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EXPERIMENTAL DETERMINATION OF LOCAL AND MEAN COEFFICIENTS OF HEAT TRANSFER FOR TURBULENT FLOW IN PIPES

By I. T. Aladyev

Translation

"Eksperimental'noe Opredelenie Lokal'nykh i Srednikh Koeffitsientov Teplootdachi Pri Turbulentnom Techenii Zhidkosti v Trubakh." Izvestiya Akademii Nauk SSSR, Otdelenie Tekhnicheskikh Nauk, no. 11, 1951.

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EXPERIMENTAL DETERMINATION OF LOCAL AND MEAN COEFFICIENTS OF HEAT TRANSFER FOR TURBULENT FLOW IN PIPES*

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A large number of papers have been devoted to the investigation of heat transfer in the flow through pipes. Many of these papers are directly or indirectly concerned with the study of the dependence of the heat-transfer coefficient on the length of the pipe. These investigations have been analyzed in detail in a number of papers (refs. 8, 9, and others). A short resume of only the most important investigations concerned with the study of the effect of the pipe length on the mean and local heat-transfer coefficients is given here for the case of turbulent flow of a fluid.

The earliest work is that of Stanton (ref. 1). He determined the mean heat-transfer coefficient in vertical pipes of different diameters when they were heated or cooled by water. As a result of this investigation, it was found that the mean heat-transfer coefficients for pipes with the ratios $l/d = 31.6$, 41.6, and 62.4 are practically the same in magnitude; somewhat higher values were obtained only for $l/d = 33.8$ (l is the length and d the internal diameter of the pipe in meters). The results of this investigation thus do not permit drawing conclusions as to the effect of the pipe length on the heat-transfer coefficient.

In 1910, Rietschel (ref. 2) conducted an investigation of heat transfer by heating the air flowing through horizontal pipes. A brass pipe of 119-millimeter diameter and of 1985-millimeter length and five different pipe boilers of different diameters were tested. The number of pipes in the boiler varied from 3 to 53. In the tests, the over-all flow of air through the boiler was measured, and the flow of air through each pipe was determined by dividing the total flow by the number of pipes. The correctness of the results obtained by such method of measuring the quantity of air discharged is open to serious doubts.

On the basis of these and also some of his own data, Nusselt (ref. 4) proposed a method of taking into account the effect of the pipe

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length by introducing in the basic equation, of the type $Nu = cRe^{n}Pr^{m}$, the ratio $(l/d)^{-0.054}$. This method received wide application and is still used at the present time.

In 1930, Burbach (ref. 3) undertook a special study of the effect of the pipe length on the heat transfer. His method differed from the method of the preceding investigators and permitted, according to the author, the determination of the local values of α_x. An analysis of the method of Burbach showed, however, that it was erroneous. The error consisted in the assumption that the temperature of the wall at the center part of the pipe is the same as at the end of a pipe of half 2943 the length. Because of the heat conductivity along the wall, however, this assumption is far from being true. For this reason the results of Burbach were in error.

In 1931, Lawrence and Sherwood (ref. 5) conducted a new investigation of the effect of the pipe length on the heat-transfer coefficient. The heat transfer for heated water flowing through horizontal pipes of four different lengths was studied. The ratio l/d in these tests changed with the shortening of the pipe. Notwithstanding the change in l/d from 59 to 224, the authors did not find any appreciable effect of the length of the pipe on the heat-transfer coefficient. In 1936, a detailed investigation of the heat transfer of airplane radiators was published by Maryamov (ref. 6). The author determined the coefficients of heat transfer of radiators of various constructions. By computing the coefficient of heat transfer from the water to the wall according to the formula of Ten-Bosch (ref. 9) and by neglecting the thermal resistance of the radiator wall, the author was able to determine the average value for the entire radiator. Then, applying the assumption of the hydrodynamic theory of heat exchange as regards the relation between the heat transfer and the resistance and making a number of assumptions, the author indirectly determined the effect of the depth of the radiator, that is, the effect of the length of the pipe on the heat-transfer coefficient. For taking account of this effect the following equation was proposed:

$$Nu = 0.25 \frac{d}{l} RePr\left[1 - 0.96 \exp\left(- 0.015 \frac{l}{d}\right)\right] \qquad (1)$$

This equation shows a still greater effect of pipe length on the heat-transfer coefficient than does the equation of Nusselt. The latter equation and that of Maryamov show that the degree of the effect of l/d on the heat-transfer coefficient α does not depend on Re.

In 1937, Rubinshtein (ref. 7) proposed a new detailed investigation of the variation of the coefficient of heat transfer with the length of the pipe. This investigation was conducted by a method based on the analogy between the phenomena of heat exchange and diffusion. The tests were conducted in both the presence and the absence of hydrodynamic stabilization. As a result of the investigation, the change of the local and mean heat-transfer coefficients with the pipe length was established.

The preceding summary of the principal experimental investigations on the change of the heat-transfer coefficient along the length of a pipe shows that a variety of results have been obtained by different authors. The available data do not permit formulation of a sufficiently reliable method for taking into account the effect of the pipe length on the heat-transfer coefficient; in recent years, a number of authors (ref. 8 and others) have therefore recommended that this effect be completely neglected. In view of the theoretical and practical significance of the problem, the present author undertook an extensive investigation of the changes of the local and mean heat-transfer coefficients along the pipe length, some results of which are described herein.

1. EXPERIMENTAL SETUP

The essential scheme of the test setup is shown in figure 1. Water (distillate) from the tank (1) enters a high-pressure gear pump (2) driven by a constant current motor. The pump discharges the water into a heat exchanger indirectly or, for small discharge quantities, through the intermediate tank (4) which together with the regulating tank (6) permits a fine regulation of the discharge and the maintenance of a constant value. On issuing from the heat exchanger the water enters a tank (6) and then, depending on the position of the three-way cock (7), enters either a measuring vessel or returns to the collecting tank (1). The heating is effected by slightly superheated steam.

The construction of the heat exchanger is shown at the top of figure 1, and its principal geometric characteristics are given in the following table:

TABLE I.

