NACA TM 1303

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1303

RESISTANCE OF CASCADE OF AIRFOILS IN GAS STREAM AT SUBSONIC VELOCITY

By L. G. Loitsianskii

Translation

Soprotivlenie reshetki profilei v gazovom potoke s dokriticheskimi skorostiami, Prikladnaia Matematika i Mekhanika, vol. XIII, no. 2, 1949.

NACA

Washington

September 1951

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1303

RESISTANCE OF CASCADE OF AIRFOILS IN GAS STREAM AT SUBSONIC VELOCITY*

By L. G. Loitsianskii

A method of computing the resistance of a cascade of airfoils in a viscous compressible gas flow is described.

The case of an incompressible gas is considered in reference 1 and appears herein only as a simple particular case of the general theory of resistance of a cascade in a compressible gas.

The investigation was restricted to subsonic velocities (that is, when the local velocity of sound is nowhere reached on the airfoil surface) because the required assumption of isentropic flow, that is, the absence of shock waves in any region of the motion, is valid only under these conditions.

The second reason for the restriction to relatively small values of Mach number is the possibility under this assumption of applying a lift formula analogous to the well-known Joukowsky formula (reference 2) and of thus assigning a definite meaning to the term "cascade resistance" or, more accurately, the "resistance of an airfoil in cascade."

The resistance formula can be derived for an isolated airfoil, as is known, by applying the momentum theorem between two parallel cross sections of the flow at an infinite distance upstream and downstream of the airfoil. In the problem of cascade resistance, difficulty is encountered, namely, the absence of an external potential flow downstream of the cascade where the boundary layers (wakes) from the individual airfoils merge. This essential difficulty, which is expressed quantitatively in the impossibility of employing the boundary-layer (wake) equation up to a plane at an infinite distance, can be circumvented by introducing the plane of merging of the boundary layers (wakes) and by establishing relations between the gas dynamic elements in this plane and in the plane at infinity downstream of the cascade.

---

*Soprotivlenie reshetki profilei v gazovom potoke s dokriticheskimi skorostiami, Prikladnaia Matematika i Mekhanika, vol. XIII, no. 2, 1949

An essential assumption of the present investigation is that a small degree of nonhomogeneity of the flow exists in the section of the aerodynamic wake of the cascade where the boundary layers from the individual airfoils, considered as layers of finite thickness, merge; the larger powers of the small velocity differences may then be neglected. The same assumption was made in the investigation of cascade resistance in an incompressible gas (reference 1) and was subsequently confirmed experimentally. The plane of merging of the boundary layers is then assumed to be the control surface required for the application of the momentum theorem and in the case of the isolated airfoil is taken to be the plane at an infinite distance downstream of the airfoil. It is evident that when the relative pitch of the cascade is increased, this plane will be farther and farther away from the axial plane of the cascade and in the limit, for a relative pitch equal to infinity, that is, in the case of an isolated airfoil, will go to infinity. This assumption may evidently be made for cascades with moderate solidities, a case that corresponds in practice to turbine and compressor cascades.

Any method of calculating the boundary layer in a compressible gas may be used to compute the characteristic thicknesses of the layer and to estimate the effect of the compressibility of the gas on the external flow. The solution of the proposed problem reduces to a straightforward and direct form that is independent of the method of computation.

1. Resistance of airfoil in cascade. Joukowsky force as component of total force exerted by incompressible fluid on airfoil. - For two-dimensional flow of a real fluid, the resistance (or drag) of an isolated cylindrical wing of infinite span referred to unit length of the wing is the component of the total force exerted by the fluid on the wing in the direction of the velocity of the approaching flow at infinity, or, in other words, of the velocity component of motion of the wing in an incompressible medium.

This definition is invalid in the case of an airfoil in a two-dimensional cascade, because in this more general case there is no unique velocity direction at infinity upstream and downstream of the cascade and there are no considerations by which preference is to be given to any particular direction for determining in this direction the resistance component of the total force acting on the wing. In this case, the problem is to determine what may be termed resistance.

An isolated wing of finite span is now considered. In this case, as also in the case of an airfoil in cascade, for each section of the wing, in view of the presence of vortex systems (films) shed from the wing and passing downstream to infinity, two velocities different in magnitude and direction exist at infinity upstream and downstream of the wing. For ideal flow about a wing of finite span in accordance with the scheme of lifting lines, the total pressure force of the flow at a given section of the wing is known to be perpendicular to the velocity of the flow at the corresponding point of the section under consideration on the lifting line. This velocity, which represents half the vector sum of the velocities at infinity upstream and downstream of the wing, is assumed at the section considered as the effective velocity of flow; the angle between the chord of the wing section and the direction of the effective velocity is considered as the effective angle of attack, and so forth.

For a two-dimensional infinite cascade of airfoils, a similar assumption is made with the difference only that in the theory of the wing of finite span the effective velocity differs slightly from the velocity of the approaching flow; whereas in the case of the cascade the jump is of the same order as the geometrical angle of attack.

In the aerodynamics of a wing of finite span, the profile drag is the difference between the head resistance, which is represented by the component of the total force exerted by the real (viscous) gas flow in the direction of the velocity at infinity upstream of the wing, and the induced drag, which is the component in the same direction of the effective lift force.

For a small difference between the directions of the effective velocity and the velocity of the approaching flow, this definition of the profile drag of a wing section differs by small terms of higher order from the true profile drag, strictly defined as the vector difference between the total force exerted by the real flow on the wing section and the effective lift force for a real fluid.

In the case of the two-dimensional cascade, it is natural to assume for the profile drag R' the difference between the vector of the total force R (fig. 1) and the Joukowsky force R_j (in the terminology of reference 1) which for an incompressible gas is given by

$$R_j = \rho V_m \Gamma \tag{1.1}$$

acting in the direction perpendicular to the fictitious velocity at infinity V_m determined as

$$V_m = \frac{1}{2} (V_{1\infty} + V_{2\infty}) \tag{1.2}$$

where $V_{1\infty} (u_{1\infty}, v_{1\infty})$ and $V_{2\infty} (u_{2\infty}, v_{2\infty})$ are the vector velocities at infinity upstream and downstream of the cascade, ρ is the density of the fluid, and Γ is the circulation determined by the equation

$$\Gamma = (v_{2\infty} - v_{1\infty})t \tag{1.3}$$

where t is the pitch of the cascade.

Introducing the concept of the vector pitch t, which is equal in length to the magnitude of the pitch t and directed at right angles to the axis of the cascade downstream of the flow, gives the Joukowsky force by the following vector equation (reference 2):

$$R_j = \rho V_m \times (t \times V_d) \tag{1.4}$$

where the following vector

$$V_d = V_{2\infty} - V_{1\infty} \tag{1.5}$$

gives the vector change of velocity produced by the cascade. The preceding formulas are valid not only for the flow of an ideal incompressible fluid but also for a viscous fluid.

