	----	----	----	-----	----	----	----	----	-----
.575	6.07	----	----	----	----	-----	----	----	----	----	-----
.600	5.70	4.31	3.46	2.91	2.49	2.20	1.76	1.44	1.19	.98	.83
.700	4.45	3.36	2.[illegible]	2.27	1.93	1.70	1.32	1.09	.89	.72	-----
.800	3.54	2.68	2.[illegible]	1.77	1.53	1.36	1.05	.84	.67	.55	.45
1.000	2.39	1.81	1.44	1.2[illegible]	1.03	.89	.68	.55	.44	.36	.30

[a]Computed by energy solution (reference 1).

TABLE V

CALCULATED MINIMUM VALUES OF k_F FOR CHANNEL AND Z-SECTIONS

b_W/b_F \ t_W/t_F	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0	[a]1.288	-----	-----	-----	-----	[a]1.288	-----	-----	-----	-----	[a]1.288
.2	-----	-----	-----	-----	-----	[a]1.111	-----	-----	-----	-----	-----
.4	[a].695	-----	-----	-----	-----	-----	-----	-----	-----	-----	[a]1.234
.6	-----	-----	-----	-----	-----	[a].962	-----	-----	-----	-----	-----
.8	[a].621	-----	-----	-----	-----	-----	-----	-----	-----	-----	[a]1.204
1.0	.598	0.650	0.706	0.772	0.836	.890	0.985	1.074	1.134	1.161	1.192
1.250	.567	.617	.676	.726	.791	.870	.964	1.056	1.110	1.140	1.152
1.429	.545	.592	.655	.712	.767	.832	.934	1.045	1.110	1.143	-----
1.667	.513	.559	.611	.671	.727	.792	.911	1.015	1.099	1.137	1.195
1.739	.502	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
1.786	.501	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
1.818	.489	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
1.905	.451	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2.000	.409	.491	.543	.598	.657	.725	.864	.990	1.084	1.147	1.185
2.105	-----	.475	-----	-----	-----	-----	-----	-----	-----	-----	-----
2.222	-----	.441	.489	.542	.606	-----	-----	-----	-----	-----	-----
2.500	.261	.351	.420	.473	.530	.598	.748	.906	1.049	1.131	1.194
2.857	-----	.270	.333	.386	.442	.503	.644	.807	.982	1.104	1.183
3.333	.146	.198	.247	.295	.342	.397	.515	.662	.828	1.013	1.174
4.000	-----	.136	.171	.206	.242	.286	.377	.489	.621	.772	.939
4.400	[a].083	-----	-----	-----	-----	[a].236	-----	-----	-----	-----	[a].799
4.800	-----	-----	-----	-----	-----	[a].199	-----	-----	-----	-----	[a].681
5.200	[a].059	-----	-----	-----	-----	[a].170	-----	-----	-----	-----	[a].587
5.600	-----	-----	-----	-----	-----	[a].146	-----	-----	-----	-----	[a].508
6.000	[a].044	-----	-----	-----	-----	[a].127	-----	-----	-----	-----	[a].444

[a]Computed by energy solution (reference 1).

TABLE VI

CALCULATED MINIMUM VALUES OF k FOR RECTANGULAR TUBES

b/h \ t_b/t_h	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0	[a]7.01	----	----	----	----	[a]7.01	----	----	----	----	[a]7.01
.050	[a]5.13	----	----	----	----	[a]6.45	----	----	----	----	-----
.075	[a]4.88	----	----	----	----	-----	----	----	----	----	-----
.100	[a]4.72	----	----	----	----	[a]6.09	----	----	----	----	[a]6.85
.125	[a]4.62	----	----	----	----	-----	----	----	----	----	-----
.200	4.44	4.68	4.94	5.21	5.42	5.66	6.02	6.31	6.48	6.62	6.71
.300	4.30	----	----	----	----	-----	----	----	----	----	-----
.400	4.21	4.41	4.62	4.82	5.05	5.30	5.72	6.05	6.29	6.45	6.59
.500	4.12	----	----	----	----	-----	----	----	----	6.43	6.57
.600	3.94	4.12	4.30	4.52	4.77	5.03	5.52	5.92	6.20	6.41	6.56
.650	3.82	----	----	----	----	-----	----	----	----	----	-----
.661	3.76	----	----	----	----	-----	----	----	----	----	-----
.670	3.68	----	----	----	----	-----	----	----	----	----	-----
.700	3.38	3.86	4.08	----	----	-----	----	----	----	6.40	6.55
.750	-----	3.66	----	----	----	-----	----	----	----	----	-----
.780	-----	3.50	----	----	----	-----	----	----	----	----	-----
.800	2.56	3.42	3.75	4.04	4.36	4.64	5.28	5.79	6.15	6.39	6.54
.820	-----	3.27	----	----	----	-----	----	----	----	----	-----
.850	-----	3.05	----	----	----	-----	----	----	----	----	-----
.900	-----	2.75	3.29	3.69	----	-----	----	----	----	6.39	6.54
1.000	1.64	2.23	2.75	3.22	3.61	4.00	4.81	5.56	6.10	6.39	6.56

[a]Computed by energy solution (reference 1).