Segment	1	2	3	4	5	6	7	8	9	10	11	12	Total
Distance to center of segment, mm	2.45	7.25	11.95	19.00	28.40	48.10	78.10	117.70	177.70	248.10	323.20	488.10	
Length of segment, mm	4.9	4.7	4.7	9.4	9.4	30.0	30.0	49.2	70.7	70.2	80.0	235.0	599.0

The heat exchanger consists of three coaxially arranged pipes: the innermost one (5) of 15.0-10.2 millimeter diameter, the middle one (inner jacket) (12) of 52-50 millimeter diameter, and the outer one (outer steam sleeve) (13) of 72-70 millimeter diameter.

In the lower half of the inner jacket, at the distances shown in table 1, are soldered 10 diaphragms (8) forming 12 segments for the condensate. The diaphragms at 0.5 millimeter along the height do not reach the inner pipe. Plates of thin copper foil (of 0.2 mm thickness) are soldered on the outer surface of the inner tube. This construction assured the flow of all the condensate formed over a given part of the pipe into the corresponding segment and did not require a constant accurate setting of the heat exchanger in the horizontal position. From the segments the condensate flowed into the measuring vessel. To guard against the loss of heat to the outside, the annular space between the middle and outer pipe was likewise heated with steam. 2943

The construction of the inlet and the outlet of the heat exchanger is clearly shown in figure 1. The inlet, which assured a uniform distribution of the temperature and inlet conditions, approximated the apparatus generally used in most commercial heat exchangers. At the outlet a mixer (15) was mounted which assured good mixing of the water before its temperature was measured. In order to reduce the loss of heat through heat conductivity between the heat exchanger and the mounting, thick textolite disks (8 mm thick) were used. The wall temperatures were measured by means of thermocouples located in the wall at a distance of 0.3 millimeter from the inner surface. All the thermocouples were placed along one generator line of the pipe at distances of 5, 50, 120, 210, 450, and 595 millimeters from the inlet section. The thermocouples were of doubly insulated (enamel and silk) copper and constantan wire 0.1 millimeter in diameter.

The change of the temperature of the water in the heat exchanger was measured by 10-junction differential thermocouples located in thick-walled glass tubes filled with paraffin. The tubes were passed through packing glands into the inlet and outlet sections of the heat exchanger in such manner that the thermocouple junctions were located at the inlet section and immediately behind the mixer. The junctions of single thermometers that were used to measure the absolute magnitudes of the temperature of the fluid at the inlet and outlet of the heat exchanger were also placed in the glass tubes.

The electromotive force of the thermocouples was measured by a potentiometer ("Etalon" type PN-2) with a mirror galvanometer which assured an accuracy of measurement up to 0.0005 millivolt corresponding to about 0.015°. The cold junctions of the thermocouples were at all times situated in melting ice.

The test was conducted in the following manner: After the required water flow was established in the inner jacket of the heat exchanger, slightly superheated (by 2° - 5°) steam was admitted. First, in order to remove the air, the steam was admitted in large excess. The amount was then decreased and was established such that only slight vaporization occurred through the pipes that conducted away the condensate. At the same time the steam was admitted to the steam jacket of the heat exchanger. The steady thermal state of the system was attained after about 1 to 1.5 hours. The temperature of the water at the inlet and outlet and the temperature of the steam varied during the test by no more than 0.1°. At the start of the test, the temperature of the water, its change in the heat exchanger, and the temperature of the steam were measured. Then, 12 flasks fixed to a single base were simultaneously placed under all the condensate pipes through which the condensate from the sections of the inner jacket flowed and the time was noted. The flasks were provided with special cocks and were located 10 to 20 millimeters from the ends of the condensate pipes so as to permit complete collection of the condensate and conduction of the steam issuing with the condensate to the atmosphere.

After 5 or 6 days of operation the run was discontinued, the water was changed, and the inner surface of the investigated pipe of the heat exchanger was cleaned. Generally, it was found to be entirely clean, but nevertheless this procedure was regularly repeated.

2. PROCEDURE IN EVALUATING TEST DATA

In the experimental investigation of the processes of heat exchange, the required magnitude is the heat-transfer coefficient, which may be determined from the heat-balance equation which for an element of the pipe has the form

$$dQ = \alpha_x \Delta t_x dF = G_f c_p dt_x \tag{2}$$

where Q is the amount of heat absorbed or given out per unit time in kilocalories per hour, α_x is the local value of the heat-transfer coefficient in kilocalories per square meter per $^{\circ}C$, Δt_x is the temperature difference in a section a distance x from the inlet section of the pipe in $^{\circ}C$, F is the area of an element of the pipe in square meters, G_f is the weight of fluid discharged in kilograms per hour, c_p is the specific heat of the fluid at constant pressure in kilocalories per kilogram per $^{\circ}C$, and dt_x is the change of temperature of the fluid over the length element dx at distance x from the inlet section of the pipe in $^{\circ}C$.

Equation (2) may be solved for the local values of α_x

$$\alpha_x = \frac{G_f c_p dt_x}{\Delta t_x dF} \tag{3}$$

Similarly, from the heat-balance equation for a pipe of length l, the average values of the heat-transfer coefficient α may be obtained

$$\alpha = \frac{G_f c_p \delta t}{\Delta t \pi d l} \tag{4}$$

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where Δt is the mean temperature difference for a pipe of length l in $^{\circ}C$ and δt is the change in the temperature of the fluid in the pipe of length l in $^{\circ}C$.

From equations (3) and (4) it follows that for determining the local and mean values of the heat-transfer coefficients it is necessary and sufficient to know, in addition to the weight of fluid discharged, the change in the temperature of the pipe wall and of the fluid along the length of the pipe. As previously mentioned, the temperature variation of the pipe wall was measured directly in the tests with the aid of thermocouples. A number of typical curves, representing the change in wall temperature along the pipe, are shown in figure 2. In computing the mean temperature difference, the temperature of the pipe wall was taken to be the mean integrated value for the element or section of the pipe under consideration.

