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THERMODYNAMIC DESIGN OF DOUBLE-PANEL, AIR-HEATED WINDSHIELDS FOR ICE PREVENTION

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RESTRICTED BULLETIN

THERMODYNAMIC DESIGN OF DOUBLE-PANEL, AIR-HEATED WINDSHIELDS FOR ICE PREVENTION

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SUMMARY

Equations are developed for the design of double-panel, air-heated windshields capable of preserving visibility during flight under icing conditions.

The use of the equations is illustrated by calculations for the design of a typical windshield.

INTRODUCTION

The investigation of the problems of ice prevention by the National Advisory Committee for Aeronautics has included a study of various means for preserving visibility through the airplane windshield during icing conditions. Intensive development of a double-panel, air-heated windshield for the prevention of ice has been undertaken in thic connection at the Ames Aeronautical Laboratory of the National Advisory Committee for Aeronautics. Several windshields of this type have been constructed and tried repeatedly in icing conditions. As noted in reference 1, the device appears to be satisfactory and, since the completion of the tests, installations (which have given satisfactory service) have been made on both commercial and military aircraft. The present bulletin is concerned with the thermal and pressure relations which must be considered in the design of such a windshield.

SYMBOLS

The symbols used throughout this bulletin, their meaning, and the relation of units employed in their evaluation are as follows:

A cross-sectional area of air gap normal to flow direction, sq ft

b gap width, ft

C thermal conductance of outer panel, Btu/(hr)(sq ft)($^{\circ}$F)

c_p specific heat of circulating air at constant pressure, Btu/(lb)($^{\circ}$F)

D inside diameter of pipe, ft

D_e equivalent duct diameter, ft

d gap thickness, ft

f friction factor, no units

G weight velocity, lb/(hr)(sq ft)

g acceleration due to gravity, 41.8×10^7, ft/(hr)(hr)

h heat-transfer coefficient of surface, Btu/(hr)(sq ft)($^{\circ}$F)

K thermal conductivity of windshield material, Btu/(hr)(sq ft)($^{\circ}$F/in.)

k thermal conductivity of fluid flowing, Btu/(hr)(sq ft)($^{\circ}$F/ft)

l thickness of windshield material in direction of heat flow, in.

m hydraulic radius, ft

N gap length in direction of air flow, ft

P_{av} average absolute pressure of circulating air, lb/sq ft

q heat-transfer rate per unit area, Btu/(hr)(sq ft)

R gas constant for air, 53 ft-lb/(lb)($^{\circ}$F abs.)

S surface area of windshield, sq ft

T_{av} average temperature of circulating air, $^{\circ}$F

T_i temperature of circulating air entering gap, $^{\circ}F$

t_i inner-surface temperature of outer panel, $^{\circ}F$

t_o outer-surface temperature of outer panel, $^{\circ}F$

v_{av} average specific volume of circulating air, cu ft/lb

W weight rate of circulating air, lb/hr

w weight rate of circulating air per foot of windshield width, lb/(hr)(ft)

μ coefficient of viscosity of fluid flowing, lb/(hr)(ft)

ΔP pressure drop of circulating air through gap, lb/sq ft

ΔT temperature drop of circulating air through gap, $^{\circ}F$

Δv change in specific volume of air, cu ft/lb

DEVELOPMENT OF DESIGN EQUATIONS

Although continued effort is being made to determine the convective heat-transfer coefficient from the surface of a windshield during icing conditions, reliable data have not yet been established. The need for the heat-transfer data for the outer surface is obviated by assuming a rate of heat flow through the panel and an outer-surface temperature which are based on the experience of extensive flight tests with several airplanes. The heat flow rate employed, as is suggested in reference 1, is 1000 Btu per hour per square foot. In order to prevent ice formation, it is apparent that the outer-surface temperature must be above 32° F and, therefore, a value of 50° F is assumed which provides a small margin of safety.

Practical design considerations limit the temperature of the circulating heated air which may be employed. If the temperature is below 100° F, the pressure drop required to deliver 1000 Btu per hour per square foot to a surface, the outer temperature of which is 50° F, becomes greater than the available dynamic pressure of the air stream. While a blower may be employed to increase the available pressure drop, this complication may be avoided in most installations. The strength of glass, plastics, and other windshield materials is reduced at temperatures above

200° F and, for this reason, the inlet temperature of the circulating air has been limited to 200° F. The development of the pressure and temperature relations, therefore, has been based upon an assumed circulating-air temperature range of 100° F to 200° F.

