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REMARK ON THE THEORY OF LIFTING SURFACES

By Aldo Muggia

Translation of "Sulla teoria delle superfici portanti."
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REMARK ON THE THEORY OF LIFTING SURFACES*

By Aldo Muggia

SUMMARY

First of all, a brief synopsis of the Weissinger method, as it applies to a rectangular wing, is set forth, in order to show how lifting surface theory is applied in this simple case and to show that his idealization of the vortex system is justifiable in this particular instance. By building on this framework and merely adding a few approximations and unrestrictive understandings, it is demonstrated how the same sort of vortex system can be devised, and can find sanction, for the treatment of the lift problem presented by any thin wing of arbitrary plan form.

1. To begin with, let attention be directed to the aerodynamics of a lifting surface (which is a suitable idealization of an actual wing) having the simple physical property that it departs but slightly from the flat surface S, which is the projection of the lifting surface on the xy-plane. Furthermore, this surface is to be considered immersed in an incompressible fluid of density ρ and to have a free-stream pressure and velocity denoted by p_∞ and V_∞, respectively. This impinging stream is assumed to be directed along and have the same sense as the positive x-axis. In the right-handed Cartesian coordinate system employed here (see fig. 1) it will be assumed that the z-axis is in the direction of the vertically downward pointing vector, as illustrated.

On the basis of the above-stated hypotheses, it is legitimate to assume that the perturbations to the free-stream uniform velocity V_∞ that are produced by the presence of the lifting surface, will be small. Consequently, it follows that the local pressure p will be an harmonic function (a solution to Laplace's equation) of the position-coordinates x, y, z, and, in addition, the overpressure at such a point will be dependent upon the potential φ describing the behavior of the local incremental velocities, through means of the relation

$$p - p_\infty = -\rho V_\infty \frac{\partial \varphi}{\partial x}$$

*"Sulla teoria delle superfici portanti." Atti della Accademia delle Scienze di Torino, vol. 87, 1952-1953. Introduced by Carlo Ferrari, Active National Member, at the Session of 13 May 1953.

The pressure jump occasioned by passage from the underside to the topside of the surface will be a certain unknown function of the points of S; call it $f(x,y)$. The value of this function must become zero along the edge of the surface S, and thus one may write that the overpressure is given by

$$p(x,y,z) - p_\infty = \frac{1}{4\pi}\iint_S \frac{f(x',y')z\,dx'dy'}{r^3}$$

where

$$r = \sqrt{(x - x')^2 + (y - y')^2 + z^2}$$

Upon invoking the stipulation that one must have $\varphi = 0$ at an infinite distance upstream from the surface, it is seen that the sought potential will have the form

$$\varphi(x,y,z) = -\frac{z}{4\pi\rho V_\infty}\int_{-\infty}^{x} dx \iint_S \frac{f(x',y')dx'dy'}{r^3}$$

Now let it be assumed, for convenience' sake, that the equation denoting the leading edge of the lifting surface is to be taken as $x = x_1(y)$ and that for the trailing edge as $x = x_2(y)$, while, likewise, as is customary, the semispan is to be denoted by b. Then, upon carrying through a few obvious transformations it is possible to rewrite the expression giving the perturbation potential as

$$\varphi(x,y,z) = -\frac{z}{4\pi\rho V_\infty}\int_{-b}^{b} dy' \int_{x_1(y')}^{x_2(y')} \frac{dx'}{r^3}\int_{x_1(y')}^{x'} f(\bar{x},y')d\bar{x} - \frac{z}{4\pi\rho V_\infty}\int_{-b}^{b} dy' \int_{x_2(y')}^{+\infty} \frac{dx'}{r^3}\int_{x_1(y')}^{x_2(y')} f(\bar{x},y')d\bar{x}$$

Furthermore, let the following conventions be hypothesized:

Take S' to be the wake region of the xy-plane; ie., the region which lies downstream of the S region and lying in between the two straight boundaries $y = \pm b$.

Then make the definitions that on the wing region S

$$m(x,y) = \frac{1}{\rho V_\infty}\int_{x_1(y)}^{x} f(x,y)dx$$

while on the wake region S'

$$m(y) = \frac{1}{\rho V_\infty}\int_{x_1(y)}^{x_2(y)} f(x,y)dx$$

Consequently, it may be seen that the perturbation potential is now expressible as

$$\varphi(x,y,z) = -\frac{z}{4\pi}\iint_{S+S'} \frac{m\ dx'dy'}{r^3}$$

which merely says that the sought value of φ is the potential which describes the behavior of a distribution of doublets of strength m per unit area in the $S + S'$ region.