The variation of the temperature of the water along the length of the pipe could be determined by computation because the quantity of heat given off and absorbed by the individual segments was known. These computations permitted constructing curves showing the variation of the temperature of the water along the length of the pipe for each test. The most typical curves are shown in figure 3. In making these computations, the quantity of heat q_l, in kilocalories per hour, given by the steam to an element of the pipe of length l_i was determined from the amount of the condensate g_{ki}, in kilograms per hour, formed in the corresponding segment according to the following equation:

$$q_i = g_{ki} r \tag{5}$$

where r is the latent heat of steam formation in kilocalories per kilogram. The total quantity of heat imparted by the steam to the water was computed as the sum of the heats imparted to all 12 segments of the pipe according to the equation

$$Q_k = \sum_{1}^{12} g_{ki} r + Q_s \tag{6}$$

where Q_s is the superheat of the steam. (The superheat of the steam was several tenths of a percent of the total quantity of heat taken up by the pipe.) On the other hand, the same quantity of heat could be determined from the change in the heat content of the water in the heat exchanger

$$Q_f = G_f c_p \delta t \tag{7}$$

2943 The second method of determining Q was the more accurate one since the amount of water discharged and the variation of its temperature were determined with great accuracy. Although the difference between Q_k and Q_f did not exceed 3 percent, the heat value in the evaluation of the test data was that given by Q_f. In correspondence with this, the heat quantities computed by equation (5) were corrected by the value Q_f where the difference between Q_f and Q_k was distributed between the parts in proportion to their areas.

After the curves for the temperature variation of the fluid and of the wall along the pipe were obtained, the values of the heat-transfer coefficients were computed by equation (3) for individual short elements of the pipe of length Δl_i, and these were assumed to be the local coefficients. The chosen lengths of the elements Δl_i varied from 2 millimeters at the inlet of the pipe to 40 millimeters at its end. Depending on requirements, the heat-transfer coefficients were computed for 14 - 7 elements at different distances from the inlet of the pipe, the distances being reckoned from the inlet to the midpoint of the element. In the same manner, the mean values of the heat-transfer coefficients for pipes of different lengths were determined.

In accordance with the most recent data in the literature (refs. 8 and 10), the determining temperature was taken to be the arithmetical mean of the temperature of the fluid with the Pr characteristic raised to the 0.4 power.

3. LOCAL VALUES OF HEAT-TRANSFER COEFFICIENT

The test data on the local values of the heat-transfer coefficient are shown in figure 4, which shows that the heat-transfer coefficient drops with increasing distance from the inlet section until x/d attains a definite value which is a function of Re_x and decreases with increase in the latter. For $x \approx 40d$, however, the heat-transfer coefficient attains a practically constant value for all Re_x. It is seen also from figure 4 that the effect of x/d on α_x decreases with increasing Re_x.

It is of interest to compare these results with those obtained by other investigators. From the brief review given at the beginning of this paper, it follows that such comparison is possible only with the results of Y. M. Rubinshtein (ref. 7). The results of the latter, evaluated to correspond with the procedure here assumed, are shown in figure 5. Comparison of figure 5 with figure 4 shows the identity of character and the degree of dependence of α_x on x/d and Re_x 2943
obtained in these two investigations. This agreement in the results obtained by the method of the diffusion analogy and by the direct method confirms the possibility of studying the phenomena of heat transfer by the method of analogy with diffusion in those cases where the temperature difference differs considerably from an infinitely small value. The succeeding tests showed, however, the limitations of the method of diffusion analogy. The latter gives considerable error if the forces of gravity are comparable with those of inertia, which is the case for the laminar flow of a fluid.

Analysis of the test data established the fact that the effect of the ratio x/d and of Re_x can be taken into account with the aid of an exponential function with negative exponent. The results of this analysis are presented in figure 6. The equation of the curve passing through the mean values of the test data is of the following form:

$$Nu_x = 0.044\ Re_x^{0.8}\ Pr_x^{0.4} \left(\frac{x}{d}\right)^{-\frac{2.25}{Re_x^{0.3}}} \tag{8}$$

This equation holds for $x/d \leq 40$. For the sections of the pipe which are at a distance $x > 40d$ from the inlet section, the local values of the heat-transfer coefficient practically do not depend on x/d and according to figure 4 may be determined from the equation

$$Nu_x = 0.0156\ Re_x^{0.86}\ Pr_x^{0.4} \tag{9}$$

The values of the numerical factors appearing before Re_x in equations (8) and (9) hold only for the conditions applying to the present investigation.

The method described for taking into account the effect of x/d and Re_x on the local values of the heat-transfer coefficient is inconvenient and complicated for practical purposes. In order to simplify the computation of this effect, another method was applied; the method is based on the proportionality of α_x for any section of the pipe to the value of the local heat-transfer coefficient $\alpha_{0,x}$ for a section at a distance $x > 40d$ from the inlet section of the pipe; that is, it was assumed that

$$\alpha_x = k_x \alpha_{0,x} \tag{10}$$

where k_x is a factor taking into account the change in α_x along the pipe and equal to unity for $x/d \geq 40$.

The results of the computations of the factor k_x from the test data of the present investigation are given in the following table:

TABLE II.

