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TECHNICAL MEMORANDUM 1330

THEORY OF DYNAMIC CREEP

By A. A. Predvoditelev and B. A. Smirnov

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An analysis is given of the causes of the increase in the creep of a material under dynamic loads. A theory of dynamic creep is proposed based on the after-effect theory of Becker.

The term creep is applied to the phenomenon of the deformation of a metal as a function of time without any increase in the stress. The creep of metals is generally investigated at constant stress. The characteristic curve obtained is shown in figure 1. There is no doubt that the testing for creep at constant load is of great importance. Under real conditions, however, metals are almost always subjected to the action of dynamic loads which can essentially change the creep of the metal. The few investigations that have been made in this direction show that the curves of dynamic creep are similar to the creep curves at constant loads and reveal the same three stages distinguishable on figure 1 except that the creep of the metals increases considerably.

Attempts to explain the increase in the creep for dynamic loads only by the nonlinearity of the dependence of the deformation on the load are not always satisfactory, because the creep under dynamic loads is greater than the creep corresponding to the static load equal to the maximum load acting during the cycle.

In a theoretical treatment of dynamic creep three circumstances must be taken into account:

1. The instantaneous creep rate depends not only on the magnitude of the instantaneous stress but also on the rate of change of the stress. This is particularly important for plastic materials because their deformation is strongly influenced particularly by the rate of deformation.

2. Variable stresses have an important effect on various metallurgical changes. The cyclic stresses may accelerate the aging, the recrystallization, etc.

*"K teorii dinamicheskoi polzuchesti." Vestnik Moskovskogo Univ., Phys., vol. 8, no. 8, 1953, pp. 79-86.

3. Dynamic loads, in contrast to static, may facilitate or give rise to slip or other forms of lattice deformation, which differ from static deformation.

The neglect of these circumstances can lead to incorrect results and a large difference between theory and experiment.

In the theory of dynamic creep presented herein, an attempt is made to take into account those circumstances which are associated with the presence of dynamic loads, namely the dependence of the instantaneous creep rate not only on the magnitude but also on the rate of change of the load. The basic idea is that the presence of a dynamic load gives rise to a certain finite rate of deformation which in turn can in an essential way facilitate slip deformation.

The effect of vibrations on the metallurgical changes is very difficult to estimate, and therefore the present theory omits this factor for the present.

The theory of dynamic creep represents an extension of the theory of after-effect of Becker (ref. 1) to dynamic loads. According to this theory, the reason for the after-effect phenomena is the nonuniformity of the metallic structure. The nonuniformity of the material is understood in the sense that the individual grains may differ greatly from each other in their plastic limit, modulus of elasticity, shape, and so forth and therefore may possess a varying capacity for deformation. Under these conditions the applied stress produces different deformations of the grains. The weak particles will be more strongly deformed and will therefore carry a smaller load than they would have to carry if the load were uniformly distributed among all particles. This gives rise to an overstressing in the neighboring grains and leads to the additional deformation of the body.

As a result of his consideration of such a model of deformation of a polycrystalline body Becker arrived at the expression

$$\varepsilon = \frac{1}{E}\left[\sigma_0 + \lambda_0 \sum_{i=1}^{N} (\sigma_0 - \sigma_i)\right] \tag{1}$$

where σ_0 is the initial stress, uniform over all grains, σ_i the stress in the i^{th} grain at the time instant t, λ is a coefficient expressing the relation between the local deformation of the i^{th} grain and the macroscopic deformation of the body.

In computing the magnitude σ_i Becker assumes that the plastic weakening of the i^{th} particle is proportional to the stress σ_i and the time

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$$d\sigma_i = -\rho_{0i}\sigma_i\,dt \qquad (2)$$

where ρ_{0i} is a certain coefficient of slip (or a magnitude inverse to the internal friction of the material) for the i^{th} grain. In its dimensions ρ_{0i} is inversely proportional to the time and according to the Maxwell point of view $\tau_{0i} = \frac{1}{\rho_{0i}}$ should denote the relaxation time.

In applying the theory of Becker to dynamic loading, two observations must be made. The first one refers to the coefficient of slip ρ.