This system of doublets may be replaced by a system of vortices, spread over the same $S + S'$ region, by having recourse to the equivalency principle between doublets and vortices. The local strength of this equivalent vortex distribution, per unit distance, will have x- and y-components given by the following expressions:

In the S region,

$$\gamma_x = \frac{\partial m}{\partial y} = \frac{1}{\rho V_\infty}\int_{x_1(y)}^{x} \frac{\partial f}{\partial y}\,dx$$

and

$$\gamma_y = -\frac{\partial m}{\partial x} = -\frac{f(x,y)}{\rho V_\infty}$$

while in the S' region,

$$\gamma_x = \frac{\partial m}{\partial y} = \frac{1}{\rho V_\infty}\int_{x_1(y)}^{x_2(y)} \frac{\partial f}{\partial y}\,dx$$

and

$$\gamma_y = -\frac{\partial m}{\partial x} = 0$$

The above-described system of vortices will induce, at any arbitrary general point of the S region, a certain velocity, the vertical component of which will be given by the integral

$$v_z = \frac{1}{4\pi\rho V_\infty}\int\int_S \frac{\partial f}{\partial y'}\frac{1}{y - y'}\left(1 + \frac{r}{x - x'}\right)dx'dy'$$

Now this vertical component of the induced velocity must be of such a magnitude that $\frac{v_z}{V_\infty} = \frac{\partial z}{\partial x}$ holds true, in order that the velocity vector representing the total flow at the point in question shall be tangent to the wing surface. Thus one now has obtained an integrodifferential equation to work with for the determination of the unknown function, $f(x,y)$.

2. The exact solution of this equation will, however, entail rather formidable difficulties, and for this reason it is best to fall back upon a much simpler approximate procedure for attaining the desired result. To this end, let the situation in regard to the simple rectangular wing be examined first of all (refs. 1 and 2). In this case one may let

$$x_1(y) = -\frac{l}{2} \quad \text{and} \quad x_2(y) = +\frac{l}{2}$$

The angle of attack for any profile section situated at a spanwise distance y from the center line of the wing may be denoted by $\alpha(y)$, where this symbol is meant to denote the true aerodynamic angle of attack of the profile in question, measured from the angle of attack for zero lift. Thus it follows that the governing integrodifferential equation may be written as

$$\alpha(y)V_\infty = \frac{2V_\infty}{\pi l}\int_{-l/2}^{l/2}\frac{\partial z}{\partial x}\sqrt{\frac{\frac{l}{2} + x}{\frac{l}{2} - x}}\,dx = \frac{2}{\pi l}\int_{-l/2}^{l/2} v_z\sqrt{\frac{\frac{l}{2} + x}{\frac{l}{2} - x}}\,dx$$

$$= \frac{1}{2\pi^2\rho V_\infty l}\int_{-l/2}^{l/2}\sqrt{\frac{\frac{l}{2} + x}{\frac{l}{2} - x}}\,dx\int\int_S \frac{\partial f}{\partial y'}\frac{1}{y - y'}\left(1 + \frac{r}{x - x'}\right)dx'dy'$$

Now make the approximation that the value of $|x - x'|$, that enters into the expression for r, is to be replaced by its average, which is simply $l/2$ in this case. Making this substitution, and changing the order of integration will result in the following simplified form for the integrodifferential equation:

$$\alpha(y)V_\infty = \frac{1}{2\pi l}\int_{-b}^{b}\frac{d\Gamma}{dy'}\frac{dy'}{y - y'}\left[\frac{l}{2} + \sqrt{\left(\frac{l}{2}\right)^2 + (y - y')^2}\right]$$

where

$$\Gamma(y') = \frac{1}{\rho V_\infty}\int_{-l/2}^{l/2} f(x',y')dx' = -\int_{-l/2}^{l/2}\gamma_y(x',y')dx'$$

is the circulation function for the velocity distribution around the profile that is situated at the spanwise location denoted by y'.

The above-derived approximate equation equates the vertical velocity component $\alpha(y)V_\infty$ to the induced velocity produced at the point $\left(\frac{l}{4},y,0\right)$ by action of a special system of vortices, which may be considered as derived from the actual distribution of vortices by concentration of all the bound vortices along the quarter-chord line.