Re_x \ x/d	0.5	1.0	2.0	5.0	10	20	30	40
10^4	2.04	1.65	1.46	1.29	1.18	1.10	1.04	1.0
2×10^4	1.78	1.45	1.36	1.23	1.15	1.08	1.03	1.0
5×10^4	1.50	1.34	1.26	1.17	1.11	1.06	1.02	1.0
10^5	1.28	1.20	1.15	1.10	1.06	1.02	1.01	1.0
10^6	1.12	1.10	1.08	1.05	1.03	1.01	1.00	1.0

MEAN VALUES OF HEAT-TRANSFER COEFFICIENT

The test data on the mean values of the heat-transfer coefficient are presented in figure 7 from which it is seen that, with decrease in the relative length of the pipe, the mean value x of the heat-transfer coefficient α increases. For pipes of length $l \geq 50d$, however, α practically ceases to depend on the length of the pipe. This result agrees qualitatively with the results obtained in most of the previous investigations, while quantitatively it agrees with the results of Lawrence and Sherwood (ref. 5) and those of Y. M. Rubinshtein (ref. 7). It is also seen from figure 7 that the effect of l/d on α decreases with increasing Re. This result does not agree with the results obtained by Nusselt (ref. 4) and Maryamov (ref. 6), according to whom the degree of the effect of l/d on the heat-transfer coefficient α does not depend on Re. The result agrees, however, with that of Rubinshtein (ref. 7). The latter's data on the mean heat-transfer coefficients were evaluated according to the procedure assumed by us. It was then found that the character of the dependence of α on Re and l/d is similar to that obtained in the author's tests. This is particularly well seen in figure 8, which shows the dependence of the exponent of Re on l/d according to the tests of the author and of Rubinshtein.

For the purpose of reducing the obtained results to a unique relation, the test data on the mean values of the heat-transfer coefficient were evaluated by a procedure similar to that employed in Section 3 for the local values of α_x. The results of this evaluation are shown in figure 9. The curve passing through the mean values of the test data has the following equation:

$$Nu = 0.124\ Re^{0.7}\ Pr^{0.4} \left(\frac{l}{d}\right)^{-\frac{3.1}{Re^{0.35}}} \quad (11)$$

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This equation holds for pipes of length $l \leq 50d$. For pipes of length $l < 50d$ the mean value of the heat-transfer coefficient for any Re is practically independent of the length of the pipe and may be determined from the equation

$$Nu = 0.031\ Re^{0.8}\ Pr^{0.4} \quad (12)$$

The numerical values of the factors in front of Re in equations (11) and (12) are valid only under the conditions of the present investigation. For the purpose of simplifying the computation of the effect of the length of the pipe on the mean value of the heat-transfer coefficient, the same method was used as for the case of the local values α_x; that is, it was assumed that

$$\alpha = k\alpha_0 \quad (13)$$

where k is a factor that takes into account the effect of the length of the pipe on the mean values of the heat-transfer coefficients and is equal to unity for $l/d \geq 50$ and α_0 is the mean value of the heat-transfer coefficient for pipes of length $l \geq 50d$.

The results of the computations of the factor k from test data obtained in the present investigation are given in the following table:

TABLE III.

Re \ l/d	0.5	1.0	2.0	5.0	10	15	20	30	40	50
10^4	1.81	1.65	1.50	1.34	1.23	1.17	1.13	1.07	1.03	1.0
2×10^4	1.63	1.51	1.40	1.27	1.18	1.13	1.10	1.05	1.02	1.0
5×10^4	1.42	1.34	1.27	1.18	1.13	1.10	1.08	1.04	1.02	1.0
10^5	1.34	1.28	1.22	1.15	1.10	1.075	1.06	1.03	1.02	1.0
10^6	1.17	1.14	1.11	1.08	1.05	1.13	1.03	1.02	1.01	1.0

The factor k may also be determined from the equation

$$k = 5.22\ Re^{-1/8} \left(\frac{l}{d}\right)^{-\frac{2}{Re^{0.3}}} \tag{14}$$

CONCLUSIONS

1. In the steady turbulent flow of a fluid in a pipe, the local values of the heat-transfer coefficient decrease along the length of the pipe practically up to a section at the distance $x \approx 40d$ from the inlet section of the pipe. This distance is a function of the Reynolds number Re_x and decreases with increase in the latter, but starting from $x \approx 40d$ the local heat-transfer coefficients for any Re_x do not depend on x and may be computed by equation (9).

The values of the local heat-transfer coefficients at a distance $x < 40d$ may be determined by equation (8) or by multiplying the local values α_x, determined by equation (9), by the factor k_x, taken from table II.

2. For a steady turbulent flow of a fluid in a pipe, the mean values of the heat-transfer coefficient decrease with increase in the relative length of the pipe. The length of the thermally stabilized part for mean values of the heat-transfer coefficient is a function of Re and decreases with increase in the latter; but starting from $l \approx 50d$ for all Re, the mean values of the heat-transfer coefficient practically do not depend on l/d and may be computed by equation (12). The values of the mean heat-transfer coefficients for pipes with $l/d < 50$ may be determined by equation (11) or by multiplying α, determined from equation (12), by the factor k, taken from table III.

The author wishes to acknowledge the valuable suggestions he received in writing this paper from Academician M. V. Kirpichev and Correspondent member of the Soviet Academy of Science M. A. Mikheev.

