# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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## HEAT TRANSFER, DIFFUSION, AND EVAPORATION

By Wilhelm Nusselt

**Translation**

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# HEAT TRANSFER, DIFFUSION, AND EVAPORATION[1]

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Although it has long been known that the differential equations of the heat-transfer and diffusion processes are identical, application to technical problems has only recently been made. In 1916 it was shown (ref. 1) that the speed of oxidation of the carbon in iron ore depends upon the speed with which the oxygen of the combustion air diffuses through the core of gas surrounding the carbon surface. The identity previously referred to was then used to calculate the amount of oxygen diffusing to the carbon surface on the basis of the heat transfer between the gas stream and the carbon surface. Then in 1921, H. Thoma (ref. 2) reversed that procedure; he used diffusion experiments to determine heat-transfer coefficients. Recently Lohrisch (ref. 3) has extended this work by experiment. A technically very important application of the identity of heat transfer and diffusion is that of the cooling tower, since in this case both processes occur simultaneously. A relation obtained in the course of such an analysis was given by Lewis (ref. 4) and checked by Robinson (ref. 5), Merkel (ref. 6), and Wolff (ref. 7).

In the following it will be shown that more accurate equations must be substituted for those used in the previous investigations of the relation between the quantity of matter exchanged by diffusion and the quantity of heat transferred by conduction.

A rigid body having a uniform surface temperature $T_w$ is cooled by an air stream having a speed $w_0$ and a temperature $T_0$. According to Fourier, a quantity of heat

$$d^2Q = -\lambda \frac{\partial T}{\partial n} dF\, dt \qquad (1)$$

then flows into the air stream in the direction $n$ normal to the surface element through a surface element $dF$ in the time $dt$. In this

[1]"Wärmeübergang, Diffusion und Verdunstung." Z.a.M.M., Bd. 10, Heft 2, Apr. 1930, pp. 105-121.

equation, $\frac{\partial T}{\partial n}$ is the instantaneous temperature gradient at the surface in the direction of the normal, and $\lambda$ is the thermal conductivity of air. If equation (1) is applied to an element of space in a gas stream, the following differential equation is obtained for the temperature field in an air stream:

$$\gamma c_p \frac{dT}{dt} = \lambda \nabla^2 T \tag{2}$$

where[2]

$\gamma$ density of air

$c_p$ specific heat of unit mass

$\lambda$ thermal conductivity of air

Therefore, the Navier-Stokes equations of motion involving u, v, and w are to be understood.

For the diffusion problem, air can be treated approximately as a homogeneous body, since the molecular weights of its components nitrogen and oxygen are only slightly different. Air is now considered to be mixed with the diffusing gas, for example, ammonia or water vapor. The concentration of the water vapor at an arbitrary point in the air stream is designated c; that is, for example, there are c kilograms of ammonia in 1 cubic meter of air. In the similarity experiment of Thoma (ref. 2), the surface of the rigid body was made of blotting paper saturated with concentrated phosphoric acid. Ammonia is absorbed very actively by such a surface, so that the partial pressure and, hence, the concentration of ammonia is very small. In a cooling tower the diffusion stream proceeds in an outward direction from the water drops. The vapor pressure and, hence, the water-vapor concentration at the surface of the drop is accordingly dependent upon the water temperature. If $p_w$ is the partial pressure of the diffusing gas at the surface of the body and $p_0$ that in the air stream, then the driving force of the diffusion stream is the partial-pressure difference $p_w - p_0$.

[2]The following nomenclature is used herein in the case of variables dependent upon time and the three coordinates x, y, and z:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z}$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

If now, again, n is a running coordinate representing the normal to the surface element dF, then the quantity of vapor diffusing through the surface dF is given by the basic relation of Fick:

$$d^2G_1 = -k \frac{\partial c}{\partial n} dF\, dt \tag{3}$$

in which

c vapor concentration, kg/cu m

$\partial c/\partial n$ gradient of vapor concentration in direction normal to surface

k diffusion coefficient, sq m/sec

If an element of space is taken in the air stream, exchange of vapor between such an element and its environment occurs partly through diffusion and partly through streaming (sensible motion). This fact leads to the differential equation:

$$\frac{dc}{dt} = k\nabla^2 c \tag{4}$$

The agreement between equations (1) and (3) as well as between (2) and (4) is immediately recognizable. Accompanying a calculation of temperature field from diffusion field or vice versa, there must also exist, however, an equivalence of boundary conditions. The temperatures $T_w$ and $T_0$ of the heat transfer correspond to the gas concentrations $c_w$ and $c_0$ of diffusion. But, while during heat transfer the gas velocity is zero at the surface of the cooled body, there exists at that point, in the case of diffusion, a finite gas-velocity component normal to the body surface. This difference is easy to see if a one-dimensional diffusion process is studied in a tube under steady-state conditions. Suppose a tube $l$ meters long is filled with air and ammonia. Suppose also that by certain experimental means the concentrations at the ends of the tube are held at different, although constant, values. Then a quantity $G_1$ kilograms per hour of ammonia diffuses through the tube in one direction, and a quantity of air $G_2$ in the other direction. If x is a running coordinate, $c_1$ the ammonia, and $c_2$ the air concentration, then

$$G_1 = -k_1 \frac{dc_1}{dx} \tag{5}$$

and

$$G_2 = k_2 \frac{dc_2}{dx} \tag{5a}$$

where $k_1$ and $k_2$ are the diffusion coefficients. If the partial pressures of ammonia and of air are $p_1$ and $p_2$, respectively, then

$$G_1R_1T = -k_1 \frac{dp_1}{dx} \tag{6}$$

and

$$G_2R_2T = k_2 \frac{dp_2}{dx} \tag{6a}$$

Integration of equations (6) and (6a) yields[3]

$$G_1R_1T = -k_1p_1 + C_1 \tag{7}$$

and

$$G_2R_2T = k_2p_2 + C_2 \tag{7a}$$

hence a linear variation of partial pressures. If gravitational influence is now ignored, the total pressure $p_0$ in the tube is constant. Hence,

$$p_1 + p_2 = p_0 \tag{8}$$

It then follows from equations (7) and (7a) that

$$k_1 = k_2 = k \tag{9}$$

and

$$G_1R_1T = G_2R_2T = Vp_0 \tag{10}$$

that is, the same volume, evaluated at the total pressure, diffuses in both directions.[4] Moreover, with the notation of figure 1,

$$V = \frac{k}{l} (p_1' - p_1'') = \frac{k}{l} (p_2'' - p_2') \tag{11}$$

and

$$G_1 = \frac{k}{l} (c_1' - c_1'') \tag{12}$$

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[3]These are obviously in error; x lacking.