A desirable simplification of the design problem can be achieved by the assumption that the thermal losses through the inner windshield panel to the cabin interior are small and may be neglected. Such an assumption has been made in the development of the equations. Furthermore, it also has been assumed that the dimensions of the windshield, the allowable pressure drop in the circulating air, and the physical properties of the windshield materials are known and fixed.

The heat-transfer rate per unit area through the windshield outer panel is given by equation (38), page 25, reference 2. In the symbols of the present bulletin, this equation becomes

$$q = C\,(t_i - t_o) \tag{1}$$

Substituting the design assumptions,

$$q = 1000 \text{ Btu/(hr)(sq ft)}$$

and $t_o = 50^\circ$ F,

and transposing, the inner-surface temperature of the outer panel is given by

$$t_i = \frac{1000 + 50\ C}{C} \tag{2}$$

Equation (2) is plotted in figure 1 and shows the variation of t_i with C, the outer-panel thermal conductance.

If the outer panel is made of laminations of different materials, the thermal conductance for the panel is determined by

$$C = \frac{1}{\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2} + \cdots \dfrac{l_n}{K_n}} \tag{3}$$

where l_1/K_1, l_2/K_2, and so forth are for successive laminations. It is recommended that values of thermal conductivities (K) be obtained from the manufacturers of the windshield materials.

The temperature drop of the circulating air in passing through the gap is given by

$$\Delta T = \frac{qS}{W\ c_p}$$

For air at 150° F, the mean allowable temperature of the circulating air (c_p) equals 0.24 Btu per pound per degree Fahrenheit and, assuming q equals 1000 Btu per hour per square foot, the above equation becomes

$$\Delta T = \frac{1000\ S}{0.24\ W} = \frac{4167\ S}{W} \qquad (4)$$

Equation (4) is plotted in figure 2 and shows ΔT as a function of S, the surface area of the windshield, for various values of W, the circulating-air weight rate necessary to supply the heat required for ice prevention.

The heat-transfer coefficient for the inner surface of the outer panel may be determined by equation (11), page 169, reference 2, developed for flow in pipes.

$$\frac{hD}{k} = 0.0225 \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{c_p\mu}{k}\right)^{0.4}$$

For air in the temperature range of this analysis $\frac{c_p\mu}{k} = 0.73$; hence

$$\frac{hD}{k} = 0.02 \left(\frac{DG}{\mu}\right)^{0.8} \qquad (5)$$

But

$$G = \frac{W}{A}$$

Substituting in equation (5) and transposing gives

$$h = 0.02 \frac{k}{D} \left(\frac{DW}{\mu A}\right)^{0.8} \qquad (6)$$

Although equation (5) was developed for flow in pipes, it can be applied, with reasonable accuracy, to small rectangular ducts by the substitution of the equivalent duct diameter (D_e) for the pipe diameter (D). By definition, the equivalent duct diameter is equal to four times the hydraulic radius (m) and the hydraulic radius equals the duct cross-sectional area divided by the wetted perimeter. Thus, if the gap thickness is d and the gap width is b,

$$D_e = 4m = \frac{4bd}{2b + 2d} \qquad (7)$$

Thickness of gap will be small compared to width; therefore only slight error will be involved in dropping the term "2d" from the denominator of the fraction in equation (7). Then,

$$D_e = \frac{4bd}{2b} \doteq 2d$$

and,

$$m = \frac{D_e}{4} = \frac{2d}{4} = \frac{d}{2}$$

Substituting this value of D_e in equation (6) gives

$$h = 0.01 \frac{k}{d} \left(\frac{2W}{\mu b}\right)^{0.8} \qquad (8)$$

But

$$\frac{W}{b} = w$$

Therefore, equation (8) becomes:

$$h = 0.01 \frac{k}{d} \left(\frac{2w}{\mu}\right)^{0.8} \qquad (9)$$

and, evaluating the physical constants (k and μ) within the temperature range considered in this analysis, equation (9) becomes:

$$h = \frac{0.0032\ w^{0.8}}{d} \qquad (10)$$

Equation (10) is plotted in figure 3 and shows the variation of h with d, the gap thickness, for various values of w, the circulating-air weight rate per unit gap width.