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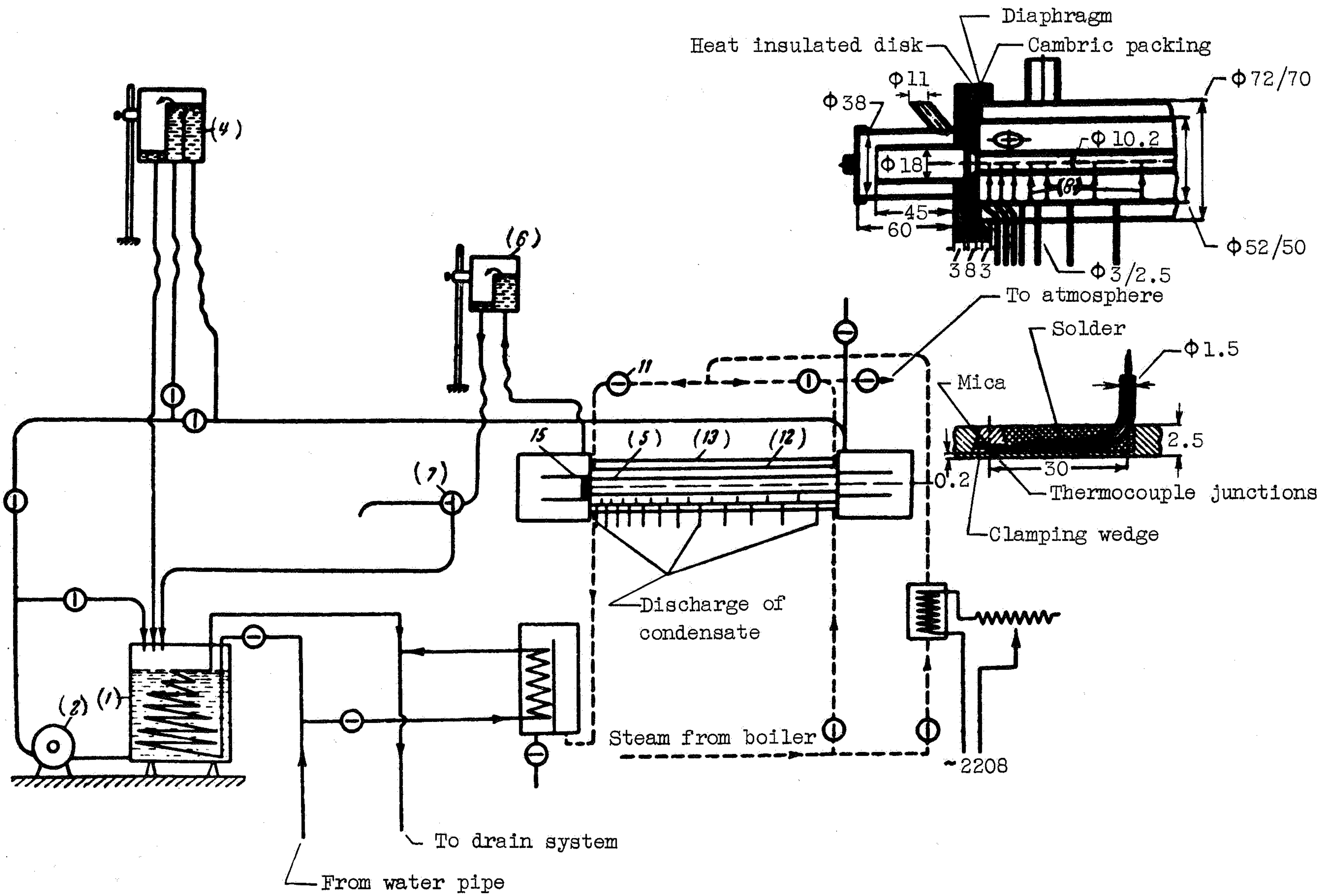

Figure 1. - Scheme of experimental setup.

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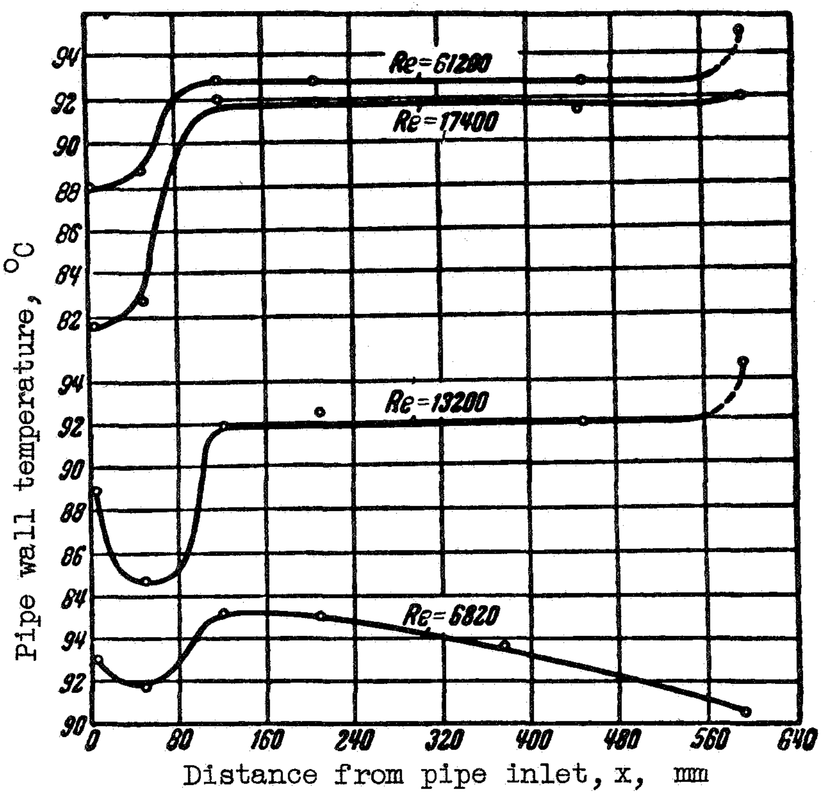

Figure 2. - Curves of variation of the wall temperature along pipe.