[4]$k_1$ and $k_2$ are equal, but it does not follow from this argument.

as well as

$$G_2 = \frac{k}{l} (c_2'' - c_2') \tag{12a}$$

Therefore, according to this example, the partial pressure and the concentration of ammonia vary linearly along the tube axis. Now, the experimental situation in the case of the cooling tower and in the diffusion research of Thoma (ref. 2) was otherwise. In the case of the tube, the boundary conditions are different at one tube end. If that end is closed with blotting paper saturated with phosphoric acid, there is set up, for the case of linear (axial) diffusion, the equivalent of Thoma's experiment; or, if that end is closed and the bottom covered with water, a situation corresponding to that of the cooling tower is obtained. Further, Stefan (ref. 8) and Winkelmann (ref. 9) determined the diffusion coefficient k for the diffusion of water vapor in air with this arrangement. It will now be shown that in this instance linear variation of the partial pressure along the tube axis is not attained in the case of the stationary diffusion stream. Once again, air and ammonia, or water vapor, are counterdiffusing. Since, however, one end is impermeable to air, no transport of air along the tube can take place. Because of the gradient of partial pressure of air, air must diffuse in the axial direction. This molecular air transport must work against a convective air transport; that is, a sensible flow of gas in the direction of the tube axis must occur. This flow will be represented by the symbol u. Then, for the flow of air the equation is

$$G_2 = 0 = uc_2 + k \frac{dc_2}{dx} \tag{13}$$

or, after introduction of the partial pressure,

$$up_2 = - k \frac{dp_2}{dx} \tag{13a}$$

For the ammonia stream the corresponding equations are

$$G_1 = uc_1 - k \frac{dc_1}{dx} \tag{14}$$

and

$$G_1 R_1 T = up_1 - k \frac{dp_1}{dx} \tag{14a}$$

Now again

$$p_1 + p_2 = p_0 \tag{8}$$

It follows then from equations (13a) and (14a) that

$$u = - \frac{k}{p_0 - p_1} \frac{dp_1}{dx} \tag{15}$$



If equation (15) is substituted for $u$ in equation (14a), the following expression is obtained for the partial pressure of ammonia:

$$G_1 R_1 T = - \frac{kp_0}{p_0 - p_1} \frac{dp_1}{dx} \tag{14b}$$

the solution of which, with regard to the boundary condition $p_1 = p_1'$ at $x = 0$, is

$$\frac{p_0 - p_1}{p_0 - p_1'} = e^{\frac{G_1 R_1 T}{kp_0} x} \tag{16}$$

Therefore, no linear variation of partial pressure along the tube axis occurs in this case, but rather a logarithmic. If $p_1 = p_1''$ at $x = l$,

$$G_1 = \frac{kp_0}{R_1 T l} \ln \frac{p_0 - p_1''}{p_0 - p_1'} \tag{17}$$

This equation, given earlier by Stefan (ref. 8), shows that the rate of gas diffusion through the tube is no longer proportional to the partial-pressure gradient. First, if $p_1$ is small compared with $p_0$, it is again true that

$$G_1 = \frac{k}{R_1 T} \frac{p_1' - p_1''}{l} = k \frac{c_1' - c_1''}{l} \tag{18}$$

From equations (15) and (16) it follows that

$$u = G_1 \frac{R_1 T}{p_0} = V_0 \tag{19}$$

That is, the gas velocity $u$ is identical with the volume of ammonia gas streaming through a unit of surface of the tube cross section per unit time, as measured at the total pressure $p_0$. Hydrodynamically, therefore, there exist in the ammonia problems sinks of strength $u$ corresponding to evaporation from the source of water. Since, in the case of heat transfer, the velocity is at that point zero, an exact similarity between heat transfer and diffusion is nonexistent in the experiment of Thoma. Only at very small values of $G_1$, hence at low partial pressures of ammonia, can $u$ be taken as zero. Only then does similarity of the boundary conditions also exist. This will be discussed later. Similarity requires, however, identity of the hydrodynamic equations as well. In the experiment of Thoma, the temperature, in the case of the diffusion work, is uniform throughout; while, in the case of heat transfer, it varies. In this instance, the air density varies with temperature. In the case of diffusion, the gas densities vary with the concentrations. Similarity obtains, therefore, only if



$$\frac{T_w}{T_0} = \frac{\gamma_w}{\gamma_0} \tag{20}$$

In equation (20) $\gamma_w$ and $\gamma_0$ are the densities of the ammonia-air mixture at the wall and in the free gas stream. Further, the variation of viscosity with temperature and ammonia concentration must still be considered. Since, however, as was shown previously, similarity is possible only at small ammonia concentrations, the experiment gives the heat-transfer coefficients only at very small temperature differences.

The similarity conditions are more favorable in the case of the cooling tower. In this instance, a hot water surface is cooled by a cold dry air stream. Accordingly, heat flows from the water to the air, and simultaneously the water that diffuses into the air evaporates.[5] In this instance, heat transfer and diffusion are, accordingly, superposed on each other under the same stream conditions. The same hydrodynamic equations and boundary conditions are associated with equations (2) and (4).

In the following discussion, the relations that follow from similarity considerations in the case of Thoma's experiment and in the case of the cooling tower are derived.

[5]Stated precisely as written by Nusselt.

## I. MODEL EXPERIMENT OF THOMA

A body having a surface F and temperature $T_w$ is cooled by an air stream having a speed $w_0$ and a temperature $T_0$. The heat loss of the body has the value Q kilocalories per hour. A diffusion experiment is now carried out by using a body of the same shape. The air stream has the same speed $w_0$ and is mixed with ammonia to the concentration $c_0$. Through the diffusion process, which occurs at the body surface, the ammonia concentration has already attained the value $c_w$. The diffusion experiment shows that the surface absorbs G kilograms of ammonia per hour. What relation exists between Q and G? This question is discussed in the following on the basis of different theories.



### Similarity Theory

The similarity theory of reference 10 leads to the following expression for the rate of heat exchange:

$$Q = \lambda L(T_w - T_0)\, f\left(\frac{Lw_0\gamma}{g\eta}\,,\ \frac{\eta c_p g}{\lambda}\right) \tag{21}$$

where

$\lambda$ thermal conductivity of air

L dimension of body

f initially unknown function of two dimensionless fractions, dependent on shape of body

$\gamma$ density of air

$\eta$ viscosity of air

$c_p$ specific heat of air

For diffusion, correspondingly,

$$G = kL\,(c_0 - c_w)\, f\left(\frac{Lw_0\gamma}{g\eta}\,,\ \frac{\eta g}{\gamma k}\right) \tag{22}$$

From these is obtained the relation sought:

$$\frac{Q}{G} = \frac{\lambda}{k} \frac{f\left(\frac{Lw_0\gamma}{g\eta}, \frac{\eta c_p g}{\lambda}\right)}{f\left(\frac{Lw_0\gamma}{g\eta}, \frac{\eta g}{\gamma k}\right)} \frac{T_w - T_0}{c_0 - c_w} \tag{23}$$

It follows from most of the experimental work that the function f may be represented as a product of two functions, that is,

$$f = b\left(\frac{Lw_0\gamma}{g\eta}\right)^m \left(\frac{\eta c_p g}{\lambda}\right)^n \tag{24}$$

where the constants are dependent on the shape of the body. Reference 11 shows that, for the flow of gases through a cylindrical tube,

$$\left.\begin{aligned} m &= 0.786 \\ \text{and} \qquad & \\ n &= 0.85 \end{aligned}\right\} \tag{25}$$

Two reasons then occur to set the two exponents equal to each other. If it is assumed that the velocity profile over the cross section is independent of density, the velocity components u, v, and w are proportional, in the differential equation of heat conduction, to the stream velocity at the center $w_0$. In this equation, the fraction $w\gamma c_p/\lambda$ as a factor can be taken out. In the function f it must then follow that m = n. Since $\eta c_p g/\lambda$ varies only slightly among the different gases, the influence of the magnitude of n on Q is slight. On that account an m = n was chosen, and in that manner a very simple kind of equation was obtained. Such a choice was expressly limited to gases, because it appeared, since the experimentation of Stanton with water had given a greater value for m than Nusselt had found for gases, that the power form is valid only in a narrow range of values of $\lambda/\eta c_p\gamma$. From the more recent work of Sonnecken and Stender (see ref. 12) were obtained, depending on the experimental conditions, values of m between 0.72 and 0.91 and values of n between 0.35 and 0.50. Merkel (ref. 13) gives for the same research m = 0.87 and n = m/2 = 0.435. Rice (ref. 14) proposes for flow in smooth tubes m = 5/6 = 0.83 and n = 1/2 = 0.50. Lately Schiller and Burbach (ref. 15) have again grappled with this question. They find

that the Nusselt relation for gases with $m = n$ is also experimentally confirmed for water and, further, support theoretically the equality of the exponents. This important question will be discussed later.