3. The line of reasoning followed in section 2 is only valid for the case of rectangular wings, but the result obtained is known to hold true even for other cases, as has already been pointed out by Weissinger. The wider generality of this result may be established upon the basis of the following considerations:

Let it be assumed that the bound vortices are concentrated along a line denoted by $x = x_0(y)$, as illustrated in figure 2. The precise way in which this line is to be selected will be explained in full later on. It may be remarked here, however, that the acceptance of this representation for the bound vortices is equivalent to replacement of the actual distribution of doublets in the S region by means of a special kind of doublet system, consisting of null doublets everywhere upstream of the $x = x_0(y)$ line and a distribution of doublets of strength (per unit area) described by means of the $m(y)$ function throughout the S_1 region (and in the S' region as well), where the S_1 region is that area of the wing which lies downstream of the $x = x_0(y)$ dividing line. The potential which describes this new redistribution of doublets is expressible as

$$\varphi^*(x,y,z) = -\frac{z}{4\pi}\iint_{S_1+S'}\frac{m(y')dx'dy'}{r^3}$$

and thus the error in this approximate expression for the potential of the flow is given by the difference in the two potentials, i.e., this difference is

$$\varphi(x,y,z) - \varphi^*(x,y,z) = -\frac{z}{4\pi\rho V_\infty}\iint_S \frac{dx'dy'}{r^3}\int_{x_1(y')}^{x'} f(\bar{x},y')d\bar{x} +$$

$$\frac{z}{4\pi\rho V_\infty}\iint_{S_1} \frac{dx'dy'}{r^3}\int_{x_1(y')}^{x_2(y')} f(\bar{x},y')d\bar{x}$$

$$= -\frac{z}{4\pi\rho V_\infty}\int_{-b}^{b} dy'\int_{x_1(y)}^{x_2(y')} f(\bar{x},y')d\bar{x}\int_{\bar{x}}^{x_2(y')}\frac{dx'}{r^3} +$$

$$\frac{z}{4\pi\rho V_\infty}\int_{-b}^{b} dy'\int_{x_1(y')}^{x_2(y')} f(\bar{x},y')d\bar{x}\int_{x_0(y')}^{x_2(y')}\frac{dx'}{r^3}$$

$$= -\frac{z}{4\pi\rho V_\infty}\int_{-b}^{b}\frac{1}{(y - y')^2 + z^2}\left\{\int_{x_1(y)}^{x_2(y')} f(x',y')\frac{x - x'}{\sqrt{(x - x')^2 + (y - y')^2 + z^2}}dx' -\right.$$

$$\left.\frac{x - x_0(y')}{\sqrt{\left[x - x_0(y')\right]^2 + (y - y')^2 + z^2}}\int_{x_1(y')}^{x_2(y')} f(x',y')dx'\right\}dy'$$

Now let attention be focussed upon that region of space which is composed of all points which have quite small absolute values for the vertical coordinate $|z|$ and which when projected upon the xy-plane fall within the S region; let this portion of space be labeled the Σ control volume. For points within Σ, therefore, one may replace the φ function with the φ^* approximation (and thus it will be permissible to substitute $\frac{\partial\varphi^*}{\partial z}$ for $\frac{\partial\varphi}{\partial z}$ on the surface of the wing) provided the quantity standing within the curlicue brackets is of small enough size.

Further, it is to be noted that for wings with sufficiently large spans, the value of $\sqrt{(x - x')^2 + (y - y')^2 + z^2}$ does not vary to any marked degree as one ranges over the values of x' of interest, provided the distances $\sqrt{(y - y')^2 + z^2}$ remain large enough, while on the other hand, if the distances $\sqrt{(y - y')^2 + z^2}$ are small, then the value that $|x - x'|$ takes on in the Σ control zone can be represented to good approximation by use of its average value $\frac{1}{2}l(y)$ where the chord distribution function $l(y)$ is defined as

$$l(y) = x_2(y) - x_1(y)$$

Thus, in analogy to what was done in the case of the rectangular wing, it will be legitimate to make the approximation that

$$\sqrt{(x - x')^2 + (y - y')^2 + z^2} \cong \sqrt{\left[x - x_0(y')\right]^2 + (y - y')^2 + z^2}$$

provided the point with coordinates (x,y,z) is so situated that it makes the relation

$$\left|x - x_0(y)\right| = \frac{1}{2} l(y)$$

hold true.

If this is true, it follows, in consequence, that

$$\int_{x_1(y')}^{x_2(y')} f(x',y')(x - x')dx' \cong \left[x - x_0(y')\right] \int_{x_1(y')}^{x_2(y')} f(x',y')dx'$$

from which one obtains the desired definition for $x_0(y')$ as

$$x_0(y') \cong \frac{\int_{x_1(y')}^{x_2(y')} x'f(x',y')dx'}{\int_{x_1(y')}^{x_2(y')} f(x',y')dx'} = \frac{\int_{x_1(y')}^{x_2(y')} x'\gamma_y(x',y')dx'}{\int_{x_1(y')}^{x_2(y')} \gamma_y(x',y')dx'}$$

The interpretation of the relationship just deduced is as follows: The proper $x_0(y')$ abscissa coordinate to choose at each profile section through the wing is the one which corresponds to the location of the barycenter of the circulation for that section. In other words, it is the barycenter of the moments of the vector forces $f\bar{i}$ taken about the points $P(x',y')$, where $\bar{i}$ is the unit vector in the direction of the x-axis and where $\bar{P}$ is the radius vector out to any arbitrary general point in the S region at which the circulation-function value is $\gamma_y(x',y')$.