The profile drag R' is as follows (reference 1):

$$R' = p't \tag{1.6}$$

where p' is the pressure loss in the cascade determined by the equation

$$p' = \left(p_{1\infty} + \frac{1}{2}\rho V_{1\infty}^2\right) - \left(p_{2\infty} + \frac{1}{2}\rho V_{2\infty}^2\right)$$
$$= \left(p_{1\infty} + \frac{1}{2}\rho v_{1\infty}^2\right) - \left(p_{2\infty} + \frac{1}{2}\rho v_{2\infty}^2\right) \tag{1.7}$$

The total force R is equal to the sum

$$R = R_j + R' = \rho V_m \times (t \times V_d) + p't \tag{1.8}$$

2. Resistance of two-dimensional cascade in real gas flow at subsonic velocities. - The expression for the total force R of the interaction of the flow with a two-dimensional cascade at subsonic velocities may be represented in the following form (reference 2):

$$R = (p_{1\infty} - p_{2\infty})t + \rho_{1\infty}(V_{1\infty} \cdot t)V_{1\infty} - \rho_{2\infty}(V_{2\infty} \cdot t)V_{2\infty} \tag{2.1}$$

where $p_{1\infty}$, $\rho_{1\infty}$ and $p_{2\infty}$, $\rho_{2\infty}$ are the pressures and densities upstream and downstream of the cascade, respectively. The equivalent expressions

$$\rho_{1\infty} V_{1\infty} \cdot t = \rho_{2\infty} V_{2\infty} \cdot t \tag{2.2}$$

evidently express the rate of mass flow per second through the section of the flow parallel to the axis of the cascade and equal in length to the pitch.

As was shown (reference 2) also in the case of a compressible gas for Mach numbers not too near unity, the lift force of an airfoil in cascade in an ideal gas flow may be represented in the form of equation (1.4), provided that for the density ρ is taken the arithmetical mean density ρ_m equal to

$$\rho_m = \frac{1}{2}(\rho_{1\infty} + \rho_{2\infty}) \tag{2.3}$$

The following approximate expression of the Bernoulli theorem is employed:

$$p_{1\infty} - p_{2\infty} = \rho_m V_m \cdot V_d = \frac{1}{2}\rho_m (V_{2\infty}^2 - V_{1\infty}^2) \tag{2.4}$$

This equation is valid with an accuracy to tenth parts of the square of the difference of squares of the Mach numbers at infinity upstream and downstream of the cascade.

In the case of the real (compressible and viscous) gas,

$$p_{1\infty} - p_{2\infty} = \frac{1}{2} \rho_m \left(V_{2\infty}^2 - V_{1\infty}^2\right) + p' \tag{2.5}$$

where p' characterizes the losses in the cascade due to the internal friction in the gas; an equation may be obtained (reference 2) analogous to equation (1.8)

$$R = R_j + R' = \rho_m V_m \times (t \times V_d) + p't \tag{2.6}$$

where p' is determined by the expression

$$p' = p_{1\infty} - p_{2\infty} - \frac{1}{2} \rho_m \left(V_{2\infty}^2 - V_{1\infty}^2\right) \tag{2.7}$$

The problem of determining the profile drag force R' equal to

$$R' = p't \tag{2.8}$$

thus reduces to finding the losses p' which depend on the shape of the airfoil in the cascade and the character of the flow about the airfoil.

3. Introduction of intermediate plane; relation between gas dynamic elements in this plane and corresponding values at infinite distance from cascade. - In addition to the planes 1∞ and 2∞ that were employed in the analysis of the incompressible fluid, (reference 1) an intermediate plane 2 is introduced for the compressible gas (fig. 2); plane 2 is located where the boundary layers (wakes) from the individual airfoils merge. The hydrodynamic and thermal boundary layers in the wake downstream of the cascade are hereinafter assumed to have the same pattern.

The following assumptions with regard to the motion of the gas near plane 2 are necessary: By definition of the position of plane 2, no individual boundary layers exist in the flow downstream of this plane; the aerodynamic and thermal wakes of the airfoils are, however, maintained and depressions in the velocity or total-pressure curves and also depressions or peaks in the temperature curves result. A fundamental property of the boundary layer is that the pressure transverse to the wake is the same at all points of a given normal section of the wake; that is, no pressure drop in the distribution curve occurs in this section. The pressure along the wake changes sharply in the immediate neighborhood of the trailing edge of the airfoil and is gradually equalized as the distance from the trailing edge is increased.

Two sections of the wake are passed through the point of intersection of plane 2 with the axis of the wake; one section lies in plane 2 and includes the y-axis (fig. 3), and the second section lies in plane 2' normal to the axis of the wake and includes the y'-axis.

The following magnitudes are introduced:

$$\left.\begin{aligned}
\Delta_u &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{u_2 - u}{u_2}\,dy \\
\Delta_\rho &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{\rho_2 - \rho}{\rho_2}\,dy \\
\Delta_v &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{v_2 - v}{v_2}\,dy \\
\Delta_p &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{p_2 - p}{p_2}\,dy \\
\Delta_V &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{V_2 - V}{V_2}\,dy \\
\Delta_T &= \frac{1}{t}\int_{y_0}^{y_0+t} \frac{T_2 - T}{T_2}\,dy
\end{aligned}\right\} \qquad (3.1)$$

which characterize the mean relative deviations of the hydrodynamic elements of the flow at the points of section 2 of the wake from the values of these elements at the boundaries of the wake at the points of intersection of the boundary layers.

Section 2 will be assumed at such distance from the cascade that the differences $u_2 - u$, . . ., and also their mean relative values Δ_u, . . . may be considered small magnitudes, the higher powers and the products of which may be neglected. Moreover, the velocities at different points of section 2 are assumed parallel and in a general direction coinciding with that of the velocity at infinity behind the cascade. It follows at once that

$$\Delta_u = \Delta_v = \Delta_V \tag{3.2}$$

Comparison with analogous mean relative deviations in section 2' gives the magnitudes

$$\Delta_u' = \frac{1}{t'}\int_{y_0}^{y_0'+t'} \frac{u_2' - u'}{u_2'}\, dy', \ . \ . \ . \qquad (t' = t \cos \beta_{2\infty}) \tag{3.3}$$

In the subsequent discussion, it will be assumed that, for a sufficient distance of planes 2 and 2' from the axial plane of the cascade, all the magnitudes (3.3) and so forth are correspondingly equal to the magnitudes (3.1); that is,

$$\Delta_u' = \Delta_u \qquad \Delta_\rho' = \Delta_\rho \ . \ . \ . \tag{3.4}$$

This additional assumption may be justified as a consequence of the assumption of a small degree of variation of the gas dynamic elements near plane 2 and behind it downstream of the flow.

In accordance with the fundamental property of the wake $\Delta_p' = 0$, the following equation may be obtained:

$$\Delta_p = 0 \tag{3.5}$$

Because of the smallness of the magnitudes Δ_u, Δ_ρ, . . . , the gas dynamic magnitudes in the intermediate plane 2 are easily shown to be connected with the corresponding values of these magnitudes in the plane 2∞ by relations that are analogous to the case of the incompressible gas.

For this purpose, a segment of a flow tube is assumed between sections 2 and 2∞, where a length equal to the pitch t is taken for the transverse dimension of the tube in the direction parallel to the axis of the cascade.