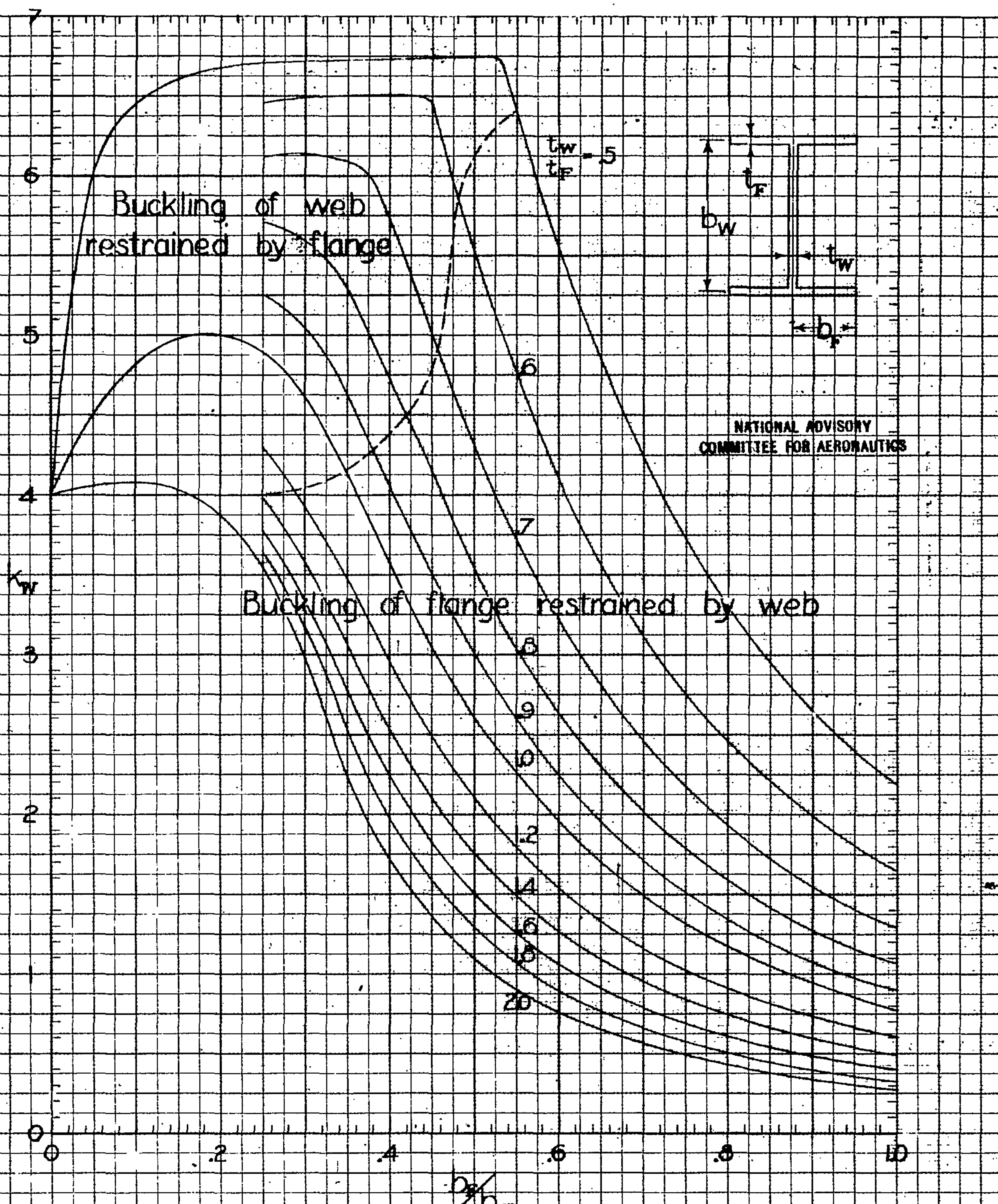

Figure 1. - Values of k_w for centrally loaded columns of I-section.