Thus, to close approximation, one may select the x_0 abscissae values according to the rule

$$x_0(y') = x_1(y') + \frac{1}{4} l(y')$$

while, when applying the boundary condition, it will be necessary, in addition, to make use of the potential values (or the induced velocity values) which appertain to the locations (x,y,0) for which it is true that

$$x(y) = x_0(y) + \frac{1}{2} l(y) = x_1(y) + \frac{3}{4} l(y)$$

This result, which has been deduced by aid of the above-mentioned list of specific observations and series of approximations, may be arrived at by examination of the general equations applying to lifting surfaces. This result is important, for example, in those cases where one wishes to obtain the distribution of circulation (and thus of the lift) which exists out along the span of swept wings (refs. 3 and 4).

Translated by R. H. Cramer
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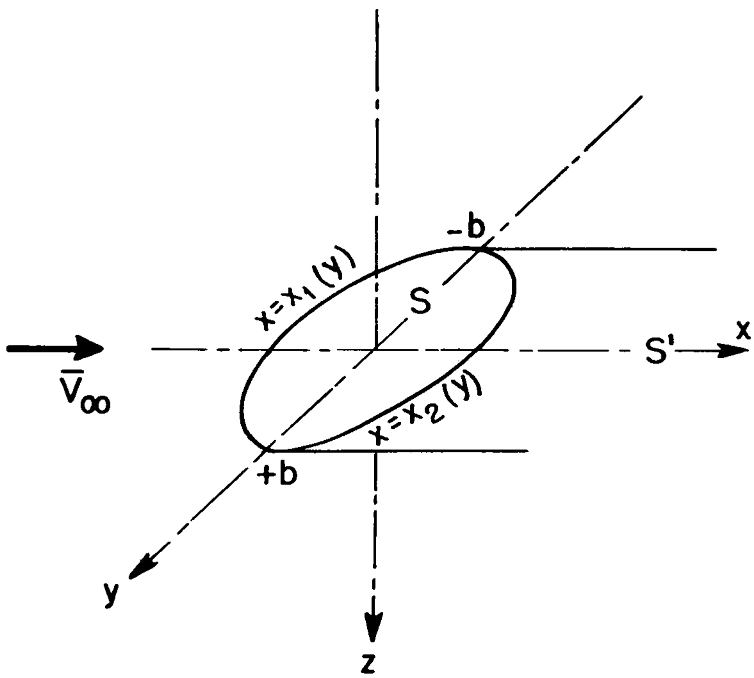

Figure 1.- Orientation of coordinate axes and definitions of integration areas.

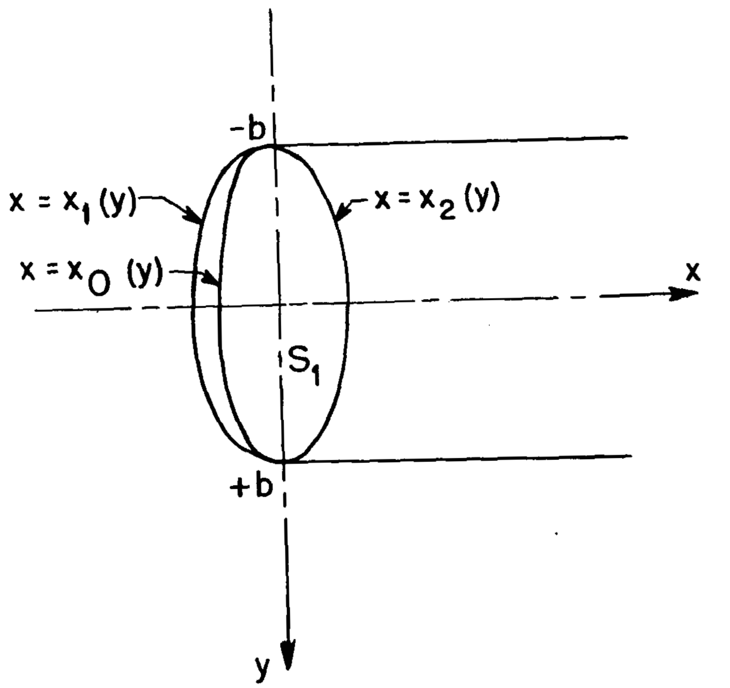

Figure 2.- Location of the bound vortex line and areas of integration.