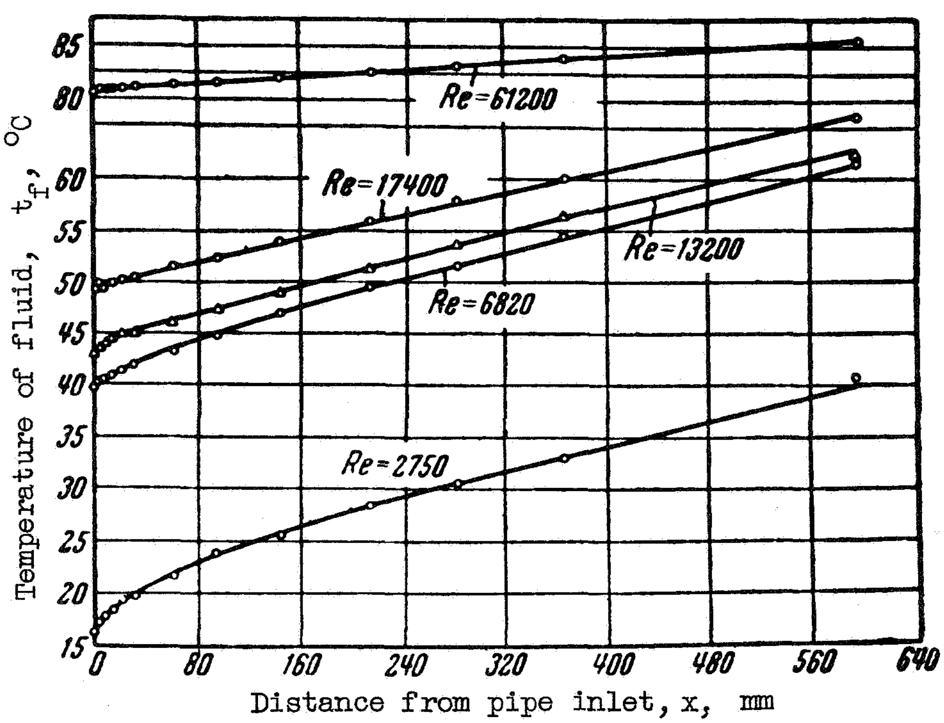

Figure 3. - Curves of variation of temperature of fluid along pipe.

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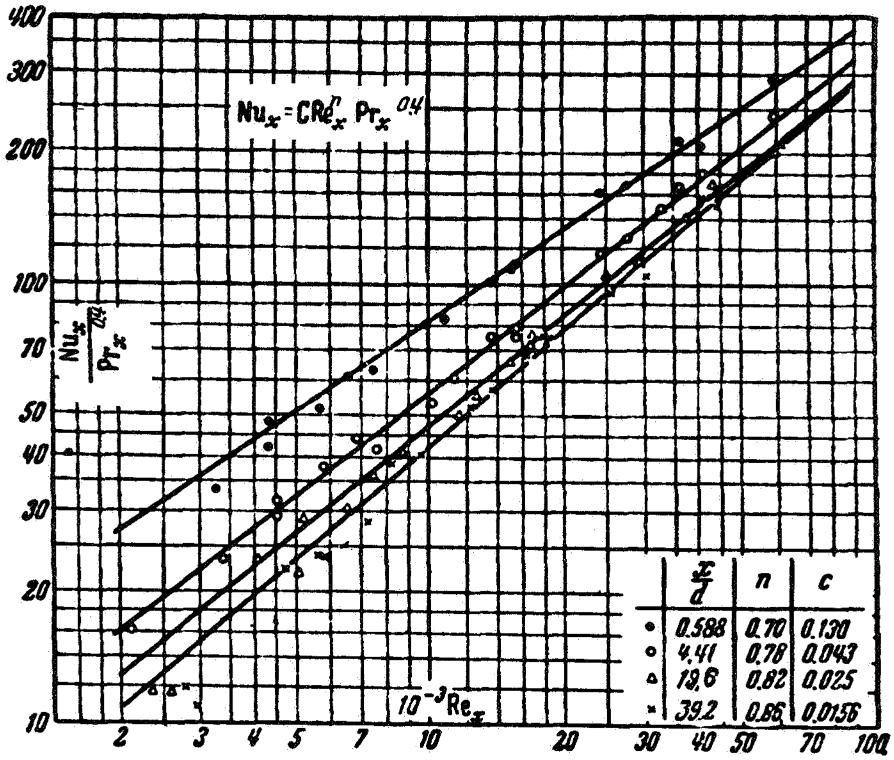

Figure 4. - Local values of heat-transfer coefficient.

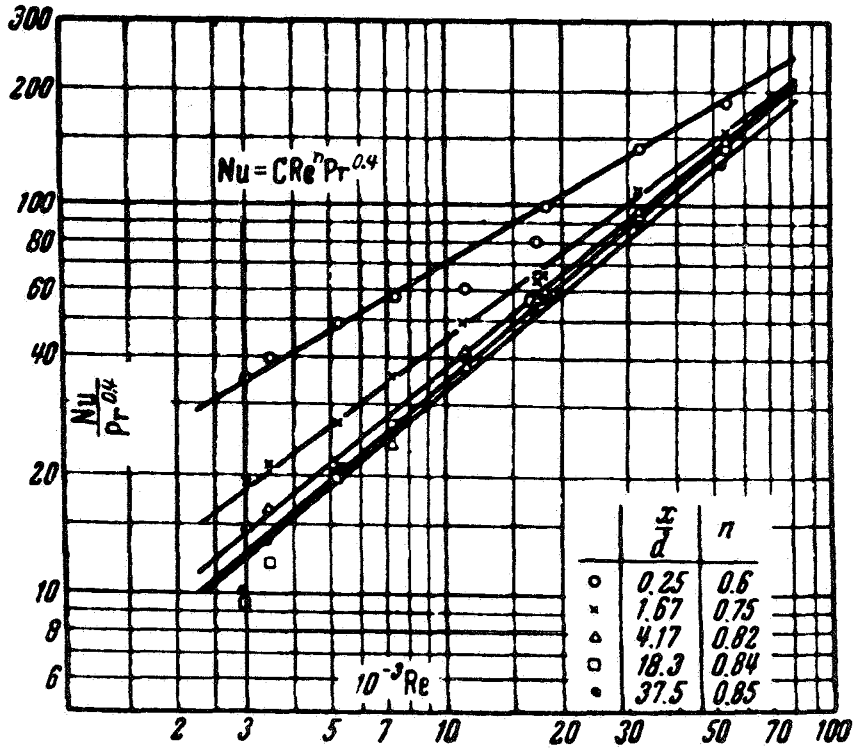

Figure 5. - Local values of heat-transfer coefficient. Data from reference 7.