In establishing the size of the gap and the quantity of the circulating air, consideration must be given to the allowable pressure drop through the gap, as well as to the thermal requirements. The pressure drop through the gap is calculated from equation (35), page 130, reference 2. In the symbols of this bulletin, it becomes

$$\Delta P = \frac{G^2 \ \Delta v}{g} + \frac{fNG^2 \ v_{av}}{2gm}$$

The first fraction may be omitted for this analysis since Δv, the change in specific volume of the air passing through the gap, is negligible. The pressure drop per foot of windshield length then becomes

$$\frac{\Delta P}{N} = \frac{fG^2 \ v_{av}}{2gm} \tag{11}$$

The value of the friction factor (f) is given by equation (4), page 111, reference 2, as

$$f = \frac{0.049}{\left(\frac{DG}{\mu}\right)^{0.2}}$$

Substituting this value of f in equation (11) leads to the expression

$$\frac{\Delta P}{N} = 0.049 \left(\frac{\mu}{D}\right)^{0.2} \left(\frac{G^{1.8} \ v_{av}}{2gm}\right) \tag{12}$$

But

$$G = \frac{W}{A} = \frac{wb}{bd} = \frac{w}{d}$$

$$D = 2d$$

and

$$m = \frac{d}{2}$$

Then,

$$\frac{\Delta P}{N} = 5.56 \times 10^{-11} \ \frac{w^{1.8} \ v_{av}}{d^3} \tag{13}$$

In equation (13), according to the universal gas law, the average specific volume of the circulating air is determined by

$$v_{av} = \frac{RT_{av}}{P_{av}}$$

Assuming an average absolute pressure of 2118 pounds per square foot and average absolute temperature of 150° plus 460° F, or 610° F, the above equation becomes

$$v_{av} = \frac{53(610)}{2118} = 15.3 \text{ cubic feet per pound}$$

Substituting this value of v_{av} in equation (13) leads to the expression

$$\frac{\Delta P}{N} = 8.51 \times 10^{-10} \frac{w^{1.8}}{d^3} \tag{14}$$

Equation (14) is plotted in figure 4 and shows the variation of $\Delta P/N$ with d for various values of w. It is of importance to note that figure 4 applies to sea-level conditions only, since the value of P_{av} used to evaluate v_{av} was 2118 pounds per square foot, absolute. In order to find $\Delta P/N$ at any other design altitude, multiply the value of $\Delta P/N$ obtained in figure 4 by the ratio $2118/P$, where P equals the absolute static pressure at design altitude, pounds per square foot.

APPLICATION OF DESIGN EQUATIONS

Before the design equations and the relations shown by the curves of figures 1 to 4 can be applied to the solution of a windshield design problem, certain data relating to the materials and dimensions of the windshield, and to temperature, pressure, and quantity of the circulating air, must be known or assumed. For a typical case these data are as follows:

1. Materials of outer panel: Assumed to be lucite between two sheets of glass

2. Windshield dimensions: Width (equals b, gap width) assumed to be 1.5 feet, and length (equals N,

gap length) assumed to be 2.0 feet. Thickness of lucite (l_1) assumed to be 0.031 inch, and thickness of each sheet of glass (l_2) assumed to be 0.125 inch

3. Thermal conductivity of materials in outer panel: For lucite, K_1 is 1.25 Btu/(hr)(sq ft)(°F/in.), and for glass, K_2 is 6.0 Btu/(hr)(sq ft)(°F/in.)

4. Allowable pressure drop through air gap (ΔP): 10 pounds per square foot

5. Available temperature of circulating air (T_i): Assumed to be 200° F

6. A tentative value of W, the weight rate for the circulating air, must be assumed. Since a value of W is determined (through use of the design equations and curves) which, for an acceptable solution, must agree with the tentative value, this may change as successive approximations of the final solution are made.

The problem then resolves itself into finding what value of d, in conjunction with the correct value of W (found by successive approximations) will provide a heat-transfer rate of 1000 Btu per hour per square foot through the windshield outer panel and maintain the outer surface at a temperature of 50° F without exceeding either the available circulating-air temperature (T_i) or the allowable circulating air pressure drop (ΔP). The solution of the air-heated windshield design problem proceeds in the following manner:

1. Calculate C, using equation (3) and the known, or assumed, data regarding the thickness and thermal conductivity of the materials.