Equations (23) and (24) lead to

$$\frac{Q}{G} = \frac{\lambda}{k}\left(\frac{\gamma c_p k}{\lambda}\right)^n \frac{T_w - T_0}{c_0 - c_w} \tag{23a}$$



If the heat-transfer coefficient $\alpha$ is calculated with the use of

$$Q = \alpha F(T_w - T_0) \tag{26}$$

then

$$\alpha = \frac{G}{F}\frac{\lambda}{k}\left(\frac{\gamma c_p k}{\lambda}\right)^n \frac{1}{c_0 - c_w} \tag{27}$$

## Relation of Thoma-Lohrisch

In the derivation of his relation whereby the heat-transfer coefficient may be calculated from a diffusion experiment, Thoma proceeds on the basis that the following condition holds:

$$k = \frac{\lambda}{c_p \gamma} \tag{28}$$

In that circumstance the two differential equations (2) and (4) are interchangeable. The temperature field is then proportional to the concentration field, and it follows that the relation

$$\frac{\frac{\partial T}{\partial n}}{\frac{\partial c}{\partial n}} = \frac{T_0 - T_w}{c_0 - c_w} \tag{29}$$

holds at any arbitrary point in the fields. From equations (1) and (3) the relation sought follows immediately:

$$\frac{Q}{G} = \frac{\lambda}{k}\frac{T_w - T_0}{c_0 - c_w} \tag{30}$$

which, using equation (28), can be also written

$$\frac{Q}{G} = c_p \gamma \frac{T_w - T_0}{c_0 - c_w} \tag{30a}$$

If the heat-transfer coefficient $\alpha$ is calculated with equation (26), there is obtained

$$\alpha = \frac{G}{F} \gamma c_p \frac{1}{c_0 - c_w} \tag{31}$$

Equation (28) is not valid in the diffusion experiment with ammonia in air, however.

According to Thoma, it is considered that

$$\frac{\lambda}{\gamma c_p k} = 1.25 \tag{28a}$$

In this case, Thoma uses equation (30a) as well as (31). As a correction factor he multiplies their right sides by the constant of equation (28a). He therefore obtains

$$\frac{Q}{G} = \frac{\lambda}{\gamma c_p k} \gamma c_p \frac{T_w - T_0}{c_0 - c_w} \tag{32}$$

from which equation (30) is obtained. Then it follows that

$$\alpha = \frac{G}{F} \frac{\lambda}{k} \frac{1}{c_0 - c_w} \tag{32a}$$

Equation (27) assumes this form if $n = 0$.

On the basis of this equation (32a), Thoma derives, from similarity and impulse considerations, the equation for heat transfer:

$$Q = g c_p \eta L (T_w - T_0) \varphi \left( \frac{L w_0 \gamma}{g \eta} \right) \tag{33}$$

If this equation is written in the form

$$Q = \lambda L(T_w - T_0) \frac{c_p \eta g}{\lambda} \varphi\left(\frac{Lw_0 \gamma}{g\eta}\right) \tag{33a}$$

it is recognized as a special case of equation (21), to the extent that

$$f\left(\frac{Lw_0 \gamma}{g\eta}, \frac{\eta c_p g}{\lambda}\right) = \frac{\eta c_p g}{\lambda} \varphi\left(\frac{Lw_0 \gamma}{g\eta}\right) \tag{34}$$

is valid. From equation (33) it follows that

$$\alpha = \frac{g c_p \eta L}{F} \varphi\left(\frac{Lw_0 \gamma}{g\eta}\right) \tag{35}$$

which may also be written

$$\alpha = \lambda \frac{g c_p \eta}{\lambda} \frac{L}{F} \varphi\left(\frac{Lw_0 \gamma}{g\eta}\right) \tag{35a}$$

Thoma now uses equation (35a) to derive equation (32a) from (31). The author cannot follow Thoma's reasoning, because if $f$ is replaced in equation (23) by Thoma's relation of equation (34), the following is obtained:

$$\frac{Q}{G} = \gamma c_p \frac{T_w - T_0}{c_0 - c_w} \tag{30a}$$

and

$$\alpha = \frac{G}{F} \gamma c_p \frac{1}{c_0 - c_w} \tag{31}$$

Thoma's superaddition leads back, therefore, via equation (34), to equation (31) and does not give equation (32a), which was used by Thoma.

Hitherto it has always been assumed that an alteration of a field is effective only in the vicinity of a body. If the whole stream cross section is affected, then another definition of $\alpha$ is necessary. Set

$$dQ = \alpha dF(T_w - T) \tag{36}$$

wherein $T$ is the temperature in the center of the stream cross section that encounters the body surface element $dF$.

Moreover, the equation

$$Q = V_0 \gamma c_p (T_2 - T_0) \tag{37}$$

applies, in which $V_0$ is the air volume swept out by the body in unit time, and $T_2 - T_0$ the temperature rise of that air caused by the hot body. From equations (36) and (37) it follows[6] that

$$\alpha = \frac{V_0 \gamma c_p}{F} \ln \frac{T_w - T_0}{T_w - T_2} \tag{38}$$

If

$$\frac{T_2 - T_0}{T_w - T_0} = \xi \tag{39}$$

then

$$\alpha = \frac{V_0 \gamma c_p}{F} \ln \frac{1}{1 - \xi} \tag{38a}$$

In the diffusion work for the same air volume $V_0$, there is obtained

$$\varepsilon = \frac{c_0 - c_2}{c_0 - c_w} \tag{40}$$

provided $c_2$ is the ammonia concentration behind (downstream of) the body. For the first special case, which is presupposed by the validity of equation (28), there would be

$$\xi = \varepsilon \tag{41}$$

and hence the heat-transfer coefficient sought:

[6]It follows only if certain additional assumptions are made.

$$\alpha = \frac{V_0 \gamma c_p}{F} \ln \frac{1}{1 - \varepsilon} \tag{42}$$

In the general case, Thoma and Lohrisch, as in the preceding, follow equation (28a) and put

$$\alpha = \frac{\lambda}{\gamma c_p k} \frac{V_0 \gamma c_p}{F} \ln \frac{1}{1 - \varepsilon} \tag{43}$$



hence,

$$\alpha = \frac{V_0}{F} \frac{\lambda}{k} \ln \frac{1}{1 - \varepsilon} \tag{43a}$$

The correct relation is obtained as in the following. The temperature $T_2$ is eliminated between equations (37) and (38), and

$$\alpha = \frac{V_0 \gamma c_p}{F} \ln \frac{1}{1 - \frac{Q}{V_0 \gamma c_p (T_w - T_0)}} \tag{44}$$