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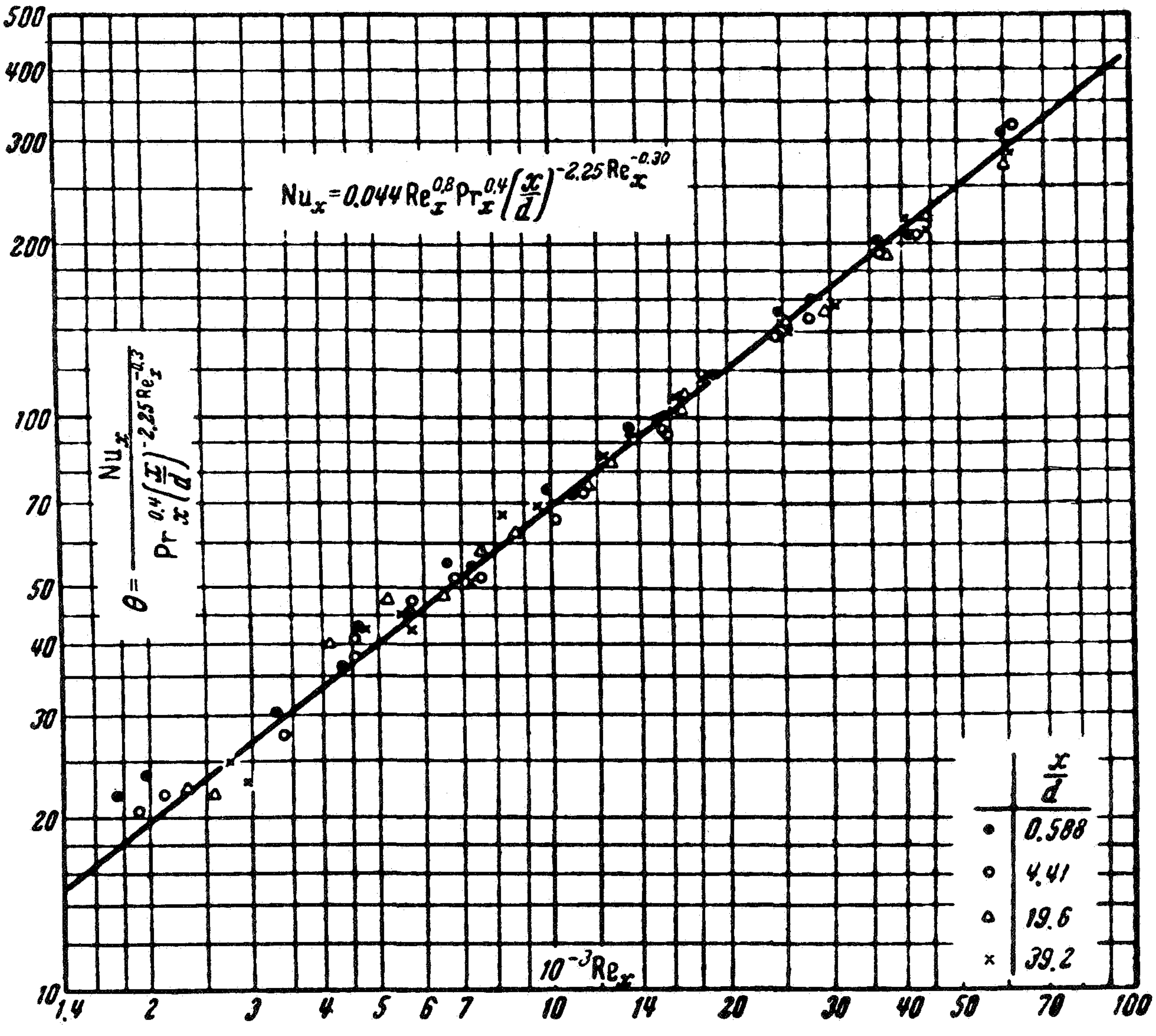

Figure 6. - Local values of heat-transfer coefficient for sections at distance $x \leq 40d$ from inlet section of pipe.

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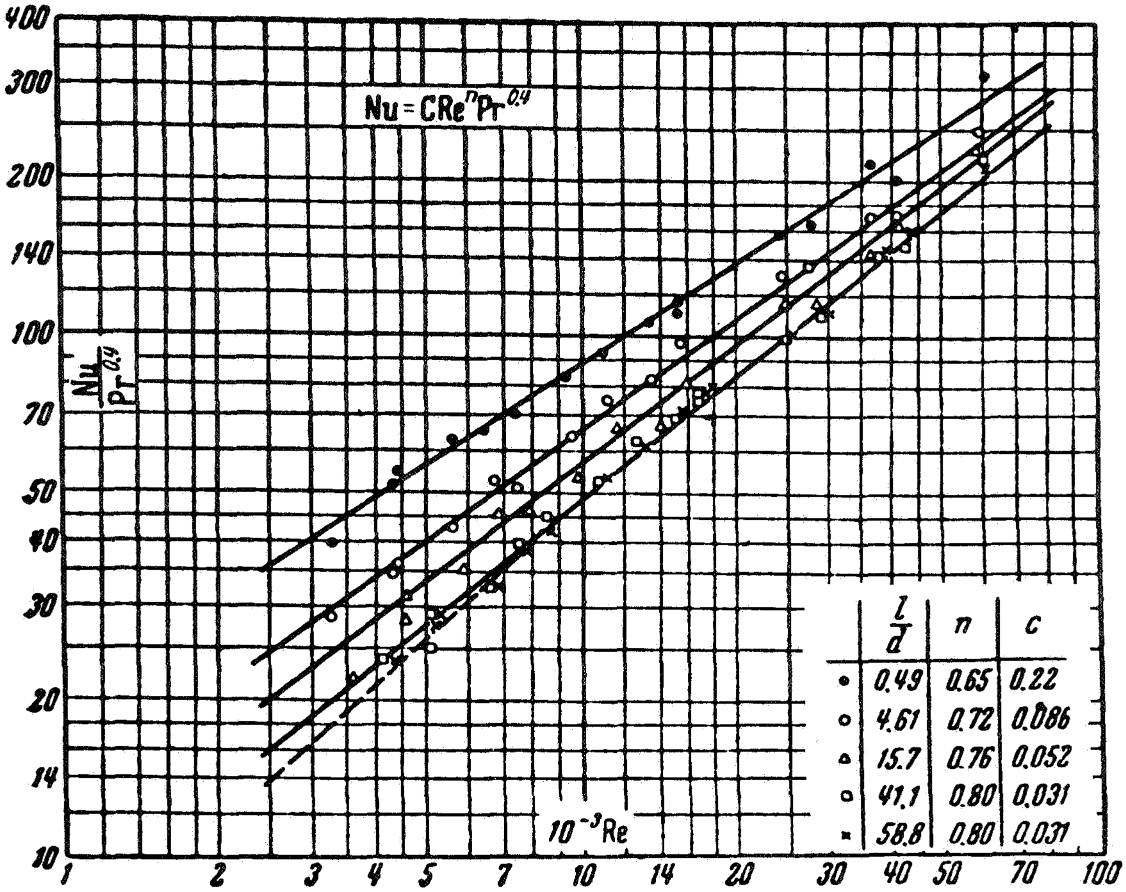

Figure 7. - Mean values of heat-transfer coefficient.