2. Refer to figure 1 and, using the computed value of C, obtained t_i.

3. From the known, or assumed, values of ΔP and N, determine $\Delta P/N$.

4. Assuming a tentative circulating-air weight rate (W_1) determine w (equals W_1/b), a tentative value of the circulating-air weight rate per foot of windshield width.

5. Refer to figure 4 and, using the computed values of $\Delta P/N$ and w, obtain d.

6. Refer to figure 3 and, from the values of d and w previously determined, obtain h.

7. From h, t_i, and q (equal to 1000 Btu per hour per square foot), calculate the average circulating-air temperature in the gap (T_{av}) by the fundamental relation q equals $h(T_{av} - t_i)$, or T_{av} equals $(1000 + ht_i)/h$.

8. With the assumption of a uniform temperature gradient in the gap, knowing T_{av} and T_i, calculate ΔT from the relation ΔT equals $2(T_i - T_{av})$.

9. From b and N, determine the surface area of the windshield (S).

10. Refer to figure 2 and, from S and ΔT, obtain a second circulating-air weight rate (W_2).

11. Refer back to step 4 to see if the assumed value of W_1 agrees with the value obtained in step 10, W_2. If the difference between W_1 and W_2 is less than 10 percent of W_1 then the design may be considered complete; if not, then assume another weight rate (W_3) halfway between W_1 and W_2 and repeat procedure beginning with step 4.

    This process should be continued until the difference between the last assumed and final resultant values of W is less than 10 percent of the former.

For purposes of illustration, an example of an application of the design procedure is presented, using the data assumed for a typical windshield installation.

1. $$C = \frac{1}{\frac{l_1}{K_1} + 2\left(\frac{l_2}{K_2}\right)} = \frac{1}{\frac{0.031}{1.25} + 2\left(\frac{0.125}{6}\right)}$$

$$= 15.1 \text{ Btu/(hr)(sq ft)}(^\circ\text{F})$$

2. From figure 1, $t_i = 115^\circ$ F

3. $\Delta P/N = 10/2 = 5$ lb/(sq ft)(ft)

4. Assuming $W_1 = 450$ lb/hr

$$w = \frac{W_1}{b} = \frac{450}{1.5} = 300 \text{ lb/(hr)(ft)}$$

5. From figure 4, $d = 0.017$ ft

6. From figure 3, $h = 18.2$ Btu/(hr)(sq ft)($^\circ$F)

7. $$T_{av} = \frac{1000 + h\,t_i}{h} = \frac{1000 + 18.2(115)}{18.2} = 170^\circ \text{ F}$$

8. $\Delta T = 2(T_i - T_{av}) = 2(200 - 170) = 60^\circ$ F

9. $S = bN = 1.5 \times 2 = 3$ sq ft

10. From figure 2, $W_2 = 210$ lb/hr

11. Since the difference between W_1 and W_2 is greater than 10 percent of W_1, a second assumption will be made. As a second approximation, assume W_3 to be halfway between 450 pounds per hour and 210 pounds per hour, or 330 pounds per hour and, returning to step 4, repeat the design process.

4. $$w = \frac{330}{1.5} = 220 \text{ lb/(hr)(ft)}$$

5. $d = 0.014$ ft

6. $h = 17$ Btu/(hr)(sq ft)($^\circ$F)

7. $$T_{av} = \frac{1000 + 17(115)}{17} = 174^\circ \text{ F}$$

8. $\Delta T = 2(200 - 174) = 52^\circ$ F

10. $W_4 = 250$ lb/hr

The difference between W_3 and W_4, 80 pounds per hour, is still too large. The process was continued and satisfactory agreement was reached at a weight rate of approximately 260 pounds per hour. The corresponding gap thickness was 0.012 foot (approximately 5/32 inch) and the temperature drop through the gap was 48° F.