$$\frac{\sigma_{cr}}{\eta} = \frac{k_w \pi^2 E t_w^2}{12(1-\mu^2) b_w^2}$$

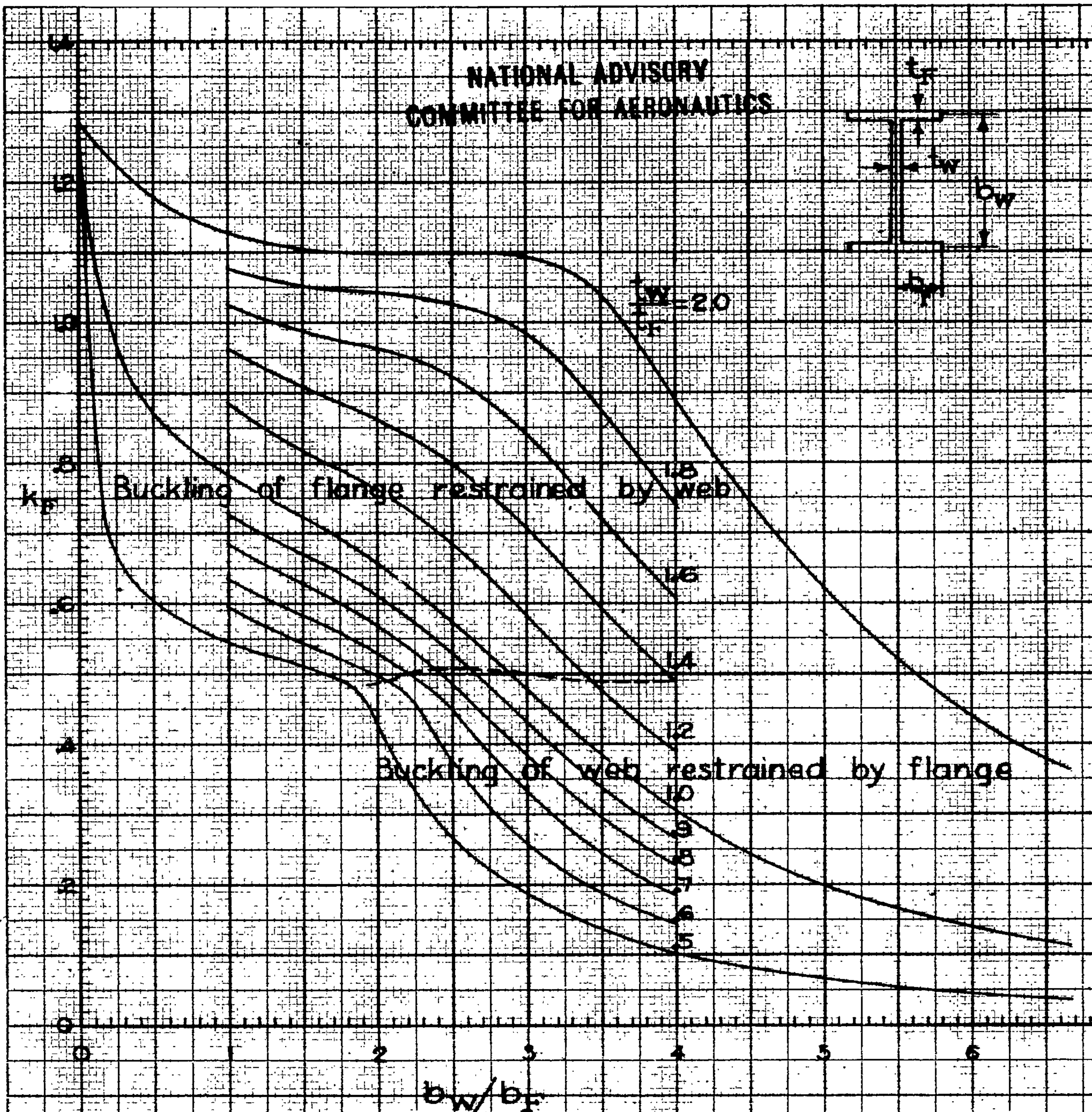

Figure 2.- Minimum values of k_F for centrally loaded columns of I-section.

$$\frac{\sigma_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1-\mu^2) b_F^2}$$

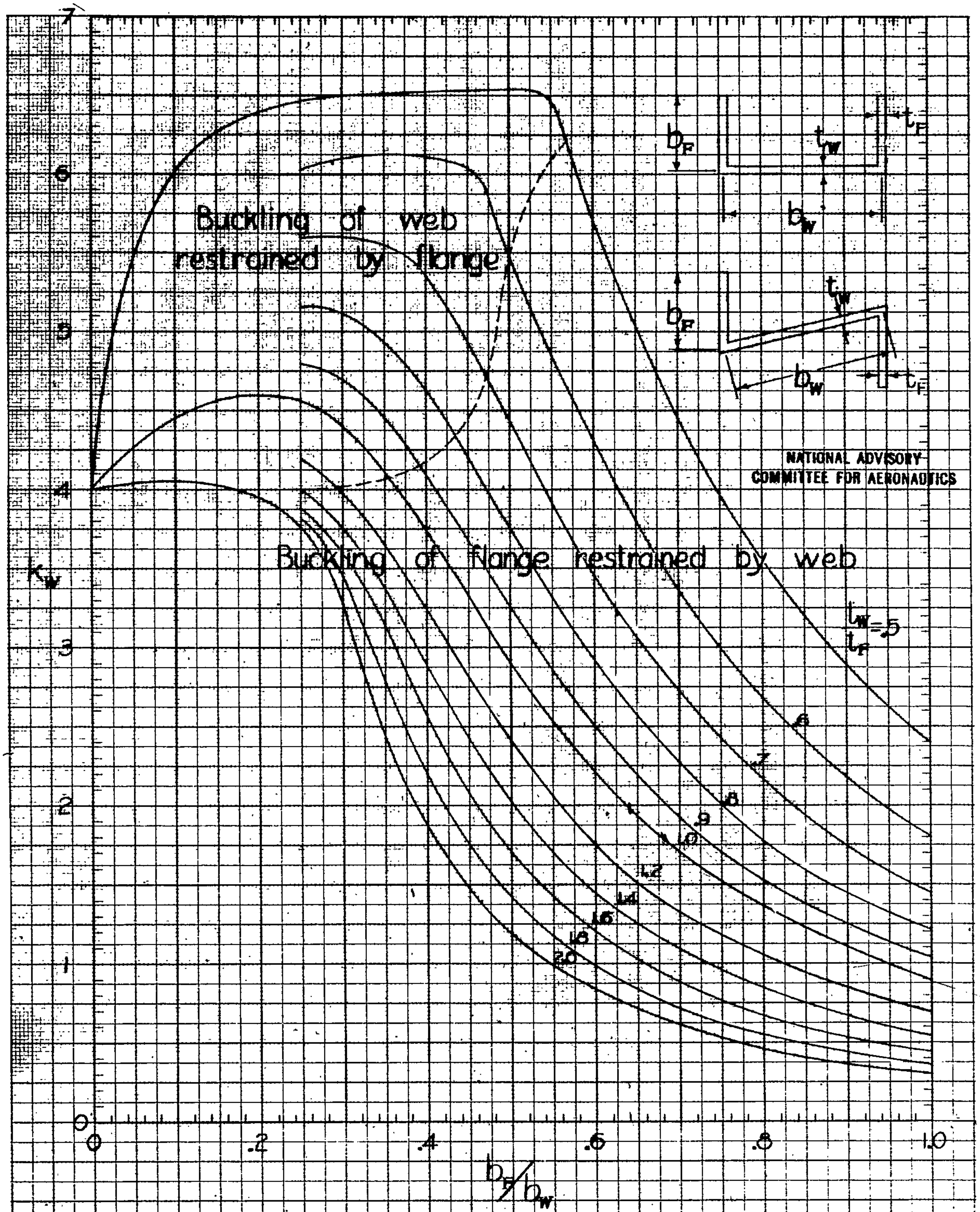

Figure 3. - Values of k_w for centrally loaded columns of channel section and Z-section.