Numerous experimental investigations (refs. 2 and 3) of the effect of the rate of deformation on the microstructure of a polycrystalline metal show that high rates of deformation intensify the slip process in each of its grains. Furthermore, the phenomenon of twinning is known to occur under the action of variable stresses, impacts, and so forth. This indicates that the coefficient of slip ρ, which figures in the theory of Becker, must not be considered as constant but as a certain function of the rate of deformation: $\rho = \rho_0 f(|\dot{\varepsilon}|)$. It should be noted that the function $f(|\dot{\varepsilon}|)$ may characterize not only magnitude of the slip in the monocrystal but may formally include also the increase in the density of the slip zone on increasing the deformation rate.

For a clear representation of the dependence of the coefficient of slip on the velocity a simple, although rough, analogy may be drawn with dry friction. The slip in the crystal may be imagined as similar to motion with friction which, as is known, depends strongly on the velocity, the friction at rest being greater than the friction in motion. An analogous hypothesis had been earlier applied by Y. I. Frenkel (ref. 4) for the consideration of the problem of discontinuous creep which is observed on monocrystals at small stresses.

What are the physical causes which facilitate slip on increasing the rate of deformation? Under cyclic loads, as is known, hysteresis phenomena are always observed which indicate the dissipation of the energy of deformation. The amount of dissipated energy is expressed by the area of the hysteresis loop. Undoubtedly a certain part of this work is transformed into heat energy. At large rates of deformation, the heat energy does not have time to dissipate, so that a disturbance of the isothermal process is obtained and the specimen undergoes an increase in temperature.

The effect of static and dynamic compression on the diffusion of a K_α doublet was studied by N. Davidenkov and Y. Terminasov (ref. 5) who showed that the transition from static to impact loads leads to certain changes in the physical nature of the plastic deformation. This is expressed in the local rise in temperature over the slip planes.

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The possibility of a local rise in temperature over the slip planes is indicated also by the data of A. Stepanov (ref. 6) who, studying the plastic deformation of monocrystals of rock salt, showed that the temperature near the plane of slip may in the process of deformation even approach the melting point. For metals this effect, on account of their considerable thermal conductivity, should of course be less pronounced although at large deformation rates it may undoubtedly play a very important part.

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There should also be mentioned the effect of Krovs-Tanavski, investigated by N. Davidenkov and I. Mirolyubov (ref. 7), indicating a local increase in temperature up to austenite conversion during impact deformations of steel. It is thus possible that local increase in temperature is capable of facilitating the process of slip under dynamic loads.

The second remark concerns the coefficient λ_0. In the theory of Becker, this coefficient connects the deformation in the grain with the total deformation of the body. In the mathematical formulation of his theory Becker assumes that the grains of the metal do not mutually affect each other, that is, the metal represents the simple sum of its individual grains. As a matter of fact, the grains act on one another through the intercrystalline layers. The deformation of a polycrystalline body proceeds in an extremely complicated manner, not only within the grain, but also within the entire formation. On loading a polycrystal, in addition to the deformation within the crystal, a characteristic slip also takes place between the grains. Thus in the deformation of a real polycrystal the following processes occur: (1) deformation within the crystals or grains, (2) displacement of the crystals with respect to each other accompanied by the rupture of the structure and the partial rupture of the bond between the grains (ref. 8). It follows that the deformation of a polycrystalline body should be characterized by two coefficients. If the slip in the cyrstallites or grains, as indicated above, is characterized by the slip coefficient ρ, the intercrystalline slip should be characterized by the coefficient λ_0 expressing the relation between the deformation in the individual grain and the entire microscopic deformation of the specimen.

We shall attempt from this point of view to evaluate the coefficient λ_0. For this purpose let us consider the elementary act of deformation occurring under a constant load. As a result of the plastic deformation of the i^{th} grain and its weakening, the remaining grains receive an additional load of the order $(\sigma_0 - \sigma_i)S_i$, where S_i is the cross-sectional area of the i^{th} grain. Because of the action of this force, further deformation of the remaining N - 1 grains will take place. Becker assumes that this additional deformation is instantaneously established and obeys Hook's law, that is, the grains do not show on their boundaries any interaction on each other. This is a very rough approximation. As a

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matter of fact, the additional deformation under the action of the force $(\sigma_0 - \sigma_i)S_i$ will occur not instantaneously but after a certain finite time interval τ as though there existed some retarding viscous forces.