After the windshield has been installed, flight tests are usually made to determine the performance. Natural icing conditions are necessary for a final check, but an estimate of the ice-prevention capability of the installation can be made from flights in non-icing conditions. If instrumentation can be installed to measure the outer-surface temperature, the adherence of the system to the design assumptions can be established. If the surface temperature cannot be measured, a flight in 40° F air will probably result in a windshield outer-surface temperature above 50° F, and a measured heat-transfer rate through the outer panel of more than 1000 Btu per hour per square foot indicates, under these conditions, a satisfactory design. It should be remembered that the heat-transfer requirement of 1000 Btu per hour per square foot of surface area is based upon the flight speeds of reference 1 and may not be conservative above 200 miles per hour. Further investigations of the design requirements are being carried out by AAL, using air-heated windshields under natural icing conditions, and should result in a better understanding of the problem.

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REFERENCES

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2. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 1933.

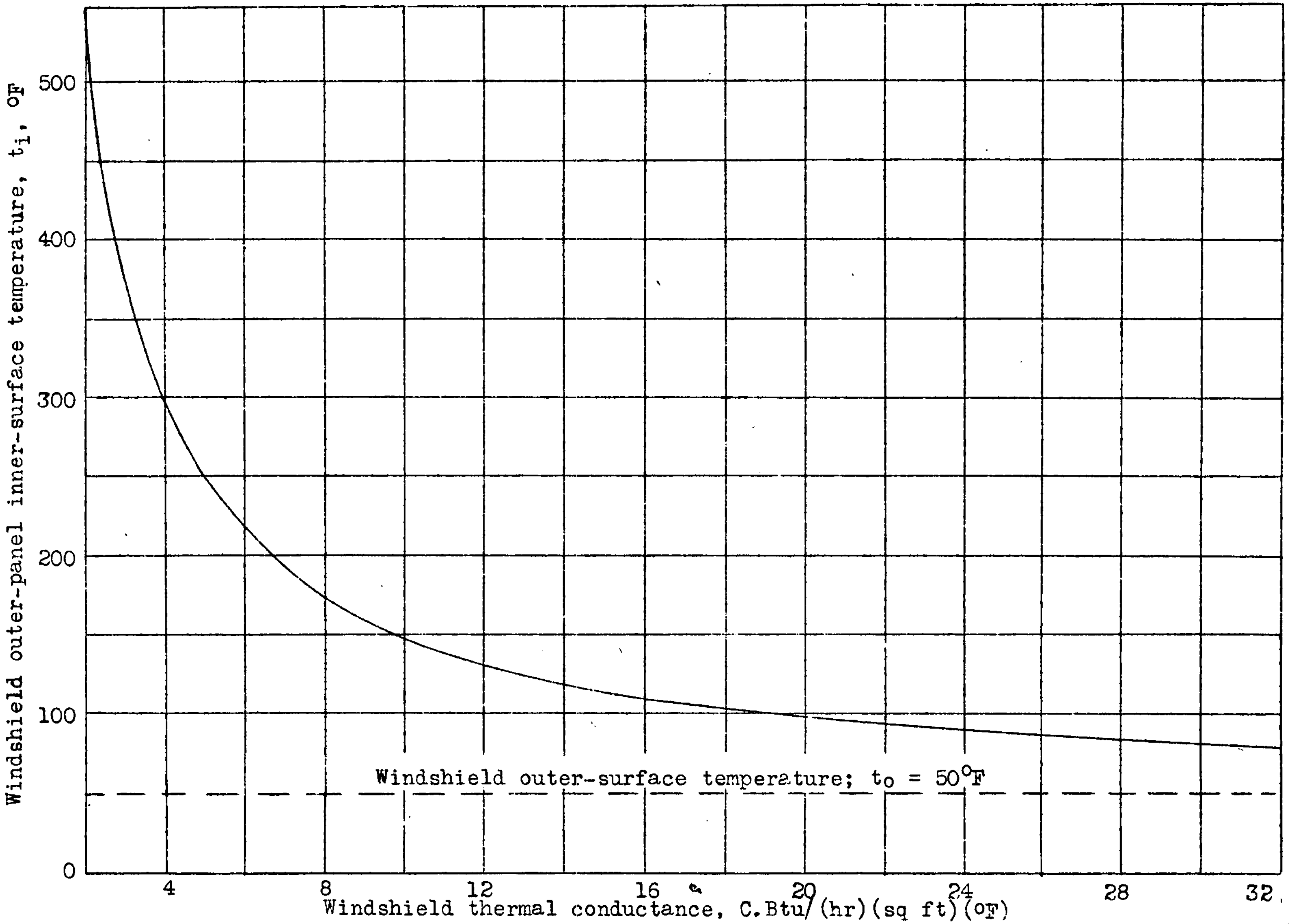

Figure 1.- Variation of air-heated windshield inner-surface temperature with thermal conductance for a windshield outer panel, assuming a heat transfer of 1,000 Btu per hour per square foot and a windshield outer-surface temperature of 50°F.