Application of the theorem of the conservation of mass then yields

$$\int_{y_0}^{y_0+t} \rho u \, dy = \int_{y_0}^{y_0+t} \left[\rho_2 - (\rho_2 - \rho)\right]\left[u_2 - (u_2 - u)\right] dy = \rho_{2\infty} u_{2\infty} t$$

Expanding the brackets and neglecting the product $(\rho_2 - \rho)(u_2 - u)$ as a small quantity of higher order gives the following equation:

$$\int_{y_0}^{y_0+t} \left[\rho_2 u_2 - u_2(\rho_2 - \rho) - \rho_2(u_2 - u)\right] dy = \rho_{2\infty} u_{2\infty} t$$

From this expression, the following relation is obtained in the notation of equation (3.1):

$$\rho_2 u_2 (1 - \Delta_\rho - \Delta_u) = \rho_{2\infty} u_{2\infty}$$

or, with the same degree of accuracy,

$$\rho_2 u_2 = \rho_{2\infty} u_{2\infty} (1 + \Delta_\rho + \Delta_u) \tag{3.6}$$

The momentum theorem in the projection on the x-axis applied to the same segment of the flow tube gives

$$\int_{y_0}^{y_0+t} p\,dy + \int_{y_0}^{y_0+t} \rho u^2 dy - p_{2\infty} t - \rho_{2\infty} u_{2\infty}^2 t = 0$$

This equation may be written in the form

$$\int_{y_0}^{y_0+t} \left[p_2 - (p_2 - p)\right] dy + \int_{y_0}^{y_0+t} \left[\rho_2 - (\rho_2 - \rho)\right] \left[u_2 - (u_2 - u)\right] dy$$

$$= p_{2\infty}t + \rho_{2\infty}u_{2\infty}^{2}t$$

If the smallness of the differences $p_2 - p$, $\rho_2 - \rho$, and $u_2 - u$ is taken into account, the following expression may be obtained:

$$p_2(1 - \Delta_p) + \rho_2 u_2^2(1 - \Delta_\rho - 2\Delta_u) = p_{2\infty} + \rho_{2\infty}u_{2\infty}^2 \tag{3.7}$$

With the aid of equations (3.5) and (3.6) and the same approximation, the following equation may be written:

$$p_2 + \rho_{2\infty}u_{2\infty}u_2(1 - \Delta_u) = p_{2\infty} + \rho_{2\infty}u_{2\infty}^2 \tag{3.8}$$

The mementum theorem is now applied in the projection to the y-axis, which gives

$$\int_{y_0}^{y_0+t} \rho uv \; dy = \rho_{2\infty}u_{2\infty}v_{2\infty}t$$

or

$$\frac{1}{t}\int_{y_0}^{y_0+t} \left[\rho_2 - (\rho_2 - \rho)\right] \left[u_2 - (u_2 - u)\right] \left[v_2 - (v_2 - v)\right] dy = \rho_{2\infty}u_{2\infty}v_{2\infty}$$

Rejection of small terms of higher order leads to the equation

$$\rho_2 u_2 v_2(1 - \Delta_\rho - \Delta_u - \Delta_v) = \rho_{2\infty}u_{2\infty}v_{2\infty}$$

or, according to equation (3.6),

$$\left.\begin{aligned} v_2(1 - \Delta_v) &= v_{2\infty} \\ v_2 &= v_{2\infty}(1 + \Delta_v) \end{aligned}\right\} \tag{3.9}$$

The assumption of parallel directions of the velocity vectors in sections 2 and 2∞, with the aid of equation (3.2), yields

$$\left.\begin{aligned} u_2 &= u_{2\infty}(1 + \Delta_u) \\ V_2 = V_{2\infty}(1 + \Delta_V) &= V_{2\infty}(1 + \Delta_u) \end{aligned}\right\} \tag{3.10}$$

Equation (3.8) then gives

$$p_2 = p_{2\infty} \tag{3.11}$$

On the basis of equation (3.10), there also follows from equation (3.6)

$$\rho_2 = \rho_{2\infty}(1 + \Delta_\rho) \tag{3.12}$$

Finally, from the Clapeyron equation,

$$\frac{p}{\rho} = \frac{p_2 - (p_2 - p)}{\rho_2 - (\rho_2 - \rho)} = RT = R\left[T_2 - (T_2 - T)\right]$$

or, when the smallness of the differences is accounted for,

$$\frac{p_2}{\rho_2}\left(1 - \frac{p_2 - p}{p_2} + \frac{\rho_2 - \rho}{\rho_2}\right) = RT_2\left(1 - \frac{T_2 - T}{T_2}\right)$$

From this equation, $\Delta_p - \Delta_\rho = \Delta_T$ is obtained by integration, or,

$$\Delta_\rho = - \Delta_T \tag{3.13}$$

Conversely, the same Clapeyron equation in planes 2 and 2∞ yields, by equation (3.11)

$$R(T_{2\infty} - T_2) = \frac{p_{2\infty}}{\rho_{2\infty}} - \frac{p_2}{\rho_2} = \frac{p_{2\infty}}{\rho_{2\infty}} \frac{\rho_2 - \rho_{2\infty}}{\rho_2} = \frac{p_{2\infty}}{\rho_{2\infty}} \Delta_\rho$$

or, by equation (3.13),

$$T_{2\infty} - T_2 = T_{2\infty}\Delta_\rho = - T_{2\infty}\Delta_T$$

that is,

$$T_2 = T_{2\infty}(1 + \Delta_T) \tag{3.14}$$

4. Relation of fictitious wake thicknesses to magnitudes Δ and Δ'. Expression of profile drag in terms of fictitious wake thicknesses. - The momentum equation in the wake behind the airfoil of the cascade will now be employed; the equation contains the following fictitious wake thicknesses defined (reference 3) as integrals over section 2': displacement thickness $\delta_2{}^*$ and loss-of-momentum thickness $\delta_2{}^{**}$,

$$\left.\begin{aligned} \delta_2{}^* &= \int_{y_0'}^{y_0'+t'} \left(1 - \frac{\rho V}{\rho_2 V_2}\right) dy' \\ \delta_2{}^{**} &= \int_{y_0'}^{y_0'+t} \frac{\rho V}{\rho_2 V_2} \left(1 - \frac{V}{V_2}\right) dy' \end{aligned}\right\} \tag{4.1}$$

When these thicknesses are connected with the magnitudes Δ,

$$\left.\begin{aligned}
\delta_2{}^* &= \int_{y_0'}^{y_0'+t} \left\{1 - \frac{\left[\rho_2 - (\rho_2 - \rho)\right]\left[V_2 - (V_2 - V)\right]}{\rho_2 V_2}\right\} dy' \\
&= \int_{y_0'}^{y_0'+t} \frac{\rho_2 - \rho}{\rho_2}\, dy' + \int_{y_0'}^{y_0'+t'} \frac{V_2 - V}{V_2}\, dy' \\
&= t'(\Delta_\rho' + \Delta_u') = (\Delta_\rho + \Delta_u)t \cos \beta_{2\infty}
\end{aligned}\right\} \tag{4.2}$$

$$\left.\begin{aligned}
\delta_2{}^{**} &= \int_{y_0'}^{y_0'+t'} \frac{\left[\rho_2 - (\rho_2 - \rho)\right]\left[V_2 - (V_2 - V)\right]}{\rho_2 V_2} \frac{V_2 - V}{V_2}\, dy' \\
&\approx t'\Delta_V' = t\Delta_u \cos \beta_{2\infty}
\end{aligned}\right\} \tag{4.3}$$