We shall make a qualitative estimate of the coefficient λ_0. The additional force $(\sigma_0 - \sigma_i)S_i$ produces a flow of the material. If K^o_{bd} is the internal friction of the material we may write

$$K^o_{bd}\overset{\cdot}{\varepsilon} \approx (\sigma_0 - \sigma_i)S_i$$

whence we find $\varepsilon \sim \frac{t}{K^o_{bd}}(\sigma_0 - \sigma_i)S_i$ or $\varepsilon \sim t_0\rho^o_{bd}(\sigma_0 - \sigma_i)S_i$ where ρ^o_{bd} characterizes the slip between the grains. As a result of the strengthening the flow does not continue for an infinitely long time but ceases and equilibrium is again restored. Thus $\varepsilon \sim \tau\rho^o_{bd}(\sigma_0 - \sigma_i)$ where τ is the time of flow.

On the other hand let us turn to equation (1). From the latter it follows that in plastic deformation of the i^{th} grain an additional deformation is obtained of the order

$$\varepsilon = \frac{\lambda_0}{E}(\sigma_0 - \sigma_i)$$

Comparing the two last equations we can conclude that $\lambda_0 \sim \rho^o_{bd}$, that is, λ_0 has the sense of a certain coefficient of slip. Earlier it was shown that the slip coefficient should depend on the rate of deformation. Hence the dynamic coefficient λ should be expressed by the relation $\lambda \sim \tau\rho^o_{bd}f(|\dot{\varepsilon}|)$ or $\lambda = \lambda_0 f(|\dot{\varepsilon}|)$, which was to be expected since ρ and λ are similar in their physical significance and characterize the processes of slip. The difference between them lies in the fact that ρ characterizes the intercrystalline slip while λ characterizes the intercrystalline slip.

In the light of what was said above the plastic unloading in the i^{th} grain should be expressed not by formula (2) but by the relation

$$d\sigma_{in} = -\rho_0 f(|\dot{\varepsilon}|)\sigma_i\,dt$$

With a varying load there must be added to the plastic change of stress in the i^{th} grain the further elastic change

$$d\sigma_{iy} = \frac{dP_i}{S_i} = d\sigma$$

where dP_i is the part of the total load assumed by the n^{th} grain. The total change of stress over the interval dt will be

$$d\sigma_i = -\rho_0 f(|\dot{\varepsilon}|)\sigma_i\, dt + \frac{dP_i}{S_i}$$

where $f(|\dot{\varepsilon}|)$, generally speaking, is a function of the time. Whence we obtain the required relation

$$\sigma(t) - \sigma_i(t) = \int_0^t \sigma(\zeta)\rho_0\, f(|\dot{\varepsilon}|)\, \exp \left[\int_0^\zeta \rho_0\, f(|\dot{\varepsilon}|)\, d\alpha - \int_0^t \rho_0\, f(|\dot{\varepsilon}|)\, d\beta\right] d\zeta$$

To compute the entire sum appearing in expression (1) it is necessary to be given the distribution function $F(\rho)$ for the coefficients $\rho = \rho_0\, f(|\dot{\varepsilon}|)$. Then

$$\varepsilon(t) = \frac{1}{E}\left[\sigma(t) + \lambda \int_0^\infty F[\rho_0\, f(|\dot{\varepsilon}|)] \int_0^t \sigma(\zeta)\rho_0\, f(|\dot{\varepsilon}|) \times \exp\left(\int_0^\zeta \rho_0\, f(|\dot{\varepsilon}|)d\alpha - \int_0 \rho_0\, f(|\dot{\varepsilon}|)d\beta\right)d\zeta d\rho\right]$$

To obtain the concrete expression $\varepsilon(t)$ we must know $F(\rho)$. Notwithstanding the fact that $F(\rho)$ uniquely determines the after-effect function, it is in no way determined by the experimentally observed after-effect function. There is therefore great arbitrariness in the choice of the function $F(\rho)$. As is shown by experiment, the after-effect function is of the order $1/\zeta$. In order to satisfy this condition, it is sufficient to take the distribution function in the form proposed by Becker:

$$F(\rho) = \frac{c}{\rho} \qquad r_0\, f(|\dot{\varepsilon}|) \leqslant \rho \leqslant Rf(|\dot{\varepsilon}|)$$

where r and R are the minimum and maximum coefficients of slip in the grains for the deformation rate $\dot{\varepsilon} = 0$.