$$\frac{\sigma_{cr}}{\eta} = \frac{k_w \pi^2 E t_w^2}{12(1-\mu^2) b_w^2}$$

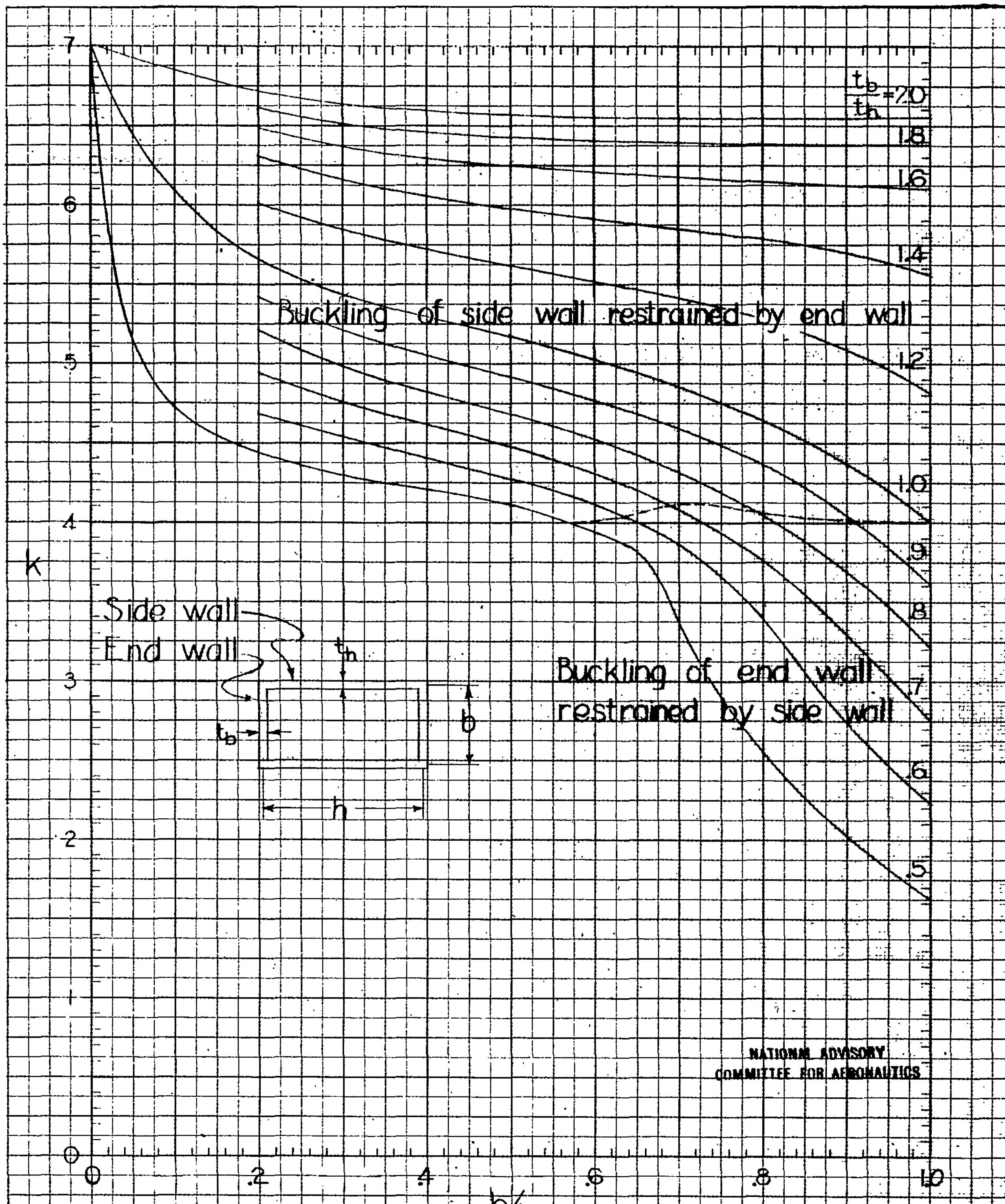

Figure 5.- Values of k for centrally loaded columns of rectangular tube section.