The profile drag will now be determined; the magnitude p' must first be found. In equation (2.7), p' is expressed as a small difference between two large magnitudes and is therefore unsuitable either for experimental or for approximate theoretical determination of p'. In order to eliminate this defect, equation (2.7) is rewritten in the form

$$p' = p_{1\infty} - p_{2\infty} - \frac{1}{4}\left(\rho_{1\infty} + \rho_{2\infty}\right)\left(V_{2\infty}{}^2 - V_{1\infty}{}^2\right) \tag{4.4}$$

and the flow is considered between section 1∞ and the limits of the boundary layers that merge in plane 2. In this entire region, the flow is nonvortical so that the Bernoulli theorem may be applied without the additional term that accounts for such losses. The following expression similar to equation (2.4) may then be written:

$$p_{1\infty} - p_2 = \frac{1}{4}\left(\rho_{1\infty} + \rho_2\right)\left(V_2{}^2 - V_{1\infty}{}^2\right) \tag{4.5}$$

Because by equation (3.11) $p_2 = p_{2\infty}$, the following expression is obtained when equation (4.5) is compared with equation (4.4):

$$p' = \frac{1}{4} (\rho_{1\infty} + \rho_2) (V_2^2 - V_{1\infty}^2) - \frac{1}{4} (\rho_{1\infty} + \rho_{2\infty}) (V_{2\infty}^2 - V_{1\infty}^2)$$

When V_2 and ρ_2 are replaced by their expressions in terms of $V_{2\infty}$ and $\rho_{2\infty}$, then according to equations (3.10) and (3.12),

$$p' = \frac{1}{4} (\rho_{1\infty} + \rho_{2\infty} + \rho_{2\infty}\Delta_\rho) \left[V_{2\infty}^2(1 + 2\Delta_V) - V_{1\infty}^2\right] - \frac{1}{4} (\rho_{1\infty} + \rho_{2\infty}) (V_{2\infty}^2 - V_{1\infty}^2)$$

$$= \frac{1}{2} (\rho_{1\infty} + \rho_{2\infty}) V_{2\infty}^2\Delta_u + \frac{1}{4} \rho_{2\infty}(V_{2\infty}^2 - V_{1\infty}^2) \Delta_\rho$$

or, by equations (4.2) and (4.3),

$$p' = \rho_m V_{2\infty}^2 \frac{\delta_2^{**}}{t \cos \beta_{2\infty}} + \frac{1}{4} \rho_{2\infty}(V_{2\infty}^2 - V_{1\infty}^2) \frac{\delta_2^* - \delta_2^{**}}{t \cos \beta_{2\infty}} \tag{4.6}$$

The following magnitude is now introduced:

$$H_2 = \frac{\delta_2^*}{\delta_2^{**}} = \frac{\Delta_u + \Delta_\rho}{\Delta_u} = 1 + \frac{\Delta_\rho}{\Delta_u} \tag{4.7}$$

which is for the case of the motion of an incompressible gas; the following simple equation is then obtained:

$$p' = \left[\rho_m V_{2\infty}^2 + \frac{1}{4} (H_2 - 1) \rho_{2\infty}(V_{2\infty}^2 - V_{1\infty}^2)\right] \frac{\delta_2^{**}}{t \cos \beta_{2\infty}} \tag{4.8}$$

The formula for profile drag is immediately obtained from equation (4.8),

$$R' = p't = \left[\rho_m V_{2\infty}^2 + \frac{1}{4} (H_2 - 1) \rho_{2\infty}(V_{2\infty}^2 - V_{1\infty}^2)\right] \frac{\delta_2^{**}}{\cos \beta_{2\infty}} \tag{4.9}$$

From this expression, the profile-drag formulas for a cascade in an incompressible viscous fluid are obtained as particular cases and for the isolated airfoil in the general case.

For the case of a cascade in an incompressible fluid (ρ = constant),

$$\rho_m = \rho$$

$$\Delta_\rho = 0$$

$$H_2 = 1$$

and equations (4.8) or (4.9) are converted into

$$\left.\begin{aligned} p' &= \frac{\rho V_{2\infty}\delta_{2\infty}^{**}}{t \cos \beta_{2\infty}} \\ R' &= \frac{\rho V_{2\infty}^{2}\delta_{2\infty}^{**}}{\cos \beta_{2\infty}} \end{aligned}\right\} \tag{4.10}$$

which are identical to equations (2.12) of reference 1.

For an isolated airfoil in a viscous compressible fluid,

$$\rho_{1\infty} = \rho_{2\infty} = \rho_m = \rho_\infty$$

$$V_{1\infty} = V_{2\infty} = V_\infty$$

$$\beta_{2\infty} = 0$$

Moreover, plane 2 extends to infinity, so that

$$R' = \rho_\infty V_\infty^{2}\delta_\infty^{**} \tag{4.11}$$

Equation (4.11) is the well-known formula of the resistance theory for an isolated airfoil.

The losses and the profile drag of the cascade are expressed by equations (4.8) and (4.9) in terms of known elements at infinity ahead of and behind the cascade and in terms of the elements H_2 and δ_2^{**} referred to plane 2, the position of which remains unknown, because up to the present no reliable theory of the turbulent wake exists.

A formula will now be obtained for the profile resistance of the cascade; by the theory of the boundary layer at the airfoil, this expression makes possible the computation of the resistance of the cascade, and the dependence of the magnitudes H_2 and δ_2^{**} just mentioned on the elements of the boundary layer at the rear edge of the airfoil of the cascade can therefore be determined.

5. Establishment of relations between wake elements in section 2 and boundary-layer elements in sections at trailing edge. - A generalization is given herein of the known device of setting up relations between the elements of the boundary layer at the trailing edge of the airfoil and in the wake behind it at infinity, as proposed for the case of the isolated airfoil in the incompressible fluid by reference 4.

In this generalization, for the case of the cascade the section at the trailing edge is connected not with the plane at an infinite distance downstream of the flow but with plane 2 of the merging of the boundary layers or, more accurately, with plane 2' inclined to it by the angle $\beta_{2\infty}$. Moreover, the generalization requires passing to the compressible case.

The momentum equation for the wake behind a body may easily be derived from the general equations of the plane boundary layer in a compressible gas

$$\left.\begin{aligned} \rho V_s \frac{\partial V_s}{\partial s} + \rho V_n \frac{\partial V_s}{\partial n} &= -\frac{dp}{ds} + \frac{\partial \tau}{\partial n} \\ \frac{\partial(\rho V_s)}{\partial s} + \frac{\partial(\rho V_n)}{\partial n} &= 0 \end{aligned}\right\} \tag{5.1}$$

where for the longitudinal (coordiante s) and transverse (coordinate n) projections of the velocities, the symbols V_s and V_n are used in contrast to the velocity projections u and v connected with the axes Ox and Oy; ρ is the local density, τ the friction stress, and p the pressure on the outer boundary of the layer.