Under these conditions we obtain the integral equation:

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$$\varepsilon(t) = \frac{1}{E}\left[\sigma(t) + \lambda \int_0^t \sigma(\zeta) \int_{rf(|\dot{\varepsilon}(t)|)}^{Rf(|\dot{\varepsilon}(t)|)} \frac{\bar{\rho}_0 Nf(|\dot{\varepsilon}(\zeta)|)}{(R - r)f(|\dot{\varepsilon}(t)|)} \times \right.$$

$$\left. \exp\left(\int_0^\zeta \rho_0 f(|\dot{\varepsilon}|)d\alpha - \int_0^t \rho_0 f(|\dot{\varepsilon}|)d\beta\right) d\rho \, d\zeta\right]$$

where $\bar{\rho}_0$ is the mean value of the slip coefficient over all the grains for the deformation rate $\dot{\varepsilon} = 0$.

Representing the periodic load of frequency ν in the form of a Fourier series we obtain the following expression:

$$\varepsilon(t) = \frac{1}{E}\left[\bar{\sigma} + \sum_{n=1}^{\infty}(\alpha_n \sin 2\pi\nu\, nt + b_n \cos 2\pi\nu\, nt) + \lambda\bar{\sigma}\int_0 \Phi(\zeta,t)d\zeta + \right.$$

$$\left. \lambda\sum_{n=1}^{\infty}\int_0^t(\alpha_n \sin 2\pi\nu\, n\zeta + b_n \cos 2\pi\nu\, n\zeta)\Phi(\zeta,t)d\zeta\right]$$

where $\Phi(\zeta,t)$ is the after-effect function. By solving this equation for ε, for the given relation $\sigma(t)$, the answer may be obtained to the question of the dynamic creep of a polycrystalline material.

It is not, however, possible to solve this equation fully. The matter is made still more complicated by the fact that the form of the function $f(|\dot{\varepsilon}|)$ giving the dependence of the slip coefficient on the deformation rate remains at the present time unknown. Hence, to obtain a concrete computational expression, methods must be found for the approximate solution of the equation.

The above relation gives the possibility of qualitatively investigating the character of the solution of this integral equation.

Evidently, the required solution $\varepsilon(t)$ will consist of the instantaneous deformation $\varepsilon_{\bar{\sigma}}$ due to the action $\bar{\sigma}$ and of the deformation $\varepsilon_{\bar{\sigma}}(t)$ due to the action of $\bar{\sigma}$. On these deformations will be superposed the periodic elastic deformation $\varepsilon^{*}_{el}(t)$ due to the action of the variable component of the external force, and finally a certain after-effect deformation $\varepsilon^{*}_{inel}(t)$ due to the same variable component. Thus

$$\dot{\varepsilon}(t) = \varepsilon^{+}_{\sigma} t \varepsilon^{-}_{\sigma}(t) + \varepsilon^{*}_{el}(t) + \varepsilon^{*}_{inel}(t)$$

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and the deformation rate will be:

$$\dot{\varepsilon}(t) = \dot{\varepsilon}^{-}_{\sigma}(t) + \dot{\varepsilon}^{*}_{el}(t) + \dot{\varepsilon}^{*}_{inel}(t)$$

The after-effects are extremely small effects. Hence the deformation rate $\dot{\varepsilon}^{*}_{el}(t)$ will be many times $(10^3 - 10^4)$ as large as $\dot{\varepsilon}^{-}_{\sigma}(t)$ and $\dot{\varepsilon}_{inel}(t)$. Whence it follows that $|\dot{\varepsilon}(t)|$ appearing in the integral equation may to a first approximation be replaced simply by the rate of elastic deformation, which is easily computed since the loading law is given. For simplifying the computations, $|\dot{\varepsilon}(t)|$ may also be replaced by the average rate of elastic deformation. Such approximation is especially good in that it permits obtaining a final computational expression without the explicit form of the function $f(|\dot{\varepsilon}|)$. Thanks to this simplification the integral equation for $\varepsilon(t)$ is obtained as a solution with respect to $\varepsilon(t)$, and the deformation rate $|\overline{\dot{\varepsilon}^{*}_{el}}|$ will enter as a parameter determined by the conditions of the experiment.