$$\frac{\sigma_{cr}}{\eta} = \frac{k\pi^2 E t_h^2}{12(1-\mu^2)h^2}$$

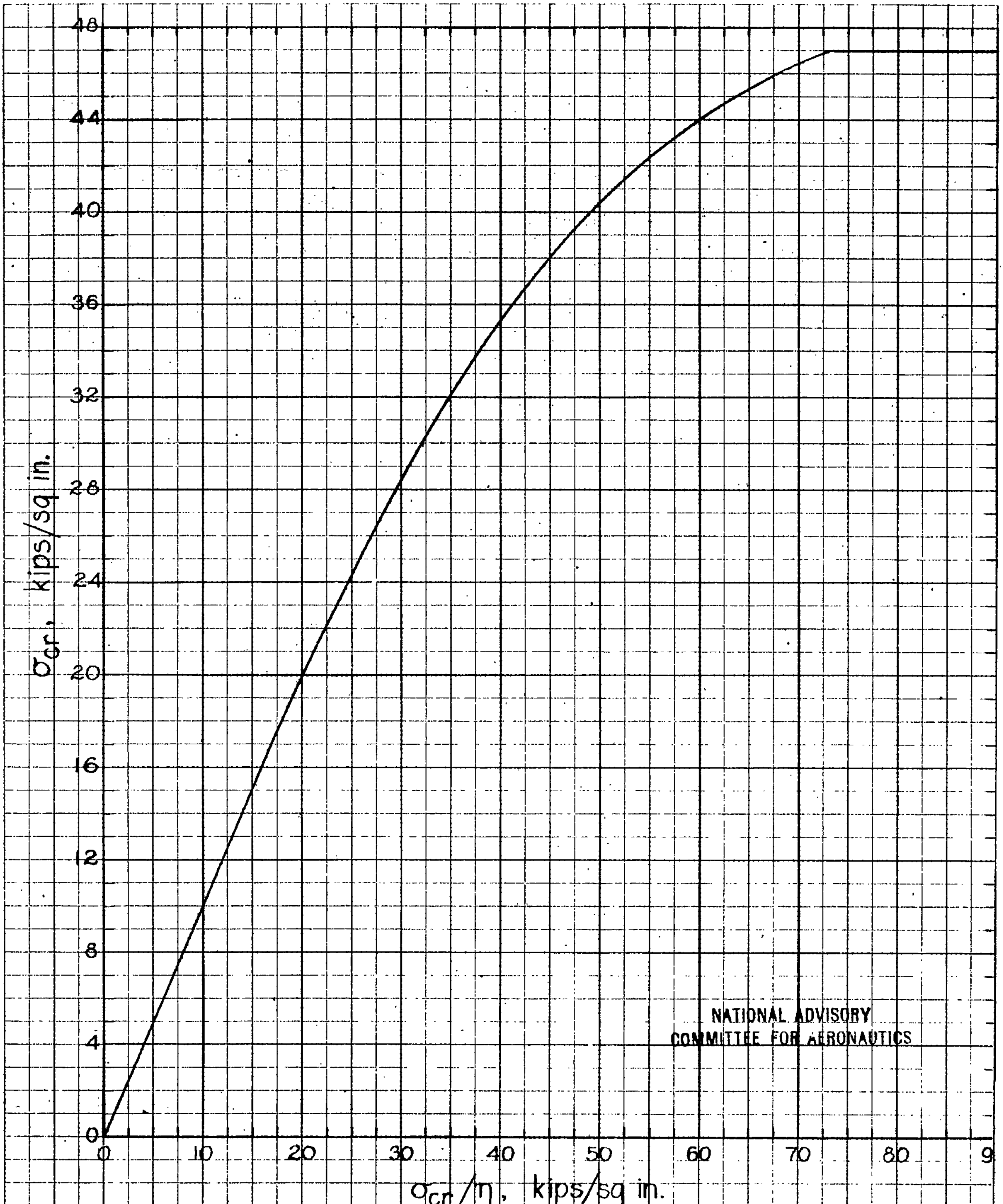

Figure 6. - Experimentally determined relationship between σ_{cr} and σ_{cr}/η for 24S-T aluminum alloy.

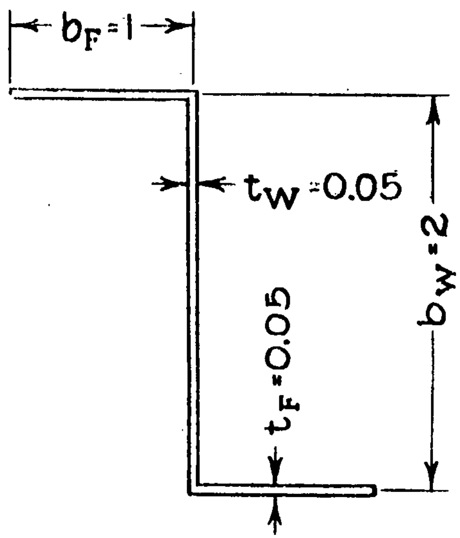

Figure 7. - Dimensions of Z-section column for illustrative problem.

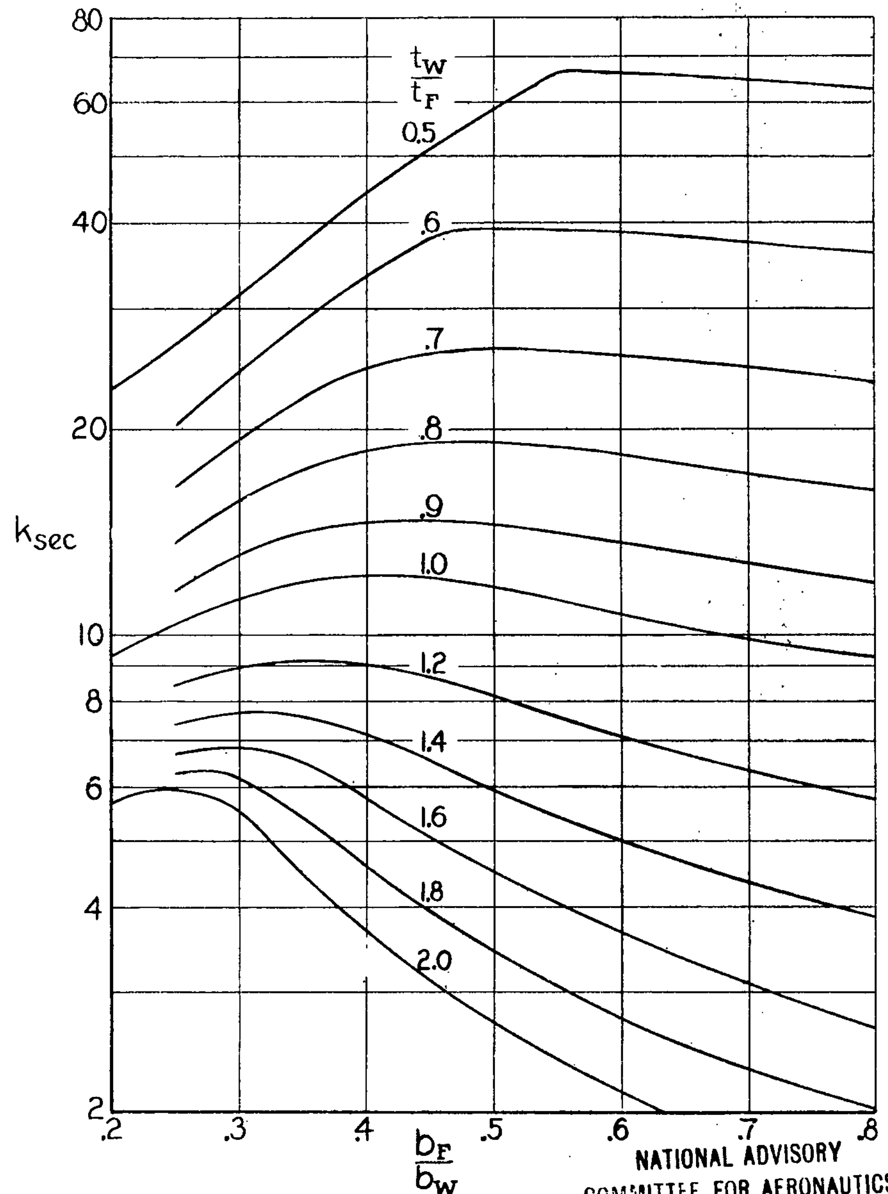

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Figure 8. – Values of k_{sec} for centrally loaded columns of channel and Z-section.