By rewriting the system (5.1) in the following form, according to the second of equations (5.1) and the general Bernoulli equation,

$$\left.\begin{aligned} \frac{\partial}{\partial s}(\rho V_s V_s) + \frac{\partial}{\partial n}(\rho V_s V_n) &= \overline{\rho}\overline{V}_s \frac{d\overline{V}_s}{ds} + \frac{\partial \tau}{\partial n} \\ \frac{\partial(\rho V_s)}{\partial s} + \frac{\partial(\rho V_n)}{\partial n} &= 0 \end{aligned}\right\} \tag{5.2}$$

where $\overline{\rho}$ and $\overline{V}_s$ denote the density and the longitudinal velocity at the outer limit of the boundary layer. Both sides of the second equation are then multiplied by $\overline{V}_s$ to yield

$$\frac{\partial}{\partial s}(\rho\overline{V}_s V_s) + \frac{\partial}{\partial n}(\rho\overline{V}_s V_n) - \rho V_s \frac{d\overline{V}_s}{ds} = 0$$

The first of equations (5.2) is then subtracted term by term from the equation just obtained; the resulting equation is then integrated, which gives

$$\frac{\partial}{\partial s}\left[\rho V_s(\overline{V}_s - V_s)\right] + \frac{\partial}{\partial n}\left[\rho V_n(\overline{V}_s - V_s)\right] + (\overline{\rho}\overline{V}_s - \rho V_s)\frac{d\overline{V}_s}{ds} = -\frac{\partial \tau}{\partial n}$$

along the normal to the section of the wake, which is considered either infinite in the usual sense of the theory of asymptotic boundary layer or finite, as is assumed in the theory of the finite thickness layer. In either case, the following relation holds:

$$\frac{d}{ds}\int_{-\infty,\delta}^{+\infty,\delta} \rho V_s(\overline{V}_s - V_s)\, dn + \frac{d\overline{V}_s}{ds}\int_{-\infty,\delta}^{+\infty,\delta} (\overline{\rho}\overline{V}_s - \rho V_s)\, dn = 0$$

The following expression is then obtained:

$$\frac{d}{ds}\left[\overline{\rho}\overline{V}_s{}^2 \int_{-\infty,\delta}^{+\infty,\delta} \frac{\rho V_s}{\overline{\rho}\overline{V}_s}\left(1 - \frac{V_s}{\overline{V}_s}\right) dn\right] + \overline{\rho}\overline{V}_s \frac{d\overline{V}_s}{ds}\int_{-\infty,\delta}^{+\infty,\delta}\left(1 - \frac{\rho V_s}{\overline{\rho}\overline{V}_s}\right) dn = 0$$

By expanding the parentheses and introducing the notation of reference 4,

$$\left.\begin{aligned} \delta^* &= \int_{-\infty,\delta}^{+\infty,\delta} \left(1 - \frac{\rho V_s}{\overline{\overline{\rho V}}_s}\right) dn \\ \delta^{**} &= \int_{-\infty,\delta}^{+\infty,\delta} \frac{\rho V_s}{\overline{\overline{\rho V}}_s} \left(1 - \frac{V_s}{\overline{\overline{V}}_s}\right) dn \end{aligned}\right\} \tag{5.3}$$

the required momentum equation is finally obtained.

$$\frac{\partial \delta^{**}}{ds} + \left(\frac{2}{\overline{\overline{V}}} \frac{d\overline{V}}{ds} + \frac{1}{\overline{\rho}} \frac{d\overline{\rho}}{ds}\right) \delta^{**} + \frac{1}{\overline{\overline{V}}} \frac{d\overline{V}}{ds} \delta^* = 0 \tag{5.4}$$

In such form, the momentum equation for the compressible gas differs from the corresponding equation for the incompressible gas only in the term $\overline{\rho}^{-1} d\overline{\rho}/ds$ (and, of course, in the definitions of the magnitudes δ^* and δ^{**}). If the momentum equation for the incompressible gas is considered for the case of axial symmetrical motion, the term $\overline{\rho}^{-1} d\overline{\rho}/ds$, which expresses the effect of the variable density of the gas, may be taken equivalent to the term that takes into account the transverse curvature.

In addition to the momentum equation, the heat equation is considered; it can be easily set up by a method analogous to the preceding method from the known heat equation of the boundary layer.

$$\left(\rho V_s \frac{\partial}{\partial s} + \rho V_n \frac{\partial}{\partial n}\right) \left(i + \frac{V_s}{2}\right) = \frac{\partial q}{\partial n} \tag{5.5}$$

where $\sigma = \mu c_p/\lambda$ is the Prandtl number.

$$i = Jc_pT$$

$$q = \mu \frac{\partial}{\partial n}\left(\frac{i}{\sigma} + \frac{V_s^2}{2}\right)$$

The value of q is given in the case of the laminar boundary layer; equation (5.5) holds also for the turbulent layer, but in this case q would be expressed in a different form.

The so-called temperature of adiabatic stagnation T^* is now considered; it is given by

$$T^* = T + \frac{{V_s}^2}{2Jc_p} \tag{5.6}$$

By means of the continuity equation, the following system of equations may be set up:

$$\left.\begin{aligned}\frac{\partial}{\partial s}(\rho V_s T^*) + \frac{\partial}{\partial n}(\rho V_n T^*) &= \frac{1}{Jc_p}\frac{\partial q}{\partial n}\\ \frac{\partial}{\partial s}(\rho V_s \overline{T}^*) + \frac{\partial}{\partial n}(\rho V_u \overline{T}^*) &= 0\end{aligned}\right\} \tag{5.7}$$

In the second equation of the system, the stagnation temperature $\overline{T}^*$ at the outer limit of the boundary layer, which is constant (because the external flow is isentropic), is taken under the sign of the derivatives in the continuity equation.

Subtracting one equation of the system (5.7) from the other and successively integrating over the cross section of the wake gives

$$\left.\begin{aligned}\frac{d}{ds}\int_{-\infty,\delta}^{+\infty,\delta} \rho V_s(\overline{T}^* - T^*)dn &= 0\\ \int_{-\infty,\delta}^{+\infty,\delta} \rho V_s(\overline{T}^* - T^*)dn &= \text{constant}\end{aligned}\right\} \tag{5.8}$$

The fictitious thickness of the wake is now introduced

$$\theta = \int_{-\infty,\delta}^{+\infty,\delta} \frac{\rho V_s}{\overline{\rho V_s}}\left(1 - \frac{T^*}{\overline{T}^*}\right)dn \tag{5.9}$$

which may be termed the thickness of the energy loss; equation (5.8) may then be rewritten as

$$\overline{\rho V T^*}\theta = \text{constant} \tag{5.10}$$

Equation (5.4) is again considered. After each side is divided by δ^{**}, the expression is integrated along the wake from section k at the trailing edge of the airfoil to plane 2', previously defined. The result is

$$\ln\left(\frac{\delta_2^{**}}{\delta_k^{**}}\right) = \ln\left(\frac{\rho_k V_k^2}{\rho_2 V_2^2}\right) - \int_{(k)}^{(2)} \frac{\delta^*}{\delta^{**}} \frac{d \ln V_s}{ds} ds \tag{5.11}$$