Replacing, for simplicity of computation, the lower limit of integration in the after-effect function $\Phi(\zeta,t)$ by zero and making a change of variable $\zeta = t - \zeta$, equation (3) may be transformed into the form:

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$$\varepsilon(t) = \frac{1}{E}\left\{\bar{\sigma} + \sum_{n=1}^{\infty} (a_n \sin 2\pi\nu\ nt + b_n \cos 2\pi\nu\ nt) + \right.$$

$$\frac{\lambda\bar{\rho}_0 N}{(R - r)}\ \bar{\sigma} \int_0^t \left[\frac{1 - e^{-Rf(\overline{|\dot{\varepsilon}|})\xi}}{\xi}\right] d\xi\ +$$

$$\frac{\lambda\bar{\rho}_0 N}{(R - r)} \sum_{n=1}^{\infty} \sin 2\pi\nu\ nt \int_0^t (a_n \cos 2\pi\nu\ n\xi + b_n \sin 2\pi\nu\ n\xi)$$

$$\left[\frac{1 - e^{-Rf(\overline{|\dot{\varepsilon}|})\xi}}{\xi}\right] d\xi\ -$$

$$\frac{\lambda\bar{\rho}_0 N}{(R - r)} \sum_{n=1}^{\infty} \cos 2\pi\nu\ nt \int_0^t (a_n \sin 2\pi\nu\ n\xi + b_n \cos 2\pi\nu\ n\xi$$

$$\left.\left[\frac{1 - e^{-Rf(\overline{|\dot{\varepsilon}|})\xi}}{\xi}\right] d\xi\right\}$$

For the comparison of experimental with theoretical data the obtained relation must be averaged over a period since the existing types of test machines do not determine any instantaneous deformations occurring in the course of a cycle but determine only an integral effect.

It may be shown that the last two integrals practically do not depend on the time since they very rapidly tend to their limiting values (they depend on the time only the first few seconds; the time is negligible in comparison with the duration of the creep test). Hence, the last two components after averaging over a period vanish, and the following expression is obtained:

$$\nu(t) = \frac{1}{E}\left\{\bar{\sigma} + \frac{\lambda\bar{\rho}_0 N}{R - r}\,\bar{\sigma}\int_0^t\left[\frac{1 - e^{-Rf(\overline{|\dot{\varepsilon}|})\xi}}{\xi}\right]d\xi\right\}$$

where

$$\bar{\sigma} = \nu\int_0^{1/\nu}\sigma(t)dt$$

Carrying out the integration and eliminating the instantaneous deformation yield

$$\varepsilon(t) = \frac{\lambda_0\bar{\rho}_0 N}{E(R - r)}\,\overline{f(|\dot{\varepsilon}|)}\bar{\sigma}\left[0.5772 + \ln Rf(\overline{|\dot{\varepsilon}|})t - Ei(- Rf(\overline{|\dot{\varepsilon}|})t)\right] \tag{4}$$

where $Ei(-\alpha t) = \int_\infty^{\alpha t} e^{-\xi}/\xi \, d\xi$ the values of which are obtained from tables.

The form of relation (4) resembles the expression obtained by Becker but differs from it in the presence of a rate factor.

The obtained relation explains the increase in the creep of a material under dynamic loads and likewise explains the experimentally observed increase in dynamic creep with increase in frequency and amplitude. In fact, both an increase in frequency and an increase in amplitude lead to an increase in the deformation rate and, as follows from the theory, the increase in the deformation rate should lead to an increase in dynamic creep.

In conclusion it is necessary to remark that the theory gives the possibility of obtaining also the temperature dependence of the dynamic creep if the corresponding temperature characteristics of the modulus of elasticity E and the relaxation time τ are explicitly given in the equation.

A discussion of the obtained results and their comparison with various experimental data will be presented in a subsequent paper.

Translated by S. Reiss
National Advisory Committee
for Aeronautics

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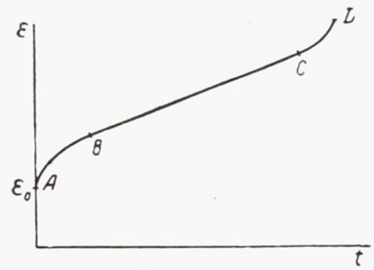

Figure 1.

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