Now, according to equation (23a)

$$\frac{Q}{T_w - T_0} = \frac{\lambda}{k} G \left( \frac{\gamma c_p k}{\lambda} \right)^n \frac{1}{c_0 - c_w} \tag{23a}$$

If it is noted that

$$G = V_0 (c_0 - c_2) \tag{45}$$

then

$$\frac{Q}{T_w - T_0} = \frac{\lambda}{k} V_0 \left( \frac{\gamma c_p k}{\lambda} \right)^n \frac{c_0 - c_2}{c_0 - c_w} \tag{46}$$

is obtained, and, therefore, from equations (44) and (40),

$$\alpha = \frac{V_0 \gamma c_p}{F} \ln \frac{1}{1 - \left( \frac{\lambda}{\gamma c_p k} \right)^{1-n} \varepsilon} \tag{47}$$

This equation should therefore replace equation (43a) of Thoma-Lohrisch. According to Thoma's research, $\varepsilon$ is in the vicinity of 0.25. If the numerical value of equation (28a) is inserted in equation (47) and, further, the values $n = m = 0.6$, equation (47) yields a value of $\alpha$ some 11 percent smaller than Thoma's equation (43a).

An additional inaccuracy arises also in the calculation of $\varepsilon$ according to equation (40). Thoma, in that equation, put $c_w = 0$, since he assumed that in consequence of the strong absorption of ammonia by phosphoric acid the partial pressure of ammonia in that region is zero. Since, however, the ammonia must diffuse through the boundary layer, a finite vapor pressure of ammonia must exist at the surface. It can naturally be quite small, but it is necessary, first, to measure it once. Lohrisch also employed Thoma's experimental technique in the case of water vapor, to the extent that he saturated with water the blotting paper comprising the body surface. In the calculation of $\varepsilon$, he assumed that the vapor pressure at the body surface corresponds to the water temperature. In section II, concerning evaporation in a cooling tower, it is shown that the vapor pressure at a body surface is smaller than the saturation pressure.



## Boundary-Layer Theory

In its most primitive and simplest form, this theory supposes that an air stream flowing past a body may be considered to consist of two contiguous but sharply demarcated portions, namely, the boundary layer adjacent to the body surface, and the balance of the air stream. The one is associated with a laminar flow, the other with a turbulent, which in the first approximation is treated as a potential flow. Then it is assumed that in the latter a complete equalization of temperature or of ammonia concentration occurs. Figure 2 exhibits this distribution of temperature $T$ and concentration $c$ in the case of heat transfer or of a similarity experiment. The width of the boundary layer is indicated by $y$. For heat transfer, then,

$$Q = \frac{\lambda F(T_w - T_0)}{y} \tag{48}$$

and for diffusion

$$G = \frac{kF(c_0 - c_w)}{y} \tag{49}$$

Accordingly, there is obtained on the basis of elementary boundary-layer theory

$$\frac{Q}{G} = \frac{\lambda}{k} \frac{T_w - T_0}{c_0 - c_w} \tag{50}$$

and also

$$\alpha = \frac{G}{F} \frac{\lambda}{k} \frac{1}{c_0 - c_w} \tag{51}$$



which is identical with the equation (32b) of Thoma and with equation (27) when $n = 0$.

## Impulse Theory

Impulse theory is a carry-over of gas-kinetic considerations into the domain of turbulent motions of a fluid. In its simplest form it is assumed, as an explanation of heat transfer, that a volume of gas $V$ having the temperature $T_0$ moves from the turbulent fluid stream to the wall, where it is heated to a temperature $T_w$ and then brought back to the core of the fluid stream. It therefore takes from the wall the heat

$$Q = V\gamma c_p(T_w - T_0) \tag{52}$$

which is given up to the fluid. It must at the same time be true that

$$\alpha = \frac{V\gamma c_p}{F} \tag{53}$$

The volume $V$ contains in the case of diffusion $c_0$ kilograms of ammonia. On impact against the body surface, ammonia is absorbed until the concentration is $c_w$. Hence,

$$G = V(c_0 - c_w) \tag{54}$$

From equations (52) and (54) it follows that

$$\frac{Q}{G} = \gamma c_p \frac{T_w - T_0}{c_0 - c_w} \tag{55}$$

and, accordingly[7],

$$\alpha = \frac{G}{F} \gamma c_p \frac{1}{c_0 - c_w} \tag{56}$$

which follows also from equation (27) when $n = 1$.



Comparison of Boundary-Layer and Impulse Theories

It will now be assumed that in the turbulent core neither perfect equalization of speed nor, therefore, of temperature or concentration is attained. On the core side of the boundary layer, the temperature $T_e$ and concentration $c_e$ are then, respectively, different from $T_0$ and $c_0$. Within the boundary layer, exchange occurs in accordance with boundary-layer theory. In the free gas stream, impulse theory applies.



Then, in the boundary layer,

$$Q = \frac{\lambda}{y} F(T_w - T_e) \tag{57}$$

and

$$G = \frac{\lambda}{y} F(c_e - c_w) \tag{58}$$

Hence,

$$\frac{Q}{G} = \frac{\lambda}{k} \left( \frac{T_w - T_e}{c_e - c_w} \right) \tag{59}$$

In the turbulent stream, on the other hand,

$$Q = V \gamma c_p (T_e - T_0) \tag{60}$$

and

$$G = V(c_0 - c_e) \tag{61}$$

therefore,

$$\frac{Q}{G} = \gamma c_p \frac{T_e - T_0}{c_0 - c_e} \tag{62}$$

[7]Equation (56) is identical with (31).

It can now be assumed that, approximately,

$$\frac{T_w - T_e}{T_w - T_0} = a \tag{63}$$

that is, is equal to a constant.[8] Also,

$$\frac{c_e - c_w}{c_0 - c_w} = b \tag{64}$$



From equations (63) and (64) it follows that

$$\frac{T_e - T_0}{T_w - T_0} = 1 - a \tag{63a}$$

and

$$\frac{c_0 - c_e}{c_0 - c_w} = 1 - b \tag{64a}$$

In that case, equations (59) and (62) become

$$\frac{Q}{G} = \frac{\lambda}{k}\frac{a}{b}\frac{T_w - T_0}{c_0 - c_w} \tag{59a}$$

and

$$\frac{Q}{G} = \gamma c_p \frac{1 - a}{1 - b}\frac{T_w - T_0}{c_0 - c_w} \tag{62a}$$

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[8]Recently, Prandtl (ref. 16) has given for this quantity the value

$$a = \frac{\frac{gc_p\eta}{\lambda}\, e^{\sqrt[8]{\frac{g\eta}{w_0 d\gamma}}}}{1 + \left(\frac{gc_p\eta}{\lambda} - 1\right) e^{\sqrt[8]{\frac{g\eta}{w_0 d\gamma}}}}$$

in which the value of the parameter e is uncertain. It lies between 1.1 and 1.75.