The notation of equation (4.7) is used for the ratio of the fictitious wake thicknesses,

$$H = \frac{\delta^*}{\delta^{**}} \tag{5.12}$$

and it is noted that equation (5.11) is integrated to completion if the magnitude H is replaced by some average value; for example, the following relation may be set up:

$$H = H_{cp} = \frac{1}{2}(H_2 + H_k) \tag{5.13}$$

By this simplification, the following expression is immediately obtained:

$$\ln\left(\frac{\delta_2^{**}}{\delta_k^{**}}\right) = \ln\left(\frac{\rho_k V_k^2}{\rho_2 V_2^2}\right) - \frac{1}{2}(H_2 + H_k) \ln \frac{V_2}{V_k}$$

or finally,

$$\frac{\delta_2^{**}}{\delta_k^{**}} = \frac{\rho_k}{\rho_2}\left(\frac{V_k}{V_2}\right)^{2 + \frac{1}{2}(H_2+H_k)} \tag{5.14}$$

This equation connects δ_2^{**} and δ_k^{**}, but does not explicitly contain δ_2^*; the exponent on the right contains the magnitude H_2, which is equal to the ratio δ_2^*/δ_2^{**}. From equations (4.8) and (4.9) previously derived, equation (5.14) serves as one of the equations for expressing the two unknowns δ_2^{**} and H_2 entering in the equations for the losses in the cascade and the resistance in terms of the elements of the boundary layer at the trailing edge of the airfoil.

The second equation is obtained by use of equation (5.10), which may be rewritten as follows:

$$\rho_2 V_2 T_2^* \theta_2 = \rho_k V_k T_k^* \theta_k$$

or, because of the isentropic character of the motion outside the boundary layer, $T_2^* = T_k^*$; the expression then becomes

$$\rho_2 V_2 \theta_2 = \rho_k V_k \theta_k \tag{5.15}$$

In this equation a new unknown quantity θ_2 appears to enter; because of the small degree of nonhomogeneity of the fields of hydrodynamic elements in planes 2 or 2', however, this term can actually be expressed in terms of the previous unknowns. When the small degree of nonhomogeneity and the formulas relating the elements in planes 2 and 2∞ (derived in section 3) are accounted for, equation (5.14) and then equation (5.15) are transformed. By equation (5.14),

$$\left.\begin{aligned}\frac{\delta_2^{**}}{\delta_k^{**}} &= \frac{\rho_k}{\rho_{2\infty}(1 + \Delta_\rho)}\left[\frac{V_k}{V_{2\infty}(1 + \Delta_u)}\right]^{2+\frac{1}{2}(H_2+H_k)} \\ &= \frac{\rho_k}{\rho_{2\infty}}\left(\frac{V_k}{V_{2\infty}}\right)^{2+\frac{1}{2}(H_2+H_k)}(1 - \Delta_\rho)\left\{1 - \left[2 + \frac{1}{2}(H_2 + H_k)\right]\Delta_u\right\} \\ &= \frac{\rho_k}{\rho_{2\infty}}\left(\frac{V_k}{V_{2\infty}}\right)^{2+\frac{1}{2}(H_2+H_k)}\left\{1 - \Delta_\rho - \left[2 + \frac{1}{2}(H_2 + H_k)\right]\Delta_u\right\}\end{aligned}\right\} \tag{5.16}$$

Because in this section everything will be expressed in terms of the unknowns δ_2^{**} and H_2, equations (4.2) and (4.3) are applied; the following expression is then obtained:

$$\left.\begin{aligned}\frac{\delta_2^{**}}{\delta_k^{**}} = \frac{\rho_k}{\rho_{2\infty}}\left(\frac{V_k}{V_{2\infty}}\right)^{2+\frac{1}{2}H_k+\frac{1}{2}H_2}\left[1 - (H_2 - 1)\frac{\delta_2^{**}}{t'} - \right.\\ \left.\left(2 + \frac{H_k}{2} + \frac{H_2}{2}\right)\frac{\delta_2^{**}}{t'}\right]\end{aligned}\right\} \tag{5.17}$$

or

$$\delta_2^{**} = \delta_k^{**}\frac{\rho_k}{\rho_{2\infty}}\left(\frac{V_k}{V_{2\infty}}\right)^{2+\frac{1}{2}H_k+\frac{1}{2}H_2}\left[1 - \left(1 + \frac{1}{2}H_k + \frac{3}{2}H_2\right)\frac{\delta_2^{**}}{t'}\right]$$

For a first approximation, the subtrahend in the brackets on the right may be neglected in comparison to unity to obtain

$$\delta_2^{**} = \delta_k^{**}\frac{\rho_k}{\rho_{2\infty}}\left(\frac{V_k}{V_{2\infty}}\right)^{2+\frac{1}{2}H_k+\frac{1}{2}H_2} \tag{5.18}$$

The second fundamental equation (5.15) is similarly transformed. With the chosen degree of accuracy,

$$\theta_2 = \int_{-\infty,\delta}^{+\infty,\delta}\frac{\rho V_s}{\rho_2 V_2}\left(1 - \frac{T^*}{T_2^*}\right)dn = \int_{-\infty,\delta}^{+\infty,\delta}\frac{\rho V_s(T_2^* - T^*)}{\rho_2 V_2 T_2^*}dn$$

$$= \int_{-\infty,\delta}^{+\infty,\delta}\frac{\rho_2 V_2(T_2^* - T^*)}{\rho_2 V_2 T_2^*}dn = \int_{-\infty,\delta}^{+\infty,\delta}\frac{T_2^* - T^*}{T_2^*}dn$$

Therefore,

$$\theta_2 = \int_{-\infty,\delta}^{+\infty,\delta} \frac{(T_2 - T)dn}{T_2 + V_2^2/2Jc_p} + \int_{-\infty,\delta}^{+\infty,\delta} \frac{V_2^2/2Jc_p - V_s^2/2Jc_p}{T_2 + V_2^2/2Jc_p} dn$$

$$= \frac{1}{1 + V_2^2/2Jc_pT_2} \int_{-\infty,\delta}^{+\infty,\delta} \frac{T_2 - T}{T_2} dn +$$

$$\int_{-\infty,\delta}^{+\infty,\delta} \frac{V_2^2/2Jc_p - \left[V_2^2 - 2V_2(V_2 - V)\right]/2Jc_p}{T_2 + V_2^2/2Jc_p} dn = \frac{1}{1 + V_2^2/2Jc_pT_2} \Delta_T t' +$$

$$\frac{V_2^2/2Jc_p}{T_2^2 + V_2^2/2Jc_p} \Delta_u t' = \left(\frac{T_{2\infty}}{T_{2\infty}^*} \Delta_T + \frac{V_{2\infty}^2}{Jc_pT_{2\infty}^*}\right) \Delta_u t'$$

$$= \left(\frac{V_{2\infty}^2}{Jc_pT_{2\infty}^*} \Delta_u - \frac{T_{2\infty}}{T_{2\infty}^*} \Delta_\rho\right) t'$$

or, by equations (4.2) and (4.3),

$$\theta_2 = \left[\frac{V_{2\infty}^2}{Jc_pT_{2\infty}^*} - (H_2 - 1) \frac{T_{2\infty}}{T_{2\infty}^*}\right] \delta_2^{**} \tag{5.19}$$