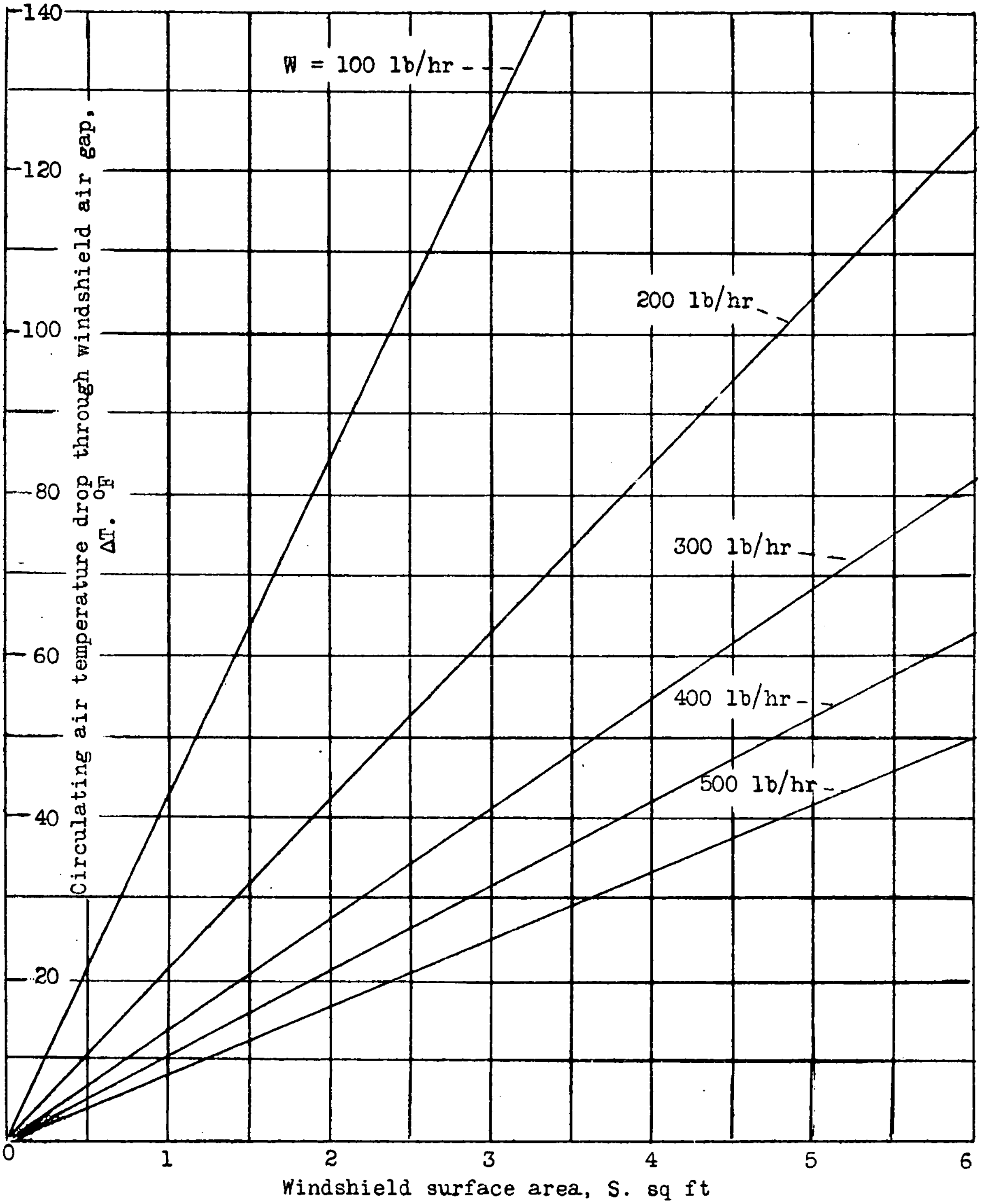

Figure 2.- Circulating air temperature drop through the windshield air gap as a function of windshield surface area, at various circulating air weight rates, necessary to supply the heat required for ice-prevention.