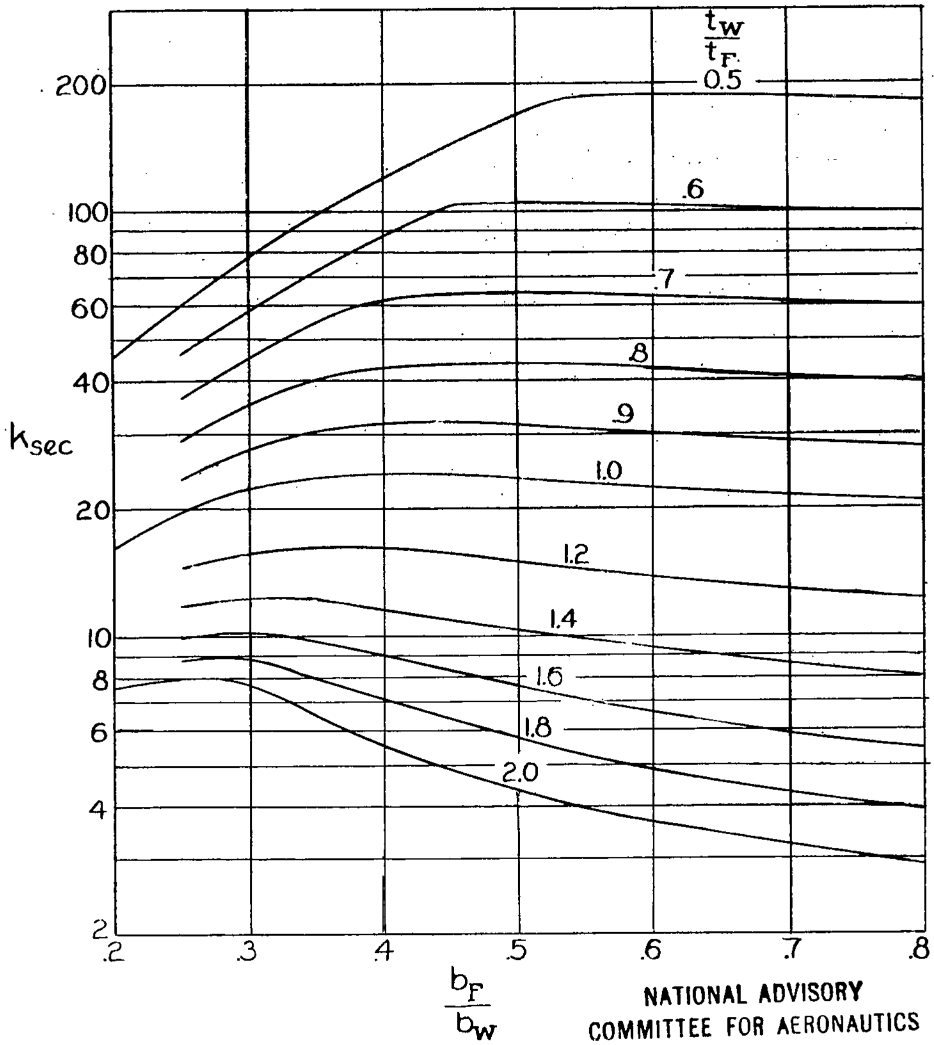

Figure 9.– Values of k_{sec} for centrally loaded columns of I-section.

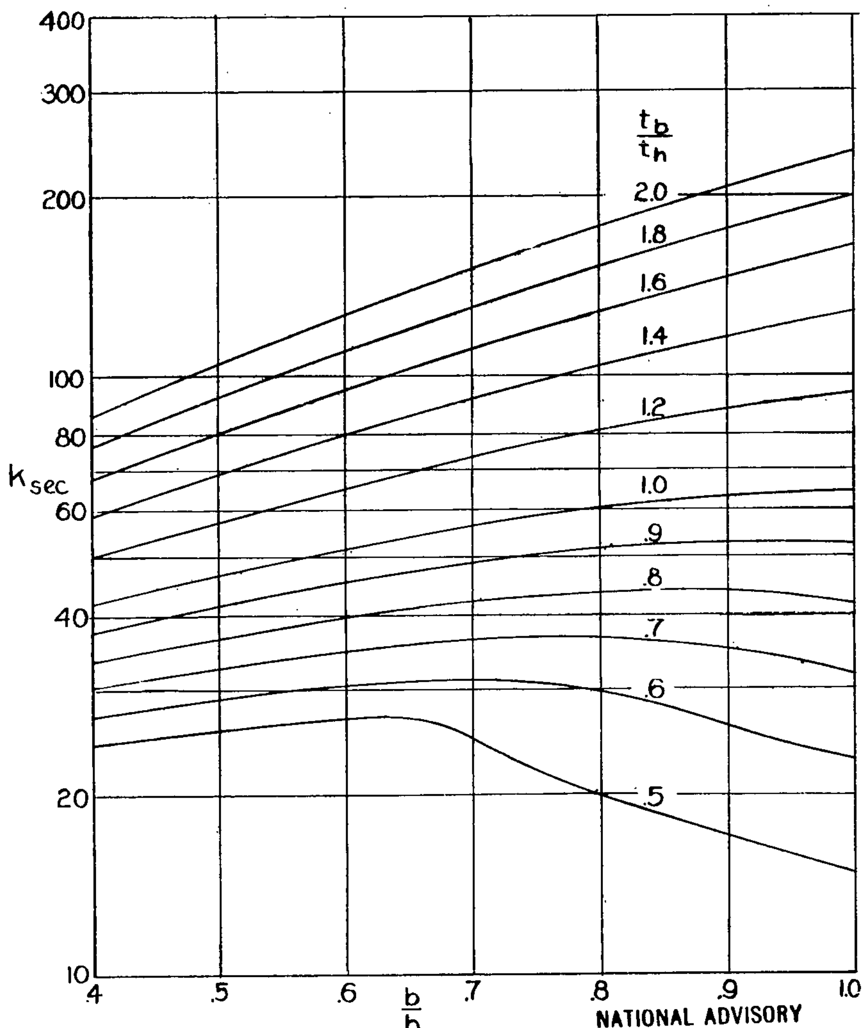

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Figure 10.– Values of k_{sec} for centrally loaded symmetrical rectangular tubes.