The term θ_2/t', which by equation (5.19) is proportional to δ_2^{**}/t', is a small quantity of the first order; with the assumed order of approximation, equation (5.15) may be transformed into

$$\rho_{2\infty}V_{2\infty}\left[\frac{V_{2\infty}^2}{Jc_pT_{2\infty}^*} - (H_2 - 1) \frac{T_{2\infty}}{T_{2\infty}^*}\right] \delta_2^{**} = \rho_k V_k \theta_k$$

or,

$$\rho_{2\infty}V_{2\infty} \left[\frac{V_{2\infty}^2}{Jc_p} - (H_2 - 1)T_{2\infty}\right] \delta_2^{**} = \rho_k V_k T_{2\infty}^* \theta_k \tag{5.20}$$

The system of equations (5.18) and (5.20) gives the required system of equations for determining the two unknown magnitudes δ_2^{**} and H_2 as functions of the parameters of the boundary layer, of the external flow near the trailing edge of the airfoil, and of the density, velocity, and temperature at infinity behind the cascade.

For the solution of this system of equations, it is noted that the unknowns are readily separated if equations (5.18) and (5.20) are divided one by the other. This procedure yields

$$\left(\frac{V_k}{V_{2\infty}}\right)^{\frac{1}{2}H_2} \frac{V_{2\infty}^2/Jc_p - (H_2 - 1)T_{2\infty}}{T_{2\infty}^*} = \frac{\theta_k}{\delta_k^{**}}\left(\frac{V_{2\infty}}{V_k}\right)^{1+\frac{1}{2}H_k}$$

or, when the Mach number $M_{2\infty}$ is introduced at infinity behind the cascade,

$$\left.\begin{aligned} M_{2\infty} &= \frac{V_{2\infty}}{a_{2\infty}} = \frac{V_{2\infty}}{\sqrt{kRT_{2\infty}}} = \frac{V_{2\infty}}{\sqrt{(k - 1)Jc_pT_{2\infty}}} \\ \left(\frac{V_k}{V_{2\infty}}\right)^{\frac{1}{2}H_2} & \frac{(k - 1)M_{2\infty}^2 - (H_2 - 1)}{\frac{1}{2}(k - 1)M_{2\infty}^2 + 1} = \frac{\theta_k}{\delta_k^{**}}\left(\frac{V_{2\infty}}{V_k}\right)^{1+\frac{1}{2}H_k} \end{aligned}\right\} \tag{5.21}$$

This transcendental equation in H_2 may be solved by one of the approximate methods in any concrete case. According to equation (5.20),

$$\delta_2^{**} = \frac{1 + \frac{1}{2}(k - 1)M_{2\infty}^2}{1 + (k - 1)M_{2\infty}^2 - H_2} \frac{\rho_kV_k}{\rho_{2\infty}V_{2\infty}} \theta_k \tag{5.22}$$

The terms δ_2^{**} and H_2 referring to section 2, the location of which is unknown, are thus eliminated and expressed in terms of the magnitudes δ_k^{**} and θ_k either measurable or computable by any method of boundary-layer theory and in terms of the velocity, density, and temperature at the outer limit of the boundary layer near the trailing edge of the air foil and at infinity behind the cascade. The terms p' and R' may then be obtained with little difficulty by equations (4.8) and (4.9).

6. Approximate formulas for computing losses and resistances in cascade. - At these relatively small subsonic Mach numbers considered herein, the nonisothermal character of the flow in the wake behind the airfoil of the cascade can occur mainly through the heating or the cooling of the surface of the airfoil and not through the internal transformations of kinetic energy into heat.

In order to verify this fact, equations (5.21) is employed. The following notation is introduced for briefness:

$$\left.\begin{aligned} \frac{k-1}{2} M_{2\infty}^{\;2} &= m \\ \frac{H_2 - 1}{2} &= \epsilon \\ \frac{3 + H_k}{2} &= \mu \end{aligned}\right\} \tag{6.1}$$

Equation (5.21), which is transcendental relative to ϵ, then assumes the form

$$\frac{m - \epsilon}{1 + m} = \frac{1}{2} \frac{\theta_k}{\delta_k^{**}} \left(\frac{V_{2\infty}}{V_k}\right)^{\mu+\epsilon} \tag{6.2}$$

The unknown magnitude ϵ is now expanded into a series in powers of the small parameter m. (For air the value of m at $M_{2\infty} < 0.7$ does not exceed 0.1)

$$\epsilon = \epsilon_0 + \epsilon_1 m + \epsilon_2 m^2 + \ldots \tag{6.3}$$

Substituting this series in equation (6.2) gives

$$\left[- \epsilon_0 + (1 - \epsilon_1)m - \ldots\right](1 - m - \ldots) = \frac{1}{2} \frac{\theta_k}{\delta_k^{**}} \left(\frac{V_{2\infty}}{V_k}\right)^{\mu+\epsilon_0} \left(\frac{V_{2\infty}}{V_k}\right)^{\epsilon_1 m+\ldots} \tag{6.4}$$

or

$$- \epsilon_0 + (1 + \epsilon_0 - \epsilon_1)m - \ldots = \frac{1}{2} \frac{\theta_k}{\delta_k^{**}} \left(\frac{V_{2\infty}}{V_k}\right)^{\mu+\epsilon_0} \left[1 + \epsilon_1 \ln \left(\frac{V_{2\infty}}{V_k}\right) m + \ldots\right]$$

By equating coefficients, the following equation is obtained:

$$-\epsilon_0 = \frac{1}{2}\frac{\theta_k}{\delta_k^{**}}\left(\frac{V_{2\infty}}{V_k}\right)^{\mu+\epsilon_0} \tag{6.5}$$

for determining ϵ_0. Because of the assumption previously made on the small heat transfer from the surface of the airfoil in the cascade, the quantity ϵ_0 is considered small for $M_{2\infty} = 0$. The following equation, accurate to small quantities of the second order, is then obtained:

$$-\epsilon_0 = A_k\left(\frac{V_{2\infty}}{V_k}\right)^{\epsilon_0} = A_k\left[1 + \epsilon_0 \ln \frac{V_{2\infty}}{V_k}\right] \tag{6.6}$$

where

$$A_k = \frac{1}{2}\frac{\theta_k}{\delta_k^{**}}\left(\frac{V_{2\infty}}{V_k}\right)^{\frac{1}{2}(3+H_k)} \tag{6.7}$$

From equation (6.6),

$$\epsilon_0 = -\frac{A_k}{1 + A_k \ln (V_{2\infty}/V_k)} \tag{6.8}$$

The ratio $V_{2\infty}/V_k$ generally differs little from unity; hence, the natural logarithm of this ratio is small so that $\epsilon_0 = -A_k$ may be written without great error.

By equating the coefficients of m to the first power, in equation (6.4), the expression $\epsilon_1 = 1 + \epsilon_0 = 1 - A_k$ is obtained with the same degree of accuracy.