From the last equations,

$$\frac{a}{b} = a - (1-a)\frac{\gamma c_p k}{\lambda} \tag{65}$$

There is obtained, therefore, the desired relation:

$$\frac{Q}{G} = \frac{\lambda}{k}\left[a + (1-a)\frac{\gamma c_p k}{\lambda}\right]\frac{T_w - T_0}{c_0 - c_w} \tag{66}$$



If this is compared with equation (23a), it is seen that the two functions can be distinguished only with respect to the dependency on the fraction $\gamma c_p k/\lambda$. Since $n > 0$ and $0 < a < 1$, both relations lead to increasing values [of $Q/G$] with increasing values of $\gamma c_p k/\lambda$. In the case of gases, only a small range of variation of this fraction, in the vicinity of unity, occurs and is, therefore, of significance. If agreement of the two relations is demanded, there is obtained as the connection between two constants the expression[9]

$$n = 1 - a \tag{67}$$



Then, if $n = 0.4$, $a = 0.6$.

## II. EVAPORATION OF WATER

Stefans should be credited with having first recognized that the evaporation of water is a problem of diffusion. At the same time, he developed the theory of diffusion. It is necessary to distinguish among several different cases in connection with diffusion.

Consider first a quiet surface of water at the same temperature as the overlying air. If the relative humidity of the air is less than 100 percent, water evaporates; that is, superheated water vapor diffuses into the air from the water surface. Since, under the same conditions, water is lighter than air, an air-streaming occurs. Above the water a rising current of air develops that sucks dry air over the water surface. Through that mechanism the evaporation is increased. If a wind blows over the water surface, a further increase in evaporation occurs as a consequence of turbulence. If the water temperature differs from

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[9]This, from $a + (1-a)(1+\Delta) = (1+\Delta)^n$,

or $a + 1 + \Delta - a - a\Delta = 1 + n\Delta + \ldots$etc.

the air temperature, an intrinsic influencing of the evaporation occurs, while because of the resulting heat exchange the convection is influenced.

## Evaporation in Still Air and in Uniform Temperature Field

Above a surface of water of area $F$ having a representative dimension $L$, a layer of air exists, the density of which, at some distance away, is $\gamma_0$ and the specific humidity of which is $c_0$. At the water surface the humidity content of the air is $c_w$. At any point above the [water] plane the vapor content of the air increases as a consequence of evaporation; it becomes lighter and suffers a lift $Z$ in the amount

$$Z = (c - c_0)\left(\frac{\mu}{\mu_1} - 1\right) \tag{68}$$

where $\mu$ is the apparent molecular weight of dry air and $\mu_1$ that of damp air. For the diffusion field, diffusion equation (4) then applies. The Navier-Stokes equations of motion are required for the determination of the velocity components appearing therein (eq. 4); in these equations, the lift $Z$ appears as an external force acting in the direction of negative gravity; and, therefore,

$$\frac{\gamma_0}{g}\frac{dw}{dt} = Z + \eta\nabla^2 w \tag{69}$$

In this expression, the air density $\gamma_0$ can be assumed constant. A similarity consideration leads to the expression for the concentration gradient at the water surface:

$$-\frac{dc}{dz} = \frac{c_w - c_0}{L}\Phi\left[\frac{L^3\gamma\left(\frac{\mu}{\mu_1} - 1\right)(c_w - c_0)}{g\eta^2}, \frac{g\eta}{k\gamma}\right] \tag{70}$$

The vapor mass evaporating per unit time from the water surface $G$ is calculated according to equation (14). If the value of $u$ according to equation (19) and the following value of $c_1$

$$c_w = \frac{p_w}{RT} \tag{71}$$

are substituted in equation (14), then

$$G = \frac{-kF}{1 - \frac{p_w}{p}} \frac{dc}{dz} \tag{72}$$

where $p$ is the total pressure.



If the concentration gradient is now replaced by its equivalent according to equation (70), the rate of vapor evaporation becomes

$$G = k \frac{(c_w - c_0)F}{\left(1 - \frac{p_w}{p}\right)L} \Phi \left[ \frac{L^3 \gamma \left(\frac{\mu}{\mu_1} - 1\right)(c_w - c_0)}{g\eta^2} , \frac{\eta g}{k\gamma} \right] \tag{73}$$

Evaporation in Wind and in Uniform Temperature Field

Over a water surface a wind passes whose speed at some distance from water has the uniform value $w_0$. The air and water temperatures are identical. Gravitational influences can accordingly be ignored if the airspeed exceeds several meters per second. Therefore, the observations and formulas of section I apply (ref. 17). It follows that, by equation (22), the rate of water evaporation is

$$G = kL(c_w - c_0) f \left( \frac{Lw_0\gamma}{g\eta} , \frac{\eta g}{\gamma k} \right) \tag{22}$$

If the assumption is now made that the velocity $u$ arising from the evaporation normal to the water surface can be ignored, the function $f$ of equation (22) can be taken over from the corresponding heat-transfer problem in accordance with equation (21). The results of reference 17 should be considered here. A copper plate heated electrically to 50° C, and having a dimension on a side of 0.5 meter is cooled by an air stream having a temperature of 20° C. For $w > 5$ meters per second, the following value was obtained:

$$\alpha = 6.14 w_0^{0.78} \text{ kcal}/(\text{m}^2)(\text{hr})(^\circ\text{C}) \tag{74}$$

It follows that in equation (24), if $m = n$,

$$f = 0.065 \left( \frac{w\gamma c_p L}{\lambda} \right)^{0.78} \tag{75}$$

In equation (22) there is obtained, accordingly,

$$f = 0.065 \left(\frac{w_0 L}{k}\right)^{0.78} \tag{75a}$$

Therefore, the expression for the amount of water evaporating from a water area of F square meters in an hour becomes

$$G = 39F \left(\frac{k}{L}\right)^{0.22} w_0^{0.78} (c_w - c_0) \text{ kg/hr} \tag{76}$$

where k is the diffusion coefficient according to the research of Mache (ref. 18):

$$k = \frac{0.078}{p} \left(\frac{T}{273}\right)^{1.89} \text{sq m/hr} \tag{77}$$

in which the total pressure p is to be used.

## Evaporation with Heat Exchange in Still Air

It will now be assumed that the temperature $T_w$ of the water surface is different from the temperature $T_0$ of the air. Hence, heat transfer occurs in addition to diffusion. In this instance, both processes are coupled through the resulting air stream to the extent that air-lifting is caused by both the lesser specific gravity of the water vapor and the heating of the air. Instead of equation (68) of section II, the lift Z per cubic meter has the value

$$Z = \gamma_0 r(T - T_0) + (c - c_0)\left(\frac{\mu}{\mu_1} - 1\right) \tag{68a}$$

Here T and c are the temperature and specific humidity at any place in the field, $\gamma_0$ is the specific gravity of dry air at a great distance from the water surface at which the conditions $p_0$ and $T_0$ prevail, and r is the coefficient of expansion of air:

$$r = 1/T \tag{78}$$

In the case of excesses of temperature that are not too great,

$$r = 1/T_0 \tag{78a}$$

approximately. Then

$$Z = \frac{\gamma_0}{T_0}(T - T_0) + (c - c_0)\left(\frac{\mu}{\mu_1} - 1\right) \tag{68b}$$

becomes the value of the body force in the equation of motion (69). Moreover, equation (2) of heat conduction and equation (4) of diffusion must be used. With the abbreviations

$$\left.\begin{aligned} B &= \frac{L^3\gamma_0^2(T_w - T_0)}{g\eta^2 T_0} \\ C &= \frac{\lambda}{gc_p\eta} \\ E &= \frac{L^3\gamma_0\left(\frac{\mu}{\mu_1} - 1\right)(c_w - c_0)}{g\eta^2} \\ \text{and} \\ D &= \frac{k\gamma_0}{g\eta} \end{aligned}\right\} \tag{79}$$

similarity considerations lead to the relations

$$G = kL(c_w - c_0)\,\Phi_1\,(B,E,C,D) \tag{80}$$

and

$$Q = \lambda L(T_w - T_0)\,\Phi_2\,(B,E,C,D,) \tag{80a}$$

where $\Phi_1$ and $\Phi_2$ are initially unknown functions of the variables B, E, C, and D.