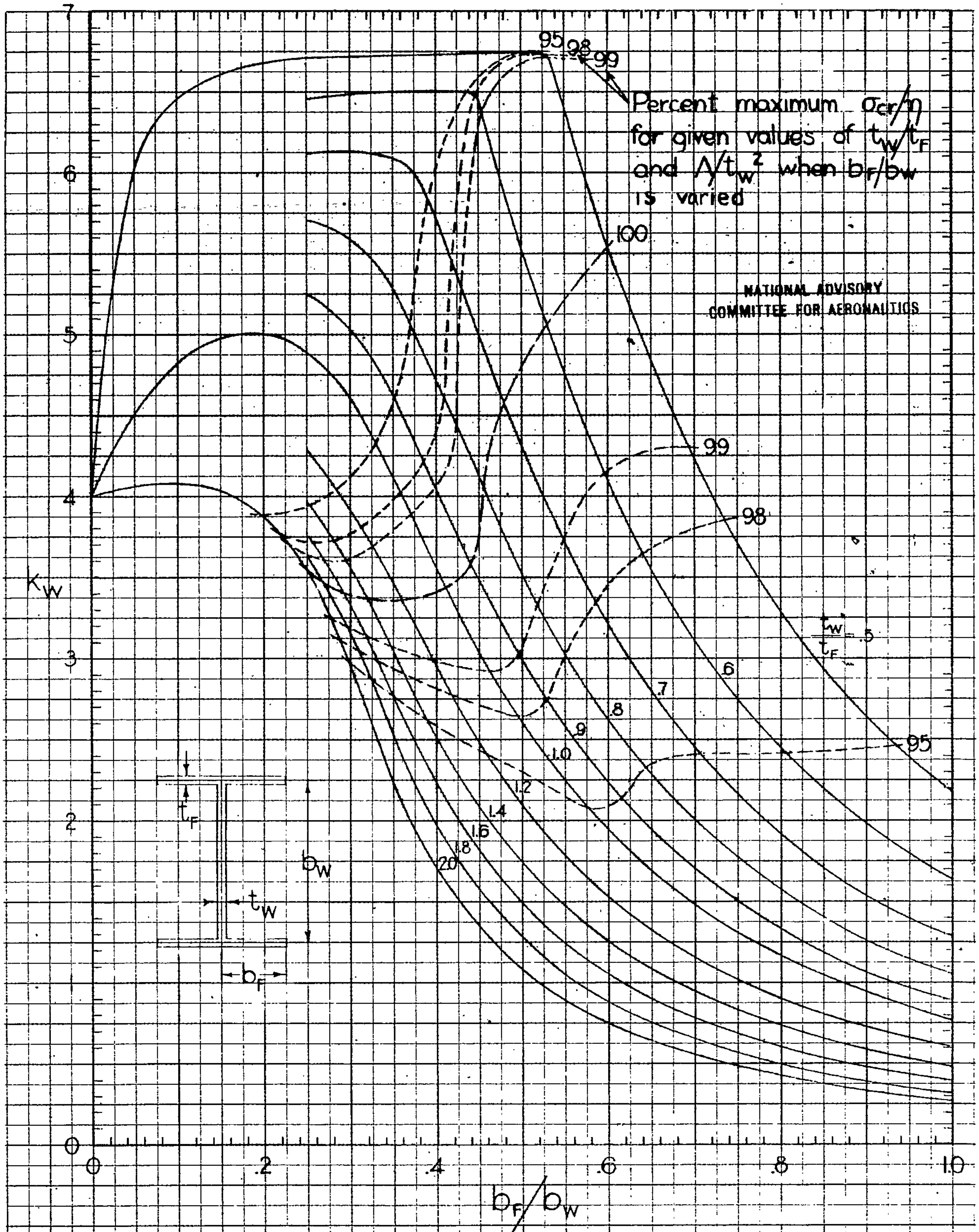

Figure 11.- Values of k_W for centrally loaded columns of I-section. $\frac{\sigma_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1-\mu^2) b_W^2}$