The approximate equation is then

$$\epsilon = -A_k + (1 + A_k)m \approx m - A_k \tag{6.9}$$

Equation (5.22) is now employed and in the new notation has the form

$$\delta_2^{**} = \frac{1 + m}{2(m - \epsilon)} \frac{\rho_k V_k}{\rho_{2\infty} V_{2\infty}} \theta_k \tag{6.10}$$

According to equations (6.11) and (6.7),

$$\delta_2^{**} = \frac{\rho_k V_k}{\rho_{2\infty} V_{2\infty}} \frac{\theta_k}{2A_k} = \delta_k^{**} \frac{\rho_k}{\rho_{2\infty}} \left(\frac{V_k}{V_{2\infty}}\right)^{\frac{1}{2}(5+H_k)} \tag{6.11}$$

If it is assumed that at the trailing edge $H_k = 1.4$, equations (6.11) and (6.7) assume the form

$$\left.\begin{aligned} \delta_2^{**} &= \delta_k^{**} \frac{\rho_k}{\rho_{2\infty}} \left(\frac{V_k}{V_{2\infty}}\right)^{3.2} \\ A_k &= \frac{1}{2} \frac{\theta_k}{\delta_k^{**}} \left(\frac{V_{2\infty}}{V_k}\right)^{2.2} \end{aligned}\right\} \tag{6.12}$$

From equation (4.8), an approximate formula for the losses is readily obtained:

$$\left.\begin{aligned} p' &= \left[\rho_m V_{2\infty}^2 + \frac{1}{2} \epsilon \rho_{2\infty} (V_{2\infty}^2 - V_{1\infty}^2)\right] \frac{\delta_2^{**}}{t \cos \beta_{2\infty}} \\ &= \rho_m V_{2\infty}^2 \frac{\rho_k}{\rho_{2\infty}} \left(\frac{V_k}{V_{2\infty}}\right)^{3.2} \frac{\delta_k^{**}}{t \cos \beta_{2\infty}} \left[1 + \frac{1}{2} (m - A_k) \frac{\rho_{2\infty}}{\rho_m} \frac{V_{2\infty}^2 - V_{1\infty}^2}{V_{2\infty}^2}\right] \end{aligned}\right\} \tag{6.13}$$

and therefore a corresponding approximate formula for the resistance differing from the right side of the previous equation only in the factor t.

The further possible simplifications of equation (6.13) are connected with the choice of devices for computing the characteristics of the boundary layer at the surface of the airfoil in the cascade and for taking into account the effect of the compressibility on the external flow.

Translation by S. Reiss
National Advisory Committee
for Aeronautics

REFERENCES

1. Loitsianskii, L. G.: Resistance of a Cascade of Airfoils in a Viscous Incompressible Fluid. Prik. Mat. i Mek., T. XI, No. 4, 1947. (Translation available on loan from NACA Headquarters.)

2. Loitsianskii, L. G.: Generalization of the Joukowski Formula to the Case of an Airfoil of a Cascade in a Compressible Gas Stream with Subsonic Velocities. NACA TM 1304, 1950.

3. Loitsianskii, L. G.: Inverse Effect of the Boundary Layer on the Pressure Distribution on the Surface of a Body in a Real Fluid. Prik. Mat. i Mek., T. XI, No. 2, 1947.

4. Squire and Young: Computation of the Profile Drag of a Wing. Collection of articles on the problem of Maximum Speed of an Airplane. N. Oborongiz, 1941, pp. 100-126.

5. Loitsianskii, L. G.: Resistance of a Cascade of Airfoils in a Gas Flow with Subsonic Velocities. Prik. Mat. i Mek., T. XIII, No. 2, 1949, pp 171-186.

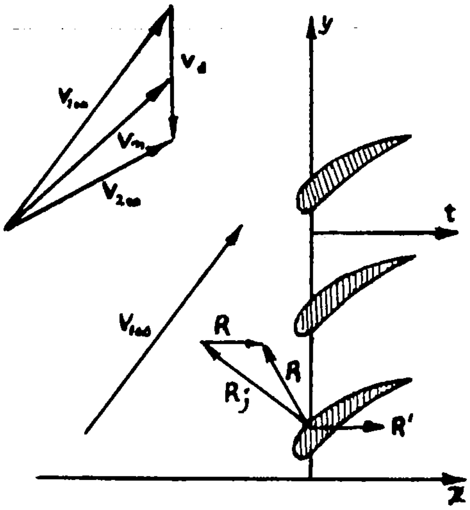

Figure 1.

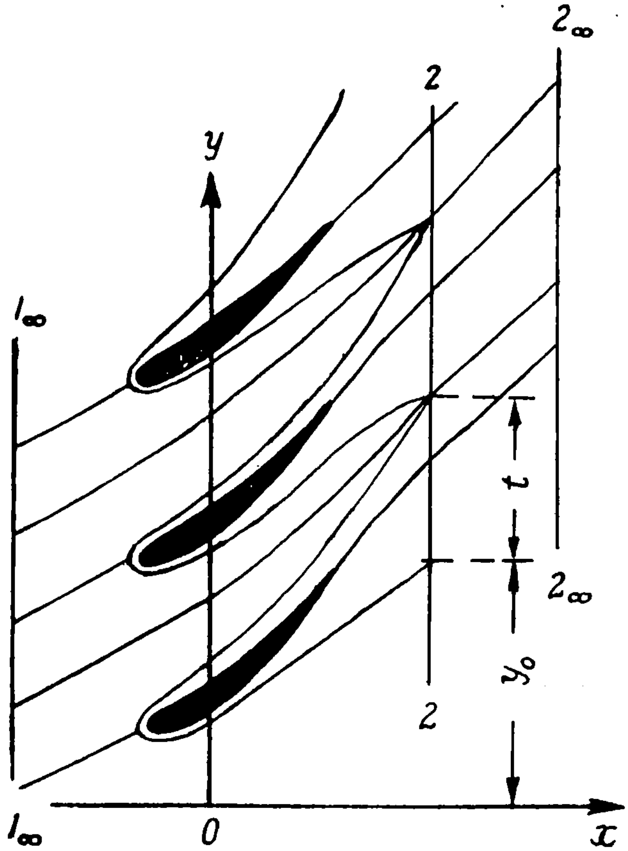

Figure 2.

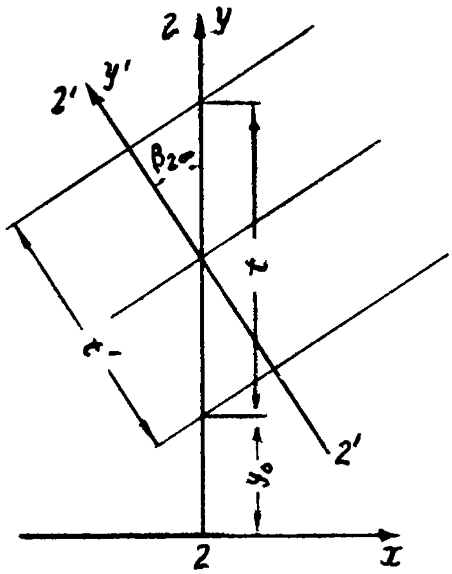

Figure 3.