The formulation is significantly simpler and clearer if it is assumed that equation (28) applies, that is,

$$k = \frac{\lambda}{c_p\gamma_0} \tag{28}$$

for then

$$C = D \tag{28a}$$

In this case T and c are proportional to each other. Equation (29) applies and, then,

$$\frac{Q}{G} = \frac{\lambda(T_w - T_0)}{k(c_w - c_0)} = \gamma_0 c_p \frac{T_w - T_0}{c_w - c_0} \tag{81}$$

The exchanged heat and evaporated moisture stand, therefore, in a very simple relation to each other. If the values of G and Q according to equations (80) and (80a) are substituted in equation (81),

$$\Phi_1 = \Phi_2 \tag{81a}$$

which is valid only if equation (28) obtains. For the determination of the function $\Phi$ there is introduced, as a matter of expediency, a new dependent variable:

$$\Omega = T - T_0 + \frac{T_0}{\gamma_0}\left(\frac{\mu}{\mu_1} - 1\right)(c - c_0) \tag{82}$$

which then leads to the differential equations

$$\left.\begin{aligned} \frac{\gamma_0}{g}\frac{dw}{dt} &= \frac{\gamma_0}{T_0}\Omega + \eta\nabla^2 w \\ \text{and} \qquad \gamma_0 c_p \frac{d\Omega}{dt} &= \lambda\nabla^2\Omega \end{aligned}\right\} \tag{83}$$

from which, through consideration of similarity involving the gradient at the water surface, it follows that

$$-\frac{\partial\Omega}{\partial n} = \frac{T_w - T_0 + \frac{T_0}{\gamma_0}\left(\frac{\mu}{\mu_1} - 1\right)(c_w - c_0)}{L} f\left[(B+E),\ C\right] \tag{84}$$

This gradient can also be calculated by the use of equation (82), from which it follows that

$$\frac{\partial\Omega}{\partial n} = \frac{\partial T}{\partial n} + \frac{T_0}{\gamma_0}\left(\frac{\mu}{\mu_1} - 1\right)\frac{\partial c}{\partial n} \tag{85}$$

If the divergence ("speed of expansion") u at the water surface is considered negligible, equations (1) and (3) apply:

$$Q = -\lambda F \frac{\partial T}{\partial n} \tag{1}$$

and

$$G = -kF \frac{\partial c}{\partial n} \tag{3}$$

From equations (1), (3), (81), (84), and (85):

$$\left.\begin{aligned} G &= \frac{kF(c_w - c_0)}{L} f\left[(B+E),\ D\right] \\ \text{and} \\ Q &= \frac{\lambda F(T_w - T_0)}{L} f\left[(B+E),\ C\right] \end{aligned}\right\} \tag{86}$$

The second of the equations (86) must also apply for the case $c_w - c_0 = 0$, and therefore when $E = 0$. The equation then goes over into the usual form for heat transfer. If the latter is known, the rate of evaporation G can thus be calculated.

On the basis of heat-transfer research (ref. 19), for large values of B/C, the following can be written:

$$Q = C_1 \lambda \frac{F}{L} \left(\frac{B}{C}\right)^{1/4} (T_w - T_0) = C_1 \lambda \frac{F}{L} \left[\frac{L^3 \gamma^2 c_p (T_w - T_0)^5}{\lambda \eta T_0}\right]^{1/4} \tag{87}$$

in which the coefficient $C_1$ is dependent upon the form and orientation with respect to gravity.

With equation (87), one now obtains from equation (86)

$$\begin{aligned} G &= C_1 k \frac{F}{L} \sqrt[4]{\frac{B+E}{D}} (c_w - c_0) \\ &= C_1 k \frac{F}{L} \sqrt[4]{\frac{L^3 \gamma_0 (T_w - T_0)}{k \eta T_0} + \frac{L^3 \left(\frac{\mu}{\mu_1} - 1\right)(c_w - c_0)}{\eta k}} (c_w - c_0) \end{aligned} \tag{88}$$

It should be observed, above all, that this equation is valid only if equation (28) is valid, and, hence, C = D. The values k, $\lambda$, $\gamma$, $\eta$, and $c_p$ pertain to the vapor-air mixture and, indeed, are to be taken as mean values over the whole field. The term k is given by equation (77). The thermal conductivity of air $\lambda$ is in the following form in reference 10:

$$\lambda = \frac{0.00167(1 + 0.000194T)\sqrt{T}}{1 + \frac{117}{T}} \text{ kcal/(m)(hr)(°C)} \tag{89}$$



Since the thermal conductivity of water vapor in the germane temperature range is only slightly less than that of air, the thermal conductivity of the vapor-air mixture can be assumed equal to that of air. The terms $\gamma$ and $c_p$ are to be calculated according to the relation appropriate for a mixture of gases. Thus, there are obtained

$$\left.\begin{array}{lll} \text{at} & t = 0 & 50^\circ\text{ C} \\ \dfrac{\lambda}{k\gamma c_p} & = 0.87 & 0.84 \end{array}\right\} \tag{90}$$

This fraction is different from unity for this vapor-air mixture. Accordingly, equations (80) and (80a) apply. Since equations (80) and (80a) become equations (87) and (88) in the limiting case C = D, in general, in the first approximation, the following can be written:

$$G = \frac{C_1 kF(c_w - c_0)}{L}\sqrt[4]{\frac{B+E}{D}} \tag{91}$$

and

$$Q = C_1\frac{\lambda F(T_w - T_0)}{L}\sqrt[4]{\frac{B+E}{C}} \tag{92}$$

Then the following is obtained:

$$\frac{G}{Q} = \frac{k}{\lambda}\sqrt[4]{\frac{C}{D}}\frac{c_w - c_0}{T_w - T_0} \tag{93}$$

or

$$\frac{G}{Q} = \frac{k}{\lambda}\sqrt[4]{\frac{\lambda}{kc_p\gamma_0}}\frac{c_w - c_0}{T_w - T_0} \tag{94}$$

If the temperature within the domain falls below the dewpoint, fog formation is initiated and the equations become invalid.

Equation (87), along with equations (88), (91), and (93), is applicable only at large differences of temperature and water-vapor concentration. For small differences, the function $f(B/C)$ can be expressed only by means of a graphical representation. By such a representation, it can be shown that for $B + E = 0$, $f$ approaches a constant value.

If the air temperature $T_0$ is greater than the temperature of the water surface $T_w$, it is possible, for finite values of $T_0 - T_w$ and $c_w - c_0$, that $B + E = 0$. In this case, no convective streaming occurs, but rather only a molecular transport of heat and vapor. Hence, for

$$B = -E \tag{95}$$

or

$$\frac{T_0 - T_w}{c_w - c_0} = \frac{T_0}{\gamma_0}\left(\frac{\mu}{\mu_1} - 1\right) \tag{95a}$$

there is obtained

$$G = C_2 k L(c_w - c_0) \tag{96}$$

and

$$Q = C_2 \lambda L(T_0 - T_w) \tag{97}$$

Therefore,

$$\frac{G}{Q} = \frac{k}{\lambda}\frac{c_w - c_0}{T_0 - T_w} \tag{98}$$

The coefficient $C_2$ is dependent upon the shape of the water surface.