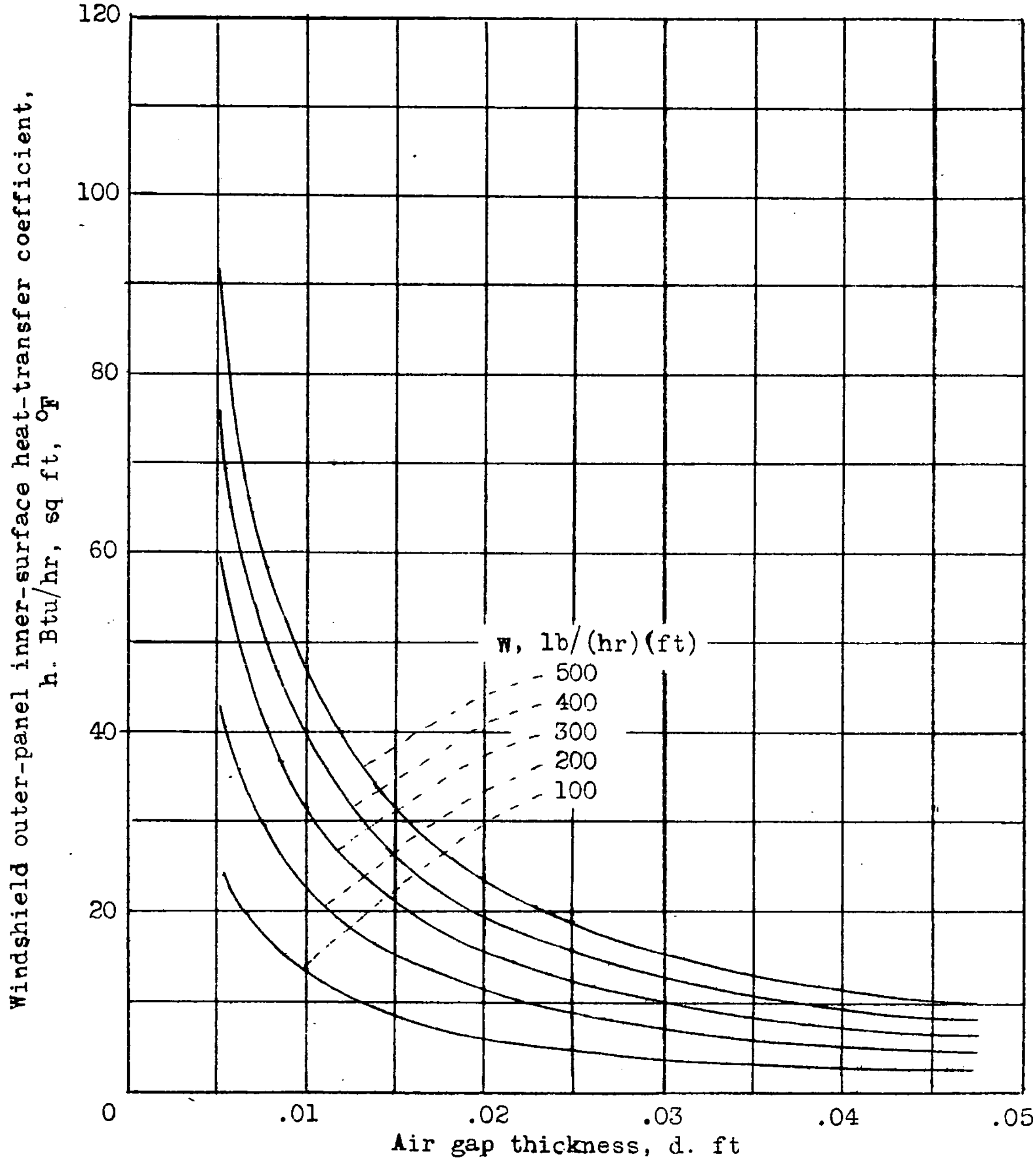

Figure 3.- Variation of air-heated windshield outer-panel inner-surface heat-transfer coefficient with air gap thickness for various circulating air weight rates per unit gap width.