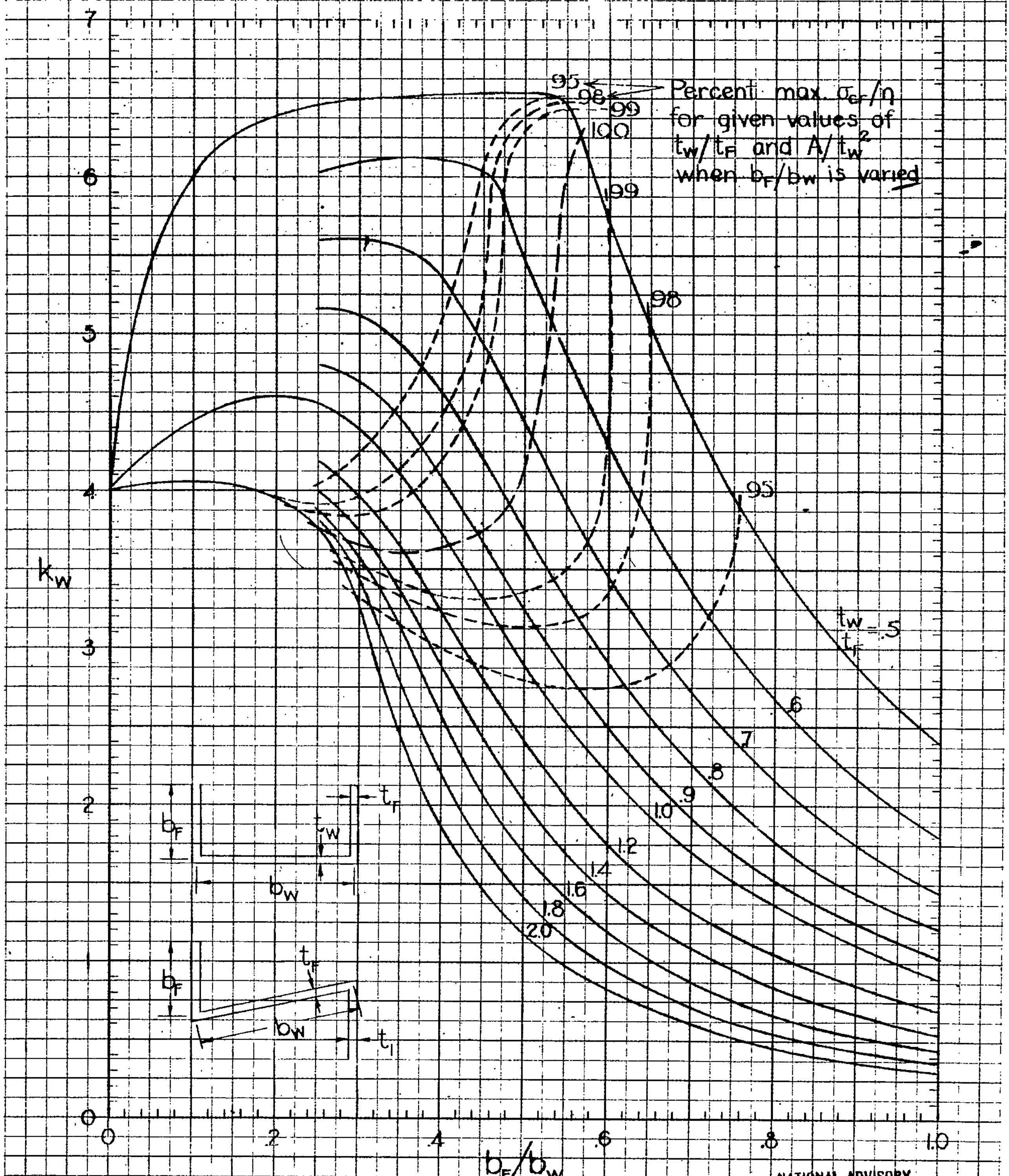

Figure 12 - Values of K_W for centrally loaded columns of channel section and Z-section.

$$\frac{\sigma_{cr}}{\eta} = \frac{K_w \pi^2 E t_w^2}{12(1-\mu^2) b_w^2}$$

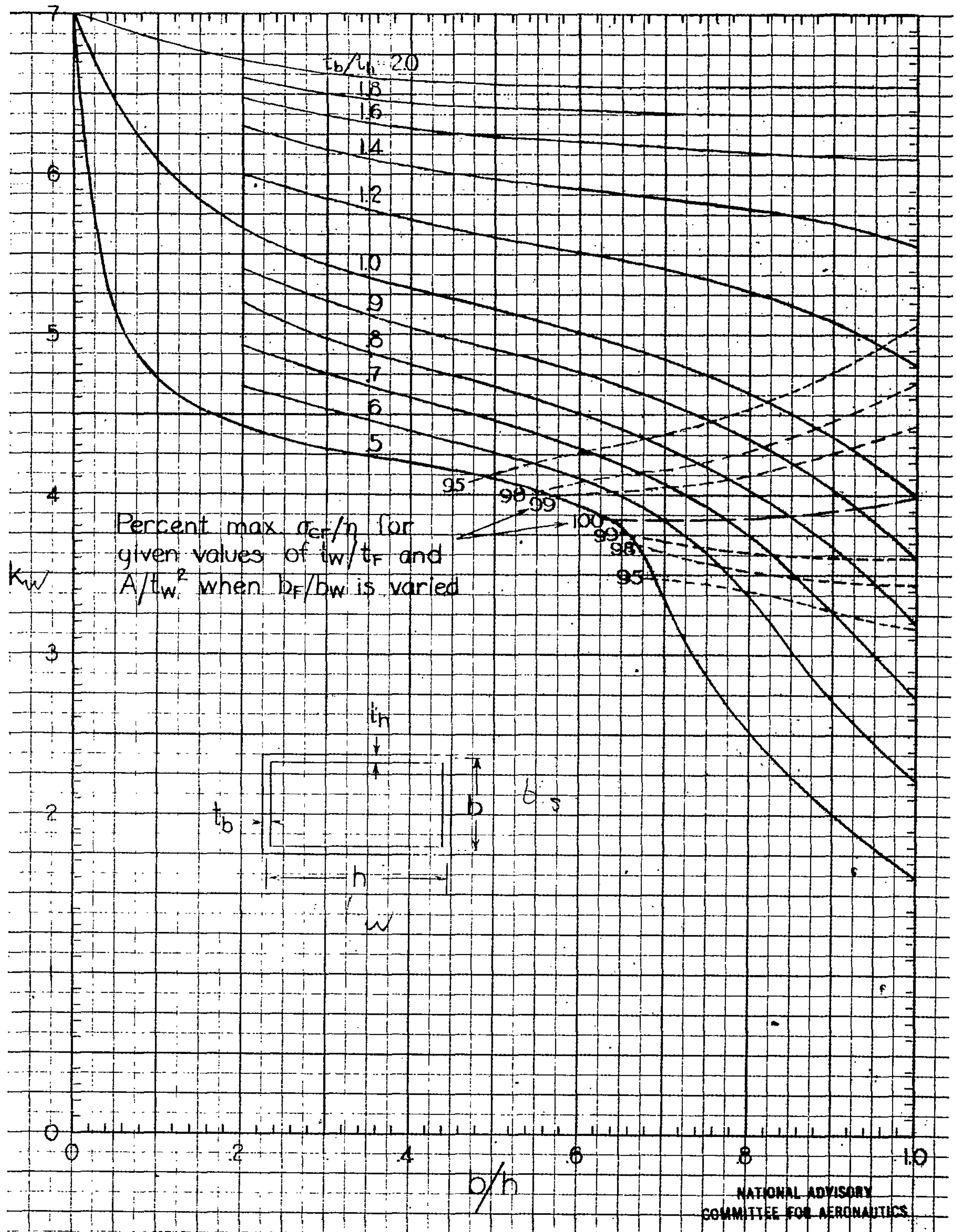

Figure 13. - Values of k for centrally loaded columns of rectangular tube section.

$$\frac{\sigma_{cr}}{\eta} = \frac{k\pi^2 E\, t_h^2}{12(1-\mu^2)h^2}$$

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