## Evaporation with Heat Exchange and Air Flow

As in the case of Evaporation in Wind and in Uniform Temperature Field, a wind having a speed $w_0$ flows by a water surface. However, the water temperature $T_w$ and the air temperature $T_0$ are now different. With a small partial pressure of water vapor assumed, equations (21), (22), and (23) of section I apply here as well. Since, however,

at 50° C the partial pressure of water vapor has already attained a value of 0.125 atmosphere, they must be modified. For the Reynolds number the following will, for brevity, be used:

$$\mathrm{Re} = \frac{Lw_0\gamma_0}{g\eta} \tag{79a}$$

On the basis of similarity theory one has again, first of all, the relations

$$-\frac{\partial T}{\partial n} = \frac{T_w - T_0}{L} f(\mathrm{Re},C) \tag{99}$$

and

$$-\frac{\partial c}{\partial n} = \frac{c_w - c_0}{L} f(\mathrm{Re},D) \tag{99a}$$

In these, in the case of a smooth water surface such as a tank,[10] L is the principal dimension; and for a water drop, it is the diameter. At the water surface there exists, further, between the rate of evaporation and the concentration gradient the following relation:

$$\frac{G}{F} = uc_w - k\frac{\partial c}{\partial n} \tag{14a}$$

where, according to equation (19),

$$u = \frac{GR_1T_w}{Fp} \tag{19a}$$

and $R_1$, in this equation, is the gas constant of water vapor, and p is the total pressure. There is then obtained

$$uc_w = \frac{Gp_w}{Fp} \tag{100}$$

in which $p_w$ is the partial pressure of water vapor at the water surface. If the value of equation (100) is used in equation (14a), the concentration gradient at the water surface becomes

$$-\frac{\partial c}{\partial n} = \frac{G\left(1 - \frac{p_w}{p}\right)}{kF} \tag{101}$$

[10] Or pond or pool.

By the same reasoning, the following applies for heat transport:

$$\frac{Q}{F} = u\gamma c_p (T_w - T_0) - \lambda \frac{\partial T}{\partial n} \tag{14b}$$

and, with equation (19a),

$$-\frac{\partial T}{\partial n} = \frac{1}{\lambda F}\left[Q - G\frac{\mu}{\mu_1} c_p(T_w - T_0)\right] \tag{102}$$

From equations (99), (99a), (101), and (102), the relation sought between Q and G is

$$\frac{Q}{G} = \left(1 - \frac{p_w}{p}\right)\frac{\lambda(T_w - T_0)f(Re,C)}{k(c_w - c_0)f(Re,D)} + \frac{\mu}{\mu_1} c_p(T_w - T_0) \tag{103}$$

If the value of f given by equation (24) is substituted in equation (103),

$$\frac{Q}{G} = \left(1 - \frac{p_w}{p}\right)\frac{\lambda}{k}\left(\frac{\gamma_0 c_p k}{\lambda}\right)^n \frac{T_w - T_0}{c_w - c_0} + \frac{\mu}{\mu_1} c_p(T_w - T_0) \tag{103a}$$

From equations (101) and (99a) there is obtained, with equation (24),

$$G = \frac{bkF(c_w - c_0)}{\left(1 - \frac{p_w}{p}\right)} \frac{Re^m}{LD^n} \tag{104}$$

and, at the same time, from equation (103a),

$$Q = \frac{bF(T_w - T_0)}{L}\left[\frac{\lambda}{C^n} + \frac{\mu c_p k(c_w - c_0)}{\mu_1\left(1 - \frac{p_w}{p}\right)D^n}\right]Re^m \tag{105}$$

If $c_w = c_0$, no evaporation occurs and equation (105) becomes equation (24).

Vapor Pressure at Water Surface

It is natural to assume that, at the evaporating water surface, the vapor pressure is equal to the pressure at the saturation value corresponding to the water temperature $T_w$, and that, therefore, $p_w = p_s$. At the same time, the vapor concentration at the water surface then becomes

$$c_w = \gamma'' = \frac{1}{v''} \tag{106}$$

that is, it is equal to the saturation density. That this, however, is not the case has already been conjectured by Winkelmann (ref. 9) and then demonstrated by Mache (ref. 18), who found, on the basis of a thorough research on evaporation in a cylindrical tube, that the following relation exists between the rate of evaporation and the pressure in question:

$$\frac{G}{F} = K_0(p_s - p_w) \tag{107}$$

that is, the vapor pressure over the water surface, during evaporation, is always smaller than the saturation pressure corresponding to the temperature of the water surface. The coefficient $K_0$ is a temperature function that unfortunately has not yet been precisely determined. If the density of vapor instead of the partial pressure is introduced in equation (107),

$$\frac{G}{F} = \beta_1(c'' - c_w) \tag{108}$$

where $c''$ is the saturation concentration of water vapor at the water temperature $T_w$, and $\beta_1$ is a constant dependent upon temperature, which, according to the researches of Mache, assumes values dependent upon the water temperature as indicated in the following table of values:

| $t_w$, °C | $K_0$, 1/hr | $\beta_1$, m/hr |
|---|---|---|
| 92.4 | 0.0086 | 148 |
| 87.8 | .0080 | 133 |
| 82.1 | .0084 | 108 |
| 27.5 | .084 | 925 |

Unfortunately, it is precisely in the technically important temperature

range between 0° and 50° C that only a single experimental value is available.

## Application to Psychrometer of August

For many technical applications of the diffusion relations previously developed, it is appropriate to calculate the evaporation coefficient on the basis of the heat-transfer analog, as has already been done in the treatment of burning and vaporization of the carbon in iron ore (ref. 1). Set

$$G = \beta F(c'' - c_0) \text{ kg/hr} \tag{109}$$

from which the dimensions of the evaporation coefficient are

$$\beta = \text{m/hr} \tag{110}$$

In the case of August's psychrometer, evaporation takes place from a moist thermometer in still air. In such an instance, equation (91) applies in the calculation of the mass of water evaporating per unit time. If, further, an evaporation coefficient $\beta_2$ is inserted, where

$$\beta_2 = C_1 \frac{k}{L} \sqrt[4]{\frac{B + E}{D}} \tag{111}$$

then, equation (91) becomes

$$G = \beta_2 F(c_w - c_0) \tag{91a}$$

Further, equation (108) still applies. If the unknown concentration $c_w$ is eliminated between equations (108) and (91a), the evaporation coefficient $\beta$ in equation (109) becomes

$$\frac{1}{\beta} = \frac{1}{\beta_1} + \frac{1}{\beta_2} \tag{112}$$

When the heat transferred along the stem of the thermometer is ignored, the heat balance of the wetted thermometer may be expressed as

$$Q = G \left[ r + c_p(T_w - T_s) \right] \tag{113}$$

In this expression, $T_w$ is the temperature of the wetted thermometer, $T_s$ the saturation temperature corresponding to the partial pressure

$p_w$ of the water vapor at the surface of the wetted thermometer, r the heat of vaporization at the pressure $p_w$, and $c_p$ the specific heat of the limiting curve [?] at the same pressure.