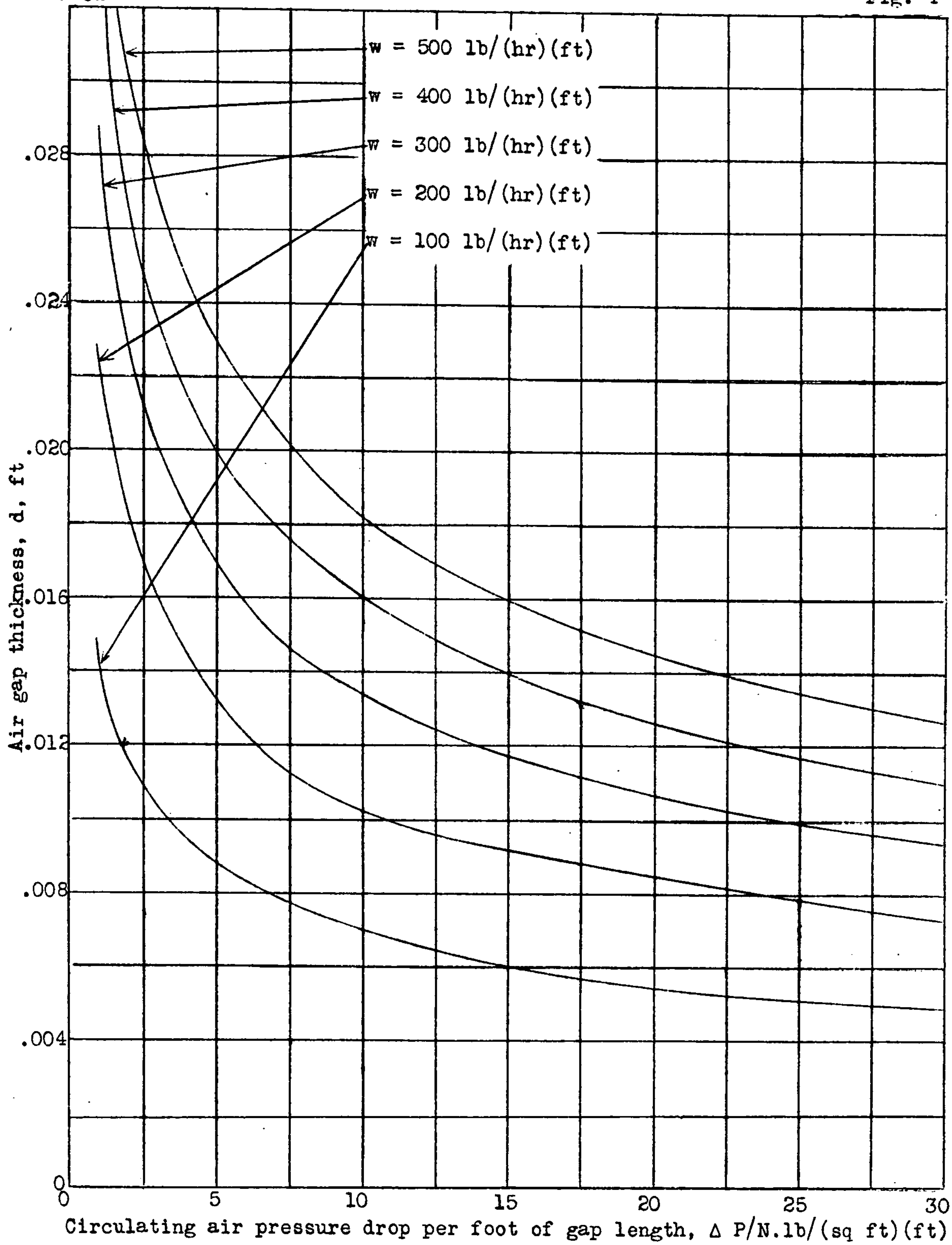

Figure 4.- Variation of circulating air pressure drop per unit length of air path with air gap thickness in an air-heated windshield at sea level for various circulating air weight rates per unit gap width.