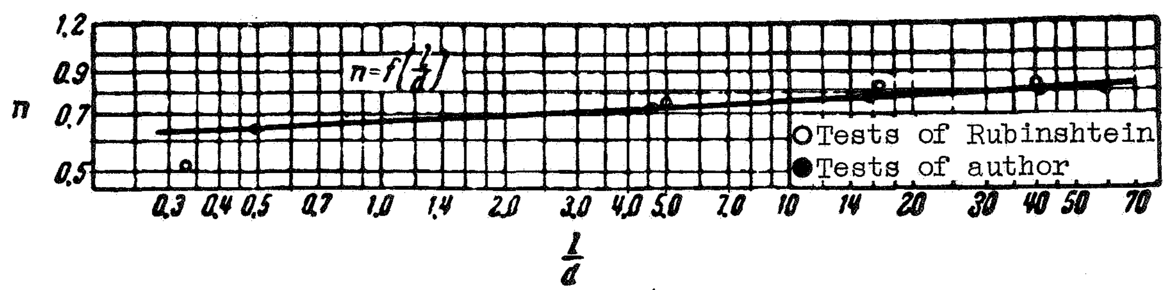

Figure 8. - Dependence of exponent of Re on l/d according to results of Rubinshtein and of author.

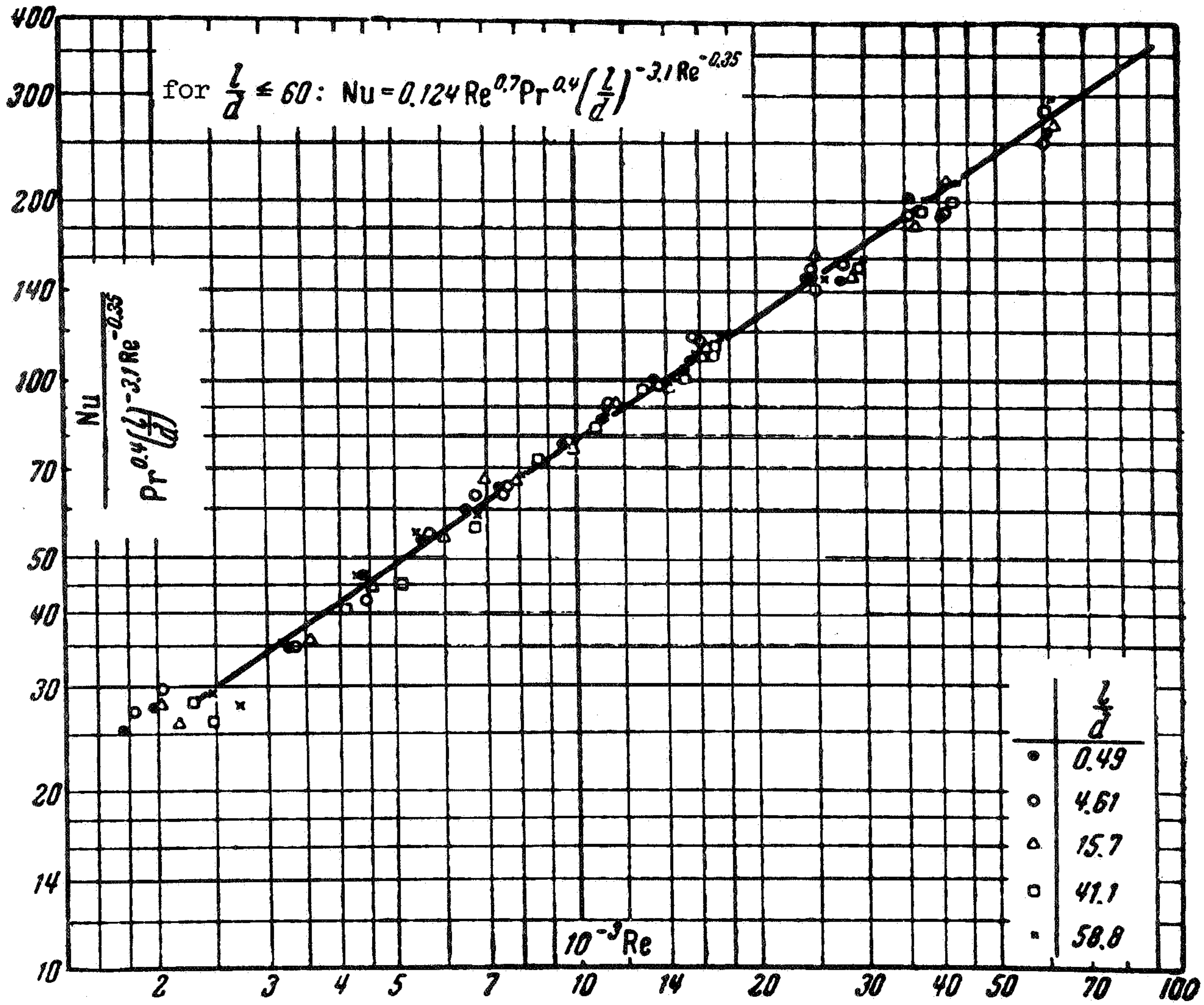

Figure 9. - Mean values of heat-transfer coefficient for pipes of length $l \leq 60$ d