If $T_0$ is the environment temperature, as measured with a dry thermometer, the heat extracted from the surroundings is

$$Q = \alpha F(T_0 - T_w) \tag{114}$$



The heat-transfer coefficient $\alpha$ consists of two parts, several terms of a sum $\alpha_b$, covering the heat conveyed by thermal conduction to the thermometer, and a portion $\alpha_s$ that gives the magnitude of heat radiation. The latter is

$$\alpha_s = C_s \frac{\left(\frac{T_0'}{100}\right)^4 - \left(\frac{T_w}{100}\right)^4}{T_0 - T_w} \tag{115}$$

where $C_s$ is the radiation coefficient of water, that is,[11]

$$C_s = 3.35 \text{ kcal}/(m^2)(^\circ C^4) \tag{116}$$

and $T_0'$ is the mean temperature of the fixed body surrounding the wetted thermometer with which radiation is exchanged. It is certainly approximately equal to the ambient-air temperature $T_0$, yet surely not quite precisely equal. Herein, under certain conditions, exists a not unimportant source of error in psychrometry. This source of error can successfully be eliminated (as was communicated to the author by Dipl.-Ing. Kaissling) by surrounding the wet thermometer by a radiation shield, which consists, as does the wet thermometer itself, of a wetted surface.

If $T_0' = T_0$ and the attainment of room temperature is assumed, equation (115) becomes, approximately,[12]

$$\alpha_s = C_s \tag{115a}$$

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[11] Dimension time$^{-1}$ apparently missing in equation (116).

[12] This does not seem to be correct.

The heat-transfer coefficient $\alpha_b$ is dependent upon the flow conditions in the vicinity of the thermometer. If the psychrometer is hanging in a region in which the air is quiet, then, according to equation (92),

$$\alpha_b = C_1 \frac{\lambda}{L} \sqrt[4]{\frac{B + E}{C}} \tag{117}$$

in which the constant $C_1$ is dependent upon the configuration of the thermometer well. For a cylindrical well of height H,

$$C_1 = 0.83 \tag{118}$$

In equation (116 [117]), L is replaced by H.

If the wetted thermometer is placed in a current of air, there is obtained, for example, for a plate-shaped thermometer, the following relation (ref. 20):

$$\alpha_b = 0.069 \frac{\lambda}{L} \mathrm{Re}^{0.78} + 0.83 \frac{\lambda}{H} \sqrt[4]{\frac{B + E}{C}}\, e^{-0.000028\mathrm{Re}} \tag{119}$$

wherein it is supposed that the wind flows along the thermometer well in a horizontal direction.

If, in the energy equation (113), the value of G from equation (109) is inserted, and that of Q from (114), the following psychrometer formula is obtained:

$$\left[r + c_p(T_w - T_s)\right](c'' - c_0) = (\alpha_b + C_s)\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right)(T_0 - T_w) \tag{120}$$

For the diffusion constant of a plate-shaped, wetted thermometer, the following is obtained with equations (119), (111), and (117):

$$\beta_2 = 0.069 \frac{k}{L} \left(\frac{w_0 L}{k}\right)^{0.78} + 0.83 \frac{k}{H} \sqrt[4]{\frac{B + E}{D}}\, e^{-0.000028 \left(\frac{w_0 L}{k}\right)^{0.78}} \tag{121}$$

Equation (120) gives a decrease of the psychrometer constant with increase of airspeed, which has been well confirmed by the investigations of Edelmann, Sworykin, and Recknagel.

## Application to Theory of Cooling Tower

In this [apparatus], finely divided warm water trickles downward and is cooled by a rising current of cold air. If, at some point, $w_0$ is the relative speed of the water and air with respect to each other, equations (103), (104), and (105) apply for the heat transfer and evaporation. If the heat-transfer coefficient $\alpha$ and the evaporation coefficient $\beta_2$ are now evaluated according to equations (26) and (92a),

$$\frac{\alpha}{\beta_2} = \left(1 - \frac{p_w}{p}\right)\frac{\lambda}{k}\left(\frac{\gamma_0 c_p^k}{\lambda}\right)^n + \frac{\mu}{\mu_1} c_p(c_w - c_0) \tag{122}$$

and, with equation (112),

$$\frac{\alpha}{\beta} = \left(1 - \frac{p_w}{p}\right)\frac{\lambda}{k}\left(\frac{\gamma_0 c_p^k}{\lambda}\right)^n + \frac{\mu}{\mu_1} c_p(c_w - c_0) + \frac{b\lambda}{\beta_1 L}\left[\left(\frac{g c_p \eta}{\lambda}\right)^n + \frac{\mu c_p^k (c_w - c_0)}{\mu_1 \lambda \left(1 - \frac{p_w}{p}\right)}\left(\frac{g\eta}{\gamma_0 k}\right)^n\right] Re^m \tag{123}$$

Material on the technical applications of the formulas here presented will soon be published elsewhere.[13]

Translated by H. H. Lowell
National Advisory Committee
for Aeronautics

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[13] Since the transmission of the original manuscript to the editor's office on May 16, 1929, the following papers have appeared: E. Schmidt, Verdunstung und Wärmeübergang, Gesundheitsing., 1929, p. 525.; R. Mollier, Das ix-Diagramm für Dampfluftgemische, Stodolafestschr., Zürich, 1929, p. 438; H. Thiesenhusen, Untersuchungen über die Wasserverdunstungsgeschwindigkeit in Abhängigkeit von der Temperatur des Wassers, der Luftfeuchtigkeit und Windgeschwindigkeit, Gesundheitsing., 1930, p. 113.

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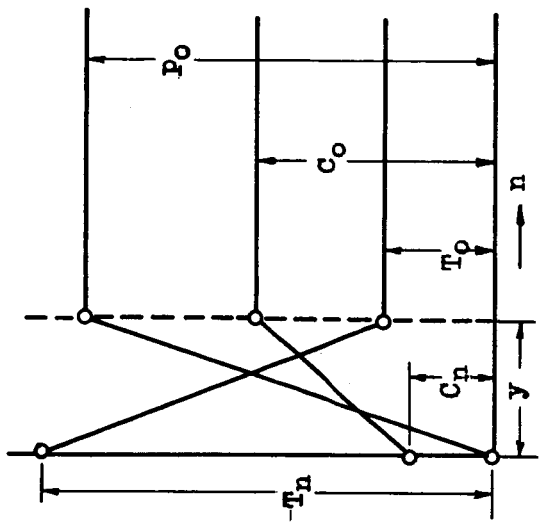


Figure 2

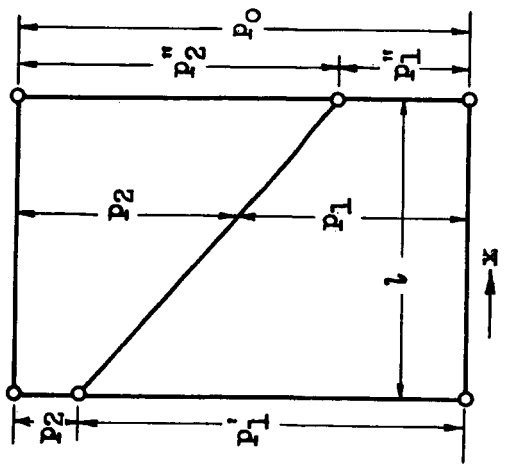


Figure 1